Capacity Theorems for the Multiple-Access Relay Channel

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Abstract

Outer bounds for the discrete memoryless multiple-access relay channel (MARC) are obtained that exploit the causal relationship between the source and relay inputs. A novel *offset encoding* technique that facilitates window decoding at the destination is presented for a decode-and-forward strategy, where the relay decodes the source messages before forwarding to the destination. A compress-and-forward strategy for the MARC and an amplify-and-forward strategy for the Gaussian MARC are also presented.

1 Introduction

The multiple-access relay channel (MARC) is a model for network topologies where multiple sources communicate with a single destination in the presence of a relay node [1]. Examples of such networks include hybrid wireless LAN/WAN networks and sensor and ad hoc networks where cooperation between the nodes is either undesirable or not possible, but one can use an intermediate relay node to aid communication between the sources and the destination. We present capacity bounds for such networks by applying and extending several known results from network information theory.

The classic single-source relay network was introduced and studied by van der Meulen [2]. Cover and El Gamal [3] developed two fundamental coding strategies for the relay channel and obtained the capacity for the physically degraded case. Recently, there has been an increased focus on networks with one or more relays as models for wireless ad hoc and sensor networks [4–10]. However, the successful deployment of any such network lies in its ability to support multiple users simultaneously and not only one. We consider here the MARC model as a specific multi-user relay network. The paper [1] (see also [11]) presents an outer bound on the capacity of the MARC using cut-sets. The paper also presents an achievable rate region for the Gaussian MARC that is extended in [6] using block Markov encoding and *backward decoding* as a *decode-and-forward* (DF) strategy for the discrete memoryless (d.m.) MARC.

In this paper, we obtain potentially tighter outer bounds on the capacity of the d.m. MARC. We also present a new code construction using *offset encoding* for the DF strategy that facilitates the more practical window decoding [10] at the destination while achieving the same

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rate region as in [6]. We also present the *compress-and-forward* (CF) strategy as an extension of [3, theorem 6] and the *amplify-and-forward* (AF) strategy for the Gaussian MARC. The paper is organized as follows. In Section 2, we define the d.m. MARC and derive outer bounds on its capacity region. In Section 3, we present an offset encoding technique for the DF strategy and a rate region for the CF strategy. We illustrate the results with two examples in Section 4 and conclude with Section 5.

2 MARC: Model and Upper Bounds

2.1 Model

The M-source discrete memoryless MARC consists of M messages W_m, $M+1$ channel inputs X_{mi} and two channel outputs $Y_{M+1,i}$ and $Y_{M+2,i}$ where $i = 1, 2, \ldots, n$ is a time index, and M message estimates $\hat{W}_m$, $m \in [1, M]$. The input X_{mi} is a function of the message W_m at the m^{th} source while $X_{M+1,i}$, the relay's input to the channel, is a causal function of its received symbols, $\mathbf{Y}_{M+1}^{i-1} = (Y_{M+1,1}, Y_{M+1,2}, \ldots, Y_{M+1,i-1})$. Finally, the n channel outputs at the destination $\mathbf{Y}_{M+2}^{n} = (Y_{M+2,1}, Y_{M+2,2}, \ldots, Y_{M+2,n})$ are used to jointly decode the messages from all M sources as $(\hat{W}_1, \hat{W}_2, \ldots, \hat{W}_M)$. The channel is assumed to be time-invariant and memoryless and is represented by the conditional probability distribution

$$p(y_{M+1}, y_{M+2}|x_1, x_2, \ldots x_M, x_{M+1}) \tag{1}$$

The message W_m is uniformly distributed in the set $[1, 2^{B_m}]$ and the W_m from all M sources are jointly independent. The capacity region $\mathcal{C}_{MARC}$ of an M-source MARC is defined as the closure of the set of rate tuples $(R_1, R_2, \ldots, R_M)$, where $R_i = B_m/n$, such that destination can decode the M source messages with an arbitrarily small positive error probability ϵ [1].

