# Proton partial widths evaluation through the $^{30}$ Si( $^{3}$ He,d) $^{31}$ P transfer reaction for understanding abundance anomalies in Globular Clusters

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**Abstract.** Some observed abundances in globular clusters have suggested the existence of multiple generations of stars within the clusters as the observations require temperature ranges higher than current stars. The  ${}^{30}\text{Si}(p,\gamma){}^{31}\text{P}$  reaction plays a key role in the synthesis of the observed abundances. The study of the  ${}^{30}\text{Si}({}^{3}\text{He,}d){}^{31}\text{P}$  transfer reaction is a tool for constraining the strengths of low-lying resonances, and the proton partial widths are the main ingredients for calculating those strengths. We present the method used for estimating the proton partial widths and their associated uncertainties.

## 1 Introduction

Globular clusters are among the oldest objects in the Universe, which makes them important sites for constraining the formation and early evolution models of galaxies. Globular clusters are mainly populated by low-mass stars in the main sequence, with some of them exhibiting anti-correlation of abundances between pairs of light elements such as C-N, O-Na, and

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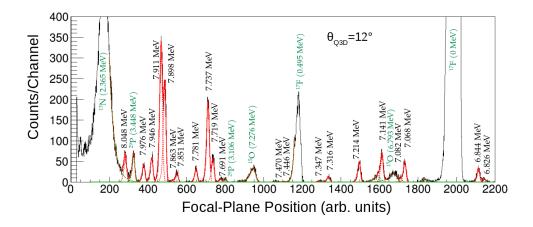
Mg-Al [1]. These observed abundance cannot originate from the clusters stars because the temperature is too low to make the observed anti-correlations, indicating that they have been partially inherited from previous generation of stars called polluters[2]. The nature of these polluters is still uncertain.

The NGC 2419 globular clusters is an interesting case since it exhibits an additional anticorrelation between Mg-K which requires a higher temperature for hydrogen burning when compared to other anticorrelations [3]. Monte Carlo network calculations suggest that the temperature for polluter candidates should range between 120 - 200 MK [4]. This range is sensitive to few proton-capture reactions, and mainly to the  $^{30}\text{Si}(p,\gamma)^{31}\text{P}$  reaction [5]. For these stellar temperature, the  $^{30}\text{Si}(p,\gamma)^{31}\text{P}$  reaction rate depends on resonances of energies  $E_r^{\text{c.m.}} = 100 - 500$  keV, where the strengths are directly proportional to the proton partial widths since the latter are much smaller than  $\gamma$ -ray partial widths.

In this proceeding we report on the analysis of the associated one-proton  ${}^{30}\text{Si}({}^{3}\text{He},d){}^{31}\text{P}$  transfer reaction and the determination of proton widths in the framework of DWBA analysis.

# 2 Data Analysis

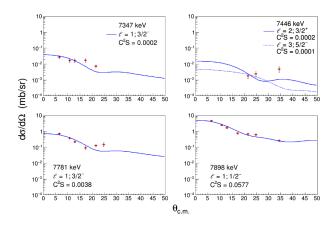
The  ${}^{30}\mathrm{Si}({}^{3}\mathrm{He},d){}^{31}\mathrm{P}$  reaction was performed at Maier-Leibnitz-Laboratorium (MLL) Germany. Ejectiles were momentum analysed using the Q3D spectrometer for different laboratory angles. Deuterons were selected using the focal-plane detectors and experimental details can be found in ref.[6] where excited states between  $E_x = 6.8$  keV MeV and  $E_x = 8.1$  keV MeV have been studied. Figure 1 shows the deuteron focal plane position spectrum obtained at an angle of  $\theta_{lab} = 12^{\circ}$ , containing both bound and unbound levels (Sp=7296 keV) of  ${}^{31}\mathrm{P}$  which were fitted with skewed Gaussians with same resolution. The red curve shows the best least-square fit of the deuteron spectrum. The experimental resolution achieved was 6.2 keV (FWHM) at  $\theta_{lab} = 12^{\circ}$ , three times better then a previous study of the same reaction [7]. The high sensitivity allowed the observation of weakly populated states at  $E_x = 7446$  keV and 7470 keV which are of interest for temperature range of polluter candidates in Globular clusters.



**Figure 1.** (Color online) Deuteron magnetic rigidity spectrum at  $\theta_{lab} = 12^{\circ}$ . Excitation energies in <sup>31</sup>P between  $E_x = 6.8$  MeV and 8.1 MeV are covered and are reported for each state. The best fit is shown as a solid line, while the individual contributions are in red/green dashed lines for <sup>31</sup>P states and contaminant peaks, respectively.

# 3 Angular distribution and DWBA analysis

The extracted deuteron yield at each spectrometer angle was used, after normalisation, to calculate the differential cross sections for the <sup>31</sup>P populated states. The latter are compared to theoretical differential cross sections computed in the Finite-Range DWBA framework, using the FRESCO code[8].



