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Using state diagrams to generate tree tensor networks of a quantum system

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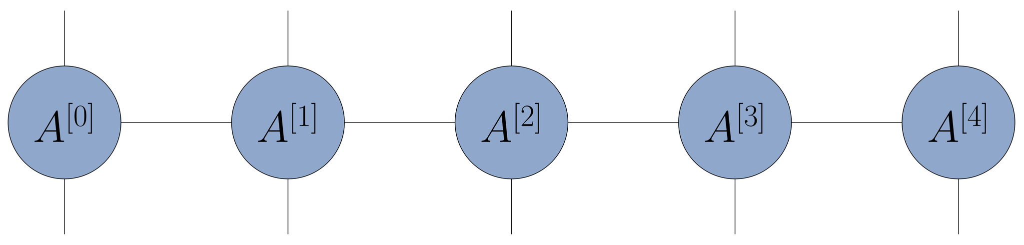
Motivation

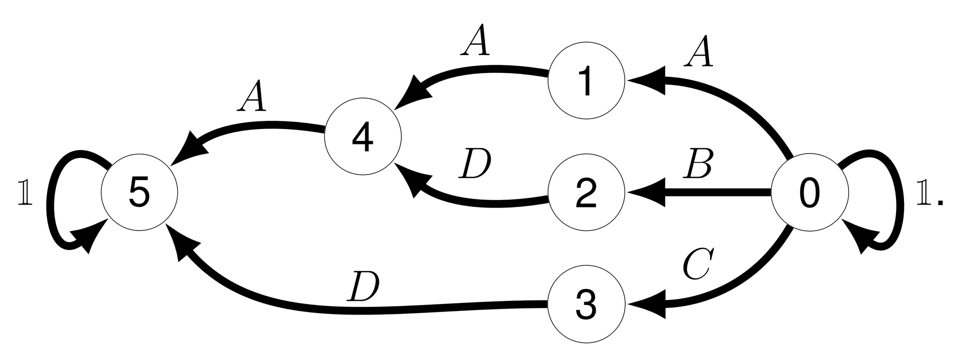
Many relevant Hamiltonians and operators have the following form

$$H = \sum_{i=1}^{K} \bigotimes_{s \in Q} A_i^{[s]},$$

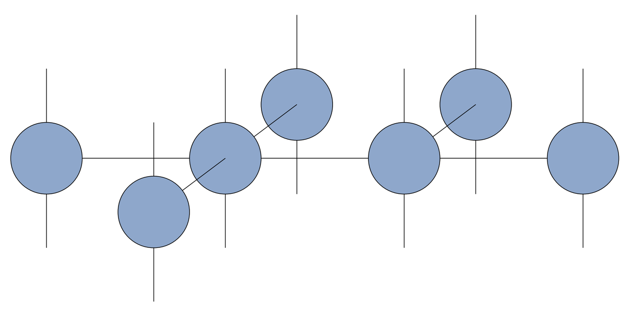
where Q is a set of small quantum systems or sites and the operator $A^{[s]}$ acts on site s.
If Q represents a 1D-chain, we can bring such an operator in matrix product operator form



using finite state automata, e.g.

