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Modeling multi-modal traffic in cities with a functional form representing the physical interactions of vehicles

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1. Introduction

Cities are complex and so are their transportation systems. As urban resources are limited, all means of transportation do not co-exist independently of each other and their degree of interactions, which in a mere economic sense are externalities, determine the optimal allocation of resources in a city. In terms of congestion externalities, the most recognizable interaction is between cars and buses (trams) on urban roads. There exist many illustrations that summarize the trade-off between the speed and space consumption of the two modes which raises the question what is the optimal share of the two of them? Smeed was among the first who analytically showed that in a more populated city, more people must use public transport to maintain the same level of average journey speed (Smeed, 1968). However, methods to analyze this trade-off beyond a simple bus-car-equivalent at large urban scale did not exist until recently the concept of the three-dimensional macroscopic fundamental diagram (3D-MFD) was introduced (Geroliminis et al., 2014).

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The 3D-MFD is able to capture the joint effects of the number of cars and buses in the network on the total flow of vehicles including many externalities and vehicle occupancies. So far, 3D-MFDs have been either estimated from observations (Geroliminis et al., 2014; Loder et al., 2017) or derived analytically in conceptual contexts (Chiabaut, 2015), but neither does a methodology exist that predicts the shape of the 3D-MFD from physical properties nor does a functional form exists which is based on physically meaningful parameters of the network and traffic operations. However, we use recent advances in the field of the MFD to propose a functional form for the 3D-MFD that can be derived from the properties of the network and the multi-modal traffic operations.

The 3D-MFD is defined first by creating a three-dimensional shape out of planes which parameters reflect physical characteristics of the network and the multi-modal traffic operations. Secondly, a concave functional form is then obtained by using the smooth approximation of the minimum operator. Among others, the parameters of the plane reflect the influence of traffic signals (Daganzo and Geroliminis, 2008; Daganzo and Lehe, 2016) and bus operations (Boyaci and Geroliminis, 2011; Guler and Menendez, 2014) on the system's capacity and vehicular flow. It is clear that the proposed method shares ideas with the variational theory used for the estimation of the MFD (Daganzo and Geroliminis, 2008; Leclercq and Geroliminis, 2013) and that any derived three-dimensional *cuts* can be used for our equation as well, but we aim to define the planes as simple as possible so that they can be parametrized in any case. Lastly, we apply the methodology to two empirical data sets of London and Zurich.

The new proposed functional form can be used to estimate the 3D-MFD for any urban environment with basic information available and subsequently used in multi-modal modeling that includes externalities beyond a simple bus-car-equivalent.

2. Methodology

In this functional representation of multi-modal traffic in cities, we start from the trapezoidal shape introduced for a highway fundamental diagram (Daganzo, 1994). Eqn. 1 defines the flow of vehicles q as a function of the vehicle density k using the minimum operator where the arguments are characterized by the free flow speed u_o, the capacity of the street Q, and wave speed w and jam density κ.

$$q(k) = \min\left(u_o k; Q; w(\kappa - k)\right) \tag{1}$$

By using the smooth approximation of the minimum operator, this trapezoidal shape can be used to describe a city's MFD with a concave relationship (Ambühl et al., 2017). Here, in the 3D-MFD, the accumulation of cars N_{cars} and buses N_{bus} is related to the total flow of vehicles or passengers Q_{tot} (Geroliminis et al., 2014). To adopt the smooth approximation of Eqn. 1 for the 3D-MFD, we first define plane i in the N_{cars}, N_{bus} and Q_{tot} space as given in Eqn. 2 with a vector defining a point on the plane with elements $(car_{0,i} \quad bus_{0,i} \quad q_{0,i})$ and a normal vector with elements $(n_{car,i} \quad n_{bus,i} \quad n_{q,i})$

$$\left(\begin{pmatrix} N_{car} \\ N_{bus} \\ Q_{tot} \end{pmatrix} - \begin{pmatrix} car_{0,i} \\ bus_{0,i} \\ q_{0,i} \end{pmatrix}\right) \cdot \begin{pmatrix} n_{car,i} \\ n_{bus,i} \\ n_{q,i} \end{pmatrix} = 0 \tag{2}$$

After solving Eqn. 2 for Q_{tot} we obtain Eqn. 3, that describes Q_{tot} as a function of both modes vehicle accumulations.

$$Q_{tot}(N_{car}, N_{bus}) = q_{o,i} + \left(n_{car,i}\left(N_{car} - car_{0,i}\right) + n_{bus,i}\left(N_{bus} - bus_{0,i}\right)\right)/n_{q,i} \tag{3}$$

Similar to Eqn. 1 with linear functions, we define a set of planes J that provide a physical limit for the flow for a given combination of accumulations. The 3D-MFD with $Q_{tot}(N_{car}, N_{bus})$ is then defined as the minimum flow from each of the J planes resulting flows for a combination of N_{car} and N_{bus} as shown in Eqn. 4.

$$Q_{tot}(N_{car}, N_{bus}) = \min\left(Q_{1,tot}; Q_{2,tot}; \dots; Q_{J,tot}\right) \quad (4)$$

We then apply the smooth approximation of the minimum operator to Eqn. 4 to obtain a concave relationship as shown in Eqn. 5.

$$Q_{tot}(N_{car}, N_{bus}) = -\lambda \ln\left(\exp\left(-\frac{Q_{1,tot}}{\lambda}\right) + \exp\left(-\frac{Q_{2,tot}}{\lambda}\right) + \cdots + \exp\left(-\frac{Q_{J,tot}}{\lambda}\right)\right) \quad (5)$$

We define the basic set of planes consisting of $J = 5$ planes that define the physical boundaries resulting from the pure extent of the vehicles and simple truism in the 3D-MFD. The first two planes ensure that there are no negative accumulations of cars and buses. The next two planes describe that the joint jam accumulation of cars and buses does not exceed the physical limits of the network. The last basic plane limits the total flow to the capacity of the network. However, with more planes added to the system, the physical accuracy of describing the 3D-MFD with this functional form increases. Last, as the function allows for negative values of flow, the planes and parameters must be chosen to avoid the function to resolve in negative values.

3. Summary

This paper discusses the choice and calculation of parameters for the five basic planes as well as additional planes that describe traffic operations in mixed conditions. Moreover, we discuss how the design of the networks influence the planes and subsequently the 3D-MFD. This includes bus lanes, vehicle technology (bus and tram), the design of intersections (influence on capacity of public transport priority), stop locations and headways of the public transport vehicles.

Finally, we apply our proposed methodology to empirical data sets of car and public transport operations from London (UK) and Zurich (CH). The functional form for the 3D-MFD can be used in modeling of urban traffic systems and multi-modal control of traffic in urban areas. We can use this functional form to discuss how changes to the infrastructure affects the performance of the system and finally to quantify how far traffic operations in cities are beyond the optimal point.

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