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Roadmap for geopotential-based height systems

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1 Objective

- To determine geopotential differences between points of known position in the International Terrestrial Reference Frame (ITRF, Altamimi et al. 2023) to replace the traditional geodetic levelling at large spatial scales.

2 Requirements

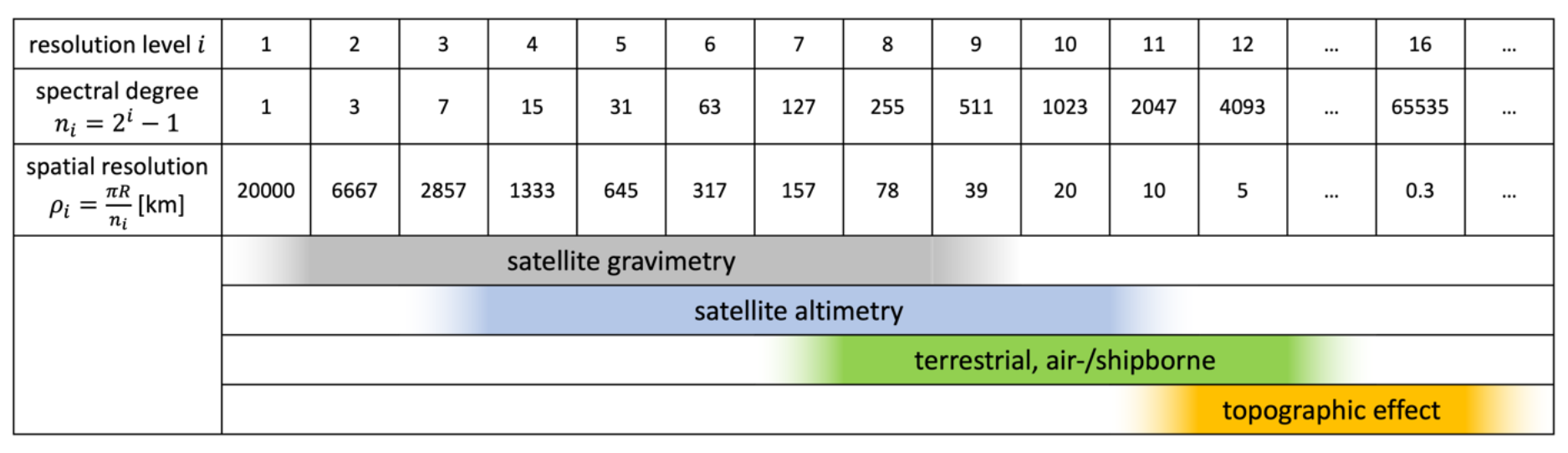
- Precise positioning (usually by Global Navigation Satellite System - GNSS - techniques) in the ITRF (hereafter referred to as the *geometric component*) and
- an optimal geopotential model based on the combination of a satellite Global Gravity Model (GGM) complemented by surface gravity data (terrestrial, airborne, marine) and gravity signals derived from topography models (hereafter referred to as the *physical component*).

3 Recommendations for the geometric component

- Determine the ITRF coordinates following the Conventions of the International Earth Rotation and Reference Systems Service – IERS (Petit and Luzum, 2010).

4 Recommendations for the physical component

- Determine the geopotential model (geoid or quasi-geoid model) by solving a Gravity Boundary Value Problem (GBVP) based on the combination of satellite gravity data, surface gravity data and topographic effects, see https://doi.org/10.1007/s00190-021-01481-0, https://doi.org/10.1007/s00190-021-01567-9.
- Ensure an optimal combination of the gravity data according to the spectral and spatial resolution offered by different measurement techniques (Fig. 1).
- Optimal data combination can be achieved by a spectral combination through a multi-resolution representation (Fig. 2), where different types of gravity measurements are included in the estimation model at the resolution level of their highest spectral sensitivities to contribute with maximum gravity information, see https://doi.org/10.1007/s00190-021-01481-0, https://doi.org/10.1007/s00190-021-01567-9. .



resolution level i	1	2	3	4	5	6	7	8	9	10	11	12	...	16	...
spectral degree $n_i = 2^i - 1$	1	3	7	15	31	63	127	255	511	1023	2047	4093	...	65535	...
spatial resolution $\rho_i = \frac{\pi R}{n_i}$ [km]	20000	6667	2857	1333	645	317	157	78	39	20	10	5	...	0.3	...

