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Information Sharing and Coordination in Supply Chains :

Game Theoretic Models under Uncertainty and Asymmetric Information

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Abstract

Supply chain members hardly ever make economic decisions under certainty. The key to improving decision-making lies in capturing more information on uncertainty and understanding the impact of such uncertainty on supply chain performance. Information sharing among supply chain members has gained significant attention as a means of better predicting different types of uncertainties. While each supply chain member holds distinctive sets of private information that both directly and indirectly influence the decision-making processes of others, revealing private information can be used opportunistically by the other supply chain members. Hence, knowing under which conditions to reveal one's information plays a pivotal role in keeping competitiveness in supply chains. Further, with the existence of demand uncertainty, whether some players can benefit from an increasing uncertainty under the Stackelberg game is not well understood.

This thesis initially investigates the implications of information asymmetry and uncertainty in supply chain decision-making processes. More specifically, the first two problems consider optimal information-sharing decisions between supply chain members under the existence of bilateral asymmetric information. In the last problem, we address the impact of demand information uncertainty on pricing decisions in the Stackelberg game under different power structures while assuming symmetric information. A central contribution to the existing literature that addresses *unilateral* information sharing is the focus on the implications of *bilateral* (mutual) information sharing: 1) capacity and demand information sharing between a supplier and a retailer and 2) market information exchange between an online platform and a seller. Further, beyond the two-player game, by analyzing channel efficiency under a three-player game, we present the impact of different decision sequences on supply network coordination.

Based on game theoretic frameworks, this thesis proposes both analytical models (Chapter 3 and 4) and a numerical model (Chapter 5) and provides managerial insights. Hence, this thesis gives guidance to managers in the era of the information explosion on how to exchange their private information and to understand the impact of demand uncertainty in decision-making.

The first problem investigates an information-sharing problem between a supplier and a retailer whose private information is capacity and demand, respectively. We analytically show that a supplier reveals a moderate capacity within a range of upper and lower thresholds, whereas a retailer only shares demand information that exceeds a certain threshold. Further, we investigate the impact of variability in prior beliefs on the ex-ante information exploration decision.

The second problem addresses another information exchange problem between a platform and a seller wherein both possess noisy signals (information) on market uncertainty. Our research outlines the circumstances in which it is beneficial for the platform to unilaterally disclose private market information to the seller. Additionally, we describe the situations in which the platform and the seller engage in a mutual exchange of their private information.

The third problem considers the optimal pricing decisions of a retailer and a manufacturer under demand uncertainty. We investigate the impact of power structures and markup schemes on channel efficiency and leader's advantages. For each player's equilibrium decision and expected profit, we examine the impact of demand uncertainty under different demand functions. Furthermore, we introduce a supply chain network where two manufacturers supply an identical product to a retailer.

Keywords: bilateral information sharing; Bayesian update; stochastic dominance; explorable uncertainty; risk-aversion; Stackelberg game; power structure; pricing decision

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Acronyms

ACI	Advance Capacity Information.
ADI	Advance Demand Information.
BNE	Bayes-Nash equilibrium.
CE	Channel Efficiency.
CV	Coefficient of Variation.
DM	Decision Maker.
Domi-manu	Dominant Manufacturer.
Domi-reta	Dominant Retailer.
DV	Decision Variable.
ELS	Economic Lot-Sizing.
EMC	Expected Margin Commitment.
FOC	First-Order Condition.
FSD	First-Order Stochastic Dominance.
IGFR	Increasing Generalized Failure Rate.
LA	Leader's Advantage.
LSR	Lost Sales Rate.
MILP	Mixed-Integer Linear Programming.

OEM	Original Equipment Manufacture.
S-Domi-manu	Dominant Global Manufacturer with Local Manufacturer.
S-Domi-reta	Dominant Retailer with Local Manufacturer.
SCM	Supply Chain Management.
SOC	Second-Order Condition.
SSD	Second-Order Stochastic Dominance.

Chapter 1.
Introduction

1.1 Motivation

Nowadays, companies have unprecedented access to vast amounts of data and can efficiently share precise information with their partners within the supply chain. As both competition and collaboration are intensified within horizontal and vertical supply chains, the decision regarding information sharing has grown increasingly complex (Shen et al. 2019a). Furthermore, supply chain members often possess different sets of private information such as demand, costs, product quality, capacity, and inventory. While such private information is seldom available to other members of the supply chain, it exerts a direct or indirect influence on the performance of each member (Kostamis and Duenyas 2011). Within a decentralized supply chain, the presence of asymmetric information can result in significant efficiency losses. Consequently, the effective utilization of available information has become an indispensable factor for companies striving to maintain their competitiveness (Eymann 2016).

While empirical evidence underscores the potential benefits of information sharing as a means for coordinating supply chains, strategic information sharing remains a challenging decision for supply chain members (Chen 2003). Despite the well-known advantages of information sharing, such as enhanced visibility, reduced bullwhip effect, and improved resource allocation (Lee et al. 1997), active information exchange among the industry partners is hard to achieve in practice. Several primary obstacles hinder this process: 1) the possibility of opportunistic behavior by their partners, 2) the risk of information leakage to competitors, and 3) the erosion of their bargaining power (Lotfi et al. 2013). Given the inherent trade-off between the efficiency gained from information transparency as a whole supply chain and the loss from the opportunistic behavior of partners, firms often exhibit reluctance when it comes to disclosing their private information (Li 2002).

Due to its significant but challenging nature, information sharing problems have attained considerable attention in the supply chain management (SCM) area Chen (2003), Shen et al. (2019a), Vosooghidizaji et al. (2020). The prevailing body of literature in this field investigates how to make the party that possesses the private information reveal it through contractual mechanisms such as a menu of contracts and side payments. Nonetheless, only a limited number of studies propose strategies for information exchange when other supply chain members also opt to share or withhold their private information. Consequently, there is little understanding of bilateral (multilateral) information sharing in SCM. A substantial portion of the problems is typically modeled under a Stackelberg game framework, in which a single leader decides on either optimal price or quantity in the context of unilateral information asymmetry (Kostamis and Duenyas 2011). However, it remains unclear whether the current mechanisms can be effective when confronted with bilateral asymmetric information.

This dissertation mainly addresses such bilateral information asymmetry problems and sharing decisions in supply chain management under different contexts. We aim to develop a decision support model and provide insights into strategic information sharing in supply networks. In the first project, we develop an information-sharing problem between a retailer with private demand information and a supplier with private capacity information in a supply chain under bilateral asymmetric information. We incorporate Bayesian learning-based information updates. Here, we assume the private demand and capacity information is deterministic (perfect information). In the second project, we switch our focus to a platform business. With growing attention to platforms' common marketplaces in recent years, we investigate mutual demand information exchange problems between a platform and an individual seller. In this project, market demand is uncertain, and the private demand signals possessed by the platform and the seller are noisy (imperfect information).

Even though seamless information sharing can be made among the supply chain members, information sharing becomes more valuable under uncertain environments (i.e., random demands and yield rates). Hence, understanding the impact of demand uncertainty on different players who set wholesale or selling prices within a supply chain (or network) is of relevance. In particular, it is unclear whether a certain player under a sequential game can actually benefit from increasing demand uncertainty under different power structures, demand functions, and markup schemes. As supply chain members often need to make economic decisions under uncertain demand information (Lee et al. 1997), our third project specifically concentrates on how demand uncertainty affects supply chain performance. In this study, we examine a scenario in which demand is subject to randomness, but both a retailer and manufacturers possess symmetric information. Our investigation delves into the effects of demand uncertainty and the sequence of the game on pricing decisions, expected profits, channel efficiency, and leader's advantages in the supply chain.

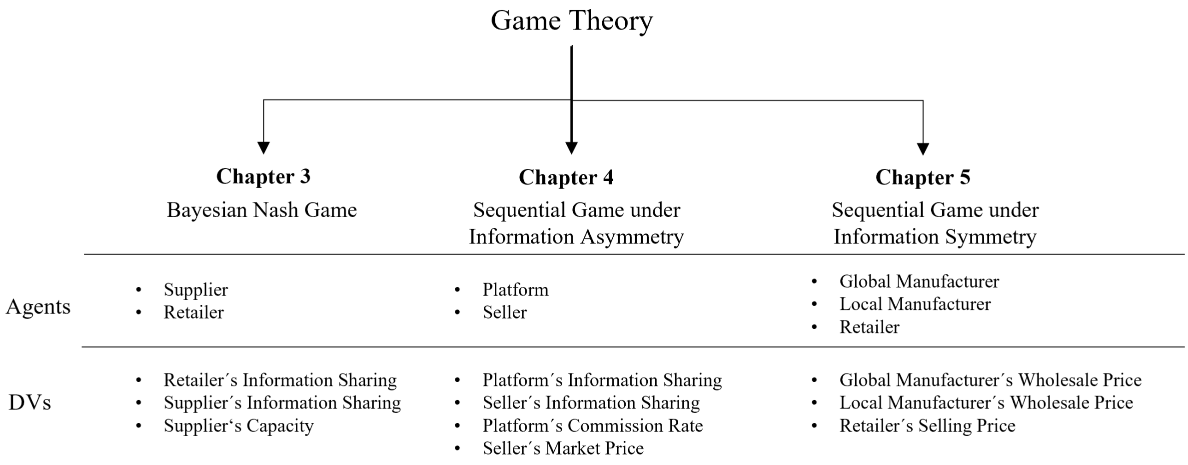


Figure 1.1: Structure of Dissertation

1.2 Contribution and research questions

In spite of the considerable amount of research conducted on the topic of information sharing within the realm of supply chain management, it has primarily taken on a one-way direction among members of the supply chain. Furthermore, the scope of shared information typically revolves around demand information (Zhang and Chen 2013). Built upon the prior studies concerning information sharing (Ha and Tang 2017), we have developed game theoretic models that address situations characterized by bilateral asymmetric information. These models aim to explore the optimal decisions regarding information sharing among participants within a supply chain context. To be more specific, we aim to address the following research questions (RQs):

RQ 1.1) If a self-interested supplier and a retailer decide whether to reveal their capacity and demand information, respectively, what is an optimal revelation decision knowing that the other also decides whether to share or not?

RQ 1.2) How does a player's risk aversion impact the information sharing decisions of supply chain members?

While there are some existing studies that touch upon the topic of bilateral or multilateral information sharing, these analyses predominantly focus on information exchange within horizontal supply chain members (Hyndman et al. 2013). Our model, in contrast, diverges from these previous models in one significant aspect: we explore vertical supply chain information sharing, where both the retailer and supplier can mutually share distinct sets of their private information, such as demand and capacity information. Also, the decision to share information while accounting for risk aversion holds practical significance, especially in light of the reluctance exhibited by companies to share information due to uncertain outcomes and the potential for opportunistic behavior by informed companies (Chen 2003).

Furthermore, focusing on a platform's growing common marketplace, the existing literature extensively examines the advantages of online platforms engaging in unilateral information sharing. In this aspect, our analysis takes it a step further by considering both players' decisions regarding mutual information exchange. Moreover, much of the literature on platform retailing focuses on studying how the platform's commission rate decision influences the choice of channel formats, such as wholesale and agency models. However, there is limited understanding of how the seller's information sharing impacts the platform's commission rate. This aspect holds significant practical importance given the recent trend among large platform companies to vary their commission rates in market-specific ways (i.e., based on product categories and regions). For this reason, the market signals from individual sellers could be invaluable in determining the optimal commission rates for the platform. Therefore, we investigate the conditions under which both the platform and the seller benefit from exchanging each other's demand information. To do so, we construct the following research questions:

RQ 2.1) When the commission rate is exogenously fixed, would sharing the platform's private signal with the seller unilaterally be mutually beneficial?

RQ 2.2) When the commission rate is endogenously determined, would exchanging private signals bilaterally between the platform and the seller be mutually beneficial?

Even when there is no information asymmetry among supply chain members, another common challenge in decision-making stems from uncertainty regarding market demand. While the impact of demand uncertainty on a single decision maker has been widely studied, how each supply chain member's profit is affected by demand uncertainty under various decision sequences (power structure) and markup schemes remains unclear. Further, although the impact of power structure on channel efficiency has been widely investigated, most of the focus has been given to a single retailer and a manufacturer supply chain. In this study, we also consider a local manufacturer who fulfills the shortage of a single retailer while competing with a global manufacturer.

As a result, we aim to identify the impact of the power structure and markup scheme on channel efficiency and the influence of demand uncertainty on each player's profit under Stackelberg games. In this context, we pose the research questions below:

RQ 3.1) Does a sequence of the game (power structure) lead to different channel efficiencies and leader's advantages under stochastic price-dependent demand functions?

RQ 3.2) Does a high demand uncertainty always reduce the players' expected profits in a supply chain or supply network?

1.3 Outline

Chapter 2 reviews related literature, stating prior contributions on information sharing in supply chain management, pricing decisions under competition, and the impact of power structure in supply chains. The following three main chapters of this thesis are based on three working papers.

In Chapter 3, we investigate a bilateral information sharing problem between a supplier endowed with private capacity information and a retailer possessing private demand information. We derive the conditions for the retailer to share demand information and the supplier to reveal capacity information under the existence of information sharing costs and bilateral information asymmetry. We incorporate Bayesian updating from the signals of players' remaining silent. Further, we investigate the joint information sharing rule and ex-ante information exploration decisions under demand and capacity uncertainty. Finally, the impact of risk aversion on the players' information sharing decisions is explored. Chapter 3 is based on a working paper, Lee and Minner (2022).

In Chapter 4, we examine information exchange decisions between an online platform and a seller who uses the platform's common marketplace. We construct an analytical model under a game theoretic framework and consider a situation in which both players possess noisy signals that capture market uncertainty. We highlight certain conditions that can benefit both players by mutually exchanging private demand information voluntarily and compare how the benefit of unilateral sharing from the platform to the seller differs from the bilateral information exchange. This chapter is available as a working paper under Lee et al. (2023), co-authored by Christopher S. Tang (University of California, Los Angeles) and Stefan Minner (Technical University of Munich).

With symmetric information, we address the Stackelberg pricing games under different power structures in Chapter 5. We apply different leader setups and markup schemes to investigate how varying power structure in the game affects the channel efficiency and leader's advantages. We extend the analysis to supply network sequential games. A global manufacturer fulfills a retailer's regular order, while a local manufacturer fulfills the retailer's shortages. We conduct numerical experiments to observe the impact of power structure, demand functions, and markup schemes on the players' profits. Chapter 5 is accessible as a working paper, Lee and Minner (2023).

Chapter 6 summarizes contributions and insights. It also discusses the limitations of the presented work and provides opportunities for further research.

Chapter 2.
Related literature

This chapter reviews literature about information sharing problems in the context of supply chain management (Section 2.1), retailing pricing/commission decisions (Section 2.2), as well as supply chain power structure (Section 2.3).

2.1 Information sharing in supply chain management

Information sharing in supply chain management has been extensively studied. Chen (2003), Ha and Tang (2017), Shen et al. (2019a) give general reviews. This section focuses on the literature on asymmetric information and information sharing, advance information sharing, and explorable uncertain information.

2.1.1 Asymmetric information in supply chains

Asymmetric demand information: Demand is the most analyzed type of asymmetric information based on signaling or screening games to design contracts while ensuring credible information sharing. Among many, for signaling, Cachon and Lariviere (2001) propose two capacity compliance contracts: 1) forced compliance and 2) voluntary compliance to overcome the credibility issue of demand information sharing. For a screening game, Akan et al. (2011) apply a menu of contracts to analyze a service provider who invests in capacity before the demand is realized and an outsourcing company with private information on the customer arrival rate. They prove that using a two-part tariff allows the service provider to observe truthful customer demand information from the outsourcing company.

Özer and Wei (2006) present both signaling (advance purchase) and screening (capacity reservation) mechanisms. While a manufacturer possesses precise demand forecast information, a supplier sets capacity based on limited demand information. They demonstrate that the degree of information asymmetry and the profit margin of each player are critical factors influencing the efficiency of the adopted contract. Xiao and Xiao (2020) investigate optimal compensation schemes of a firm that sells a product to a sales agent with private demand information. They propose a menu of linear compensation schemes based on forecast accuracy and incorporate supply and demand mismatch costs.

Gal-Or et al. (2008) study demand information sharing in a vertical supply chain. They examine different characteristics of demand information possessed by a manufacturer and two competing retailers. While the retailers have more accurate sales data, the manufacturer has a better overview of market trends, such as demand correlations. They demonstrate that as the retailers can infer the manufacturer's demand information through his wholesale price, without information sharing, the manufacturer sets a lower price, similar to the result from Li and Zhang (2008). They find that although information sharing benefits the manufacturer, he shares only with the retailer endowed with a noisier signal if sharing information incurs costs.

Even though the theoretical predictions demonstrate that a simple wholesale price may underperform compared to the elaborate contract mechanisms, they find that the complexity of implementing those contracts leads to lower efficiency compared to theoretical expectations. For example, Kalkanci et al. (2011), Schiffels and Voigt (2021) compare the efficiency of complex contract mechanisms such as a quantity discount contract or a nonlinear capacity reservation contract to a wholesale price contract in lab experiments under asymmetric demand information. Chu et al. (2017) propose a contract where the manufacturer makes both capacity and wholesale price decisions while the retailer shares private demand information via cheap talk. They find that misrepresenting the retailer's demand as higher than the actual value leads to both higher capacity and wholesale price, reducing the retailer's incentive to misrepresent information.

Without involving any financial mechanisms, Chu and Lee (2006) examine when the retailer benefits from sharing demand information using a Bayesian game. They find that the retailer shares demand information when the observed demand is high, or the information-sharing cost is low. In particular, the retailer's benefit from disclosing demand information is a possible increase in profit by letting the supplier set a better capacity due to shared information. However, such a benefit needs to justify the cost of sharing. Although a few papers are concerned with bi/multilateral information sharing, the analysis is limited to sharing among horizontal supply chain members (Hyndman et al. 2013) and demand information sharing in a vertical supply chain (Zhang and Chen 2013).

In the context of the platform's information sharing, Zhang and Zhang (2020) investigate an e-tailer's incentive to share demand information with suppliers who can use such information to expand their channel to offline. They show that if the e-tailer is a selling agency, he shares the information with the supplier if the offline setup cost is relatively low or high. On the other hand, under high accuracy of demand information, the e-tailer remains silent to avoid channel competition. Tsunoda and Zennyo (2021) explore an optimal demand information sharing policy for a platform where the platform competes with an offline retailer selling the same product from a common supplier. While the supplier sets a wholesale price, the platform charges a commission rate. They examine the impact of the platform's information sharing on the supplier's optimal channel decision and show that sharing enables Pareto improvement among the supplier, retailer, and platform.

Li et al. (2021) investigate a platform's optimal demand information sharing decision where he can share either with a manufacturer, a reseller, or both. While the reseller orders from the manufacturer under a wholesale price-only contract, they both sell the products via a common platform. They show that if the demand is highly uncertain and the competition is intense in the market, the platform shares only with the manufacturer. This is because the double marginalization can be mitigated by a signaling effect that makes the reseller infer the information from the manufacturer's pricing decisions and eventually reduces her price. Liu et al. (2021) investigates a platform's optimal sharing decision with multiple sellers competing under Cournot competition. Their results show that sharing demand information is always an equilibrium. Whereas the platform will share truthfully if he could select a subgroup of sellers to share its information, he will share noisy information with high uncertainty if he has to share the information with all sellers.

Zha et al. (2022) study an online platform's incentive to share demand information with a manufacturer and a retailer. While considering two roles for the platform where it acts as a reseller or an agency, they focus on the questions of 1) with whom to share and 2) when to share. They demonstrate that the platform's optimal sharing policy is not influenced by the accuracy of its private information. Further, they show that in equilibrium, the platform always has the incentive to share its demand information at least with one player. Similarly, Zhong et al. (2023) consider a platform that decides whether to share demand information with a manufacturer and a retailer (seller). They compare optimal sharing decisions under two settings: 1) the platform with encroachment and 2) the platform without encroachment. They demonstrate that the platform, as a reseller, always has an incentive to share its information with the seller but not with the manufacturer.

Liu et al. (2023) focus on a supply chain composed of a platform, two competing original equipment manufacturers (OEMs), and a common supplier. They consider different power structures in the game where the leader is either the OEMs or the supplier and investigate the platform's demand information sharing decision. When the common supplier is a leader, the platform benefits the most from sharing demand information with the common supplier only. However, when the OEMs are the leader, the platform information sharing depends on the competition intensity of the two OEMs. Especially when the competition intensity is low, he shares only with the OEMs, while under high competition, sharing with all (i.e., the supplier and OEMs) is an equilibrium decision.

Asymmetric cost information: Quantity discounts and a menu of contracts are the most frequently applied mechanisms to achieve supply chain coordination under asymmetric cost information. Corbett and De Groote (2000) present an optimal quantity discount for an economic lot-sizing (ELS) problem where the buyer's holding cost is unknown to the supplier. Corbett (2001) develops a menu of contracts under a (Q, r) policy with consignment stock, considering two settings where 1) a supplier has a private setup cost information, and 2) a retailer has private backorder penalty cost information. He demonstrates that under asymmetric information, the player cannot induce jointly optimal behavior, and the supplier sets a lower price than with full information. Cakanyıldırım et al. (2012) study a retailer who offers a menu of contracts consisting of the order amount and the revenue share to the supplier whose production cost is private information. The optimal menu of contracts reduces information rent of the supplier as long as the supplier's cost is not significantly under or overestimated by the retailer.

Kayış et al. (2013) consider a manufacturer and a supplier with private production cost information. The manufacturer decides whether to delegate the procurement tasks to a supplier. Comparing the quantity discount contract to the price-only contract, the manufacturer benefits from delegation if he believes the supplier has a high cost or if the manufacturer has a very high level of uncertainty about the suppliers' costs. Kim and Netessine (2013) investigate a manufacturer and a supplier with private cost information. They compare an expected margin commitment (EMC) and a menu of contracts. If collaboration brings a significant reduction in production cost and demand variability, the EMC is preferred to the menu of contracts. Davis et al. (2022) consider an OEM and two suppliers whose costs are private information. They compare two power structures where 1) the OEM has similar bargaining power to the suppliers and 2) the OEM has more substantial bargaining power. By conducting a human-subject experiment with two-part tariff contracts, they demonstrate that most of the experimental results support the normative results from their analytical model.

Even though asymmetric information and information sharing issues are addressed extensively in operations research, economics, and principal-agent theory, most of the literature focuses on mechanism designs where one supply chain member holds private information, and the other offers a contract that induces truth-telling by aligning the incentives. Among many, Cachon and Lariviere (2001) consider the case where a manufacturer shares demand information and a supplier constructs the capacity accordingly. They develop two capacity compliance regimes contract schemes that realize truthful demand forecast sharing. Özer and Wei (2006) develop both signaling (advance purchase) and screening (capacity reservation) mechanisms for truthful demand sharing. Shamir (2013) extend a single-period incentive alignment problem to multiple periods. They prove that the supplier's ability to observe demand realization enables them to offer a simple price contract and worsens the manufacturer's profit. Chu et al. (2017) propose a contract where the manufacturer makes both capacity and wholesale price decisions while the retailer shares private demand information via cheap talk. They find that misrepresenting the retailer's demand higher than the actual value leads to a higher capacity and a higher wholesale price, reducing the retailer's incentive to misrepresent the information.

2.1.2 Advance information sharing

Sharing information in advance can facilitate planning. Gallego and Özer (2001) study an optimal inventory control policy under advance demand information (ADI). They show that by using ADI under (s, S) and base-stock policies, both inventory level and costs can be reduced. Li and Zhang (2013) investigate the impact of ADI on a retailer's pricing decision. They find that while ADI increases product availability, it deters the retailer from adopting discriminatory pricing strategies for two selling seasons (preorder and normal seasons) due to strategic consumer behaviors. Jakšič et al. (2011) examine the value of advance capacity information (ACI) and show that the ACI can significantly reduce the expected cost if a supply chain is flexible enough to react to demand and capacity mismatches.

2.1.3 Explorable uncertain information

As demand uncertainty causes major challenges in decision-making, information sharing has been referred to as an effective way to overcome the unpredictable impact of decisions up to a certain extent (Megow and Schlöter 2021). In some cases, a player with an informative market signal shares its information based on financial incentives such as a two-part tariff (Zha et al. 2022). In other cases, if gathering information on demand uncertainty is a common interest of supply chain members, and the benefit of such sharing can be quantified among supply chain members Lee et al. (2000), voluntary sharing may occur. While explorable uncertainty in the context of logistics and scheduling problems (Bruce et al. 2005, Sanchez-Rodrigues et al. 2010, Simangunsong et al. 2012) are proliferating, how information-sharing decision is affected by one's explorable demand uncertainty with different private signal precisions is not fully understood.

2.1.4 Bilateral/multilateral information sharing

The literature on bilateral information sharing is relatively scarce. While information sharing itself is a rising topic in the SCM area, only a few offer information exchange strategies for scenarios where others also share/ do not share (Lee and Minner 2021). Wang et al. (2008) investigate the ramifications of sharing a retailer's private information with a supplier with prior knowledge from a subjective distribution. They elaborate on the advantages and disadvantages that the retailer faces under different contract schemes by letting the supplier improve their knowledge. However, they assume that the private information is held by the retailer only; hence, the information exchange between the supply chain members is not considered.

Zhang and Chen (2013) consider both a retailer and a supplier that possess partial demand information. Under a wholesale price contract, they find that sharing occurs based on the variance and the correlation of demand information between the companies and the other company's sharing behavior. Besides, they suggest that a revenue-sharing contract can coordinate the system while ensuring information sharing from both firms. Hyndman et al. (2013) examine the role of demand information on the capacity allocation decision when two suppliers can exchange demand information. They conduct lab experiments to compare the theoretical results to the behavioral results. They demonstrate that preplay communication (cheap-talk) results in almost the same efficiency as truthful information sharing.

Thus far, unilateral asymmetric information problems have been well investigated. However, bilateral or multilateral information sharing issues regarding how to exchange information reciprocally have not been covered, as pointed out by Vosooghidizaji et al. (2020). While information sharing is a mature topic in the SCM area, only a few offer information exchange strategies for scenarios where all involved players can decide to either share or not share. Wang et al. (2008) investigate the ramifications of sharing a retailer's private demand information with a supplier who has prior knowledge from a subjective distribution. They elaborate on the advantages and disadvantages of the retailer under different contract schemes by letting the supplier improve his knowledge. However, they assume that the private information is held by the retailer only; hence, the information exchange between the supply chain members is not considered.

Further, as most of the literature concerns unilateral asymmetric information, the supply chain configuration is often monopolistic where there is a supplier and a retailer. A few noteworthy information sharing literature under non-monopolistic supply chain configurations are Cachon and Lariviere (1999), Guo et al. (2014), Araneda-Fuentes et al. (2015), Belloni et al. (2017). Cachon and Lariviere (1999) consider a single supplier and multiple retailers who compete for scarce supplier capacity. The private information in their model is the private inventory level of the retailers. They prove that a truth-telling mechanism is not universally desirable when a single supplier deals with several retailers, as truth-telling can reduce the supplier's and the retailers' profits under limited capacity.

Guo et al. (2014) investigate sharing behaviors of retailers who possess private demand information. They include two competing channels and analyze the impact of competition on the sharing strategy. They propose an information-sharing framework that suggests when to share the information with their upstream manufacturers to lower wholesale prices. However, they only consider unilateral private information. Araneda-Fuentes et al. (2015) propose a capacity reservation contract with discount and penalty conditions between a retailer and two manufacturers. They investigate under which circumstances the manufacturers achieve capacity coordination, leading to a maximum joint expected profit.

2.2 Retail pricing and markup decision

2.2.1 Price-setting Newsvendor

The price-setting newsvendor attained considerable attention in operations research (see Simon et al. 1989 and DeYong 2020). Petruzzi and Dada (1999) establish the optimal pricing solution by assuming an increasing hazard rate and introducing a stocking factor. They provide analytical properties of optimal prices by separating riskless profit from uncertainty-relevant expected profit under stochastic demand. Kocabıyıkoğlu and Popescu (2011) introduce a novel concept called the elasticity of lost sales rate (LSR). This new concept enables them to deliver structural properties of price and quantity decisions under stochastic demand.

Jadidi et al. (2016) consider a newsvendor retailer with an option to decrease the selling price in the middle of the product lifecycle to prevent the demand from decreasing sharply. They find that the price adjustment benefits the retailer in general; however, the manufacturer prefers the buy-back contract over the retailer-driven two-price policy. Schulte and Sachs (2020) study the price-setting newsvendor by assuming that stochastic demand follows a discrete probability distribution (e.g., Poisson demand). They show that neglecting the discrete nature of demand in the pricing decision leads to a significant profit loss, and such a negative impact is exaggerated when the demand rate is small.

Markup price settings: Each player's profit varies because of the power structure or demand function of the game and the price setting of downstream players. There are two ways that a retailer typically sets her price: absolute and percentage markup (Arcelus and Srinivasan 1987). Based on these two schemes, von Ungern-Sternberg et al. (1994) figure out that percentage markup leads to higher downstream profits. However, their analysis is limited to a deterministic iso-elastic demand function. One interesting observation in their work is if the retailer uses a combined price scheme, both absolute and percentage markups, then the manufacturer's price strictly increases in the absolute markup and decreases in the percentage markup. Therefore, they demonstrate that if the retailer is a leader of the game, she may prefer to apply only percentage markup because it is rational to set absolute markup to zero and percentage markup as high as possible.

While markup schemes are decided by retailers based on wholesale prices they receive from suppliers, little justification is provided for the selection of a specific markup scheme (i.e., percentage or absolute) in the literature. Wang et al. (2013) state that absolute markup is widely used in the agricultural industry or luxurious products such as jewelry, while in consumer retailing, percentage markup is common practice. Wang et al. (2016) extend their previous work by including a competition framework where two substitutable retailers are the leaders of the game, having dedicated suppliers as the followers. They claim that the percentage markup under competition leads to a prisoner's dilemma, which contradicts the conventional belief that the percentage markup benefits the retailer under the Domi-reta game.

Canyakmaz et al. (2022) consider a percentage markup for a retailer who encounters stochastic price volatility under a Poisson process. They demonstrate that as inventory increases, the optimal markup decreases, whilst the optimal base stock level decreases as markup increases. Wang et al. (2023) focus on two retailers' optimal markup choice decisions (absolute or percentage) and a supplier's wholesale price decision between retailer-specific or uniform wholesale prices under a Domi-reta game. They demonstrate that while both retailers always prefer percentage markup over absolute markup, the supplier opts for a uniform wholesale price as the uniform pricing can mitigate the market power of two retailers, especially if the competition between the retailers is high.

Stochastic price-dependent demand functions: Most of the literature concerned with stochastic contract design approaches the problems numerically because of the inherent complexity of stochastic models and tractability. However, some papers interpret the system behavior of the supply chain under stochastic demand with an analytical solution (Cachon 2003). Young (1978) mentions necessary restrictions to obtain an optimal solution under stochastic demand and systematically compares the optimal price under the deterministic demand and stochastic demand. Lau and Lau (2003) do not incorporate stochastic demand. Nevertheless, this paper contributes to the field of research regarding price-sensitive demand functions. They investigate the most often applied price-dependent demand functions, such as linear and iso-elastic, and summarize the optimal solutions under each demand function. Chiu et al. (2011) also analyze the solution properties under the additive and multiplicative price-dependent demand functions and argue that a manufacturer can achieve channel coordination under a dominant manufacturer (Domi-manu) game by employing channel rebate and return policy.

