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RESEARCH PAPER

# Interdisciplinary design optimization of compressor blades combining low- and high-fidelity models

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## Abstract
Multidisciplinary design optimization has great potential to support the turbomachinery development process by improving designs at reduced time and cost. As part of the industrial compressor design process, we seek for a rotor blade geometry that minimizes stresses without impairing the aerodynamic performance. However, the presence of structural mechanics, aerodynamics, and their interdisciplinary coupling poses challenges concerning computational effort and organizational integration. In order to reduce both computation times and the required exchange between disciplinary design teams, we propose an *inter*- instead of *multi*disciplinary design optimization approach tailored to the studied optimization problem. This involves a distinction between main and side discipline. The main discipline, structural mechanics, is computed by accurate high-fidelity finite element models. The side discipline, aerodynamics, is represented by efficient low-fidelity models, using Kriging and proper-orthogonal decomposition to approximate constraints and the gas load field as coupling variable. The proposed approach is shown to yield a valid blade design with reasonable computational effort for training the aerodynamic low-fidelity models and significantly reduced optimization times compared to a high-fidelity multidisciplinary design optimization. Especially for expensive side disciplines like aerodynamics, the multi-fidelity interdisciplinary design optimization has the potential to consider the effects of all involved disciplines at little additional cost and organizational complexity, while keeping the focus on the main discipline.

**Keywords** Multidisciplinary design optimization · Multi-fidelity methods · Kriging · Proper-orthogonal decomposition · Turbomachinery

## 1 Introduction

Reduced development costs and decreased emissions are two of the goals set by the aerospace industry and the European Commission (2011) in the Flightpath 2050. Reduced development costs imply shorter design cycles with less iterations between the disciplines, motivating the use of multidisciplinary approaches. Decreased emissions can be achieved by either innovative concepts or by optimizing existing aircraft components.

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One of the main aircraft engine components is the compressor, responsible for raising the pressure level of the intake air, while it passes through several rows of rotor and stator blades, see Fig. 1. The compressor development involves several engineering disciplines, notably aerodynamics and structural mechanics.

### 1.1 Aero-structural optimization problem statement

As part of the industrial aero-structural compressor design process, we seek for the best possible blade geometry. After the initial design optimization by the aerodynamic department, the second step is a structural optimization of individual blades. However, design changes that are beneficial from the structural mechanics point of view may counteract the aerodynamic performance and disturb the aerodynamic integration with the neighboring stages. Structural blade design

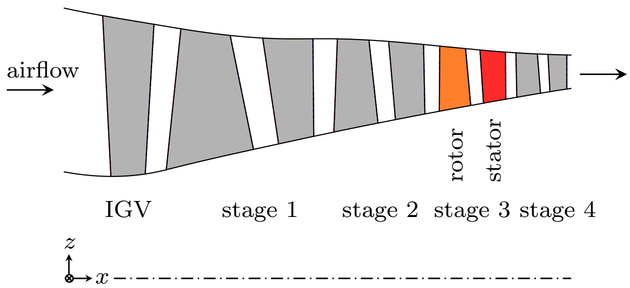


**Fig. 1** High-pressure compressor frontblock annulus geometry illustration with inlet guide vane (IGV) and four stages of rotor and stator blades. The third-stage rotor and stator blades, highlighted in orange and red, are the subject of the treated optimization problem. The arrows indicate the airflow in $x$-direction

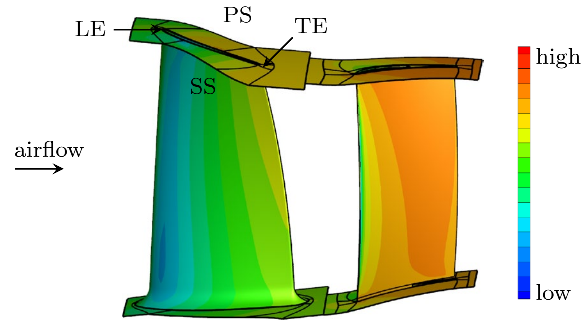


**Fig. 2** Pressure field on the third-stage rotor and stator blade surface and annulus walls as a result of the CFD simulations. The four profile sides pressure side (PS), suction side (SS), leading edge (LE), and trailing edge (TE) are indicated on the rotor blade

problems should thus include aerodynamic constraints and aero-structural coupling.

A frequently arising optimization problem is the minimization of stresses in the compressor blades without impairing the aerodynamic performance. It is herein studied for the third-stage rotor blade, highlighted in orange in Fig. 1. Figure 2 shows the qualitative pressure field on the third-stage surfaces, as a result of CFD simulations with the initial geometry. It induces aerodynamic loads, the gas loads, which in turn affect the stress field, depicted in Fig. 3 for the rotor suction side.

The blade design problem can be treated either by an iterative process between disciplinary design teams or in a multidisciplinary design optimization (MDO) (Sobieszczanski-Sobieski and Haftka 1997). Papageorgiou et al. (2018) reviewed recent advancements and challenges in MDO of aerial vehicles and provided a roadmap including nine fundamental elements, among others computational efficiency and organizational integration.

For the above-mentioned aero-structural compressor blade design task, these two elements are particularly challenging and inhibit the application of MDO as a standard tool in the industrial development process. First, computing times can be prohibitively long if expensive aerodynamic simulations are performed in addition to the structural analyses in every optimization iteration. Interdisciplinary coupling additionally increases computational cost and complexity. It is therefore often neglected or strongly simplified at the expense of model accuracy. Second, the integration into company structures is usually difficult, as disciplinary department boundaries impede multidisciplinary developments. The aim for a practical optimization approach thus is as little disciplinary interdependence as possible and as much as needed for a useful optimization result. In other words, a multidisciplinary feasible solution should be obtained while maintaining the division of work between responsible discipline-focused company units.

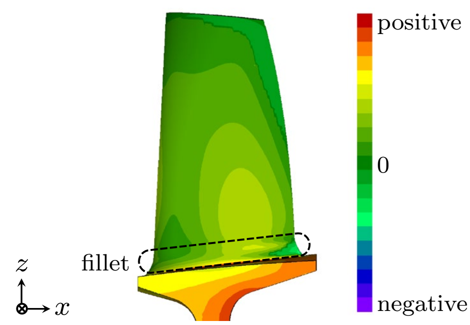


**Fig. 3** Maximum principal stress field on the rotor blade suction side as a result of the FE simulations. The stresses depend on the rotor blade geometry and are induced by centrifugal forces, thermal loads, and gas loads. The latter result from the aerodynamic pressures in Fig. 2. The dashed line indicates the fillet region

### 1.2 Multi-fidelity approaches

The first challenge, the computational efficiency, is commonly tackled by parallelized computations and multi-fidelity methods. An additional option is adjoint methods for efficient gradient-based optimization, not discussed herein, as used among others by He et al. (2020) in an uncoupled aero-structural compressor blade optimization. Multi-fidelity methods (Viana et al. 2014) speed up computations by complementing a comparatively small number of high-fidelity simulations, here finite element (FE) and computational fluid dynamics (CFD) simulations, with a large amount of low-fidelity model evaluations. The latter are cheaper to evaluate, while providing useful information within the design domain. They are also called surrogate models or metamodels. Before outlining our ideas, we review recent literature on the application of low-fidelity models in aero-structural optimization of aircraft engine blades.

The most widely used low-fidelity models are data-fit models. Cuciumita et al. (2021) approximated a maximum stress constraint by a radial basis function (RBF) model. Aissa and Verstraete (2019) proposed a bounded Kriging model as robust method for surrogate-assisted MDO of compressor blades. Neural networks were recently used by Ghalandari et al. (2019) to model aerodynamic performance and stress levels, or by Vanti et al. (2018) in an uncoupled aeroelastic optimization.

Another type of low-fidelity models are reduced-order models, including proper-orthogonal decomposition (POD). Benamara et al. (2017) proposed multi-fidelity non-intrusive POD to predict isentropic efficiency and pressure ratio. Instead of approximating the response variables, Zhang et al. (2018) replaced the 30 original design parameters in the MDO by only four POD coefficients to decrease computation time.

