CHAIR OF ELECTRONIC DESIGN AUTOMATION

TECHNISCHE UNIVERSITÄT MÜNCHEN

Research Project

Analysis of Wavelength Usage with Multicast in Wavelength-Routed Optical Networks-on-Chip

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I confirm that this research project is my own work and I have documented all sources and material used.

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Abstract

There are two categories into which optical Network-on-Chips (ONoCs) are divided: active NoCs and passive NoCs. For the passive NoCs, the wavelength routing concept is applied. This is done by introducing the so-called Micro-Ring-Resonators (MRRs). MRRs are used to direct an incoming wavelength to a destination. Such a passive network is often called a wavelength-routed Network-on-Chip (WRONoC). The multicast concept will be explored to optimize the network's performance. Multicast refers to using a single wavelength from one sender to multiple receivers, and as a method, it can be applied to different types of networks, such as a mesh or a ring network instance. The network structure for this exploration will be a ring NoC containing some shortcuts. The given network is modeled as a graph. This graph will consist of nodes representing communication tiles and edges representing the tiles' connections in the given ring network. Additionally, some edges will be representing the shortcut connections. Consequently, all of the proposed algorithms in this report are applied to a graph. These will be Dijkstra's Algorithm and the K-means Algorithm.

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1 Introduction

Multiprocessor Systems-on-Chip (MPSoCs) are increasingly recognized as a leading solution for handling intensive computational tasks. However, the high demands for efficient communication within MPSoCs create a need for new data transmission methods that provide both high bandwidth and low latency [XTS21]. In recent years, Optical Network-on-Chip (ONoC) has emerged as a promising solution for such requirements. By using optical signals instead of traditional electronic signals, ONoC achieves high bandwidth and low latency.

ONoCs are divided into two groups: active ONoCs and passive (also known as wavelength-routed) ONoCs. The active ONoCs have a so-called control system that is used to manage the routing in real time during communication [XTS21]. For the wavelength-routed Network-on-Chips, no control system exists for routing, unlike for an active ONoC. The paths are predefined at the design stage [Li+20], and the signals are transmitted accordingly. Micro-Ring-Resonators (MRRs) are used to facilitate the traffic in this sort of communication. MRRs are selectively activated based on a set of specific wavelengths. If the wavelength of the transmitted signal matches the one from the MRR, then the signal is coupled. Coupling, in this sense, means that the signal is directed in a different direction from the one in which it was being transmitted until reaching the MRR. MRRs are placed strategically very often in WRONoC topologies to enable all sorts of communication. The main goal of this report is the application of the multicast method to minimize the wavelength usage in the network. Multicast in this context would be the multiple use of the same wavelength from a single communication node. To achieve this, all network nodes will consist of multiple senders while having only one receiver. If the nodes had multiple senders as well as multiple receivers, the method would be considered a broadcast instead of multicast. Each sender of a transmitting node sends the same wavelength to different receivers at the same time, thus lowering the number of used wavelengths for the communication of the whole system.

For the task of feeding the senders of a node with the same signal as in the singlesender scenario, the power distribution network (PDN) and splitters are used in the design [Ort+17]. In some sense, the power distribution network is similar to a clock distribution network in terms of sequential design. While the clock distribution network focuses on distributing the clock signal to all the D-Flip Flops of the design, the PDN distributes to the nodes the power needed to transmit optical signals. With multiple senders for each node, the PDN needs to feed each node a larger amount of power. If, for instance, each sending node has two senders, the PDN distributes a signal with the wavelength $2 \times \lambda$ to the node, and this is split into a wavelength λ for each sender. The power loss for this operation is in the vicinity of 3 dB. One of the common methods to implement such splitters is the Y-branch [Zha+19].

2 Background

In this work, multicast will be explored for the case of a ring network. The proposed methods should be applied to the already existing XRing tool [Zhe+23]. XRing [Zhe+23] should be applied to obtain the network (as shown in Figure 3.1), including possible shortcuts. However, the obtained output is not a graph in the mathematical sense [Wes01] since it still contains MRRs in the shortcut (see Figure 3.1).