2.2.2 Omnichannel retailer pricing decision

Caro et al. (2020), Cai and Lo (2020), Hübner et al. (2022) give a comprehensive overview of the recent studies on omnichannel retailing models. Gu and Tayi (2017) present an analytical model to investigate the benefit of a retailer's usage of an offline channel as a showroom while extending assortment options only via an online channel. The retailer sets optimal prices for two products it offers. They demonstrate that having exclusive access via the online channel yields a higher profit than offering both products in the omnichannel. While this study provides a product offering strategy for a retailer who operates both off-/online channels, the substitutability between the products is not considered.

Unlike Gu and Tayi (2017), who adapt an online channel as a means of extending assortment, Shi et al. (2018) focus on the purchasing strategy of buy-online-and-pick-up-in-store via omnichannel. A retailer sets a pre-order discount rate through an online channel under a finite horizon. Further, the retailer uses the pre-order information in making inventory decisions for its offline channel, while the substitution between the channels is neglected. They show the retailer's benefit of offering a pre-ordering discount depends on demand uncertainty and production costs. Similarly, Zhang et al. (2018) optimize a retailer's price and inventory for its omnichannel. They consider a single product and present the value of the omnichannel strategy compared to the online channel-only strategy. They argue that the profit from the omnichannel strategy is not always greater than the online channel strategy as the optimal price decreases as they operate both channels. Ishfaq and Bajwa (2019) propose mixed-integer linear programming (MILP) to maximize the profit of online channels while deciding on the price of online channel products. They investigate the profitability of different fulfillment options for the sales incurred via online channels. Using the outer approximation approach, they conclude that store-based fulfillment leads to worse performance than fulfillment from the distribution centers.

2.2.3 Platform retailing and commission rate decision

Conventionally, there are two formats of a platform as an online marketplace: 1) agency model and 2) reselling model. In the first format, the platform only offers a common marketplace as an e-tailer and charges a commission rate to manufacturers (sellers), using a revenue-sharing scheme. In the reselling model, the platform buys products from the manufacturers directly and charges selling prices to its customers. Abhishek et al. (2016) show that if the platform is an agent, the selling prices of manufacturers are lower than when the platform is a reseller. Further, they demonstrate that if offering the platform's online channel reduces the manufacturers' direct channel demands, the platform prefers to be an agency. While platforms often charge proportional commission rates to the sellers' profit in the agency model, Jia (2016) incorporates auction theory to decide an optimal service fee for the platform. He introduces two types of payments: 1) a final value fee related to the final trading price between the seller and the buyer and 2) a reserve fee based on the seller's reserve price. They argue that using either of the fee policies can obtain an optimality condition while, in general, the fee policy shows nonlinearity in the seller's price.

Wang et al. (2019b) propose a contract mechanism, 'cost-sharing joint commission', to overcome efficiency loss from a decentralized supply chain between a manufacturer and a platform. While the platform is the leader and decides a commission rate to charge to the manufacturer first, they investigate how adapting fairness concerns (i.e., the fairness of the income distribution between a giant e-commerce platform and a manufacturer) affects the manufacturer's equilibrium selling price and the platform's commission rate decisions. Zennyo (2020) considers two competing suppliers with different market sizes and a monopolistic platform. Both suppliers can opt for either wholesale price or agency contracts when using a platform's common marketplace, while the platform decides on an optimal commission rate. He demonstrates that when the substitutability is low, the platform sets a lower commission rate to induce the suppliers to prefer the agency contract over the wholesale price contract.

Hasiloglu and Kaya (2021) consider two sellers using a common online platform. While the online platform charges a commission rate, and the sellers decide on both service level and selling prices, they state that when competition is high between the sellers, the platform's commission rate increases. Tsunoda and Zennyo (2021) examine the impact of the platform's information-sharing decision on a supplier's channel choice via the platform (i.e., wholesale and agency models). Similar to Hasiloglu and Kaya (2021), they show that the platform sets a lower commission rate so that the supplier opts for the agency over the wholesale model. Further, sharing information induces the supplier to choose the agency model. Whilst previous works focus on the supplier's choice between either the wholesale or the agency models, Ha et al. (2022b) present conditions under which a manufacturer can also operate with the platform's dual channel (i.e., agency and reselling channels). They show that introducing the agency model reduces the wholesale price, and the manufacturer's operational flexibility under dual channels makes the platform increase the service effort.

2.3 Power structure in supply chains

2.3.1 Power dominance under sequential games

Under a sequential game, the leader who can decide first is considered to possess more power in the supply chain (Moorthy 1988). Lee and Staelin (1997) consider dominant retailer power. They study the relationship between a manufacturer's and retailer's equilibrium price decisions and introduce the concept of vertical strategic interaction under different price-sensitive demand models.

Dukes et al. (2006) mention that channel dominance should be precisely examined considering the growing power of retailers such as Amazon, and a Domi-reta contract design game should be more widely investigated. Lau and Lau (2005) study the system behavior in the Domi-reta game with stochastic price-dependent demand functions in combination with asymmetric demand information. They show that when demand uncertainty is high, the manufacturer, being a leader, has a higher channel efficiency by charging his wholesale price than enforcing a close-to-retailer price. Similarly, Raju and Zhang (2005) study the Domi-reta game and suggest two contract design mechanisms that can coordinate channel inefficiency: quantity discount and two-part tariffs.

Shi et al. (2013) consider a retailer and a manufacturer and the impact of different power dominance on the players. They demonstrate that a retailer being a leader under a linear demand brings higher channel efficiency, while under an iso-elastic demand, the manufacturer as a leader results in higher channel efficiency. They also show that lower demand uncertainty increases the manufacturer's profit while the retailer benefits only when demand follows an iso-elastic function. Luo et al. (2017) study a retailer and two manufacturers offering differentiated products under horizontal and vertical competition. They show that no dominance among the players yields the highest channel efficiency, while the manufacturer who announces a wholesale price first makes a lower profit as the competing manufacturer learns from the pricing decision and takes over bargaining power.

Zhao et al. (2022) study two competing supply chains where each supply chain consists of a risk-neutral manufacturer and a risk-averse retailer. They consider the Domi-manu game and investigate under which conditions the players prefer a revenue-sharing contract over a wholesale price-only contract. They show if the demand uncertainty is high and the price competition is moderate, the wholesale price contract is preferred by the manufacturers. Wang et al. (2023) focus on two retailers' optimal markup choice decisions (absolute or percentage markups) and a supplier's wholesale price decision between retailer-specific or uniform wholesale prices under a Domi-reta game. They demonstrate that while both retailers always prefer percentage markup over absolute markup, the supplier opts for a uniform wholesale price as the uniform pricing can mitigate the market power of two retailers, especially if the competition between the retailers is high.

2.3.2 Selling to the Newsvendor with upstream competition

Lariviere (2006) study a decentralized supply chain where a supplier decides on a wholesale price, and then a Newsvendor retailer sets an optimal quantity. With the condition of increasing IGFR, they use the concept of price elasticity to derive an optimal wholesale price decision of the manufacturer concerning the retailer's order quantity. McGuire and Staelin (1983) explore a supply network involving two competing manufacturers, each faced with the choice of distributing their products independently or through dedicated retailers. They illustrate that in cases where the substitutability between the products of these two manufacturers is high, the manufacturers prefer to distribute their products through decentralized retailers. Conversely, when product substitutability is low, the manufacturers opt to directly offer their products to customers, bypassing the need for exclusive retail intermediaries.

Choi (1991) considers two competing manufacturers and a retailer for both Domi-manu and Domi-reta games. He presents the equilibrium price of each player and explores linear and nonlinear deterministic demands. Without a strictly dominating power of one player, all supply chain members can benefit from higher profits. Also, he argues that when the manufacturers' products are easily substitutable, having a common retailer reduces their profits. Li et al. (2010) consider a retailer, two competing manufacturers with unreliable supplies, and a spot market manufacturer who is perfectly reliable. They investigate the retailer's optimal sourcing strategy while the manufacturers set their prices. By assuming uniformly distributed demand, they find the optimal order quantities from different suppliers.

Huang and Xu (2015) considers a retailer with a dual-sourcing option and a spot market that can be used as a backup production for any shortage. Using a two-stage dynamic programming model, the retailer decides whether to take a single or dual-sourcing strategy in the first stage. In the second stage, the retailer sets the emergency order quantity from the backup manufacturer. They demonstrate that when two manufacturers' reliability is similar and the backup production price is low, the retailer uses both dual-sourcing and backup production.

Chapter 3.

Capacity and demand information sharing under bilateral asymmetric information in supply chains

3.1 Introduction

In industries such as automotive and semiconductor, demand and capacity information are commonly exchanged among companies to establish a single point of truth. While a substantial amount of research has been conducted on unilateral information sharing, how companies make bilateral sharing decisions is not well understood. This study focuses on a newsvendor-type supply chain, where a supplier possesses private capacity information and a retailer holds private demand information. We analyze voluntary information sharing rules under two settings: 1) decentralized information sharing, and 2) joint sharing decisions under locally known demand and capacity information. Furthermore, we explore the ex-ante information exploration decisions of a single decision-maker. Finally, we investigate sharing decisions under risk aversion.

Our analysis yields three main insights. First, when demand and capacity information is shared voluntarily, there are significant differences in information sharing rules between the retailer and the supplier depending on whether they make sharing decisions jointly or individually. Under the joint sharing decision, both the retailer and the supplier disclose either relatively high or low demand and capacity information, respectively. However, if sharing is decentralized, a supplier reveals a moderate capacity within a range of upper and lower thresholds, whereas a retailer only shares demand information that exceeds a certain threshold. Second, when making an ex-ante information exploration decision, high variability in the prior demand distribution makes a decision-maker more likely to explore both demand and capacity information. Lastly, if any player in a supply chain is risk-averse, the retailer's demand information is more likely to be revealed to the supplier. However, the retailer's risk aversion makes the supplier less likely to share capacity information, as the supplier anticipates that the retailer is more likely to share demand information unilaterally.

3.1.1 Problem setting

Companies are often reluctant to reveal their information because of information leakage, opportunistic behavior of competitors, and the loss of bargaining power (Gümüş 2017). However, if disclosing private information can prevent an informed party from making poor planning decisions that could consecutively harm one's profit, such disclosure may occur (Shang et al. 2016). For instance, an aerospace manufacturer, Boeing, informed their suppliers that they forecast a demand increase in their aircraft valued at $6.8 trillion, expecting a high increase in suppliers' delivery (Reuters 2019). Further, in the semiconductor industry, where the production time is relatively long, manufacturers regularly exchange capacity information with their buyers to mitigate potential shortages (Jakšič et al. 2011). As these examples show, companies may be willing to disclose their private demand and capacity information in their own interest voluntarily if doing so is more beneficial than keeping the uncertainty by remaining silent. Further, even though it is known that information sharing can be done efficiently by using financial mechanisms to incentivize information sharing, companies often exchange private information, even without elaborate mechanisms, repetitively (Shamir and Shin 2016).

Besides unilateral information sharing, bilateral information exchange becomes important as industrial partners are encouraged to share their information more actively (McKinsey & Company 2020). For example, Continental AG, an automotive parts manufacturer, shows a long history of exchanging demand forecast information with OEMs and major first-tier suppliers. In return, the suppliers reciprocally report their capacity levels to detect supply chain problems at an early stage (Continental AG 2014). Recently, German automotive industry firms, including BMW AG, Mercedes-Benz AG, Schaeffler AG, and Siemens AG, formed a data exchange network where upstream and downstream members voluntarily share capacity and demand information bilaterally. The collaboration (with a budget of 230 million euros until mid-2024) aims to achieve a "single point of truth" and to reduce the risk of demand and capacity mismatch (Catena-X 2021). Moreover, during the pandemic, many governmental health institutes exchanged their demand information with vaccine manufacturers. In return, those pharmaceutical companies allowed the government organizations to follow their available capacity levels precisely. Such early information sharing played an essential role in mitigating the risk of capacity investments in vaccine development and potential shortage issues (Druedahl et al. 2021). As shown in the examples, more companies see the necessity of bilateral information exchange (PricewaterhouseCoopers GmbH 2018).

In practice, such reciprocal exchange of information is mainly executed under a mutual agreement without specific contractual mechanisms in a voluntary manner. While many companies are now more encouraged to exchange private information with their industry partners, it is unclear what the strategic way of revealing one's information is in a reciprocal sharing decision. Moreover, even if clear operational incentives exist to disclose one's private information, exchanging information and adopting a new system is often effort-intensive and costly. Especially, implementation costs such as technology maintenance, employee training, or auditing service for small companies may not compensate for the expected benefit from sharing information (Kannisto et al. 2020). Hence, companies need to be aware of under which circumstances it is worth disclosing or withholding the information (Guo et al. 2014).

3.1.2 Modeling approach and contribution

In our model, a retailer's demand and a supplier's capacity information-sharing decisions are made either unilaterally or mutually. Information sharing causes the costs of utilizing data exchange and auditing services in interaction. As this costly sharing action enables the players to verify shared information, we assume that information is truthfully shared whenever it is shared. Especially, sharing costs with an auditing service can detect false information reports automatically. Such a truthful sharing assumption is typically applied in the literature in which the main focus is to investigate binary sharing decisions (Gal-Or 1985b, Chu and Lee 2006). Moreover, Natarajan and Kostamis (2013), Ren et al. (2010) demonstrate that exchanging non-verifiable information still validates the purpose of sharing information, and a wrong announcement can often be verified and punished.

We apply a Bayesian game where both players simultaneously decide whether to reveal the information voluntarily. Another focus of our study is signal-based prior updating. If one remains silent, even if the information is not explicitly shared, the other player receives the signal that remaining silent makes the player better off. The other player can then use this informative signal to update the prior belief on the information.

We investigate four settings, including two benchmark models: 1) decentralized ex-post information sharing, 2) joint ex-post information sharing (Benchmark 1), 3) complete ex-ante information exploration (Benchmark 2), and 4) sharing decisions under risk-aversion. Under the ex-post setting, the retailer and the supplier are locally informed about demand and capacity information, respectively, when making information sharing decisions. In the ex-ante setting, information exploration decisions from a single decision-maker are made based on prior beliefs regarding demand or capacity information.

In the decentralized setting, the players are self-interested and maximize their own expected profit by revealing (withholding) private information. In doing so, they consider the impact of their own sharing decision on the other player's sharing decision and incorporate a signal-based prior update from remaining silent. The decentralized ex-post sharing decisions are particularly relevant as companies can decide on which privately known information to be revealed. In the first benchmark, a vertically integrated retailer and a supplier are locally informed about their demand and capacity information, respectively. While voluntarily revealing and maintaining the information system incurs the cost of sharing, the players make sharing decisions as a team to avoid a significant mismatch in the joint expected profit.

In the second benchmark, we investigate the case where a single decision-maker has prior beliefs on demand and capacity and makes an information exploration decision ex-ante, incurring the cost of exploring information. Demand and capacity vary due to changing economic factors, consumer preferences, and supply disruptions (Eymann 2016). As such, exploring more information on demand and capacity is helpful even if the companies possess prior beliefs from their historical data. However, such exploration entails costs such as conducting market research, implementing forecast tools, and conducting focused group studies. In this context, this analysis shows the value of possessing certain information for a company. Along our study, we assume the prior beliefs of demand and capacity are common knowledge and remain the same in the four settings for comparison purposes.

3.1.3 Organization

The remainder of this chapter is organized as follows: Section 3.2 presents the information-sharing decision problems and model setup. Section 3.2.1 contains the results from decentralized information sharing. In Section 3.2.2 and 3.2.3, two benchmark cases are presented (Joint sharing and Ex-ante sharing). In Section 3.2.4, information sharing under risk-aversion is investigated. Section 3.3 provides concluding remarks and future research directions. The appendix contains all the proofs.

3.2 Model formulation and equilibrium analysis

We investigate a newsvendor problem under uncertain demand and cost information to make sharing decisions. A supplier (he) and a retailer (she) are both risk-neutral and maximize expected profits. The supplier's wholesale price w and the retailer's market price p are exogenously given. As this chapter focuses on players' intrinsic motivation to reveal their private information voluntarily, we do not consider financial incentives such as price discounts. The supplier's capacity cost per unit c is private information unless he shares the information with the retailer, where $p > w > c$. Information sharing incurs a fixed cost u_R for the retailer and u_S for the supplier.

By incurring information-sharing costs, we assume that this costly action entails systematic auditing and enables the players to verify the truthfulness of shared information. Hence, the players transmit only correct information whenever they share. Another justification for the assumption of truthful information sharing is that once the information-sharing decision is made, the companies transmit and exchange the information on a regular basis. With a standard technology (e.g., cloud computing and blockchain network platform) implemented at a cost, data is often transmitted automatically, and misrepresentation of the data is not possible (Ha et al. 2022a).

Private information and prior beliefs. The retailer has private market demand information d. The supplier endowed with private capacity investment cost c possesses information about their capacity with respect to the capacity investment cost, $K(c)$. As the newsvendor supplier's capacity $K(c)$ corresponds to his capacity investment cost c, knowing c is equivalent to knowing $K(c)$. Although perfectly known private information is a strong assumption, this can guide how supply chain members proactively decide information flows as one can control what it knows, as well as whether the other should know (Anand and Goyal 2009). Further, the demand information possessed by the retailer is more precise than that of the supplier due to the proximity to the market and the capacity information from the supplier is more accurate than the retailer can project (Klein and Rai 2009). By assuming the information from each player can be obtained through a highly accurate database, we consider that d and $K(c)$ are observed perfectly by each player, and sharing decisions are made ex-post after demand and capacity are locally revealed, similar to the ex-post information sharing by Natarajan and Kostamis (2013). Throughout this chapter, we use the terms "share," "disclose," and "reveal" interchangeably to avoid confusion.

The prior belief on the capacity cost c, and the prior belief on demand d are common knowledge. Both prior beliefs are continuously distributed with probability density $g(c)$ and cumulative distribution function $G(c)$ for capacity cost on the interval $C \in (0, w)$ and $f(d)$ and $F(d)$ for demand on the interval $D \in [0, \infty)$. If the retailer shares her market demand information, the supplier uses the information to set the capacity $K = d$. Otherwise, the newsvendor-type supplier sets the capacity based on c and the prior belief on D. If the supplier does not share capacity information, the retailer expects the supplier's capacity level to be determined based on the prior belief on C.

Base model structure. We first introduce the (expected) profits of the players under different sharing decisions. For ease of exposition, we present each player's expected profit without considering prior updating in this section. However, we will expand upon the updating case in Section 3.2.1. Each player can share (S) or not share (NS) its private information. These two actions lead to four sharing cases between the supplier and the retailer: Case I (S, S), Case II (NS, S), Case III (S, NS), and Case IV (NS, NS), where the first position in the parentheses represents the supplier's sharing decision while the second position denotes the retailer's sharing decision. Under symmetric information (Case I), the profits π_S^I for the supplier and π_R^I for the retailer are

$$\pi_S^I = (w - c) \cdot d - u_S \quad \text{and} \quad \pi_R^I = (p - w) \cdot d - u_R. \tag{3.1}$$

In Case II, the supplier does not share capacity information, while the retailer shares demand information. The sets of information available to the players are S[d, c] and R[d, $C \sim g(c)$]. When the supplier receives the demand information d, there is no incentive to deviate from the capacity level $K = d$, and the supplier's capacity cost prior becomes irrelevant to the retailer.

$$\pi_S^{II} = (w - c) \cdot d \quad \text{and} \quad \pi_R^{II} = (p - w) \cdot d - u_R \tag{3.2}$$

Note that Case II dominates Case I as the players obtain the same profits while the supplier does not incur information sharing cost u_S. In Case III, the supplier shares c (equivalent to capacity information), while the retailer withholds demand information. In such case, the supplier has S[$D \sim f(d)$, c] and the retailer R[d, c]. The supplier sets the capacity based on the prior demand distribution and bears the information sharing cost u_S. The supplier's expected profit is

$$\mathbb{E}[\pi_S^{III}] = (w - c) \cdot K - w \cdot \int_0^K (K - d)\, f(d) \mathrm{d}d - u_S. \tag{3.3}$$

As the retailer knows what the supplier knows (i.e., demand prior distribution and capacity cost), she can anticipate the optimal capacity level $K^* = F^{-1}(\frac{w-c}{w})$. The profit of the retailer is

$$\pi_R^{III} = (p - w) \cdot \min\{d, K^*\}. \tag{3.4}$$

In Case IV, where both withhold private information, the supplier has S[$D \sim f(d)$, c], but now the retailer holds the information set R[d, $C \sim g(c)$]. The supplier faces a similar situation to Case III, where he sets the capacity based on the prior demand belief but without sharing capacity information. The supplier's expected profit is

$$\mathbb{E}[\pi_S^{IV}] = (w - c) \cdot K - w \cdot \int_0^K (K - d)\, f(d)\mathrm{d}d \tag{3.5}$$

The retailer does not know the capacity of the supplier and only sees expected capacity $K^*(c)$ based on the capacity cost prior distribution $C \sim g(c)$. We define c^* as the capacity cost that makes the supplier's optimal capacity equal to the retailer's known demand level, $K^*(c^*) = d$. With $c^* = w \cdot (1 - F(d))$, if $c^* \leq c$, then $K^*(c) \leq d$. The retailer's expected profit is

$$\mathbb{E}[\pi_R^{IV}] = (p - w) \cdot \left(d \cdot \int_0^{c^*} g(c)dc + \int_{c^*}^{w} K^*(c) \cdot g(c)dc \right) = -(p - w) \cdot \left(\int_{c^*}^{w} \frac{\partial K^*(c)}{\partial c} \cdot G(c)dc \right). \tag{3.6}$$

Similar to Cases I and II, if the retailer does not share demand information, the supplier's expected profit of remaining silent dominates the expected profit from sharing $\mathbb{E}[\pi_S^{III}] < \mathbb{E}[\pi_S^{IV}]$ for any positive information sharing cost $u_S > 0$.

3.2.1 Decentralized information sharing decision

When a retailer and a supplier make bilateral information-sharing decisions decentrally, the players are self-interested and maximize their local expected profit. Hence, the players share only if remaining silent is worse than disclosing the information with the cost. Since the players strategically decide whether to disclose private information after observing d and c, withholding information itself gives the other player an informative signal. As mentioned before, observing c is equivalent to the supplier's optimal capacity $K^*(c)$ from (3.3) or (3.5).

The sequence of events is as follows: 1) the players make information-sharing decisions after locally observing private information d and $K^*(c)$; 2) each player receives a signal in case the other player remains silent, and updates the prior distribution if the information is withheld. As the sharing rules and the signal may change after the other player's prior update, the players obtain equilibrium sharing decisions that no longer influence one's best response; and 3) Once the information-sharing decisions are made, the supplier sets his capacity level based on either received demand information from the retailer or the updated prior demand.

We present a Bayes-Nash equilibrium (BNE) information sharing rule for each player under bilateral asymmetric information. In doing so, we incorporate a *signaling effect* that enables the other player to update the prior belief when withholding private information (i.e., the retailer withholding information makes the supplier update the demand prior and the supplier concealing information makes the retailer update the capacity cost prior). The BNE solution in our analysis represents a steady-state equilibrium where the impact of remaining silent on the prior is implicitly considered, while the updated demand and cost priors also exhibit steady-state distributions.

When each player decides whether to share the locally observed private information d and $K^*(c)$ simultaneously and bilaterally, the retailer and the supplier conjecture how their own sharing (or withholding) would impact the other's information structure and reciprocal sharing decision upon his or her decisions. When the supplier decides to share capacity information, his expected profit from disclosing the information is determined by the sharing policies (S, S) and (S, NS). On the other hand, if the supplier remains silent, the retailer updates her prior belief regarding the supplier's capacity information, and the supplier's expected profit is determined by (NS, S) and (NS, NS). While remaining silent leaves a signal for the retailer to update the cost prior, the supplier can derive the expected profits under sharing and not sharing, considering such possible sharing policies upon his decision.

Similarly, when the retailer decides whether to share demand information, the expected profit from sharing is determined by (S, S) and (NS, S), while the expected profit from withholding demand information is defined by (S, NS) or (NS, NS). If the retailer remains silent, the supplier updates the prior belief on the retailer's demand information. We shall denote the terms that are influenced by any prior updates with a wide-hat notation ($\widehat{\cdot}$). Based on the above process of reaching equilibrium, we present the following sharing rule for each player and the resulting sharing policy.

Proposition 3.1. *There exists a unique demand threshold level* $\widehat{TH}^R$ *for the retailer that makes her disclose demand information, anticipating the supplier's demand prior update.*

$$\widehat{TH}^R = \frac{u_R}{p-w} + \int_0^w \widehat{K}^*(c) \cdot g(c)dc$$

Proposition 3.1 implies that if the retailer's demand information $d = \widehat{TH}^R$, she is indifferent between revealing and withholding demand information as the expected profit from remaining silent is equal to revealing demand information $\mathbb{E}\left[\widehat{\pi}_R^{IV}\right] = \pi^{II}$. The condition represents that when the retailer's demand d is higher than the expected capacity under the capacity cost prior, she reveals her demand information, incurring the cost of sharing.

Note that when the retailer shares, she knows $\pi_R^I = \pi_R^{II} = (p-w) \cdot d - u_R$, regardless of the supplier's best response. Once the demand information is transmitted to the supplier, the capacity information becomes irrelevant as $K^* = d$, and the supplier's sharing decision does not influence the profit of the retailer. This represents the fact that the retailer has the power to immediately avoid both demand and capacity uncertainty by revealing her private demand information. Further, as the supplier's expected profit under demand uncertainty exhibits $\mathbb{E}\left[\widehat{\pi}_S^{IV}\right] > \mathbb{E}\left[\widehat{\pi}_S^{III}\right]$, if the retailer remains silent, her expected profit under remaining silent is induced from $\mathbb{E}\left[\widehat{\pi}_R^{IV}\right]$.

Proposition 3.2. *If the supplier's expected overage cost is higher than incurring the sharing cost u_S for any $K^*(c)$, there exists a cost range $\widehat{TH}_{lb}^S < c < \widehat{TH}_{ub}^S$ that makes the supplier disclose capacity information. The lower and upper thresholds within the range, $\widehat{TH}_{lb}^S$ and $\widehat{TH}_{ub}^S$ satisfy:*

$$\left(F(\widehat{TH}_{s:ns}) - F(TH_{s:s})\right) \cdot \left(w \cdot \int_0^{K^*(c)} F(d)\mathrm{d}d + \frac{u_R \cdot (w-c)}{p-w}\right) = u_S \tag{3.7}$$

where, $TH_{s:s} = \frac{u_R}{p-w} + K^*(c)$ *and* $\widehat{TH}_{s:ns} = \frac{u_R}{p-w} - \int_{c^*}^{w} \frac{\partial K^*(c)}{\partial c} \cdot \widehat{G}(c)dc$

Unlike the retailer, the supplier faces two cost thresholds $\widehat{TH}_{lb}^S$ and $\widehat{TH}_{ub}^S$ which make his expected profit under sharing equivalent to remaining silent. Proposition 3.2 shows that the supplier's incentive to reveal capacity information is from 1) the probability that the retailer overestimates the supplier's capacity, $F(\widehat{TH}_{s:ns}) - F(TH_{s:s})$ and 2) the expected profit from avoiding overage costs. Especially, the supplier noticing that the retailer has a high capacity expectation has a higher incentive to reveal his capacity information as the retailer is more likely to reveal demand information reciprocally. Especially, $F(\widehat{TH}_{s:ns}) - F(TH_{s:s}) > 0 \equiv TH_{s:s} < \widehat{TH}_{s:ns}$ affecting the sharing decision of the supplier implies that the supplier reveals his capacity information to induce the retailer's demand sharing reciprocally.

From the retailer's probability to overestimate the supplier's capacity, $F(\widehat{TH}_{s:ns})$ is constant in c, while $F(TH_{s:s})$ monotonically decreases in c. The interpretation is straightforward: the probability of the retailer overestimating the supplier's capacity increases as the lower capacity the supplier has, $\frac{\partial F(\widehat{TH}_{s:ns}) - F(TH_{s:s})}{\partial c} > 0$. On the other hand, the savings from the expected overcapacity $\left(w \cdot \int_0^{K^*(c)} F(d)\mathrm{d}d + \frac{u_R \cdot (w-c)}{p-w}\right)$ decreases in c as $\frac{\partial K^*(c)}{\partial c} < 0$. Hence, the supplier's benefit from sharing follows a concave function in c, and his sharing rule depends on a range. As the supplier's expected gain from sharing is not monotonic in c, we present Figure 3.1 to illustrate (3.7).

The intuition behind the supplier's sharing behavior is as follows. Suppose the supplier has a relatively high cost $c \geq \widehat{TH}_{ub}^S$. Then, the supplier's capacity is relatively low and the retailer is more likely to share the demand from knowing $K^*(c)$. However, the supplier's profit margin, $w - c$, does not cover the information sharing cost u_S, making $\lim_{c \to w} \mathbb{E}\left[\pi_S\right] = 0$ for any sharing cases. On the other hand, if the supplier has a considerably low cost $c \leq \widehat{TH}^S_{lb}$, the resulting large capacity of the supplier cannot encourage the retailer to share demand information reciprocally. As a relatively large capacity the supplier has, he anticipates a lower probability of the retailer's reciprocal sharing $\lim_{c\to 0} F(\widehat{TH}_{s:ns}) - F(TH_{s:s}) < 0$. Hence, the supplier's incentive to reveal $K^*(c)$ also reduces. Lastly, the larger the information sharing cost u_S, the smaller the cost range the supplier benefits from sharing, making the supplier less likely to share.

Theorem 3.1. *In the decentralized case, there exists a unique Bayesian Nash Equilibrium for each player to share their information.*

1. *if* $\widehat{TH}^S_{lb} \leq c \leq \widehat{TH}^S_{ub}$ *and* $d < \widehat{TH}^R$, *a supplier unilaterally shares* $K^*(c)$ *(S, NS).*
2. *if* $c < \widehat{TH}^S_{lb}$ *or* $c > \widehat{TH}^S_{ub}$ *and* $d \geq \widehat{TH}^R$, *a retailer unilaterally shares* d *(NS, S).*
3. *if* $\widehat{TH}^S_{lb} \leq c \leq \widehat{TH}^S_{ub}$ *and* $d \geq \widehat{TH}^R$, *both mutually share* d *and* $K^*(c)$ *(S, S).*
4. *if* $c < \widehat{TH}^S_{lb}$ *or* $c > \widehat{TH}^S_{ub}$ *and* $d < \widehat{TH}^R$, *both mutually remain silent (NS, NS).*

where, $\widehat{TH}^R$ *from Proposition 3.1, and* $\widehat{TH}^S_{lb}$, $\widehat{TH}^S_{ub}$ *from Proposition 3.2.*

The resulting policy has two implications: 1) the retailer with high demand information d shares voluntarily; however, under a low d she remains silent. 2) when the supplier has extreme capacity information (i.e., $K^*(c)$ is high or low), he is more likely to remain silent in both cases, as depicted in Figure 3.2. For this reason, mutual information sharing (S, S) between the two players occurs when the supplier keeps a moderate capacity while the retailer has a relatively high demand. Note that both players' sharing rules incorporate the updated prior beliefs from one's remaining silent ($\widehat{TH}^R$, $\widehat{TH}^S_{lb}$, and $\widehat{TH}^S_{ub}$). Hereafter, we describe in detail how the prior distributions are updated and a few insights drawn from such updates.

