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Review of different calculation approaches for the mean coefficient of friction in ISO 6336

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Abstract

Important goals in gear design are high load carrying capacity, good noise, vibration, and harshness (NVH) performance and high efficiency. Regarding the load carrying capacity, the international series of standards ISO 6336 [13–15] is the state of the art for its calculation.

To ensure reliable calculation of the load carrying capacity of gears and the temperatures occurring during operation, knowledge of the friction in the gear mesh is crucial. Currently, various approaches exist in the literature for calculating the mean coefficient of friction, which weight the influencing variables to varying degrees.

In this publication, the empirical approaches for calculating the mean coefficient of friction given in the international series of standards ISO 6336 [13–15] are to be analyzed in terms of their origin and validated ranges, systematically compared, and contrasted. These calculation approaches are mainly covered in the parts ISO/TS 6336-20 [14], ISO/TS 6336-21 [15], and ISO/TS 6336-22 [13], which address the calculation of the scuffing load carrying capacity according to the flash and integral temperature method and the calculation of the micropitting load carrying capacity, respectively. Additionally, ISO/TR 14179-2 [12] which describes a calculation approach for the thermal load carrying capacity will be included in this review. Besides the analysis of their origin, exemplary comparative calculations for various applications are intended to show possible differences between the various calculation approaches and enable a quantitative evaluation.

The overall, long-term goal is to merge and standardize the various calculation approaches for the mean coefficient of friction in the international series of standards ISO 6336.

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Überblick über verschiedene Berechnungsansätze für die mittlere Verzahnungsreibungszahl nach ISO 6336

Zusammenfassung

Neben der Tragfähigkeit, dem NVH-Verhalten und den Herstellungskosten stellt der Wirkungsgrad eine zentrale Zielgröße in der Auslegung moderner Zahnradgetriebe dar. Hinsichtlich der Tragfähigkeit ist die internationale Normenreihe ISO 6336 [13–15] Stand der Technik für deren Berechnung.

Um eine zuverlässige Berechnung der Tragfähigkeit von Getrieben und der im Betrieb auftretenden Temperaturen zu gewährleisten, ist die Kenntnis der Reibung im Zahneingriff entscheidend. Derzeit existieren in der Literatur verschiedene Ansätze zur Berechnung der mittleren Verzahnungsreibungszahl, die die Einflussgrößen unterschiedlich stark gewichten.

In dieser Publikation sollen die in der internationalen Normenreihe ISO 6336 [13–15] angegebenen empirischen Ansätze zur Berechnung der mittleren Verzahnungsreibungszahl hinsichtlich ihrer Herkunft und validierten Bereiche analysiert, systematisch verglichen und gegenübergestellt werden. Diese Berechnungsansätze werden hauptsächlich in den Teilen ISO/TS 6336-20 [14], ISO/TS 6336-21 [15] und ISO/TS 6336-22 [13] behandelt, die sich mit der Berechnung der Fresstragfähigkeit nach der Blitz- und Integraltemperaturmethode bzw. der Berechnung der Graufleckentragfähigkeit befassen. Zusätzlich wird die ISO/TR 14179-2 [12], die einen Berechnungsansatz für die thermische Tragfähigkeit beschreibt, in die Betrachtung einbezogen. Neben der Analyse ihrer Herkunft sollen exemplarische Vergleichsrechnungen für verschiedene Anwendungen mögliche Unterschiede zwischen den verschiedenen Berechnungsansätzen aufzeigen und eine quantitative Bewertung ermöglichen.

Das übergeordnete, langfristige Ziel ist die Zusammenführung und Vereinheitlichung der verschiedenen Berechnungsansätze für die mittlere Verzahnungsreibungszahl in der internationalen Normenreihe ISO 6336.

1 Introduction

In industrial practice, the load carrying capacity of gears is usually calculated according to international standards such as ISO 6336, which contains separate calculation methods for the different types of damage to cylindrical gears. Some calculation methods require knowledge of the mean gear coefficient of friction, e.g. the calculation of the scuffing load carrying capacity according to the flash and integral temperature method (ISO/TS 6336-20 [14] respectively ISO/TS 6336-21 [15]) or the calculation of the micropitting load carrying capacity (ISO/TS 6336-22 [13]).

Due to the historical development of the standard, different calculation approaches for the mean gear coefficient of friction are used in the respective parts. These are each based on experimental investigations that were carried out at different times and with different focuses in research. As a result, the approaches take into account partly different influencing factors with different weighting. However, the basic structure of the approaches is comparable.

Within this publication, the calculation approaches for the mean gear coefficient of friction according to ISO 6336 and ISO/TR 14179-2 are compared and contrasted with each other. Exemplary calculations based on several example applications given in ISO/TR 6336-30 [16] and on a typical test gear set for lubricants demonstrate the differences between the methods.

2 State of the art

In the international series of standards ISO 6336, there are existing different approaches for calculating the mean gear coefficient of friction μ_m. They were derived from different experimental investigations and with respect to one particular gear damage type.

ISO/TS 6336-20:2022 [14] describes the calculation of the scuffing load carrying capacity of cylindrical gears using the flash temperature method, which was developed by Blok [1, 2]. Currently, this document has the status of a Technical Specification, which is not a full standard but intended to become one in the future. It is based on the withdrawn ISO/TR 13989-1 [10], which was applicable not only to cylindrical gears, but also to bevel and hypoid gears.