2.1 Graph theory

A non-empty set of vertices and edges is considered to be a graph. Two important attributes for each node in the following algorithms will be the color of the node and also the degree. The degree of a node in a graph is considered to be the number of incident vertices to this specific node. Two vertices/nodes are referred to as incident if an edge connecting these with one another exists. This also means that the degree of a node can be calculated by counting the number of incoming/outgoing edges with respect to the node under inspection.

Edges are themselves divided into two categories: the directed edges and the nondirected edges. For the purpose of this paper, all the graphs contain non-directed edges, which enable communication in both directions between two incident nodes. Each edge will also hold a specific attribute, which will be referred to as the weight of the edge. This should represent the cost of transitioning from one node to another incident node. In a ring network without considering shortcuts, all nodes are one hop away from the incident nodes, and all nodes would have a degree equal to two. For the shortcut, on the other hand, the weight will be considered differently. This will be explained in more detail in the Methodology 3 part of the report.

2.2 Shortcut construction, XRING

The XRing tool [Zhe+23] takes advantage of both the clockwise and counter-clockwise directions for the communication between the nodes of the network. This will also be the case for the proposed extension. Also, the so-called switching elements are

introduced for the XRing tool. These can be divided into two sub-groups: the parallel switching elements (PSEs) and the cross switching elements (CSEs) [Zhe+23]. A PSE consists of an MRR that couples the incoming signal with 180 degrees, while a CSE couples the incoming signal with either 90 degrees or 270 degrees. In both cases, the MRR couples the signal only if the wavelengths/frequencies of the signal and the MRR match one another. Otherwise, the MRR is not activated at all, and no coupling takes place.

3 Methodology

The XRing tool [Zhe+23] does not provide the mathematical graph model directly (as discussed in Background 2, but a good enough approximation is obtained, as shown in the following figure:



Figure 3.1: 8-node ring network and the corresponding shortcut construction [Zhe+23].

The only part that is not a graph from this figure is the so-called shortcut. The inner structure of the shortcut is also presented for the specific location in the network. As it can be seen in this figure, the shortcut consists of four MRRs placed in such a way that the following communications are enabled: 2-6, 2-7, 3-6, 3-7, 6-2, 6-3, 7-2, and 7-3. For the total of eight communications, four of them are coupled from the MRRs, namely 2-6, 3-7, 6-2, and 7-3. For these four, a CSE is applied since a 90-degree coupling takes place in all cases. For the rest of the communications, no coupling is needed since the output of the sending node is directly connected to the input of the receiving node. An important observation for this particular structure is that no connection exists between nodes 2 and 3 or nodes 6 and 7 in the given shortcut structure. This is expected to happen since 2 and 3 (and also 6 and 7) are connected in the ring network directly to one another without using the shortcut.

For each edge representing a connection in the ring network, the weight will be considered to be equal to one, while for a shortcut node, it will be considered as two. This assumption was made after considering the geographical structure of the shortcut, as shown in the following figure:



Figure 3.2: Weight of a shortcut edge obtained from Figure 3.1.



Figure 3.3: Assumption of the weights for the shortcut edges.

At this point, the network is fully converted to a graph (see Figure 3.3), and thus, a graph algorithm can be applied to solve the given problem. Even though the addressed problem is to apply multicast to the network, this can be translated into a different form. As mentioned earlier, each node needs to send signals to different receivers using the same wavelengths. Therefore, there are a few points that need to be cleared:

- How many senders will each node have?
- Is there a difference between a node with degree 2 and another one with degree 4 with respect to the number of senders per node?
- How will the signals be transmitted?

3.1 Assumption of no shortcut

For simplicity reasons and to also give the main idea of the proposed approach, the case of no shortcut in the network shall be examined. The graph in this scenario does not contain any node with a degree of 4, only with a degree of 2. In other words, each node is incident to 2 neighboring nodes. As also mentioned in the XRing paper [Zhe+23], the direction in which a signal can be sent can vary between different communications. Sometimes, the clockwise direction is used to send a signal, while other times, the counter-clockwise direction is used to send a signal. Since the task is to apply multicast to the given graph, then each node will have multiple senders. These senders will be placed in the clockwise and counter-clockwise directions of the transmitting node. Hence, this approach suggests that both directions should be used at the same time.