2.2 Upper Bounds on Capacity of d.m. MARC

Let $S = \{1, 2, \ldots, M\}$ be a set of source indices and define $\mathbf{X}_{(G)} = \{X_m : m \in G \subseteq S\}$. Let G^c be the complement of G in S. Define $\mathbf{Y} \triangleq (Y_{M+1}, Y_{M+2})$, $\mathbf{X} \triangleq (X_1, X_2, \ldots, X_M)$, and $\mathbf{V} \triangleq (V_1, V_2, \ldots, V_M)$. The capacity region $\mathcal{C}_{MARC}$ is shown in [11] to be contained in the union of the set of rate tuples $(R_1, R_2, \ldots, R_M)$ that satisfy

$$\sum_{m \in G} R_m \leq \min\left(I(\mathbf{X}_{(G)}; \mathbf{Y}|\mathbf{X}_{(G^c)}, X_{M+1}, U), I(\mathbf{X}_{(G)}, X_{M+1}; Y_{M+2}|\mathbf{X}_{(G^c)}, U)\right) \tag{2}$$

for all $G \subseteq S$, where the union is over all input distributions $p(u) \cdot \left(\prod_{i=1}^{M} p(x_i|u)\right) \cdot p(x_{M+1}| x_1, x_2, \ldots, x_M, u)$ and U has an alphabet $\mathcal{U}$ of size $|\mathcal{U}| \leq 2^{M+1} - 2$.

We present a (potentially) tighter bound on $\mathcal{C}_{MARC}$ by taking into account the causal relationship between the source and relay inputs.

Theorem 1 *$\mathcal{C}_{MARC}$ is a subset of the union of the sets of M-tuples $(R_1, R_2, \ldots, R_M)$ satisfying*

$$\sum_{m \in G} R_m \leq \min \begin{pmatrix} I(\mathbf{X}_{(G)}; \mathbf{Y}|\mathbf{X}_{(G^c)}, \mathbf{V}_{(G^c)}, X_{M+1}, U), \\ I(\mathbf{X}_{(G)}, X_{M+1}; Y_{M+2}|\mathbf{X}_{(G^c)}, \mathbf{V}_{(G^c)}, U), \\ H(\mathbf{X}_{(G)}|\mathbf{V}_{(G)}, U) \end{pmatrix} \tag{3}$$

for all $G \subseteq S$ where the union is over all probability distributions $p(u)\cdot\left(\prod_{i=1}^{M} p(v_i|u)p(x_i|v_i,u)\right)\cdot p(x_{M+1}|v_1,v_2,\ldots,v_M,u)$.

Proof. The proof follows along the lines of the cut-set bounds of [12, theorem 14.10.1] that are modified to take into account the independent sources. Additionally, instead of eliminating the past source inputs, we set $V_{mi} \triangleq \mathbf{X}_m^{i-1}$, $m \in [1,M]$, $i \in [1,n]$, to obtain the bounds

$$\sum_{m\in G} R_m \leq \frac{1}{n}\sum_{i=1}^{n} \left(\begin{array}{c} H(\mathbf{Y}_i|\mathbf{Y}^{i-1},\mathbf{X}^i_{(G^c)},X_{M+1,i}) - \\ H(\mathbf{Y}_i|\mathbf{Y}^{i-1},\mathbf{X}^i_{(S)},X_{M+1,i}) \end{array} \right) \tag{4}$$

$$\leq \frac{1}{n}\sum_{i=1}^{n} I(\mathbf{Y}_i;\mathbf{X}_{(G)i}|\mathbf{X}_{(G^c)i},\mathbf{V}_{(G^c)i},X_{M+1,i}) \tag{5}$$

Similarly, we have

$$\sum_{m\in G} R_m \leq \frac{1}{n}\sum_{i=1}^{n} I(Y_{M+2,i};\mathbf{X}_{(G)i},X_{M+1,i}|\mathbf{X}_{(G^c)i},\mathbf{V}_{(G^c)i}) \tag{6}$$

Lastly, we quantify the dependence of X_{mi} on V_{mi} in a manner similar to [13] as

$$R_m \leq \frac{1}{n}\sum_{i=1}^{n} H(X_{m,i}|V_{m,i}) \tag{7}$$

The joint distribution of the random variables (r.v.'s) $\mathbf{X}_i$,$\mathbf{V}_i$, $\mathbf{Y}_i$, and $X_{M+1,i}$ can then be written as

$$p(\mathbf{X}_i,\mathbf{V}_i,X_{M+1,i},\mathbf{Y}_i) = \left(\prod_{m=1}^{M} p(V_{mi})p(X_{mi}|V_{mi})\right)\cdot p(X_{M+1,i}|\mathbf{V}_i)p(\mathbf{Y}_i|\mathbf{X}_i,X_{M+1,i}) \tag{8}$$

Finally, we simplify (5), (6), and (7) using a time-sharing r.v. U to obtain the bounds in (3) for an input distribution $p(u)\cdot\left(\prod_{m=1}^{M} p(v_m|u)p(x_m|v_m,u)\right)\cdot p(x_{M+1}|v_1,v_2,\ldots,v_M,u)$. ∎