The flash temperature method is based on the assumption that a maximum contact temperature along the path of contact is most relevant regarding scuffing damage. Within the calculation, the coefficient of friction μ_m is needed to determine the occurring flash temperature. If no further information is available, it can be estimated acc. to ISO/TS 6336-20:2022 [14], method C as following:

$$\mu_m = 0.060 \cdot \left(\frac{w_{Bt}}{v_{g\Sigma C} \cdot \rho_{\mathrm{relC}}}\right)^{0.2} \cdot X_L \cdot X_R \quad (1)$$

$$w_{Bt} = K_A \cdot K_v \cdot K_{B\beta} \cdot K_{B\alpha} \cdot K_{mp} \cdot \frac{F_t}{b} \quad (2)$$

$$X_L = 0.6 \ldots 1.5 \cdot \eta_{\mathrm{oil}}^{-0.05} \quad (3)$$

$$X_R = \left(\frac{Ra_1 + Ra_2}{2}\right)^{0.25} \quad (4)$$

It can be seen that the coefficient of friction depends on the transverse unit load w_{Bt}, the sum of tangential velocities $v_{g\Sigma C}$ and the transverse equivalent radius of curvature at pitch point ρ_{relC}, the type of lubricant (range 0.6 ...1.5) and its dynamic viscosity (X_L, see Eq. 3) and the tooth flank surface roughness Ra of pinion and wheel (X_R, see Eq. 4). The factor K_{mp} in Eq. 2 accounts for the maldistribution in multiple-path transmissions.

The original source of Eq. 1 is not stated in ISO/TS 6336-20 [14] but the application of Eq. 1 is limited to a maximum circumferential velocity of $v_t = 50\,m/s$. For higher values than 50 m/s, μ_m shall be calculated with $v_t = 50\,m/s$. Furthermore, it has to be noted that Eq. 1 is different from the respective equation for μ_m acc. to DIN 3990-4 [3] which is the basis of ISO 6336 in many cases.

The integral temperature method is described in the Technical Specification ISO/TS 6336-21:2022 [15]. This method is mainly based on the work of Michaelis [18] and was standardized for the first time in DIN 3990-4 [3] and ISO/TR 13989-2 [11], respectively. It is assumed that an average temperature on the tooth flank is decisive for scuffing. The general approach for the coefficient of friction is given in Eq. 5. It was derived from experimental gear power loss tests of Michaelis [18], Ohlendorf [19] and Eiselt [8] with mineral oils.

$$\mu_{mC} = 0.045 \cdot \left(\frac{w_{Bt} \cdot K_{B\gamma}}{v_{\Sigma C} \cdot \rho_{\mathrm{redC}}}\right)^{0.2} \cdot \eta_{\mathrm{oil}}^{-0.05} \cdot X_L \cdot X_R \quad (5)$$

Equation 5 differs from Eq. 1 at several positions. The constant of 0.045 is smaller, which is due to fitting on experimental data. Furthermore, Eq. 5 contains an additional factor $K_{B\gamma}$, which captures the influence of the helix angle on the scuffing load carrying capacity. This helical load factor $K_{B\gamma}$ depends only on the total contact ratio. X_L in ISO/TS 6336-21:2022 [15] does not include the influence of the oil viscosity because it is instead explicitly mentioned in the approach. The surface roughness is taken into account by the factor X_R, which is defined as follows:

$$X_R = 2.2 \cdot \left(\frac{Ra}{\rho_{\mathrm{redC}}}\right)^{0.25} \text{with } Ra = \frac{(Ra_1 + Ra_2)}{2} \quad (6)$$

Hence, the roughness factor X_R depends not only on the arithmetic mean roughness Ra but also on the equivalent radius of curvature at the pitch point ρ_{redC}, which is calculated in the normal section. This is different to the flash temperature method acc. to ISO/TS 6336-20:2022 [14], as it is already stated in ISO/TS 6336-21:2022 [15].

Equation 5 can be applied in a velocity range of $1\,m/s \le v_t \le 50\,m/s$. For higher circumferential speeds than 50 m/s, v_t has to be set to $v_t = 50\,m/s$. Furthermore, the specific normal tooth load w_{Bt} must be $> 150\,N/mm$. For lower values, it has to be set to $w_{Bt} = 150\,N/mm$.

Additionally, ISO/TS 6336-21 [15] offers an alternative equation for the calculation of μ_{mC} which represents test results within a range of $a = 91.5\,mm$ to 200 mm::

$$\mu_{mC} = 0.048 \cdot \left(\frac{F_{bt}/b}{v_{\Sigma C} \cdot \rho_{\mathrm{redC}}}\right)^{0.2} \cdot \eta_{\mathrm{oil}}^{-0.05} \cdot X_L \cdot Ra^{0.25} \quad (7)$$

This alternative approach was adapted by Schlenk [22] who investigated the size influence on the scuffing load carrying capacity. Therefore, experimental tests on gears with center distances $a = 91.5\,mm$ and $a = 200\,mm$ were performed. The equation is validated for mineral and synthetic oils. In the approach, the load is defined divergently compared to Eqs. 1 and (5). Instead of the nominal transverse circumferential load F_t, the nominal transverse load in plane of action F_{bt} is used. Additionally, the load factors K are not considered. The roughness influence is taken into account in the same manner as in ISO/TS 6336-20:2022 [14]. It should be considered that Eq. 7 has the same speed and load limits for application as Eq. 5. [15].

Besides scuffing, also the micropitting load carrying capacity requires the calculation of the contact temperature in the gear mesh during operation. Therefore, the mean gear coefficient of friction acc. to ISO/TS 6336-22:2018 [13] is calculated as follows:

$$\mu_m = 0.045 \cdot \left(\frac{K_A \cdot K_v \cdot K_{H\alpha} \cdot K_{H\beta} \cdot F_{bt} \cdot K_{B\gamma}}{b \cdot v_{\Sigma,C} \cdot \rho_{n,C}}\right)^{0.2} \cdot \left(10^3 \cdot \eta_{\theta \mathrm{oil}}\right)^{-0.05} \cdot X_R \cdot X_L \quad (8)$$

This equation shows a similar structure as Eq. 5 from ISO 6336-21:2022 [15]. The roughness factor X_R is defined identically to ISO/TS 6336-21:2022 [15] (see Eq. 6; $\rho_{n,C}$ (normal section) instead of ρ_{redC} (transverse section)) and the lubricant factor X_L takes into account the oil type, but not the dynamic viscosity $\eta_{\theta oil}$, which is considered instead in the main equation. The factor 10^3 is due to the used units. For the load, the nominal transverse load in plane of action F_{bt} is used.

Equation 8 acc. to ISO/TS 6336-22:2018 [13] has no limitations in application regarding speed and specific tooth load.