In the above turbomachinery references, static aero-structural coupling is mostly neglected. In aircraft wing optimization, Coelho et al. (2009) used reduced-order models to approximate pressure loading and displacement field. The low-fidelity models enabled coupling at little additional computational cost, enhancing the accuracy of the disciplinary high-fidelity results.

The second challenge, the organizational integration of MDO, is a much less active research area (Papageorgiou et al. 2018). Lian and Liou (2006) are one of the few references to address both challenges together for a statically coupled aero-structural compressor optimization. They used genetic algorithms solely based on quadratic response surface models. These low-fidelity models were in turn based on high-fidelity samples with unidirectional coupling via the aerodynamic pressure field, realized by sequential CFD and FE simulations. Hu et al. (2016) employed a collaborative optimization (CO) strategy for a bidirectionally coupled aero-structural turbine blade optimization. The distributed CO strategy guarantees disciplinary autonomy for better organizational integration. Its commonly prohibitive computational expense was reduced by quadratic response surface models, based on data from medium-fidelity aerodynamic and structural analyses and high-fidelity fluid–structure interaction simulations.

The above literature references have in common that they treat both disciplines as equally important. They do not exploit the fact that industrial aero-structural optimizations are often performed with focus on monodisciplinary objectives, and include multidisciplinary coupling and constraints merely not to interfere with the respective other discipline.

### 1.3 Proposed approach

This work addresses the question of how low-fidelity models can be employed to improve the aero-structural optimization of compressor blades with regard to both computational efficiency and organizational integration.

For the compressor blade optimization problem stated in the beginning, we propose a multi-fidelity optimization process which is *inter*- instead of *multi*disciplinary. This involves a distinction between main and side discipline. The main discipline is structural mechanics, as it is the objective and focus of the optimization problem. The side discipline is aerodynamics. The main discipline is represented by a high-fidelity model. Additionally, the side discipline and the coupling from side to main discipline is taken into account by low-fidelity models only. Thereby, the main discipline is evaluated as accurately as possible, while the side discipline is computed only as accurately as necessary for a useful optimization result.

The benefit of the proposed approach is shown by comparison to a common uncoupled and purely high-fidelity MDO. Despite a significant computation speed-up, the approach provides a valid blade design that is not only structurally optimized, but also respects aerodynamic effects and constraints.

The paper is structured as follows: In Sect. 2, multi-fidelity and multidisciplinary optimization approaches are outlined, followed by the employed low-fidelity modeling methods in Sect. 3. The proposed interdisciplinary optimization process is presented in Sect. 4. Afterward, in Sect. 5, the computational set-up and concrete optimization problem of the compressor blade application are defined. The results are presented in Sect. 6 and the paper is concluded by a summary and outlook in Sect. 7.

## 2 Multi-fidelity and multidisciplinary optimization methods

In the present context, optimization (Martins and Ning 2021) uses numerical methods to seek the overall best design of an engineering system. It can yield an improved system performance at reduced time and cost compared to a conventional development process. However, the correct problem formulation is crucial and requires expertise in both the involved engineering discipline(s) and numerical optimization.

A standard optimization problem is to minimize an objective function by varying the design variables within their prescribed bounds subject to equality and inequality constraints. For compressor blade design, common objectives are minimum stresses or maximum efficiency, the design parameters define the blade geometry, and constraints concern for example mass flow, pressure ratio, surge margin, stresses, and eigenfrequencies.

In every optimization iteration, objectives and constraints need to be evaluated. With aerodynamics and structural mechanics as the most influential disciplines in compressor blade design, this usually entails thousands of CFD and FE simulations.

### 2.1 Multi-fidelity optimization processes

Expensive high-fidelity simulations can be partly or fully replaced by efficient low-fidelity model evaluations within the optimization. Models of various fidelities are combined in multi-fidelity optimization (MFO) processes, either by adaptation, fusion, or filtering (Peherstorfer et al. 2018; Khatouri et al. 2022).

In adaptive MFO, the low-fidelity model is improved by high-fidelity results at new sampling points during the optimization. Fusion strategies combine the outputs of models of different fidelities. Filtering means that the high-fidelity model is only evaluated if the low-fidelity output meets a predefined criterion. Note that all of these approaches combine models of multiple fidelities with the same output parameters and thereby differ from the approach we propose in Sect. 4.

### 2.2 Multidisciplinary optimization processes

Just like models of various fidelities are embedded in a multi-fidelity optimization process, the presence of several disciplines allows different approaches for the efficient organization of the disciplinary analyses and optimization methods. MDO processes, or architectures, were extensively reviewed by Martins and Lambe (2013) and compared by Tedford and Martins (2009) and Gray et al. (2013) based on benchmark problems. They classify MDO architectures as monolithic or distributed.

The former treat the MDO problem in a single optimization and are therefore simple to implement and well-suited for small problems. However, they are inefficient for problems with a large number of disciplines. Here, distributed architectures may perform better. They decompose the MDO problem into disciplinary optimization subproblems which are coordinated by a system-level optimization subproblem. Thereby, they try to mimic the structure of industrial development teams and strive for independence between the disciplines. Although the ideas of distributed architectures seem promising, they exhibit a slow convergence for many problems (Martins and Ning 2021).

The basic monolithic MDO architecture is the multidisciplinary feasible (MDF) approach. It treats the MDO as a common disciplinary optimization problem, with the difference that objective and constraints are computed by multidisciplinary analyses in every optimization iteration. MDF has the advantage that it makes use of established optimization and multidisciplinary analysis methods. The optimization problem remains as small as possible and is thus also suited for gradient-free methods. Moreover, the multidisciplinary analysis ensures physical compatibility after every optimization iteration. This is especially useful for engineering and industry applications, where the aim usually is finding a better design, rather than a mathematical optimum, and optimizations are often terminated prematurely. The main disadvantage is the computational effort and often slow convergence of the multidisciplinary analyses in case of strong interdisciplinary coupling.

With regard to the use of low-fidelity models, concurrent subspace optimization (CSSO) is an interesting distributed approach. It was introduced by Sobieszczanski-Sobieski (1989) and extended to the version considered herein by Sellar et al. (1996). In CSSO, the coordinating system-level subproblem is optimized based on low-fidelity models. Each disciplinary subproblem is optimized based on high-fidelity models for the corresponding discipline and low-fidelity models for all other disciplines. The disciplinary subproblem results are used to update the low-fidelity models. Despite the fast low-fidelity model evaluations, the slow convergence hinders its efficient application.

## 3 Low-fidelity models

Before starting the interdisciplinary optimization process, low-fidelity models of the aerodynamic quantities of interest must be generated. According to Peherstorfer et al. (2018), low-fidelity models can be divided into three categories: data-fit models, projection-based models, and simplified models. While data-fit models are purely mathematical response surfaces, projection-based models, hereinafter referred to as reduced-order models, represent a system by its most important eigenmodes and thereby provide a certain physical interpretability. Simplified models, for example with a coarser mesh, are based on the original problem physics. However, their evaluation often takes a lot longer than for the other two categories and they may be difficult to combine with the high-fidelity models; that is why they are omitted in this work.

The process of data-fit and reduced-order model generation followed for the optimization problem herein is illustrated as a flowchart in Fig. 4. First, a set of input points in the design space is selected, also called design of experiments (DoE) (Giunta et al. 2003). Popular approaches are Monte Carlo sampling, Latin hypercube sampling (LHS), and low discrepancy sequences. Then, the corresponding outputs are computed by high-fidelity simulations, here CFD for aerodynamics.