Updated equilibrium prior distributions. From Theorem 3.1, if the retailer withholds private information, the supplier updates the prior belief on demand. For example, even if the supplier does not know under which d the retailer decides to remain silent, at least he can conjecture if the retailer remains silent, it was due to $d \leq \widehat{TH}^R$. As $D \in (0, \infty)$, the signal makes the supplier truncate the prior, $\widehat{f}(d)/\widehat{F}(d)$ based on the threshold level. The equilibrium demand prior in steady-state is

$$\widehat{f}(d) = \begin{cases} 0 & d > \widehat{TH}^R \\ \frac{f(d)}{F(\widehat{TH}^R)} & 0 < d \leq \widehat{TH}^R \end{cases} \tag{3.8}$$

Figure 3.1: Supplier's Capacity Sharing Rule

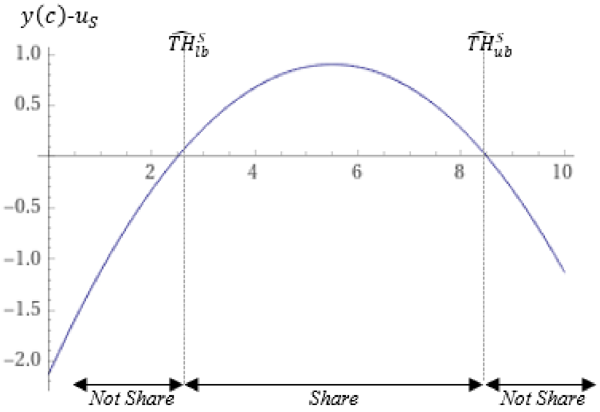
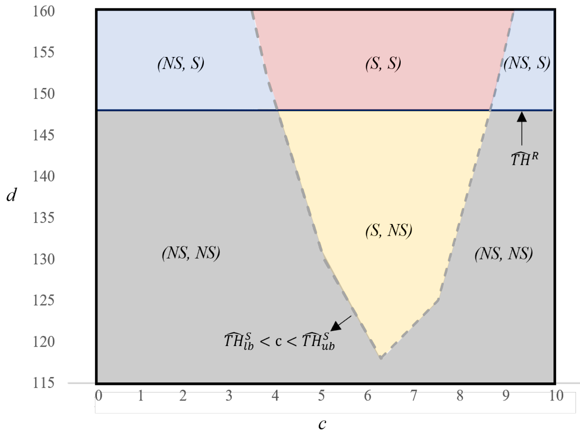


Figure 3.2: Sharing Decision: Bayes Nash Equilibrium



$$\textit{where, } y(c) = \left(F(\widehat{TH}_{s:ns}) - F(TH_{s:s})\right) \cdot \left(w \cdot \int_0^{K^*(c)} F(d)\mathrm{d}d + \tfrac{u_R \cdot (w-c)}{p-w}\right),$$
$$p = 15,\ w = 10,\ u_S = 5,\ u_R = 25,\ D \sim U(100, 400),\ C \sim U(0, w)$$

Lemma 3.1. *The demand prior $F(d)$ exhibits first-order stochastic dominance (FSD) over the updated demand prior $\widehat{F}(d)$ in a steady-state with regards to the retailer's demand information sharing threshold, $\widehat{TH}^R$. Hence, after updating the prior belief, it is optimal for the supplier to set a lower capacity level, $\widehat{K}^*(c) \le K^*(c)$.*

Lemma 3.1 shows that the supplier's demand prior update directly affects the retailer's expected profit of remaining silent as $\mathbb{E}\left[\widehat{\pi}_R^{IV}\right]$ depend on the supplier's capacity level under demand prior. As the updating behavior of the supplier leads to a reduced capacity level $\widehat{K}^*(c) \le K^*(c)$, the retailer's incentive to share demand information increases to avoid capacitated demand fulfillment.

Another implication from Lemma 3.1 is that the retailer compares the profit π_R^{II} that eliminates the mismatch between demand and capacity by sharing d to the expected profit $\mathbb{E}[\widehat{\pi}_R^{IV}]$ containing both demand and capacity uncertainty. Hence, her sharing decision is influenced by two-directional asymmetric information. This bilateral asymmetric information has a countervailing effect on the retailer's threshold demand level $\widehat{TH}^R$ as the supplier's expectation on updated demand prior $\widehat{F}(d)$ decreases, the capacity level decreases while the retailer's low expectation on the cost prior $G(c)$ increases the expected capacity.

Similarly, in case the supplier remains silent, the retailer receives a signal that the supplier's capacity cost follows $c \le TH_{lb}^S$ or $c \ge TH_{ub}^S$. The retailer updates the prior cost distribution to $\widehat{g}(c)/\widehat{G}(c)$ if the supplier remains silent. Hence, the steady-state equilibrium capacity sharing range for the supplier to share $\widehat{TH}_{lb}^{S}$ and $\widehat{TH}_{ub}^{S}$ are derived based on the equilibrium prior cost distribution.

$$\widehat{g}(c) = \begin{cases} 0 & \widehat{TH}_{lb}^{S} < c < \widehat{TH}_{ub}^{S} \\ \frac{g(c)}{1-G\left(\widehat{TH}_{ub}^{S}\right)+G\left(\widehat{TH}_{lb}^{S}\right)} & 0 \leq c \leq \widehat{TH}_{lb}^{S} \vee \widehat{TH}_{ub}^{S} \leq c \leq w \end{cases} \tag{3.9}$$

Lemma 3.2. *The cost prior $G(c)$ exhibits second-order stochastic dominance (SSD) over the updated cost prior $\widehat{G}(c)$ in a steady-state with respect to the supplier's capacity information sharing range, $\widehat{TH}_{lb}^{S} < c < \widehat{TH}_{ub}^{S}$. Hence, the retailer's expectation of capacity increases under the cost prior update,*

$$\mathbb{E}[K^*(c)] = \int_0^w K^*(c)g(c)dc \leq \int_0^w K^*(c)\widehat{g}(c)dc = \mathbb{E}[\widehat{K}^*(c)].$$

Lemma 3.2 demonstrates that the retailer has a higher expectation of the supplier's capacity when she updates the cost prior $\widehat{G}(c)$. Recall from Proposition 3.2 that the supplier's incentive to withhold capacity information reduces when the retailer's expectation of his capacity is relatively higher than his private capacity information, $F(\widehat{TH}_{s:ns}) > F(TH_{s:s}) \equiv \widehat{TH}_{s:ns} > TH_{s:s}$. Combining the results from Lemma 3.1 and 3.2, we observe that each player's remaining silent leads to unfavorable prior updates from the uninformed player (i.e., the supplier reduces the optimal capacity, and the retailer anticipates a higher expected capacity from the supplier). Therefore, to prevent such updating, both players are more likely to reveal their information.

Sensitivity analysis of demand and cost prior distributions. Higher profit margins $p-w$ and $w-c$ in the decentralized setting make information sharing more likely as the economic loss from mismatching demand and capacity for both players is high. Further, higher information sharing costs, u_S and u_R make sharing less likely. In this section, we investigate the impact of the expectations and the uncertainty (variability) of prior beliefs on each player's sharing decision.

Proposition 3.3. *For random demand D_1 and D_2 with prior distributions $F_1(d)$ and $F_2(d)$,*

1. *if the supplier has a higher expectation of demand prior $\mathbb{E}[D_1] > \mathbb{E}[D_2]$, both players are less likely to share demand as $\mathbb{E}_1\left[\pi_S^{IV}\right] \geq \mathbb{E}_2\left[\pi_S^{IV}\right]$, and $\mathbb{E}_1\left[\widehat{\pi}_R^{IV}\right] \geq \mathbb{E}_2\left[\widehat{\pi}_R^{IV}\right]$.*

2. *if the supplier has a more variable demand prior distribution* $\mathrm{Var}(D_1) > \mathrm{Var}(D_2)$*, both players are more likely to share demand and cost information as* $\mathbb{E}_1\left[\pi_S^{IV}\right] \le \mathbb{E}_2\left[\pi_S^{IV}\right]$*, and* $\mathbb{E}_1\left[\widehat{\pi}_R^{IV}\right] \le \mathbb{E}_2\left[\widehat{\pi}_R^{IV}\right]$*.*

Proposition 3.3 imposes that when the expected market demand is high, the retailer knows that the supplier sets a high capacity. Hence, the retailer's incentive to reveal private information to alert the supplier about her relatively high demand level reduces, $\widehat{TH}_1^R \ge \widehat{TH}_2^R$. Although the retailer's expectation of the capacity increases, the supplier's expected profit from withholding information further increases. Therefore, a higher expectation of demand prior reduces both players' incentive to share information.

On the other hand, a high variance in the demand prior reduces the players' expected profits under demand uncertainty. From Proposition 3.1, the retailer's expected profit from remaining silent is composed of two parts where she expects her demand to be lower than the supplier's capacity $\int_0^{c^*} d \cdot g(c)dc$ and expects capacitated demand $\int_{c^*}^{w} K^*(c) \cdot g(c)dc$. For $c \in (c^*, w)$, the retailer sees a lower expected capacity from the supplier with a more variable demand prior, $\mathbb{E}_1[\widehat{K}^*] \le \mathbb{E}_2[\widehat{K}^*]$. Hence, when the supplier expects a more variable market demand, the retailer is more likely to reveal demand information in her own interest, setting $\widehat{TH}_1^R \le \widehat{TH}_2^R$.

Further, the supplier's expected profit is higher under the less variable demand at any capacity level as the overage and underage costs are reduced. In other words, the expected profit of remaining silent under a highly variable prior distribution is smaller $\mathbb{E}_1\left[\pi_S^{IV}\right] \le E_2\left[\pi_S^{IV}\right]$. Hence, as the variability in demand prior increases, both players are more likely to share their private information.

Proposition 3.4. *For random cost* C_1 *and* C_2 *with prior distributions* $G_1(c)$ *and* $G_2(c)$*,*

1. *if the retailer has a higher expectation of cost prior* $\mathbb{E}[C_1] > \mathbb{E}[C_2]$*, the retailer is more likely to share demand and the supplier is more reluctant to share cost information as* $\mathbb{E}_1\left[\pi_R^{IV}\right] \le \mathbb{E}_2\left[\pi_R^{IV}\right]$*, and* $\widehat{TH}_{1s:ns} \le \widehat{TH}_{2s:ns}$*.*

2. *if the retailer has a more variable cost prior distribution* $\mathrm{Var}(C_1) > \mathrm{Var}(C_2)$*, the retailer is more likely to share demand and the supplier is more reluctant to share cost information as* $\mathbb{E}_1\left[\pi_R^{IV}\right] \le \mathbb{E}_2\left[\pi_R^{IV}\right]$*, and* $\widehat{TH}_{1s:ns} \le \widehat{TH}_{2s:ns}$*.*

The main insight from Proposition 3.4 is as follows: when the retailer has a higher cost expectation $\mathbb{E}[C_1] > \mathbb{E}[C_2]$ or a more variable cost prior $\mathrm{Var}(C_1) > \mathrm{Var}(C_2)$, her expectation on the supplier's capacity reduces $\mathbb{E}_1[K^*] < \mathbb{E}_2[K^*]$. A low expected capacity of the retailer incentivizes her to reveal demand information to prevent capacitated demand fulfillment. On the other hand, recall from Proposition 3.2 that the supplier has the incentive to reveal capacity information when the retailer's expectation of his capacity is relatively high. Even though the cost prior does not impact the supplier's expected profit function directly, it does implicitly influence the supplier's incentive to share as the cost prior update defines the retailer's expectation of the supplier's capacity without sharing information, $\widehat{TH}_{s:ns}$. Therefore, the retailer's higher expectation of the supplier capacity under $\mathbb{E}[C_1]$ and $\mathrm{Var}(C_1)$ makes the supplier less likely to reveal capacity information. This is because the unilateral demand revelation from the retailer allows the supplier to save both capacity mismatch costs and the information sharing cost without incurring his own cost u_S for sharing.

As managerial interpretation from the prior sensitivity analysis, the main motivation for the players to exchange information is to avoid demand and capacity mismatches. For a self-interested retailer, the supplier's increasing capacity based on a higher demand expectation is not of concern, while a reduced expected profit due to the variability in demand prior gives an alert to the retailer to disclose demand information. The supplier knows that the retailer has the incentive to share if the expectation or variability of the cost prior is relatively high, while the supplier's capacity level does not change. Therefore, in case the retailer is more likely to share due to the prior cost distribution, the supplier's incentive decreases to reveal capacity information (Propositions 3.4). On the other hand, the supplier's more variable demand prior increases both underage and overage, which reduces both players' incentive to remain silent (Propositions 3.3). Hence, the high variability in demand prior makes the players more likely to share as the optimal capacity decision changes, while the variability in cost prior only makes the retailer more likely to share.

3.2.2 Benchmark case I: joint information sharing decision

We now investigate a benchmark where a retailer and a supplier optimize toward a joint (firm) expected profit instead of being independent and self-interested companies. In this vein, the players aim to avoid mismatches in the expected profit of the total supply chain by revealing private information. The analysis in this section can be analogous to many common practices where the companies voluntarily share information to avoid future demand and capacity mismatch. In case the expected profit under no information shows a high mismatch due to expected overage and underage compared to the expected profit by revealing certain information, such sharing can occur. Under the joint benchmark, since the players make information-sharing decisions based on vertically integrated supply chain profit rather than self-interest, remaining silent does not result in prior updating.

We first show the players' information sharing rules and investigate the impact of variability in the prior beliefs on the optimal sharing decisions of the players. The information sharing structure of the benchmark case is as follows: 1) both retailer and supplier are locally informed about d and $K^*(c)$; 2) the players decide whether to share private information based on the mismatch in the expected joint profit and the sharing costs; 3) If the retailer shares demand information, the supplier sets his capacity level to the shared demand information. In the supply chain, the retailer makes an order shortly before the selling season, and the supplier must secure capacity prior to the order in each selling season; and 4) Finally, the demand is fulfilled from the supplier's capacity set based on the sharing decision between two players.

If both players do not reveal any information, the expected profit based on both prior distributions is

$$\mathbb{E}\left[\pi^{IV}\right] = \int_0^w \left[(p-c)\cdot K(c) - p\int_0^{K(c)} (K(c)-d)f(d)dd\right] g(c)dc. \tag{3.10}$$

The expected profit under mutually not sharing information, $\mathbb{E}\left[\pi^{IV}\right]$ is used as a reference expected profit for the players to make sharing decisions. Although $\mathbb{E}\left[\pi^{IV}\right]$ depends purely on the prior beliefs on demand and capacity, the variability in such information impacts each player's incentive to share. We consider the cost prior $C \sim (0, w)$ to compare the benchmark sharing decisions with decentralized cases, even if there is no transfer price w in the total supply chain. If the retailer reveals demand information d, the integrated market profit is $\pi^{II} = (p - \mathbb{E}[c]) \cdot d$ as $d = K^*(c)$. If the supplier shares the optimal capacity $K^*(c) = F^{-1}(\frac{p-c}{p})$, the integrated market expected profit is

$$\mathbb{E}\left[\pi^{III}\right] = (p-c)\cdot K^*(c) - p\int_0^{K^*(c)} (K^*(c)-d)f(d)dd. \tag{3.11}$$

Lemma 3.3. *In the benchmark where the players make information sharing decisions considering the joint expected profit, the information sharing policy follows*

1. *if* $d > TH^R_{UB}$ *or* $d < TH^R_{LB}$ *and* $TH^S_{LB} \le c \le TH^S_{UB}$, *a retailer shares d (NS, S).*
2. *if* $c > TH^S_{UB}$ *or* $c < TH^S_{LB}$ *and* $TH^R_{LB} \le d \le TH^R_{UB}$, *a supplier shares* $K^*(c)$ *(S, NS).*
3. *if* $c > TH^S_{UB}$ *or* $c < TH^S_{LB}$ *and* $d > TH^R_{UB}$ *or* $d < TH^R_{LB}$, *both share d and* $K^*(c)$ *(S, S).*
4. *if* $TH^R_{LB} \le d \le TH^R_{UB}$ *and* $TH^S_{LB} \le c \le TH^S_{UB}$, *both remain silent (NS, NS).*

where, the thresholds are obtained by
$\mathbb{E}\left[\pi^{IV}\right] - \mathbb{E}\left[\pi^{III}(TH^S_{UB})\right] = u_S, \quad \mathbb{E}\left[\pi^{III}(TH^S_{LB})\right] - \mathbb{E}\left[\pi^{IV}\right] = u_S,$

$TH_{LB}^{R} = \int_0^w \left[K(c) - \frac{p}{p-c}\int_0^{K(c)}(K(c)-d)f(d)dd - \frac{u_R}{p-c}\right] g(c)dc$, *and*
$TH_{UB}^{R} = \int_0^w \left[K(c) - \frac{p}{p-c}\int_0^{K(c)}(K(c)-d)f(d)dd + \frac{u_R}{p-c}\right] g(c)dc.$

Lemma 3.3 states that demand or capacity information is shared to reduce uncertainty if their values are relatively high or low. As depicted in Figure 3.3, the players share the information even though sharing incurs costs u_R and u_S to reduce the mismatches in the expected profit. If the value of preventing over/underestimating expected profits by revealing the information is insufficient to cover the sharing costs, both players remain silent. Such sharing rules reflect the rationale behind forming a demand and capacity information exchange network in practice. As the common objective is to avoid a mismatch between demand and capacity, companies with an extreme value of private information are willing to report their information and alert their partners.

Based on Lemma 3.3 and Theorem 3.1, the sharing decisions under the benchmark and the decentralized setting differ. One of the differences in the information sharing rules is that demand information is revealed only when it is relatively high in the decentralized setting whereas, under the benchmark, the retailer reveals both relatively high or low demand information. More importantly, the supplier's capacity information sharing rule shows a reversed interpretation: in the benchmark, the supplier with either a significantly low or high capacity does reveal capacity information. In a decentralized setting, however, the supplier withholds the information in such cases as his incentive to reveal the information reduces from the low profit margin or the retailer's less likeliness of reciprocal sharing.

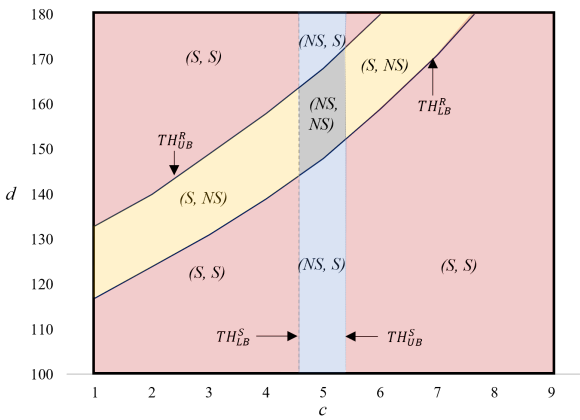
Proposition 3.5. *The variability in prior beliefs has the following impact on the range of no sharing thresholds for the retailer, $TH_{UB}^{R} - TH_{LB}^{R}$ and for the supplier, $TH_{UB}^{S} - TH_{LB}^{S}$ in joint sharing decision.*

1. *A supplier with high demand variability is more likely to reveal capacity information as the expected mismatch from underage and overage increases in a higher demand variability. Hence, the increased expected profit mismatch makes incurring the cost of sharing u_S more likely, making $TH_{UB}^{S} - TH_{LB}^{S}$ smaller.*

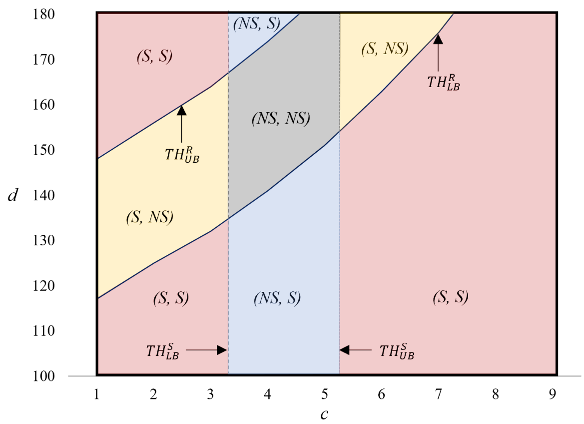
2. *A retailer with high cost (capacity) variability is more likely to reveal demand information as the retailer's marginal value of incurring the cost of sharing $\int_0^w \frac{u_R}{p-c} \cdot g(c)dc$ reduces in an increasing cost variability, making her remaining silent range $TH_{UB}^{R} - TH_{LB}^{R}$ smaller.*

The implications from Proposition 3.5 are twofold: 1) the retailer with more uncertain cost prior is more likely to reveal demand information as she observes a higher expected unfulfilled demand under capacity uncertainty, and 2) the supplier with highly uncertain demand prior is more likely

Figure 3.3: Benchmark Sharing Decisions
where, $D \sim U(100, 300)$, $C \sim U(0, w)$



(a) $p = 20$, $w = 10$, $u_S = 50$, $u_R = 150$



(b) $p = 20$, $w = 10$, $u_S = 150$, $u_R = 250$

to reveal capacity information to alert the retailer on the anticipated mismatch under demand uncertainty. For instance, in the automotive industry, downstream members not knowing the upstream capacity information reveal demand information mainly to let the suppliers prepare for the upcoming demand level. On the other hand, the supplier's active capacity revelation is to notify downstream members of the anticipated shortage or overstock issues.

While an increasing retailer's remaining silent region $TH^R_{UB} - TH^R_{LB}$ denotes that the retailer's incentive to share demand information decreases and is characterized by the retailer's expected marginal sharing cost relative to the profit margin ($\int_0^w \frac{u_R}{p-c} \cdot g(c)dc$) as shown in Lemma 3.3. Interpreting $TH^R_{UB} - TH^R_{LB}$ as the retailer's relative reluctance to incur the cost of sharing, as she anticipates a highly variable cost from the supplier, her relative reluctance to incur the information-sharing cost reduces as the expected profit margin is higher when revealing demand information for $c \in (0, w)$. Hence, she is more likely to share under increasing variability in cost prior.

The supplier's incentive to reveal the capacity information increases as the variability of demand prior increases. In joint decision-making, the supplier's expected profit decreases in an increasing variability of demand prior for any given c as the expected over-/underage costs increase. As such, the supplier has more instances where the expected profit loss from the over-/underage is greater than the information sharing cost u_S and is worth revealing the capacity information to the retailer. Lastly, both players are less likely to share the information as the sharing costs, u_R and u_S, increase as shown in Figure 3.3b.

When comparing Proposition 3.5 to the effect of variability in decentralized information sharing (Section 3.2.1), it can be observed that the players with greater variability in prior beliefs are more likely to disclose their own information in both settings. This suggests that when there is greater variability in the information possessed by each player regarding the other's information, there is an increased incentive for a player to disclose their own information when they make joint sharing decisions.

The information sharing policies under Lemma 3.3 and Theorem 3.1 exhibit important differences. Following the optimal sharing rules under these two settings, Table 3.1 illustrates the ex-post profits, which represent the profits realized based on the sharing decisions and the corresponding capacity of the supplier. As Lemma 3.3 demonstrated, we observe that under the joint sharing decision, low or high demands (d_L and d_H) or costs (c_L and c_H) are revealed by the retailer and the supplier. On the other hand, in the decentralized setting, the retailer with high demand ($d > 350$), and the supplier with a moderate cost ($5.57 < c < 7.9$) reveal their information, respectively.

More interestingly, under certain circumstances, the optimal sharing policy in the decentralized setting brings a higher ex-post profit, as highlighted in Table 3.1. Such cases occur 1) when the supplier has relatively high capacity and the joint decision leads to mutual sharing (S, S), or 2) the players' private demand and capacity are close to the other's expectation ($d_M = \mathbb{E}[D]$ and $c_M = \mathbb{E}[C]$). Note that the retailer's unilateral demand sharing (NS, S) is an ex-post dominant strategy over mutual sharing ($\pi_R^I + \pi_S^I < \pi_R^{II} + \pi_S^{II}$). However, under the benchmark, the players consider a joint profit margin, $p - c$, making the supplier more likely to share capacity while not knowing the retailer's sharing decision as $\frac{u_S}{p-c} < \frac{u_S}{w-c}$.

3.2.3 Benchmark case II: ex-ante information acquisition decision

In many cases, a company does not know precise demand or capacity information, as tracking and maintaining such information comes at costs by conducting market research. A decision maker (DM), who wants to avoid a significant loss from not knowing certain information, may need to decide ex-ante whether to explore certain information, incurring the cost of doing so or not. If the prior belief, containing information uncertainty, has little variability, or the cost of procuring additional information (i.e., u_R for demand information from the retailer and u_S for capacity information from the supplier) is too high, the DM does not have the incentive to explore more information.

This part differentiates from the analysis introduced earlier. We explore a single decision maker's ex-ante information exploration decision, focusing on the value of having certain information by

Table 3.1: Ex-post Efficiency Comparison between Benchmark 1 and Decentralized Sharing where, $p = 10$, $w = 8$, $u_R = 250$, $u_S = 100$, $D \sim U(100, 400)$, and $C \sim U(0, w)$

	Joint Sharing		Decentralized Sharing			
	Sharing Policy	π	Sharing Policy	π_R	π_S	$\pi_R + \pi_S$
$c_L = 2$, $d_L = 100$	(S, S)	550	(NS, NS)	300	150	450
$\mathbf{c_L = 2}$**,** $\mathbf{d_M = 250}$	**(S, S)**	**1900**	**(NS, NS)**	**750**	**1350**	**2100**
$\mathbf{c_L = 2}$**,** $\mathbf{d_H = 400}$	**(S, S)**	**3250**	**(NS, S)**	**950**	**2400**	**3350**
$c_M = 4$, $d_L = 100$	(NS, S)	450	(NS, NS)	300	-200	100
$\mathbf{c_M = 4}$**,** $\mathbf{d_M = 250}$	**(NS, S)**	**1500**	**(NS, NS)**	**750**	**1000**	**1750**
$c_M = 4$, $d_H = 400$	(NS, S)	2550	(NS, S)	950	1600	2550
$c_H = 6$, $d_L = 100$	(S, S)	150	(S, NS)	300	-350	-50
$c_H = 6$, $d_M = 250$	(S, S)	900	(S, NS)	525	250	775
$c_H = 6$, $d_H = 400$	(S, S)	1650	(S, S)	950	700	1650

committing the ex-ante to centrally explore information. The question of concern is: under which conditions would the DM explore demand and capacity information? Recall that in §3.2.1 and §3.2.2, each player can voluntarily transmit his own private information to the other player. Comparing these two structures with the current information exploration decision allows us to observe when demand or capacity information is needed from the uninformed party and, in contrast, when the party endowed with certain private information reveals such information.

From Figure 3.4, there are three possible choices: 1) do not explore any information on d and $K^*(c)$, 2) explore demand information, and 3) explore capacity information which can further lead to additional exploration of d depending on the capacity exploration results.

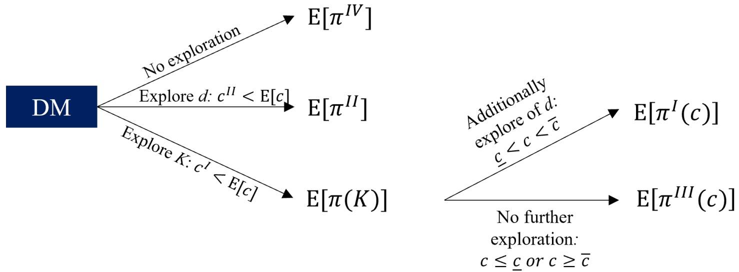


Figure 3.4: Ex-ante Information Exploration Structure

If the DM does not explore any information, the ex-ante expected profit under the prior demand and cost distributions for K is

$$\mathbb{E}[\pi^{IV}] = \int_0^w \left\{ (p-c)K - p\int_0^K F(d)dd \right\} g(c)dc. \tag{3.12}$$

Without exploring demand and capacity information, the capacity based on the first order condition on K from (3.12) yields

$$\frac{\partial \mathbb{E}[\pi^{IV}]}{\partial K} = \int_0^w \{(p-c) - pF(K)\} g(c)dc = 0 \Leftrightarrow K = F^{-1}\left(\frac{p-\mathbb{E}[c]}{p}\right).$$

If the DM decides to explore demand information, the capacity information is no longer relevant as $d = K$, while the cost prior affects the expected profit. The ex-ante expected profit for exploring demand information is

$$\mathbb{E}[\pi^{II}] = \int_0^\infty \int_0^w (p-c)\cdot d \cdot g(c)f(d)dcdd - u_R = (p-\mathbb{E}[c])\cdot \mathbb{E}[d] - u_R. \tag{3.13}$$

If exploring demand information does not occur (due to a high exploration cost u_R or a small uncertainty in the demand prior), the DM can also decide to explore capacity information.

For capacity exploration, the ex-ante expected profit is

$$\begin{aligned}\mathbb{E}[\pi(K)] = &\int_0^{\underline{c}} \left\{ (p-c)K(c) - p\int_0^{K(c)} F(d)dd \right\} g(c)dc + \int_{\underline{c}}^{\overline{c}} ((p-c)\cdot\mathbb{E}[d] - u_R)\, g(c)dc \\ &+ \int_{\overline{c}}^{w} \left\{ (p-c)K(c) - p\int_0^{K(c)} F(d)dd \right\} g(c)dc - u_S.\end{aligned} \tag{3.14}$$

If the DM makes a decision of capacity exploration, his ex-ante expected profit $\mathbb{E}[\pi(K)]$ is composed of the expected profit $\mathbb{E}[\pi^{III}(c)]$ from exploring $K^*(c)$ only and 2) the expected profit $\mathbb{E}[\pi^{I}(c)]$ of exploring d in addition to $K^*(c)$ incurs both costs u_R and u_S, where

$$\mathbb{E}[\pi^{III}(c)] = (p-c)K(c) - p\int_0^{K(c)} F(d)dd - u_S \quad \text{and} \quad \mathbb{E}[\pi^{I}(c)] = (p-c)\cdot\mathbb{E}[d] - u_R - u_S.$$

Denote $\underline{c}$ and $\overline{c}$ as the costs that satisfy $\mathbb{E}[\pi^{I}(c)] = \mathbb{E}[\pi^{III}(c)]$ for $c \in (0, w)$. If $0 \leq c \leq \underline{c}$, the DM has sufficient capacity $K^*(c)$ with a high profit margin $p - c$ that leads to a higher expected profit under capacity exploration $\mathbb{E}[\pi^{III}(c)] \geq \mathbb{E}[\pi^{I}(c)]$. Then, the DM does not explore demand information additionally. Hence, the information decision is terminated after exploring $K^*(c)$. If $\underline{c} < c < \overline{c}$, the DM encounters a moderate capacity and a profit margin which leads to an additional exploration of demand information to avoid capacitated demands, $\mathbb{E}[\pi^{III}(c)] < \mathbb{E}[\pi^{I}(c)]$. Lastly, for $\overline{c} \leq c \leq w$, then a small profit margin $p - c$ and the DM does not explore d additionally as his ex-ante expected gain from exploring d does not offset the additional cost u_R, making $\mathbb{E}[\pi^{III}(c)] \geq \mathbb{E}[\pi^{I}(c)]$. Note that, unlike the direct exploration of d in (3.13), if the DM additionally explores demand information, the DM's expected profit $\mathbb{E}[\pi^{I}(c)]$ depends on c explored from capacity information, $K^*(c)$.