Fig 1. Spectral degree n_i and spatial resolution ρ_i of gravity data obtained from different observation techniques, see https://doi.org/10.1007/s00190-022-01670-5, https://doi.org/10.3390/rs12101617.

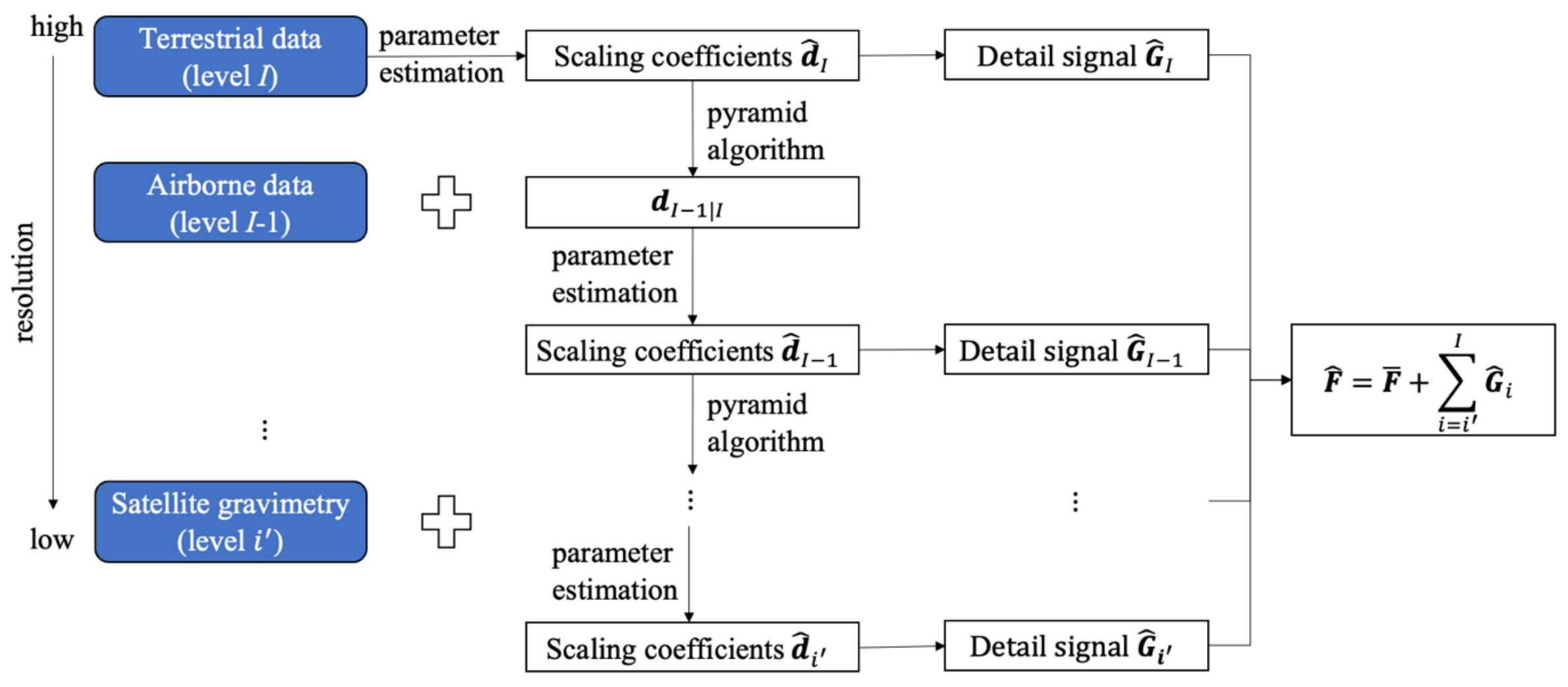


Fig 2. Spectral combination based on a multi-resolution representation. Terrestrial data are included at the highest resolution level I for estimating the coefficients $\hat{\boldsymbol{d}}_I$, which are used to compute the detail signal $\hat{\boldsymbol{G}}_I$ of level I by wavelet functions. These estimated coefficients are transformed to $\boldsymbol{d}_{I-1|I}$ of the next lower level I-1 by applying a low-pass filtering (pyramid algorithm), and $\boldsymbol{d}_{I-1|I}$ is then updated by the lower-resolution airborne data introduced at level I-1. Continuing this process until the lowest level, all data sets are introduced into the scheme at the resolution level of their highest sensitivities. The final gravity functional $\hat{\boldsymbol{F}}$ is obtained by summing up the detail signals $\hat{\boldsymbol{G}}_i$ with the reference model $\bar{\boldsymbol{F}}$, which is the long-wavelength component from a global gravity model, see https://doi.org/10.1007/s00190-022-01670-5, https://doi.org/10.3390/rs12101617.

