Quark Mass Effects in JIMWLK Evolution

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Abstract. Quark mass effects in the Jalilian-Marian-Iancu-McLerran-Weigert-Leonidov-Kovner (JIMWLK) evolution appear at the next to leading order (NLO) of QCD, i.e., at $O(\alpha_s^2)$ of the QCD coupling for the JIMWLK evolution kernels. Based on the light-cone hadronic wave function approach, we obtain the analytic expressions for the JIMWLK evolution kernels, which are expressed in terms of one dimensional integrals that in principle can be numerically useful for future phenomenological applications. In comparison to the situation with massless quarks, a new divergence arises. We illustrate the origin of this divergence and how the divergence is canceled within the renormalization procedures of light-cone quantization.

1 Introduction

The high energy (or small-x) evolution of hadronic matter leads to increasing of color charge densities and eventually saturation effects [1, 2], and it is one of main topics to be studied in the coming EIC collider [3]. The already developed higher energy evolution equations include the Balitsky-Fadin-Kuraev-Lipatov (BFKL) [1, 2, 4–6] equation and the nonlinear generalizations: the Balitsky- Kovchegov (BK) and the JIMWLK equation [7–13]. The NLO JIMWLK equation has been derived with massless quarks [14, 15]. Even though it has not been studied in phenomenology, we expect it to be important, since the applications of the NLO BK equation have shown to be phenomenologically relevant already in describing HERA data [16].

In this work [17], we go a step further and derive the JIMWLK equation with massive quarks. Note that the quark mass effects come from massive quark loops and so can only appear beyond the LO in α_s. Even at HERA, the contribution of heavy quark mass are considerable [18], and the quark mass effects in high energy evolution are expected to be even more important in future experiments. The derivations we carry out follow closely those of Ref. [15]. In particular, the hadronic light-cone (LC) wave function approach has been used, the pedagogical details of which can be found in Ref [19], and we have checked step by step to reproduce the corresponding results with massless quarks. The same as in Ref. [15], our derivations are based on LC-quantization, which leads to a spurious divergence which does not appear in the case with massless quarks. We show how this divergence is canceled when the proper renormalization procedures are followed for the LC quantization.

This report is based on the results obtained in Ref. [17] and is organized as follows: In Section 2, we review the LC quantization and LC wave functions, and how to use them to

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extract JIMWLK evolution kernels. In Section 3, we sketch the derivation of NLO JIMWLK evolution kernels and deal with the new divergence that appears due to the introduction of quark masses. Section 4 is a brief conclusion.

2 Review of the LC wave function approach to extract the JIMWLK equations

We will be working in the LC coordinate, with $x^{\pm} = x^0 \pm x^3$. LC quantization means "equal LC time" commutators of quantum operators are imposed on a plane with fixed x^+, instead of fixed x^0 in the usual equal time quantization procedure. Also, LC gauge is imposed, i.e., $A^{a+} = 0$ for gauge fields, so that only the traverse degrees of freedoms (d.o.f.) of gauge fields appear in the Hamiltonian. The fermion d.o.f.s are projected out with $\psi_{\pm} = \Lambda_{\pm}\psi$, where $\Lambda_{\pm} \equiv \gamma^0\gamma^{\pm}/2$ with γ^{μ} the Dirac matrices, and ψ_+ is the dynamical d.o.f. that appears in the Hamiltonian (ψ_- is related to ψ_+ through equations of motion (EoM)).

When quark masses are considered, the LC Hamiltonian reads

$$H_0 \equiv \int dx_+ d^2\mathbf{x} \left[\frac{1}{2}\left(\partial_i A_j^a\right)^2 + \psi_+^{\dagger} \frac{-\partial_i\partial_i + m^2}{i\partial^+}\psi_+ \right], \tag{1}$$