Lemma 3.4. *A decision maker has an ex-ante incentive to explore information.*

1. *if* $\mathbb{E}[c] \leq c^{I} \leq c^{II}$ *or* $\mathbb{E}[c] \leq c^{II} \leq c^{I}$, *no information is explored.*
2. *if* $c^{II} \leq \mathbb{E}[c] \leq c^{I}$ *or* $c^{II} \leq c^{I} \leq \mathbb{E}[c]$, *demand information is explored.*
3. *if* $c^{I} \leq \mathbb{E}[c] \leq c^{II}$ *or* $c^{I} \leq c^{II} \leq \mathbb{E}[c]$, *capacity information is explored.*
4. *After capacity exploration, if* $\underline{c} < c < \overline{c}$, *demand information is explored additionally.*

with the thresholds obtained by $\mathbb{E}[\pi(c^{I})] = \mathbb{E}[\pi(K)]$, *and* $\mathbb{E}[\pi(c^{II})] = \mathbb{E}[\pi^{II}]$ *from* (3.13) *and* (3.14) *where,* $\mathbb{E}[\pi(c)] = (p - c)K(c) - p\int_0^{K(c)} F(d)dd$.

The threshold costs c^{I}, and c^{II} make ex-ante expected profit for capacity exploration $\mathbb{E}[\pi(K)]$ and demand exploration $\mathbb{E}[\pi^{II}]$ equal to a newsvendor profit, $\mathbb{E}[\pi(c)]$. It is noteworthy that c^{I} and c^{II} are the derived values for decision support in terms of the different magnitude of costs for exploring information. (c^{I}, and c^{II} are not the costs that the DM can access in the ERP system.) Hence, c^{I} and c^{II} are analogous to the "investment costs" for the DM to expect in exploring certain information. Defining the costs c^{I}, and c^{II} that make the ex-ante expected profit under capacity and demand exploration equivalent to $\mathbb{E}[\pi(c^{I})]$ and $\mathbb{E}[\pi(c^{II})]$ enables us to compare the exploration decision rules in terms of the expected cost for exploring information (i.e., the higher the expected investment costs c^{I} and c^{II} are, the less likely the DM explores certain information).

While both exploration decisions are made ex-ante, $c^{II} < c^{I}$ implies the case where the cost of demand exploration is relatively lower than the cost of capacity exploration. Hence, if $c^{II} \leq \mathbb{E}[c] \leq c^{I}$ or $c^{II} \leq c^{I} \leq \mathbb{E}[c]$, the DM explores demand information directly. The rationale behind the DM for exploring capacity ex-ante from Lemma 3.4 is that if the (expected) capacity is low, it signals the DM that he needs to explore demand to avoid capacitated fulfillment as long as incurring the cost of exploration offsets the expected profit by exploration. On the other hand, if the (expected) capacity level is sufficiently high, the DM would not need to explore further information. Anticipating these cases, the DM obtains c^I, which makes the ex-ante profit under capacity exploration equivalent to the newsvendor profit. If $K(c^I)$ is sufficiently large ($\mathbb{E}[c] \geq c^{II} \geq c^I$ or $c^{II} \geq \mathbb{E}[c] \geq c^I$), the DM has the incentive to explore capacity information.

The last case from Lemma 3.4 denotes that the DM has the incentive to explore demand information additionally as $\mathbb{E}[\pi^{III}(c)] < \mathbb{E}[\pi^I(c)]$. As Figure 3.4 illustrates, once the DM explores capacity information, he may find it beneficial to further explore d depending on the explored value on $K^*(c)$. A significantly high or low c both reduce the DM's incentive to explore d additionally as a high profit margin along with a large capacity $K^*(c)$ ensures a high ex-ante profit without knowing demand and a low profit margin does not compensate the additional cost of demand exploration. We now explore the impact of the expectation and the variability of prior demand distribution on the DM's information exploration decision.

Proposition 3.6. *For random demand D_1 and D_2 with prior distributions $F_1(d)$ and $F_2(d)$,*

1. *if the DM has a higher expectation of demand prior distribution $\mathbb{E}[D_1] > \mathbb{E}[D_2]$, he is less likely to explore demand information while the decision on capacity exploration is not affected ($c_1^I = c_2^I$, $c_1^{II} > c_2^{II}$, and $\mathbb{E}_1[c] = \mathbb{E}_2[c]$).*

2. *if the DM has a more variable demand prior distribution $\mathrm{Var}(D_1) > \mathrm{Var}(D_2)$, he is more likely to explore both demand and capacity information ($c_1^I < c_2^I$, $c_1^{II} < c_2^{II}$, and $\mathbb{E}_1[c] = \mathbb{E}_2[c]$).*

Similarly, the expectation and the variability of prior cost distribution show the following effect on the DM's information exploration decision:

Proposition 3.7. *For random cost C_1 and C_2 with prior distributions $G_1(c)$ and $G_2(c)$,*

1. *if the DM has a higher cost expectation $\mathbb{E}[C_1] > \mathbb{E}[C_2]$, he is more likely to explore capacity information as $\mathbb{E}[C_1] - c_1^I > \mathbb{E}[C_2] - c_2^I$. For demand information, the DM is more likely to explore if $c_2^{II} < \frac{p}{2}$ as $\mathbb{E}[C_1] - c_1^{II} > \mathbb{E}[C_2] - c_2^{II}$.*

2. *if the DM has a more variable cost prior distribution $\mathrm{Var}(C_1) > \mathrm{Var}(C_2)$, he is more likely to explore capacity information, whilst the decision on the demand exploration is not affected ($c_1^I < c_2^I$, $c_1^{II} = c_2^{II}$, and $\mathbb{E}[C_1] = \mathbb{E}[C_2]$).*

The implications from Propositions 3.6 and 3.7 are 1) although a DM has a high demand expectation, interestingly, his ex-ante incentive to explore capacity information does not increase.

This is because although capacity information is unknown, the optimal capacity is set relative to the expectation of demand $\mathbb{E}[D]$ and the general ex-ante expected profit under both demand and capacity uncertainty increases relative to $\mathbb{E}[D]$. Further, 2) the DM's incentive to explore demand information is independent of increasing variability in the capacity cost prior. The demand variability impacts the ex-ante expected profits for no exploration and capacity exploration. However, the capacity cost variability only affects the ex-ante profit of capacity exploration. Hence, a high demand uncertainty encourages the DM to explore both information, while capacity uncertainty makes the DM more likely to explore capacity information only.

Recall from Lemma 3.4, the DM's ex-ante incentive for capacity exploration is to get a signal whether to explore demand or not, in case exploring demand information initially is too costly $c^I < c^{II}$. Specifically, anticipating the instances where he may need to additionally explore d, a higher variability in cost prior makes the DM's ex-ante profit under capacity exploration increases for $0 < c < \underline{c}$ and $\bar{c} < c \leq w$. Hence, although the expected profit under demand exploration is not influenced by the variability of the cost prior, the DM's incentive to explore capacity increases as the prior cost distribution has high uncertainty.

The following presents comparisons of how variability in demand and capacity prior distributions impacts the information exploration and revelation decisions in the decentralized setting: 1) In the case of highly variable demand prior, the decision maker has a greater incentive to explore both demand and capacity information. Additionally, the incentive of the retailer and supplier to reveal such information increases (Proposition 3.3). 2) When the cost (capacity) prior is highly variable, the decision maker is more likely to explore capacity information, while the supplier's incentive to reveal this information decreases. However, the retailer is more likely to disclose the capacity information (Proposition 3.4), while the incentive to explore demand information remains unchanged.

3.2.4 Information sharing decision of risk-averse agents

Assume that the players maximize their expected utility functions $\mathbb{E}[U(\pi)]$ with $U'(\pi) > 0$ and $U''(\pi) < 0$. Eeckhoudt et al. (1995) prove that a risk-averse newsvendor who maximizes expected utility sets a lower capacity than a risk-neutral newsvendor. In the context of information sharing, we pose the following question: If the supplier or retailer are risk-averse, would they be more willing to share their information to avoid uncertainty or more reluctant to reveal the information compared to the risk-neutral supplier or retailer?

Sharing decision under risk-aversion is relevant in practice, considering that companies are reluctant to share information due to uncertain outcomes and the risk of informed companies' opportunistic behavior (Chen 2003). On the other hand, information sharing enables agents to make decisions without uncertainty (e.g., prior beliefs). While previous work incorporates risk attitude under uncertainty, our setting under sharing decisions differs from these works. Under the sharing decision, withholding information results in keeping uncertain information of the other player and may indirectly influence one's expected utility, whereas sharing information may let the players obtain utilities with certainty. We denote a risk-averse player i as a superscript $i(A)$ and a risk neutral player i as a superscript $i(N)$. We delineate two findings observed when the retailer and the supplier are risk-averse, respectively.

Lemma 3.5. *If a retailer is risk-averse, she is more likely to share demand information, while a supplier facing a risk-averse retailer is less likely to reveal capacity information as the retailer's reduced expectation of capacity decreases the supplier's incentive to share capacity information, leading to* $\widehat{TH}^{R(A)} \leq \widehat{TH}^{R(N)}$ *and* $\widehat{TH}^{R(A)}_{s:ns} \leq \widehat{TH}^{R(N)}_{s:ns}$.

A retailer's incentive to disclose demand information is driven by avoiding the capacity under-investment of the supplier. Lemma 3.5 implies that if the retailer is risk-averse, her utility from avoiding demand uncertainty by sharing demand upfront is always higher than if she is risk neutral. This is because the risk-averse retailer's incentive to remain silent, keeping the demand uncertainty in the supply chain, decreases as $\mathbb{E}[U(\widehat{\pi}_R^{IV})] < U(\mathbb{E}[\widehat{\pi}_R^{IV}])$. Therefore, a risk-averse retailer sets a lower sharing threshold, $\widehat{TH}^{R(A)} < \widehat{TH}^{R(N)}$.

As shown in Proposition 3.2, the supplier has a higher incentive to reveal his capacity when the retailer's expected capacity from the supplier is significantly higher than his capacity information, denoted as $F(\widehat{TH}_{s:ns}) - F(TH_{s:s})$. However, when the supplier facing a risk-averse retailer knows that the retailer is more likely to reveal demand as her expected utility of remaining silent based on the expectation of capacity reduces. Subsequently, the supplier is more reluctant to share. Considering that the retailer's unilateral demand sharing is the best case for the supplier, the result that the risk-averse retailer's more likeliness to share makes the supplier less likely to reveal capacity information follows intuition.

Lemma 3.6. *In case a supplier is risk-averse, a retailer is more likely to reveal demand information due to the risk-averse supplier's reduced capacity* ($\widehat{K}^{S(A)} < \widehat{K}^{S(N)}$)*. However, the risk-averse supplier is less likely to share capacity information, if the expected utility under demand uncertainty is significantly low, sufficing* $\delta^{S(A)} < \delta_{u_S}$*. where,* $\delta_{u_S} = \mathbb{E}\left[U(\pi_S^{IV})\right] - \mathbb{E}\left[U(\pi_S^{III})\right]$ *and* $\delta^{S(A)} = (1 - F(TH_{s:s})) \cdot \left(U(\pi_S^{I}) - \mathbb{E}\left[U(\pi_S^{III})\right]\right) - (1 - F(\widehat{TH}_{s:ns})) \cdot \left(U(\pi_S^{II}) - \mathbb{E}\left[U(\pi_S^{IV})\right]\right)$.

Lemma 3.6 implies that a risk-neutral retailer facing a risk-averse supplier anticipates a lower optimal capacity level from the supplier as $\mathbb{E}[U(\pi_S^{IV})] < U(\mathbb{E}[\pi_S^{IV})])$. This reduced capacity for any given c makes the retailer's expected sales reduce when remaining silent. Therefore, the retailer, in her own interest, has more incentive to reveal demand information if the supplier is risk-averse as $\widehat{TH}^R$ decreases. In contrast to the impact of risk-aversion on the retailer's sharing decision in Lemma 3.5, a risk-averse supplier does not have a strict likeliness to share more or less. The main reason is that the supplier's capacity disclosure still entails demand uncertainty while the retailer's demand revelation leads to $d = K^*$ and brings a certain utility $U(\pi_R^{II})$. Hence, the supplier's sharing decision is made under a trade-off between 1) the "expected" utility of remaining silent and 2) the "expected" gain from reciprocally receiving the retailer's demand information.

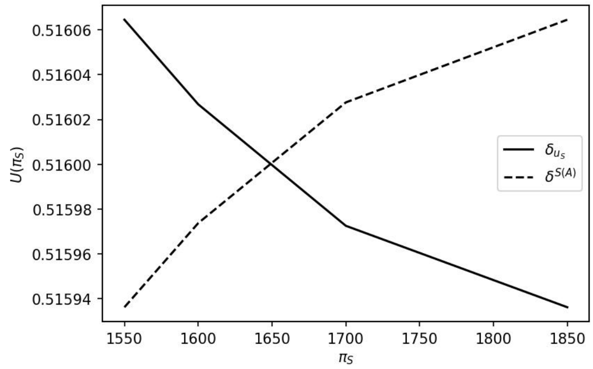
While the risk-neutral supplier's loss from disclosing capacity information is constant to $\mathbb{E}\left[\pi_S^{IV}\right] - \mathbb{E}\left[\pi_S^{III}\right] = u_S$, the risk-averse supplier's expected utility of disclosing information δ_{u_S} decreases as the expected utility under demand prior is higher due to $U'(\pi) > 0$ and $U''(\pi) < 0$. Hence, if the expected utility of receiving demand information by sharing capacity information $\delta^{S(A)}$ cannot outweigh the investment for proactive sharing δ_{u_S}, the risk-averse supplier is less likely to share.

Figure 3.5 illustrates graphically such a trade-off relationship between the expected utility gain from sharing $\delta^{S(A)}$ and the loss from revealing the information δ_{u_S}. It is shown that $\delta^{S(A)}$ increases as the supplier's expectation on demand prior increases as under a high expected demand prior, the saving from the expected overage decreases significantly after receiving the retailer's demand information. In the meantime, δ_{u_S} decreases as the supplier's marginal utility for incurring the cost of sharing decreases as the expectation on demand prior increases. Therefore, for $\delta_{u_S} \geq \delta^{S(A)}$, the risk-averse supplier's reluctance towards taking the risk of sharing increases. Hence, he is more likely to withhold information. On the other hand, for $\delta_{u_S} < \delta^{S(A)}$, the risk-averse supplier is more likely to reveal his capacity information.

The interpretation is that if the supplier has a high expectation on demand prior, the marginal differences in the expected utility to choose between sharing and withholding information δ_{u_S} is relatively small. Hence, he is more likely to share capacity information than the risk-neutral supplier. On the other hand, under a low expectation of the retailer's demand, the supplier's expected utility under demand uncertainty is low. Hence, the risk-averse supplier sees a higher utility loss by disclosing capacity information with the information-sharing cost.

The risk-averse supplier's sharing rule is somewhat counter-intuitive. A risk-averse supplier, under a higher expected utility by remaining silent, is more likely to disclose the capacity information. This is because a risk-averse supplier with a significantly low expected utility under demand uncertainty does not know whether the retailer would share demand information reciprocally or not, while the incentive of his capacity revelation significantly lies in receiving demand information. Hence, the supplier, being risk-averse and having a low expected utility under demand prior, is more reluctant to disclose capacity information as the marginal utility of sharing cost is relatively high.

Figure 3.5: Risk-averse Supplier's Sharing Decision Trade-off
($p = 20$, $w = 10$, $u_S = 150$, $u_R = 250$, $D \sim U(100, 300)$, $C \sim U(0, w)$, and $U(x) = \frac{x^{1-0.25}}{1-0.25}$)



3.3 Conclusions and future research

Motivated by the recent emphasis on forming information exchange networks in various industries, we proposed a stylized model as an initial attempt to analyze the intrinsic value for a retailer and a supplier to exchange their demand and capacity information. We analytically showed that under the benchmark case, both players share their information if demand or capacity is significantly low or high. On the other hand, in the decentralized setting, a supplier shares capacity information if the capacity level is intermediate, and the retailer discloses demand information only when the information is above a certain threshold. While the retailer's incentive to reveal demand information increases as she anticipates capacitated fulfillment issues. The supplier's incentive to reveal capacity information decreases if he expects a high probability of the retailer's unilateral sharing.

Moreover, in the benchmark case, the players are more likely to reveal their information if the other's prior belief is highly variable. On the other hand, we found that in decentralized sharing, an increasing demand variability induces the retailer to be more likely to share demand. However, if the retailer expects highly variable capacity from the supplier associated with increasing variability in the cost prior, surprisingly, the supplier is more likely to withhold capacity information as an increased probability of a retailer's unilateral demand sharing demotivates a supplier to reveal capacity information. We further showed that when the retailer and the supplier make information sharing decisions, the incentive to withhold private information decreases if doing so leads to the other's signal-based prior update.

The sensitivity analysis shows that if the supplier has a high expectation of demand prior, both players are more likely to remain silent. If the retailer has a high expectation of cost prior, she is more likely to disclose demand information, while the supplier is more reluctant to share capacity information. By extending our analysis to the case where the players are risk-averse, we demonstrated that the major sharing rules from the base model under risk neutrality continue to hold. However, the retailer's risk-aversion makes her more likely to reveal demand information, while the risk-averse supplier may even be more likely to withhold capacity information.

Our stylized analytical model has several limitations that require further investigations in the future. First, our analysis assumes that each player's private information is deterministic. Therefore, future research could investigate sharing strategies for stochastic demand and cost forecast information combined with imperfect signals. Second, our model does not incorporate the possibility of misrepresenting private information and focuses on non-monetary and voluntary information-sharing decisions. However, if an informing party's sharing benefits an informed party, optimal splitting of information-sharing costs could encourage the informing party to reveal information and reduce mutually remaining silent cases. Hence, developing sharing mechanisms without the assumption of truthful sharing can be of interest in the future to optimize the transfer payment of information-sharing costs.

Lastly, our model focuses on a single-period decision between a retailer and a supplier. As information sharing is often continuous with multiple partners, one may incorporate partial information sharing in multilateral sharing decisions, such as sharing a certain range of private information among multiple retailers and suppliers to obtain further insights. Moreover, multi-period information sharing with dynamically evolving private information and Bayesian learning over the periods to integrate the question of timing for sharing is not well understood. Ultimately, there are many under-investigated research avenues in the area of voluntary information exchange among supply chain members with different sets of private information.

Appendix

Proof of Proposition 3.1 Based on the supplier's (expected) profits $\pi_S^{II} > \pi_S^I$ and $\mathbb{E}[\widehat{\pi}_S^{IV}] > \mathbb{E}[\widehat{\pi}_S^{III}]$, the condition for the retailer to share demand information with the initial prior follows

$$\pi_R^{II} - \mathbb{E}[\pi_R^{IV}] > 0 \Leftrightarrow d \cdot (1 - G(c^*)) - \int_{c^*}^{w} K^*(c) \cdot g(c) dc - \frac{u_R}{p - w} > 0.$$

Define TH^R as an equilibrium demand level for the retailer that makes $\pi_R^{II} = \mathbb{E}[\pi_R^{IV}]$, we have

$$TH^R = \frac{u_R}{p - w} - \int_{c^*}^{w} \frac{\partial K^*(c)}{\partial c} \cdot G(c) dc.$$

The retailer's Bayesian equilibrium sharing decision considers the supplier's demand prior update after remaining silent ($d < TH^R$). For an updated demand threshold $\widehat{TH}^R$, if it reaches an equilibrium state, the prior demand is no longer truncated as $\widehat{f}(d) = \frac{f(d)}{\widehat{F}(\widehat{TH}^R)}$ for $d \in (0, \widehat{TH}^R)$, making $c^* = w \cdot (1 - F(\widehat{TH}^R)) = 0$. Therefore, the retailer's decision rule, adapting the prior demand distribution converged to a steady state holds the following condition

$$\widehat{TH}^R = \frac{u_R}{p - w} + \int_{0}^{w} \widehat{K}^*(c) \cdot g(c) dc.$$

Moreover, we prove that $\widehat{TH}^R$ is a unique demand sharing threshold that fulfills $\pi_R^{II} - \mathbb{E}\left[\widehat{\pi}_R^{IV}\right] = 0$ by showing $\frac{\partial \pi_R^{II} - \mathbb{E}\left[\widehat{\pi}_R^{IV}\right]}{\partial d} > 0$ for $d \in (0, \infty)$, $\pi_R^{II} - \mathbb{E}\left[\widehat{\pi}_R^{IV}\right] < 0$ for $d = 0$ and $\pi_R^{II} - \mathbb{E}\left[\widehat{\pi}_R^{IV}\right] > 0$ for $d = \infty$.

$$\pi_R^{II} - \mathbb{E}\left[\widehat{\pi}_R^{IV}\right] \Leftrightarrow \int_{c^*}^{w} \left(d - \widehat{K}^*(c)\right) \cdot g(c) dc - \frac{u_R}{p - w}$$

While $d - \widehat{K}^*(c) > 0$ for any random $\widehat{K}^*(c))$ for $c \in (c^*, w)$, we define $H(d) = d - \widehat{K}^*(c)$. From the derivative rule for product,

$$\frac{\partial \pi_R^{II} - \mathbb{E}\left[\widehat{\pi}_R^{IV}\right]}{\partial d} = -G'(c^*) \cdot H(d) + (1 - G(c^*)) \cdot H'(d).$$

As $G'(c^*) = \frac{\partial G(c^*)}{\partial c^*} \cdot \frac{\partial c^*}{\partial d} < 0$ based on the chain rule ($\because \frac{\partial c^*}{\partial d} = -\widehat{f}(d) \cdot w < 0$) and $H'(d) > 0$, $\frac{\partial \pi_R^{II} - \mathbb{E}\left[\widehat{\pi}_R^{IV}\right]}{\partial d} > 0$. Further, for $d = 0$, $\pi_R^{II} = -u_R < 0$ and $\mathbb{E}\left[\widehat{\pi}_R^{IV}\right] = 0$. For $d = \infty$, $\pi_R^{II} = \infty > 0$ and $\mathbb{E}\left[\widehat{\pi}_R^{IV}\right] > 0$. Hence, we have $\pi_R^{II} - \mathbb{E}\left[\widehat{\pi}_R^{IV}\right] < 0$ for $d = 0$ and $\pi_R^{II} - \mathbb{E}\left[\widehat{\pi}_R^{IV}\right] > 0$ for $d = \infty$. Since $\frac{\partial \pi_R^{II} - \mathbb{E}\left[\widehat{\pi}_R^{IV}\right]}{\partial d} > 0$, we have a unique $\widehat{TH}^R$ and as $\frac{\partial \widehat{K}^*(c)}{\partial c} < 0$, the demand threshold level is always positive $\widehat{TH}^R > 0$. □

Proof of Proposition 3.2 If the supplier shares capacity information, there exists a threshold demand $TH_{s:s}$, which makes the retailer indifferent between sharing and not sharing private demand information, $\pi_R^I = \pi_R^{III}$.

$$TH_{s:s} = \frac{u_R}{p-w} + K^*$$

On the other hand, if the supplier withholds capacity information, a threshold demand level $TH_{s:ns}$ that makes the retailer's profit from sharing demand information equal to the expected profit of remaining silent $\pi_R^{II} = \mathbb{E}\left[\pi_R^{IV}\right]$ follows:

$$TH_{s:ns} = \frac{u_R}{p-w} - \int_{c^*}^{w} \frac{\partial K^*(c)}{\partial c} \cdot G(c)dc.$$

When the supplier reveals the information, the retailer with $d \leq TH_{s:s}$ remains silent and the reciprocal sharing occurs with probability $1 - F(TH_{s:s})$. Similarly, when the supplier remains silent, the retailer unilaterally shares with the probability, $1 - F(TH_{s:ns})$. Using the retailer's probabilistic reactions on sharing, the supplier's expected profits for sharing ($\mathbb{E}\left[\pi_S(S)\right]$) and remaining silents ($\mathbb{E}\left[\pi_S(NS)\right]$) are

$$\begin{aligned} \mathbb{E}\left[\pi_S(S)\right] &= (1 - F(TH_{s:s})) \cdot \pi_S^I + F(TH_{s:s}) \cdot \mathbb{E}\left[\pi_S^{III}\right], \text{ and} \\ \mathbb{E}\left[\pi_S(NS)\right] &= (1 - F(TH_{s:ns})) \cdot \pi_S^{II} + F(TH_{s:ns}) \cdot \mathbb{E}\left[\pi_S^{IV}\right]. \end{aligned} \tag{3.15}$$

The condition for the supplier to share capacity information is

$$\mathbb{E}[\pi_S(S)] - \mathbb{E}[\pi_S(NS)] > 0 \Leftrightarrow (F(TH_{s:ns}) - F(TH_{s:s})) \cdot \left(\pi_S^{II} - \mathbb{E}[\pi_S^{IV}]\right) > u_S,$$

using the fact that $\pi_S^{II} - \pi_S^I = u_S$, $\mathbb{E}[\pi_S^{IV}] - \mathbb{E}[\pi_S^{III}] = u_S$ and $\pi_S^I - \mathbb{E}[\widehat{\pi}_S^{III}] = \pi_S^{II} - \mathbb{E}[\widehat{\pi}_S^{IV}]$. Based on (3.2) and (3.5), if the capacity cost c fulfills the following condition, the supplier shares capacity information.

$$(F(TH_{s:ns}) - F(TH_{s:s})) \left\{ (w-c) \cdot (d - K^*(c)) + w \cdot \int_0^{K^*(c)} (K^*(c) - d) \cdot f(d) \mathrm{d}d \right\} > u_S$$

Given that $F(TH_{s:ns}) - F(TH_{s:s}) > 0 \Leftrightarrow TH_{s:ns}^R \geq TH_{s:ns}^R \geq K^*$, for $F(TH_{s:ns}) - F(TH_{s:s})) > 0$ the supplier's minimum economic gain from revealing capacity information becomes

$$\frac{(w-c) \cdot u_R}{p-w} + w \cdot \int_0^{K^*(c)} (K^*(c) - d) \cdot f(d) \mathrm{d}d.$$

While $\frac{\partial^2 \pi_S^{II}}{\partial c^2} = 0$, we can reformulate $-\mathbb{E}[\pi_S^{IV}]$ as

$$-\mathbb{E}[\pi_S^{IV}] = \int_0^{K^*} c \cdot (K^* - d) f(d)dd + \int_{K^*}^{\infty} (w - c) \cdot (d - K^*) f(d)dd.$$

Using $F(K^*) = \frac{w-c}{w}$ and integration by parts,

$$-\mathbb{E}[\pi_S^{IV}] = (w - c) \cdot \mathbb{E}[D] - p \cdot \int_0^{K^*} df(d)dd = (w - c) \cdot (\mathbb{E}[D] - K^*) + w \cdot \int_0^{K^*} F(d)dd.$$

From the chain rule, $\frac{\partial w \cdot \int_0^{K^*} F(d)dd}{\partial c} = w \cdot F(K^*)dd \cdot \frac{\partial K^*}{\partial c} = (w - c) \cdot \frac{\partial K^*}{\partial c}$ ($\because F(K^*) = \frac{w-c}{w}$). Accordingly, the first-order and the second-order conditions follow

$$-\frac{\partial\, \mathbb{E}[\pi_S^{IV}]}{\partial c} = K^* - \mathbb{E}[D] \quad and \quad -\frac{\partial^2\, \mathbb{E}[\pi_S^{IV}]}{\partial c^2} = \frac{\partial K^*}{\partial c} < 0.$$

Therefore,

$$\frac{\partial^2 \pi_S^{II} - \mathbb{E}[\pi_S^{IV}]}{\partial c^2} < 0.$$

Note that the cost prior update $\widehat{G}(c)$ does not impact the supplier's expected profits in $\mathbb{E}[\pi_S^{IV}]$ directly. However, the retailer's decision rule on $TH_{s:ns}$ influences the unilateral sharing probability $1 - F(\widehat{TH}_{s:ns})$ as

$$-\int_{c^*}^{w} \frac{\partial K^*(c)}{\partial c} \cdot G(c)dc < -\int_{c^*}^{w} \frac{\partial K^*(c)}{\partial c} \cdot \widehat{G}(c)dc \Leftrightarrow TH_{s:ns} < \widehat{TH}_{s:ns}.$$

As $(F(\widehat{TH}_{s:ns}) - F(TH_{s:s})) \cdot \left(\pi_S^{II} - \mathbb{E}[\pi_S^{IV}]\right) < 0$ for $c = 0$ and $c = w$, while $\frac{\partial^2 \pi_S^{II} - \mathbb{E}[\pi_S^{IV}]}{\partial c^2} < 0$ and $\frac{\partial F(\widehat{TH}_{s:ns}) - F(TH_{s:s})}{\partial c} > 0$, there exist two roots $0 \le c = TH_{lb}^S$ and $c = TH_{ub}^S \le w$ that fulfill $(F(TH_{s:ns}) - F(TH_{s:s})) \cdot \left(\pi_S^{II} - \mathbb{E}[\pi_S^{IV}]\right) = u_S$ if for any given $c \in (0, w)$,

$$(F(\widehat{TH}_{s:ns}) - F(TH_{s:s})) \left(\frac{(w - c) \cdot u_R}{p - w} + w \cdot \int_0^{K^*(c)} F(d)\mathrm{d}d \right) > u_S. \qquad \square$$

Proof of Theorem 3.1 Based on the findings from Propositions 3.1 and 3.2, we obtain each player's equilibrium sharing rule. Under a Bayesian Nash setting, the players make sharing decisions while not knowing the other's expected profit associated with asymmetric private information. Hence, the resulting information policy combines each player's equilibrium sharing decision. As the supplier's sharing incurs while not knowing the retailer's action profile if $\widehat{TH}_{lb}^S \le c \le \widehat{TH}_{ub}^S$, while the retailer's sharing occurs in case $d \ge \widehat{TH}^R$, the mutual sharing is an optimal policy if each player's d and $K^*(c)$ fulfill such conditions. Reversely, for $d < \widehat{TH}^R$ and $\widehat{TH}_{lb}^S > c \vee c > \widehat{TH}_{ub}^S$, mutually remaining silent becomes a sharing policy. Similarly, the unilateral sharing from each player can be derived from Propositions 3.1 and 3.2. □