5 Practical aspects

Depending on the availability of surface gravity data, we consider three cases, see https://doi.org/10.1007/s00190-021-01481-0:

a) For *regions without (or with very few) surface gravity data*,
  - Use a GGM extended with the gravity signals derived from topography models.
  - Expected average accuracy: 4.0 m^2s^{-2} (~ 0.4 m).
  - Validate available GGMs against GNSS/levelling to select the most appropriate one.

b) For *regions with some surface gravity data, but with poor data coverage or low data quality*,
  - Standardise the existing data as much as possible (gravity data linked to absolute gravity stations and all positions in the ITRF).
  - Evaluate gravity datasets to identify/quantify/remove systematic errors in the gravity surveys (Fig. 3).
  - Expected average accuracy: 0.5 m^2s^{-2} (0.05 m) to 1.0 m^2s^{-2} (0.10 m) .

c) In *regions with good surface gravity data coverage and quality*,
  - High resolution regional geopotential models with accuracy to the cm level usually available.
  - GGMs also achieve accuracies of around 0.2 m^2s^{-2} (0.02 m) to 0.6 m^2s^{-2} (0.06 m), see Fig. 4.

6 Topography effects

- Use topographic density information instead of a standard density in residual terrain modelling (RTM) calculations and for synthetic gravity data.
- Using the model UNB TopoDens (Sheng et al. 2019) results in differences in the dm-level for height anomalies above degree 300 (see Fig. 5) and various cm above degree 2160 in mountainous regions (see Fig. 6).
- To minimise the effect of possible uncertainties in the density model and to ensure reliable results, improve the coverage of surface gravity data.

7 Validation and quality assessment

- Derive an uncertainty estimate from the estimation model based on error propagation (Fig. 7).
- To avoid over-optimistic estimation, ensure external evaluation using independent data; i.e. GNSS/levelling data of high quality, see https://doi.org/10.1515/jogs-2019-0008.
- If not GNSS/levelling available, establish two or three high accuracy GNSS/levelling lines with extensions of about 200 km or 300 km.
- Maintain and secure these GNSS/levelling lines to provide a stable reference for monitoring, validation and independent determination of height differences.
- Evaluate the differences between GNSS/levelling and geopotential models using correlation with distance, height, and orientation (azimuth), see Fig. 7c and 8.