3 Coding Strategies

3.1 Decode-and-Forward Strategy

An achievable strategy for the white Gaussian MARC is presented in [1] by extending the code construction in [3, theorem 5] to multiple sources. The strategy, called decode-and-forward, is extended to the d.m. MARC in [6] (see also [8]) using a combination of regular Markov encoding at the sources and relay and *backward* decoding [14] at the destination. The resulting rate region is the set of M-tuples $(R_1,R_2,\ldots,R_M)$ that, for all $G \subseteq S$, satisfy

$$\sum_{m\in G} R_m \leq \min \left(\begin{array}{c} I(\mathbf{X}_{(G)};Y_{M+1}|\mathbf{X}_{(G^c)},\mathbf{V}_{(S)},X_{M+1}), \\ I(\mathbf{X}_{(G)},X_{M+1};Y_{M+2}|\mathbf{X}_{(G^c)},\mathbf{V}_{(G^c)}) \end{array} \right) \tag{9}$$

for an input distribution $\left(\prod_{m=1}^{M} p(v_m)p(x_m|v_m)\right)\cdot p(x_r|v_1,v_2,\ldots,v_M)$.

While regular Markov encoding simplifies codebook design, backward decoding has the disadvantage that the destination decodes only after the entire block of B messages from each

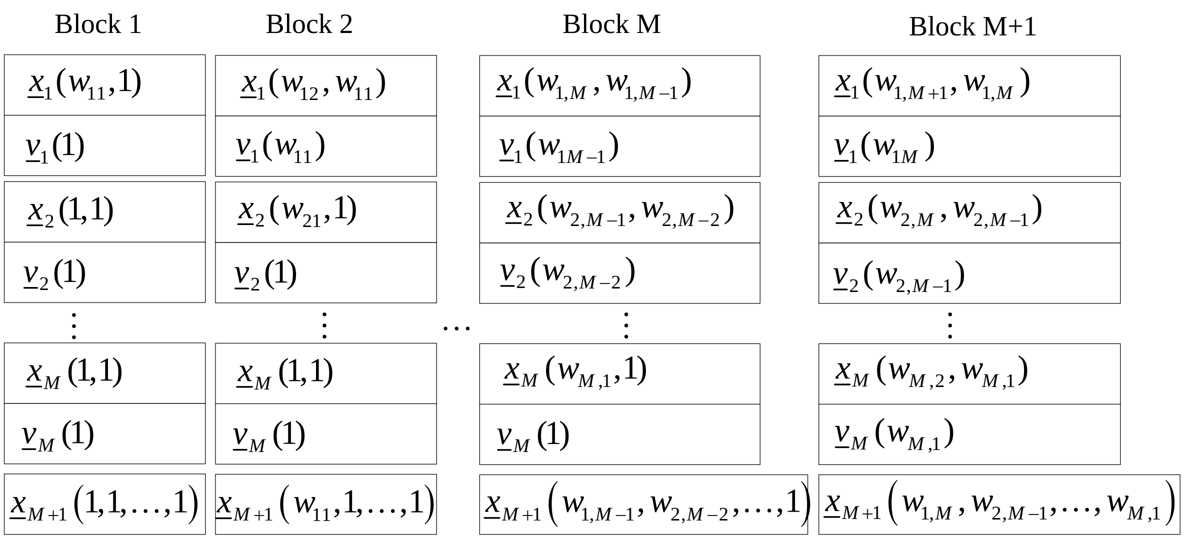

Figure 1: Offset encoding for a M-MARC with offset order $\pi = (1, 2, \ldots, M)$

source is received over $B+1$ blocks. Recently, for single-source multi-relay channels, Xie and Kumar [10] combined regular Markov encoding with a more practical moving *window decoding* at the destination. For the MARC, however, a straightforward application of this technique fails to achieve all rates of (9). To fix this, we introduce an *offset encoding* scheme that offsets the M source transmissions by one block per user. The advantage of this approach is that we achieve the rates of (9) with a delay of $M+1$ transmit blocks per message rather than $B+1$, as is required for backward decoding. Note that M is usually much smaller than B.

The code construction uses regular block Markov encoding as in [8] such that the m^{th} source uses the codewords $\underline{x}_m(w_m, s_m)$, $\underline{v}_m(s_m)$ in each block $b \in [1, B]$ where w_m is the new message sent in the block and s_m is the message from the previous block with $s_m, w_m \in [1, 2^{nR_m}]$. In each such block, the relay sends the codeword $\underline{x}_{M+1}(s_1, s_2, \ldots, s_M)$. Let π denote some offset order (permutation) of the source indices. Without loss of generality, we choose $\pi = (1, 2, \ldots, M)$. Then the m^{th} source, $m \in [1, M]$, sends the messages $(w_{m1}, w_{m2}, \ldots, w_{mB})$, each of nR_m bits, over B consecutive blocks, starting from the m^{th} block, using the codewords described above. The resulting message to codeword mapping for an M-source MARC with offset encoding is shown in Fig. 1.

