Relay vs. User Cooperation in Multiaccess Networks

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***Abstract*— The performance of user-cooperation in a multiaccess network is compared to that of using a wireless relay. Using the total transmit and processing power consumed at all nodes as a cost metric, the outage probabilities achieved by dynamic decode-and-forward (DDF) and amplify-and-forward (AF) are compared for the two networks. A high SNR outage analysis in conjunction with area-averaged numerical simulations is used to show that user and relay cooperation can achieve a maximum diversity of K and 2 respectively for a K-user multiaccess network under both DDF and AF. However, by accounting for the energy costs of cooperation it is shown that relay cooperation is more energy efficient than user cooperation, i.e., it achieves *coding* (SNR) *gains* that override the diversity advantage of the latter.**

I. Introduction

Cooperation results when nodes in a network share their power and bandwidth resources to mutually enhance their transmissions and receptions. Cooperation can be induced in several ways. We compare two approaches to inducing cooperation in a multiaccess channel (MAC) comprised of K sources and one destination. First, we allow source nodes to forward data for each other and second, we introduce a wireless relay node when cooperation between the sources nodes is either undesirable or not possible. We refer to networks employing the former approach as *user cooperative networks* and those employing the latter as *relay networks*.

There are important differences between user cooperative and relay networks that are not easy to analyze from an information-theoretic point of view. For example, in cooperative networks one likely needs economic incentives to induce cooperation. On the other hand, hierarchical networks incur infrastructure costs (see [1]). While incentives and infrastructure costs are important elements that need to be considered in comparing the two networks, we use the total transmit and processing power consumed for both cooperative and non-cooperative transmissions in each network as a cost metric for our comparisons. To this end, we model the processing power as a function of the transmission rate, and hence, the transmit signal-to-noise ratio (SNR). We also introduce *processing scale factors* to characterize the ratio of the energy costs of processing relative to that for transmission. By accounting for both the transmit and processing power (energy) costs, we identify the processing factor regimes where the two cooperative approaches are energy efficient.

We consider single-antenna half-duplex nodes and constrain the source nodes in both networks to time-duplex their transmissions. Thus, in the relay network, each source coooperates with the relay over two-hops. For the user cooperative network, for $K > 2$ we consider both two-hop and multi-hop cooperative schemes. We compare the outage performance of the two networks as a function of the total transmit SNR for the cooperative strategies of *dynamic* decode-and-forward (DDF) [2] and amplify-and-forward (AF). We present upper and lower bounds on the outage probability of DDF and AF for both networks and compare their outage performance via a *coding (SNR) gain* [3]. For single-antenna nodes, the maximum DDF and AF diversity for two-hop relaying is 2 [2]. For the two-hop user cooperative network, we show that, if relay selection is allowed, AF achieves a maximum diversity of 2. Further, except for a *clustered* geometry where the maximum diversity approaches K, DDF also achieves a maximum diversity of 2 [4, Chap. 4]. On the other hand, when users cooperate using a K-hop scheme, both DDF [2] and AF [3], [4] achieve a maximum diversity of K.

The coding gains achieved are in general a function of the transmission parameters and network geometry. In an effort to generalize such results, we present an *area-averaged* numerical comparison. Specifically, we consider a sector of a circular area with the destination at the center, a fixed relay position, and the users randomly distributed in the sector. We remark that this geometry encompasses a variety of centralized network architectures ranging from wireless LAN and cellular to sensor networks. Our analytical and numerical results demonstrate the effect of processing power in cooperation and are summarized by the following observations: i) user cooperation can achieve higher diversity gains than relay cooperation but at the expense of increased complexity and ii) relay cooperation achieves larger coding gains when we account for the energy costs of cooperation thus dramatically diminishing the effect of the diversity gains achieved by user cooperation.

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This paper is organized as follows. In Section II, we present the network and channel models and develop a power-based cost metric. In Section III, we present the outage approximations for the DDF and AF strategies for both networks. In Section IV, we present the numerical results. We conclude in Section V.

II. Channel and Network Models

A. Network Model

Our networks consist of K users (source nodes) numbered $1, 2, \ldots, K$ and a destination node d. For the relay network there is one additional node, the relay node r. The input and output alphabets of node k are $\mathcal{X}_k$ and $\mathcal{Y}_k$, respectively. We impose a *half-duplex* constraint on every node, i.e., each node can be in one of two modes, *listen* (*L*) or *transmit* (*T*) (*LoT*). We write $\mathcal{K} = \{1, 2, \ldots, K\}$ for the set of users and for the relay network, we write $\mathcal{T} = \mathcal{K} \cup \{r\}$ for the set of transmitters.

Let $X_{k,i} \in \mathcal{X}_k$ be the input of node k at time i, $i = 1$, $2, \ldots, n$. We model the wireless multiaccess networks under study as additive Gaussian noise channels with fading. For such channels, the output of node m at time i is

$$Y_{m,i} = \begin{cases} \left(\sum_{k \neq m} h_{m,k,i} X_{k,i} \right) + Z_{m,i} & M_{m,i} = L \\ 0 & M_{m,i} = T \end{cases} \quad (1)$$

where the $Z_{m,i}$ are independent, proper, complex, zero-mean, unit variance Gaussian noise random variables, $M_{m,i}$ is the half-duplex mode at node m, and $h_{m,k,i}$ is the fading gain between transmitter k and receiver m at time i. Note that for both networks as well as the (non-cooperative) MAC, $X_{d,i} = 0$, i.e., $M_{d,i} = L$, for all i. Further, for the relay network and the MAC, we also have $Y_{k,i} = 0$, i.e., $M_{k,i} = T$, for all i and for all $k \in \mathcal{K}$. We assume that the transmitted signals in both networks are constrained in power as

$$\sum_{i=1}^{n} E \left| X_{k,i} \right|^2 \leq n P_k \quad k \in \mathcal{T}. \quad (2)$$

We assume that the modes $M_{k,i}$ for all k, are shared, as needed, between all nodes with negligible overhead. Finally, we use the usual notation for entropy and mutual information [5], [6] and take all logarithms to the base 2 so that our rate units are bits. We write random variables (e.g. W_k) with uppercase letters and their realizations (e.g. w_k) with the corresponding lowercase letters. Finally, throughout the sequel we use the words user and source interchangeably.