ISO/TR 14179-2:2001 [12] deals with the thermal load carrying capacity of gears. This Technical Report can generally be applied to cylindrical gears, bevel and hypoid gears. Equation 9 refers to the corresponding calculation of the

Table 1 Comparison of the different approaches for calculating the mean gear coefficient of friction (μ_m is proportional to the mathematical terms given in the table each; $K_i = K_A \cdot K_v \cdot K_{H\alpha} \cdot K_{H\beta}$)

Approach	Constant	Load	Sum velocity	Equivalent radius of curvature	Surface roughness	Viscosity
ISO/TS 6336-20:2022 [14]	0.060	$\left(\frac{K_i\,K_{mp}\,F_t}{b}\right)^{0.2}$	$\left(v_{g\Sigma C}\right)^{-0.2}$	$(\rho_{\mathrm{relC}})^{-0.2}$ (transverse section)	$Ra^{0.25}$	$\eta_{\mathrm{oil}}^{-0.05}$
ISO/TS 6336-21:2022 Formula (1) [15]	0.045	$\left(\frac{K_i\,F_t\,K_{B\gamma}}{b}\right)^{0.2}$	$(v_{\Sigma C})^{-0.2}$	$2.2 \cdot (\rho_{\mathrm{redC}})^{-0.45}$ (normal section)	$Ra^{0.25}$	$\eta_{\mathrm{oil}}^{-0.05}$
ISO/TS 6336-21:2022 Formula (8) [15]/ ISO/TR 14179-2:2001 [12]	0.048	$\left(\frac{F_{bt}}{b}\right)^{0.2}$	$(v_{\Sigma C})^{-0.2}$	$(\rho_{\mathrm{redC}})^{-0.2}$ (normal section)	$Ra^{0.25}$	$\eta_{\mathrm{oil}}^{-0.05}$
ISO/TS 6336-22:2018 [13]	0.045	$\left(\frac{K_i\,F_{bt}\,K_{B\gamma}}{b}\right)^{0.2}$	$(v_{\Sigma})^{-0.2}$	$2.2 \cdot (\rho_{n,C})^{-0.45}$ (normal section)	$Ra^{0.25}$	$\eta_{\Theta\mathrm{oil}}^{-0.05}$

mean gear coefficient of friction which is based on the approach of Schlenk [22]:

$$\mu_{mz} = 0.048 \cdot \left(\frac{F/b}{v_\Sigma \cdot \rho}\right)^{0.2} \cdot \eta_{\mathrm{oil}}^{-0.05} \cdot X_L \cdot Ra^{0.25} \tag{9}$$

In case of cylindrical gears, the force F is substituted by F_{bt}, which is the nominal transverse load in plane of action. K factors are not included in the equation and are therefore set to 1.0. The equivalent radius of curvature ρ is calculated in the normal section. The surface roughness Ra is taken into account as like for the flash temperature method acc. to ISO/TS 6336-20:2022 [14]. Equation 9 is identical to Eq. 7 with the exception of the K factors.

Equation 9 can be applied in a velocity range of $1\,\mathrm{m/s} \leq v_t \leq 50\,\mathrm{m/s}$. For higher circumferential speeds than 50 m/s, v_t has to be set to $v_t = 50\,\mathrm{m/s}$. Furthermore, the specific normal tooth load w_{Bt} must be $>150\,\mathrm{N/mm}$. For lower values, it has to be set to $w_{Bt} = 150\,\mathrm{N/mm}$. This is analogous to Eq. 5.

Table 1 gives an overview of all investigated approaches for calculating the mean gear coefficient of friction.

The state of the art regarding the calculation of the mean gear coefficient of friction shows for different parts of the international standard ISO 6336 respectively ISO/TR 14179-2 different calculation equations. The approaches are comparable with respect to their general structure and identical input parameters are considered. However, there are different definitions for the load, the section plane and the weighting of the factors. The substantial discrepancies can be identified for the constant, the consideration of the influence of the helix angle and of the equivalent radius of curvature.

The mean gear coefficient of friction is defined with different symbols in the standards. In the following, μ_{mC} is used as a consistent symbol but the values for the mean gear coefficient of friction are calculated acc. to each approach.

3 Results on comparative calculations

To demonstrate the differences between the different approaches for the mean gear coefficient of friction, exemplary calculations were performed. Therefore, three examples (1, 4 and 6) were taken from ISO/TR 6336-30 [16]. Additionally, the FZG test gears type C were investigated, which are used in standardized oil test procedures [6]. The input data for the calculations are summarized in Table 2.

In all following exemplary calculations for the mean gear coefficient of friction, the load factors K except for the helical load factor $K_{B\gamma}$ were set to 1.0.

The calculus of variations were performed only for mineral oils of ISO VG 320 (examples 1, 4, 6 from ISO/TR 6336-30 [16]) and ISO VG 100 (FZG test gears type C) without any additives. The lubricant factor (without the influence of the viscosity) was therefore set to $X_L = 1.0$ for all calculations. The kinematic viscosity at oil temperature was calculated acc. to DIN 51563 [5]. The oil density at oil temperature was calculated acc. to DIN 51757 [4]. The kinematic viscosity at 40 °C and 100 °C and the density at 15 °C were taken from the oil database of the calculation program 'WTplus' which is developed by FZG [20, 21]. The values in the database are similar to the values documented in the FVA reference oil catalogue [17].

Surface roughness, rotational speed and the lubricant temperature were varied to investigate the influence on the calculated values of the mean gear coefficient of friction. By varying the oil temperature, the operating viscosity of the oil is changed indirectly. It should be added that the varied parameters and operation conditions can partly lead to unrealistic combinations of operating parameters. The example calculations shall only demonstrate the general influence on the calculated mean gear coefficient of friction.