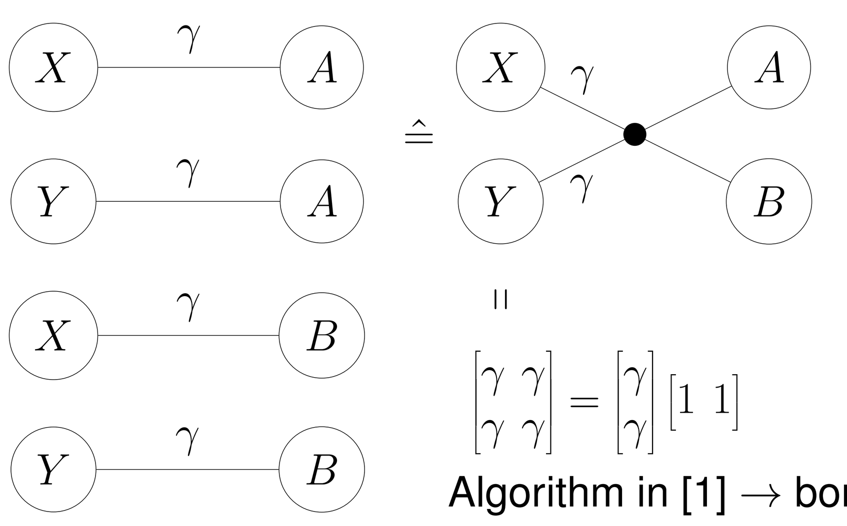


However, if Q or the operator have a tree structure it can be advantageous to use a tree tensor network operator



In this case the basic automaton method fails. Therefore we considered state diagrams and developed an algorithm to obtain a state diagram that corresponds to a given operator.

Special Case



$$\begin{bmatrix}\gamma & \gamma\\ \gamma & \gamma\end{bmatrix} = \begin{bmatrix}\gamma\\ \gamma\end{bmatrix}\begin{bmatrix}1 & 1\end{bmatrix}$$

Algorithm in [1] → bond dim 2.

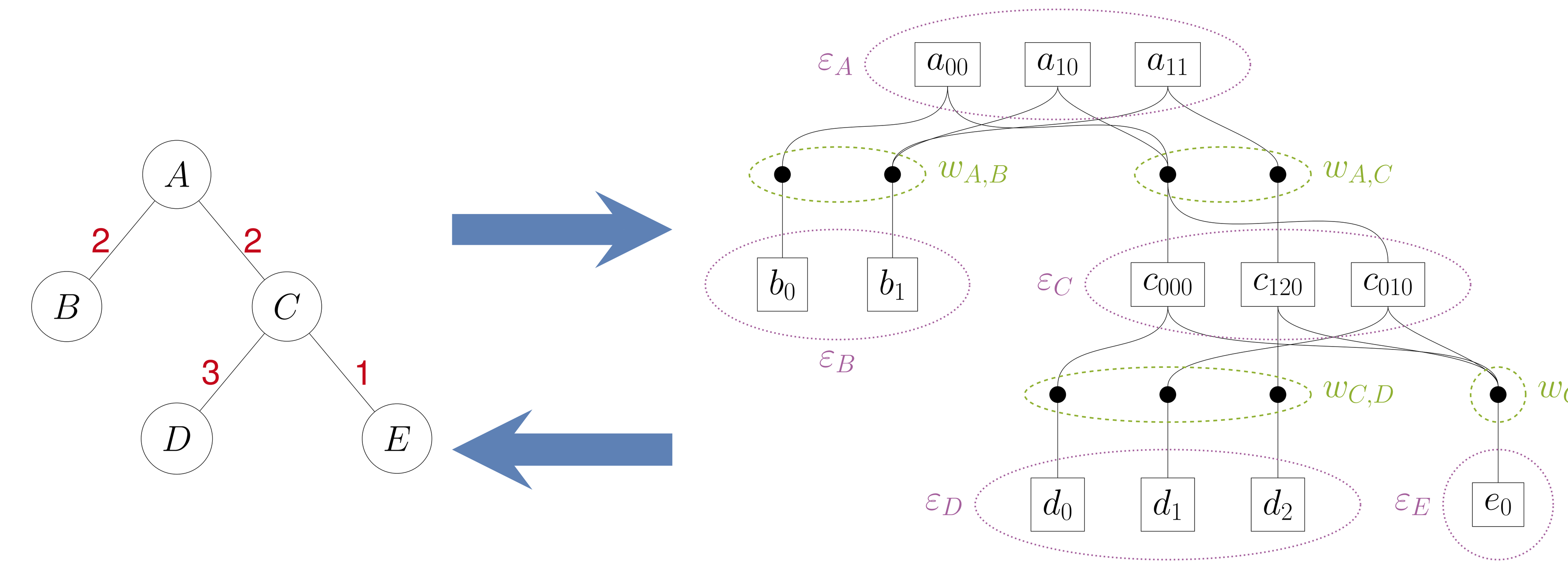
Reference

Richard M. Milbradt, Qunsheng Huang, Christian B. Mendl; *State Diagrams to determine Tree Tensor Network Operators*; SciPost Phys. Core 7, 036 (2024)