Proof of Lemma 3.1 After updating the demand prior, $\widehat{F}(d)$ follows

$$\widehat{F}(d) = \int_0^d \frac{f(d)}{F(\widehat{TH}^R)} dd.$$

As $0 \le F(\widehat{TH}^R) \le 1$,

$$\widehat{F}(d) = \frac{F(d)}{F(\widehat{TH}^R)} \ge F(d) \quad for \quad d \in (0, \infty). \tag{3.16}$$

Given that $F(d) < \widehat{F}(d)$, for any capacity level K, $F(K) \le \widehat{F}(K)$. Reversely, for a critical fractile $\frac{w-c}{w}$,

$$\widehat{F}^{-1}(\frac{w-c}{w}) \le F^{-1}(\frac{w-c}{w}) \Leftrightarrow \widehat{K}^* \le K^*.$$ □

Proof of Lemma 3.2 Denoting $\tau = 1 - G(\widehat{TH}^S_{ub}) + G(\widehat{TH}^S_{lb})$ and based on (3.9), the cumulative distribution of updated cost prior $\widehat{G}(c)$ in a steady-state is defined as

$$\widehat{G}(c) = \begin{cases} \frac{G(c)}{\tau} & 0 < c < \widehat{TH}^S_{lb} \\ \frac{G(\widehat{TH}^S_{lb})}{\tau} & \widehat{TH}^S_{lb} \le c \le \widehat{TH}^S_{ub} \\ \frac{G(c)}{\tau} - \frac{G(\widehat{TH}^S_{ub}) - G(\widehat{TH}^S_{lb})}{\tau} & \widehat{TH}^S_{ub} < c < w \end{cases}$$

For a non-decreasing function $G(c)$ and $\widehat{G}(c)$,

$$\begin{aligned} &\int_0^w \widehat{G}(c) - G(c)dc \\ &= \frac{1}{\tau}\int_0^{\widehat{TH}^S_{lb}} G(c)dc + \frac{1}{\tau}\int_{\widehat{TH}^S_{lb}}^{\widehat{TH}^S_{ub}} G(\widehat{TH}^S_{lb})dc + \frac{1}{\tau}\int_{\widehat{TH}^S_{ub}}^{w} G(\widehat{TH}^S_{lb}) + G(c) - G(\widehat{TH}^S_{ub})dc \\ &- \left(\int_0^{\widehat{TH}^S_{lb}} G(c)dc + \int_{\widehat{TH}^S_{lb}}^{\widehat{TH}^S_{ub}} G(c)dc + \int_{\widehat{TH}^S_{ub}}^{w} G(c)dc\right). \end{aligned}$$

Defining $S(c) = \int_0^c G(c)dc$ for ease of exposition,

$$\begin{aligned} &\int_0^w \widehat{G}(c) - G(c)dc \\ &= \frac{S(\widehat{TH}^S_{lb})}{\tau} + \frac{G(\widehat{TH}^S_{lb})}{\tau}(w - \widehat{TH}^S_{lb}) + \frac{S(w) - S(\widehat{TH}^S_{ub})}{\tau} - \frac{G(\widehat{TH}^S_{ub})}{\tau}(w - \widehat{TH}^S_{ub}) - S(w). \end{aligned}$$

Using the property $\widehat{G}(\widehat{TH}_{lb}^{S}) = \widehat{G}(\widehat{TH}_{ub}^{S}) \Leftrightarrow \frac{G(\widehat{TH}_{lb}^{S})}{\tau} = \frac{G(\widehat{TH}_{lb}^{S})}{\tau}$ as $\widehat{g}(\widehat{TH}_{lb}^{S}) = \widehat{g}(\widehat{TH}_{ub}^{S}) = 0$,

$$\int_0^w \widehat{G}(c) - G(c)dc$$
$$= S(w)(\frac{1}{\tau} - 1) + \frac{G(\widehat{TH}_{lb}^{S})}{\tau}\left(\widehat{TH}_{ub}^{S} - \widehat{TH}_{lb}^{S}\right) - \frac{S(\widehat{TH}_{ub}^{S}) - S(\widehat{TH}_{lb}^{S})}{\tau}.$$

As $\frac{S(\widehat{TH}_{ub}^{S}) - S(\widehat{TH}_{lb}^{S})}{\tau} - \frac{G(\widehat{TH}_{lb}^{S})}{\tau}\left(\widehat{TH}_{ub}^{S} - \widehat{TH}_{lb}^{S}\right) \geq \int_{\widehat{TH}_{lb}^{S}}^{\widehat{TH}_{ub}^{S}} G(c) - G(\widehat{TH}_{lb}^{S})dc$, the following condition holds

$$S(w)(\frac{1}{\tau} - 1) \geq \frac{S(\widehat{TH}_{ub}^{S}) - S(\widehat{TH}_{lb}^{S})}{\tau} - \frac{G(\widehat{TH}_{lb}^{S})}{\tau}\left(\widehat{TH}_{ub}^{S} - \widehat{TH}_{lb}^{S}\right).$$

Therefore,

$$\int_0^w \widehat{G}(c) - G(c)dc \geq 0.$$

Moreover, from the second order stochastic dominance,

$$-\int_{c^*}^{w} \frac{\partial K(c)}{\partial c} \cdot G(c)dc \leq -\int_{c^*}^{w} \frac{\partial K(c)}{\partial c} \cdot \widehat{G}(c)dc \Leftrightarrow TH_{s:ns} \leq \widehat{TH}_{s:ns} \Leftrightarrow \mathbb{E}[K^*(c)] \leq \mathbb{E}[\widehat{K}^*(c)]. \qquad \square$$

Proof of Proposition 3.3 a) To analyze the impact of high expectation on demand prior, we apply the first-order stochastic dominance between two random variables D_1 and D_2, where

$$\mathbb{E}[D_1] > \mathbb{E}[D_2] \Leftrightarrow (F_1(d) - F_2(d)) \leq 0 \quad \text{for all } d, \text{ with strict inequality at some } d \in (0, \infty).$$

Equivalently, a stochastically larger demand prior leads to

$$F_1^{-1}(\frac{w-c}{w}) \geq F_2^{-1}(\frac{w-c}{w}) \Rightarrow TH_1^R \geq TH_2^R.$$

After the prior update,

$$\widehat{F}_2(\widehat{TH}_2^{R}) = \frac{F_2(\widehat{TH}_2^{R})}{F_2(\widehat{TH}_2^{R})} = 1 \geq \widehat{F}_1(\widehat{TH}_2^{R}) = \frac{F_1(\widehat{TH}_2^{R})}{F_1(\widehat{TH}_1^{R})}.$$

Since $F_2(d) - F_1(d) \geq 0$, the stochastic order of demand prior holds after updating the priors $\widehat{F}_2(d) > \widehat{F}_1(d)$ for $d \in (0, \widehat{TH}_2^{R})$, making

$$\widehat{F}_1^{-1}(\frac{w-c}{w}) > \widehat{F}_2^{-1}(\frac{w-c}{w}) \Rightarrow \mathbb{E}_1[\widehat{\pi}_R^{IV}] > \mathbb{E}_2[\widehat{\pi}_R^{IV}]$$

From (3.5), $\frac{\partial \mathbb{E}[\pi_S^{IV}]}{\partial \mathbb{E}[D]} > 0$. Subsequently,

$$\mathbb{E}_1[\pi_S^{IV}] \geq \mathbb{E}_2[\pi_S^{IV}] \Leftrightarrow \pi_S^{II} - \mathbb{E}_1[\pi_S^{IV}] \leq \pi_S^{II} - \mathbb{E}_2[\pi_S^{IV}].$$

b) Secondly, more variable demand prior holds for $\mathrm{Var}(D_1) > \mathrm{Var}(D_2) \Leftrightarrow \int_0^d (F_1(d) - F_2(d))\, dd \geq 0$ for all d, with strict inequality at some$d \in (0, \infty)$.

We denote the expected sales under low and high variability as follows and show that the inequality relationship holds for the two expected sales terms under different variability in demand priors:

$$\int_0^{c_2^*} d \cdot g(c)dc + \int_{c_2^*}^{w} K_2^*(c) \cdot g(c)dc > \int_0^{c_1^*} d \cdot g(c)dc + \int_{c_1^*}^{w} K_1^*(c) \cdot g(c)dc. \tag{3.17}$$

For $d < \mathbb{E}_1[D] = \mathbb{E}[D_2]$, $F_2(d) < F_1(d)$ and for $d \geq \mathbb{E}_1[D] = \mathbb{E}[D_2]$, $F_2(d) \geq F_1(d)$. For c^*, equal to $w \cdot (1 - F(d))$, if $d < \mathbb{E}[D]$, then $c_1^* < c_2^*$ and if $d \geq \mathbb{E}[D]$, then $c_1^* \geq c_2^*$. Therefore, for any $0 \leq d < \mathbb{E}[D]$, as $c_2^* > c_1^*$ we can reformulate (3.17) to

$$\int_{c_1^*}^{c_2^*} (d - K_1^*(c))\, g(c)dc + \int_{c_2^*}^{w} (K_2^*(c) - K_1^*(c)) \cdot g(c)dc > 0. \tag{3.18}$$

For $c \in (c_1^*, w)$, we have $d > K_1^*(c)$, accordingly, the first term in (3.18) is positive. Further, for $c \in (c_2^*, w)$, the expected capacity level under the low variable prior is higher than the expected capacity level under the high variable prior as by definition of $\mathrm{Var}(D_1) > \mathrm{Var}(D_2)$,

$$\int_0^d F_1(d) - F_2(d)dd > 0 \Rightarrow \int_{c_2^*}^{w} F_2^{-1}(\frac{w-c}{c}) \cdot g(c)dc > \int_{c_2^*}^{w} F_1^{-1}(\frac{w-c}{c}) \cdot g(c)dc.$$

Therefore, the second term in (3.18) is also positive. As the left hand side in (3.18) is always positive, the above inequality (3.18) holds for $0 \leq d < \mathbb{E}[D]$. Furthermore, for $\mathbb{E}[D] \leq d \leq \infty$, as $c_2^* \leq c_1^*$ the expected sales inequality can be rearranged to

$$\int_{c_2^*}^{c_1^*} (K_1^*(c) - d)\, g(c)dc + \int_{c_2^*}^{w} (K_1^*(c) - K_2^*(c)) \cdot g(c)dc > 0. \tag{3.19}$$

As before, the expected capacity level under $F_2(d)$ is higher than $F_1(d)$, $\mathbb{E}[K_2^*(c)] > \mathbb{E}[K_1^*(c)]$ for $c \in (c^*, w)$, making the second term in (3.19) stays positive. Also for $c \in (c_2^* < c < c_1^*)$ we have $d < K_1^*(c)$; hence the left hand side in (3.19) is again positive. Since for any c^*,

$$\int_0^{c_2^*} d \cdot g(c)dc + \int_{c_2^*}^{w} K_2^*(c) \cdot g(c)dc > \int_0^{c_1^*} d \cdot g(c)dc + \int_{c_1^*}^{w} K_1^*(c) \cdot g(c)dc \equiv \mathbb{E}_2\left[\pi_R^{IV}\right] > \mathbb{E}_1\left[\pi_R^{IV}\right].$$

We further show that if $\mathrm{Var}(D_1) > \mathrm{Var}(D_2)$, the variability is still larger even after prior updating $\mathrm{Var}(\widehat{D}_1) > \mathrm{Var}(\widehat{D}_2)$. For two prior distributions $F_1(d)$ and $F_2(d)$ with the differences only in variability, $\mathbb{E}_1[D] = \mathbb{E}[D_2]$, if $\widehat{TH}_1^R \in (0, \mathbb{E}[D])$, it is easy to see that $\widehat{D}_2 \succeq_{st} \widehat{D}_1$ as $\widehat{F}_1(x) - \widehat{F}_2(x) \geq 0 \Rightarrow \int_0^d \widehat{F}_1(d) - \widehat{F}_2(d)dd > 0$.

For $\widehat{TH}_1^R \in (\mathbb{E}[D], \widehat{TH}_2^R)$, as $F_1(\widehat{TH}_1^R) < F_2(\widehat{TH}_1^R)$

$$\int_0^d \widehat{F}_1(d) - \widehat{F}_2(d)dd = \int_0^d \frac{F_1(d)}{F_1(\widehat{TH}_1^R)} - \frac{F_2(d)}{F_2(\widehat{TH}_2^R)} dd \geq 0.$$

Therefore, from $\mathrm{Var}(\widehat{D}_1) > \mathrm{Var}(\widehat{D}_2)$

$$\widehat{TH}_1^R \leq \widehat{TH}_2^R \Leftrightarrow \mathbb{E}_1\left[\widehat{\pi}_R^{IV}\right] \leq \mathbb{E}_2\left[\widehat{\pi}_R^{IV}\right].$$

For the expected profit of the supplier, we show that for any capacity level of K, the supplier's expected profit under the less variable updated demand prior distribution is higher than the more variable updated demand prior distribution $\mathbb{E}_1\left[\pi_S^{IV}\right] \leq \mathbb{E}_2\left[\pi_S^{IV}\right]$. Defining $\pi_S^{IV} = -c \cdot K^* + w \cdot \min(d, K^*)$,

$$\mathbb{E}_1\left[\pi_S^{IV}\right] = \int_0^\infty \pi_S^{IV} \cdot f_1(d)dd \quad and \quad \mathbb{E}_2\left[\pi_S^{IV}\right] = \int_0^\infty \pi_S^{IV} \cdot f_2(d)dd$$

Using integration by parts, we can reformulate the expected profit to

$$\begin{aligned}
&\int_0^\infty \pi_S^{IV} \cdot f_1(d)dd = [\pi_S^{IV} \cdot F_1(d)]_0^\infty - \int_0^\infty \pi_S^{'IV} \cdot F_1(d)dd = \pi_S^{IV}(\infty) - \int_0^\infty \pi_S^{'IV} \cdot F_1(d)dd \\
&= \pi_S^{IV}(\infty) - [\pi_S^{'IV} \cdot S_1(d)]_0^\infty + \int_0^\infty \pi_S^{''IV} \cdot S_1(d)dd \quad where, \quad S(x) = \int_0^x F(x)dx \\
&= \pi_S^{IV}(\infty) - \pi_S^{'IV}(\infty) \cdot S_1(\infty) + \int_0^\infty \pi_S^{''IV} \cdot S_1(d)dd.
\end{aligned}$$

Analogous from $\mathbb{E}_1\left[\pi_S^{IV}\right]$,

$$\mathbb{E}_2\left[\pi_S^{IV}\right] = \pi_S^{IV}(\infty) - \pi_S^{'IV}(\infty) \cdot S_2(\infty) + \int_0^\infty \pi_S^{''IV} \cdot S_2(d)dd.$$

Using the extended form, we have

$$\mathbb{E}_2\left[\pi_S^{IV}\right] - \mathbb{E}_1\left[\pi_S^{IV}\right] = -\pi_S^{'IV}(\infty) \cdot (S_2(\infty) - S_1(\infty)) + \int_0^\infty \pi_S^{''IV} \cdot (S_2(d) - S_2(1))\, dd$$

As by definition $S_2(d) \leq S_1(d)$ (=$\mathrm{Var}(D_1) > \mathrm{Var}(D_2)$), $\pi_S^{'IV} \geq 0$ and $\pi_S^{''IV} = 0$ in d, we have $\mathbb{E}_2\left[\pi_S^{IV}\right] - \mathbb{E}_1\left[\pi_S^{IV}\right] \geq 0$. □

Proof of Proposition 3.4 a) We use the first-order stochastic dominance between two random variables C_1 and C_2 for cost prior distributions. A stochastically larger cost prior $C_1 \succeq_{st} C_2$ denotes a higher expectation on the cost prior $\mathbb{E}[C_1] > \mathbb{E}[C_2]$. We first show the impact of stochastically larger cost prior distribution on the retailer's sharing decision. The higher expectation on the capacity cost the retailer has $C_1 \succeq_{st} C_2$, she expects a lower expected capacity $\mathbb{E}[\widehat{K}_2^*(c)] \geq \mathbb{E}[\widehat{K}_1^*(c)]$ and a lower expected profit from remaining silent $\mathbb{E}_1\left[\widehat{\pi}_R^{IV}\right] \leq \mathbb{E}_2\left[\widehat{\pi}_R^{IV}\right]$.

For the supplier, a reduced benefit of the retailer's remaining silent implies $\widehat{TH}_{1s:ns} \leq \widehat{TH}_{2s:ns}$. While the cost prior does not influence the decision rule of the retailer for mutual sharing, $TH_{s:s}$ and the expected profit of the supplier, a decreasing threshold for the retailer's unilateral sharing $\widehat{TH}_{1s:ns} \leq \widehat{TH}_{2s:ns}$ (ie., the retailer's lower expectation of capacity) makes the supplier's incentive to reveal capacity information decrease, $F(\widehat{TH}_{1s:ns}) - F(TH_{s:s}) < F(\widehat{TH}_{2s:ns}) - F(TH_{s:s})$.

b) If $\mathrm{Var}(C_1) > \mathrm{Var}(C_2)$, the more variable cost prior leads to a lower expected capacity $\mathbb{E}[\widehat{K}_2^*(c)] \geq \mathbb{E}[\widehat{K}_1^*(c)]$ for $c \in (c^*, w)$. As the lower expected capacity means the retailer's expected profit of remaining silent reduces $\mathbb{E}_1\left[\widehat{\pi}_R^{IV}\right] \leq \mathbb{E}_2\left[\widehat{\pi}_R^{IV}\right]$. The demand threshold also reduces accordingly $\widehat{TH}_1^R \leq \widehat{TH}_2^R$. Similar to the impact of an increasing expectation on the cost prior, the reduced incentive from the retailer under a more variable cost prior $\mathbb{E}_1\left[\pi_R^{IV}\right] \leq \mathbb{E}_2\left[\pi_R^{IV}\right]$ induces the supplier to remain silent as $F(\widehat{TH}_{1s:ns}) - F(TH_{s:s}) < F(\widehat{TH}_{2s:ns}) - F(TH_{s:s})$. $\square$

Proof of Lemma 3.3 a) For $\frac{\partial \pi^{II}}{\partial d} > 0$ and $\frac{\partial \mathbb{E}[\pi^{IV}]}{\partial d} = 0$, let $d = TH_{UB}^R$ be the demand level that makes $\pi^{II} - \mathbb{E}\left[\pi^{IV}\right] = u_R$ and $d = TH_{LB}^R$ be the demand level that makes $\mathbb{E}\left[\pi^{IV}\right] - \pi^{II} = u_R$. Formally,

$$TH_{UB}^R = \int_0^w \left[K(c) - \frac{p}{p-c}\int_0^{K(c)} (K(c)-d)f(d)dd\right] g(c)dc + \frac{u_R}{p-c}$$

$$TH_{LB}^R = \int_0^w \left[K(c) - \frac{p}{p-c}\int_0^{K(c)} (K(c)-d)f(d)dd\right] g(c)dc - \frac{u_R}{p-c}$$

Using the threshold demand levels, $|\pi^{II} - \mathbb{E}\left[\pi^{IV}\right]| > u_R$ is equivalent to

$$\pi^{II} - \mathbb{E}\left[\pi^{IV}\right] > u_R \Leftrightarrow d > TH_{UB}^R \quad and \quad \mathbb{E}\left[\pi^{IV}\right] - \pi^{II} > u_R \Leftrightarrow d < TH_{LB}^R.$$

Similarly, **b)** For $\frac{\partial \mathbb{E}[\pi^{III}]}{\partial c} < 0$ ($\because \frac{\partial K(c)}{\partial c} < 0$) and $\frac{\partial \mathbb{E}[\pi^{IV}]}{\partial c} = 0$, let $c = TH_{LB}^S$ be the capacity investment cost that makes $\mathbb{E}\left[\pi^{III}\right] - \mathbb{E}\left[\pi^{IV}\right] = u_S$ and $c = TH_{UB}^S$ be the capacity investment cost that makes $\mathbb{E}\left[\pi^{IV}\right] - \mathbb{E}\left[\pi^{III}\right] = u_S$. As $\frac{\partial \mathbb{E}\left[\pi^{III}\right]}{\partial c} < 0$, there exist $c = TH_{LB}^S$ and $c = TH_{UB}^S$ for $c \in (0, w)$.

From the cost thresholds TH_{UB}^S and TH_{LB}^S,

$$\pi^{III} - \mathbb{E}\left[\pi^{IV}\right] > u_S \Leftrightarrow c < TH_{LB}^S \quad and \quad \mathbb{E}\left[\pi^{IV}\right] - \pi^{III} > u_S \Leftrightarrow c > TH_U^S. \qquad \square$$

Proof of Proposition 3.5 We define random variables D_1 and D_2, with demand prior distributions $F_1(d)$ and $F_2(d)$, in which $Var(D_1) > Var(D_2)$ and $\mathbb{E}[D_1] = \mathbb{E}[D_1]$. By showing that the expected profit $\mathbb{E}\left[\pi^{III}\right]$ reduces as the variability in demand prior increases, we induce $TH_{UB}^S - TH_{LB}^S$ decreases in the demand prior variability.

$$\mathbb{E}_2\left[\pi^{III}\right] - \mathbb{E}_1\left[\pi^{III}\right] = (p-c)\cdot(K_2^* - K_1^*) + p\left(\int_0^{K_1^*} F_1(d)dd - \int_0^{K_2^*} F_2(d)dd\right). \tag{3.20}$$

For any critical ratio $\frac{p-c}{p}$, we have

$$\int_0^{K_1^*} F_1(d)dd \geq \int_0^{K_2^*} F_2(d)dd$$

from the second order stochastic dominance for D_2 over D_1.

If $\frac{p-c}{p} < 0.5 \Leftrightarrow K_1^* < K_2^*$, $\mathbb{E}_2\left[\pi^{III}\right] - \mathbb{E}_1\left[\pi^{III}\right] > 0$. Further, for $\frac{p-c}{p} \geq 0.5 \Leftrightarrow K_1^* \geq K_2^*$, as

$$\int_0^{\mu} \left(F_1(d) - F_2(d)\right) dd \geq \int_{K_2^*}^{K_1^*} \left(F_2(d) - F_1(d)\right) dd$$

(3.20) holds for

$$p\left(K_2^* - \int_0^{K_2^*} F_2(d)dd\right) - p\left(K_1^* - \int_0^{K_1^*} F_1(d)dd\right) - c\cdot(K_2^* - K_1^*) > 0 \Leftrightarrow \mathbb{E}_2\left[\pi^{III}\right] > \mathbb{E}_1\left[\pi^{III}\right].$$

As $\frac{\partial^2 \mathbb{E}\left[\pi^{III}\right]}{\partial c^2} > 0$ and $\frac{\partial \mathbb{E}\left[\pi^{III}\right]}{\partial c} > -K(c)$,

$$TH_{2LB}^S - TH_{1LB}^S < TH_{2UB}^S - TH_{1UB}^S \Leftrightarrow TH_{1UB}^S - TH_{1LB}^S < TH_{2UB}^S - TH_{2LB}^S.$$

b) From Lemma 3.3,

$$TH_{UB}^R - TH_{LB}^R = \int_0^w \frac{u_R}{p-c} g(c)dc = \frac{u_R}{p-w} - \int_0^w \frac{u_R}{(p-c)^2} G(c)dc$$

For random variables C_1 and C_2, with cost prior distributions $G_1(c)$ and $G_2(c)$, if the retailer has a more variable capacity cost prior distribution $Var(C_1) > Var(C_2)$, while $\mathbb{E}[C_1] = \mathbb{E}[C_1]$, then the second-order stochastic dominance holds for C_2. By definition, we have

$$\int_0^w \frac{u_R}{(p-c)^2} G_1(c)dc > \int_0^w \frac{u_R}{(p-c)^2} G_2(c)dc \Leftrightarrow TH^R_{1UB} - TH^R_{1LB} < TH^R_{2UB} - TH^R_{2LB}. \qquad \square$$

Proof of Lemma 3.4 We first show $\mathbb{E}[\pi(c)] = (p-c)K(c) - p\int_0^{K(c)} F(d)dd$ conditional on c has $\frac{\partial \mathbb{E}[\pi(c)]}{\partial c} < 0$ and $\frac{\partial^2 \mathbb{E}[\pi(c)]}{\partial c^2} > 0$. Based on $F(K(c)) = \frac{p-c}{p}$ and the chain rule,

$$\frac{\partial \mathbb{E}[\pi(c)]}{\partial c} = \frac{\partial K(c)}{\partial c}\{(p-c) - pF(K(c))\} - K(c) = -K(c) < 0 \text{ and } \frac{\partial^2 \mathbb{E}[\pi(c)]}{\partial c^2} = -\frac{\partial K(c)}{\partial c} > 0. \tag{3.21}$$

These first- and second-order conditions describe that the expected profit marginally decreases in c while the function is convex in c, implying that a different level of c is analogous to the level of expected profit (e.g., a higher c has a lower $\mathbb{E}[\pi(c)]$). **a)** Without information exploration, from the single optimal capacity $K = F^{-1}(\frac{p-\mathbb{E}[c]}{p})$, the expected profit is

$$\mathbb{E}[\pi^{IV}] = \int_0^w \left\{(p-c)K - p\int_0^K F(d)dd\right\} g(c)dc = (p-\mathbb{E}[c])K(\mathbb{E}[c]) - p\int_0^{K(\mathbb{E}[c])} F(d)dd.$$

Hence, $\mathbb{E}[c]$ is the cost which makes $\mathbb{E}[\pi^{IV}] = \mathbb{E}[\pi(\mathbb{E}[c])]$.

b) Similarly, define c^{II} that satisfies

$$\mathbb{E}[\pi^{II}] = \mathbb{E}[\pi(c^{II})] \Leftrightarrow K(c^{II}) = \frac{p-\mathbb{E}[c]}{p-c^{II}}\mathbb{E}[d] + \frac{p}{p-c^{II}}\int_0^{K(c^{II})} F(d)dd - \frac{u_R}{p-c^{II}}.$$

From two equivalent points of $\mathbb{E}[c]$ and c^{II}, if $c^{II} < \mathbb{E}[c]$, we can induce

$$c^{II} < \mathbb{E}[c] \Leftrightarrow \mathbb{E}[\pi(c^{II})] > \mathbb{E}[\pi(\mathbb{E}[c])] \Leftrightarrow \mathbb{E}[\pi^{II}] > \mathbb{E}[\pi^{IV}].$$

c) While $\frac{\partial \mathbb{E}[\pi^{IV}]}{\partial c} = 0$ and $\frac{\partial \mathbb{E}[\pi^{II}]}{\partial c} = 0$, we define c' as a cost that makes

$$\frac{\partial \mathbb{E}[\pi(c)]}{\partial c'} = -\mathbb{E}[d].$$

As $\frac{\partial \mathbb{E}[\pi^I]}{\partial c} = -\mathbb{E}[d]$ and $\mathbb{E}[\pi(c)] = \mathbb{E}[\pi^{III}] + u_S$, if $\mathbb{E}[\pi^I(c')] - \mathbb{E}[\pi^{III}(c')] < 0$, the DM has no incentive to explore $K(c)$ as for any given c, $\mathbb{E}[\pi^I(c')] < \mathbb{E}[\pi^{III}(c')]$, consecutively making $\mathbb{E}[\pi(K)] \leq \mathbb{E}[\pi^{IV}]$.

d) If $\mathbb{E}[\pi^I(c')] - \mathbb{E}[\pi^{III}(c')] \geq 0$, there exist a range $\underline{c} < c < \overline{c}$ that makes $\mathbb{E}[\pi^I(c)] > \mathbb{E}[\pi^{III}(c)]$ and the expected profit of exploring capacity information follows (3.14).

Based on $\underline{c} < c < \bar{c}$ and $\frac{\partial \mathbb{E}[\pi(K)]}{\partial c} = 0$, c^I can be defined, satisfying $\mathbb{E}[\pi(K)] = \mathbb{E}[\pi(c^I)]$. As $\mathbb{E}[\pi(c)]$ monotonically decreases in c, the DM can make information exploration decision by comparing the costs c^I, c^{II}, and $\mathbb{E}[c]$ that represents the ex-ante expected profit under capacity, demand information and no exploration, respectively. □

Proof of Proposition 3.6 a) For two random variables D_1 and D_2 with demand prior distributions $F_1(d)$, and $F_2(d)$, $\mathbb{E}_1[D] > \mathbb{E}_2[D]$. The ex-ante expected profits have

$$\frac{\partial \mathbb{E}[\pi^I]}{\partial \mathbb{E}[D]} = p - c, \quad \frac{\partial \mathbb{E}[\pi(c)]}{\partial \mathbb{E}[D]} = \frac{\partial \mathbb{E}[\pi^{III}]}{\partial \mathbb{E}[D]} = p - c - p\frac{\partial \int_0^K F(d)dd}{\partial \mathbb{E}[D]}, \text{ and } \frac{\partial \mathbb{E}[\pi^{II}]}{\partial \mathbb{E}[D]} = p - \mathbb{E}[C].$$

While the marginal expected profit for demand exploration $\frac{\partial \mathbb{E}[\pi^{II}]}{\partial \mathbb{E}[D]}$ is independent of c, $\mathbb{E}[\pi^I]$ and $\mathbb{E}[\pi^{III}]$ show that the expected profit increases in $\mathbb{E}[D]$ while the marginal increase differs in c. From $\frac{\partial \mathbb{E}[\pi(\mathbb{E}[C])]}{\partial \mathbb{E}[D]} < \frac{\partial \mathbb{E}[\pi^{II}]}{\partial \mathbb{E}[D]}$, for any $c^{II} < \mathbb{E}[C]$, $\frac{\partial \mathbb{E}[\pi(c^{II})]}{\partial \mathbb{E}[D]} < \frac{\partial \mathbb{E}[\pi^{II}]}{\partial \mathbb{E}[D]}$ as $p - c^{II} < p - \mathbb{E}[C]$. Therefore, the following inequality holds

$$\mathbb{E}_1[\pi(c_2^{II})] > \mathbb{E}_1[\pi(c_1^{II})] \Leftrightarrow \mathbb{E}[C] - c_2^{II} > \mathbb{E}[C] - c_1^{II}.$$

The cost that makes the ex-ante between no exploration and capacity exploration c^I depends on $\underline{c}$ and $\bar{c}$. Since $\frac{\partial \mathbb{E}[\pi^I]}{\partial \mathbb{E}[D]} = \frac{\partial \mathbb{E}[\pi^{III}]}{\partial \mathbb{E}[D]}$, $\underline{c}_1 = \underline{c}_2$ and $\bar{c}_1 = \bar{c}_2$. Further, based on $\frac{\partial \mathbb{E}[\pi^{III}]}{\partial \mathbb{E}[D]} = \frac{\partial \mathbb{E}[\pi]}{\partial \mathbb{E}[D]}$, $c_1^I = c_2^I$. Hence, $\mathbb{E}[C] - c_1^I = \mathbb{E}[C] - c_2^I$.

b) For $Var(D_1) > Var(D_2)$. The newsvendor expected profit function is $\mathbb{E}[\pi] = (p - c) \cdot K - p \int_0^K F(d)dd$.