B. Relay Network

We model the relay network as a Gaussian multiaccess relay channel (MARC) with $K+1$ inputs $X_{k,i}$, $k \in \mathcal{T}$, and two outputs $Y_{r,i}$ and $Y_{d,i}$ given by (1). We consider a time-duplexed model where each source transmits over the channel for a period $T = 1/K$ of the total time (see Fig. 1). Further, the transmission period of source k, for all k, is subdivided into two slots such that the relay listens in first slot and transmits in the second slot. We denote the time fractions for the two slots as θ_k and $\overline{\theta}_k = 1 - \theta_k$ for user k such that $\theta_k = \Pr(M_r = L) = 1 - \Pr(M_r = T)$. The time-duplexed two-hop scheme for the MARC is illustrated in Fig. 1 for user 2 where $\mathcal{C}_2 = \{r\}$ denotes the set of nodes that cooperate with user 2. We remark that the time-duplex multiaccess (TDMA) model considered here simplifies the analysis for the MARC to that for single-source relay channel and henceforth we refer to this model as a TD-MARC. Note that Fig. 1 also includes the slotting schemes for a MAC and a MAC with time-duplexed sources (TD-MAC).

C. Cooperative Network

We model the cooperative network as a Gaussian MAC-GF [7]. In general, there is a combinatorial explosion in the number of ways one can duplex K sources over their half-duplex states. We present two schemes that allow each user to be aided by an arbitrary number of users, up to K. In both schemes the users time-duplex their transmissions; the two schemes differ in the manner the period T is further sub-divided between the transmitting and the cooperating users.

We first consider a *two-hop scheme* such that the period over which user k, for all k, transmits is sub-divided into two slots. In the first slot only user k transmits while in the second slot both user k and the set $\mathcal{C}_k \subseteq \mathcal{K} \backslash \{k\}$ of users that cooperate with user k transmit. This is shown in Fig. 1 for user 2 and $\mathcal{C}_2 = \{3, 4\}$. We remark that this scheme has the same number of hops as the TD-MARC except now user k can be aided by more than one user in $\mathcal{C}_k$. We write θ_k and $1 - \theta_k$ to denote the time fractions associated with the first and second slots of user k such that $\theta_k = \Pr(M_j = L) = 1 - \Pr(M_j = T)$ for all $j \in \mathcal{C}_k$.

We also consider a *multi-hop* scheme where the total transmission time for source k is divided into L_k slots, $1 \leq L_k \leq K$, where $L_k = |\mathcal{C}_k| + 1$. Specifically, in each time-slot, except the first slot where only user k transmits, one additional user cooperates in the transmission until all L_k users transmit in slot L_k. We denote the l^{th} time fraction for user k as $\theta_{k,l}$, $l = 1, 2, \ldots, L$ (see Fig. 1 for user 2 with $\mathcal{C}_2 = \{3, 4\}$). We refer to this model as a MAC-GF with time-duplexed sources or simply a TD-MAC-GF.

Remark 1: For AF we assume equal length slots and consider symbol-based two-hop and multi-hop schemes.

D. Cost Metric: Total Power

We use the total power consumed by all the nodes in each network as a cost metric for comparisons. Observe that in addition to its transmit power a node also consumes power in processing, i.e., in encoding and decoding its transmissions and receptions, respectively. Further, in addition to its own transmission and processing costs, a node that relays consumes additional power in encoding and decoding packets for other nodes. We account for this by introducing processing costs for (device and protocol) overhead, encoding, and decoding as a function of the transmission and reception rates. For e.g., a relay node that uses DDF consumes power for overhead, encoding, and decoding costs while a relay using AF only has overhead costs. We model these costs by defining encoding

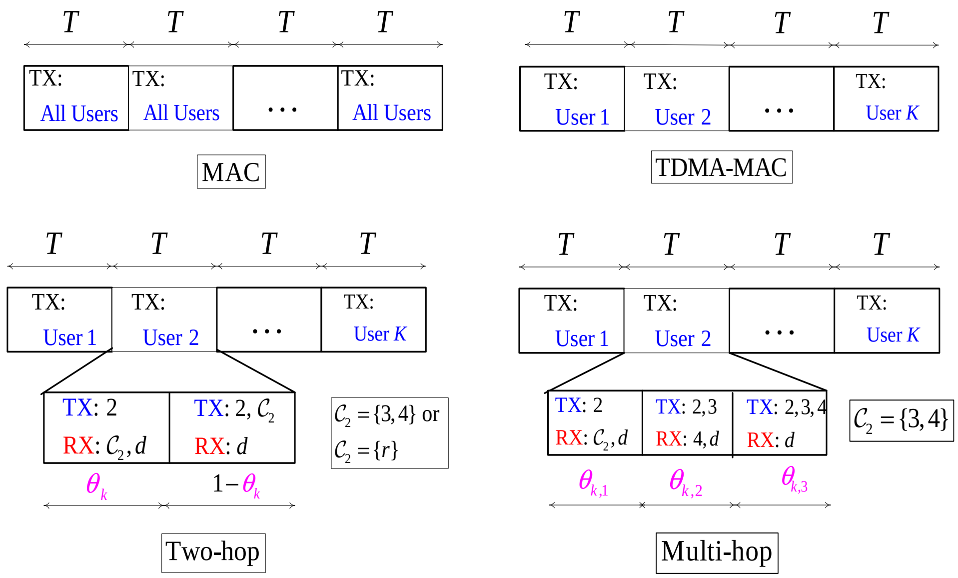

Fig. 1. Time-duplexed transmission schemes for the MARC, the MAC-GF, and the MAC.