Fix the input distribution as in (9). The relay decodes the source messages at the end of each block. Then, for reliable decoding at the relay, we require

$$\sum_{m \in G} R_m \leq I(\mathbf{X}_{(G)}; Y_{M+1} | \mathbf{X}_{(G^c)}, \mathbf{V}_{(S)}, X_{M+1}) \qquad (10)$$

for all $G \subseteq S$. The destination uses a sliding window of length $M+1$ to jointly decode the M source messages $(w_{1b}, w_{2b}, \ldots, w_{M,b})$, $b \in [1, B]$. Thus, the set of b^{th} messages from the sources in $G \subseteq S$, $\mathbf{w}_{(G),b}$, are decoded using the blocks $[b, b+M]$ as $\mathbf{w}_{(G),b} = \bigcap_{j=b}^{b+M} \mathbf{W}_{(G),b}^{(j)}$, where $\mathbf{W}_{(G),b}^{(j)}$ is the set of b^{th} messages decoded in the j^{th} block from sources in G [10].

For the sake of simplicity, we first consider $M = 2$; the offset order is now $\pi = [1, 2]$. The message pair (w_{1b}, w_{2b}) is decoded reliably as described above using blocks $[b, b+2]$ if

$$R_1 \leq I(X_1; Y_4|X_2, V_1, V_2, X_3) + I(V_1, X_3; Y_4|X_2, V_2) \tag{11}$$

$$= I(X_1, X_3; Y_4|X_2, V_2) \tag{12}$$

$$R_2 \leq I(X_2; Y_4|X_1, V_1, V_2, X_3) + I(V_2; Y_4) \tag{13}$$

$$R_1 + R_2 \leq I(V_2; Y_4) + I(X_2, V_1, X_3; Y_4|V_2) + I(X_1; Y_4|X_2, V_1, V_2, X_3) \tag{14}$$

$$= I(X_1, X_2, X_3; Y_4) \tag{15}$$

The sum rate can also be written as

$$R_1 + R_2 \leq I(V_2, X_2; Y_4) + I(X_1, X_3; Y_4|X_2, V_2) \tag{16}$$

resulting in a corner point where source 1 achieves its maximum rate in (12) if the rate achieved by the second source at this point, $I(V_2, X_2; Y_4)$, is less than its maximum in (13). Observe that we have

$$I(V_2, X_2; Y_4) = I(V_2; Y_4) + I(X_2; Y_4|V_2) \tag{17}$$

$$\leq I(V_2; Y_4) + H(X_2|V_2) - H(X_2|Y_4, X_1, V_1, V_2) \tag{18}$$

$$= I(V_2; Y_4) + I(X_2; Y_4|X_1, V_1, V_2, X_3) \tag{19}$$

where (19) results from the Markov relationship $X_2 \rightarrow V_2 \rightarrow X_3$ and independence of sources. Thus, for the offset order $\pi = [1, 2]$, we obtain a corner point where the first source in the offset order is decoded after the second and achieves its maximum rate. For $\pi = [2, 1]$, we similarly obtain the corner point where source 2 is decoded last and achieves its maximum rate. The above two corner points are the same as those resulting from the second bound in (9), obtained via backward decoding, for $M = 2$. The non-corner points are achieved by time-sharing, and thus, for $M = 2$, we obtain the same rate bounds at the destination as backward decoding.

Extending the analysis to $M > 2$ sources, we write the set G as $G = \cup_{k=1}^{M+1} G_k$,where $G_k = \{g_k, g_{k-1}\}$ with $g_k = k$ if the k^{th} source belongs to G and $g_{k-1} = k-1$ if the $(k-1)^{th}$ source belongs to G; if the k^{th} ($(k-1)^{th}$) source does not belong to G, g_k (g_{k-1}) is set to $\emptyset$. Thus, $G_1 = \{g_1\}$ and $G_{M+1} = \{g_M\}$. Then the rate bounds using window decoding are given as