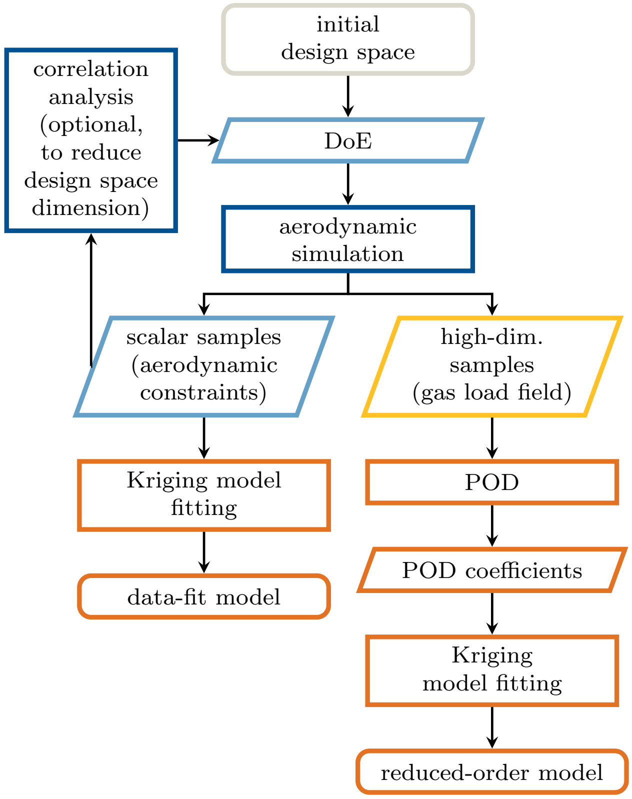


**Fig. 4** Flowchart of the low-fidelity model generation process. The high-fidelity aerodynamic model, CFD, is approximated by a data-fit model, Kriging, and a reduced-order model, consisting of POD and Kriging

Based on the resulting data set, a correlation or sensitivity analysis (Saltelli et al. 2007), for instance in form of Spearman correlation coefficients (Spearman 1904), can provide a better understanding of the problem at hand and help to identify variables with negligible influence on the quantities of interest. Omitting these variables during the low-fidelity model generation can allow for more accurate models with respect to the remaining variables, especially in case of limited sampling budget.

The CFD results serve as a common training base for both kinds of low-fidelity models. We use a data-fit model, Kriging, to approximate the scalar aerodynamic constraints. A reduced-order model, consisting of POD and Kriging, approximates the gas load field for aero-structural coupling. Both are explained in what follows.

### 3.1 Data-fit model: Kriging

Popular data-fit models are polynomial response surface models, moving least-squares, RBF interpolation, Kriging, or support vector regression, summarized by Forrester et al. (2008). We use Kriging (Krige 1951; Matheron 1963; Sacks et al. 1989), because it does not assume a certain problem structure and can thus yield accurate predictions for various function forms.

Kriging is also known as Gaussian process (GP) regression (Rasmussen and Williams 2006). It predicts an unknown output by interpolation between the values in its vicinity. The response is considered as a random variable, contrary to its actual properties as observed response obtained by deterministic computations. This takes into account the actual uncertainty of the prediction.

Since the method is very popular, we only briefly recapitulate the main steps of Kriging model fitting and prediction. To start with, the a-priori GP is defined as sum of mean and covariance function. In ordinary Kriging, used herein, the mean is an unknown constant. The covariance is specified by kernel functions. For modeling physical quantities, the Matérn kernels are in general preferred over the squared exponential kernel, because the latter are unrealistically smooth (Rasmussen and Williams 2006). Then, the model is fitted to the training data by optimizing the hyperparameters, that is the unknown mean and the kernel parameters, commonly using maximum likelihood estimation (MLE). The resulting *a-posteriori* GP can be used to compute the most probable output values and the prediction uncertainty at new input points.

### 3.2 Reduced-order model: POD + Kriging

While data-fit models approximate scalar quantities, like the efficiency, reduced-order models approximate multi-dimensional outputs, like the gas load field. The underlying idea is to extract the most important features, or modes, of the physical field and then represent the field as a linear combination of these modes. The resulting lower-dimensional system representation is much easier to handle.

The most popular method for obtaining the lower-dimensional representation is POD. It is similar to principal component analysis (PCA) from the field of statistics. Non-intrusive snapshot POD, developed by Sirovich (1987), is employed in this work and will be outlined in what follows. Case studies of snapshot POD in both aerodynamics and structural mechanics were presented by Swischuk et al. (2019). An overview of non-intrusive reduced-order models was provided by Yu et al. (2019).

Following the right branch of Fig. 4, the lower-dimensional outputs, that is the POD coefficients, are approximated by Kriging models, like in the previous subsection. The procedure of reduced-order model generation and prediction is summarized hereinafter. For more details and underlying equations of snapshot POD, we refer to the above references.

The starting point of POD is the snapshot matrix containing $n$ samples of $m$-dimensional simulation results. This matrix is projected onto the subspace spanned by the dominant modes. Mathematically, this is achieved using thin singular value decomposition (SVD). The resulting matrices can then be truncated after the first $k$ modes, loosing as little information as possible for a given reduced rank $k$. The reduced rank $k$ can either be specified in advance or chosen such that the proportion of energy, or variance, captured by the first $k$ modes should be above or equal to a given threshold $\kappa$, typically $\kappa \geq 90\%$. In case of computational meshes with thousands of nodes, the energy usually decreases rapidly with increasing rank and $k \ll m$. As lower-dimensional representation of the high-dimensional samples, the POD coefficients are computed by projecting the snapshot vectors onto the first $k$ POD modes.

Afterward, a Kriging model can be trained to approximate the POD coefficients as a function of the original input. Note that the number of degrees of freedom to be modeled decreases from $m$ nodal values to $k \ll m$ coefficients due to the model order reduction.

For the low-fidelity model prediction, the POD coefficients are predicted by the Kriging models. The full-order physical field can then be approximated as a linear combination of POD coefficients and modes.

## 4 Proposed interdisciplinary optimization process

The goal of this work is a fast and industrially integrable aero-structural optimization approach for the compressor blade design problem stated in Sect. 1.1. For this purpose, we combine the methods explained in the two previous sections. We propose a combination of a simple MDF architecture and a multi-fidelity approach inspired by the CSSO disciplinary subproblems. The resulting interdisciplinary optimization (IDO) process can be categorized as monolithic architecture and is illustrated as a flowchart in Fig. 5.

The underlying idea is the distinction between main and side discipline, leading to an *inter*- rather than *multi*disciplinary optimization problem. It follows the concept of a disciplinary design team, that maintains an interdisciplinary exchange with its neighboring teams, but does not have fully multidisciplinary competences.

The main discipline is the focus of the aforementioned design team and the optimization objective, here structural mechanics to minimize stresses. It is evaluated by a high-fidelity FE model. While the side discipline, aerodynamics, is neither the optimization objective, nor the expertise of the design team, it needs to be considered for a useful optimization result. Therefore, the side discipline is treated as a low-fidelity model, which completely replaces the high-fidelity CFD simulations. It thus generates very little additional computational effort in each optimization iteration. Instead of exchanging large amounts of data, complex CFD models, and simulation tools plus the corresponding licenses, only

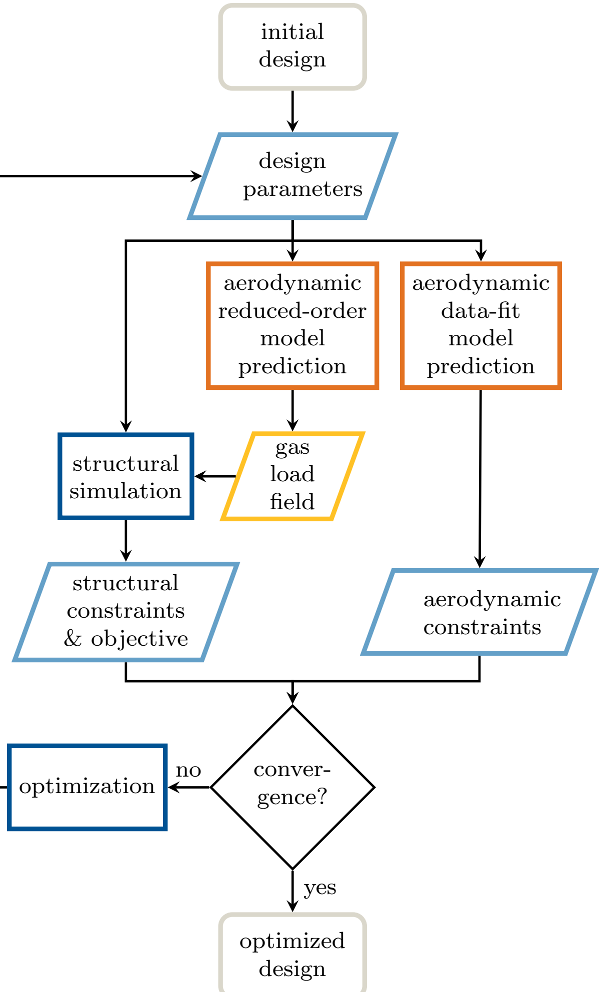


**Fig. 5** Flowchart of the proposed multi-fidelity IDO process. The structural mechanics simulations are based on high-fidelity FE models, the aerodynamic simulations are replaced by low-fidelity models

simple low-fidelity models need to be provided by the aerodynamic design team.

