

# Proton partial widths evaluation through the $^{30}\text{Si}(^3\text{He},d)^{31}\text{P}$ transfer reaction for understanding abundance anomalies in Globular Clusters

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**Abstract.** Some observed abundances in globular clusters have suggested the existence of multiple generations of stars within the clusters as the observations require temperature ranges higher than current stars. The  $^{30}\text{Si}(p,\gamma)^{31}\text{P}$  reaction plays a key role in the synthesis of the observed abundances. The study of the  $^{30}\text{Si}(^3\text{He},d)^{31}\text{P}$  transfer reaction is a tool for constraining the strengths of low-lying resonances, and the proton partial widths are the main ingredients for calculating those strengths. We present the method used for estimating the proton partial widths and their associated uncertainties.

## 1 Introduction

Globular clusters are among the oldest objects in the Universe, which makes them important sites for constraining the formation and early evolution models of galaxies. Globular clusters are mainly populated by low-mass stars in the main sequence, with some of them exhibiting anti-correlation of abundances between pairs of light elements such as C-N, O-Na, and

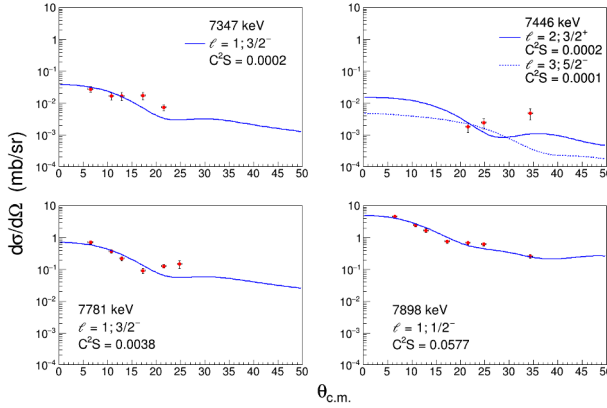
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### 3 Angular distribution and DWBA analysis

The extracted deuteron yield at each spectrometer angle was used, after normalisation, to calculate the differential cross sections for the  $^{31}\text{P}$  populated states. The latter are compared to theoretical differential cross sections computed in the Finite-Range DWBA framework, using the FRESKO code[8].



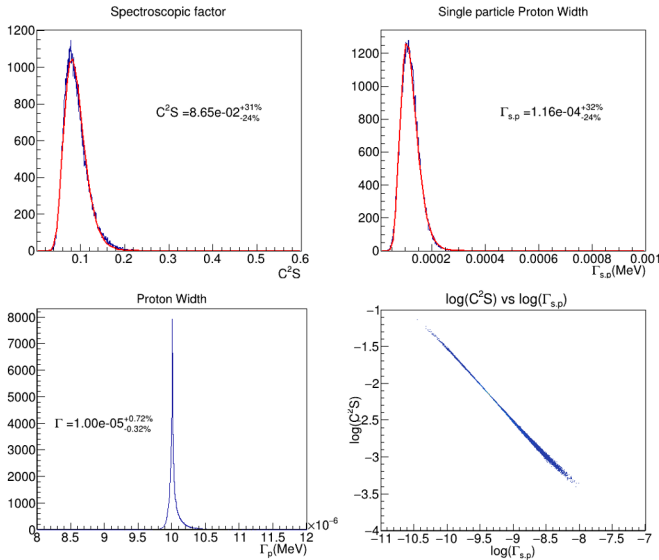
**Figure 2.** (Color online) Angular distributions of  $^{31}\text{P}$  levels that dominate the thermonuclear reaction rate in the  $^{30}\text{Si}(p,\gamma)^{31}\text{P}$  reaction. Blue curves are finite-range DWBA calculations normalized to experimental data points in red.