$$\sum_{m\in G} R_m \leq \begin{cases} \begin{pmatrix} I(X_{g_2}; Y_{M+2}|\mathbf{X}_{(S\setminus G_2\cup\{1\})}, \mathbf{V}_{(S)}, X_{M+1}) \\ +\Sigma_{k=3}^{M} I(X_{g_k}, V_{g_{k-1}}; Y_{M+2}|\mathbf{X}_{(A_k)}, \mathbf{V}_{(B_k)}) \end{pmatrix} & g_1 = \emptyset \\ \begin{pmatrix} I(X_{(G_1)}; Y_{M+2}|\mathbf{X}_{(S\setminus G_1)}, \mathbf{V}_{(S)}, X_{M+1}) \\ +I(X_{g_2}, V_{g_1}, X_{M+1}; Y_{M+2}|\mathbf{X}_{(S\setminus G_2)}, \mathbf{V}_{(S\setminus\{g_1\})}) \\ +\Sigma_{k=3}^{M} I(X_{g_k}, V_{g_{k-1}}; Y_{M+2}|\mathbf{X}_{(A_k)}, \mathbf{V}_{(B_k)}) \end{pmatrix} & g_1 \neq \emptyset \end{cases} \tag{20}$$

where

$$A_k = \begin{cases} [k+1, M] & g_k \neq \emptyset \\ [k, M] & g_k = \emptyset \end{cases}, \; B_k = \begin{cases} [k, M] & g_{k-1} \neq \emptyset \\ [k-1, M] & g_{k-1} = \emptyset \end{cases} \quad k \in [3, M] \tag{21}$$

By expanding the expressions in (20), it can be seen that for all sets G of the form $[1, k]$, $k \in [1, M]$, the above bound simplifies to

$$\sum_{m=1}^{k} R_m \leq I(\mathbf{X}_{(G)}, X_{M+1}; Y_{M+2}|\mathbf{X}_{(G^c)}, \mathbf{V}_{(G^c)}) \tag{22}$$

$$= \left(I(X_k, V_k; Y_{M+2}|\mathbf{X}_{([k+1,M])}, \mathbf{V}_{([k+1,M])}) + I(\mathbf{X}_{(G_1)}, X_{M+1}; Y_{M+2}|\mathbf{X}_{[k,M]}, \mathbf{V}_{[k,M]})\right) \tag{23}$$

where $G_1 \cup \{k\} = G$ and $G_1 = [1, k-1]$. For $G = \{1\}$, we obtain the bounds

$$R_1 \leq I(X_1, X_{M+1}; Y_{M+2} | \mathbf{X}_{([2,M])}, \mathbf{V}_{([2,M])}) = R_{1,\max} \tag{24}$$

for the chosen input distribution. We now show that the sum-rate bound for any G of the form $[1, k]$, $k \in [2, M]$, results in a corner point achievable by successive decoding such that source 1 achieves $R_{1,\max}$. We refer to such a corner point as *associated* with source 1.

From (20), the bound on the k^{th} source rate R_k, $k \in [2, M]$, is

$$R_k \leq \begin{cases} \left(I(X_2; Y_{M+2} | \mathbf{X}_{[3,M]}, \mathbf{V}_{[1,M]}, X_{M+1}) + I(V_2; Y_{M+2} | \mathbf{X}_{[3,M]}, \mathbf{V}_{[3,M]})\right) & k = 2 \\ \left(I(X_k; Y_{M+2} | \mathbf{X}_{[k+1,M]}, \mathbf{V}_{[k-1,M]}) + I(V_k; Y_{M+2} | \mathbf{X}_{[k+1,M]}, \mathbf{V}_{[k+1,M]})\right) & k \geq 3 \end{cases} \tag{25}$$

Comparing (23) with (25) for each k and using the Markov relation $X_k \to V_k \to X_{M+1}$, we have

$$I(X_k, V_k; Y_{M+2} | \mathbf{X}_{[k+1,M]}, \mathbf{V}_{[k+1,M]}) = \left(\begin{array}{c} I(V_k; Y_{M+2} | \mathbf{X}_{[k+1,M]}, \mathbf{V}_{[k+1,M]}) + \\ H(X_k | V_k) - H(X_k | Y_{M+2}, \mathbf{X}_{[k+1,M]}, \mathbf{V}_{[k,M]}) \end{array} \right) \tag{26}$$

$$\leq R_{k,\max}(\pi) \tag{27}$$

where $R_{k,\max}(\pi)$ is the maximum single-user rate achieved by the k^{th} source, $k > 1$, for an offset order π, in (25). Thus, for each k chosen successively in increasing order from $[2, M]$, a sum-rate corner point for the first k sources in π is achieved since the rate requirement on the k^{th} source at the sum-rate point is smaller than its maximum achievable $R_{k,\max}(\pi)$. Applying the same argument for all k from $[2, M]$, we finally obtain an M-source sum-rate corner point associated with source 1. Further, at this corner point, the sources are decoded successively in the reverse offset order starting with the last source in π such that source 1 achieves $R_{1,\max}$.