Nevertheless, the aerodynamic influence is considered via both constraints and coupling variables. The constraints are represented by data-fit models, here Kriging, the high-dimensional coupling variables by reduced-order models combining POD and Kriging, see Sect. 3. The effect of the aerodynamics on the structure is represented by unidirectional coupling, transferring the aerodynamic load field as a boundary condition for the static structural analysis. The rapid approximation enables a concurrent coupling, meaning that it is computed during the structural simulation set-up and requires no additional time, contrary to common sequential computations. Cross-coupling, in our case a bidirectional exchange of aerodynamic loads and structural displacements, would make the sampling much more complex and expensive. Approaches for POD and interpolation

(POD + I) model generation for cross-coupled multidisciplinary analyses were presented by Coelho et al. (2009) and Berthelin et al. (2022). Herein, unidirectional coupling via the aerodynamic loads is assumed to yield sufficiently accurate results, because the low-fidelity model is not fully accurate by definition. This also resolves any multidisciplinary analysis convergence issues.

The most widely used approach for aero-structural turbomachinery blade optimization is a MDF architecture without coupling, as employed in most references in Sect. 1.2. The proposed approach differs in that the aerodynamic simulation is replaced by the low-fidelity models (orange building blocks in Fig. 5), enabling a speed up and a coupling via the gas load field (yellow block in Fig. 5). Contrary to common MFO strategies, as presented by Peherstorfer et al. (2018), in the proposed approach the multiple fidelities are not combined within one discipline, but separated between the disciplines. In particular, adaptation is omitted, because it would hinder the interdisciplinary applicability.

To summarize, the main discipline structural mechanics is evaluated as accurately as possible, including updated coupling via gas loads, while the side discipline aerodynamics is computed only as accurately as necessary for a useful optimization result. The IDO is much faster than a complete MDO and better integrable into company structures, because complex high-fidelity CFD models are no longer required in the optimization. Provided sufficiently accurate surrogate models, it fulfills the aim of an industrial MDO, which is an improved design, rather than a mathematical optimum.

## 5 Compressor problem set-up

To illustrate the proposed approach, it is applied to a concrete compressor rotor blade design problem. More precisely, we seek to optimize the rotor blade geometry of a compressor stage so as to minimize the stresses in the fillet, while fulfilling a number of structural, aerodynamic, and geometrical constraints. The simulation set-up and optimization problem are specified in what follows.

### 5.1 Computational set-up

Our model represents the third stage of a high-pressure compressor in a next-generation turbofan aircraft engine. It is based on a model from the *Clean Sky 2* (Clean Aviation 2021) project by the European Union.

Geometry parameterization and generation are carried out using *AirFoil Designer pdesk* (atech GmbH 2022) and are explained in Sect. 5.2.

The FE simulations including meshing are performed in the open-source program *CalculiX* (Dhondt 2004). They consist of a static analysis at cruise conditions, and a dynamic analysis at red line conditions to prevent resonance. The structural analyses are carried out for the rotor blades only, which are a critical component in the considered problem. A structural analysis takes on average 6 min on 4 CPUs.

For the CFD simulations, the mesh is generated using *AutoGrid* (Cadence 2022). Then, the unsteady RANS flow solver *TRACE* (German Aerospace Center 2022) is employed, which is specialized on compressor and turbine components. The aerodynamics are computed for cruise conditions, that is at the aero design point (ADP). Here, the entire stage is of interest, to investigate the interaction between rotor and stator. One CFD simulation takes on average 33 min on 8 CPUs.

A fully coupled multidisciplinary analysis would require several sequential iterations of FE and CFD simulations to ensure compatibility of structural displacements and aerodynamic pressures on the blade surface. This would take a prohibitively long time in each optimization iteration. Therefore, the structural displacements are neglected in the CFD model, also because the aerodynamics are only treated as side discipline. The pressure field is obtained from a 2D flow solver, which is only capable of estimating a 1D radial distribution of the total pressure loads. It represents an approximation of the difference between PS and SS pressures and does not account for 3D blade geometry changes. The resulting loads are then projected only onto the blades' PS surface using a heuristic tool to produce the 2D gas load distribution. The gas load field is thus considered constant and only applied to the PS surface unless replaced by the reduced-order model, which is based on 3D CFD samples. Thermal loads are applied in a similar manner.

The computational effort of the CFD is considerably higher than for the FE simulations. This further motivates the use of aerodynamic low-fidelity models in an optimization focusing on structural mechanics. For each design evaluation herein, FE and CFD simulations are run in parallel. Additionally, 20 designs are evaluated in parallel to speed up computations in both sampling and optimization.

The optimizations are run using *AutoOpti* (Siller et al. 2009), a program tailored to the multidisciplinary optimization of turbomachinery components, which can handle high-dimensional non-linearly constrained problems. It is based on an efficient global optimization (Jones et al. 1998) approach, combining evolutionary algorithms, adaptive Kriging surrogate models, and an expected improvement infill criterion. However, our data-fit models for the side discipline are not directly integrated into the optimizer for the sake of modularity. The optimizer is not the focus of this work and can theoretically be exchanged with other gradient-free methods.

## 5.2 Optimization problem definition

Our optimization problem represents the second step in the industrial blade design process, after the initial aerodynamic design step. We focus on structural mechanics as the main discipline, but also consider aerodynamic aspects, in order to reduce the number of subsequent interdisciplinary iterations. The optimization problem is defined in Table 1.

We seek for a rotor blade geometry, which minimizes the maximum static principal stress $\sigma_{\mathrm{I,\,max}}$ in the fillet, that is the transition between blade hub and disk in a blisk (blade integrated disk), indicated in Fig. 3. Low maximum stresses in the fillet region are crucial for robust compressor designs. Structural constraints are the maximum static principal stresses on the four blade sides PS, SS, LE, and TE, indicated in Fig. 2. Moreover, the first two eigenfrequencies are constrained to prevent resonance. Aerodynamic constraints refer to the mass flow, the isentropic efficiency, and an incidence criterion at five radial positions. The equality constraint for the mass flow, in order to maintain the initial operating point, is implemented as narrow two-sided inequality constraint. In addition, the maximum blade thickness for the upper part of the blade is constrained to avoid extremely thin profiles. This is particularly important to prevent significant changes in the higher frequency torsion modes of the blade, which are not explicitly set as structural constraints. The constriction at eleven radial positions is limited for good manufacturability.

The design variables are rotor blade geometry parameters, as illustrated in Fig. 6.