Since $p \cdot F(K^*) = (p - c)$ under the optimal decision, we can rewrite the expected profit as

$$\mathbb{E}[\pi] = p \cdot F(K^*) \cdot K^* - p \int_0^{K^*} F(d)dd = p \left\{ F(K^*) \cdot K^* - \int_0^{K^*} F(d)dd \right\}.$$

For $Var(D_1) > Var(D_2)$ and $\mathbb{E}[D_1] = \mathbb{E}[D_2]$, the random variable on the second order stochastic dominance, D_2 is second-order stochastically dominant over D_1 and by definition $\int_0^x [F_1(d) - F_2(d)]dd \geq 0$. Using this property,

$$\mathbb{E}[\pi] = p \left\{ \int_0^{K^*} [F(K^*) - F(d)]dd \right\}.$$

While $F_1(K_1^*) = F_2(K_2^*)$, for the critical ratio $\frac{p-c}{p} < 0.5$, then $K_1^* < K_2^*$. Further, for $\frac{p-c}{p} \geq 0.5 \Leftrightarrow K_1^* \geq K_2^*$, as $\int_0^{\mu} (F_1(d) - F_2(d))\, dd \geq \int_{K_2^*}^{K_1^*} (F_2(d) - F_1(d))\, dd$,

$$\int_0^{K_1^*} F_1(d)dd \geq \int_0^{K_2^*} F_2(d)dd.$$

Note that for most commonly used demand distributions such as normal, lognormal, beta, gamma, weibull and uniform, the condition, $\int_0^{K_2^*} F_1(d) - F_2(d)dd \geq 0$ holds based on Theorem 1 from Ridder et al. (1998). By comparing the expected profit under two distributions,

$$\mathbb{E}[\pi_1] - \mathbb{E}[\pi_2] = p\left\{\int_0^{K_1^*} (\frac{p-c}{p} - F_1(d))dd - \int_0^{K_2^*} (\frac{p-c}{p} - F_2(d))dd\right\}.$$

As $\int_0^K [F_1(d) - F_2(d)]dd \geq 0$ and $\int_0^{\mu} (F_1(d) - F_2(d))\, dd \geq \int_{\mu}^{K} (F_2(d) - F_1(d))\, dd$,

$$\int_0^{K_1^*} (\frac{p-c}{p} - F_1(d))dd - \int_0^{K_2^*} (\frac{p-c}{p} - F_2(d))dd < 0 \Leftrightarrow \mathbb{E}[\pi_1] < \mathbb{E}[\pi_2].$$

From this finding, it is easy to observe that c^{II} which makes the newsvendor expected profit, $\mathbb{E}[\pi(c^{II})] = \mathbb{E}[\pi^{II}]$ reduces. $\mathbb{E}[\pi^{II}] = (p - \mathbb{E}[c])\mathbb{E}[d] - u_R$ remains the same in changing demand prior variability. Given that $\frac{\partial \mathbb{E}[\pi(c)]}{\partial c} < 0$ and $\mathbb{E}[\pi_1(c)] < \mathbb{E}[\pi_2(c)]$, the equivalent cost for demand exploration reduces under a more variable demand prior $c_1^{II} < c_2^{II}$. Therefore,

$$\mathbb{E}[c] - c_1^{II} > \mathbb{E}[c] - c_2^{II}.$$

From Lemma 3.4, $\frac{\partial^2 \mathbb{E}[\pi(c)]}{\partial c^2} > 0$. While $\underline{c}$ and $\overline{c}$ are defined by $\mathbb{E}[\pi^I] = \mathbb{E}[\pi^{III}(c)]$ for $\mathbb{E}[\pi^I(c')] - \mathbb{E}[\pi^{III}(c')] \geq 0$, $\mathbb{E}[\pi^I] = (p - c)\mathbb{E}[d] - u_R - u_S$ is not affected by demand variability. Hence, an increasing variability reduces the expected profit from $\mathbb{E}[\pi^{III}(c)]$, making $\underline{c}_1 < \underline{c}_2 < c < \overline{c}_2 < \overline{c}_1$. Therefore, $\mathbb{E}[\pi_1(K)] - \mathbb{E}[\pi_2(K)]$ can be rearranged to:

$$\begin{aligned}\mathbb{E}[\pi_1(K)] - \mathbb{E}[\pi_2(K)] = &\int_0^{\underline{c}_1} \left\{(\mathbb{E}[\pi_1^{III}(c)] - \mathbb{E}[\pi_2^{III}(c)]\right\} g(c)dc + \int_{\underline{c}_1}^{\underline{c}_2} \left(\mathbb{E}[\pi^I] - \mathbb{E}[\pi_2^{III}(c)]\right) g(c)dc \\ &+ \int_{\overline{c}_2}^{\overline{c}_1} \left\{\mathbb{E}[\pi^I] - \mathbb{E}[\pi_2^{III}(c)]\right\} g(c)dc + \int_{\overline{c}_1}^{w} \left\{\mathbb{E}[\pi_1^{III}(c)] - \mathbb{E}[\pi_2^{III}(c)]\right\} g(c)dc\end{aligned}$$

While $\mathbb{E}[\pi_1^{III}(c)] < \mathbb{E}[\pi_2^{III}(c)]$, $\mathbb{E}[\pi^I] < \mathbb{E}[\pi_2^{III}(c)]$ for $c \in (\underline{c}_1, \underline{c}_2)$ and $c \in (\overline{c}_2 < \overline{c}_1)$. Therefore,

$$\mathbb{E}[\pi_1(K)] < \mathbb{E}[\pi_2(K)] \Leftrightarrow c_1^I < c_2^I \Leftrightarrow \mathbb{E}[c] - c_1^I > \mathbb{E}[c] - c_2^I. \qquad \square$$

Proof of Proposition 3.7 a) The expectation of the cost prior affects the ex-ante expected profit for demand exploration as $\frac{\partial \mathbb{E}[\pi^{II}]}{\partial \mathbb{E}[C]} = -\mathbb{E}[D]$. For $c_2^{II} = \frac{p}{2}$, $\frac{\partial \mathbb{E}[\pi(c_2^{II})]}{\partial c_2^{II}} = -\mathbb{E}[D]$. Therefore, for $c_2^{II} < \frac{p}{2}$, we have

$$\frac{\partial \mathbb{E}[\pi(c_2^{II})]}{\partial c_2^{II}} < \frac{\partial \mathbb{E}[\pi^{II}]}{\partial \mathbb{E}[C]} \Leftrightarrow \frac{\partial \mathbb{E}[\pi(c_2^{II})]}{\partial c_2^{II}} < -\mathbb{E}[D].$$

Hence, $\frac{p}{2} < c_2^{II}$, then $\mathbb{E}_1[C] - \mathbb{E}_2[C] < c_1^{II} - c_2^{II}$ as a marginal decrease in demand exploration is $\frac{\partial \mathbb{E}[\pi(c^{II})]}{\partial c^{II}} < -\mathbb{E}[D]$, while $\frac{\partial \mathbb{E}[\pi^{II}]}{\partial \mathbb{E}[C]} = -\mathbb{E}[D]$. On the other hand, for $\frac{p}{2} \geq c_2^{II}$ we have $\frac{\partial \mathbb{E}[\pi(c^{II})]}{\partial c^{II}} \geq -\mathbb{E}[D]$. Therefore, $\mathbb{E}_1[C] - \mathbb{E}_2[C] \geq c_1^{II} - c_2^{II}$ for any increasing expectation of cost prior distribution $\mathbb{E}[C]$. The cost that makes the ex-ante profit between no exploration and capacity exploration equivalent c^I depends on $\mathbb{E}[\pi^I]$, and $\mathbb{E}[\pi^{III}]$. As none of them are affected by the expectation on the cost prior distribution $\mathbb{E}[C]$, $\underline{c}$ and $\overline{c}$ and c^I remain the same under $G_1(c)$, and $G_2(c)$ while $\mathbb{E}_1[C] > \mathbb{E}_2[C]$.

b) For two random variables C_1 and C_2 with cost prior distributions $G_1(d)$, and $G_2(d)$, $Var(C_1) > Var(C_2)$. The expected profit $\mathbb{E}[\pi(c)]$, $\mathbb{E}[\pi^{III}]$, $\mathbb{E}[\pi^{II}]$ and $\mathbb{E}[\pi^I]$ are not affected by the cost prior $G(c)$. Therefore, c^I, $\underline{c} < c < \overline{c}$ and $\mathbb{E}[c]$ remain unchanged in an increasing cost prior variability. However, the expected profit for exploring capacity information $\mathbb{E}[\pi(K)]$ incorporates the capacity uncertainty. Based on the second-order stochastic dominance $\int_0^c [G_1(c) - G_2(c)]dc \geq 0$, we can compare $\mathbb{E}[\pi_1(K)]$ and $\mathbb{E}[\pi_2(K)]$ as

$$\begin{aligned}\mathbb{E}[\pi_1(K)] - \mathbb{E}[\pi_2(K)] = &\int_0^{\underline{c}} \mathbb{E}[\pi^{III}(c)]g_1(c)dc - \int_0^{\underline{c}} \mathbb{E}[\pi^{III}(c)]g_2(c)dc \\ &+ \int_{\underline{c}}^{\overline{c}} \mathbb{E}[\pi^I(c)]g_1(c)dc - \int_{\underline{c}}^{\overline{c}} \mathbb{E}[\pi^I(c)]g_2(c)dc \\ &+ \int_{\overline{c}}^{w} \mathbb{E}[\pi^{III}(c)]g_1(c)dc - \int_{\overline{c}}^{w} \mathbb{E}[\pi^{III}(c)]g_2(c)dc.\end{aligned}$$

As $\int_0^x \mathbb{E}[\pi^{III}(c)]g(c)dc = \mathbb{E}[\pi^{III}(x)]G(x) - \int_0^x \frac{\partial \mathbb{E}[\pi^{III}(c)]}{\partial c} G(c)dc$ and both expected profits decreases in c , $\frac{\partial \mathbb{E}[\pi^{III}(c)]}{\partial c} < 0$ and $\frac{\partial \mathbb{E}[\pi^{I}(c)]}{\partial c} < 0$,

$$-\int_0^x \frac{\partial \mathbb{E}[\pi^{III}(c)]}{\partial c} G_1(c)dc > -\int_0^x \frac{\partial \mathbb{E}[\pi^{III}(c)]}{\partial c} G_2(c)dc \Leftrightarrow \mathbb{E}[\pi_1(K)] > \mathbb{E}[\pi_2(K)] \Leftrightarrow c_1^I < c_2^I.$$

Therefore, $Var(C_1) > Var(C_2)$ leads to $c_1^I < c_2^I$, $c_1^{II} = c_2^{II}$, and $\mathbb{E}[c_1] = \mathbb{E}[c_2]$. □

Proof of Lemma 3.5 If the retailer is risk-averse, the (expected) utility functions $U(x) = \frac{x^{1-\beta}}{1-\beta}$

for each sharing decision follow

$$U\left(\pi_R^I\right) = U\left(\pi_R^{II}\right) = \frac{((p-w)\cdot d - u_R)^{1-\beta}}{1-\beta} \text{ and } U\left(\widehat{\pi}_R^{III}\right) = \frac{\left((p-w)\cdot \min\left[d, \widehat{K}^*\right]\right)^{1-\beta}}{1-\beta}$$

$$\mathbb{E}\left[U\left(\widehat{\pi}_R^{IV}\right)\right] = \int_0^{c^*} \left(\frac{((p-w)\cdot d)^{1-\beta}}{1-\beta}\right)\cdot g(c)dc + \int_{c^*}^{w} \left(\frac{\left((p-w)\cdot \widehat{K}^*(c)\right)^{1-\beta}}{1-\beta}\right)\cdot g(c)dc.$$

We first show that the gain from the retailer's sharing is non-decreasing in d using the first-order condition. Further, by showing that $\pi_R^{II}(\widehat{TH}^{R(N)}) - \mathbb{E}[\widehat{\pi}_R^{IV}(\widehat{TH}^{R(N)})] < U(\pi_R^{II}(\widehat{TH}^{R(N)})) - \mathbb{E}[U(\widehat{\pi}_R^{IV}(\widehat{TH}^{R(N)}))]$, we derive that $\widehat{TH}^{R(A)} \leq \widehat{TH}^{R(N)}$. The expected profit gained from the retailer's sharing follows

$$\pi_R^{II} - \mathbb{E}[\widehat{\pi}_R^{IV}] = (p-w)\left\{d - \int_0^{\widehat{c}^*} d\cdot g(c)dc - \int_{\widehat{c}^*}^{w} \widehat{K}^*(c)\cdot g(c)dc\right\} - u_R.$$

Given that $\widehat{c}^* = w(1-\widehat{F}(d))$ monotonically decreases in d and $d - \widehat{K}^*(c)$ increases for $c \in (\widehat{c}^*, w)$,

$$\frac{\partial \pi_R^{II} - \mathbb{E}[\widehat{\pi}_R^{IV}]}{\partial d} \geq 0.$$

As the utility function $U(\pi)$ has $\frac{\partial U(\pi)}{\partial \pi} > 0$, $\frac{\partial U(\pi_R^{II}) - \mathbb{E}[U(\widehat{\pi}_R^{IV})]}{\partial d} \geq 0$ is analogous from the expected profit function analysis. As $\pi_R^{II} = \mathbb{E}[\widehat{\pi}_R^{IV}]$ when $d = \widehat{TH}^{R(N)}$, from the risk-averse utility function we obtain $U(\pi_R^{II}) = U(\mathbb{E}[\widehat{\pi}_R^{IV}])$. Further, based on Jensen's inequality,

$$\mathbb{E}[U(\widehat{\pi}_R^{IV})] < U(\mathbb{E}[\widehat{\pi}_R^{IV}]).$$

Therefore,

$$U(\pi_R^{II}(\widehat{TH}^{R(N)})) - \mathbb{E}[U(\widehat{\pi}_R^{IV}(\widehat{TH}^{R(N)}))] \Leftrightarrow U(\mathbb{E}[\widehat{\pi}_R^{IV}(\widehat{TH}^{R(N)})]) - \mathbb{E}[U(\widehat{\pi}_R^{IV}(\widehat{TH}^{R(N)}))] > 0.$$

As $U(\pi_R^{II}(\widehat{TH}^{R(N)})) - \mathbb{E}[U(\widehat{\pi}_R^{IV}(\widehat{TH}^{R(N)}))] > \pi_R^{II}(\widehat{TH}^{R(N)}) - \mathbb{E}[\widehat{\pi}_R^{IV}(\widehat{TH}^{R(N)})] = 0$,

$$\widehat{TH}^{R(A)} < \widehat{TH}^{R(N)}.$$

When a supplier faces a risk-averse retailer, the risk-aversion of the retailer only influences the unilateral sharing decision $\widehat{TH}_{s:ns}$. Similar to the risk-averse retailer's equilibrium sharing rule $\widehat{TH}^{R(A)} < \widehat{TH}^{R(N)}$, the supplier also anticipates the risk-averse retailer's reaction under unilateral sharing as $\widehat{TH}_{s:ns}^{R(A)} < \widehat{TH}_{s:ns}^{R(N)}$. □

Proof of Lemma 3.6 If the supplier is risk-averse, the (expected) utility functions, $U(x) = \frac{x^{1-\beta}}{1-\beta}$, for each sharing decision follow

$$U\left(\pi_S^I\right) = \frac{((w-c)\cdot d - u_S)^{1-\beta}}{1-\beta} \text{ and } U\left(\pi_S^{II}\right) = \frac{((w-c)\cdot d)^{1-\beta}}{1-\beta}$$

$$\mathbb{E}\left[U\left(\pi_S^{III}\right)\right] = \int_0^K \left(\frac{(-c\cdot K + w\cdot d - u_S)^{1-\beta}}{1-\beta}\right) f(d)dd + \int_K^\infty \left(\frac{((w-c)\cdot K - u_S)^{1-\beta}}{1-\beta}\right) f(d)dd$$

$$\mathbb{E}\left[U\left(\pi_S^{IV}\right)\right] = \int_0^K \left(\frac{(-c\cdot K + w\cdot d)^{1-\beta}}{1-\beta}\right) f(d)dd + \int_K^\infty \left(\frac{((w-c)\cdot K)^{1-\beta}}{1-\beta}\right) f(d)dd$$

If the supplier is risk-averse, the expected utility of sharing capacity information while the risk-neutral retailer does not reciprocally share demand information is

$$\mathbb{E}\left[U\left(\pi_S^{III}\right)\right] = \int_0^K U\left(-c\cdot K + w\cdot d - u_S\right) f(d)dd + \int_K^\infty U\left((w-c)\cdot K - u_S\right) f(d)dd.$$

We denote $\pi_S^- = -c\cdot K + w\cdot d$ and $\pi_S^+ = (w-c)\cdot K$. The first-order conditions of the expected profit and the expected utility with respect to the capacity K are

$$\frac{\partial \mathbb{E}\left[\pi_S(K)\right]}{\partial K} = -c\cdot \int_0^K \pi_S^{'-} \cdot f(d)dd + (w-c)\cdot \int_K^\infty \pi_S^{'+} \cdot f(d)dd = 0 \quad \textit{and}$$

$$\frac{\partial \mathbb{E}[U(\widehat{\pi}_S^{III}(K))]}{\partial K} = -c\cdot \int_0^K U'(\pi_S^-)\pi_S^{'-} \cdot f(d)dd + (w-c)\cdot \int_K^\infty U'(\pi_S^+)\pi_S^{'+} \cdot f(d)dd.$$

As $U'(\pi_S) > 0$, $\pi_S^{'-} > 0$ and $\pi_S^{'+} < 0$, $K^{S(A)} < K^{S(N)}$. The lowered capacity affects the threshold demand level for the retailer's sharing decision because the cost c^* which makes the optimal capacity equal to the demand level, reduces $c^{*S(A)} < c^{*S(N)}$. Consecutively, the retailer who remains silent has reduced expected sales and profits if the supplier is risk-averse $\mathbb{E}\left[\pi_{R|S(N)}^{IV}\right] > \mathbb{E}\left[\pi_{R|S(A)}^{IV}\right]$. Therefore,

$$\widehat{TH}^{R|S(A)} < \widehat{TH}^{R|S(N)}.$$

On the other hand, the risk-averse supplier's sharing condition follows

$$(1-F(TH_{s:s}))\cdot U\left(\pi_S^I\right) + F(TH_{s:s})\cdot \mathbb{E}\left[U(\pi_S^{III})\right] > (1-F(\widehat{TH}_{s:ns}))\cdot U\left(\pi_S^{II}\right) + F(\widehat{TH}_{s:ns})\cdot \mathbb{E}\left[U(\pi_S^{IV})\right] \tag{3.22}$$

We can rearrange the inequality as below:

$$(1-F(TH_{s:s}))\left(U\left(\pi_S^I\right) - \mathbb{E}\left[U(\pi_S^{III})\right]\right) - (1-F(\widehat{TH}_{s:ns}))\left(U\left(\pi_S^{II}\right) - \mathbb{E}\left[U(\pi_S^{IV})\right]\right)$$
$$> \mathbb{E}\left[U(\pi_S^{IV})\right] - \mathbb{E}\left[U(\pi_S^{III})\right]$$

We denote

$$\delta^{S(A)} = (1 - F(TH_{s:s}))\left(U\left(\pi_S^I\right) - \mathbb{E}\left[U(\pi_S^{III})\right]\right) - (1 - F(\widehat{TH}_{s:ns}))\left(U\left(\pi_S^{II}\right) - \mathbb{E}\left[U(\pi_S^{IV})\right]\right)$$

as the expected utility from the economic gain by revealing capacity information. Further, $\delta_{u_S} = \mathbb{E}\left[U(\pi_S^{IV})\right] - \mathbb{E}\left[U(\pi_S^{III})\right]$ denotes the expected utility loss by incurring the cost of sharing. For $c = \widehat{TH}_{lb}^S$ or $c = \widehat{TH}_{ub}^S$, making $\pi_S^I - \mathbb{E}\left[\pi_S^{III}\right] = u_S$, the following inequality holds based on Jensen's theorem.

$$U\left(\pi_S^I\right) - \mathbb{E}\left[U(\pi_S^{III})\right] > U\left(\pi_S^I - u_S\right) - \mathbb{E}\left[U(\pi_S^{III})\right] > U\left(\pi_S^I - u_S\right) - U\left(\mathbb{E}\left[\pi_S^{III}\right]\right) = 0 \quad (3.23)$$

As $\pi_S^{II} - \pi_S^I = u_S$ and $\mathbb{E}\left[\pi_S^{IV}\right] - \mathbb{E}\left[\pi_S^{III}\right] = u_S$,

$$U\left(\pi_S^{II}\right) - \mathbb{E}\left[U(\pi_S^{IV})\right] > U\left(\pi_S^{II} - u_S\right) - \mathbb{E}\left[U(\pi_S^{IV})\right] > U\left(\pi_S^{II} - u_S\right) - U\left(\mathbb{E}\left[\pi_S^{IV}\right]\right) = 0 \quad (3.24)$$

Based on $U'(\pi) > 0$ and $U''(\pi) = 0$, $U\left(\pi_S^I\right) - \mathbb{E}\left[U(\pi_S^{III})\right] > U\left(\pi_S^{II}\right) - \mathbb{E}\left[U(\pi_S^{IV})\right] > 0$. Therefore, a risk-averse supplier has a higher utility from the economic gain which incentivizes him to reveal capacity information and to avoid demand uncertainty. However, $U'(\pi) > 0$ and $U''(\pi) < 0$ implies the marginal utility reduces in an increasing profit π. Hence, while $\delta_{u_S} = u_S$ for a risk-neutral supplier, the marginal utility of incurring the cost of sharing for a risk-averse supplier δ_{u_S} also changes depending on the expected profit under demand uncertainty. Specifically, as the supplier's expected profit under demand uncertainty increases, δ_{u_S} decreases as $\frac{\partial^2 \delta_{u_S}}{\partial \mathbb{E}[\pi_S]^2} < 0$.

To summarize, although the risk-averse supplier has a higher utility from revealing capacity than the risk-neutral supplier based on (3.23) and (3.24), the utility of incurring the sharing cost δ_{u_S} can increase (decrease) if the expected profit under demand uncertainty is relatively low (high) compared to the risk-neutral supplier's u_S. Specifically, δ_{u_S} can offset the increased utility from sharing $\delta^{S(A)}$. Hence, if $\delta_{u_S} > \delta_{u_S}$, the risk-averse supplier is more reluctant to share than the risk-neutral supplier. □

Table 3.2: Summary of Notation

Overview of notation

Notation	Description
p	Market price of retailer
w	Wholesale price of supplier
c	Capacity investment cost of supplier
u_S/u_R	Information sharing cost of supplier and retailer
d	Market demand
K	Supplier capacity
K^*	Optimal supplier capacity
$\widehat{K}^*$	Optimal supplier capacity after demand prior update
$f(d)/F(d)$	Demand prior probability density and cumulative distribution function
$g(d)/G(d)$	Cost prior probability density and cumulative distribution function
$\widehat{f}(d)/\widehat{F}(d)$	Updated demand prior probability density and cumulative distribution function
$\widehat{g}(c)/\widehat{G}(c)$	Updated cost prior probability density and cumulative distribution function
π_S^i/π_R^i	Supplier and retailer profit for sharing case i where, $i \in (I, II, III, IV)$
$\widehat{\pi}_S^i/\widehat{\pi}_R^i$	Supplier and retailer profit after prior update for sharing case i where, $i \in (I, II, III, IV)$
$c^* = w \cdot (1 - F(d))$	Capacity investment cost that makes the optimal capacity equal to demand level d
$\widehat{c}^* = w \cdot (1 - \widehat{F}(d))$	Capacity investment cost that makes the optimal capacity equal to demand level d after prior udpate
TH_{LB}^R/TH_{UB}^R	Demand information sharing threshold in benchmark
TH_{LB}^S/TH_{UB}^S	Capacity information sharing threshold in benchmark
$\widehat{TH}^R$	Equilibrium demand information sharing threshold of the retailer in decentralized setting
$\widehat{TH}_{lb}^S/\widehat{TH}_{ub}^S$	Equilibrium capacity information sharing threshold of the supplier in decentralized setting
$\widehat{TH}_{s:ns}$	Threshold demand of the retailer when capacity information is not shared
$\widehat{TH}_{s:ns}^j$	Threshold demand of the retailer when capacity information is not shared under the risk preferences where, $j \in (RA, RN)$
$TH_{s:s}$	Threshold demand of the retailer when capacity information is shared
$TH_{s:s}^j$	Threshold demand of the retailer when capacity information is shared under the risk preferences j where, $j \in (RA, RN)$

Chapter 4.

Implications of sharing and exchanging market information between an online platform and a seller

4.1 Introduction

When a seller offers products through its own direct sales channel and an external online platform, both the seller and the platform can acquire private and yet correlated *market signals* that can be used to improve demand forecasts. This observation motivates us to examine *if and under which conditions* these two parties should share or exchange their private market signals. We examine two settings by using a game theoretic framework. First, we consider the case when the platform's commission rate is *exogenously* fixed in advance so that the platform cannot make use of the seller's private information. Hence, the crux of the issue is to examine if and when the platform should "share" its private signal with the seller so that the seller can leverage this information to make better pricing decisions. Second, when the platform can determine its commission rate *endogenously*, we investigate whether and the circumstances in which it is beneficial for the platform and the seller to "exchange" their private signals *bilaterally* so that the platform can choose its commission rate and the seller can set its price to maximize their own profits by leveraging additional signals.

Our equilibrium analysis yields the following results. First, when the platform's commission rate is exogenous, the platform should share its signal with the seller. Second, when the commission rate is endogenously determined by the platform, the results are more nuanced. Specifically, exchanging private signals between both parties can result in a lower commission rate when the seller's signal is precise, but it does not help the seller earn a higher profit. When the market potential of the seller's direct sales demand is sufficiently large, the platform does not gain from bilateral information exchange because the seller's dominant power in the direct sales channel induces the platform to reduce the commission rate. However, if the market potential of the seller's online demand is sufficiently large, bilateral information exchange can result in a win-win situation.

4.1.1 Problem setting

To expand market reach, brands (sellers) such as Adidas and Under Armour often sell their products through their own "direct" channels (i.e., online and offline stores) as well as external online "platforms" such as Amazon. As a result, different channels observe private and yet correlated "market signals" such as consumer preference trends and consumer price sensitivity (Hübner et al. 2022). Although both the seller and the platform observe market signals from different sources, no party possesses complete information about the market. Consequently, each party possesses valuable information about the market that can help the other party to make better decisions (i.e., commission rate decisions for the platforms and pricing decisions for the sellers) (Amazon 2021).

These observations motivate us to examine *if and under which conditions* these two parties should "share" or "exchange" their private market signals in two different contexts. First, when the platform's sales commission rate is fixed and pre-specified in advance, the platform cannot exploit the seller's market signal to improve its revenue by adjusting its commission rate. This scenario is common in practice, as seen with the standard 20% commission rate of Uber Eats for each delivery order facilitated through its platform (UberEats 2023). Nevertheless, the seller can make use of the platform's signal to improve its demand forecast so that it can make better pricing decisions. In this setting, should the platform "share" its market signal *unilaterally* with the seller for a fee? Second, when the platform's sales commission rate is endogenously determined, should the platform and the seller "exchange" their own private signals *bilaterally* so that the platform can choose its commission rate and the seller can set its price to maximize their profits by leveraging additional market signals?

Sharing information *unilaterally* is more prevalent because platforms' commission rates are often pre-specified for all sellers. As the seller's sales can improve the platform's commissions, platforms are eager to share their market signals with their sellers unilaterally. For instance, Amazon announced that, on average, 53.3% (1.01 million) of its sellers use Fulfillment by Amazon (FBA). Based on the collected data from FBA, Amazon offers a program known as *Product*

Opportunity Explorer, where it provides information such as customers' behavior, sellers listing catalog, and (anonymized) sales data to its sellers for free (Amazon 2023). Similarly, Uber Eats (a food delivery platform partnered with more than 870,000 restaurants worldwide) provides information on customer retention rates and the number of competitors in the local area to its sellers (restaurants) for free (Guszkowski 2020). Moreover, Uber Eats offers a service for an extra fee that is known as the *Webshop Online Ordering Solution*, where the restaurants can get more insights into customer purchasing behavior (UberEats 2023).

Exchanging information of market signals *bilaterally* between platforms and sellers are emerging, probably because various platforms are changing their commission rates for different sellers under different arrangements. For instance, Uber Eats has announced its intention to exercise greater control over its commission rate frameworks. The platform plans to create new rate models tailored to specific regions for its sellers, leveraging input data sourced from a multitude of restaurants across the U.S. beginning in 2020 (Bloomberg 2022). When the platform can choose its commission rate and the seller can determine its selling price, there is an incentive for both parties to explore bilateral exchanges of market information. For example, Alibaba (a Chinese e-commerce company) and Casio (a Japanese electronics manufacturing corporation) cooperated by mutually exchanging omnichannel data from both online and direct sales channels using Alibaba's cloud service. The collaboration aimed to increase overall sales and reduce inventory turnover rates for Casio. As a result of the information exchange, Casio's store visits increased to 3.17 million, and the online store views increased to 1.7 million using the omnichannel data solution provided by Alibaba Cloud. As Casio's sales increase, Alibaba's commission revenue also increases (Alibaba 2021).

4.1.2 Modeling approach and contribution

While unilateral information sharing and bilateral information exchanges are becoming more common in practice, the impact of this practice on the performance of a seller and an online platform remains unclear (Mishra et al. 2009, Hyndman et al. 2013, Zhang and Chen 2013). To fill this research gap, we pose the following research questions: (1) When the commission rate is exogenously fixed, would sharing the platform's private signal with the seller unilaterally be mutually beneficial? (2) When the commission rate is endogenously determined, would exchanging private signals bilaterally between the platform and the seller be mutually beneficial? Upon information exchange, the platform's signal influences the seller's pricing decision, while reciprocally, the seller provides her information. This mutual exchange of information influences the platform's commission rate decision, leveraging the signals exchanged between the parties.

By using a game theoretic framework, our equilibrium analysis yields the following results. First, when the platform's commission rate is exogenous, the platform's information sharing is mutually beneficial for both the seller and the platform. Second, when the commission rate is endogenously determined by the platform, the results are more nuanced. Specifically, we find that exchanging private signals between both parties can result in a lower commission rate when the seller's signal is precise, but it does not help the seller earn a higher profit while the platform benefits from exchanging information. This is because the surplus generated by information exchange is primarily garnered by the platform, as he benefits from 1) imposing a more advantageous commission rate by observing the seller's exact price reaction function without information asymmetry (the seller's private signal) and 2) obtaining more accurate demand forecasting facilitated by the information exchange.

When the market potential of the seller's direct sales channel is sufficiently large, the platform does not gain from bilateral information exchange as the seller primarily focuses on extracting profit from the direct sales channel, becoming less reactive to the platform's commission rate decision based on information exchange. However, if the market potential of the seller's online channel is sufficiently large, then bilateral information exchange can result in a win-win situation. In particular, although the seller's signal is imprecise when the online market potential is large, the platform's benefit from observing the seller's price reaction function symmetrically increases. On the other hand, the primary advantage for the seller in sharing information lies in acquiring more precise demand forecasts, fostering a mutually beneficial outcome.

