Information Theory for Dispersion-Free Fiber Channels with Distributed Amplification

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***Abstract*—Information theory is developed for dispersion-free fiber channels with distributed optical amplification (OA). Autocorrelation functions are derived and used to characterize the spectral broadening of propagating signals, to upper bound the output power of bandlimited receivers, and to upper bound the output entropy and capacity of receivers with white noise. The output power of bandlimited receivers is shown to scale at most with the square-root of the average signal launch power if the OA bandwidth is held fixed. The capacity thus scales at most as one-half the logarithm of the launch power. However, in practice the OA bandwidth should exceed the propagating signal bandwidth to compensate attenuation. It is shown that there is a power threshold beyond which this is not possible and the model loses practical relevance. Nevertheless, for the mathematical model an upper bound on capacity is developed when the OA bandwidth scales as the square-root of the launch power, in which case the capacity scales at most as the inverse fourth root of the launch power.**

I. Introduction

One obstacle to understand the capacity of optical fiber is that nonlinearity and distributed optical amplification (OA) cause spectral broadening that is difficult to characterize. One approach to make progress is to study simplified models that retain the essential features of spectral broadening. The model studied here is dispersion-free fiber with distributed OA.

There are two existing approaches to proceed. The first is by Mecozzi [1] who derived the *per-sample* statistics of the channel, including the channel conditional probability distribution. Turitsyn et al. [2], and Yousefi and Kschischang [3], rederive this distribution with other methods. They further argue that, for large launch power P, the *per-sample* capacity is the same as the capacity of an additive white Gaussian noise (AWGN) channel with a direct detection receiver, i.e., capacity is closely approached with intensity modulation, and grows as $\frac{1}{2}\log P$ for large P. Refined results were recently posted in [4], [5].

A second approach considers the entire received *waveform*. Tang [6], [7] studied the auto- and crosscorrelation functions of the channel input and output signals when the input signals are Gaussian and stationary, and in particular when the input signals are sinc pulses with complex and circularly symmetric Gaussian modulation. The autocorrelation function defines the signal power spectral density (PSD) that lets one study spectral broadening. Tang used the PSD to evaluate Pinsker's capacity lower bound [8] for wavelength division multiplexing (WDM) and per-channel receivers without cooperation.

II. OA Bandwidth and Waveform Receivers

The per-sample model is attractive because one has closed-form expressions for the statistics. However, the model has several limitations. First, the per-sample statistics do not capture spectral broadening, and this tempts one to consider only the *launch* signal bandwidth rather than the *propagating* signal bandwidth[1]. The propagating signal bandwidth W grows with the launch power P and a practical requirement is that the OA bandwidth B exceed W to compensate attenuation, i.e., one requires $B \geq W$. However, we show that there is a P beyond which B does not exceed W and the model loses practical relevance.[2] The growth of W is due to signal-noise mixing that cannot be controlled by waveform design.

Second, a per-sample receiver has infinite bandwidth while practical receivers are bandlimited. In other words, a per-sample analysis takes limits in a particular order: first the receiver bandwidth is made infinite and then P is made large. However, for a given system the receiver bandwidth is fixed, and changing the order of limits (first P is made large) can change the capacity scaling.

Third, the per-sample model ignores correlations in the received waveform, and this can lead to suboptimal receivers. In fact, we show that a *three-sample* receiver achieves infinite capacity for *any* P for the model studied in [1], [2], [3]. The per-sample rate $\frac{1}{2}\log P$ thus underestimates capacity.[3] This issue will also appear for the nonlinear Schrödinger equation (NLSE) with dispersion, nonlinearity, and distributed noise.

The result may be understood as follows: the noise in the model of [1] has limited bandwidth, and by sending signal energy in the noise-free spectrum one achieves large rate, cf. [10, Thm. 5]. The reader may expect that an obvious fix is to add white (thermal or electronic) noise to the channel or receiver models. However, the per-sample capacity is then zero. This conundrum shows that reasonable and precise noise

[1]There are many reasonable definitions for bandwidth. We use a common one, namely the length of the frequency range centered at the carrier frequency that contains a specified fraction of the signal power.

[2]The short article [9] also argues that the model of [1] may be impractical for large P. The arguments are based on qualitative and empirical observations concerning spectral broadening and signal-noise mixing.

[3]The potential for capacity increase was noted in [3, Sec. VIII] but without recognizing the extent of the effect, i.e., that the noise model is unreasonable. Hence, the main conclusions in [3, Sec. VIII] should be treated with caution, namely that the capacity of dispersion-free fiber grows as $\frac{1}{2}\log P$, and that a potential peak of spectral efficiency curves is due to deterministic effects only, and not due to signal-noise mixing.

models, device models, and spectral analyses are needed when analyzing capacity, e.g., see [11, Sec. IX.A-B].

Based on these observations, we conclude that one must study the waveform model, and not only the per-sample model. We proceed by studying *two-sample* statistics for the waveform channel. We additionally place practical constraints on the OAs, transmitter, and receiver. First, we model the receiver as performing *projections* with white noise, e.g., due to thermal noise. Second, the receiver has either finite bandwidth or finite time resolution. Finally, we study OA bandwidth that grows with the propagating signal bandwidth. The detailed derivations for the results can be found in [12].

III. Organization

The talk is organized as follows. We begin by reviewing AWGN channels and the fiber models under study. We next point out limitations of the latter models for capacity analyses and present receiver models that remedy these problems somewhat. We develop bounds on the output power of our receiver models for general waveforms. The bounds show that the model loses practical relevance beyond some power threshold because the OA bandwidth no longer exceeds the propagating signal bandwidth.

Although the model is not practical at large signal launch power, we use the power bounds to establish capacity bounds. Our approach is to scale the OA bandwidth at the same rate as the spectral broadening when the OA bandwidth does not scale (the actual scaling must be faster and does not catch up to the spectral broadening). We show that the capacity scales at most as the inverse fourth root of the launch power. This bound is likely far above the true capacity but it establishes the presence of a nonlinear Shannon limit under certain practical device and OA requirements.

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