The free design parameters are the blade angles $\alpha$, the stagger $\beta_S$, the wedge angles $\gamma$, and the distance wedge $l$. These parameters define the 2D blade profiles for each of the three radial airfoil sections at hub, mid, and tip by setting the four base points. The profiles are then constructed by spline

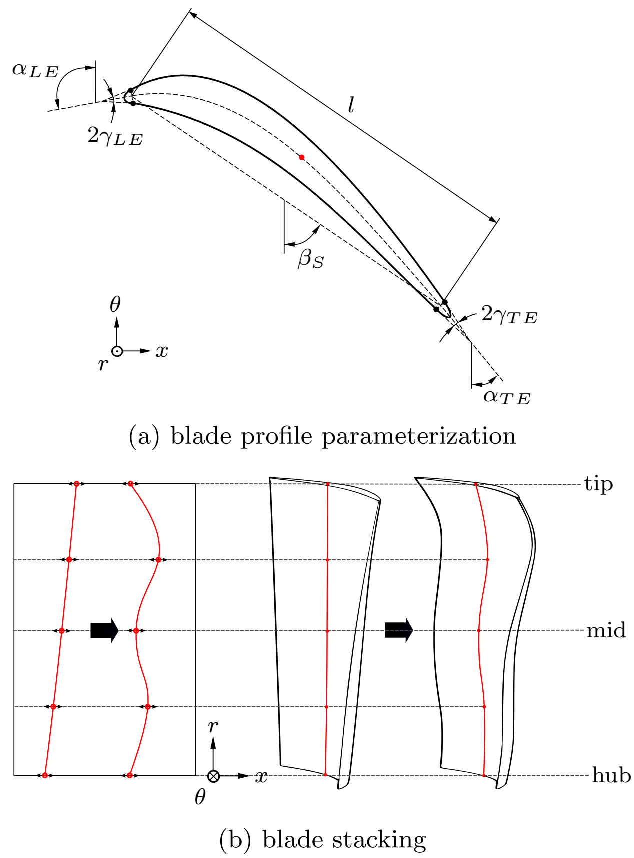


**Fig. 6** Blade geometry parameterization. Modified from Arsenyev (2018). The parameters in **a** at blade hub, mid, and tip (see **b**) are the design variables of the optimization problem

**Table 1** Interdisciplinary compressor rotor blade optimization problem with main discipline structural mechanics

| Objective | Type | Discipline |
|---|---|---|
| Max. static principal stress $\sigma_{\mathrm{I,\,max}}$ in the fillet | Minimization | Structural mechanics |
| **Constraints** | **Type** | **Discipline** |
| Max. static principal stress $\sigma_{\mathrm{I,\,max}}$ PS, SS, LE, TE | Upper bound | Structural mechanics |
| Eigenfrequency $f$ 1, 2 | Lower & Upper bound | Structural mechanics |
| Mass flow $\dot{m}$ | Equality constraint (= baseline) | Aerodynamics |
| Isentropic efficiency $\eta_{\mathrm{is}}$ | Lower bound (= baseline) | Aerodynamics |
| Incidence criterion slice 1–5 | Upper bound (= baseline) | Aerodynamics |
| Max. blade thickness near tip | Lower bound | Geometry |
| Constriction slice 1–11 | Lower bound | Geometry |
| **Design variables (each for rotor hub, mid, tip)** | **Range (relative to baseline)** | **Discipline** |
| Blade angle $\alpha$ LE, TE | ±5% | Geometry |
| Stagger $\beta_S$ | ±4% | Geometry |
| Wedge angle $\gamma$ LE, TE | ±10% | Geometry |
| Distance wedge $l$ | ±5% | Geometry |

interpolation between these base points, which also separate the four sides PS, SS, LE, and TE. The resulting 2D profiles are stacked in radial direction to obtain the 3D blade geometry. The profiles' position in space is defined by the axial and circumferential location of their centers (highlighted in red), which are fixed within the optimization. The blade shape between the predefined profiles is interpolated, for instance by fourth-order splines defined by five radial control points as shown in Fig. 6b. In this work, the three control points at hub, mid, and tip allow for second-order splines to interpolate the design variables. Finally, the transition between blade and disk is smoothed by a fillet with fixed radius. The stator blade geometry is generated analogously and remains constant throughout the optimization, which is common for a structural optimization.

In total, the optimization problem sums up to 26 response variables (one objective plus 25 constrained variables) and 18 design variables.

## 6 Results

In the following, data-fit models of the aerodynamic constrained variables and reduced-order models of the aerodynamic loads are generated to replace the expensive CFD simulations in the optimization. Multi-fidelity IDO results are validated and compared to purely high-fidelity MDO results as a reference.

### 6.1 Data-fit model generation

The proposed approach involves data-fit models that predict the aerodynamic constrained variables: the mass flow, the efficiency, and the five incidence criteria. For the Kriging model generation, we follow the flowchart in Fig. 4, taking the left branch. The prediction accuracy is estimated by 10-fold cross-validation. The aim is a NRMSE < 5% (normalized by the respective variable range) and a correlation coefficient $R^2 > 0.9$.

The initial design space is the one used later in the optimization, see Table 1. The sampling points are obtained by LHS. For a reasonable sampling effort, the number of CFD simulations is limited to half of what would be required for a comparable high-fidelity optimization. 300 sampling points are evaluated, of which 11 fail, leaving 289 samples available for the training.

Correlations between design and response variables are analyzed based on Spearman's correlation coefficients and scatter plots. The former are shown in Fig. 15 in the appendix. The results indicate that blade angle and stagger variables correlate strongly with the quantities of interest. Furthermore, wedge angles and distance wedge at hub and tip are only weakly correlated to the aerodynamic constraints. These six design variables are therefore kept constant in the sampling.

The Kriging model implementation relies on the Python package *scikit-learn* (Pedregosa et al. 2011) that in turn builds upon the formulations by Rasmussen and Williams (2006). An isotropic Matérn 5/2 kernel is chosen, as it yields the best fit among various considered kernels.

The distribution of the errors over the response variables is shown by the gray bars in Fig. 7. Although the mean error measures are below the self-imposed limits, the prediction of the isentropic efficiency still poses a challenge.

Based on the response variable distribution in the training data, three significant outliers can be recognized in the set of 289 samples. They can be associated with very unrealistic geometries generated by the automated sampling. Removing these outliers, the errors clearly improve, especially for the isentropic efficiency and the mass flow, see the orange bars in Fig. 7. Consequently, a Kriging model with Matérn 5/2 kernel, based on the training set without outliers, is chosen as low-fidelity model for all aerodynamic constraints.

With 33 min per design evaluation and 20 evaluations in parallel, the sampling runs 8.25 h. Afterward, the aerodynamic data-fit model generation takes only about 1.5 s. Its evaluation takes less than one millisecond, instead of 33 min for a CFD simulation in each optimization iteration.

### 6.2 Reduced-order model generation

For a concurrent aero-structural coupling in the IDO, we train reduced-order models of the gas load field on the

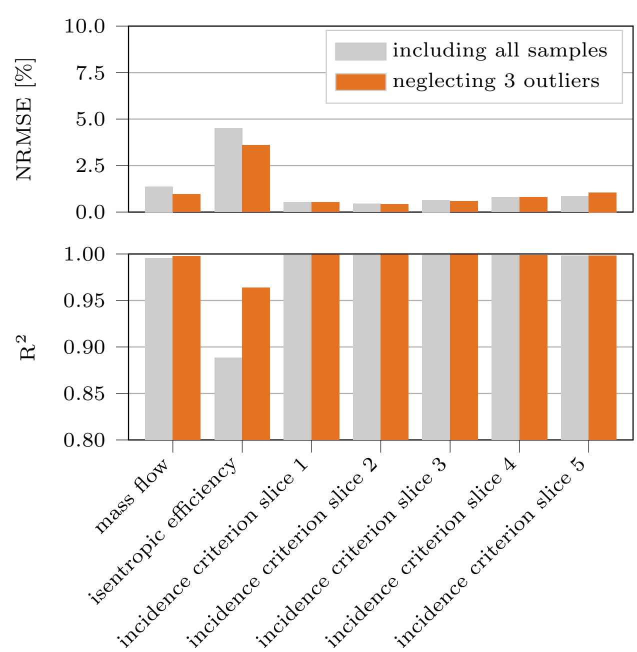


**Fig. 7** Error measures for the Kriging data-fit models with and without inclusion of three outliers in the set of 289 samples, based on a 10-fold cross-validation

blade surface, following the right branch of the flowchart in Fig. 4. First, the high-dimensional field is reduced to a low-dimensional representation using POD. Then, the POD coefficients are predicted by a Kriging model. The NRMSE (normalized by the range at the respective node) and $R^2$ are estimated using a 10-fold cross-validation based on the sampling data.