4.1.3 Organization

The remainder of this chapter is structured as follows: Model formulations can be found in Section 4.2. The equilibrium and the managerial insights derived from the model with an exogenous commission rate (Information Sharing) are presented in 4.3. 4.4 presents the equilibrium analysis and implications with an endogenized commission rate (Information Exchange). §4.5 compares equilibrium results between information exchange and no exchange and provides managerial implications. Lastly, 4.6 provides concluding remarks and outlines avenues for future research. The appendix contains all the proofs.

4.2 Model preliminaries

Consider a seller that sells a single product with a unit cost c through two separate channels: (1) a direct sales channel controlled by the seller and (2) an external online platform. As an initial attempt to examine the value of information exchange, we abstract away the decisions of all other sellers who sell similar products through the platform. The platform charges a commission rate r as a percentage of the seller's revenue generated via the online platform. Initially, we treat r as an exogenously given parameter and look at how the platform decides to share information unilaterally. In a later section, we delve into a case where r is endogenously determined by the platform, which entails bilateral information exchanges. Without loss of generality, we normalize the platform's unit cost to zero (Zha et al. 2022, Huang et al. 2018).

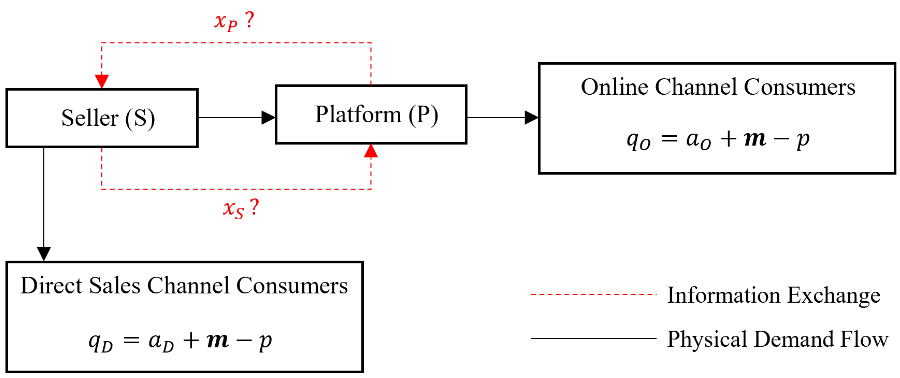
Given the commission rate r, the seller sets an optimal price p. Based on the evidence that multichannel sellers often offer identical prices across channels to avoid arbitrage or confusion (Cavallo 2017), we assume that channel-specific selling prices are not permissible.

Demand Functions q_O and q_D. As depicted in Figure 4.1, the demand generated from two separate online and direct sales channels are assumed to be linearly dependent on the selling price p:

$$q_O = a_O + m - p \quad \textit{and} \quad q_D = a_D + m - p, \tag{4.1}$$

where a_O and a_D represent the "market potential" of the online and direct sales channels, respectively.

Figure 4.1: Market Structure



Market Uncertainty m. From (4.1), we assume a_O and a_D are common knowledge; however, the customer demands q_O and q_D are subject to "market uncertainty" m, where m represents the adjustment to the market potentials, a_O and a_D, caused by uncertain consumer preference of the seller's product and/or market conditions. For tractability, we follow Gal-Or (1985b) by assuming that $m \sim N(0, \sigma)$ and denote $\nu \equiv \frac{1}{\sigma}$ as "market certainty".[1]

[1] To avoid trivial cases, a_O and a_D are assumed to be sufficiently large and the variance of the market uncertainty σ is reasonably bounded such that the seller's price p and the platform's commission rate r are ensured to be positive with a high probability (Chen and Tang 2015, Li 2002, Ha et al. 2011).

Noisy Signals x_S and x_P. The demands q_O and q_D given in (4.1) are affected by market uncertainty m, and this uncertainty hinders the seller and the platform from making better pricing p and commission rate r decisions. Hence, capturing more information about market uncertainty m to better forecast the demands can benefit both parties.

Both parties obtain their private noisy signals about m through different sources, and the private information possessed by one party can benefit the other party. Hence, an immediate question is: should the seller share her signal x_S with the platform and receive the platform's signal x_P reciprocally? To investigate the impact of exchanging information between the seller and the platform, we consider two settings of information exchanges: (1) no information exchange (N) and (2) with information exchange (W). We assume that both parties will truthfully and voluntarily exchange the private signals, consistent with the existing literature (Shang et al. 2016, Zha et al. 2022).

As each party has access to different sources of information, the seller and the platform obtain "noisy and imperfect" private and yet correlated signals about the uncertain market m. The seller can observe her private and noisy signal x_S about market uncertainty m by conducting consumer surveys and market analysis. At the same time, the platform can leverage its direct observations of online consumer behavior (browse history, click sequence, and online purchasing behavior) to deduce private and noisy signal x_P about m. Following Grossman (1981), Mendelson and Tunca (2007), the noisy signals obtained by the seller and the platform satisfy:

$$x_S = m + \varepsilon_S, \quad \text{and} \quad x_P = m + \varepsilon_P. \tag{4.2}$$

For tractability, we assume that the noise in the signals follows $\varepsilon_S \sim N(0, \tau_S)$ for the seller and $\varepsilon_P \sim N(0, \tau_P)$ for the platform that is commonly used in the economics literature (Gal-Or 1985b). τ_S and τ_P are common knowledge, hence known by both players. The noise is captured by the variance τ_S for the seller and τ_P for the platform, and we define $\nu_S \equiv \frac{1}{\tau_S}$ and $\nu_P \equiv \frac{1}{\tau_P}$ as the "precision" of private signal to simplify interpretation of results later. For instance, $\nu_S = 0$ can be interpreted as the case when the seller cannot obtain her own private signal, and $\nu_S = \infty$ corresponds to the case when the seller's private signal is perfect.

For ease of exposition, we assume that the noise ε_S, and ε_P are independent with the market uncertainty m and to each other with $\text{cov}(m, \varepsilon_S) = \text{cov}(m, \varepsilon_P) = \text{cov}(\varepsilon_P, \varepsilon_S) = 0$. However, note that the private signals x_S and x_P are "correlated" with market uncertainty m with an additive form. Therefore, the signals x_S and x_P serve as unbiased estimators of the market uncertainty m. Moreover, the seller's signal x_S and the platform's signal x_P are positively correlated as $Corr(x_S, x_P) = \frac{1}{\sqrt{1+\frac{\nu}{\nu_S}}\sqrt{1+\frac{\nu}{\nu_P}}} > 0$.

Information Exchange. Without information exchange, the seller observes only x_S, and the platform observes only x_P. However, with information exchange as shown in Figure 4.1, both the seller and the platform observe both signals (x_S, x_P) so that they can obtain a more accurate forecast about m through variance reduction, as shown in Lemma 4.1.

Lemma 4.1. *Without information exchange, each player can determine the conditional expectation and variance of market uncertainty $(m|x_S)$ and $(m|x_P)$ using $\sigma = \frac{1}{\nu}$, $\tau_S = \frac{1}{\nu_S}$, and $\tau_P = \frac{1}{\nu_P}$, where:*

$$\begin{aligned} \mathbb{E}(m|x_S) &= \frac{\sigma}{\sigma+\tau_S}x_S = \frac{\nu_S}{\nu+\nu_S}x_S \quad \text{and} \quad Var(m|x_S) = \frac{\sigma\tau_S}{\sigma+\tau_S} = \frac{1}{\nu+\nu_S} \\ \mathbb{E}(m|x_P) &= \frac{\sigma}{\sigma+\tau_P}x_P = \frac{\nu_P}{\nu+\nu_P}x_P \quad \text{and} \quad Var(m|x_P) = \frac{\sigma\tau_P}{\sigma+\tau_P} = \frac{1}{\nu+\nu_P} \end{aligned} \tag{4.3}$$

With information exchange, both the seller and the platform possess the same signals (x_S, x_P) so that the conditional expectation and variance of market uncertainty $(m|x_S, x_P)$ are:

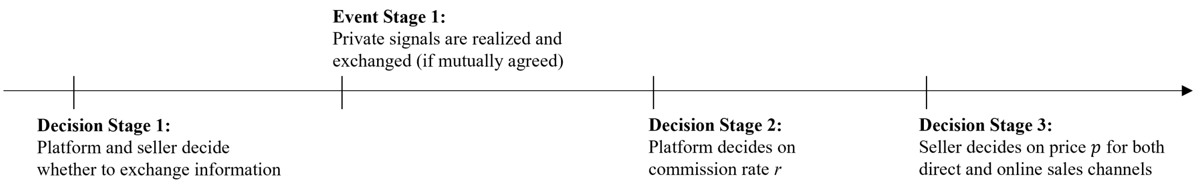
$$\begin{aligned} \mathbb{E}(m|x_S, x_P) &= \frac{\sigma\left[\tau_P x_S + \tau_S x_P\right]}{\sigma\tau_P + \sigma\tau_S + \tau_P\tau_S} = \frac{\nu_S x_S + \nu_P x_P}{\nu+\nu_S+\nu_P} \quad \text{and} \\ Var(m|x_S, x_P) &= \frac{\sigma\tau_P\tau_S}{\sigma\tau_P + \sigma\tau_S + \tau_P\tau_S} = \frac{1}{\nu+\nu_S+\nu_P} \end{aligned} \tag{4.4}$$

Lemma 4.1 shows that, for any imperfect private signal with $\tau_j > 0$ for $j \in (S, P)$, information exchange enables both parties to improve their forecast accuracy about the market conditions via variance reduction; i.e., $Var(m|x_P, x_S) < Var(m|x_S)$ and $Var(m|x_P, x_S) < Var(m|x_P)$. However, it is unclear how this reduced variance affects the seller's price decision p (in the information sharing model) and both the seller's price decision p and the platform's commission rate r (in the information exchange model). Specifically, in the *information sharing* model, when r is exogenously given or pre-specified, the platform cannot make use of the exchanged signal x_S to adjust its commission rate r. Therefore, in this setting, only the seller can use the signal x_P (shared by the platform) to adjust her selling price p. However, when the commission rate r is endogenously determined, both parties will utilize the exchanged signal to adjust their decisions. We consider this *information exchange* model in §4.4.

Sequence of Events and Decisions. Information exchange is a long-term decision that requires appropriate investment in information technology infrastructure. On the other hand, the decisions on price p and commission rate r are made after obtaining signal(s) about the market condition through direct observation or information exchange. For example, a new or seasonal product launch in the fashion industry requires additional market research to collect information on market uncertainty (m) to identify customer trends and gather competitors' information before making price decisions (McKinsey 2022). Hence, the platform and the seller commit to the information exchange policy before observing private signals.

Further, as stated by Abhishek et al. (2016), Tsunoda and Zennyo (2021), platforms served as common marketplaces are endowed with substantial power to make commission rate decisions first because online marketplaces have broader customer bases and competing sellers' information. The sequence of events for the bilateral information exchange scenario is depicted in Figure 4.2. (For the unilateral information-sharing scenario, the commission rate r is exogenously given (not a decision), and only the platform decides whether or not to share information with the seller. To avoid repetition, we omit the corresponding figure.) The events depicted in Figure 4.2 can be described as follows: 1) The seller and the platform decide whether to exchange information on the private signals (i.e., x_P and x_S) on market conditions; 2) If they agree on exchanging information, both parties exchange private signals "bilaterally" after the signals are realized; 3) The platform decides on the commission rate r to charge the seller; and 4) The seller sets the price p for both sales channels.

Figure 4.2: The Sequence of Events



Structure of Analysis. The platform earns a commission based on a proportion of the seller's revenue. Hence, the platform has an incentive to share his private signal x_P unilaterally (even if he cannot make use of the seller's signal x_S to determine his commission rate r when r is exogenously given in the information sharing model as examined in §4) to help the seller make a better price decision and increase sales. This implies that when the commission rate is fixed, the platform is effectively "passive", making no use of the exchange information x_S provided by the seller. Due to this passivity of the platform, receiving the seller's signal x_S becomes moot. However, sharing his signal x_P with the seller can impact the seller's price decision, which can yield a higher commission for the platform. In §5, for the case when the platform endogenously determines r, the platform can use the seller's signal x_S to determine its commission rate decision r so that bilateral information exchange can become meaningful in the information exchange model.

The structure of our analysis can be illustrated in Table 4.1. In each section, we analyze the equilibrium price of the seller p, the ex-ante expected profit for the seller Π_S, and the ex-ante expected revenue for the platform Π_P. Further, we compare the results on equilibrium price, commission rate and expected profit of each player with information exchange (W) to those without information exchange (N), as shown in the superscript. The notation $(\tilde{\cdot})$ denotes the result from the information exchange model (endogenous r).

Table 4.1: Structure of Analysis

Section: Setting	No Exchange (N)	With Exchange (W)
§4.3: Exogenous r	p^N, Π_S^N, and Π_P^N	p^W, Π_S^W, and Π_P^W
§4.4: Endogenous r	$\widetilde{p}^N$, $\widetilde{\Pi}_S^N$, and $\widetilde{\Pi}_P^N$	$\widetilde{p}^W$, $\widetilde{\Pi}_S^W$, and $\widetilde{\Pi}_P^W$

4.3 Information sharing: Exogenous commission rate r

4.3.1 No information sharing

We determine the seller's expected profit and optimal price when there is no information exchange.

Ex-Post Expected Profit of the Seller. By considering the expected demands from both channels q_O and q_D from (4.1) along with any given commission rate r and price p, the seller's total expected profit derived from both channels is:

$$\begin{aligned}\mathbb{E}\left(\pi_S^N \mid p; x_S, r\right) &= \underbrace{(p(1-r)-c)\cdot\mathbb{E}(q_O \mid x_S)}_{\text{Online channel profit}} + \underbrace{(p-c)\cdot\mathbb{E}(q_D \mid x_S)}_{\text{Direct channel profit}} \\ &= (p(1-r)-c)\cdot(a_O+\mathbb{E}(m \mid x_S)-p)+(p-c)\cdot(a_D+\mathbb{E}(m \mid x_S)-p)\end{aligned} \tag{4.5}$$

The expected profit function is quadratic in p, and the random variables $(m,\ x_S)$ follow a multivariate normal distribution as $m \sim N(0,\sigma)$ and $x_S \sim N(0,\tau_S)$. Hence, we can derive an optimal price p^N for any given x_S, which exhibits the following linear decision rules: $p^N = A^N + A_S^N x_S$.

Seller's Optimal Price. The optimal price p^N maximizes the seller's expected profit based on her observed signal x_S, $\mathbb{E}\left(\pi_S^N \mid p; x_S, r\right)$ as given in (4.5).

Proposition 4.1. *When there is no information sharing, the seller's optimal price p^N upon observing her signal x_S satisfies: $p^N = A^N + A_S^N x_S$, where*

$$A^N = \frac{a_O(1-r)+a_D+2c}{2(2-r)}, \quad \textit{and} \quad A_S^N = \frac{\nu_S}{2(\nu+\nu_S)}.$$

We can interpret A^N as the *base price* factoring in the commission rate r established by the platform in advance. Also, A_S^N is the *information factor* associated with the private signal of the seller x_S. Proposition 4.1 implies that, when there is no private information (e.g., when $\tau_S = \infty$ or equivalently $\nu_S = 0$), the seller sets the "base price" $p^N = A^N = \frac{a_O(1-r)+a_D+2c}{2(2-r)}$. The comparative statics of the equilibrium price can be summarized as:

$$\frac{\partial A^N}{\partial a_O} > 0, \quad \frac{\partial A^N}{\partial a_D} > 0, \quad \frac{\partial A^N}{\partial c} > 0, \quad \frac{\partial A^N}{\partial r} = \frac{-a_O + a_D + 2c}{2(2-r)^2}, \quad \frac{\partial A_S^N}{\partial \nu} < 0, \quad \text{and} \quad \frac{\partial A_S^N}{\partial \nu_S} > 0.$$

From the comparative statistics, the seller's equilibrium price p^N increases as the platform charges a higher commission rate r ($\frac{\partial A^N}{\partial r} > 0$) if the market potential from the online channel is relatively smaller compared to the direct sales channel ($a_O < a_D + 2c$). On the other hand, if the online channel market potential is relatively large ($a_O \geq a_D + 2c$), an increasing r makes the seller reduce the selling price p^N ($\frac{\partial A^N}{\partial r} < 0$). The reason for such a price decision is that while the platform's commission rate is charged to the online market revenue, the seller offers the same price p^N to both markets (q_O and q_D). In equilibrium, the seller balances between the price p^N and the demands in two markets, $q_O = a_O + m - p^N$, and $q_D = a_D + m - p^N$. Therefore, when the commission rate increases, the seller with the small online market potential $a_O \leq a_D + 2c$ reduces her own price, although the profit margin from the online market $p^N(1-r) - c$ reduces. By doing so, she ensures high demand generated from the direct sales channel and avoids significant online market demand reduction.

On the other hand, when the online market potential is high $a_O > a_D + 2c$, even if the platform charges a higher commission rate, an increasing selling price still ensures positive demands from the online market. Hence, the seller rather focuses on securing the profit margin in the online market $p^N(1-r) - c$, increasing the price. Generally, the seller sets a higher price p^N when the market potentials from two channels a_O, a_D, or the unit cost c increases. Moreover, from the comparative statics on the intrinsic market certainty ν and the information precision ν_S, we observe the following impact: (i) as the intrinsic market certainty ν increases, the seller is less responsive to the private signal x_S; and (ii) when the signal is more accurate (e.g. when ν_S is higher), the seller is more responsive towards the signal from her market research.

Ex-Post Expected Revenue of the Platform. Because the platform's commission is based on the seller's revenue from the online channel, the platform's revenue is $\pi_P = r \cdot p \cdot q_O$. Given the seller's optimal price p^N, the ex-post expected revenue of the platform is:

$$\mathbb{E}(\pi_P^N | x_P, x_S) = r \cdot \mathbb{E}\left(p^N \cdot q_O(p^N) \mid x_P, x_S\right) = r \cdot p^N(x_S) \cdot \left(a_O + \mathbb{E}\left(m \mid x_P\right) - p^N(x_S)\right). \quad (4.6)$$

Although the platform does not share, he still observes a private signal x_P which he can use for $\mathbb{E}(m \mid x_P)$ to determine his ex-post expected revenue that will be later compared to the revenue under sharing.

Ex-ante Expected Profit and Revenue Function. As the information sharing on x_P is made before the players observe their private signals x_S and x_P, let Π_S^N be the seller's ex-ante expected profit and Π_P^N be the platform's ex-ante expected revenue without information sharing:

$$\Pi_S^N \equiv \mathbb{E}_{x_S}\left\{\pi_S^N(p^N(x_S))\right\} \quad \text{and} \quad \Pi_P^N \equiv \mathbb{E}_{x_S,x_P}\left\{\pi_P^N(p^N(x_S),x_P)\right\} \tag{4.7}$$

If r is exogenously given and there is no information sharing from the platform, the seller's and the platform's ex-ante expected profit and revenue are:

$$\begin{aligned}\Pi_S^N &= (2-r)\left\{(A_S^N)^2\left(\frac{1}{\nu}+\frac{1}{\nu_S}\right)\right\} + (2-r)(A^N)^2 - c(a_O+a_D),\\ \Pi_P^N &= r\left\{\frac{A_S^N}{2\nu}\left(B_P - \frac{\nu}{\nu+\nu_P}\right) + A^N(a_O - A^N)\right\}.\end{aligned} \tag{4.8}$$

where A^N and A_S^N are given in Proposition 4.1 and $B_P = \frac{\nu_P}{\nu+\nu_P}$. By definition, the precision of each player's information is represented as $\nu_S = \frac{1}{\tau_S}$ and $\nu_P = \frac{1}{\tau_P}$. Hence, by letting $\nu_S = 0$ and $\nu_P = 0$, we can capture the effect of having no information. Specifically, each party is better off by obtaining their own private signals even without information sharing as shown in the following equations:

$$\begin{aligned}\Pi_S^N(\nu_S>0) - \Pi_S^N(\nu_S=0) &= (2-r)\frac{\nu_S}{\nu(\nu+\nu_S)} > 0, \text{ and}\\ \Pi_P^N(\nu_P>0) - \Pi_P^N(\nu_P=0) &= r\frac{\nu_S\nu_P}{2\nu(\nu+\nu_S)(\nu+\nu_P)} > 0.\end{aligned}$$

Without private information ($\nu_S = A_S^N = 0$), the seller sets the optimal price $p^N = A^N$, and the ex-ante expected profit of the seller only depends on the economic factors (i.e., a_O, a_D, c, and r), $\Pi_S^N(\nu_S = 0) = (2-r)(A^N)^2 - c(a_O + a_D)$. Possessing private information enables the seller to obtain a higher ex-ante expected profit as presented by the value of private information, $\Pi(\nu_S > 0) - \Pi(\nu_S = 0)$. Further, such benefit increases as the precision of her signal ν_S is high. Similarly, the platform's ex-ante expected revenue increases as the signal precision about his common marketplace ν_P is high or the seller sets the price p^N based on a more precise signal ν_S due to the reduced variance in market uncertainty ($\frac{\nu_P}{\nu+\nu_P}$, and $\frac{\nu_S}{\nu+\nu_S}$).

4.3.2 With information sharing

Now consider the platform shares his signal x_P unilaterally with the seller when the commission rate r is exogenously given. The seller's optimal price p^W is now determined by leveraging two signals (x_S, x_P). By comparing the results obtained under information sharing, we examine the impact of sharing the platform's signal x_P, containing the online market dynamics, on the ex-ante expected profits of both players.

Seller's Ex-post Expected Profit and Optimal Price. With information sharing, the ex-post expected profit of the seller for any given signals (x_S, x_P) satisfies:

$$\begin{aligned}\mathbb{E}\left(\pi_S^W \mid p; x_S, x_P, r\right) &= (p(1-r)-c)\cdot(a_O + \mathbb{E}(m \mid x_S, x_P) - p) \\ &\quad + (p-c)\cdot(a_D + \mathbb{E}(m \mid x_S, x_P) - p)\end{aligned} \tag{4.9}$$

The seller uses her own private signal x_S and the shared signal x_P provided by the platform to compute $\mathbb{E}(m \mid x_S, x_P)$. By considering the first order consideration associated with the seller's profit $\mathbb{E}\left(\pi_S^W \mid p; x_S, x_P, r\right)$ given in (4.9), we get:

Proposition 4.2. *With information sharing, the seller's optimal price p^W satisfies: $p^W = A^W + A_S^W x_S + A_P^W x_P$, where*

$$A^W = \frac{a_O(1-r) + a_D + 2c}{2(2-r)}, \quad A_S^W = \frac{\nu_S}{2(\nu + \nu_P + \nu_S)}, \text{ and } \quad A_P^W = \frac{\nu_P}{2(\nu + \nu_P + \nu_S)}.$$

Comparing the seller's optimal price from no information sharing p^N from Proposition 4.1, for a given r, the base price from the economic factors remains the same between the two information sharing policies ($A^W = A^N$). Moreover, for an observed signal of the seller x_S, the coefficient A_S^W is now reduced to adapt the impact of the additional signal x_P (i.e., less responsive to the signal from her own market research, $A_S^W < A_S^N$), whereas an additional coefficient A_P^W associated with the signal of the platform x_P affects the equilibrium price under information sharing.

Lemma 4.2. *Relative to no information sharing, the platform's information sharing on x_P leads to the same expected price of the seller $\mathbb{E}_{x_S,x_P}\left\{p^W(x_S, x_P)\right\} = \mathbb{E}_{x_S}\left\{p^N(x_S)\right\}$. However, the demand variance based on the price under sharing is smaller than without sharing, formally*

$$Var\left\{q_O(p^W)\right\} - Var\left\{q_O(p^N)\right\} = -\frac{3\nu_P}{4(\nu + \nu_S + \nu_P)(\nu + \nu_S)} < 0.$$

Since both equilibrium prices $p^N(x_S)$ and $p^W(x_S, x_P)$ have linearity to the private signals, x_S, and x_P (Proposition 4.1 and 4.2) and the expectation of signals is $\mathbb{E}[x_S] = \mathbb{E}[x_P] = 0$, the ex-ante expected prices for the seller are equal to the base price, $A^W = A^N$. Hence, the expected prices remain the same between two sharing policies, $\mathbb{E}_{x_S}\left\{p^N(x_S)\right\} = \mathbb{E}_{x_S,x_P}\left\{p^W(x_S, x_P)\right\}$. More interestingly, with the presence of the platform's information sharing, the variance of online market demand $Var\left\{q_O(p^W)\right\}$ is smaller than when the seller sets the price based on her own signal, $Var\left\{q_O(p^N)\right\}$. Specifically, when the intrinsic market uncertainty is high (i.e., ν is low) or the seller's signal is not precise (i.e., ν_S is low), sharing the platform's information x_P can increase forecasting accuracy on demand by reducing the variability of the information on market uncertainty. Since the seller charges the same price for the online and direct sales channels and both channels share the common market uncertainty m, the impact of the equilibrium price in the direct channel demand q_D is analogous to the online channel demand q_O as shown in Lemma 4.2.

Ex-post Platform's Expected Revenue. Define the platform's expected revenue after sharing the private signal x_P with the seller ($\mathbb{E}(\pi_P^W|x_P, x_S)$), where:

$$\begin{aligned} \mathbb{E}(\pi_P^W|x_P, x_S) &= r \cdot \mathbb{E}\left(p^W \cdot q_O(p^W) \mid x_P, x_S\right) \\ &= r \cdot p^W(x_S, x_P) \cdot \left(a_O + \mathbb{E}\left(m \mid x_P\right) - p^W(x_S, x_P)\right). \end{aligned} \tag{4.10}$$

Compared to (4.6), the seller's equilibrium price now incorporates the signal x_P shared by the platform.

Ex-ante Seller's Expected Profit and Platform's Expected Revenue. We denote Π_S^W as the seller's ex-ante expected profit and Π_P^W as the platform's ex-ante expected revenue with information sharing where:

$$\Pi_S^W \equiv \mathbb{E}_{x_S,x_P}\left\{\pi_S^W(p^N(x_S, x_P))\right\} \quad \text{and} \quad \Pi_P^W \equiv \mathbb{E}_{x_S,x_P}\left\{\pi_P^W(p^W(x_S, x_P))\right\} \tag{4.11}$$

When the commission rate r is exogenously given, the seller's ex-ante expected profit and the platform's ex-ante expected revenue associated with the case when the platform shares its information x_P with the seller can be expressed as:

$$\begin{aligned} \Pi_S^W &= (2-r)\left\{(A_S^W)^2(\frac{1}{\nu} + \frac{1}{\nu_S}) + (A_P^W)^2(\frac{1}{\nu} + \frac{1}{\nu_P}) + A_P^W A_S^W \frac{2}{\nu}\right\} + (2-r)(A^W)^2 - c(a_O + a_D) \\ \Pi_P^W &= r\left\{A^W\left(a_O - A^W\right) + \frac{A_S^W}{2\nu}\left(B_P - \frac{\nu}{\nu + \nu_P}\right) + \frac{A_P^W}{2\nu}\right\}. \end{aligned} \tag{4.12}$$

where A^W A_S^W and A_P^W are given in Proposition 4.2 and $B_P = \frac{\nu_P}{\nu+\nu_P}$. By using the expressions from (4.8) and (4.12), we can compare the ex-ante expected gain (loss) of each player when the platform shares private information, x_P.

Lemma 4.3. *By sharing x_P with the seller, the platform creates the following value:*

1. *The seller will always benefit from receiving the platform's private signal x_P from the platform because its ex-ante expected profit is always higher than the case of no information sharing $\Pi_S^W \geq \Pi_S^N$ where,*

$$\Pi_S^W - \Pi_S^N = \frac{2-r}{4}\left\{\frac{\nu_P}{(\nu+\nu_S+\nu_P)(\nu+\nu_S)}\right\} > 0$$

2. *The platform is better off by sharing his private signal x_P with the seller because its ex-ante expected revenue is always higher than the case of no information sharing $\Pi_P^W \geq \Pi_P^N$ where,*

$$\Pi_P^W - \Pi_P^N = \frac{r}{4}\frac{\nu_P(\nu+2\nu_S+\nu_P)}{(\nu+\nu_S)(\nu+\nu_P)(\nu+\nu_S+\nu_P)} > 0$$

Based on the analytical expression in Lemma 4.3, the comparative statics in the market certainty ν, and the precision terms, ν_S and ν_P show:

$$\frac{\partial \Pi_S^W - \Pi_S^N}{\partial \nu} < 0, \quad \frac{\partial \Pi_S^W - \Pi_S^N}{\partial \nu_S} < 0, \quad \frac{\partial \Pi_S^W - \Pi_S^N}{\partial \nu_P} > 0, \quad \frac{\partial \Pi_P^W - \Pi_P^N}{\partial \nu} < 0, \quad \frac{\partial \Pi_P^W - \Pi_P^N}{\partial \nu_S} < 0,$$

$$\text{and} \quad \frac{\partial \Pi_P^W - \Pi_P^N}{\partial \nu_P} = \frac{r}{4}\left\{\frac{2\nu}{(\nu+\nu_P)^2(\nu+\nu_S)} - \frac{1}{(\nu+\nu_S+\nu_P)^2}\right\}.$$

Lemma 4.3 implies that it is advantageous for both players ("win-win") when the platform shares private information. Because both players generally have higher ex-ante expected profits from the platform's sharing. Especially the ex-ante expected gain from sharing increases when the intrinsic market uncertainty is high (i.e., ν is small). However, the players' ex-ante expected gain from the platform's sharing reduces if the seller's signal x_S is precise (i.e., ν_S is large) as there is little to extract from the platform's signal.

Notably, the platform's ex-ante expected gain from his sharing shows that his gain from sharing private signal x_P decreases when the intrinsic market uncertainty is high (i.e., ν is small) and the platform has an imprecise signal (i.e., ν_P is small). This implies that when both market uncertainty and the platform's information imprecision are high, the platform prefers to remain silent. The reason is as follows: In general, the value of information sharing increases as the players face high market uncertainty. However, when the platform shares imprecise information, the seller's equilibrium price p^W becomes reactive to highly noisy x_P under sharing (otherwise, the seller's price depends on her informative signal x_S), which makes the seller's price react more variable from sharing, hence the platform's ex-ante revenue reduces.

From Lemma 4.3, the seller has a higher ex-ante expected profit under information sharing $\Pi_S^W - \Pi_S^N > 0$ and the seller prefers to receive the platform's signal x_P. Hence, when the platform charges a fixed payment K for sharing information x_P to the seller, then the optimal K^* satisfies: $K^* = \Pi_S^W - \Pi_S^N$. By charging the seller a fixed payment K^* for sharing information x_P with the seller, the platform's ex-ante expected gain increases to $\Delta_P = \Pi_P^W - \Pi_P^N + K^*$, whereas the seller's ex-ante expected gain is reduced to $0 = \Pi_S^W - \Pi_S^N - K^*$.

In summary, if the platform's commission rate r is exogenous, the platform's unilateral information sharing on x_P benefits both players under this symbiotic relationship. The reasoning behind this is that letting the seller optimize her price p incorporating both signals 1) reduces the variance of market uncertainty and increases the ex-ante expected profit of the seller. Subsequently, 2) the increased expected revenue of the seller in general brings a higher ex-ante revenue for the platform. Therefore, the platform does have the incentive to transmit what it observes from the online market to the seller. However, the platform can charge a fee K^* to extract the entire surplus from the seller.