**Figure 2.** (Color online) Angular distributions of  ${}^{31}P$  levels that dominate the thermonuclear reaction rate in the  ${}^{30}Si(p,\gamma){}^{31}P$  reaction. Blue curves are finite-range DWBA calculations normalized to experimental data points in red.

Figure 2 shows the experimental angular distributions and the best fit with FR-DWBA calculation for the resonances that contribute the most to the  $^{30}\mathrm{Si}(p,\gamma)^{31}\mathrm{P}$  reaction rates, namely  $E_r^{\mathrm{c.m.}} = 50,149,485$  and 602 keV corresponding to excited states at  $E_x = 7347,7446,7781$  and 7898 keV, respectively. The optical potential parameters for the entrance channel were obtained from a previous experimental study of the elastic scattering  $^{30}\mathrm{Si}(p,p)^{30}\mathrm{Si}$  with the same beam energy as the present work [9], and the parameters for exit channel are adapted from global deuteron potentials, set F of ref. [10]. The overlap function  $\langle ^3\mathrm{He}|\mathrm{d}\rangle$  for the Finite-Range calculations was taken from ref. [11]. A Woods-Saxon potential was used to describe the  $^{30}\mathrm{Si}+p$  wave function, with the depth adjusted to match the binding energy of each calculation. The spectroscopic factor  $C^2S_p$  represents the normalisation factor obtained when fitting the theoretical angular distribution to the experimental data points. The spectroscopic factor is used to calculate the proton partial width, which is crucial for the resonance strengths and reaction rate calculations. The proton width is defined as the product  $\Gamma_p = C^2S_p \times \Gamma_{s.p}$  where  $\Gamma_{s.p}$  is the single-particle proton width calculated as [12]:

$$\Gamma_{s.p} = \frac{\hbar^2 s}{2\mu} P_{\ell}(E_r, s) |R(s)|^2, \tag{1}$$

where  $\mu$  is the reduced mass of the  $^{30}$ Si+p system,  $P_{\ell}(E_r, s)$  is the penetrability of the centrifugal and Coulomb barrier, calculated for a radius s and associated to a transferred angular momentum  $\ell$ , and |R(s)| is the radial part of the wave function, estimated at the same radius s where it achieves an asymptotic behaviour. The uncertainties on the proton width come from (i) the optical potential parameters for entrance and exit channels (ii) the binding potential for the relative  $^{30}$ Si+p wave function, and (iii) experimental uncertainties associated with the differential cross sections. The geometry of the binding potential is poorly constrained, thus the associated uncertainty has been investigated through a Monte Carlo study where the radius and the diffuseness of the Wood-Saxon well have been varied according to a Gaussian distribution, with a full width half maximum of 25% and 35% with respect to the nominal values of r = 1.25fm and a = 0.65fm, respectively. A sample of 100000 calculations were performed using the DWUCK4 code since it uses a straightforward method for solving the

Schrödinger equation for unbound levels, in a Zero-Range approximation. The spectroscopic factor and single-particle widths obtained follow a log-normal distributions shown in Figure 3, both with an associated uncertainty of  $\approx 30\%$ . A strong correlation between  $C^2S_p$  and  $\Gamma_{s,p}$  is found and the product gives rise to an uncertainty on  $\Gamma_p$  smaller than 1%. The proton width distribution is not well described by any analytical probability distribution function (PDF), and thus the cumulative distribution function is used to extract the  $1\sigma$  uncertainty as the difference between the 0.84 and the 0.16 quantiles. The final uncertainty on the proton width is then dominated by the optical potential parameters and the statistical fit errors.

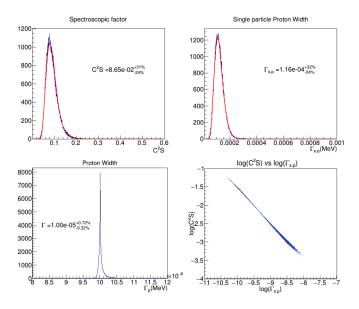


Figure 3. (Color online)
Statistical distributions for:
(top left) the spectroscopic
factor, (top right) the single
particle partial width, and
(bottom left) the proton
partial width. The red
curves are the log-normal
distributions fitted to the
data. The bottom right
panel shows the correlation
between the spectroscopic
factor and the single
particle widths, in log
scales.

## 4 Conclusions

Proton partial widths in the compound nucleus  $^{31}P$  have been extracted and the associated uncertainties estimated with a statistical method. These proton widths, obtained with the  $^{30}Si(^{3}He,d)^{31}P$  transfer reaction are key ingredients for the evaluation of the proton-capture rate  $^{30}Si(p,\gamma)^{31}P$  which is crucial for understanding the abundance anomalies in Globular Clusters. More details on the evaluation the evaluation of the  $^{30}Si(p,\gamma)^{31}P$  reaction rate can be found in ref. [6].

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