The training data originate from the same sampling as for the data-fit models and thus requires no additional run time. For each sample, the pressure field on the blade surface is interpolated onto the mesh of the structural simulations. Note that the structural mesh topology remains constant throughout all simulations. This is a prerequisite for a direct construction of the snapshot matrix from the nodal gas load values.

The implementation of the POD uses the SVD algorithm from *scipy* (Virtanen et al. 2020). It is embedded in a self-implemented reduced-order model generation, closely following the theory in Sect. 3.2.

The reduced rank $k$ is computed such that $\kappa = 99\%$ of the energy is conserved. Table 2 holds the resulting reduced ranks, compared to the original full rank, that is the number of degrees of freedom $m$, equal to the number of nodes. The POD is conducted separately for each of the four blade sides. For pressure and suction side, the rank is reduced by more than two orders of magnitude, for leading and trailing edge by a factor of more than ten. Over the whole blade surface, 4048 nodal values are reduced to 33 POD coefficients to be approximated by the data-fit model.

After the model order reduction, the POD coefficients are computed as output variables for the data-fit model generation. Here, a Kriging model with Matérn 5/2 kernel is used, as in Sect. 6.1, and all 289 samples are considered. The overall low-fidelity model error estimates, due to the reduction plus the data-fit approximation, are shown in the lower part of Table 2. The combination of high loads and a small area make the prediction on the leading edge challenging. At the other three blade sides, the self-imposed accuracy targets (NRMSE < 5%, $R^2 > 0.9$) are achieved, notably also for the highly loaded pressure side.

Based on the above accuracy estimates, the reduced-order models of the gas loads on the blade surface can be considered a large improvement in accuracy compared to the previously used constant gas loads. First, the loads predicted by the reduced-order model are applied to the entire blade surface, instead of the pressure side only. Moreover, they are based on 3D CFD, instead of 2D flow solutions, and hence take into account the blade geometries. As geometry-dependent boundary conditions in the FE analyses, the reduced-order model gas loads enable a weak aero-structural coupling.

Since the reduced-order model is generated based on the same samples as the data-fit models in the previous section, no additional sampling time is required. The reduced-order model generation takes less than 10 s. The gas load field is predicted within milliseconds and the respective input files for the FE models are written in about 0.2 s during the structural simulation set-up. Consequently, the structural analyses in the optimization can start without delay despite the unidirectional coupling.

### 6.3 Optimization studies

The proposed multi-fidelity IDO process is compared to a purely high-fidelity MDO approach as a reference, to assess the potential to solve the aero-structural compressor blade optimization problem defined in Sect. 5.2. The two approaches are specified in Table 3.

Both involve high-fidelity structural analysis via 3D FEM, as the structural behavior is the focus of the optimization and thus the main discipline. The aerodynamic constraints are computed either by high-fidelity 3D CFD or the low-fidelity data-fit model. The aero-structural coupling is realized either by a constant gas load field from 2D computations, or by the low-fidelity reduced-order model predictions based on 3D CFD. A coupling directly via high-fidelity 3D CFD gas loads would be prohibitively expensive.

Since the employed evolutionary optimization algorithm is non-deterministic, both approaches are repeated five times with the same low- and high-fidelity models. 20 individuals are evaluated in parallel for 30 optimization iterations,

**Table 2** Ranks and error measures for the reduced-order models of the gas load field on the four blade sides

| | PS | SS | LE | TE |
|---|---|---|---|---|
| Full rank $m$ | 1932 | 1932 | 92 | 92 |
| Reduced rank $k$ | 9 | 11 | 7 | 6 |
| NRMSE (%) | 3.10 | 2.73 | 6.53 | 3.62 |
| $R^2$ | 0.965 | 0.977 | 0.869 | 0.966 |

The full rank is equal to the number of degrees of freedom $m$ on the respective side. The reduced rank $k$ is obtained by snapshot POD with $\kappa = 99\%$ energy conservation, based on 289 samples. The error measures of the final model (POD + Kriging) are averaged over the associated nodes and are based on 10-fold cross-validation

**Table 3** Model fidelities in the two compared optimization approaches

| | Structural mechanics | Aerodynamics | Coupling via gas loads |
|---|---|---|---|
| MDO | High-fidelity | High-fidelity | Constant |
| IDO | High-fidelity | Low-fidelity | Low-fidelity |

MDO is the high-fidelity reference multidisciplinary optimization, IDO is the proposed multi-fidelity interdisciplinary approach which makes 600 design evaluations in total. Figure 8 shows the development of the objective function values over the number of iterations.

Both optimization approaches converge quickly and most runs achieve a stress reduction by about 40% after 20 iterations, compared to the initial blade from the predesign step. Afterward, there is only little improvement. The differences in aerodynamic model fidelity and type of gas loads do not seem to considerably affect the convergence behavior.

The optimizer itself requires a similar amount of time for both high- and multi-fidelity optimizations. Consequently, the aero-structural analysis time, illustrated in Fig. 9, determines the difference in overall computation time.

Considering 30 iterations with 33 min each, the analysis time can be reduced by more than 80% from 16.5 h to only 3 h due to the low-fidelity models in the IDO. Additionally, the 8 CPUs for the CFD simulations are no longer necessary. For a holistic consideration, the 3 h of simulation time in the multi-fidelity IDO must be offset by the sampling time, adding up to 11.25 h. Note that multiple optimizations with varying objective functions and constraints can be run with a single sampling, reducing the sampling time per optimization. Moreover, the sampling can be arbitrarily parallelized, depending on the computational resources.

Figure 10 shows the aerodynamic responses for the IDO results.

The low-fidelity data-fit model results are indicated in orange. They satisfy all optimization constraints, that is they lie inside the light gray ranges, as expected for a feasible optimization result. For validation, the low-fidelity results are recomputed by high-fidelity CFD simulations, indicated in black. They show that all aerodynamic inequality constraints are also satisfied for the high-fidelity model. The

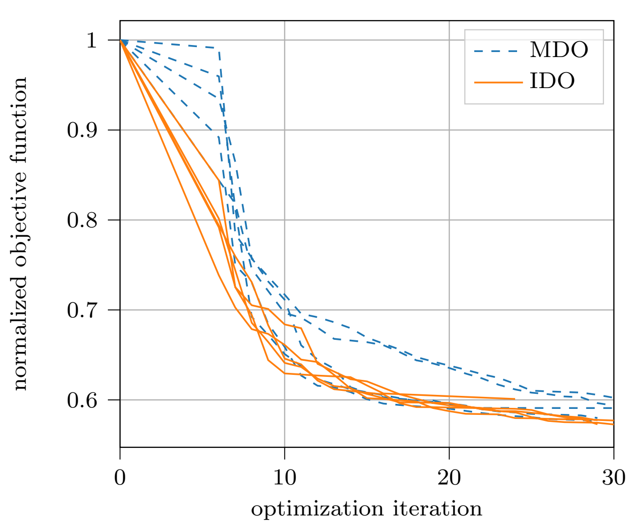


**Fig. 8** Convergence of the objective function, normalized with its initial value, as a function of the optimization iteration. Only feasible results with improved objective values are plotted

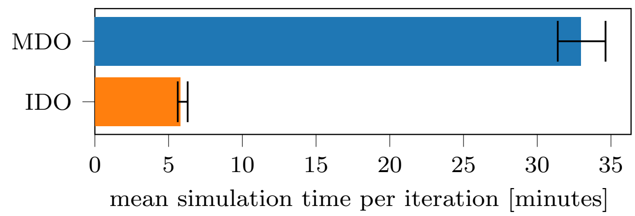


**Fig. 9** Mean simulation time per optimization iteration for the two different optimization approaches. The whiskers show minimum and maximum values of the five repetitions

relaxed equality constraint for the mass flow with very narrow bounds is slightly violated. The clear trends in the deviations between low- and high-fidelity model predictions can be explained by all runs converging to a similar region in the design space, see Fig. 12. The mean, minimum, and maximum errors are shown in Table 4 in the appendix. In summary, all deviations are within reasonable limits and do not undermine the validity of the overall optimization results.

In addition to the aerodynamic result variables, the gas loads on the rotor blade surface are predicted in the IDO approach. As an example of the prediction quality, Fig. 11 compares the CFD result and the reduced-order model prediction of the gas load field on the pressure side for an optimized blade design.