Table 2 Input data for the exemplary calculations of the mean gear coefficient of friction; with * marked parameters were varied

Parameter	Examples from ISO/TR 6336-30 [16]			FZG type C [6]
	Example 1	Example 4	Example 6	
Center distance a in mm	500		–500	91.5
Face width b in mm	100		125	14
Reference diameter d_1/d_2 in mm	141.34/856.35	136/864	180/–1188	72/108
Normal module m_n in mm	8.0		12.0	4.5
Number of teeth z_1/z_2	17/103	17/108	15/–99	16/24
Normal pressure angle α_n in °		20.0		
Base helix angle β_b in °	14.82	0.0	0.0	0.0
Surface roughness Ra in µm (*)	0.5	0.5	1.0	0.3
Nominal transverse load in plane of action F_{bt} in N	136,160	140,850	11,824	5912
Rotational speed n_1 in min^{-1} (*)	360	360	360	2250
Kinematic viscosity at 40 °C ν_{40} in mm^2/s		320		100
Oil temperature ϑ_{oil} in °C (*)		60		90
Oil density at 15 °C ρ_{15} in kg/m^3		900		880

3.1 Variation of surface roughness

In a first step, the arithmetic mean roughness Ra of pinion and wheel was varied in a spectrum between 0.05 and 1.0 µm. Figure 1 shows the resulting mean gear coefficient of friction with increasing surface roughness for example 4 taken from ISO/TR 6336-30 [16].

A degressive profile can be observed for all considered calculation approaches. The calculation acc. to ISO/TS 6336-20:2022 [14] leads to the highest values for the mean gear coefficient of friction. Furthermore, it is obvious that the calculation approach acc. to ISO/TS 6336-21:2022, p. 7, Formula (8) (see Eq. 7) [15] is identical to the approach acc. to ISO/TR 14179-2:2001 (see Eq. 9) [12]. The lowest values for the mean coefficient of friction are calculated acc. to the approaches given in ISO/TS 6336-21:2022, p. 5, Formula (1) (see Eq. 5) [15] and ISO/TS 6336-22:2018 [13].

For the Example 4 acc. to ISO/TR 6336-30 [16], it can be stated that the calculation approaches acc. to ISO/TS 6336-21:2022 [15], ISO/TS 6336-22:2018 [13] and ISO/TR 14179-2:2001 [12] lead to similar results for the mean coefficient of friction whereas the approach acc. to the flash temperature concept in ISO/TS 6336-20:2022 [14] yields approximately 25% higher values.

Analogous calculations were carried out for the FZG test gears type C, which are used as standardized test gears in the micropitting test acc. to DIN 3990-16:2020 [6] or FVA 54/7 [9], respectively, and in the gear efficiency test acc. to FVA 345 [7]. Compared to example 4 from ISO/TR 6336-30 [16], the test gears type C are much smaller (normal

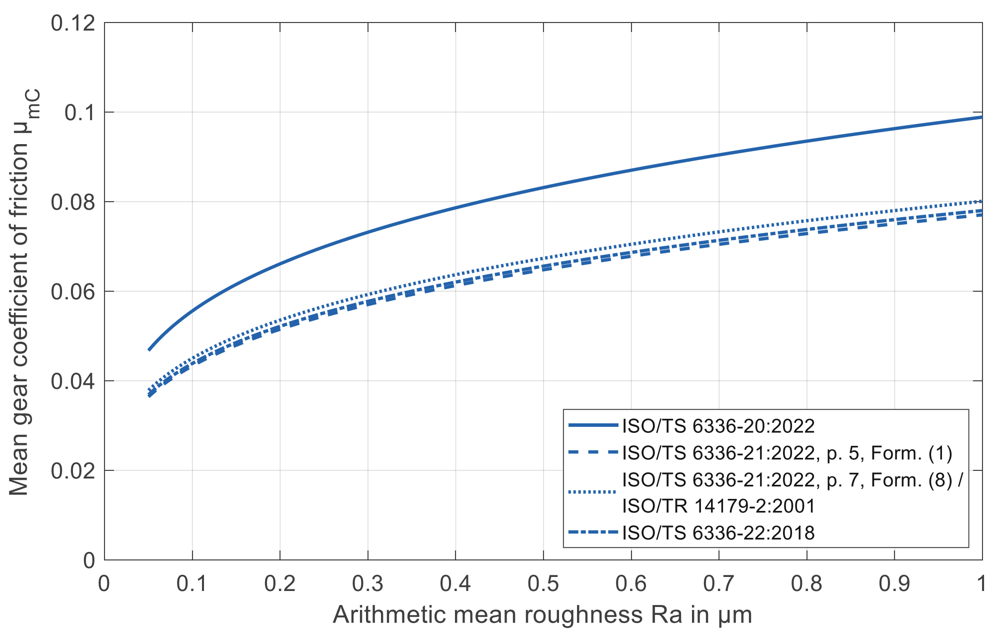

Fig. 1 Calculated values for the mean gear coefficient of friction with varying surface roughness (Example 4 from ISO/TR 6336-30 [16])

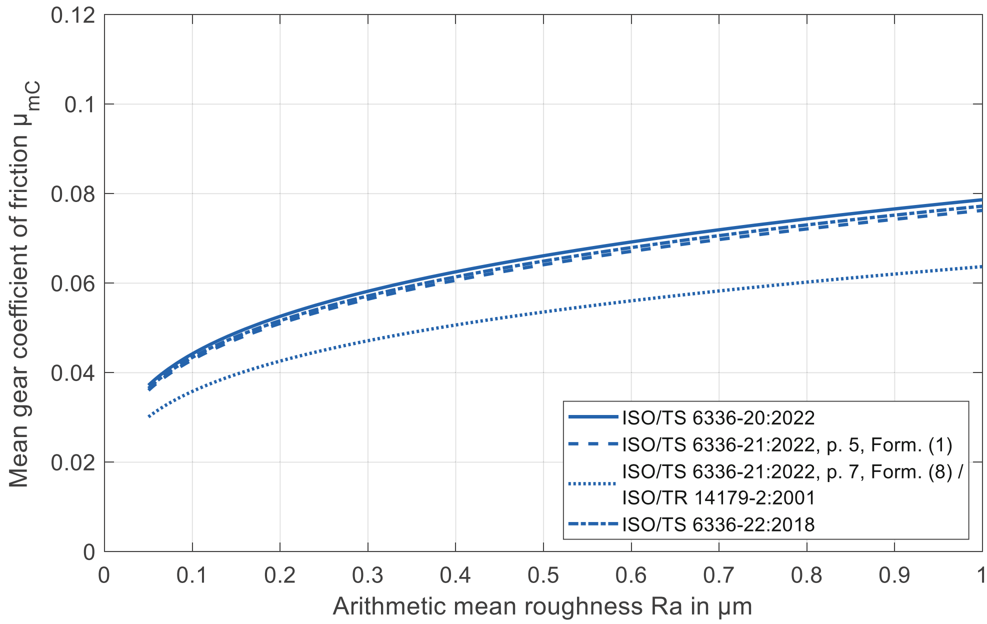

Fig. 2 Calculated values for the mean gear coefficient of friction with varying surface roughness (FZG type C gear acc. to DIN 3990-16:2020 [6])

module $m_n = 4.5$ mm) and the operating conditions differ significantly.