The remaining corner points associated with source 1 are then obtained by fixing source 1 as the first source in π while choosing all possible offset permutations of the other $M-1$ sources. Similarly, all $(M-1)!$ corner points associated with the m^{th} source, $m \in [1, M]$, are obtained using those offset permutations π where the m^{th} source is the first transmitting source. Finally, the non-corner points are achieved by time-sharing resulting in an achievable rate region at the destination as the set of rate tuples $(R_1, R_2, \ldots, R_M)$ such that

$$\sum_{m \in G} R_m \leq I(\mathbf{X}_{(G)}, X_{M+1}; Y_{M+2} | \mathbf{X}_{(G^c)}, \mathbf{V}_{(G^c)}) \tag{28}$$

for all $G \subseteq S$. Combining the bounds at the relay in (10), we then achieve the same rate region as in (9). The effective rate achieved by the m^{th} source is $R_m \cdot B\,/(B+M)$ and approaches R_m for large values of B. Thus, by offsetting the source transmissions and incurring a small delay of M blocks, we avoid the excessive delay of $B+1$ blocks associated with backward decoding while achieving the same rate region.

3.2 Compress-and-Forward Strategy

Instead of decoding the source messages, the relay can also aid the destination by forwarding a compressed version of its received signal [3, theorem 6]. The resulting *compress-and-forward* (CF) strategy [8] employs Wyner-Ziv coding [15] to exploit the correlation between Y_{M+1} and

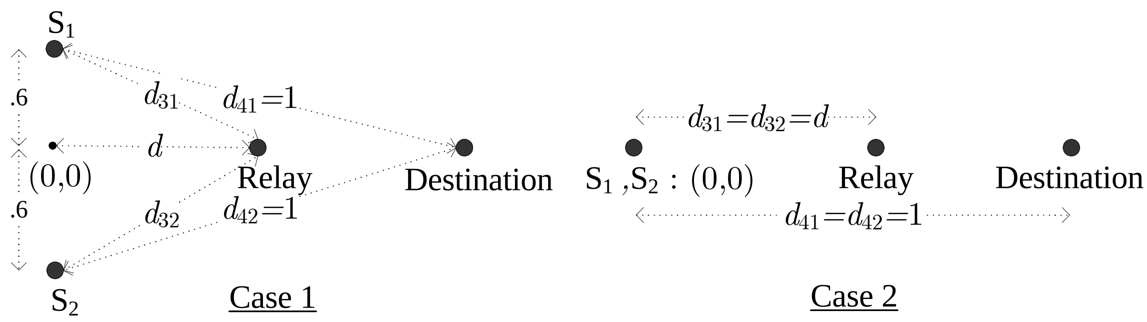

Figure 2: Two geometries for the two-source Gaussian MARC

Y_{M+2}. The strategy can also be extended to the MARC (see [11]). The relay compresses the signal Y_{M+1} received from all sources as $\hat{Y}_{M+1}$. The resulting rate region is the set of M-tuples $(R_1, R_2, \ldots, R_M)$ that, for all $G \subseteq S$, satisfy

$$\sum_{m \in G} R_m \leq I(\mathbf{X}_{(G)}; \hat{Y}_{M+1}, Y_{M+2} | \mathbf{X}_{(G^c)}, X_{M+1}) \tag{29}$$

subject to the constraint $I(X_{M+1}; Y_{M+2}) \geq I(\hat{Y}_{M+1}; Y_{M+1} | X_{M+1}, Y_{M+2})$ for the joint distribution $\left(\prod_{m=1}^{M+1} p(x_m)\right) \cdot p(\hat{y}_{M+1} | y_{M+1}, x_{M+1}) \cdot p(y_{M+1}, y_{M+2} | \mathbf{x}, x_{M+1})$.

4 Wireless Examples

We consider a two-source additive white Gaussian MARC (G-MARC) with fading such that the received signals at the relay and destination in the i^{th} interval, $i \in [1, n]$, are

$$Y_{ji} = \left(\sum_{k=1}^{j-1} h_{jki} X_{ki} \right) + Z_{ji}, \; j \in [3, 4] \tag{30}$$

where $Z_{ji} \sim \mathcal{CN}(0, 1)$ with i.i.d real and imaginary parts for all j and i. The source and relay transmit signals that are constrained in power as $\sum_{i=1}^{n} E(|X_{ki}|^2) \big/ n \leq P_k, \; k \in [1, 3]$. The parameter h_{jki} is the fading experienced by the k^{th} transmit signal at the j^{th} receiver in the i^{th} symbol and is assumed known only at the j^{th} receiver. In this analysis, analogous to [5], we consider two kinds of fading channels:

1. constant fading $h_{jki} = 1 \Big/ \sqrt{d_{jk}^{\gamma}} \; \forall i \in [1, n]$ where d_{jk} is the distance between the j^{th} receiver and the k^{th} transmitter and γ is the path-loss exponent.