and decoding factors η_k and δ_k, respectively, and write the power required to process the transmissions of node j at node k as

$$P_{k,j}^{proc} = P_{k,0}^{proc} + \left(\eta_k I_k^{enc}(j) + \delta_k I_k^{dec}(j)\right) \cdot f(R_j), \quad \text{for all } k \in \mathcal{T}, j \in \mathcal{K} \tag{3}$$

where $P_{k,j}^{proc}$ is the power required by user k to cooperate with user j, $I_k^{enc}(j)$ and $I_k^{dec}(j)$ are indicator functions that are set to 1 if user k encodes and decodes, respectively, for user j, $P_{k,0}^{proc}$ is the minimum processing power at user k which is in general device and protocol dependent, and $f(R_j)$ is a function of the transmission rate R_j at user j. The scale factors η_k and δ_k quantify the energy in Joules required at user k to encode and decode a bit, respectively. Note that for the relay node, we have $P_{r,r}^{proc} = P_{r,0}^{proc}$ which accounts for the costs of simply operating the relay. The processing cost function f in general depends on the encoding and decoding schemes used as well as the device functionality. For simplicity, we choose f as

$$f(R_k) = R_k \quad \text{for all } k. \tag{4}$$

Finally, we assume that the destination in typical multiaccess networks such as cellular or many-to-one sensor networks has access to an unlimited energy source and ignore its processing costs.

We write the total power consumed at node k, $k \in \mathcal{T}$, as

$$P_{k,tot} = \begin{cases} P_k + P_{k,k}^{proc} + \sum\limits_{j \in \mathcal{K}, j \neq k} I_k(j) P_{k,j}^{proc} & k \in \mathcal{K} \\ P_k + \sum\limits_{j \in \mathcal{K}} I_k(j) P_{k,j}^{proc} & k = r \end{cases} \tag{5}$$

where $I_k(j)$ is an indicator function that takes the value 1 if node k cooperates with node j. The first $P_{k,k}^{proc}$ term corresponds to the power used to process its own message while the second summation term accounts for the power node k incurs in cooperating with all other source nodes. Note that at high SNR, i.e., high P_k for all k, the dominating term in (5) is P_k since $P_{k,0}^{proc}$ is usually a constant and R_j can at most increase logarithmically in P_j, for all $k, j \in \mathcal{K}$. The total power consumed by all transmitting nodes in each network is given as

$$P_{tot} = \begin{cases} \sum\limits_{k \in \mathcal{K}} P_{k,tot} & \text{MAC-GF or MAC} \\ \sum\limits_{k \in \mathcal{T}} P_{k,tot} & \text{MARC.} \end{cases} \tag{6}$$

Finally, we remark that for AF, the chosen model ignores the energy costs of forwarding; i.e., $P_k^{tot} = P_k + P_{k,0}^{proc}$ for all k.

E. Fading Models

We model the fading gains as

$$h_{m,k,i} = \frac{A_{m,k,i}}{\sqrt{d_{m,k}^{\gamma}}} \tag{7}$$

where $d_{m,k}$ is the distance between the m^{th} receiver and the k^{th} source, γ is the path-loss exponent, and $A_{m,k,i}$ are jointly i.i.d. zero-mean, unit variance proper, complex Gaussian random variables. We assume that the fading gain $h_{m,k,i}$ is known only at receiver m. We assume that the fading gains are independent of the transmitted signals X_k, for all $k \in \mathcal{T}$. Finally, without loss of generality, we assume that no two nodes are co-located.

III. Geometry-Inclusive Outage Analysis

We compare the outage performance of the two cooperative networks via a limiting analysis of the outage probabilities achieved by DDF and AF. Limiting analysis of the outage probability enables the characterization of two key parameters, namely, the diversity order and the coding gains, which correspond to the slope and the intercept respectively of the log-outage vs. SNR in dB curve [3]. In [3], Laneman develops bounds on the DF and AF outage probabilities for a relay channel where the source and the relay transmit on orthogonal channels. In [2], the authors introduce the DDF strategy and show that DDF achieves the diversity-multiplexing tradeoff performance [8] of an equivalent MIMO channel in the regime of small multiplexing gains for both two-hop and multi-hop relay channels. However such an analysis does not characterize the coding gains thus neglecting the effect of geometry on the outage performance of distributed cooperative networks.

To quantify the diversity and coding gains achieved by both user and relay cooperative networks, in [4, Chap. 4], we develop geometry-inclusive upper and lower bounds on the DDF and AF outage probability for both two-hop and multi-hop cooperative schemes. We present a brief summary of the results below.