Figure 3.4: Same graph as the one in Figure 3.3, but the shortcut is ignored.

In this specific structure, since each node has a degree of 2, the number of senders per node is also set equal to 2. This means that each node can send, at most, the same wavelength to 2 different destinations at the same time. As mentioned earlier, the splitters and the PDN are the ones tasked with feeding both of the senders, so the remaining issue is to ensure that there is no overlapping of these signals until each reaches its respective destination (as shown in Figure 3.5 (a)).

In Figure 3.5 (b), an overlapping scenario is presented, and it needs to be avoided. In both cases, the node that originally sends two signals with the same wavelength is

node 1, using its senders S1 and S2.



Figure 3.5: Multiple Senders, no overlapping and overlapping.

3.2 Graph algorithms, A* and Dijkstra's

In order to solve this problem, this method suggests applying the K-means Algorithm to the given graph problem. However, a requirement of the K-means Algorithm is that all the shortest distances from each sending node to each destination node are known formerly. With these conditions, there are generally two different algorithms that could be applied:

- A* Algorithm
- Dijkstra's Algorithm

Both of these algorithms are typically used to find the shortest distance from one starting node to a destination node, but there is a difference between them. The A* Algorithm is a 1-source-1-target heuristic approach that performs better than Dijkstra's Algorithm in most scenarios, where there are multiple small obstacles along the path to the target. On the other hand, Dijkstra's Algorithm is a more suitable approach for a 1-source-multiple-targets problem since, for a starting node, the shortest distance to all possible targets is calculated, even if the specific target is set from the beginning. This algorithm is normally applied rarely in real-life problems since, for most cases, A* performs better. Only for extreme scenarios, where there are large obstacles along the

path to the destination node, is it recommended to use Dijkstra's Algorithm. For the specific task at hand, since the K-means Algorithm needs information regarding the shortest distances between each pair of communicating nodes, Dijkstra's Algorithm is preferred instead of the A* Algorithm.

The Dijkstra Algorithm takes its name from its founder, E.W.Disjkstra [TL19] [Dij59]. Two sets of nodes are declared: the processing set and the processed set. The value dist_i represents the shortest distance from the source node (s) to each possible target node (v_i). The steps that need to be followed are [TL19]:

- 1. The source node is added to the set of processing nodes.
- 2. The node with the smallest value $dist_i$ is chosen from the set of the processing nodes. This node is renamed as v_{min} and sent to the set of the processed nodes.
- 3. All neighbors (v_{next}) of v_{min} are observed:
 - a) If v_{next} is already in the set of processed nodes, ignore it.
 - b) If v_{next} is in the processing set, check if $dist_{min}$ plus the edge weight from v_{min} to v_{next} is smaller than $dist_{next}$. If this is the case, update $dist_{next} = dist_{min} + weight$ of the edge.
 - c) If v_{next} is in neither the processed or processing set of nodes, it is added to the processing set with the value of $dist_{next} = dist_{min} + weight of the edge.$
- 4. Repeat steps 2. and 3. until the target node is stored in the processed set or until all reachable nodes from the source are stored in the processed set.

Applying the Dijkstra's Algorithm on the 8-node network (having ignored the shortcut) would lead to the following results (see Table 3.1):