2. ergodic phase-fading with $h_{jki} = e^{j\theta_{jki}} \Big/ \sqrt{d_{jk}^{\gamma}}$ where θ_{jki} is uniformly distributed as $\mathbf{U}[-\pi, \pi]$.

The analysis for these models generalizes to other types of fading such as Rayleigh fading [6].

We consider the two geometries shown in Fig. 2. Case 1 has a symmetric positioning of the sources with respect to the relay and destination while case 2 is a collinear geometry with both

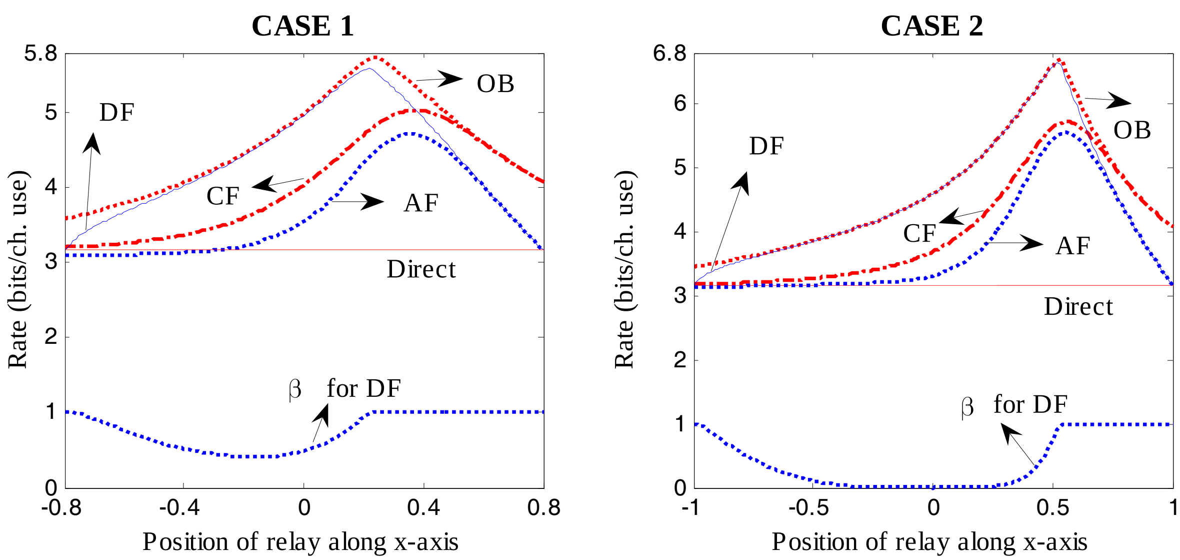

Figure 3: Sum rates for no fading GMARC with $P_1 = P_2 = P_3 = 6$ dB and $\gamma = 4$

sources at the origin and the destination a unit distance away from the origin. In both cases, the relay moves along the line connecting the destination with the origin. The sum-rates for the outer bound (OB) and the cooperative strategies are plotted as a function of the relay's position relative to the origin. The transmitter signal-to-noise ratio (SNR) is chosen as 6 dB for both sources and relay while the path-loss exponent γ is chosen as 4.

For the constant fading G-MARC, the rate region for the DF strategy is obtained using the code construction in [1] as the set of rate-tuples (R_1, R_2) that, for $G \subseteq \{1,2\}$, satisfy

$$\sum_{m\in G} R_m \leq \max_{\substack{\{\beta_m\}_{m\in G} \\ \sum_{m=1}^{M}\beta_{rm}\leq 1}} \min\left\{ C\left(\sum_{m\in G}\beta_m \frac{P_m}{d_{M+1,m}}\right), C\left(\begin{array}{c} \sum_{m\in G}\frac{P_m}{d_{M+2,m}} + \frac{P_{M+1}}{d_{M+2,M+1}} + \\ 2\sum_{m\in G}\sqrt{\bar{\beta}\beta_{rm}\frac{P_m}{d_{M+2,m}}\cdot\frac{P_{M+1}}{d_{M+2,M+1}}} \end{array}\right)\right\} \tag{31}$$