The corresponding results for the mean gear coefficient of friction are plotted in Fig. 2. A degressive rise of the mean gear coefficient of friction with increasing arithmetic mean roughness values can be observed. For this example, the approaches acc. to ISO/TS 6336-20:2022 [14] (see Eq. 1), ISO/TS 6336-21, p. 5, Formula (1) [15] (see Eq. 5) and ISO/TS 6336-22:2018 [13] (see Eq. 8) lead to comparable results. The calculated values acc. to these three approaches are considerably higher than the values calculated acc. to ISO/TS 6336-21:2022, p. 7, Formula (8) [15] and ISO/TR 14179-2:2001 [12].

3.2 Variation of rotational speed

Furthermore, the rotational speed of the gears was varied in a defined range. The rotational speed has a strong impact on the sum velocity in the gear mesh, which directly influences the formation of the lubricating film.

In Fig. 3, the mean gear coefficient of friction was calculated for Example 1 from ISO/TR 6336-30 [16]. The rotational speed is varied in a range between 50 and 3000 min^{-1}. All other input values were chosen acc. to Table 2. It can be observed that the mean gear coefficient of friction decreases with higher rotational speed values. Especially for low rotational velocities, comparably high values are calculated. The highest mean gear coefficients of friction are calculated by the method acc. to ISO/TS 6336-20:2022 [14]. The other approaches lead to similar results, which are much lower than for the approach acc. to ISO/TS 6336-20:2022 [14].

In comparison to the Example 1 (see Table 2), the results with the FZG test gears of type C show the same tendency as for the variation of the surface roughness (see Fig. 4). The highest values are calculated for the method acc. to ISO/TS 6336-20:2022 [14]. Much lower values are calculated acc. to ISO/TS 6336-21:2022, p. 7, Formula (8) [15] and ISO/TR 14179-2:2001 [12], respectively. The results of the calculations acc. to ISO/TS 6336-21:2022, p. 5, Formula (1) [15] and ISO/TS 6336-22:2018 [13] are similar to the ones acc. to ISO/TS 6336-20:2022 [13].

3.3 Variation of oil temperature

As a last varied parameter, the oil temperature is considered. By changing this parameter, the operating viscosity is influenced. The oil temperature is varied in a range from 40 °C and 140 °C.

Exemplary results for Example 6 from ISO/TR 6336-30 [16] are plotted in Fig. 5. The influence of the oil temperature and therefore the influence of the operating lubricant viscosity are smaller than the influences from surface roughness and rotational speed.

3.4 Comparative calculations regarding the gear size influence

To evaluate the influence of the gear size on the mean gear coefficient of friction, further calculations with the four known examples (see Table 2) were conducted. Therefore, input parameters were set to constant values where possible. The dynamic oil viscosity η was set to 50 mPa s, the arithmetic mean roughness Ra was chosen to 0.3 µm and the rotational speed was 1000 min^{-1}. For the load, the original values from Table 2 were taken. The sum of tangential velocities at the pitch point and the equivalent radius of

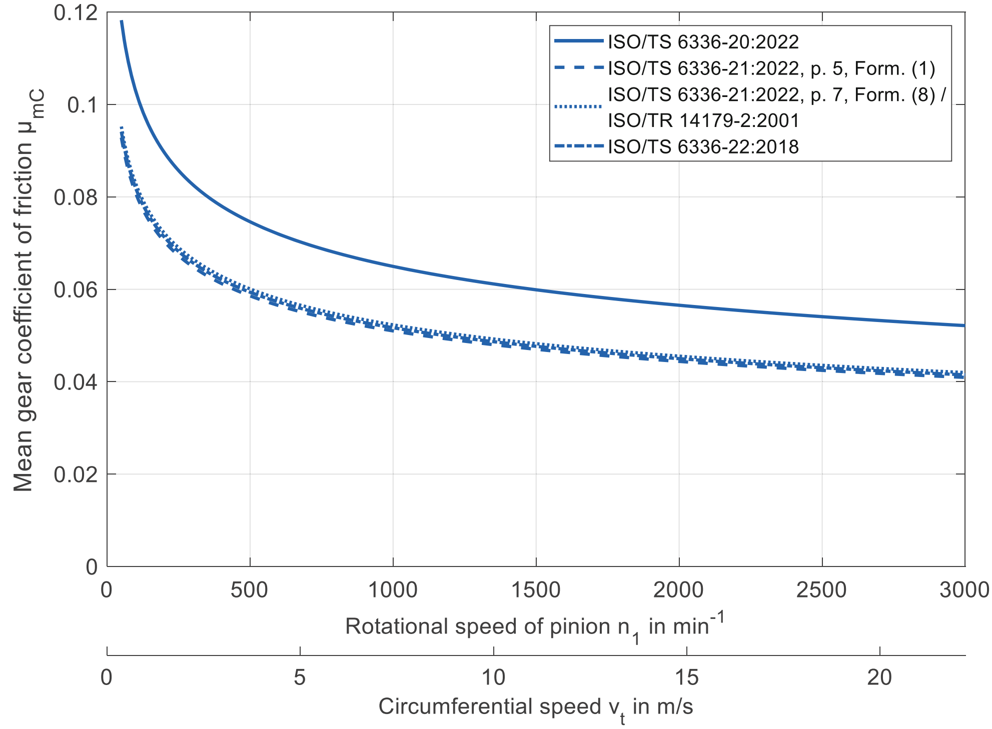

Fig. 3 Calculated values for the mean gear coefficient of friction with varying rotational speed (Example 1 from ISO/TR 6336-30 [16])