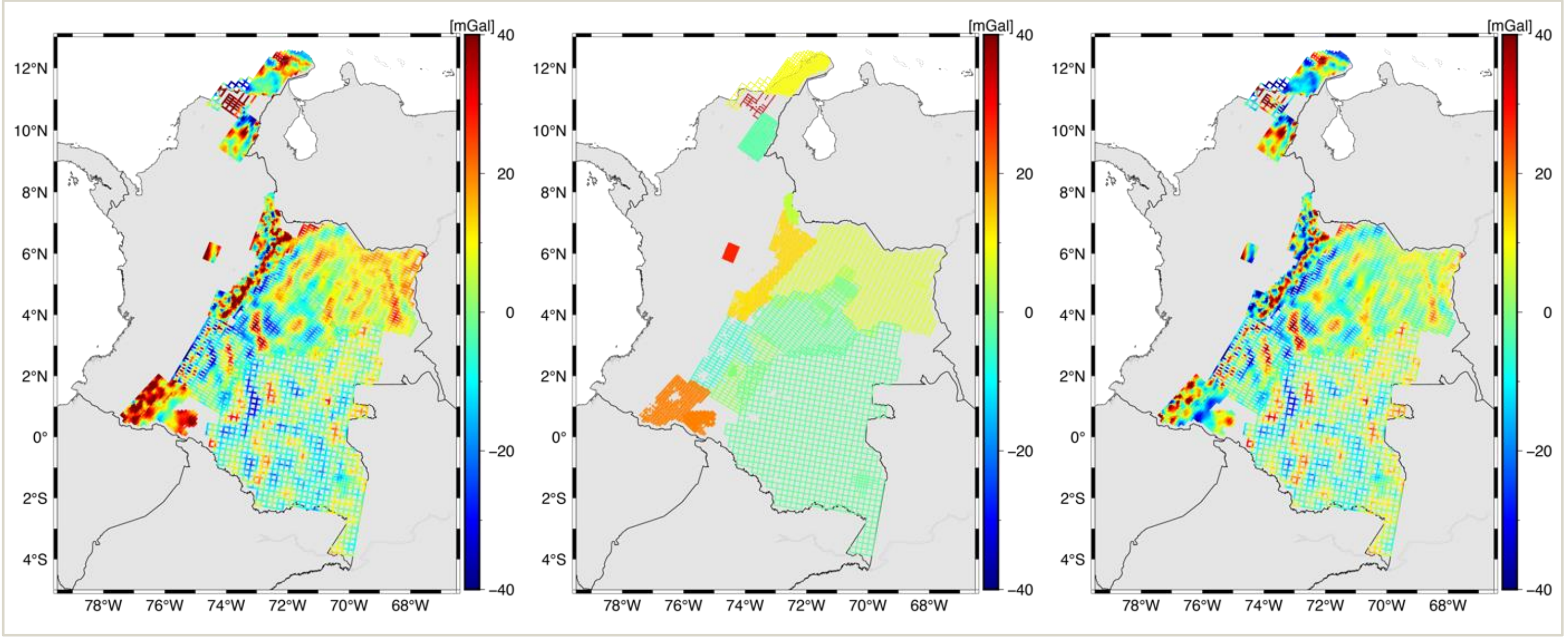


Fig 3. Evaluation of airborne gravity data: The airborne surveys are validated against the SATOP (SAtellite-TOPography combined, Zingerle et al. 2019) model in terms of gravity disturbances. The largest mean difference (left) reaches 47.32 mGal. The biases (middle) are then estimated for each airborne survey individually using Spherical Radial Basis Functions (SRBFs). After removing the biases, the largest mean difference with respect to the SATOP model is reduced to 0.31 mGal (right), see https://doi.org/10.1186/s40623-024-01981-1.

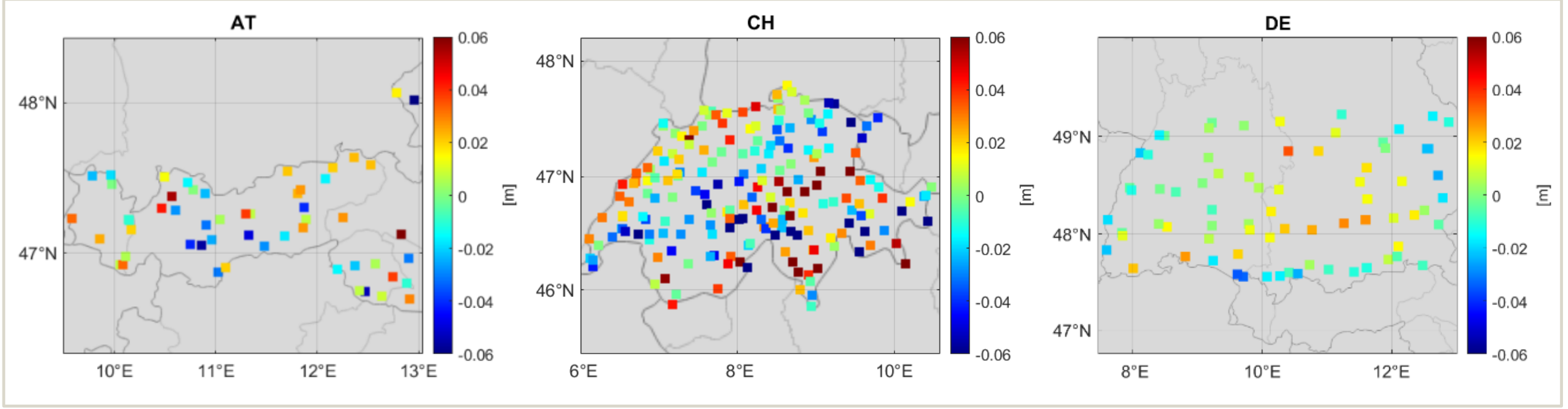


Fig 4. Differences between height anomalies obtained from the GGM EIGEN-6C4 (Förste et al. 2014) **and levelling data** in Austria (left), Switzerland (middle), and Germany (right) after applying a planar correction surface to the levelling data. The comparison shows standard deviations of 3.1 cm (Austria), 4,1 cm (Switzerland) and 1.6 cm (Germany).