Figure 2 shows the experimental angular distributions and the best fit with FR-DWBA calculation for the resonances that contribute the most to the  $^{30}\text{Si}(p,\gamma)^{31}\text{P}$  reaction rates, namely  $E_r^{\text{c.m.}} = 50, 149, 485$  and  $602$  keV corresponding to excited states at  $E_x = 7347, 7446, 7781$  and  $7898$  keV, respectively. The optical potential parameters for the entrance channel were obtained from a previous experimental study of the elastic scattering  $^{30}\text{Si}(p,p)^{30}\text{Si}$  with the same beam energy as the present work [9], and the parameters for exit channel are adapted from global deuteron potentials, set F of ref. [10]. The overlap function  $\langle ^3\text{He} | d \rangle$  for the Finite-Range calculations was taken from ref. [11]. A Woods-Saxon potential was used to describe the  $^{30}\text{Si}+p$  wave function, with the depth adjusted to match the binding energy of each calculation. The spectroscopic factor  $C^2S_p$  represents the normalisation factor obtained when fitting the theoretical angular distribution to the experimental data points. The spectroscopic factor is used to calculate the proton partial width, which is crucial for the resonance strengths and reaction rate calculations. The proton width is defined as the product  $\Gamma_p = C^2S_p \times \Gamma_{s,p}$  where  $\Gamma_{s,p}$  is the single-particle proton width calculated as [12]:

$$\Gamma_{s,p} = \frac{\hbar^2 s}{2\mu} P_\ell(E_r, s) |R(s)|^2, \quad (1)$$

where  $\mu$  is the reduced mass of the  $^{30}\text{Si}+p$  system,  $P_\ell(E_r, s)$  is the penetrability of the centrifugal and Coulomb barrier, calculated for a radius  $s$  and associated to a transferred angular momentum  $\ell$ , and  $|R(s)|$  is the radial part of the wave function, estimated at the same radius  $s$  where it achieves an asymptotic behaviour. The uncertainties on the proton width come from (i) the optical potential parameters for entrance and exit channels (ii) the binding potential for the relative  $^{30}\text{Si}+p$  wave function, and (iii) experimental uncertainties associated with the differential cross sections. The geometry of the binding potential is poorly constrained, thus the associated uncertainty has been investigated through a Monte Carlo study where the radius and the diffuseness of the Wood-Saxon well have been varied according to a Gaussian distribution, with a full width half maximum of 25% and 35% with respect to the nominal values of  $r = 1.25\text{fm}$  and  $a = 0.65\text{fm}$ , respectively. A sample of 100000 calculations were performed using the DWUCK4 code since it uses a straightforward method for solving the

Schrödinger equation for unbound levels, in a Zero-Range approximation. The spectroscopic factor and single-particle widths obtained follow a log-normal distributions shown in Figure 3, both with an associated uncertainty of  $\approx 30\%$ . A strong correlation between  $C^2S_p$  and  $\Gamma_{s,p}$  is found and the product gives rise to an uncertainty on  $\Gamma_p$  smaller than 1%. The proton width distribution is not well described by any analytical probability distribution function (PDF), and thus the cumulative distribution function is used to extract the  $1\sigma$  uncertainty as the difference between the 0.84 and the 0.16 quantiles. The final uncertainty on the proton width is then dominated by the optical potential parameters and the statistical fit errors.



**Figure 3.** (Color online) Statistical distributions for: (top left) the spectroscopic factor, (top right) the single particle partial width, and (bottom left) the proton partial width. The red curves are the log-normal distributions fitted to the data. The bottom right panel shows the correlation between the spectroscopic factor and the single particle widths, in log scales.

## 4 Conclusions

Proton partial widths in the compound nucleus  $^{31}\text{P}$  have been extracted and the associated uncertainties estimated with a statistical method. These proton widths, obtained with the  $^{30}\text{Si}(^3\text{He},d)^{31}\text{P}$  transfer reaction are key ingredients for the evaluation of the proton-capture rate  $^{30}\text{Si}(p,\gamma)^{31}\text{P}$  which is crucial for understanding the abundance anomalies in Globular Clusters. More details on the evaluation of the  $^{30}\text{Si}(p,\gamma)^{31}\text{P}$  reaction rate can be found in ref. [6].

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