for the kinetic terms and

$$\begin{aligned} H_{int} \equiv \int dx_+ d^2\mathbf{x} \Big[& -g f^{abc}\partial_i A_j^a A_i^b A_j^c + \frac{g^2}{4} f^{abc} f^{ade} A_i^b A_j^c A_i^d A_j^e \\ & -g f^{abc}\left(\partial_i A_i^a\right)\frac{1}{\partial^+}\left(A_j^b\partial^+ A_j^c\right) + \frac{g^2}{2} f^{abc} f^{ade}\frac{1}{\partial^+}\left(A_i^b\partial^+ A_i^c\right)\frac{1}{\partial^+}\left(A_j^d\partial^+ A_j^e\right) \\ & +2g^2 f^{abc}\frac{1}{\partial^+}\left(A_i^b\partial^+ A_i^c\right)\frac{1}{\partial^+}\left(\psi_+^{\dagger}t^a\psi_+\right) + 2g^2\frac{1}{\partial^+}\left(\psi_+^{\dagger}t^a\psi_+\right)\frac{1}{\partial^+}\left(\psi_+^{\dagger}t^a\psi_+\right) \\ & -2g\left(\partial_i A_i^a\right)\frac{1}{\partial^+}\left(\psi_+^{\dagger}t^a\psi_+\right) - g\psi_+^{\dagger}t^a\left(i\alpha_i\partial_i + m\beta\right)\frac{1}{i\partial^+}\left(\alpha_j A_j^a\psi_+\right) \\ & -g\psi_+^{\dagger}t^a\alpha_i A_i^a\frac{1}{i\partial^+}\left(i\alpha_j\partial_j + m\beta\right)\psi_+ - ig^2\psi_+^{\dagger}t^a t^b\alpha_i A_i^a\frac{1}{\partial^+}\left(\alpha_j A_j^b\psi_+\right)\Big] \end{aligned} \tag{2}$$

for interaction terms [20], where g is the strong coupling constant, t^a are Gell-Mann matrices for $SU(3)$ with $[t^a, t^b] = if^{abc}t^c$, $\beta \equiv \gamma^0$, and $\alpha^i \equiv \gamma^0\gamma^i$. Note that only quark-quark-gluon three point interactions have a mass dependence.

In principle, one can extract JIMWLK evolution kernel from the evolution of any suitable observable, such that

$$\frac{\delta O}{\delta Y} = -H^{JIMWLK}O \tag{3}$$

with H^{JIMWLK} the JIMWLK Hamiltonian that generates the high energy evolution with respect to rapidity Y. In practice, it is simplest just to study the forward scattering amplitude of a LC hadronic wave function

$$\Sigma \equiv \langle\psi|\hat{S} - 1|\psi\rangle \tag{4}$$

where Born-Oppenheimer approximation will be used in the calculation of the soft wave function $|\psi\rangle$, which includes modes only in a small rapidity window δY as demonstrated in Figure 1 [15, 17], and as a result one has

$$\Sigma = e^{-\delta Y H_{JIMWLK}} - 1 = -\delta Y H_{JIMWLK} + \frac{1}{2}\delta Y^2 H_{JIMWLK}^2 + O(\delta Y^3). \tag{5}$$

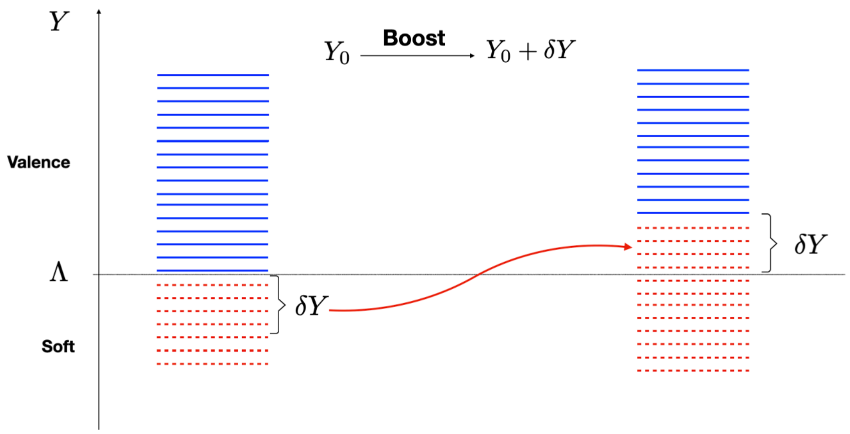

Figure 1. The longitudinal momenta of the fast moving hadrons (projectiles) are divided into valence modes and soft modes with a separation scale Λ. After a differential boost of δY, the soft modes within the range δY right below the cutoff Λ will emerge above Λ.