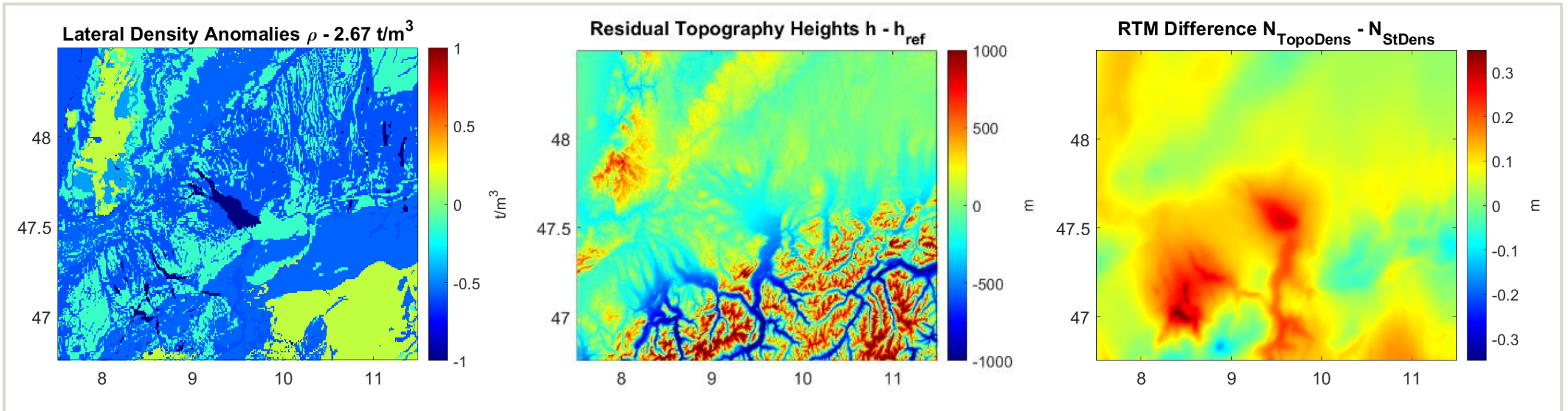


Fig 5. Anomalous density (left), **residual heights above degree 300**, i.e., spatial scales of ~70 km down to ~90 m, (middle), and the difference between the forward modelled height anomalies using density information vs. standard density (2,670 kgm-^{3}) (right)
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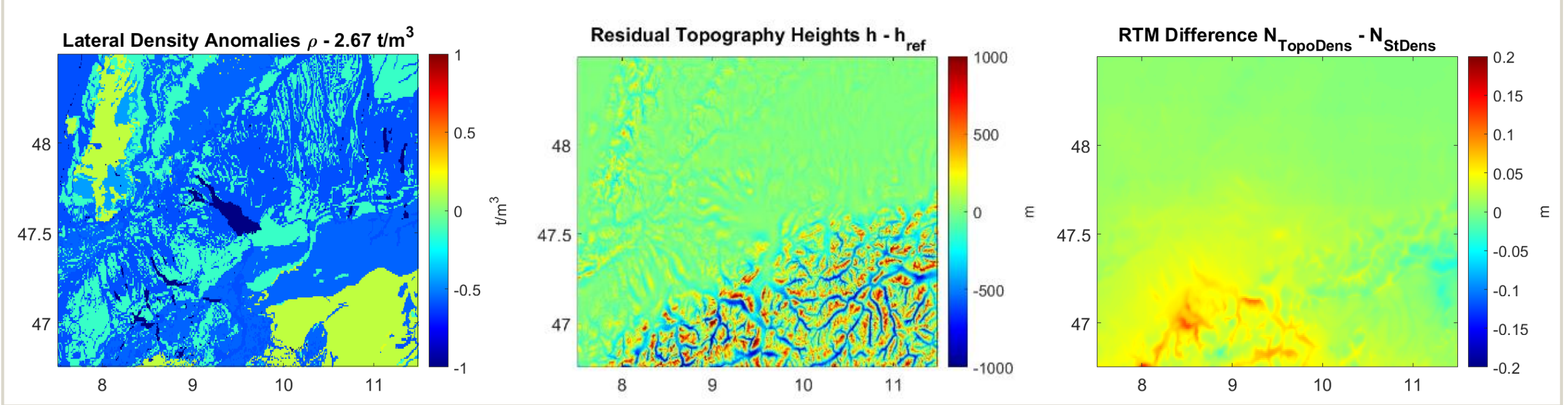