Richard M. Milbradt, Qunsheng Huang, Christian B. Mendl; *PyTreeNet: A Python Library for easy Utilisation of Tree Tensor Networks*; arXiv: 2407.13249

[1] J. Ren, W. Li, T. Jiang, Z. Shuai; J. Chem. Phys. 153, 084118 (2020)

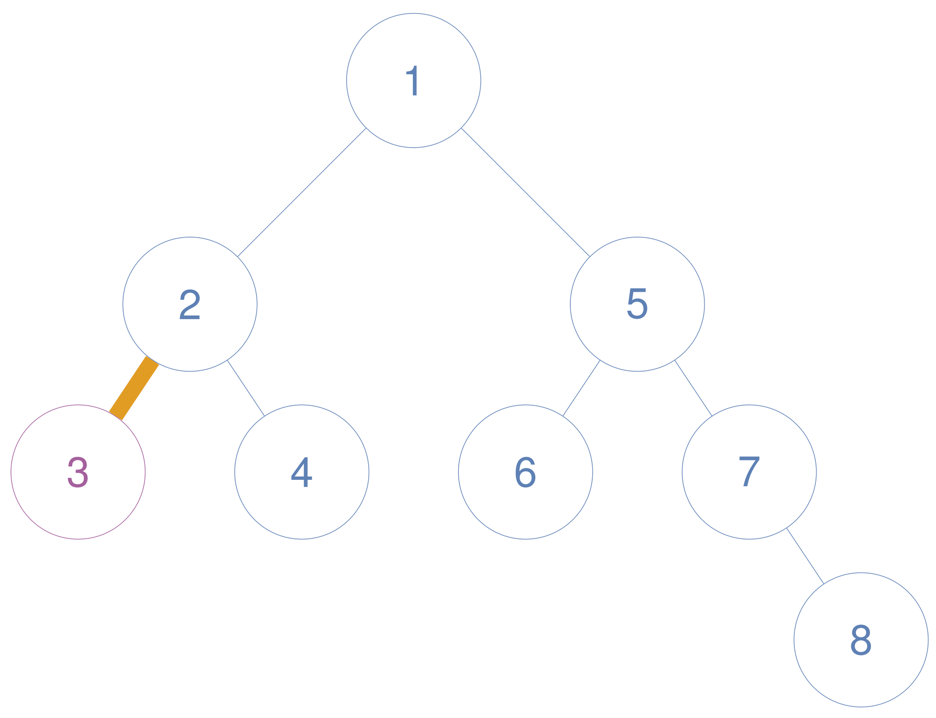
The State Diagram



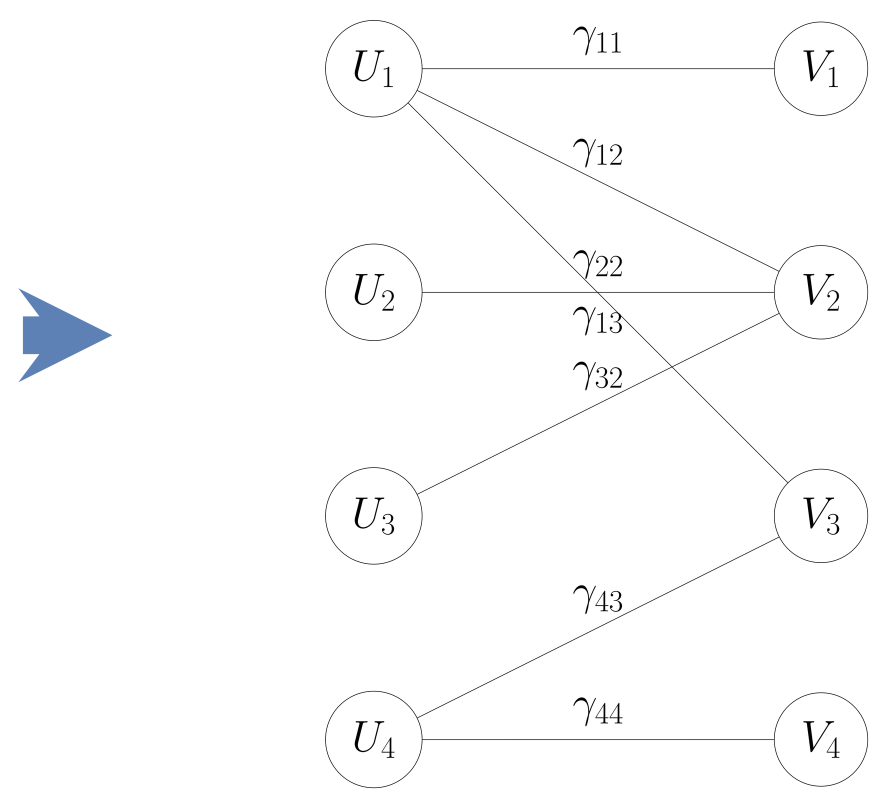
Tree Tensor Network | State Diagram

Combining Terms

Focus on a single bond:



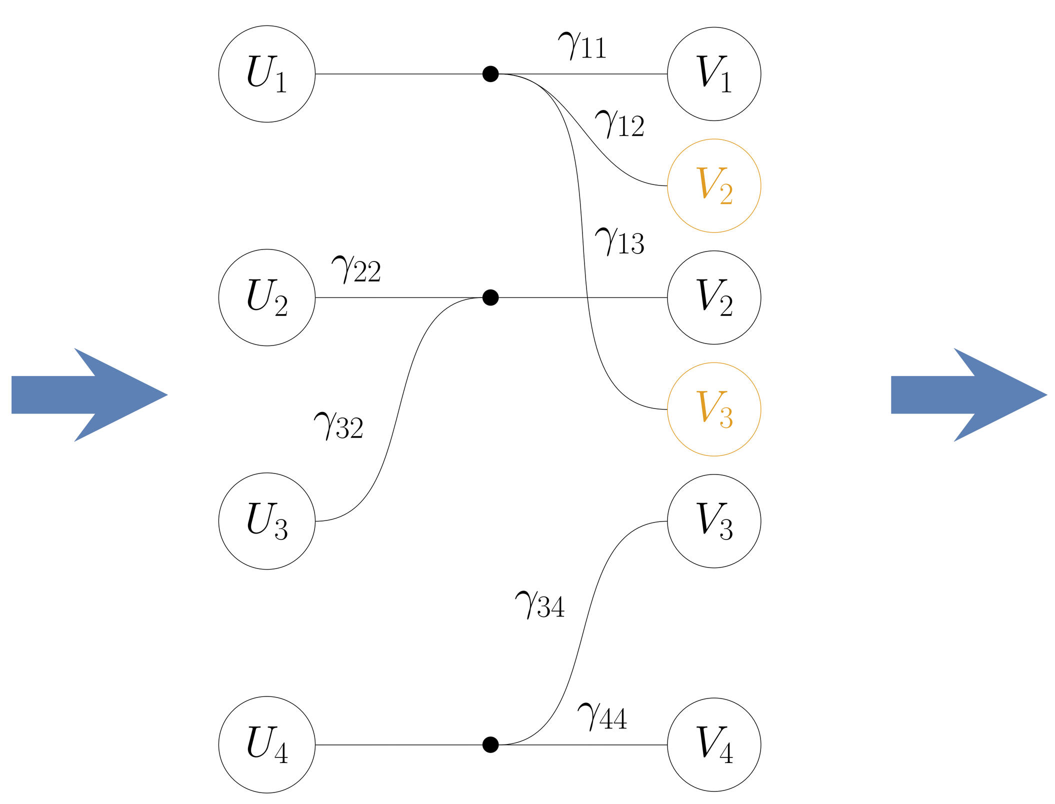
Compare Sub State Diagrams:



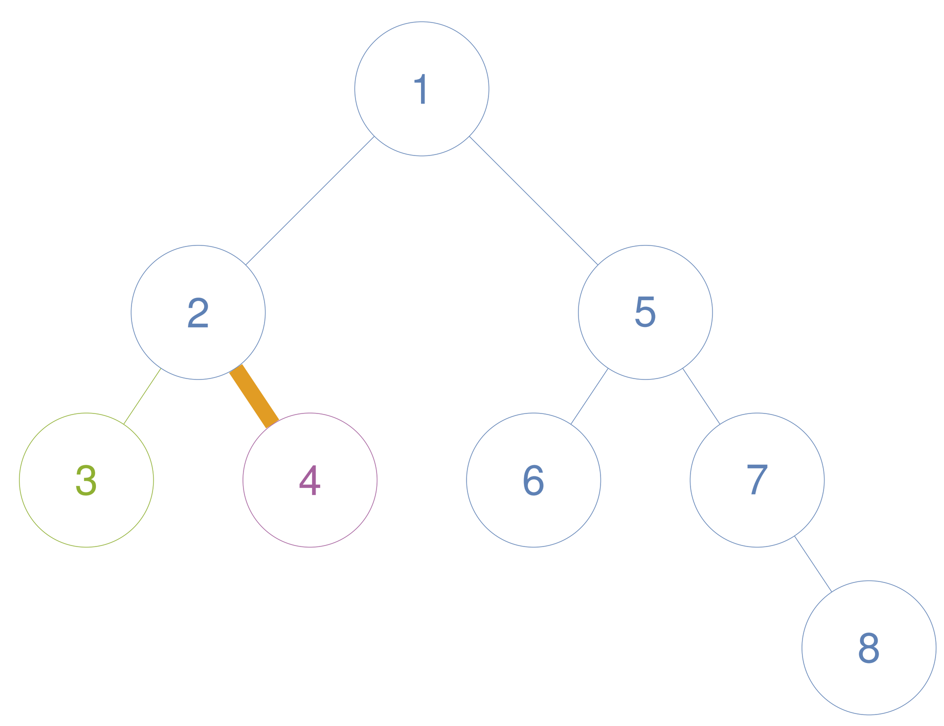
$$\Gamma = \begin{array}{c|cccc} & V_1 & V_2 & V_3 & V_4 \\ \hline U_1 & \gamma_{11} & \gamma_{12} & \gamma_{13} & 0 \\ U_2 & \gamma_{21} & \gamma_{22} & 0 & 0 \\ U_3 & 0 & \gamma_{32} & 0 & 0 \\ U_4 & 0 & 0 & \gamma_{43} & \gamma_{44} \end{array}$$

Symbolic Gaussian Elimination $=$

$$\begin{bmatrix}1\\0\\0\\0\end{bmatrix}\begin{bmatrix}\gamma_{11} & \gamma_{12} & \gamma_{13} & 0\end{bmatrix} + \begin{bmatrix}0\\ \gamma_{22}\\ \gamma_{32}\\ 0\end{bmatrix}\begin{bmatrix}0 & 1 & 0 & 0\end{bmatrix} + \begin{bmatrix}0\\0\\0\\1\end{bmatrix}\begin{bmatrix}0 & 0 & \gamma_{43} & \gamma_{44}\end{bmatrix}$$



Next Bond:



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