A. Dynamic-Decode-and-Forward

1) MARC with Time-Duplexed Sources: We consider independent Gaussian signaling at the sources and relay with transmit variance $\overline{P}_k = KP_k$ and $\overline{P}_r = P_r/\overline{\theta}_k$ at the sources and relay respectively. The outage probability for user k transmitting at a fixed rate R_k is

$$P_o^{(k)} = \Pr\left(I_d^r\left(k\right) < R_k\right) \tag{8}$$

where $I_d^r\left(k\right)$, the mutual information achieved by user k and the relay at the destination, is

$$\begin{aligned} I_d^r\left(k\right) = {} & \theta_k \log\left(1 + \left|h_{d,k}\right|^2 \overline{P}_k\right) + \\ & \overline{\theta}_k \log\left(1 + \left|h_{d,k}\right|^2 \overline{P}_k + \left|h_{d,r}\right|^2 \overline{P}_r\right) \end{aligned} \tag{9}$$

and the relay remains in the listen mode until successful decoding such that

$$\theta_k = \min\left(1, \left\lceil \frac{R_k}{\log\left(1 + \left|h_{r,k}\right|^2 \overline{P}_k\right)} \right\rceil\right). \tag{10}$$

In [4, Chap. 4], we develop the mixed distribution of θ_k, $p(\theta_k)$, and show that it can be simplified in the high SNR regime by a two-sample discrete distribution at θ_k^* and 1 where θ_k^* is the half-duplex fraction at which $p\left(\theta_k\right)$ is maximized. We use this simplification to bound $P_o^{(k)}$ as

$$P_{o,2\times1} \leq P_o^{(k)} \leq K_2 \frac{(2^R - 1)^2 d_{d,k}^\gamma d_{d,r}^\gamma}{2\overline{P}_k^2} \tag{11}$$

where

$$K_2 = \left(\frac{(2^{R/(1-\theta_k^*)} - 1)^2 \overline{P}_k}{(2^R - 1)^2 P_r / \left(1 - \theta_k^*\right)} + \frac{2d_{r,k}^\gamma}{d_{d,r}^\gamma}\right) \tag{12}$$

and $P_{o,2\times1}$ is the outage probability of a 2×1 *distributed* MIMO channel whose i^{th} transmit antenna is at a distance $d_{d,i}$, $i = k, r$, from the destination. For the assumption that the nodes are not co-located, one can use the hypoexponential distribution [9, p. 11] to write $P_{o,2\times1}$ as [4, Appendix C]

$$P_{o,2\times1} \sim \frac{(2^R - 1)^2 d_{d,k}^\gamma d_{d,r}^\gamma}{2\overline{P}_k^2} \tag{13}$$

where the approximation $f(x) \sim g(x)$ is meant in the sense of $\lim_{x\to\infty} f/g = 1$. Thus, from (11) and (13) we see that for a fixed rate transmission, the maximum diversity achieved by DDF is 2, as predicted by the diversity-multiplexing tradeoff analysis for DDF in [2, Theorem 4]. The factor K_2 upper bounds the *coding gains* by which $P_o^{(k)}$ differs from the MIMO lower bounds.

2) Time-Duplexed MAC-GF – Two-Hop Scheme: One can extend the analysis for the two-hop single relay TD-MARC to the two-hop TD-MAC-GF where more than one cooperating user in $\mathcal{C}_k$ can relay the signals of user k. We thus have

$$\theta_k = \min\left(1, \max_{j\in\mathcal{C}_k} \left\lceil \frac{R_k}{\log\left(1 + \left|h_{j,k}\right|^2 \overline{P}_k\right)} \right\rceil\right) \tag{14}$$

and the outage probability for user k is

$$P_o^{(k)} = \Pr\left(I_d^c\left(k\right) < R_k\right) \tag{15}$$

where $I_d^c\left(k\right)$ is

$$I_d^c(k) = \theta_k \log\left(1 + \left|h_{d,k}\right|^2 \overline{P}_k\right) + \tag{16}$$

$$\overline{\theta}_k \log\left(1 + \left|h_{d,k}\right|^2 \overline{P}_k + \sum_{j\in\mathcal{C}_k} \left|h_{d,j}\right|^2 \frac{\overline{P}_j}{\overline{\theta}_k}\right). \tag{17}$$

The diversity-multiplexing tradeoff for this two-hop multi-relay scheme is not yet known. However, from (14) and (16), we see that irrespective of node geometry for a fixed R_k, one can choose P_k, and hence, $\overline{P}_k$ sufficiently large such that θ_k is negligible. Thus, we can asymptotically approach the outage probability $P_{L_k\times1}$ of a $L_k \times 1$ MIMO channel for a fixed rate. In [4, Chap. 4], we show that

$$\begin{aligned} P_{o,L_k\times1} \leq P_o^{(k)} \leq {} & \frac{\left(2^{\frac{R_k}{\overline{\theta}_k^*}} - 1\right)^{L_k} \left(\overline{\theta}_k^*\right)^{L_k - 1}}{(L_k!)\left(\overline{P}_k\right)^{L_k}} \prod_{j\in\mathcal{S}_k} \frac{d_{d,j}^\gamma}{\lambda_j} \\ & + \frac{(2^{R_k} - 1)^2 d_{d,k}^\gamma \left(\sum_{j\in C_k} d_{j,k}^\gamma\right)}{\overline{P}_k^2} \end{aligned} \tag{18}$$

where $\lambda_j = \overline{P}_j / \overline{P}_k$ for all $j \in \mathcal{S}_k$, $\theta_k^* = \arg\max_{\theta_k} p(\theta_k)$, $\overline{\theta}_k^* = 1 - \theta_k^*$, and

$$P_{o,L_k\times1} \sim \frac{\left(2^{R_k} - 1\right)^{L_k}}{(L_k!)\left(\overline{P}_k\right)^{L_k}} \prod_{j\in\mathcal{S}_k} \frac{d_{d,j}^\gamma}{\lambda_j}. \tag{19}$$