Fig 6. Anomalous density (left), **residual heights above degree 2160**, i.e., spatial scales of ~10 km down to ~90 m, (middle), and the difference between the forward modelled height anomalies using density information vs. standard density (2,670 kgm-^{3}) (right).

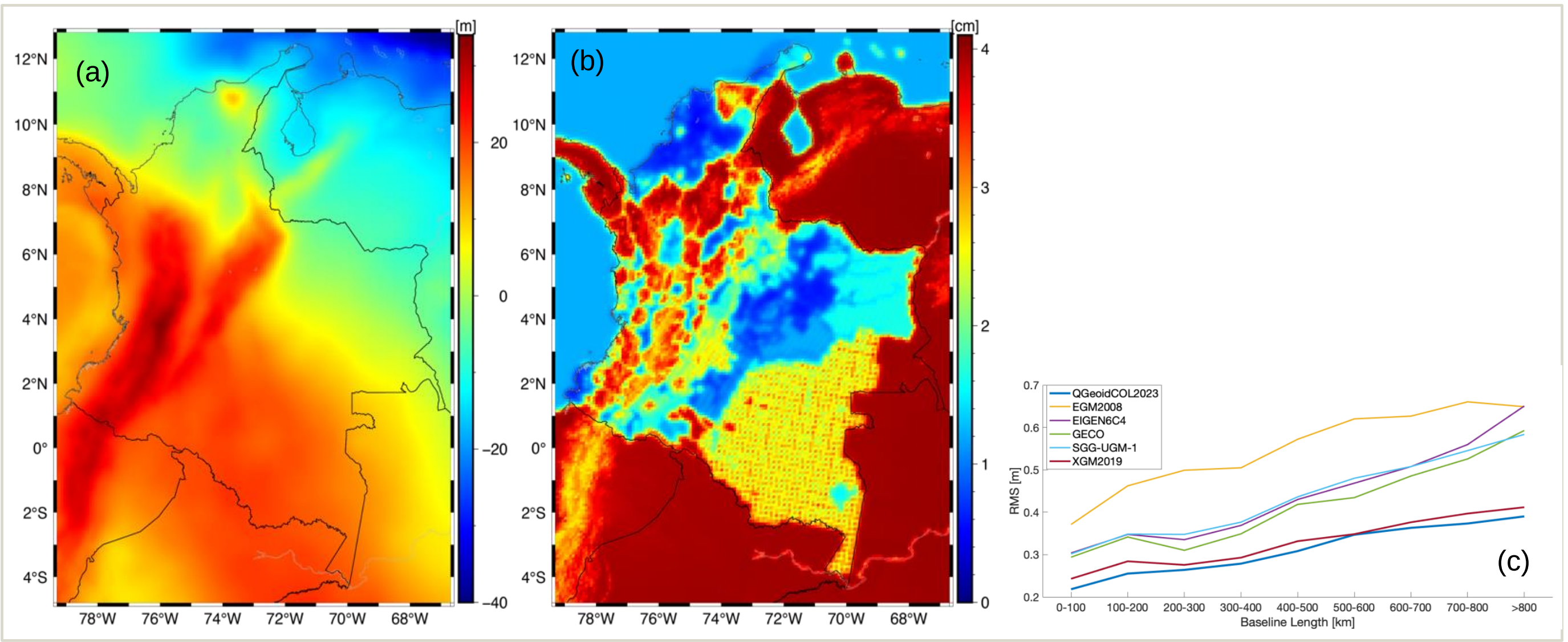

Fig 7. Formal errors and validation of regional geopotential models: (a) Regional quasi-geoid model for Colombia, (b) formal uncertainty derived from the estimation model through error propagation,
(c) Relative validation of the regional quasi-geoid model and selected GGMs in terms of RMS error with respect to GNSS/levelling data. The RMS values of all models increase when the baseline length increases, revealing the accumulation of systematic errors in levelling over long distances, see https://doi.org/10.1186/s40623-024-01981-1.

