Low–Density Parity–Check Modulation

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Two common approaches to combining coding and modulation are to employ trellis coded modulation (TCM) [1] or bit-interleaved coded modulation (BICM) [2]. We here discuss an alternative that is well–suited for iterative decoding and that employs low–density parity–check (LDPC) codes [3]. We consider three basic ideas: apply the output of the LDPC encoder directly to the modulator (see, e.g., [4]), combine the demodulator with the LDPC decoder's variable nodes, and design the LDPC degree sequences based on the *combined* detector and decoder. We call this approach *LDPC modulation*. The approach is perhaps more closely related to TCM than BICM in that the code is designed together with the modulation.

The structure of a LDPC decoder is shown in Fig. 1. The decoder has n variable nodes, an edge interleaver, and $n-k$ check nodes. The ith variable node represents the ith bit of the code word. Suppose that this bit is involved in $d_{v,i}$ parity checks so that its node has $d_{v,i}$ edges going into the edge interleaver. The edge interleaver connects the variable nodes to the check nodes, each of which represents a parity-check equation. The ith check node checks $d_{c,i}$ bits so that it has $d_{c,i}$ edges. The sets of variable and check nodes are referred to as the variable node decoder (VND) and check node decoder (CND), respectively. Iterative decoding is performed by exchanging extrinsic information between the VND and CND (see, e.g., [5]).

We consider long irregular LDPC codes for which the most important design aspect is the choice of node degrees, or the node degree sequences [6, 7]. We will design the node degrees by using extrinsic information transfer (EXIT) charts [8]. The result is coded modulations that approach capacity within about 1dB.

Multi-Antenna Examples

We illustrate how to combine the coding and modulation by considering a multi–input, multi–output (MIMO) channel where the channel is known at the receiver but not at the transmitter. Suppose there are M transmit and N receive antennas. Each transmitter symbol is thus an $M \times 1$ vector $\mathbf{s} = [s_1, ..., s_M]^T$ whose entries take on complex values in a constellation set. We consider constellations of size 2^{M_c} so that each symbol $\mathbf{s}$ carries $M \cdot M_c$ coded bits. For example, for quadrature phase-shift keying (QPSK) we have $M_c = 2$ and for 16 quadrature amplitude modulation (16-QAM) we have $M_c = 4$. The average energy per transmit symbol is limited to E_s, and we further assume that $E[||s_m||^2] = E_s/M$.

The receiver sees vectors $\mathbf{y} = \mathbf{Hs} + \mathbf{n}$ of size $N \times 1$, where $\mathbf{H}$ is the $N \times M$ channel matrix and $\mathbf{n}$ is a $N \times 1$ noise vector. We assume $\mathbf{H}$ to be known by the receiver only, and the entries of $\mathbf{n}$ to be independent, complex, zero–mean, Gaussian random variables with variance $\sigma^2 = N_0/2$ per real component. The MIMO detector performs *a posteriori* probability (APP) bit detection by considering all 2^{MM_c} possible hypotheses on $\mathbf{s}$. The detector's soft output is forwarded to the decoder, which in turn computes extrinsic information to be used as *a priori* knowledge by the detector for the next iteration.

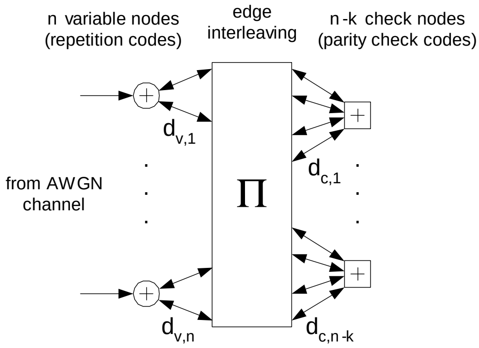

Figure 1: Structure of the decoder for a LDPC code.

Suppose the entries of $\mathbf{H}$ are independent, complex, zero–mean, Gaussian random variables with unit variance. We define the normalized signal-to-noise ratio E_b/N_0 as

$$\left.\frac{E_b}{N_0}\right|_{\text{dB}} = \left.\frac{E_s}{N_0}\right|_{\text{dB}} + 10\log_{10}\frac{N}{RMM_c}. \tag{1}$$

For a *piecewise constant* channel the matrix $\mathbf{H}$ remains unchanged over long time intervals, while for an *ergodic* channel $\mathbf{H}$ changes for every symbol $\mathbf{s}$. We consider only the ergodic model whose capacity is (see [9, 10])

$$C = E\left[\log_2 \det\left(\mathbf{I}_N + \frac{E_s}{N_0}\frac{1}{M}\mathbf{H}\mathbf{H}^\dagger\right)\right] \tag{2}$$

where $\mathbf{H}^\dagger$ denotes the complex-conjugate transpose of $\mathbf{H}$. Capacity is achieved by using Gaussian distributed symbols $\mathbf{s}$. However, in practice the entries of $\mathbf{s}$ are constrained to come from QAM constellations, in which case the capacity is reduced somewhat.