Source Node - Target Node	Dist (Distance) Value	Shortest path
1 - 2	1	1 - 2
1 - 3	2	1 - 2 - 3
1 - 4	3	1 - 2 - 3 - 4
1 - 5	1	1 - 5
1 - 6	2	1 - 5 - 6
1 - 7	3	1 - 5 - 6 - 7
1 - 8	4	1 - 2 - 4 - 8 or 1 - 5 - 6 - 7
2 - 3	1	2 - 3
2 - 4	2	2 - 3 - 4
2 - 5	2	2 - 1 - 5
2 - 6	3	2 - 1 - 5 - 6
2 - 7	4	2 - 1 - 5 - 6 - 7 or 2 - 3 - 4 - 8 - 7
2 - 8	3	2 - 3 - 4 - 8
3 - 4	1	3 - 4
3 - 5	3	3 - 2 - 1 - 5
3 - 6	4	3 - 2 - 1 - 5 - 6 or 3 - 4 - 8 - 7 - 6
3 - 7	3	3 - 4 - 8 - 7
3 - 8	2	3 - 4 - 8
4 - 5	4	4 - 3 - 2 - 1 - 5 or 4 - 8 - 7 - 6 - 5
4 - 6	3	4 - 8 - 7 - 6
4 - 7	2	4 - 8 - 7
4 - 8	1	4 - 8
5 - 6	1	5 - 6
5 - 7	2	5 - 6 - 7
5 - 8	3	5 - 6 - 7 - 8
6 - 7	1	6 - 7
6 - 8	2	6 - 7 - 8
7 - 8	1	7 - 8

3 Methodology

Table 3.1: Dijkstra's Algorithm applied on Figure 3.4.

To be noted in this table is the fact that results with the reversed source and target nodes do not appear at any entry. Since Dijkstra's Algorithm finds the shortest path from a given source to any target, then the result of the distance value and the shortest path found from a target to a source node would be exactly the same. Thus, these entries in the table are ignored. Also, for the cases where the distance value is equal to four, a choice can be made for the shortest path since multiple exist. Normally, the one that was found first has a priority compared to the rest, but for specific applications, the priority may change to another path found later.

3.3 K-Means Algorithm

The K-means Algorithm originates from the work of [Llo82]. Its objective is to group a given set of points (in this case, graph nodes) into K groups/clusters. The steps that need to be followed are [TL19]:

- 1. K means/nodes are chosen.
- 2. Each other node is assigned to the closest mean, and thus, K clusters are formed.
- 3. The means of the clusters are re-calculated.
- 4. Steps 2. and 3. are repeated until no change in the means and clusters is observed.

With the information from Dijkstra's Algorithm, the K-means Algorithm also needs another input, which is the number of clusters. As mentioned earlier, each node should have two senders, from which each sends a signal in either the clockwise direction or the counter-clockwise direction. This also means that the number of clusters with respect to each possible sending node (containing two senders) is going to be equal to 2, so the value of K is going to be set to 2 accordingly. Some other initial conditions needed to apply the K-means Algorithm are the initial means of the K clusters. So, for the two initial clusters, two means are required. This approach chooses the initial means as the two neighboring nodes of the starting node. This choice is done arbitrarily for this approach. Since the sending node sends in both directions, these two neighboring nodes are the first to be reached with respect to each sender. The K-means Algorithm then simply groups the remaining nodes into 2 clusters: the clockwise cluster or the counter-clockwise cluster. This clustering method is applied on each possible sending node, so if a graph consists of *n* nodes, the K-means Algorithm will be applied at least *n* times (even more for the network with a shortcut, as it will be explained later). The resulting clusters for each sending node indicate which receiving nodes should be reached from the sending node in the clockwise or counter-clockwise direction. The application of the K-means Algorithm for the network (without considering the shortcut) can be observed in Figure 3.6.



Figure 3.6: Application of the K-means Algorithm onto the graph from Figure 3.4 with node one as the starting node and nodes 2 and 5 as the initial means.

The 2 clusters obtained from the K-means Algorithm represent possible combinations when applying multicast. Furthermore, any overlap between signals is avoided. Each sender is connected to one of the clusters. In the best-case scenario, the senders want to send a signal to 2 nodes positioned in different clusters. That way, each sender uses either the clockwise or the counter-clockwise direction to reach the final destination. In the worst-case scenario, both target nodes are positioned in the same cluster (either the clockwise cluster or the counter-clockwise cluster). In this case, a new step must be added to the K-means Algorithm. That would be to force the next means to these two destination nodes so that one of them remains in the original clockwise or counter-clockwise cluster while the other is forced to be a new mean to the next cluster. This step could also lead to a mean re-calculation.