Note that for $L_k = 2$, our analysis simplifies to the outage analysis for the TD-MARC. Consider the case of $L_k > 2$ where the two-hop cooperative network can potentially achieve larger diversity gains than the time-duplexed relay network. Comparing the two terms in (18), we see that the first term dominates only when

$$\left(\sum_{j\in C_k} d_{j,k}^\gamma\right) \leq \frac{C_0}{\left(\overline{P}_k\right)^{L_k - 2}} \tag{20}$$

where C_0 is a constant independent of $\overline{P}_k$ and is obtained by substituting (20) in (18) and equating the two terms in the summation. Thus, to achieve the maximum diversity L_k, we need to choose P_k for all k large enough that the finite distances $d_{j,k}$ for all $j \in \mathcal{C}_k$ satisfy (20). Alternately, for a fixed P_k, for all k, we require user k and its cooperating users in $\mathcal{C}_k$ to be clustered close enough to satisfy (20). Thus, the maximum DDF diversity for a two-hop cooperative network does not exceed that of a TD-MARC except when user k and its cooperating users are *clustered*, i.e., the inter-node distances satisfy (20). We demonstrate this distance-dependent behavior in Section IV.

3) TD-MAC-GF – Multi-Hop Scheme: Let $\pi_k(\cdot)$ be a permutation on $\mathcal{C}_k$ such that user $\pi_k(l)$ begins its transmissions in the fraction $\theta_{k,l}$, for all $l = 2, 3, \ldots, L_k$. We further define $\pi_k(1) = k$. Unlike the two-hop case where the choice of θ_k is dictated by the node with the worst receive SNR, we now choose the fraction $\theta_{k,l}$, for $l = 1, 2, \ldots, L_k - 1$, small enough to ensure that at least one cooperating node, denoted as $\pi_k(l+1)$, decodes the message from user k. Note that $\theta_{k,L_k} = 1 - \sum_{l=1}^{L_k-1} \theta_{k,l}$.In general, computing the probability distribution of $p(\theta_{k,l})$ is not straightforward. In [4, Chap. 4] we show that it suffices to consider specific values of $p(\theta_{k,l})$ to obtain upper and lower bounds on $P_o^{(k)}$ as

$$P_{o,L_k\times 1} \le P_o^{(k)} \le \frac{\left(2^{\frac{R_k}{\overline{\theta}^*_{k,L_k}}} - 1\right)^{L_k} \left(\theta^*_{k,L_k}\right)^{L_k-1}}{(L_k!)\left(\overline{P}_k\right)^{L_k}} \cdot \prod_{j\in\mathcal{C}_k} \frac{d^{\gamma}_{d,j}}{\lambda_j} + \frac{(2^{R_k}-1)^{L_k}\left(\prod_{j\in\mathcal{C}_k} d^{\gamma}_{j,k}\right) d^{\gamma}_{d,k}}{\overline{P}_k^{L_k}}. \quad (21)$$

Thus, we see that DDF achieves a maximum diversity of L_k for a L_k-hop cooperative network.

B. Amplify-and-Forward

A cooperating node or a relay can amplify its received signal and forward it to the destination; the resulting AF strategy is appropriate for nodes with limited processing capabilities. We present the outage bounds for the two-hop and the L_k-hop cooperative networks. We remark that for $L_k = 2$ and $\mathcal{C}_k = \{r\}$, the two-hop analysis specializes to that for a time-duplexed MARC. We consider $\theta_k = 1/2$ and $\theta_{k,l} = 1/L_k$, $l = 1, 2, \ldots, L_k$, for the two-hop and L_k-hop schemes, respectively.

1) Two-hop User- and Relay-Cooperative Networks: Since the transmitters lack CSI, the mutual information is maximized by choosing the signals $X_{k,1}$ and $X_{k,2}$ transmitted by user k in the first and second fractions, respectively, as independent Gaussian signals [10]. Over two-hops and $(L_k - 1)$ cooperating nodes one can simplify P_{out} as (see also [11])

$$P_{out} = \Pr\left(\frac{1}{2}\log(1+S) < R_k\right) \quad (22)$$

where we write the received signal S at the destination and the scale factor c_j at the cooperating node j, for all j, as

$$S = |h_{d,k}|^2\,\overline{P}_k\left(1+\frac{1}{c_s}\right) + \frac{\overline{P}_k}{c_s^2}\left|\sum_{j\in\mathcal{C}_k}\frac{c_j}{c_s} h_{d,j}h_{j,k}\right|^2 \quad (23)$$

$$|c_j|^2 = \frac{2\overline{P}_j}{|h_{j,k}|^2\,\overline{P}_k + 1}, c_s^2 = 1 + \sum\nolimits_{j\in\mathcal{C}_k} |c_j h_{d,j}|^2. \quad (24)$$

A lower bound on P_{out} is obtained by observing that all the cooperating nodes forward a noisy signal from user k to the destination over two symbols. We thus lower bound P_{out} by the outage probability of a $L_k \times 1$ MIMO channel where all but one of the antennas transmit the same signal as

$$P_{out} \ge \Pr\left(\log\left(1 + |h_{d,k}|^2\,\overline{P}_k + \overline{P}_k\left|\sum_{j\in\mathcal{C}_k} h_{d,j}\right|^2\right) < R_k\right) \quad (25)$$

$$\sim \frac{\left(2^{R_k}-1\right)^2 d^{\gamma}_{d,k}}{2\overline{P}_k^2\left(\sum_{j\in\mathcal{C}_k} 1/d^{\gamma}_{d,j}\right)}. \quad (26)$$

Thus, we see that the maximum diversity is bounded by 2. Further, since AF achieves a maximum diversity of 2 with one cooperating node or relay [3], allowing selection of the best cooperating node, we can upper bound P_{out} by the AF outage probability of a relay channel with $|\mathcal{C}_j| = 1$. The high SNR AF bound for an orthogonal relay channel are developed in [3] and we apply it to obtain

$$P_{out} \le \frac{\left(2^{2R_k}-1\right)^2 d^{\gamma}_{d,k} \max_{j\in\mathcal{C}_k}\left(d^{\gamma}_{j,k} + d^{\gamma}_{d,j}\right)}{2\overline{P}_k^2}. \quad (27)$$

Thus, we see that the maximum diversity achievable by a two-hop AF scheme in the high SNR regime is at most 2 and is independent of the number of cooperating users in $\mathcal{C}_k$.

