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Methods for geoid determination in regions with challenging data quality and coverage

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Motivation

- At present, hundreds of local height systems exist globally, and the discrepancies between them can reach more than 2 m. Therefore, there is a rapidly increasing need for the establishment of a globally unified height reference system. A current main objective of Geodesy is the realization of the **International Height Reference System (IHRS)**, which is a geopotential-based height system. Its realization therefore relies on the precise determination of (quasi-) geoid models of high resolution.

- In regions with less developed geodetic infrastructure, the main accuracy limitation in (quasi-) geoid determination is the **low availability or quality of surface gravity data**, i.e., terrestrial, airborne, or shipborne gravity data. However, it is not realistic to systematically carrying out gravimetry in some regions due to e.g., economic constraints.

- So, it is **crucial to develop methodologies to recover as many existing gravity surveys as possible** by utilizing modern mathematical methods that allow the evaluation and refinement of gravity data acquired long ago or lacking standard procedures and metadata.

Study area and Data

- The chosen study area is Colombia (Fig. 1 left), the only South American country with coastlines on both oceans, the Pacific and the Atlantic. It is a challenging study area as the country features strong topographical gradients with elevations reaching more than 5000 m above mean sea level, and a large area of about 40% covered by the Amazon rainforest.

- The available terrestrial and airborne gravity data (see Fig. 1 right) in Colombia were collected by different organizations during a time span of 70 years since 1941, so the **gravity measurements widely contain systematic errors, outliers, and biases**.

- In the offshore area, gravity data derived from the latest release of the DTU gravity anomaly model **DTU21GRA** is included.