3.4 Network with a shortcut

However, the solved graph problem does not represent the original given problem since no shortcut was taken into consideration. If a node is part of a shortcut (as shown in Figure 3.1), then its degree is equal to 4 since this node is incident to two nodes from the original ring network and two other nodes from the shortcut connections. If the same logic is used again for these types of nodes, it would mean that the sending node would have four senders, each sending the same wavelength to 4 different directions. However, this is not enabled due to the structure of the shortcut. For instance, if node 2 is the sending node, then it is connected to nodes 1 and 3 with the edges of the ring network and also connected to nodes 6 and 7 with the help of the shortcut. Due to the structure of the shortcut, if node 2 wants to communicate with node 6, an MRR is used to couple the signal to the receiving port of node 6. In order to enable a communication path with node 7 at the same time, one would need to use a different wavelength since the MRR that couples the signal to node 6 is along the way, and it would redirect the signal originally meant for node 7 as well. In order to tackle this issue, a different approach is considered for implementing a shortcut node, as shown in Figure 3.7).



Figure 3.7: Multicast for a ring node (a) and for a shortcut node (b).

This new model treats a shortcut node as a super-node. These nodes possess four senders compared to 2 from the former observation. In order to achieve this, the application of splitters is required for such a structure before the signal reaches the senders. The senders are divided into two groups of 2 (see Figure 3.7 (b)), where each group works with different wavelengths. Each group consists of a sender incident to a ring node and one of the shortcut nodes (for instance, the shortcut node 2 is connected to the senders of group 1 with nodes 1 and 6 and senders of group 2 with nodes 3 and 7). Furthermore, for each of these sub-groups, the K-means Algorithm will be applied to minimize the usage of different wavelengths, as shown in Figure 3.8.



Figure 3.8: K-means applied onto the graph from Figure 3.3

Figure 3.8 represents the result of the original problem. For this particular example, node 1 is considered to be the original sending node. Thus, a K-means Algorithm starts with the sending node 1 and the two initial means, which are nodes 2 and 5, since these are the first ones that can be reached in the clockwise and counter-clockwise directions. The 2 clusters obtained from the K-means Algorithm on node 1 are noted with the green and red colors (node border and node number). The counter-clockwise cluster (green) consists of nodes 5 and 6, while the clockwise cluster (red) consists of nodes 2, 3, 4, 7, and 8.

Since node 2 is a shortcut node, a new K-means Algorithm is started at that location, where the structure from Figure 3.7 (b) should be considered. Node 2 is incident to nodes 1, 6, 3, and 7. The two-by-two grouping of the incident nodes of the shortcut node 2 would be for this scenario: 1 and 6 (group 1) and 3 and 7 (group 2). This is a valid approach since each group consists of a shortcut node (6 and 7, respectively) and a ring node (1 and 3, respectively).

An important observation at this point is that nodes 1 and 6 have already been visited. Consequently, one of the two halves of Figure 3.7 (b) can be simply ignored. In the worst-case scenario, node 2 might choose to use two different wavelengths to reach the two groups, but in this case, it functions as a typical ring node. Nevertheless, the K-means Algorithm is applied at this point, and the new means will be nodes 3 (in the clockwise direction) and node 7 (in the counter-clockwise direction). The new clusters formed from the application of the K-means algorithm on node 2 are distinguished with the light blue and yellow colors (node filling). The light blue clockwise cluster contains nodes 3 and 4, while the yellow counter-clockwise cluster contains nodes 7 and 8.

In conclusion, this application of the proposed method suggests that if node 1 is the sending node of the network, the following clusters are formed:

- Cluster 1 containing nodes 5 and 6.
- Cluster 2.1 containing nodes 3 and 4.
- Cluster 2.2 containing nodes 7 and 8.

Therefore, node 1, as a sending node, can send signals with the same wavelength at the same time to different receivers. A possible combination would be 1 - 6, 1 - 3, and 1 - 8. In order to complete the K-means Algorithms for this graph, the results of Dijkstra's Algorithm (see Table 3.2) were used.