2) Multi-hop Cooperative Network: We consider an L_k-hop cooperative AF protocol where only user k and user $\pi_k(l)$, $l = 1, 2, \ldots, L_k$, transmit in the l^{th} fraction, i.e., user $\pi_k(l)$ forwards in the fraction $\theta_{k,l}$ a scaled version of the signal it receives from user k in the first fraction. Note that $\pi_k(1) = k$ and $\theta_{k,l} = 1/L_k$ for all l. The destination decodes after collecting the received signals from all L_k fractions. Choosing $X_{k,l}$, for all l, as independent Gaussian signals, and denoting the resulting channel gains matrix at the destination as H, we write

$$P_{out} = \Pr\left(\frac{1}{L_k}\log\left|I + \overline{P}_k HH^{\dagger}\right| < R_k\right) \quad (28)$$

where $H^{\dagger}$ is the conjugate transpose of H. We lower bound P_{out} with the outage probability of a $L_k \times 1$ MIMO channel with i.i.d. Gaussian signaling at the L_k transmit antennas to obtain

$$P_{out} \ge P_{o,L_k\times 1} \sim \frac{\left(2^{R_k}-1\right)^{L_k}\prod_{l=1}^{L_k} d^{\gamma}_{d,\pi_k(l)}}{(L_k!)\,\overline{P}_k^{L_k}}. \quad (29)$$

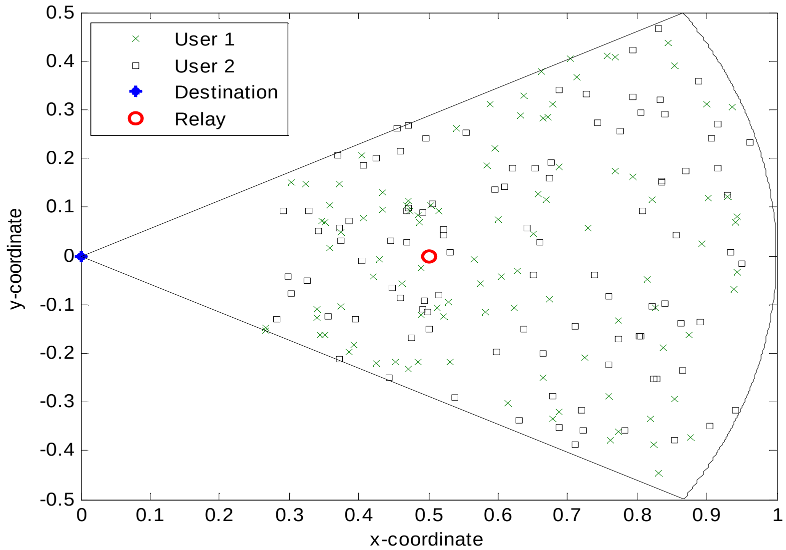

Fig. 2. Sector of a circle with the destination at the origin and 100 randomly chosen locations for a two-user MAC.

On the other hand, one can upper bound P_{out} by the outage probability of an orthogonal AF protocol where user k and its cooperating users transmit on orthogonal channels, i.e., only user $\pi_k(l)$ transmits in the fraction $\theta_{k,l}$, as developed in [3]. Thus, we have

$$P_{out} \leq \frac{\left(2^{L_k R_k}-1\right)^{L_k} d_{d,k}^{\gamma} \prod_{j \in \mathcal{C}_k}\left(d_{d,j}^{\gamma}+d_{j,k}^{\gamma}\right)}{L_k! \overline{P}_k^{L_k}}. \tag{30}$$

Comparing (29) and (30), we see that the L_k-hop AF scheme can achieve a maximum diversity of L_k in the high SNR regime at the expense of user k repeating the signal L_k times.

IV. Illustration of Results

We consider a planar geometry with the users distributed randomly in a sector of a circle of unit radius and angle $\pi/3$. We place the destination at the center of the circle and place the relay at $(0.5, 0)$ as shown in Fig. 2. The K users are distributed randomly over the sector excluding a dead zone around the destination of radius 0.3. We compute the outage probabilities for the TD-MARC, TD-MAC-GF, and TD-MAC networks assuming a random distribution of users and average the results over 100 random placements. Finally, we also average P_{out} over all time-duplexed users.

We consider a three-user MAC. We assume that all three users have the same transmit power constraint, i.e., $P_k = P_1$ for all k. For the relay we choose $P_r = f_r \cdot P_1$ where $f_r \in \{0.5, 1\}$. We set the path loss exponent $\gamma = 4$ and the processing factors $\eta_k = \delta_k = \eta$ for all k. We compare P_{out} as a function of P_{tot} for $\eta = 0.01,\ 0.5$, and 1 thereby modeling three different regimes of processing to transmit power ratios. We consider a symmetric transmission rate, i.e., all users transmit at $R = 0.25$ bits/channel use. We first plot P_{out} as a function of the transmit SNR P_1 in dB obtained by normalizing P_1 by the unit variance noise. We also plot P_{out} as a function of P_{tot} in dB where P_{tot} is given by (5) and (6). For user cooperation, we plot the outage for both the two-hop and three-hop schemes.