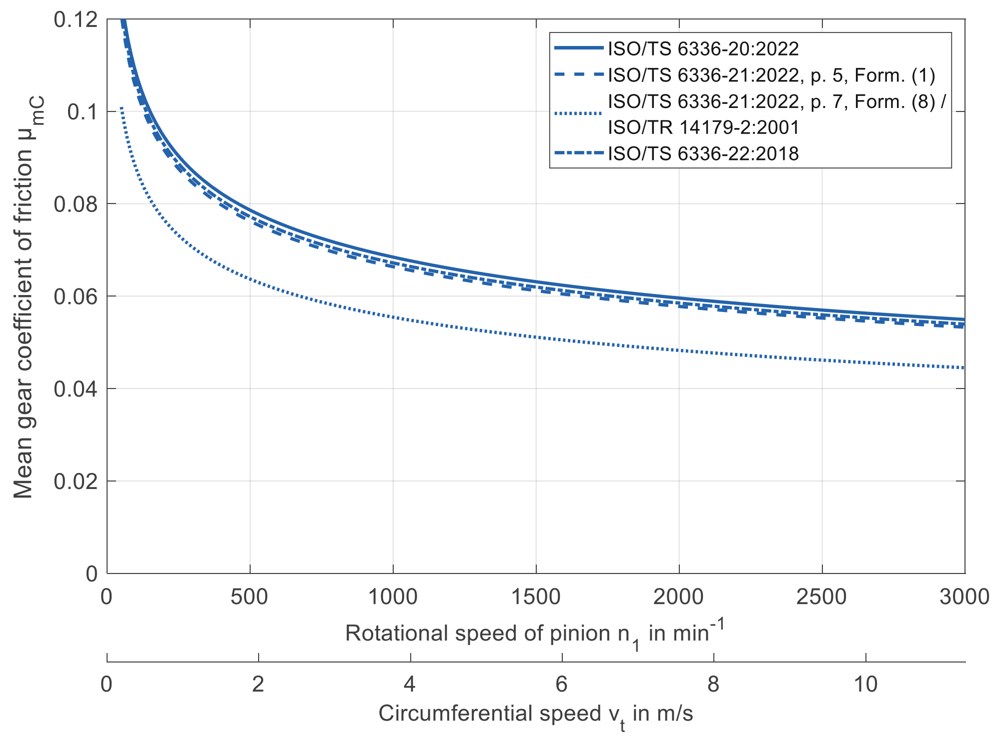

Fig. 4 Calculated values for the mean gear coefficient of friction with varying rotational speed (FZG type C gear acc. to DIN 3990-16:2020 [6])

curvature are influenced by the different dimensions of the gears.

The results of the calculations are depicted in Fig. 6. The Examples 1 and 4 from ISO/TR 6336-30 [16] (see Table 2) show comparable results for the mean gear coefficient of friction due to their similar main geometry. The internal gear (Example 6) is characterized by significantly lower values as to be expected due to the advantageous curvature and sliding conditions and the smaller specific tooth load. The values for the FZG type C test gears are in a similar range as for Examples 1 and 4.

An interesting aspect of these results is that the approaches acc. to ISO/TS 6336-21:2022, p. 5, Formula (1) [15] and ISO/TS 6336-22:2018 [13] converge with the re-

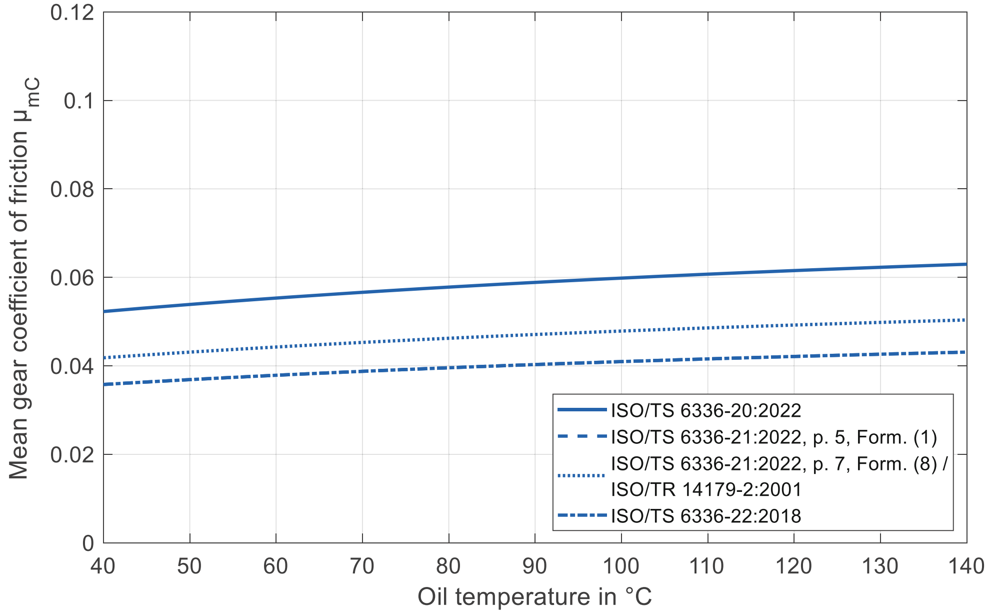

Fig. 5 Calculated values for the mean gear coefficient of friction with varying oil temperature (Example 6 from ISO/TR 6336-30 [16])