Source Node - Target Node	Dist (Distance) Value	Shortest path
1 - 2	1	1 - 2
1 - 3	2	1 - 2 - 3
1 - 4	3	1 - 2 - 3 - 4
1 - 5	1	1 - 5
1 - 6	2	1 - 5 - 6
1 - 7	3	1 - 2 - 7 or 1 - 5- 6 - 7
1 - 8	4	1 - 2 - 3 - 4 - 8 or 1 - 5 - 6 - 7 - 8 or 1 - 2 - 7 - 8
2 - 3	1	2 - 3
2 - 4	2	2 - 3 - 4
2 - 5	2	2 - 1 - 5
2 - 6	2	2 - 6
2 - 7	2	2 - 7
2 - 8	3	2 - 3 - 4 - 8 or 2 - 7 - 8
3 - 4	1	3 - 4
3 - 5	3	3 - 2 - 1 - 5 or 3 - 6 - 5
3 - 6	2	3 - 6
3 - 7	2	3 - 7
3 - 8	2	3 - 4 - 8
4 - 5	4	4 - 3 - 2 - 1 - 5 or 4 - 8 - 7 - 6 - 5 or 4 - 3 - 6 - 5
4 - 6	3	4 - 8 - 7 - 6 or 4 - 3 - 6
4 - 7	2	4 - 8 - 7
4 - 8	1	4 - 8
5 - 6	1	5 - 6
5 - 7	2	5 - 6 - 7
5 - 8	3	5 - 6 - 7 - 8
6 - 7	1	6 - 7
6 - 8	2	6 - 7 - 8
7 - 8	1	7 - 8

3 Methodology

Table 3.2: Dijkstra's Algorithm applied on Figure 3.8 including the shortcut.

3.5 Shortcut structure

Another aspect treated in this exploration is the structure of the shortcut. As mentioned earlier, a node cannot use both paths of the shortcut at the same time (an example is already demonstrated in Figure 3.8). Thus, the original four-node shortcut structure (see Figure 3.1) can be rearranged into a three-node shortcut structure, as shown in

Figure 3.9. This was inspired by the contents of the multi-frequency paper [Zhe+24], Figure 3. The proposed three-node shortcut construct (see Figure 3.9) enables the communications 2 - 6, 2 - 7, 6 - 2, and 7 - 2. Similarly, the analog structure between nodes 3, 6, and 7 can be built, which would enable the remaining four communications to complete a total of eight, as calculated earlier. The blank nodes presented in the graph represent the MRRs.



Figure 3.9: Three-node shortcut structure.

Naturally, there exists also the possibility of using a so-called GWOR router [Tan+11] for this case, which would consist of a fully connected 4x4 router containing 8 MRRs.

Instead of focusing on the specific structure of the shortcut as presented in Figure 3.1, the MILP solver will be provided with more possibilities of structures for a shortcut to choose from. After the results are obtained, they will be analyzed, and the best structure will be set as the default. The advantage of the proposed three-node shortcut structure is the number of MRRs that are used (two), which is half of the number of MRRs used for the four-node shortcut (four, see Figure 3.1).

4 Conclusion

This exploration focused on applying the multicast method for a WRONoC. The condition of using this proposed method is that the WRONoC topology has been obtained using another tool, like the XRing tool [Zhe+23], for instance. After this is fulfilled, the network is transformed into a graph, onto which graph algorithms like Dijkstra's Algorithm and K-means Algorithm are applied. The clusters obtained from the K-means Algorithm are then taken into consideration for the multicast. In the best-case scenario, this method could offer a satisfactory improvement for the wavelength usage issue. Additionally, the number of MRRs used could be reduced from the initial four introduced in the XRing tool [Zhe+23] to two if the proposed three-node shortcut is used. This could lower the power consumption compared to the existing XRing tool [Zhe+23].

4.1 Future Work

Since the topology is obtained from other tools, the MILP constraints for this method will be simply added to the existing constraints from the topology constructing tool (like XRing [Zhe+23], for instance). The constraints that need to be formulated for this approach are the ones regarding Dijkstra's Algorithm and the K-means Algorithm. Additionally, since the initial means were chosen arbitrarily for the K-means Algorithm, it should be considered if there are better results for different choices of initial means. Lastly, other structures will be analyzed for a possible application of the multicast method, like mesh, for instance.

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