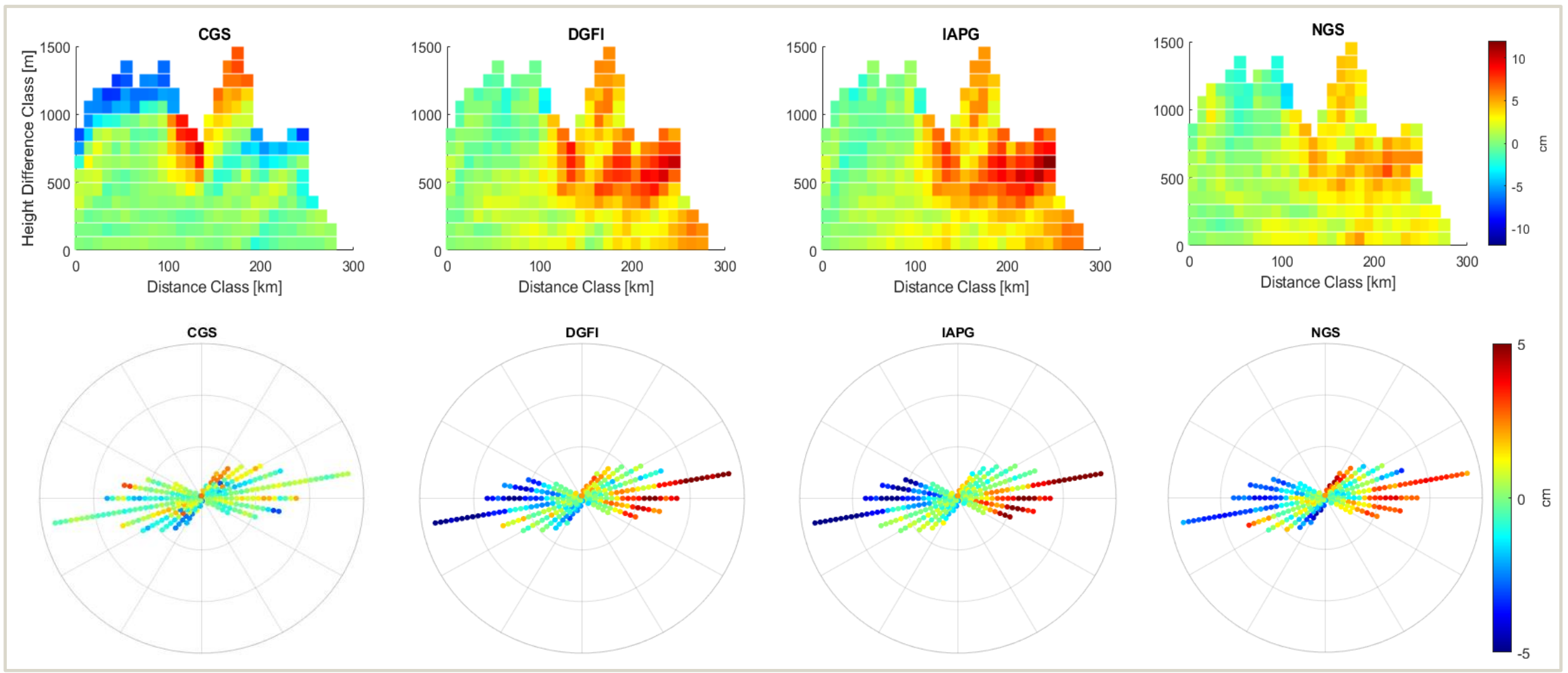

Fig 8. Differential analysis to investigate systematic errors in gravity field models and GNSS/levelling validation data. Top row: Correlations of the differences with height and distance. Bottom row: Correlations of differences with orientation and distance.