A. Outage Probability: DDF

We compare the outage probability of a three user MAC in Figs. 3 and 4. The figures clearly validates our analytical results that the two hop cooperative scheme on average does not achieve the maximum diversity gains of 3. On the other hand, the slope of P_{out} for the three-hop scheme approaches 3 but does not achieve coding gains relative to the relay network in the SNR regime shown. This SNR gain of the relay network relative to the user cooperative network is not diminished even the energy costs of cooperation are accounted for in sub-plot 2 and Fig. 4 by plotting P_{out} as a function of P_{tot}. This difference in SNR gains between user and relay cooperation is due to the fact that user cooperation increases spatial diversity at the expense of requiring users to share their power for cooperative transmissions. Observe that with increasing η, the outage curves are translated to the right. In fact, for a fixed R, the processing costs increase with increasing η, and thus, we expect the SNR gains from cooperation to diminish relative to TD-MAC, particularly in the SNR regimes of interest. This is demonstrated in Fig. 4.

B. Outage Probability: AF

In Figs. 5 and 6 we plot the two user AF outage probability for all three networks. As predicted, we see that both relay and user cooperation achieve a maximum diversity of 2 for the two-hop scheme. The three-hop user cooperative scheme achieves a maximum diversity approaching 3. However, it achieves coding gains relative to the relay network only as the SNR increases. These gains are a result of the model

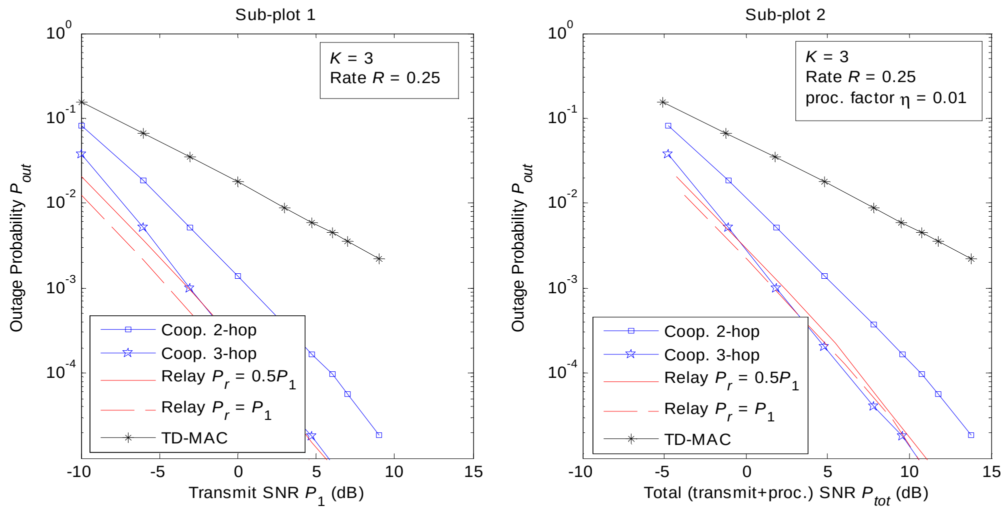

Fig. 3. Three user DDF outage probability P_{out} vs. P_1 (sub-plot 1) and vs. P_{tot} for $\eta = 0.01$ (sub-plot 2).

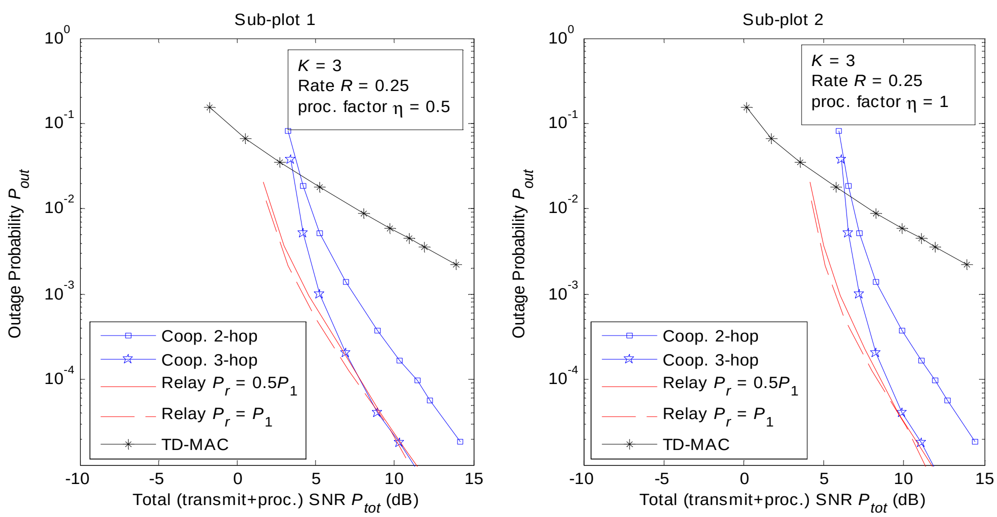

Fig. 4. Three user DDF outage probability P_{out} vs. total transmit SNR P_{tot} in dB for $\eta = 0.5$ (sub-plot 1) and $\eta = 1$ (sub-plot 2).

chosen for the processing power (only model costs of encoding and decoding) and the choice of $P_{k,0}^{proc} = 0$ for all k for the purposes of illustration. In general, $P_{k,0}^{proc} > 0$ since it models protocol and device overhead including front-end processing and amplification costs, and thus, the total processing power will scale proportionate to the number of users that a node relays for.