where $C(x) \triangleq \log(1+x)$, $\beta_m = 1 - \bar{\beta}_m$ is the fraction of power the m^{th} source allocates to its new message, and β_{rm} is the fraction of power the relay allocates to cooperating with the m^{th} source. The rate region for the CF strategy is obtained from (29) using Gaussian signaling at the sources and relay. We also consider the amplify-and-forward strategy (AF) where the relay forwards an amplified version of its received signal to the destination as $X_{M+1,i} = cY_{M+1,i-1}$, $i \in [1,n]$, where c is chosen so that $\sum_{i=1}^{n} E(|X_{M+1,i}|^2)/n \leq P_{M+1}$. This results in a multiple-access intersymbol-interference (ISI) channel at the destination, the rate region for which is given by the multi-user water-filling algorithm in [16]. The rates for the three strategies and the outer bound are plotted in Fig. 3 for both cases. The outer bounds for the G-MARC result from ignoring the entropy bounds in theorem 1 and using an entropy maximization theorem in [17] to set the auxiliary r.v.'s as Gaussian. The plots also include the optimal fraction $\beta_1 = \beta_2 = \beta$, with the two fractions taking the same value β for the symmetric geometry considered in case 1 and 2 at the maximum sum-rate point. Finally, the multiple-access sum rate resulting from direct communication between the sources and destination is plotted as a straight line independent of relay position. Observe that for the cases where the relay is physically closer to the sources or the destination, the DF and CF strategy respectively approach the outer bound.

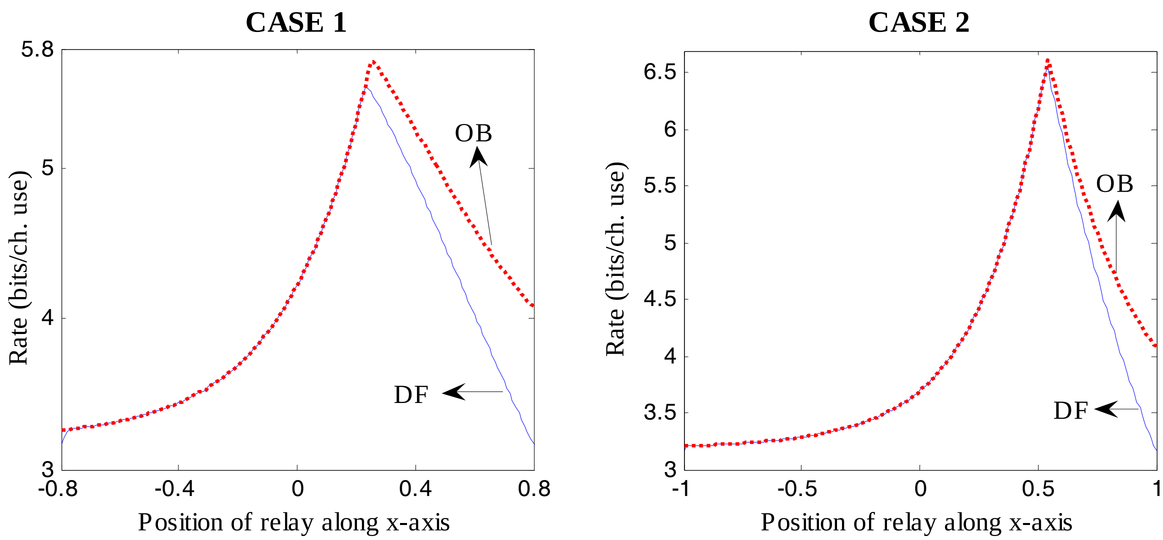

Figure 4: Sum rates for ergodic phase-fading GMARC with $P_1 = P_2 = P_3 = 6$ dB and $\gamma = 4$

In [8], it was shown that the DF strategy achieves capacity for the ergodic phase-fading MARC when the relay and sources are clustered as in [8, theorem 10]. The resulting rate region is then the set of rate-pairs (R_1, R_2) that for all $G \subseteq \{1, 2\}$ satisfy

$$\sum_{m\in G} R_m \leq \min\left\{C\left(\sum_{m\in G}\frac{P_m}{d_{M+1,m}}\right), C\left(\sum_{m\in G}\frac{P_m}{d_{M+2,m}} + \frac{P_{M+1}}{d_{M+2,M+1}}\right)\right\} \tag{32}$$

Fig. 4 clearly illustrates the capacity-achieving region for both geometries.

5 Conclusions

We obtained new outer bounds for the discrete memoryless MARC and presented a novel offset encoding technique that enables window decoding at the destination for the decode-and-forward strategy. Though the capacity of the MARC remains unknown, capacity-achieving strategies for certain wireless channels and geometries were illustrated. The analysis can be extended to bound outage capacities, a relevant performance metric for slow-fading channels. Further, a detailed analysis of the rate region for each of the strategies considered gives insight into good resource allocation and decoding techniques [18]. Finally, we can also develop capacity theorems for constrained MARCs where the relays do not transmit and receive simultaneously [11].

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