In Figure 1, the modes below Λ are too soft to participate in high energy collisions and the modes above Λ are frozen and are described with semi-classical color charge density $\rho(\mathbf{x})$. Using Old-Fashioned-Perturbation-Theory (OFPT), one can easily obtains the LO soft-wave function as

$$|\psi_g^{\rm LO}\rangle = \int_\Lambda^{e^{\delta Y}\Lambda} \frac{dk^+}{\sqrt{k^+}} \int_{\mathbf{x},\mathbf{z}} \frac{ig\,(\mathbf{x}-\mathbf{z})^i}{2\pi^{3/2}(\mathbf{x}-\mathbf{z})^2}\rho^a(\mathbf{x})\left|g_i^a(k^+,\mathbf{z})\right\rangle \tag{6}$$

which after being plugged in Eq. (5) gives rise to the LO JIMWLK equation [15, 17, 19].

3 NLO calculations JIMWLK evolution kernels with massive quarks

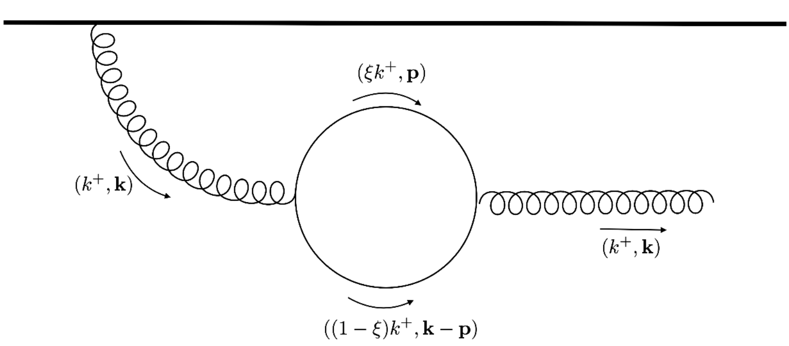

Figure 2. Quark loop correction for the single gluon emission diagram; ξ is the defined as $\xi \equiv p^+/k^+$. The horizontal thick line is the valence current.

The only diagrams contributing to NLO soft wave functions that will have quark mass effects are shown in Figure 2 and 3. It turns out that a mass dependent counter term is needed

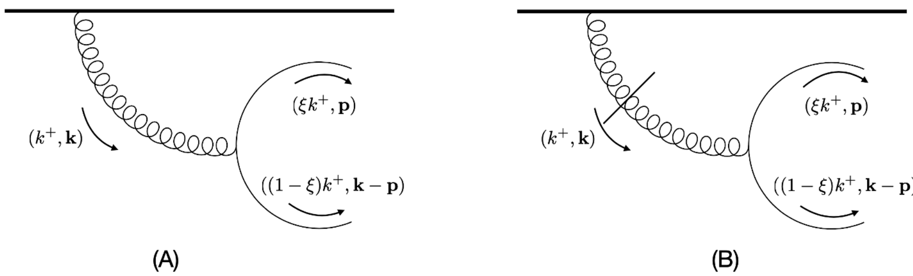

Figure 3. Quark emission diagrams, where ξ is the defined as $\xi \equiv p^+/k^+$. The horizontal thick lines are valence charge current. Diagram (A) corresponds to the interaction there point interaction H_{gqq}, and (B) to the instantaneous quark pair emission from the vertex $H_{qq\text{-inst}}$ [17].

to cancel the new divergence that will arise in Figure 2:

$$H_{gg}^{c.t.} = \frac{1}{2}\Delta (A_i^a)^2, \tag{7}$$

where

$$\begin{aligned} \Delta &= \frac{g^2}{4\pi}\int_0^1 d\xi \frac{1}{\xi(1-\xi)d}\int \frac{d^d\tilde{\mathbf{p}}}{(2\pi)^d}\frac{\left(4\xi^2-4\xi+d\right)\tilde{\mathbf{p}}^2+m^2 d}{\tilde{\mathbf{p}}^2+m^2} \\ &= \frac{g^2 m^2}{8\pi^2}\left(\frac{2}{\epsilon}+\ln\frac{\bar{\mu}^2}{m^2}+1\right) \end{aligned} \tag{8}$$

in dimensional regularization [17, 21–23]. Including the counter term contribution in Figure 2, we obtain