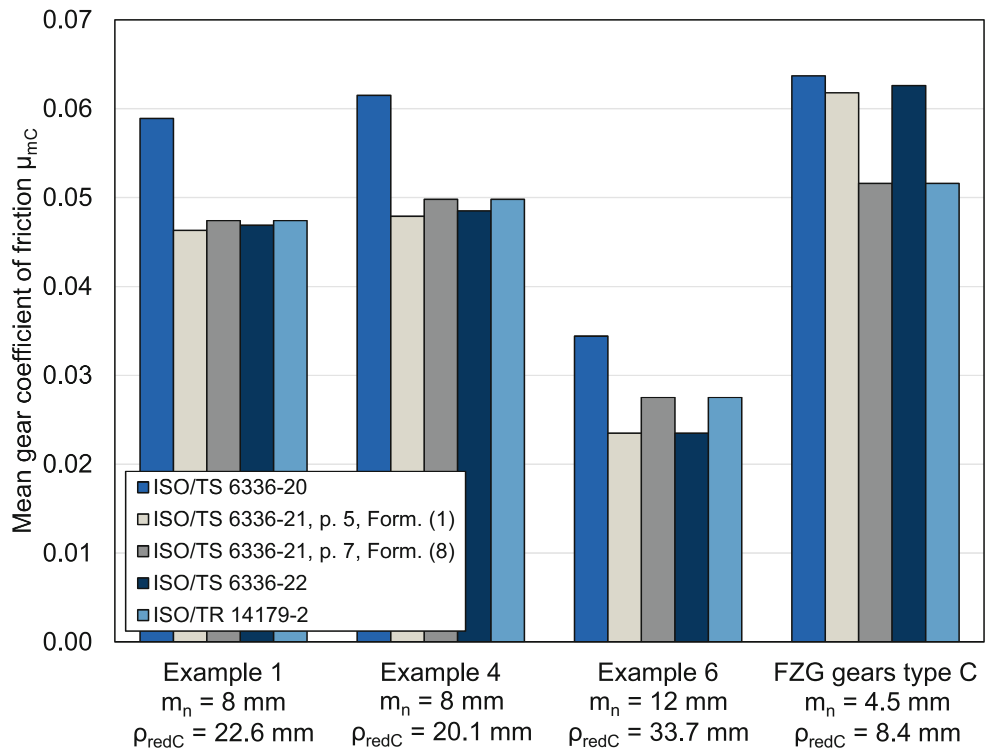

Fig. 6 Calculated values for the mean gear coefficient of friction for the four examples (see Table 2) with focus on the size influence

sults acc. to ISO/TS 6336-20:2022 [14] for smaller gear size. The difference between the approaches is minimal for the FZG type C test gears with a normal module of $m_n = 4.5$ mm. In the case of Example 6 with $m_n = 12$ mm, the calculated values acc. to ISO/TS 6336-21:2022, p. 5, Formula (1) [15] and ISO/TS 6336-22:2018 [13] have the highest relative deviation from the results acc. to ISO/TS 6336-20:2022 [14] and the values are smaller than for both other approaches acc. to ISO/TS 6336-21:2022, p. 7, Formula (8) [15] and ISO/TR 14179-2:2001 [12], respectively. The reason for the described pattern is the different weighting of the equivalent radius of curvature as it is discussed in the next chapter.

4 Discussion

The exemplary calculations and the comparison of the different approaches for calculating the mean gear coefficient of friction show some remarkable differences.

First, it can be stated that all approaches are fitted on experimental data which was generated with a specific research focus. The equations for the mean gear coefficient of friction e.g. for scuffing were derived from experimental tests and are only calibrated for each respective purpose. They shall not be used for the calculation of other types of damage or power loss calculations. The same applies for all other investigated approaches.

As a second aspect, it should be noted that all equations for the mean gear coefficient of friction are similar regarding their general structure. They consider identical physical factors as the load, the sum velocity, the equivalent radius of curvature, the oil viscosity, the lubricant type and the surface roughness. All approaches show identical tendencies regarding the roughness influence, speed influence and the influence of the oil temperature. However, the absolute values differ in most cases.

The approaches acc. to ISO/TS 6336-21:2022, p. 5, Formula (1) [15] and ISO/TS 6336-22:2018 [13] are nearly identical with except of the load. The calculation acc. to ISO/TS 6336-21:2022, p. 5, Formula (1) [15] uses the nominal circumferential load F_t whereas in ISO/TS 6336-22:2018 [13] the nominal transverse load in plane of action F_{bt} is taken into account. In both approaches, the load is additionally multiplied with the helical load factor $K_{B\gamma}$ for scuffing, which considers the increasing gear friction for an increasing total contact ratio. This factor is not included in the remaining calculation approaches. In the investigated examples, $K_{B\gamma}$ is >1 for Example 1 (see Table 2).

The approach in ISO/TS 6336-21:2022, p. 7, Formula (8) [15] is exactly identical with the one in ISO/TR 14179-2:2001 [12]. Hence, the calculated values for the mean gear coefficient of friction are the same.

The equation acc. to ISO/TS 6336-20:2022 [14] differs from the other four equations because of the constant factor of 0.06 which leads to higher results for the mean coefficient of friction. Furthermore, the calculation of the equivalent radius of curvature is different compared to the other approaches because it is performed in transverse section. In all other investigated approaches, the base helix angle is included in the calculation of the equivalent radius of curvature, which is synonymous with the normal section.

The calculation of the sum of tangential velocities at the pitch point is equal for all five approaches, although the notation is not uniform among the different standards. In ISO/TS 6336-21:2022 [15], the calculation requires the circumferential velocity v at the reference diameter. In the other approaches, the sum of tangential velocities is calculated with the pitch line velocity v_t.

The surface roughness shows a degressive influence on the mean gear coefficient of friction (see Fig. 1 and 2). The higher the roughness, the more asperities come into contact and cause higher friction in the gear mesh. Furthermore, the mean gear coefficient of friction depends reciprocally from the rotational speed, as it is shown in Fig. 3 and 4. Higher circumferential speeds affect a more advantageous lubrication due to a higher sum velocity in the gear mesh. A higher lubricating film thickness supports the separation of the surfaces and therefore, the friction is reduced. The influence of the oil temperature on the resulting mean gear coefficient of friction is comparably low, see Fig. 5. The operating oil viscosity, which is strongly influenced by the oil temperature, is weighted only with an exponent of –0.05 in all approaches.