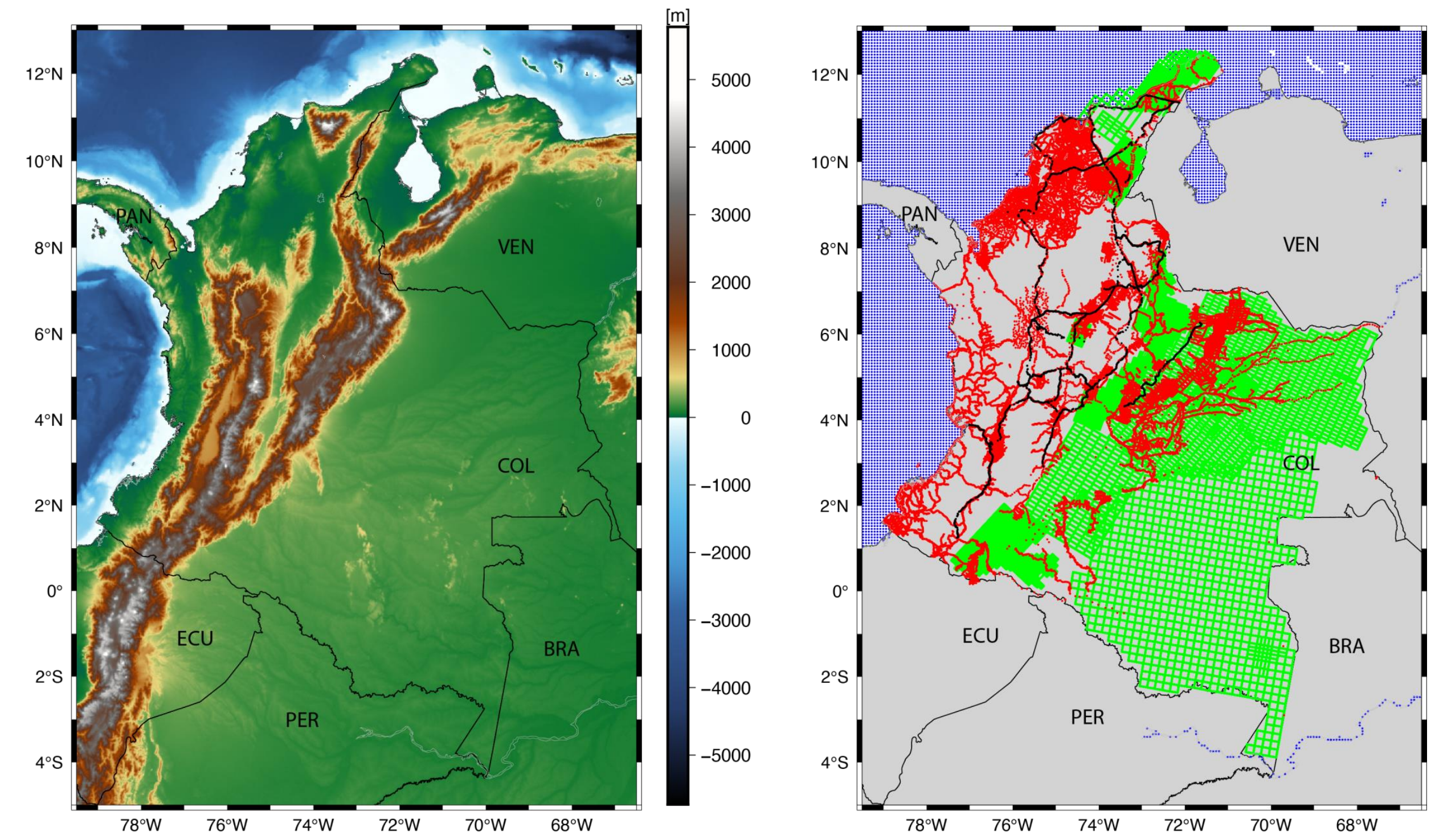

Figure 1: (left) Terrain map of the study area; (right) Available gravity data, including terrestrial (red points), airborne (green flight tracks), and altimetry (blue points) data, as well as the GPS/leveling data (black points) for validation purpose

Methodology

Data Processing

- For terrestrial data, a coordinate transformation from the local datum to the ITRF is performed; potential mistakes in the vertical and horizontal positions are ruled out; outlier values in the gravity disturbance are checked and excluded.

- For airborne data, a crossover analysis is performed; an along-track Gaussian low-pass filter is applied; an evaluation is conducted in comparison to the SATOP (SAtellite-TOPography) model, which **indicates large biases in different airborne surveys**; outlier values in the gravity disturbance are checked and excluded.

Bias estimation based on Spherical Radial Basis Functions (SRBFs)

- In the evaluation against the SATOP model, the airborne survey with the smallest difference (with a mean value of 0.14 mGal) was assumed to be free of bias and set as the reference for estimating the bias of the other airborne surveys using the following bias estimation model:

$$\begin{cases} \boldsymbol{y}_1 + \boldsymbol{e}_1 = \boldsymbol{A}_1\boldsymbol{d} & \text{with } D(\boldsymbol{y}_1) = \sigma_1^2\boldsymbol{P}_1^{-1} \\ \boldsymbol{y}_q + \boldsymbol{e}_q = \boldsymbol{A}_q\boldsymbol{d} + \boldsymbol{1}_q\Delta_{q|1}(q = 2, 3, \cdots, 17) & \text{with } D(\boldsymbol{y}_q) = \sigma_q^2\boldsymbol{P}_q^{-1} \end{cases}$$

- The bias term $\Delta_{q|1}$ can be estimated together with the coefficient vector $\boldsymbol{d}$:

$$\begin{bmatrix}\boldsymbol{y}_1\\ \boldsymbol{y}_2\\ \boldsymbol{y}_3\\ \vdots\\ \boldsymbol{y}_{17}\\ \boldsymbol{\mu}_d\end{bmatrix} + \begin{bmatrix}\boldsymbol{e}_1\\ \boldsymbol{e}_2\\ \boldsymbol{e}_3\\ \vdots\\ \boldsymbol{e}_{17}\\ \boldsymbol{e}_\mu\end{bmatrix} = \begin{array}{c} \boldsymbol{A}_1^\star \\ \begin{bmatrix} \boldsymbol{A}_1 & 0 & 0 & \dots & 0 \\ \boldsymbol{A}_2 & 1_2 & 0 & \dots & 0 \\ \boldsymbol{A}_3 & 0 & 1_3 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{A}_{17} & 0 & 0 & \dots & 1_{17} \\ \boldsymbol{I} & 0 & 0 & \dots & 0 \end{bmatrix} \\ \boldsymbol{A}_\mu \end{array} \cdot \begin{array}{c} \\ \begin{bmatrix}\boldsymbol{d}\\ \Delta_{2|1}\\ \Delta_{3|1}\\ \vdots\\ \Delta_{17|1}\\ \\ \end{bmatrix} \\ \boldsymbol{c} \end{array} \qquad D\left(\begin{bmatrix}\boldsymbol{y}_1\\ \boldsymbol{y}_2\\ \boldsymbol{y}_3\\ \vdots\\ \boldsymbol{y}_{17}\\ \boldsymbol{\mu}_d\end{bmatrix}\right) = \begin{bmatrix} \sigma_1^2\boldsymbol{P}_1^{-1} & 0 & 0 & \dots & 0 & 0 \\ 0 & \sigma_2^2\boldsymbol{P}_2^{-1} & 0 & \vdots & \vdots & 0 \\ 0 & 0 & \sigma_3^2\boldsymbol{P}_3^{-1} & \vdots & \vdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & \sigma_{17}^2\boldsymbol{P}_{17}^{-1} & 0 \\ 0 & 0 & 0 & \dots & 0 & \sigma_\mu^2\boldsymbol{P}_\mu^{-1} \end{bmatrix}$$