$$\begin{aligned} |\psi_g^1\rangle &= -\int_\Lambda^{e^{\delta Y}\Lambda} dk^+ \int \frac{d^2\mathbf{k}}{(2\pi)^2}\frac{d^d\tilde{\mathbf{p}}}{(2\pi)^d}\int_0^1 d\xi \frac{g^3\rho^a(-\mathbf{k})k_i}{2\sqrt{\pi k^+}\mathbf{k}^4}\frac{1}{\xi(1-\xi)d}\left[\left(4\xi^2-4\xi+d\right)\tilde{\mathbf{p}}^2+m^2 d\right] \\ &\times\left(\frac{1}{\xi(1-\xi)\mathbf{k}^2+\tilde{\mathbf{p}}^2+m^2}-\frac{1}{\tilde{\mathbf{p}}^2+m^2}\right)|g_i^a(k)\rangle \end{aligned} \tag{9}$$

where the second term in the second line is from the counter term in Eq. (7), without which there will be a "IR" divergence coming from the $1/\mathbf{k}^4$. With the counter term included, the second line of Eq. (9) gives rise to a factor of $\mathbf{k}^2$

$$\left(\frac{1}{\xi(1-\xi)\mathbf{k}^2+\tilde{\mathbf{p}}^2+m^2}-\frac{1}{\tilde{\mathbf{p}}^2+m^2}\right) = -\frac{\xi(1-\xi)\mathbf{k}^2}{(\xi(1-\xi)\mathbf{k}^2+\tilde{\mathbf{p}}^2+m^2)(\tilde{\mathbf{p}}^2+m^2)} \tag{10}$$

and cures the spurious divergence.

Note that, for the case with massless quarks, a factor of $\mathbf{k}^2$ will appear after the $\tilde{\mathbf{p}}$-integral and there is no problem with the divergence for the $\mathbf{k}$-integral. Also note that the divergence due to quark masses that appears in Figure 2 does not appear in Figure 3, so that except for having more complex integrals, there are no more conceptual difficulties in calculating Diagrams in Figure 3 than the same diagrams with massless quarks.

After some laborious evaluation of the diagrams in Figure 2 and 3, and combing the resulting soft wave functions with Eqs. (4) and (5), we obtain the following NLO JIMWLK evolution kernels that have have quark mass dependence:

$$K^m_{JSJ} = K'^m_{JSJ} - \frac{1}{2}\int_{\mathbf{z}'} K^m_{q\bar{q}}(\mathbf{x},\mathbf{y},\mathbf{z},\mathbf{z}') \tag{11}$$

with

$$K'^m_{JSJ} = \frac{g^4}{128\pi^5}\frac{X\cdot Y}{X^2Y^2}\left[\frac{4}{3}\ln\frac{m^2}{\bar{\mu}^2} + g_1(m|X|) + g_1(m|Y|)\right]. \tag{12}$$

and

$$\begin{aligned}
&\int_{\mathbf{z}'} K^m_{q\bar{q}}(\mathbf{x},\mathbf{y},\mathbf{z},\mathbf{z}') \\
&= \frac{g^4}{32\pi^5}\int_0^1 d\alpha\alpha(1-\alpha)\left\{\frac{2X\cdot Y}{X^2Y^2}\left(-\ln[\alpha(1-\alpha)] - 2K_0\left[\frac{m|X|}{\sqrt{\alpha(1-\alpha)}}\right] - \ln\frac{m^2}{\alpha(1-\alpha)\bar{\mu}^2}\right)\right. \\
&\quad +2F\left[\frac{m}{\sqrt{\alpha(1-\alpha)}}, -X, Y\right] + \frac{1}{X^2}\ln\left[\frac{(X-Y)^2}{Y^2}\right]\frac{m|X|}{\sqrt{\alpha(1-\alpha)}}K_1\left[\frac{m|X|}{\sqrt{\alpha(1-\alpha)}}\right] \\
&\quad \left.+\frac{1}{Y^2}\ln\left[\frac{(X-Y)^2}{X^2}\right]\frac{m|Y|}{\sqrt{\alpha(1-\alpha)}}K_1\left[\frac{m|Y|}{\sqrt{\alpha(1-\alpha)}}\right]\right\},
\end{aligned} \tag{13}$$

where $X \equiv \mathbf{x} - \mathbf{z}$, $Y \equiv \mathbf{y} - \mathbf{z}$, K_0 and K_1 are modified Bessel function of the second kind, and the functions $g_1(x)$ and $F(x, y, z)$ (the details of which can be found in Ref. [17]) are well behaved as quark mass $m \to 0$.