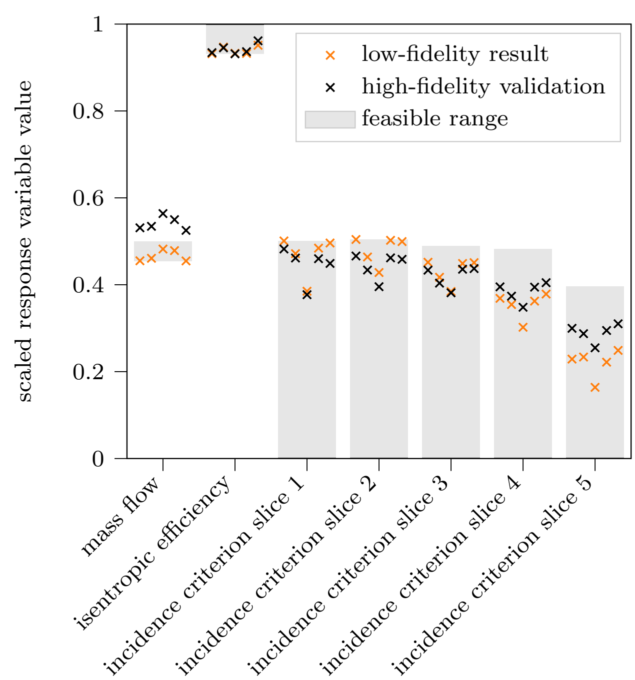


**Fig. 10** Aerodynamic response variable values for the multi-fidelity IDO results, scaled with the respective variable ranges from the sampling. The low-fidelity data-fit model results are indicated in orange, the high-fidelity CFD validations in black. The feasible ranges, that is where the optimization constraints are satisfied, are marked in light gray

Comparing the high- and low-fidelity results in Fig. 11a and b, distributions are in good agreement. The normalized error in Fig. 11c indicates that the prediction is very accurate on the majority of the blade's pressure side. The error only increases in the area of high gradients near the leading edge. The mean, minimum, and maximum errors are shown in Table 5 in the appendix. Overall, the gas load prediction can be considered a large improvement in accuracy compared to the constant loads from the 2D flow simulations. The computational effort for model generation and evaluation is negligible, as it exploits existing samples from the scalar aerodynamic response predictions.

The optimized design variables of the two approaches are shown in the parallel coordinates plot in Fig. 12.

Both yield the same trends compared to the initial design. The variation among the results of each approach is larger than the variations between the approaches. Moreover, the gas load accuracy improvement in the IDO does also have no visible effect on the optimization results. Its impact may be weakened by dominating centrifugal loads in the present case. One can conclude that the multi-fidelity approach is accurate enough to provide design proposals for industrial applications, where optimizations are stopped when improvements become minor, like between 20 and 30 iterations in the convergence plot in Fig. 8.

Figure 13 shows an optimized blade design from the proposed IDO approach in orange, in comparison to the initial geometry in gray. Starting at the blade hub, the profile becomes considerably thicker, for higher stiffness in the fillet region, which in turn reduces the stresses and thus the objective function. This is realized by an increase in the wedge angles at both LE and TE hub, that also show a strong correlation to the fillet stresses, see Figs. 12 and 15. Continuing toward the blade tip, curvature and chord length were significantly changed, which is likely to affect higher order eigenmodes and eigenfrequencies. They should thus be included for an improved problem formulation in future optimizations.

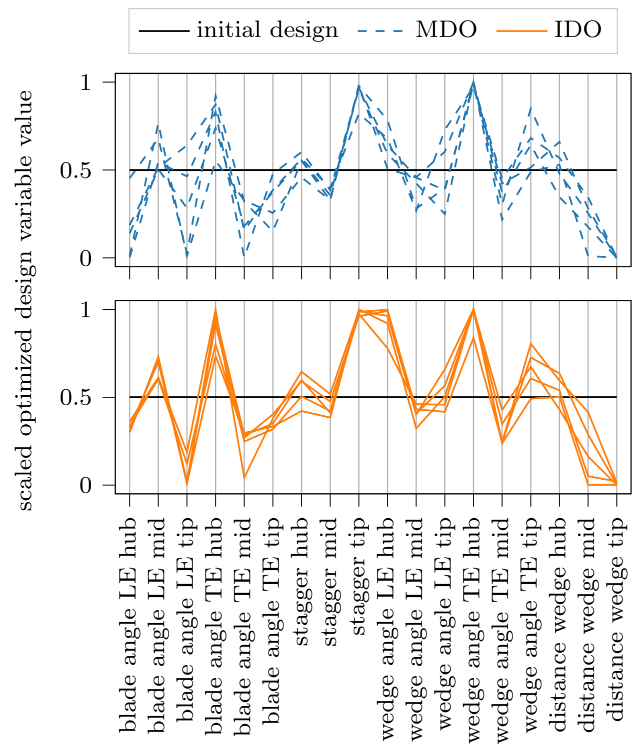


**Fig. 12** Parallel coordinates plot comparing the results of the two optimization approaches to the initial design. The design variable values are scaled with their respective range

The effect of the design changes can be observed in the stress field, notably the maximum static principal stresses. The maxima occur on the pressure side, which is shown in Fig. 14. We compare again the initial and optimized blade design from the proposed IDO approach. The maximum is shifted from the fillet to a second peak in the middle, to relieve the fillet region without exceeding the upper bound set as constraint in the optimization problem.

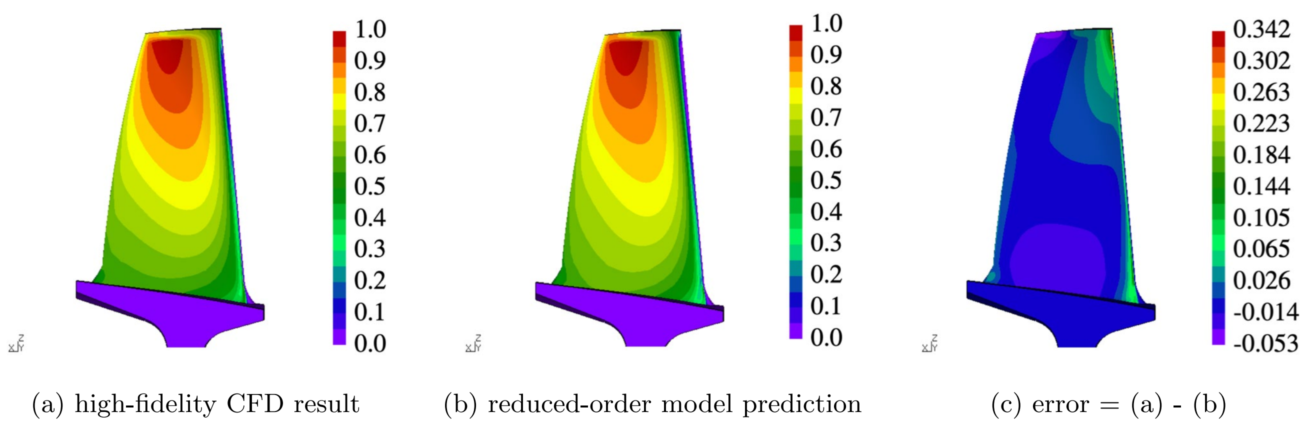


**Fig. 11** Gas load field on the pressure side of an optimized rotor blade from the proposed IDO approach, scaled (a,b), and normalized (c) with the range from the high-fidelity CFD result

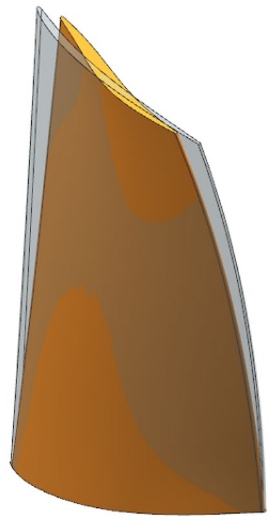

**Fig. 13** Optimized rotor blade design from the proposed IDO approach in orange. For comparison, the initial design is depicted in semitransparent gray

## 7 Conclusions

An interdisciplinary approach for the optimization of compressor blade geometries so as to minimize structural stresses without impairing the aerodynamic performance is proposed in this work. We consider structural mechanics as main discipline, aerodynamics as side discipline, and static aero-structural coupling. Combining high- and low-fidelity models, we imitate the interdisciplinary workflow of design teams in industry. The main discipline is the objective and focus of the optimization and therefore evaluated by high-fidelity FE models. The side discipline computations inside the optimization are entirely replaced by low-fidelity model evaluations with almost negligible computation time and model complexity compared to high-fidelity CFD. The scalar aerodynamic constraints are approximated by Kriging models. The gas load field is predicted using POD combined with Kriging. The latter enables a unidirectional concurrent aero-structural coupling at no additional time in each optimization iteration and thereby enhances the high-fidelity structural analysis accuracy.