V. Concluding Remarks

We compared the outage performance of user and relay cooperation in a multiaccess network using the total transmit and processing power as a cost metric for the comparison. We developed a model for processing power costs as a function of the transmitted rate, and hence, transmitted power. We considered a time-duplexed transmission model for both cooperative networks and the MAC and developed a two-hop scheme for both the relay and user cooperative network. We also presented a multi-hop scheme for the user cooperative network for the case of multiple cooperating users. We presented geometry-inclusive upper and lower bounds on the outage probability of DDF and AF to facilitate comparisons of diversity and coding gains achieved by the two cooperative approaches. We showed that the relay network achieves a maximum diversity of 2 for both DDF and AF. We also showed that a two-hop K-user cooperative network achieves a K-fold diversity gain with DDF only when the cooperating users are physical proximal and achieves a maximum diversity of 2 with AF. On the other hand, a K-hop scheme achieves a maximum diversity of K for both DDF and AF. Using area-averaged numerical results that account for the costs of cooperation, we demonstrated that the relay network achieves SNR gains that either diminish or completely eliminate the diversity advantage of the user cooperative network. Besides a fixed relay position, this difference is due to the fact that user cooperation results in a tradeoff between diversity and SNR gains as a result of sharing limited power resources between the users.

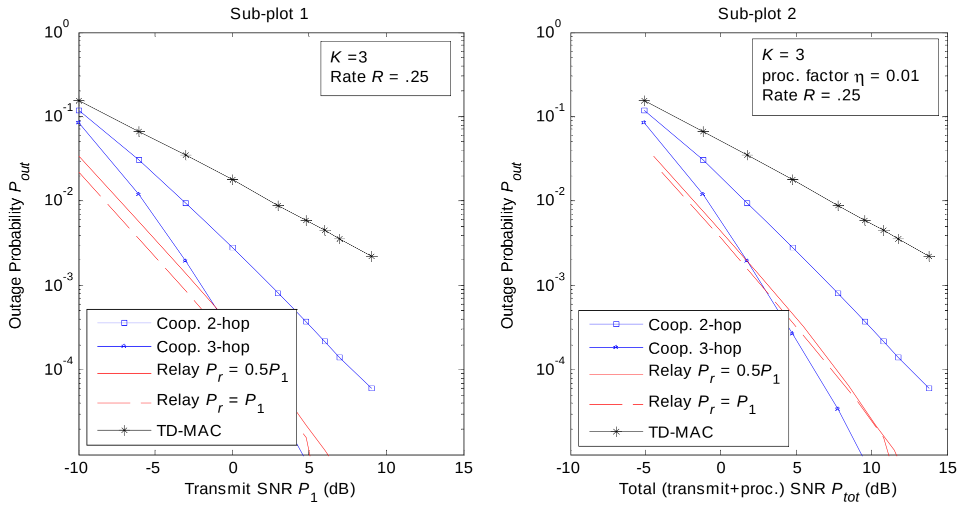

Fig. 5. Three user AF outage probability P_{out} vs. P_1 (sub-plot 1) and P_{tot} for $\eta = 0.01$ (sub-plot 2).

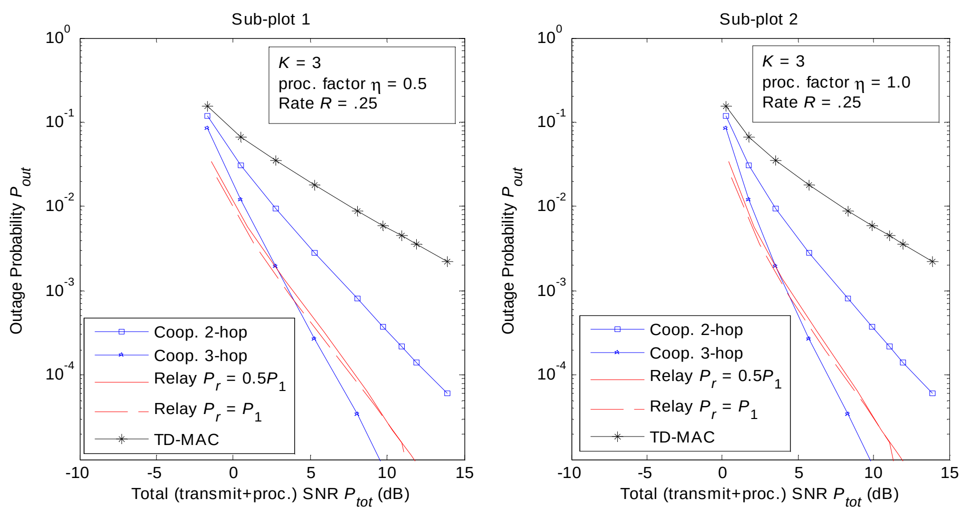

Fig. 6. Three user AF outage probability P_{out} vs. P_{tot} for $\eta = 0.5$ (sub-plot 1) and $\eta = 1$ (sub-plot 2).

In conclusion, we see that user cooperative schemes are desirable only if the processing costs associated with achieving the maximum diversity gains are not prohibitive, i.e., in the regime where they achieve positive coding gains relative to the relay and non-cooperative networks. The simple processing cost model presented here captures the effect of transmit rate on processing power. One can also tailor this model to explicitly include delay, complexity, and device-specific processing costs. Finally, one can also compare the energy-efficiency and diversity of a variety of cooperative schemes.

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