The other mass dependent evolution kernel (from diagram in Figure 3) reads $K^m_{q\bar{q}}$:

$$\begin{aligned}
&K^m_{q\bar{q}}(\mathbf{x},\mathbf{y},\mathbf{z},\mathbf{z}') = \frac{\alpha_s^2}{4\pi^4}\int_0^1 d\xi\left\{\frac{1}{Z^4\left((1-\xi)(X')^2+\xi X^2\right)\left((1-\xi)(Y')^2+\xi Y^2\right)}\right. \\
&\times\Big[\xi(1-\xi)\left(X'^2-X^2\right)\left(Y'^2-Y\right)H_4 \\
&+Z^2\Big(2Z^2\Big(2\xi(1-\xi)(1-2\xi+2\xi^2)H_4 - \xi(1-\xi)(1-2\xi)^2(H_2+H_3) - 4\xi^2(1-\xi)^2H_1\Big) \\
&-X'^2\Big((-2\xi(1-\xi)(1-2\xi)+(1-\xi))H_4 + 2\xi(1-\xi)(1-2\xi)H_2\Big) \\
&-Y'^2\Big((-2\xi(1-\xi)(1-2\xi)+(1-\xi))H_4 + 2\xi(1-\xi)(1-2\xi)H_3\Big) \\
&-X^2\Big((2\xi(1-\xi)(1-2\xi)-\xi)H_4 - 2\xi(1-\xi)(1-2\xi)H_2\Big) \\
&-Y^2\Big((2\xi(1-\xi)(1-2\xi)-\xi)H_4 - 2\xi(1-\xi)(1-2\xi)H_3\Big) - (X-Y)^2H_4\Big)\Big] \\
&\left.-4m^2\frac{\left(X'-\xi Z\right)\cdot(Y'-\xi Z)}{(X'-\xi Z)^2(Y'-\xi Z)^2}H_5\right\}.
\end{aligned} \tag{14}$$

where $X' \equiv \mathbf{x} - \mathbf{z}'$, $Y' \equiv \mathbf{y} - \mathbf{z}'$, $Z = \mathbf{z} - \mathbf{z}'$ and H_i's are functions involving modified Bessel functions (details can be found in Ref. [17]) whose massless limit can be easily obtained.

Eventually the NLO JIMWLK Hamiltonian can be written as

$$\begin{aligned}
H^{NLO}_{JIMWLK} &= \int_{\mathbf{x},\mathbf{y},\mathbf{z}} K^m_{JSJ}(\mathbf{x},\mathbf{y},\mathbf{z})\left[J^a_L(\mathbf{x})J^a_L(\mathbf{y}) + J^a_R(\mathbf{x})J^a_R(\mathbf{y}) - 2J^a_L(\mathbf{x})S^{ab}_A(\mathbf{z})J^b_R(\mathbf{y})\right] \\
&+ \int_{\mathbf{x},\mathbf{y},\mathbf{z},\mathbf{z}'} K^m_{q\bar{q}}(\mathbf{x},\mathbf{y},\mathbf{z},\mathbf{z}')\left[2J^a_L(\mathbf{x})\,\mathrm{tr}\left[S^\dagger(\mathbf{z})t^aS(\mathbf{z}')t^b\right]J^b_R(\mathbf{y}) - J^a_L(\mathbf{x})S^{ab}_A(\mathbf{z})J^b_R(\mathbf{y})\right] \\
&+ \textbf{mass independent terms}.
\end{aligned} \tag{15}$$

4 Conclusions

We report here the results of Ref. [17], where we have obtained the NLO JIMWLK equation with quark masses taken in to account. The derivation is based on the LC wave function approach (see Ref. [19]) and is a generalization of Ref. [15] to the situation with massive quarks. In addition to having more complex integrals due to the introduction of quark masses, a new divergence appears in quark loop diagrams. This new spurious divergence is due to the use of LC quantization, and is canceled through introducing a mass dependent counter term for gluon quadratic term, which are required for the renormalization procedures in LC quantization [21–23]. The evolution kernels we obtained, i.e., Eq. (11) and (14), are one dimension integrals involving Bessel functions, which in principle can be used in numerical study. To be able to efficiently use these results in numerical study, further simplifications will be preferred. However, due to the lack of the separation of quark mass scale and other scales involved in the system, the simplification is not obvious and we leave it to future study.

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