From the exemplary calculations, it can be observed that the constant factor of each equation has a strong impact on the calculated mean gear coefficient of friction. For smaller gear sizes, the values for the mean gear coefficient of friction calculated acc. to ISO/TS 6336-21:2022, p. 5, Formula (1) [15] and ISO/TS 6336-22:2018 [13] tend to approximate the values calculated acc. to ISO/TS 6336-20:2022 [14], see Fig. 6. This size influence is indirectly included in the equivalent radius of curvature. The equivalent radius of curvature itself is not equally weighted in the investigated approaches. In the equations acc. to ISO/TS 6336-20:2022 [14], ISO/TS 6336-21:2022, p. 7, Formula (8) [15] and ISO/TR 14179-2:2001 [12], the mean coefficient of friction depends from the equivalent radius of curvature in the following manner:

$$\mu_{mC} \sim \rho_{\mathrm{redC}}^{-0.2} \tag{10}$$

In contrast to this, the weighting of the equivalent radius of curvature in ISO/TS 6336-21:2022, p. 5, Formula (1) [15] and ISO/TS 6336-22:2018 [13] is as follows whereby the different sections (transverse and normal plane) have to be considered:

$$\mu_{mC} \sim \rho_{\mathrm{redC}}^{-0.45} \tag{11}$$

The different weighting is included in the roughness factor X_R. From Fig. 1 to Fig. 4 and from Fig. 6, it can be seen that smaller gears with smaller equivalent radii of curvature cause higher mean gear coefficients of friction acc. to ISO/TS 6336-21:2022, p. 5, Formula (1) [15] and ISO/TS 6336-22:2018 [13].

As a last aspect, it should be mentioned that the calculated values for the mean gear coefficient of friction for helical gears compared to similar spur gears are smaller. This is valid for all five investigated approaches. Base helix angles >0° cause higher equivalent radii of curvature and therefore lower values for the mean coefficient of friction.

5 Conclusion

In this review, the different approaches for calculating the mean gear coefficient of friction from several international standards and Technical Specifications were compared. The origin of the approaches was documented and the equations were analyzed in terms of the input parameters and their weighting. Exemplary calculations for gears of different size and different operating conditions were performed. To analyze the impact of specific input values, the surface roughness, the rotational speed and the oil temperature and therefore the operating oil viscosity were varied within typical ranges. The results acc. to the different approaches were compared.

The results show that the approaches differ in several manner although they are identical regarding their general structure. The equations for the mean gear coefficient of friction acc. to ISO/TS 6336-21:2022, p. 7, Formula (8) [15] and ISO/TR 14179-2:2001 [12] are identical. The equations acc. to ISO/TS 6336-21:2022, p. 5, Formula (1) [15] and ISO/TS 6336-22:2018 [13] are identical apart from the different input for the load (F_t vs F_{bt}). The equation acc. to ISO/TS 6336-20:2022 [14] has a higher constant factor and leads to the highest values for the mean coefficient of friction among all calculated examples. Furthermore, the equivalent radius of curvature is weighted differently for two of the five approaches (acc. to ISO/TS 6336-21:2022, p. 5, Formula (1) [15] and ISO/TS 6336-22:2018 [13]).

At current state, each equation is only valid for the dedicated international standard. Due to the slight differences between the approaches, it should be the superordinate future target to merge the equations to one single equation for calculating the mean gear coefficient of friction. At the same time, the permissible values of the individual calculations must also be adjusted. For this purpose, further experimental investigations should be carried out with the focus also outside the previous validity ranges.

6 Nomenclature

The nomenclature is shown in Table 3.

7 Indices

The indices are shown in Table 4.

Table 3 Nomenclature

a	Center distance in mm
b	Face width in mm
d	Reference diameter in mm
F	Load in N (ISO/TR 14179-2:2001 [12])
F_{bt}	Nominal transverse load in plane of action in N
F_t	Nominal circumferential load in N
K_A	Application factor
$K_{B\alpha}$	Transverse load factor for scuffing
$K_{B\beta}$	Face load factor for scuffing
$K_{B\gamma}$	Helical load factor for scuffing
$K_{H\alpha}$	Transverse load factor
$K_{H\beta}$	Face load factor
K_{mp}	Multiple-path factor
K_v	Dynamic factor
m_n	Normal module in mm
n	Rotational speed in min^{-1}
Ra	Arithmetic mean roughness in µm (newly manufactured gears)
$v_{g\Sigma C}$	Sum of tangential velocities at the pitch point in m/s (ISO/TS 6336-20:2022 [14])
v_{Σ}	Sum of tangential velocities (at the pitch point) in m/s (ISO/TR 14179-2:2001 [12])
$v_{\Sigma C}$	Sum of tangential velocities at the pitch point in m/s
w_{Bt}	Specific tooth load for scuffing in N/mm
X_L	Lubricant factor
X_R	Roughness factor
z	Number of teeth
α_n	Normal pressure angle in °
β	Helix angle in °
β_b	Base helix angle in °
η_{oil}	Dynamic viscosity at lubricant temperature in mPa s
$\eta_{\theta\text{oil}}$	Dynamic viscosity at lubricant temperature in Pa s (ISO/TS 6336-22:2018 [13])
ϑ_{oil}	Oil temperature in °C
μ, μ_m	Mean gear coefficient of friction
μ_{mz}	Mean gear coefficient of friction (ISO/TR 14179-2:2001 [12])
ν_{40}	Kinematic viscosity at 40 °C in mm^2/s
ρ_{15}	Oil density at 15 °C in kg/m^3
ρ	Equivalent radius of curvature (normal section) at the pitch diameter in mm (ISO/TR 14179-2:2001 [12])
$\rho_{n,C}$	Equivalent radius of curvature (normal section) at the pitch diameter in mm (ISO/TS 6336-22:2018 [13])
ρ_{redC}	Equivalent radius of curvature (normal section) at the pitch diameter in mm (ISO/TS 6336-21:2022 [15])
ρ_{relC}	Transverse equivalent radius of curvature in mm (ISO/TS 6336-20:2022 [14])

Table 4 Indices

1	Pinion
2	Wheel
B	Scuffing
C	Pitch point
H	Tooth flank (pitting)

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