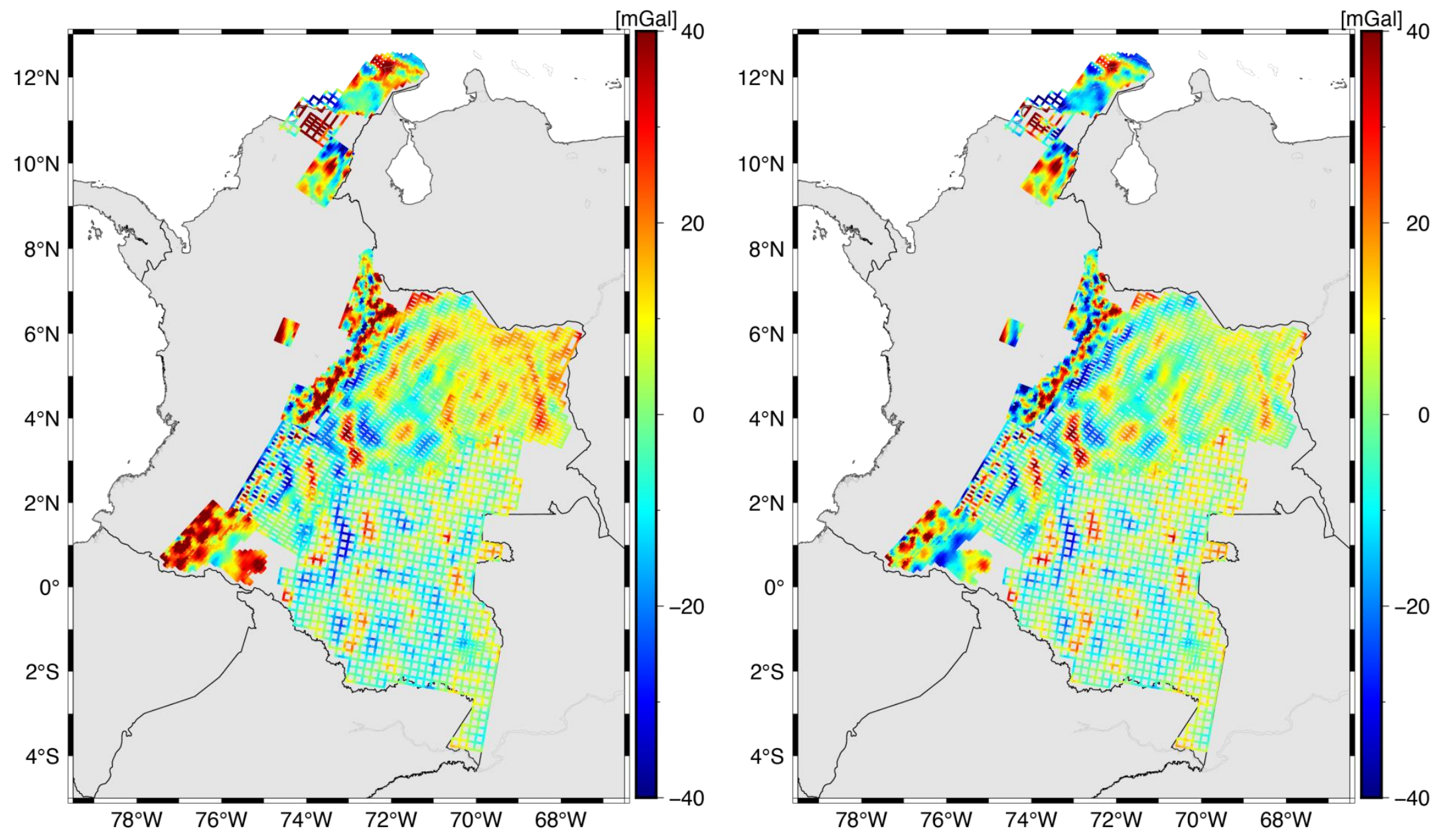

Figure 2: Differences between the airborne gravity data and the SATOP model in terms of gravity disturbance (left) before and (right) after removing the estimated bias in each airborne survey

Computation procedure

- The remove-compute-restore (**RCR**) procedure is applied using XGM2019 up to degree 719 and the topography models dV_ELL_Earth2014 (degree 720 to 2159) and ERTM2160 (degree 2160 to around degree 80,000).

- Terrestrial, airborne, and altimetry data are combined through a parameter estimation, with the relative weight estimated by the **variance component estimation** (VCE).