Consequently, aerodynamic effects of design changes can be considered at very low cost inside the optimization. Moreover, the required exchange of data and expertise between disciplinary design teams is greatly reduced, facilitating the organizational integration compared to common multidisciplinary optimization approaches.

The proposed approach is illustrated by a structural compressor blade optimization with 25 structural, aerodynamic, and geometric constraints and 18 design variables. The results show that sufficiently accurate low-fidelity models (NRMSE $< 5\%$, $R^2 > 0.95$) can be obtained at reasonable cost (300 sampling points, as opposed to 600 design evaluations in each optimization). The multi-fidelity optimizations yield valid designs. Design evaluation times in the optimization are reduced by more than 80% compared to the high-fidelity reference. This shows the great potential to reduce computation time, especially for expensive side disciplines like aerodynamics.

Based on the above results, a number of interesting directions for future research and developments can be discussed. For different problems, for example if the aerodynamic design team sets the isentropic efficiency as objective function, main and side disciplines could be reversed. Since high-fidelity aerodynamic evaluations are more expensive than structural ones, low-fidelity structural constraints would not reduce the overall simulation times for parallel disciplinary evaluations. Here, additional adaptive low-fidelity models for the main discipline may be useful. For unidirectional coupling variables, however, the low-fidelity models would still enable a concurrent coupling. The benefit in terms of organizational integration remains, independent of

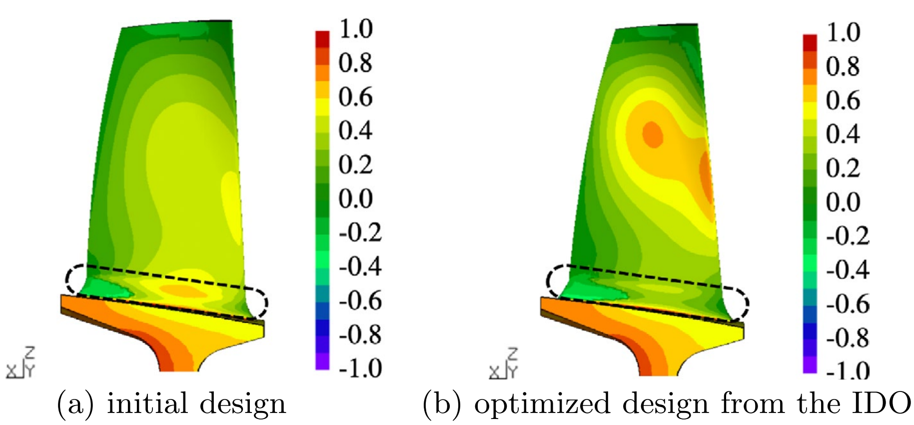


**Fig. 14** Maximum principal stress field on the rotor blade pressure side, normalized with the maximum allowed value defined in the optimization problem. In the fillet region, indicated by the dashed line, the stress peak is minimized. On the rest of the blade, the stresses stay below the maximum allowed value

the computational effort. Besides, the proposed IDO approach is readily extendable to additional side disciplines, for example thermodynamics in the compressor blade design. With regard to the organizational structure of most companies, more than one main discipline does usually not make sense. Finally, the potential of the proposed approach should be confirmed for even larger problems, for example by a multi-stage compressor blade optimization. The problem size may then require not only to decouple disciplines, but also split the design domain, for instance into single stages. Here, reduced-order models have great potential to predict physical interfaces.

## Appendix

See Fig. 15 and Tables 4 and 5.

**Table 5** Mean, minimum, and maximum errors of the reduced-order model gas load predictions for the optimized rotor blades from the multi-fidelity IDO, normalized by the respective nodal range from the sampling

| Error (%) | PS | SS | LE | TE |
|---|---|---|---|---|
| Mean | 7.88 | 5.27 | 7.93 | 12.11 |
| Min | 7.26 | 4.78 | 4.88 | 10.75 |
| Max | 8.84 | 5.65 | 17.53 | 13.97 |

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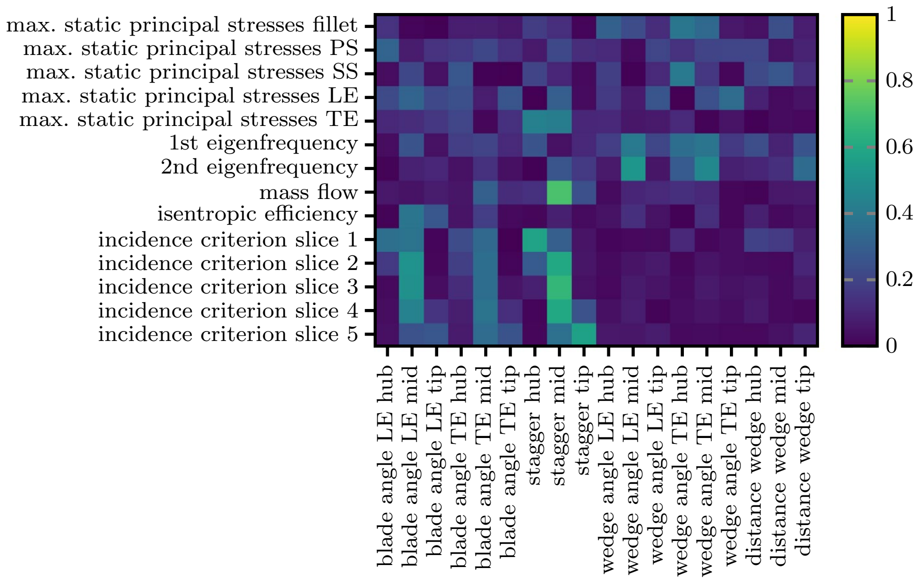


**Fig. 15** Spearman's correlation coefficients for the design and response variables of the optimization problem defined in Table 1. The coefficients indicate a relevant monotonic relationship of the structural responses to all design variables. The aerodynamic responses show a strong correlation with blade angles and stagger in the middle of the blade, but only weakly relate to wedge angles and distance wedge

**Table 4** Mean, minimum, and maximum errors of the data-fit model predictions for the optimized rotor blades from the multi-fidelity IDO, normalized by the respective variable range from the sampling

| Error (%) | Mass flow | Isentropic efficiency | Incidence criterion slice 1 | Incidence criterion slice 2 | Incidence criterion slice 3 | Incidence criterion slice 4 | Incidence criterion slice 5 |
|---|---|---|---|---|---|---|---|
| Mean | 7.46 | 0.40 | 2.16 | 3.64 | 1.24 | 3.00 | 7.00 |
| Min | 7.03 | 0.12 | 0.80 | 3.00 | 0.31 | 1.92 | 5.38 |
| Max | 8.17 | 1.03 | 4.70 | 4.08 | 1.83 | 4.60 | 9.10 |

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## Declarations

**Conflict of interest** The authors declare that they have no conflict of interest.

**Replication of results** The implementation of the low-fidelity modeling methods is described in detail and the associated Python code can be obtained from the corresponding author on reasonable request. For the purpose of a realistic industrial application, the compressor blade optimization was performed in the design environment of *MTU Aero Engines* and is therefore unfortunately subject to strict confidentiality. Different simulation models and software may produce different results, but should yield similar trends.



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