We combine the MIMO detector and the LDPC variable node decoder as in Fig. 2. The values $I_{A,VND}$ and $I_{A,DET}$ measure the *a priori* information going into the VND and MIMO detector, respectively. Similarly, the values $I_{E,DET}$ and $I_{E,VND}$ measure the extrinsic information coming out of the detector and VND, respectively. We refer to [12] for details on how to compute these values. We consider two kinds of EXIT curves: one for the CND and one for the *combined* detector and VND. We will use check–regular (or right–regular) LDPC codes, i.e., all check nodes have the same degree d_c [11]. We design the variable node degrees to fit the EXIT curve of the detector/VND to the EXIT curve of the CND. For simplicity we restrict ourselves to just three different variable node degrees so that the curve fit can be performed manually. This simple approach gives surprisingly powerful coded modulation schemes.

Fig. 3 shows simulation results with QPSK modulation, $R = 1/2$, $n = 10^5$, a random edge interleaver, and 60 iterations. Our approach turns out to be quite successful: All LDPC modulation schemes operate within about 1dB of their respective capacity limits (see Table 1). We remark that the gaps to capacity in Fig. 3 (or Table 1) can be narrowed by using more variable node degrees. The performance curves of a BICM scheme using a turbo code [13] with memory 3 constituent codes are given as references. Note that the turbo code does not interact efficiently with the MIMO detector when $M > N$, as was already pointed out in [14].

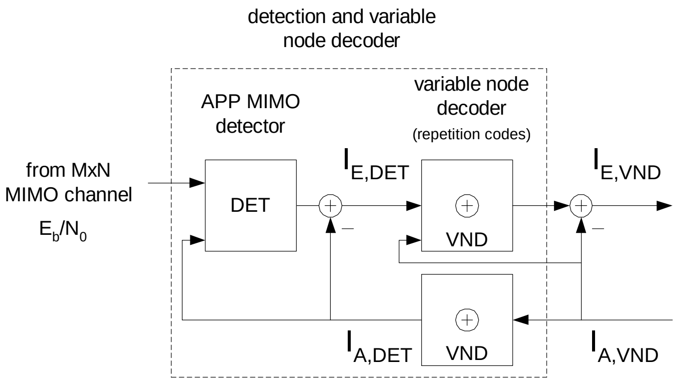

Figure 2: The combined MIMO detector and VND.

Table 1: Capacity Limits (with QPSK) and LDPC Modulation Performance

$M \times N$	E_b/N_0 [dB] Capacity	E_b/N_0 [dB] for BER $= 10^{-5}$
2×1	3.25	4.00
4×1	6.65	8.20
4×2	2.95	3.60
4×3	1.97	2.45
4×4	1.47	1.95

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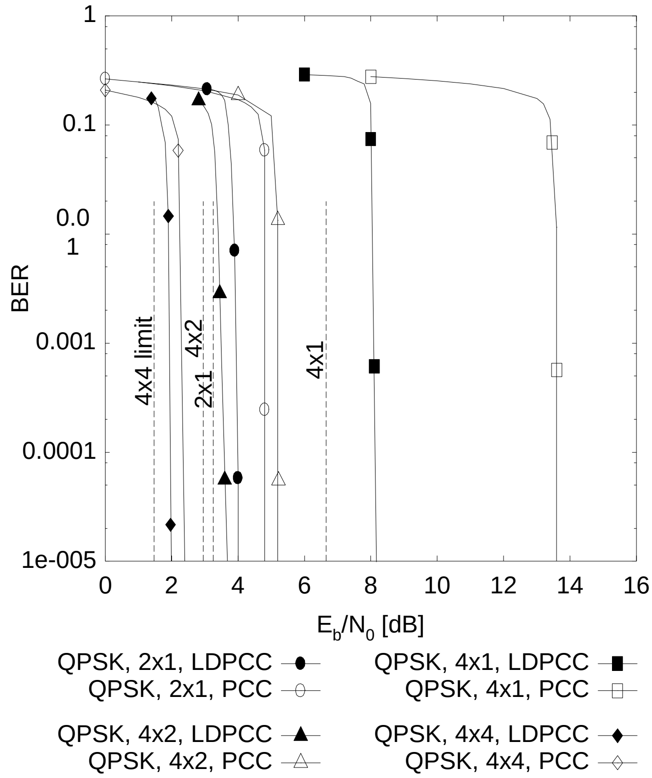

Figure 3: Bit error rates for combined MIMO detectors/decoders. The notation $M \times N$ means there are M transmit and N receive antennas. The modulation is QPSK and all codes have $R = 1/2$ and $n = 10^5$ for a total rate of M bits/use. LDPC codes: 60 iterations. Parallel concatenated (turbo) codes: memory 3, 20 iterations.

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