Results and Validation

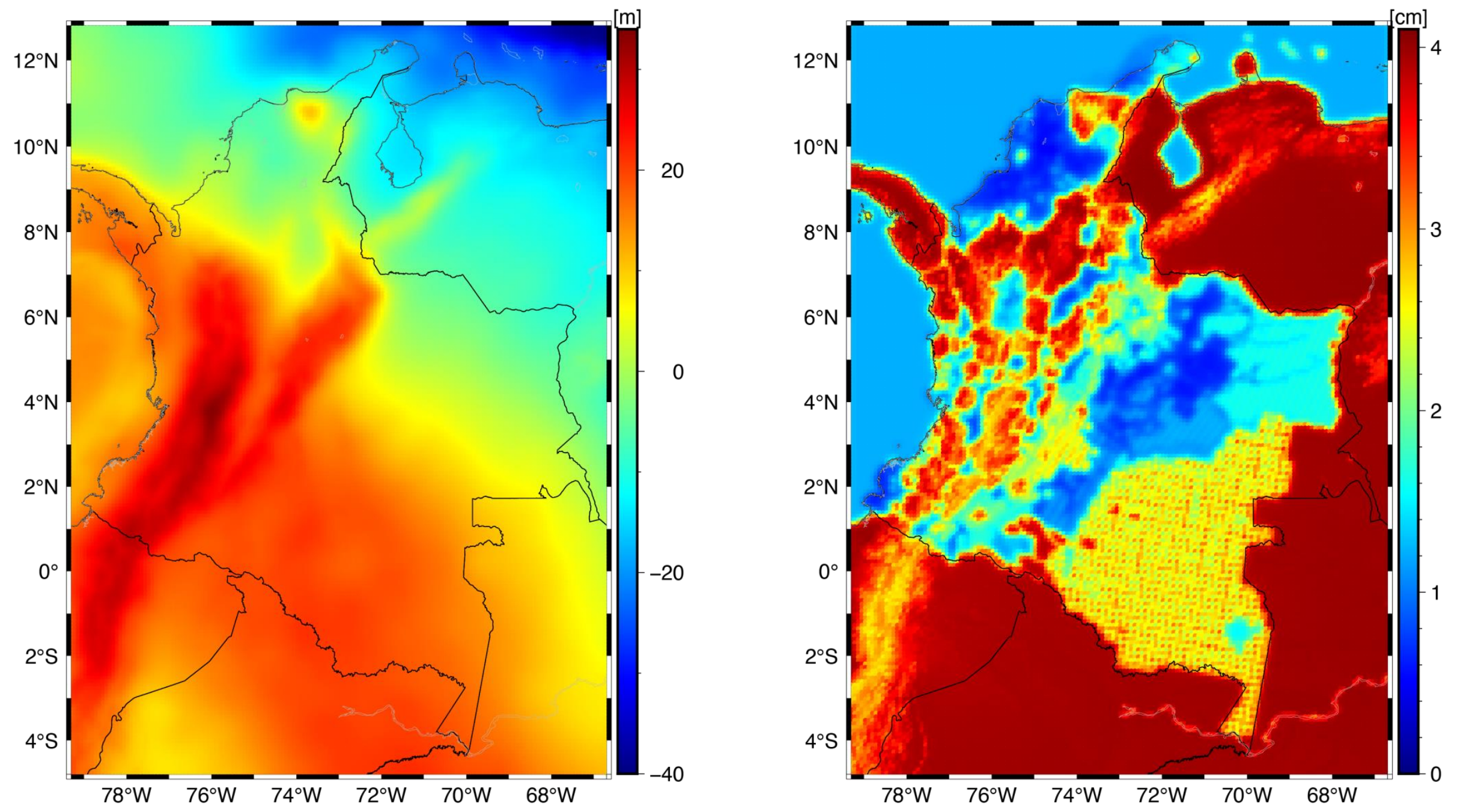

Figure 3: (left) The computed quasi-geoid model QGeoidCOL2023; (right) its standard deviation map

Table: Comparison between QGeoidCOL2023, the latest South American model, and five recent high-resolution global gravity models (GGMs) w.r.t. the GPS/leveling data (unit [cm])

	Min	Max	STD
QGeoidCOL2023	-76.06	65.21	**15.76**
QGEOID2021	-85.37	72.93	24.51
EGM2008	-80.73	83.85	28.09
EIGEN6C4	-93.29	66.80	21.10
GECO	-110.82	60.37	20.39
SGG-UGM-1	-71.03	68.28	20.93
XGM2019	-75.93	80.06	*17.86*

Discussion and Conclusion

- **QGeoidCOL2023 is the best performing quasi-geoid model** with the smallest STD value w.r.t. the GPS/leveling data. This value is **27%** smaller compared to the mean STD value given by the five high-resolution GGMs and **36%** smaller than the one delivered by QGEOID2021.

- The differences between the GPS/leveling data and the gravimetric quasi-geoid models also contain errors in the ellipsoidal and leveling height, which are not of high accuracy.

- The main accuracy limitation in quasi-geoid determination is the gravity data distribution and quality. Applying robust data processing strategies and structures to mitigate systematic errors, outliers, and biases help improving the computation accuracy of the quasi-geoid model.

- The methods and procedures developed can be **applied in other study areas with undefined or challenging data quality, facilitating the realization of the IHRS** or any geopotential-based height system.

References and Acknowledgements:

- *Liu Q., Schmidt M., Sánchez L., Moisés L., Cortez D.:* High-resolution regional gravity field modeling in data-challenging regions for the realization of geopotential-based height systems. Earth, Planets and Space, 76(1), 10.1186/s40623-024-01981-1, 2024
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