So far, our analysis predicates the assumption that the commission rate r is exogenously given in advance. Without the flexibility to change the commission rate, the platform cannot make use of the seller's private information x_S. This observation motivates us to extend the information sharing model to the case when r is endogenously determined by the platform so that the benefit of mutual information exchange can be observed.

4.4 Information exchange: Endogenous commission rate r

We examine the case when the platform can make use of the seller's private information x_S to determine his commission rate r to maximize his expected revenue. In this setting, observe from Stage 3 in the sequence of events depicted in Figure 4.2 that the platform sets the commission rate r "before" the seller decides on the selling price p in Stage 4. Hence, the platform can anticipate the seller's best response price $\tilde{p}$ for any given commission rate r, and determine his optimal commission rate r^N under no information exchange and r^W with information exchange, respectively. Hence, for any given r, we use Propositions 4.1 and 4.2 as the seller's best response functions in r without and with information exchange $\tilde{p}^N(r, x_S)$ and $\tilde{p}^W(r, x_S, x_P)$ as functions of r, where:

$$\tilde{p}^N(r, x_S) = \tilde{A}^N + \tilde{A}_S^N x_S \quad \text{and} \quad \tilde{p}^W(r, x_S, x_P) = \tilde{A}^W + \tilde{A}_S^W x_S + \tilde{A}_P^W x_P. \tag{4.13}$$

Observe that the terms $\tilde{A}^N(r) = \frac{a_O(1-r)+a_D+2c}{2(2-r)}$ and $\tilde{A}^W(r) = \frac{a_O(1-r)+a_D+2c}{2(2-r)}$ depend on r. However, the coefficients capturing the impact of private signals, $\tilde{A}_S^N = A_S^N$ (from Proposition 4.1) and $\tilde{A}_S^W = A_S^W$ as well as $\tilde{A}_P^W = A_P^W$ (from Proposition 4.2) are independent of r.

4.4.1 No information exchange

When no information exchange takes place, the platform uses its private signal x_P to set r, and the seller uses her private signal x_S to set p.

Ex-post Expected Revenue of the Platform. We now define the platform's expected revenue under his signal x_P and derive r^N that maximizes his expected revenue without information exchange. While the seller's price $\tilde{p}^N(r, x_S)$ from Proposition 4.2 depends on her own private signal x_S, the platform does not know the seller's signal x_S without information exchange. Therefore, the platform uses his expectation of the seller's optimal price $\tilde{p}^N(r, x_S)$ that depends on x_S by using his own signal x_P as:

$$\mathbb{E}(\tilde{p}^N|r, x_P) = \tilde{A}^N + \tilde{A}_S^N \mathbb{E}(x_S \mid x_P) = \tilde{A}^N + \tilde{A}_S^N \mathbb{E}(m \mid x_P) = \tilde{A}^N + \tilde{A}_S^N \frac{\nu}{\nu + \nu_P} x_P. \tag{4.14}$$

The derivation follows from our assumption that $\text{cov}(m, \epsilon_P) = \text{cov}(m, \epsilon_S) = \text{cov}(\epsilon_S, \epsilon_P) = 0$, and $\text{cov}(m, x_P) = \text{cov}(x_P, x_S) = \sigma$. As the conditional expectation of the seller's signal x_S in (4.14) is derived as $\mathbb{E}(x_S \mid x_P) = \mathbb{E}(m + \epsilon_S \mid x_P) = \mathbb{E}(m \mid x_P)$, by combining this observation and Lemma 4.1, we obtain the expression in (4.14).

By using $\mathbb{E}(\tilde{p}^N|r, x_P)$, the platform's expected revenue $\mathbb{E}(\tilde{\pi}_P^N|r; x_P)$ given his private signal x_P is:

$$\mathbb{E}(\tilde{\pi}_P^N|r; x_P) = r \cdot \mathbb{E}\left(\tilde{p}^N \cdot q_O(\tilde{p}^N) \mid r, x_P\right) = r \cdot \mathbb{E}\left(\tilde{p}^N \mid r, x_P\right) \cdot \left(a_O + \mathbb{E}\left(m \mid x_P\right) - \mathbb{E}\left(\tilde{p}^N \mid r, x_P\right)\right), \tag{4.15}$$

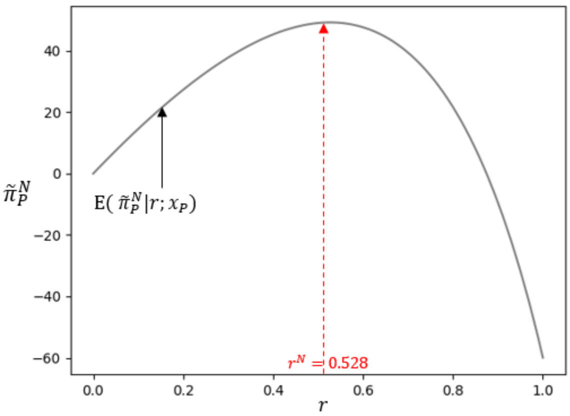
where $\mathbb{E}\left(m \mid x_P\right) = \frac{\nu_P}{\nu+\nu_P} x_P$ as given in (4.3) and $\mathbb{E}\left(\tilde{p}^N \mid r, x_P\right)$ is given in (4.14).

Platform's Optimal Commission Rate r^N. Considering the platform's revenue $\mathbb{E}(\tilde{\pi}_P^N|r; x_P)$ in (4.15), the platform's optimal commission rate r^N for any observed private signal x_P that maximizes $\mathbb{E}(\tilde{\pi}_P^N|r; x_P)$. By differentiating $\mathbb{E}(\tilde{\pi}_P^N|r; x_P)$ with respect to r, the optimal commission rate r^N is the solution to the following cubic equation:

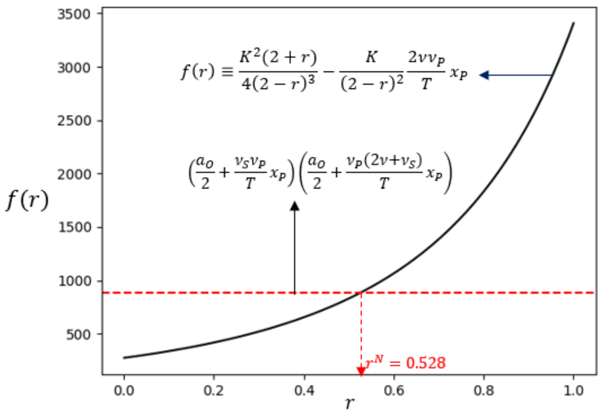
$$\frac{K^2(2+r)}{4(2-r)^3} - \frac{K}{(2-r)^2}\frac{2\nu\nu_P}{T}x_P = \left(\frac{a_O}{2} + \frac{\nu_S\nu_P}{T}x_P\right)\left(\frac{a_O}{2} + \frac{\nu_P(2\nu+\nu_S)}{T}x_P\right), \tag{4.16}$$

where $K = -a_O + a_D + 2c$ and $T = 2(\nu+\nu_P)(\nu+\nu_S)$. We illustrate the value r^N that satisfies (4.16) graphically. As shown in Figure 4.3a, the platform's expected revenue is concave for $r \in (0, 2)$, although a cubic function shows a concave-convex function in general. From Figure 4.3b that the left-hand side of (4.16) is monotonically increasing in r, the right-hand side is independent of r, so these two curves intersect at r^N.

Figure 4.3: Platform's Expected Revenue and Equilibrium Commission Rate Condition where, $a_O = 30, a_D = 60, c = 1, \nu = 1, \nu_S = 0.5,$ and $\nu_P = 0.5$



(a) Platform's Expected Revenue



(b) Optimal Commission rate Condition

Ex-ante Expected Profit and Revenue Functions. Without information exchange, the commission rate $r^N(x_P)$ that maximizes the expected revenue given in (4.15) depends on the platform's own signal x_P, whilst the seller's optimal price depends on $r^N(x_P)$ and x_S. Hence, the seller's ex-ante expected profit and the platform's ex-ante expected revenue are:

$$\tilde{\Pi}_S^N \equiv \mathbb{E}_{x_S,x_P}\left\{\tilde{\pi}_S^N\left(r^N(x_P), \tilde{p}^N(x_S)\right)\right\} \quad \text{and} \quad \tilde{\Pi}_P^N \equiv \mathbb{E}_{x_S,x_P}\left\{\tilde{\pi}_P^N\left(r^N(x_P), \tilde{p}^N(x_S)\right)\right\}. \tag{4.17}$$

Unlike the ex-ante expected revenue in (4.7) for the case of exogenous commission rate r, the platform's optimal commission rate $r^N(x_P)$ given in (4.16) depends on the platform's signal x_P, which will, in turn, affect the seller's expected profit. Hence, the seller's ex-ante expected profit is based on the expectation over x_P as well.

4.4.2 With information exchange

When the seller shares her private signal x_S with the platform in exchange for the platform's private signal x_P, both parties can leverage two signals (x_S, x_P) to obtain a more accurate forecast about the market condition m as shown in Lemma 4.1. From (4.13), the seller's best response price $\tilde{p}^W(r, x_P, x_S) = \tilde{A}^W + \tilde{A}^W_S x_S + \tilde{A}^W_P x_P$ depends on two signals (x_S, x_P) and r.

Optimal Commission Rate r^W. By substituting the seller's best response price $\tilde{p}^W(r, x_P, x_S)$ into the platform's expected revenue $\mathbb{E}(\tilde{\pi}^W_P | r; x_P, x_S)$, we get:

$$\mathbb{E}(\tilde{\pi}^W_P | r; x_P, x_S) = r \cdot \tilde{p}^W(r, x_S, x_P) \cdot \big(a_O + \mathbb{E}\left(m \mid x_P, x_S\right) - \tilde{p}^W(r, x_S, x_P)\big)\,. \tag{4.18}$$

where $\mathbb{E}\left(m \mid x_P, x_S\right)$ is given in Lemma 4.1 and $\tilde{p}^W(r, x_P, x_S)$ is stated in (4.13). By considering the first-order condition with respect to r, the platform's optimal commission rate r^W solves the following equation:

$$\frac{K^2(2+r)}{4(2-r)^3} = \left(\frac{a_O}{2} + \frac{\nu_S x_S + \nu_P x_P}{2(\nu + \nu_S + \nu_P)}\right)^2, \tag{4.19}$$

where $K = -a_O + a_D + 2c$.

Ex-ante Expected Profit and Revenue Function. Armed with the platform's optimal commission rate r^W and the corresponding seller's best response price $\tilde{p}^W\left(r^W, x_S, x_P\right)$, the seller's ex-ante expected profit and the platform's ex-ante expected revenue are:

$$\begin{aligned} \tilde{\Pi}^W_S &\equiv \mathbb{E}_{x_S,x_P}\left\{\tilde{\pi}^W_S\left(r^W(x_S, x_P), \tilde{p}^W\left(x_S, x_P\right)\right)\right\} \text{ and} \\ \tilde{\Pi}^W_P &\equiv \mathbb{E}_{x_S,x_P}\left\{\tilde{\pi}^W_P\left(r^W(x_S, x_P), \tilde{p}^W\left(x_S, x_P\right)\right)\right\}. \end{aligned} \tag{4.20}$$

4.5 Result comparison of information exchange and no exchange

Comparison of Platform's Expected Commission Rates and Prices. Based on two signals x_S and x_P, we now compare the platform's optimal commission rate $r^N(x_P)$ and $r^W(x_S, x_P)$ given in (4.16) and (4.19), and the impact of an endogenized commission rate to the seller's price. While the analytical expressions for the optimal commission rates appear to be intricate, we can derive the following outcomes related to two distinct scenarios:[2]

[2] These results can provide us some structural results that we can examine numerically for intermediate values.

Proposition 4.3. *When the commission rate r is endogenously determined by the platform, the expected optimal commission rate satisfies $\mathbb{E}[r^W(x_S, x_P)] < \mathbb{E}[r^N(x_P)]$ if the seller's signal is precise (i.e., $\nu_S \to \infty$) or the platform's signal is not precise (i.e., $\nu_P \to 0$).*

Proposition 4.3 appears to be counter-intuitive as the seller might believe that once the platform obtains the exchanged information x_S provided by the seller, the platform would exploit the seller's information by increasing the commission rate r. However, Proposition 4.3 debunks this common belief. Indeed, when the seller's private signal is considerably precise ($\nu_S \to \infty$), the platform will actually lower its commission rate by possessing the seller's precise signal. At the same time, when the platform's own private signal is imprecise ($\nu_P \to 0$), exchanging information makes him reduce the commission rate too, as depicted in Figure 4.4.

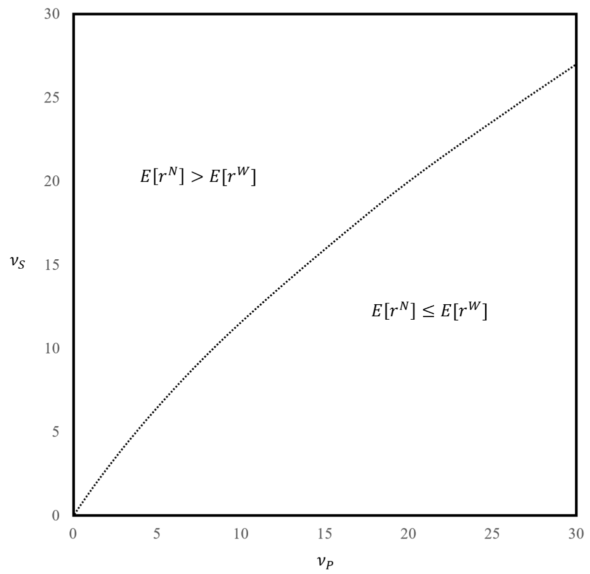


Figure 4.4: Comparison of Expected Commission Rates: $\mathbb{E}[r^W]$ vs. $\mathbb{E}[r^N]$ where, $a_O = 30$, $a_D = 60$, $c = 1$, and $\nu = 1$

These results can be interpreted based on the following reasons. First, when $\nu_S \to \infty$, receiving x_S significantly reduces the variance of demand randomness for the platform. Additionally, under the information exchange, the seller's price reaction function in x_S becomes symmetric, enabling the platform to extract greater revenue from the seller as x_S serves as an almost perfect signal for the market when $\nu_S \to \infty$. As a result, exchanging x_S and x_P leads to a lower expected commission rate compared to the no exchange case. By doing so, the platform earns a higher revenue.

Furthermore, when the platform's own signal precision is low ($\nu_P \to 0$), the platform gives little weight to adapt its imprecise signals. This means that under no information exchange, the equilibrium commission rate suffices $\frac{2+r^N}{(2-r^N)^3} = (\frac{a_O}{K})^2$ that is independent of x_P. Hence, without information exchange, if $\nu_P \to 0$, as the platform has an uninformative signal x_P he gives little reaction to x_P. However, when exchanging signals, the platform can use the seller's signal x_S to improve his forecast accuracy of market uncertainty as exhibited in Lemma 4.1 due to variance reduction. Further, the platform now incorporates the seller's relatively precise signal x_S to his commission rate decision. This reduction in variance, along with signal adaptation, lets the platform afford a lower commission rate that can entice the seller to set an optimal price $\tilde{p}^W(r, x_S, x_P)$ that increases the seller's revenue from the online channel.

From the seller's side, her pricing is influenced by two factors: the commission rate r set by the platform and the signals capturing market uncertainty. While possessing more precise market information from mutual exchange might lead her to raise her expected price, the effect of this could be tempered by the decrease in the platform's commission rate. We introduce the following proposition to demonstrate the outcomes regarding the seller's expected price.

Proposition 4.4. *Suppose $\nu_P \to 0$ or $\nu_S \to \infty$ so that $\mathbb{E}[r^W] < \mathbb{E}[r^N]$. Then the seller's expected price is* ***lower*** *under information exchange than no exchange; i.e., $\mathbb{E}[\tilde{p}^W] \leq \mathbb{E}[\tilde{p}^N]$ if and only if the online market potential is small ($a_O \leq a_D + 2c$).*

The impact of reduced expected commission rate from the platform on the seller's expected price depends on the market potential between a_O and a_D. The results imply that when the platform's information is highly imprecise (i.e., $\nu_P \to 0$) or the seller's information is significantly precise (i.e., $\nu_S \to \infty$), the platform imposes a lower expected commission rate (Proposition 4.3) in exchanging information, $\mathbb{E}[r^W] < \mathbb{E}[r^N]$. Consecutively, when the online market potential is relatively low ($a_O \leq a_D + 2c$), the seller's expected price with information exchange is also lower than without exchange, $\mathbb{E}[\tilde{p}^W] < \mathbb{E}[\tilde{p}^N]$. However, as the online market potential becomes larger, $a_O > a_D + 2c$, the lower expected commission rate from the platform without information exchange $\mathbb{E}[r^W] < \mathbb{E}[r^N]$ leads to a higher expected price from the seller $\mathbb{E}[\tilde{p}^W] \geq \mathbb{E}[\tilde{p}^N]$.

This can be explained by the seller's market power under two channels. When the online market potential a_O is small, the seller's market power against the platform in the online channel is weak. Hence, the reduced commission rate of r from the platform induces the seller to follow by decreasing the selling price. On the other hand, in case the online market potential is sufficiently large, she can afford to increase the selling price in exchanging information while keeping the benefit of having more accurate market information.

Proposition 4.3 shows that, upon possessing the exchanged information x_S, the platform will lower its commission rate under certain conditions. With a lower commission rate, will the platform earn less with information exchange? Further, from Proposition 4.4, we observe that the platform's reduced expected commission rate does not unidirectionally make the seller's expected price higher or lower but depends on the market potentials of the two channels. Hence, it is worth investigating the following questions: will the seller earn more with information exchange? Is information exchange always beneficial to both parties, only one party, or no party?

Comparison of Players' Ex-ante Expected Profits. We now investigate each player's incentive to exchange their own signals. Specifically, built upon the findings from Proposition 4.3 and 4.4, we present how the endogenized commission rate decision of the platform and the subsequent reaction of the seller's price affect the ex-ante expected profit of each player with and without information exchange.

Proposition 4.5. *When the platform's signal is not precise (i.e., $\nu_P \to 0$), or the seller's signal is precise (i.e., $\nu_S \to \infty$), exchanging information only benefits the platform ($\tilde{\Pi}_P^W > \tilde{\Pi}_P^N$), while the seller is worse off ($\tilde{\Pi}_S^W < \tilde{\Pi}_S^N$).*

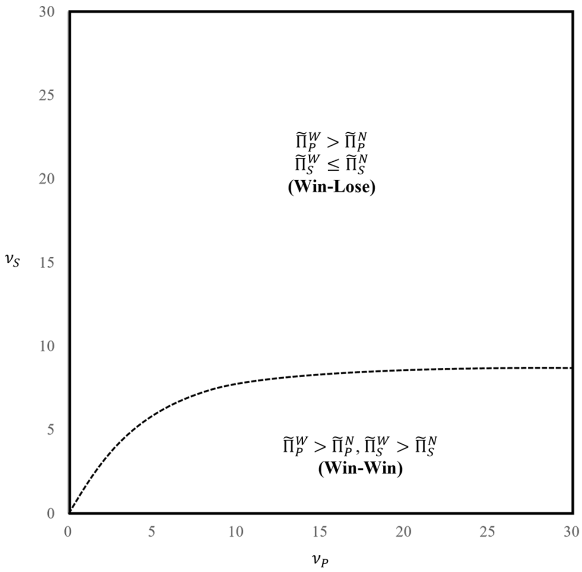
Proposition 4.5 states that for the cases in which the platform reduces the commission rate (i.e., $\nu_P \to 0$ and $\nu_S \to \infty$), such a decision makes the seller worse off while the platform benefits from the information exchange. This is because when the seller's information is precise ($\nu_S \to \infty$) or the platform's information is imprecise ($\nu_P \to 0$), the platform's benefit of exchanging information comes from two factors: 1) the reduction of the variability in market randomness and 2) information symmetry (i.e., the platform observes the seller's price reaction on his commission rate r as well as the seller's private signal x_S). Under these conditions, the reduced market uncertainty increases his revenue. Also, the platform can extract a higher online market surplus by charging a better commission rate.

On the other hand, the seller does not benefit from reducing the market variability as in both cases ($\nu_S \to \infty$ or $\nu_P \to 0$), the seller is more reactive to her own signal, regardless of information exchange, $Var(m|x_S) = Var(m|x_S, x_P)$. Further, as the platform absorbs more surplus by charging a better commission rate under information exchange, the seller loses the incentive to exchange information under these two conditions.

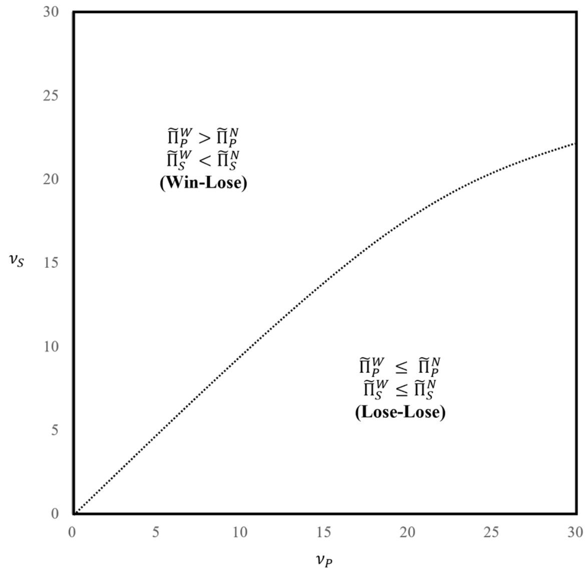
Moreover, Figure 4.5 numerically illustrates that when the seller exchanges imprecise information (i.e., $\nu_S \to 0$), the benefit of such exchange depends on the market potential of two channels. For instance, when the online market potential is relatively higher than that of the direct sales ($a_O > a_D$), a win-win scenario can happen. However, the motivation behind each player benefiting from information exchange varies. The platform gains from knowing the price reaction function $\tilde{p}^W(r, x_S, x_P)$ from the seller's sharing without information asymmetry, whilst the seller mainly benefits from better forecasting market uncertainty based on the platform's precise information. Reducing the market uncertainty can offset the seller's loss from a higher commission rate charged by the platform in the online channel with large potential ($a_O > a_D$).

On the other hand, when the seller's online market potential is relatively smaller than that of the direct sales channel ($a_O \leq a_D$), both players do not gain from the information exchange. The platform (although he can charge a better commission rate) cannot extract sufficient revenue from the seller's online channel, and the seller's benefit of obtaining a better forecast on market uncertainty disappears as there is little room to generate more revenue from the online market. As Figure 4.5 shows, the advantage of information exchange between the players fluctuates not only with the precision of private information but also with the market potential of both online and direct sales channels. In the following part, we explore how market potential influences each player's incentive to exchange information.

Figure 4.5: Ex-ante Expected Profit of the Seller $\tilde{\Pi}_S$ and Revenue of the Platform $\tilde{\Pi}_P$



(a) $a_O > a_D$ where, $a_O = 60$, $a_D = 30$, $c = 1$, and $\nu = 1$



(b) $a_O \leq a_D$ where, $a_O = 30$, $a_D = 60$, $c = 1$, and $\nu = 1$

Impact of Market Potentials. When the platform's commission rate is endogenized, the platform's optimal commission rate decision is affected not only by the private signals but also by the seller's market potential from two channels (i.e., a_O and a_D) as shown in (4.16) and (4.19). By taking the market potentials into consideration, we get:

Lemma 4.4. *The ex-ante expected profits of the seller and the platform with bilateral information exchange exhibit the following properties:*

1. *If the online channel market potential is sufficiently large ($a_O \to \infty$), then both players will benefit from information exchange; i.e., $\tilde{\Pi}_P^W > \tilde{\Pi}_P^N$ and $\tilde{\Pi}_S^W > \tilde{\Pi}_S^N$.*

2. *If the direct sales channel market potential is sufficiently large ($a_D \to \infty$), then information exchange will only benefit the seller so that $\tilde{\Pi}_S^W > \tilde{\Pi}_S^N$ and $\tilde{\Pi}_P^W = \tilde{\Pi}_P^N = 0$.*

Lemma 4.4 implies that exchanging market signals bilaterally, allowing the platform to determine its commission rate based on the seller's signal, does not necessarily have a detrimental effect on the seller's profit. This is because, in situations where market potentials are significantly large (i.e., as $a_O \to \infty$ and $a_D \to \infty$), the platform's commission rate decision becomes less responsive to the exchanged signals. This reduced responsiveness occurs because the impact of market potentials dominates the influence of signals, effectively minimizing the disparity between r^N and r^W brought by the exchange of information. Consequently, the seller enjoys the benefit of reducing market variability due to the information exchange, all the while having fewer concerns about the platform exploiting her information when it comes to deciding the commission rate.

Specifically, when the online channel has a significantly large market potential (i.e., when $a_O \to \infty$), the seller's primary revenue stream will be generated through the platform's online channel. Hence, the platform's equilibrium commission rate imposed on the seller plays a pivotal role. Nevertheless, as the seller can bring a substantial market potential for the online channel (because $a_O \to \infty$), the platform's commission rate decision places more significance on the magnitude of the online market potential a_O rather than on the exchanged information itself, making $\frac{2+r}{(2-r)^3} = 1$ hold true for both r^N and r^W as derived from (4.16) and (4.19). Consequently, as the online market potential becomes larger, the platform's commission rates and the seller's prices are fully determined by economic factors, a_O, a_D, and c, leading to $r^N = r^W$ and $\tilde{A}^N = \tilde{A}^W$ ($\tilde{p}^N = \tilde{p}^W$) as outlined in (4.13), (4.16), and (4.19). Therefore, the information exchange mainly serves to diminish the variability of market uncertainty and subsequently increases the ex-ante expected profits for both parties involved, explaining statement 1 of Lemma 4.4.

On the flip side, as the seller's direct sales channel market potential a_D is high, the seller is less influenced by the platform and its online marketplace. Although the platform is better off sharing its signal in general, when the direct sales channel market potential is significantly larger ($a_D \to \infty$) than that of the online channel, the platform knows that the seller can afford to charge a higher price to obtain a much higher profit from its direct channel. Anticipating that the seller will set a higher price, the platform is encouraged to lower its commission rate for the online channel, $r^N(x_P) = r^W(x_P, x_S) \to 0$, which essentially takes out his benefit of exchanging information. However, a lower commission rate combined with more accurate information brings the seller a higher profit (mainly generated from the direct sale channel) under the information exchange. This explains statement 2 of Lemma 4.4.

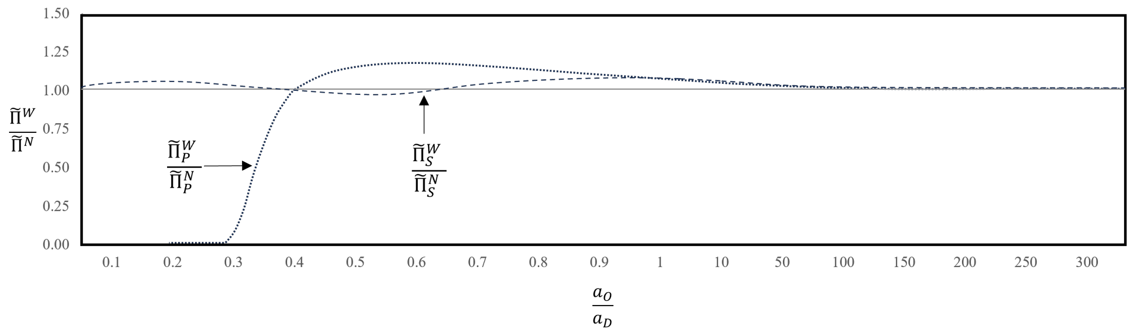
We graphically show the impact of the market potentials of both channels (a_O and a_D) on the value of bilateral information exchange between the platform and the seller in Figure 4.6. To capture these effects succinctly, we use $\frac{a_O}{a_D}$ in the horizontal axis to capture both extreme cases so that $\frac{a_O}{a_D} \to \infty$ when $a_O \to \infty$ as captured in statement 1 in Lemma 5 and $\frac{a_O}{a_D} \to 0$ as $a_D \to \infty$ as captured in statement 2 in Lemma 4.4. Also, we use the ratio $\frac{\tilde{\Pi}_j^W}{\tilde{\Pi}_j^N}$ to observe the benefit of bilateral information exchange for entity $j \in (S, P)$.

Figure 4.6a shows that as market potential experiences an upsurge in the online channel ($\frac{a_O}{a_D} \to \infty$), both players have the incentive to exchange information, demonstrated by the ratios: $\frac{\tilde{\Pi}_P^W}{\tilde{\Pi}_P^N} > 1$ and $\frac{\tilde{\Pi}_S^W}{\tilde{\Pi}_S^N} > 1$. One notable observation is that the platform's ex-ante expected revenue under information exchange increases to some extent as the online market potential of the seller becomes relatively larger than the direct sales channel. However, as the online market potential becomes considerably large ($a_O \to \infty$), the seller charges the price as high as possible as demonstrated by (4.13) and eventually the profits obtained from no exchange and exchange remain the same, making $\frac{\tilde{\Pi}_j^W}{\tilde{\Pi}_j^N}$ converge to 1 for both players.

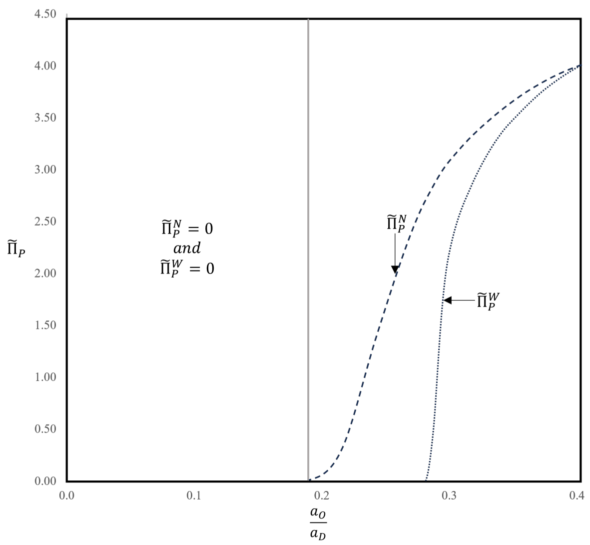
When the seller's direct sales channel market potential is substantially larger than the online channel market potential ($\frac{a_O}{a_D} \to 0$), the platform gains little as he has to reduce the commission rate close to zero and the seller's presence in the platform's common marketplace reduces. In general, when the market potential in the direct sales channel becomes significant ($a_D \to \infty$), the platform's ex-ante expected profits from both exchange cases reduce significantly as illustrated in Figure 4.6b. In particular, under the information exchange, the platform's ex-ante expected profit reduces more sharply than that of no information exchange. This implies that for $a_D \to \infty$, the commission rate under the information exchange converges to $r^W \to 0$ faster than $r^N \to 0$. Therefore, as $a_D \to \infty$, the platform's incentive to exchange information disappears because no profit can be generated by doing so.

On the other hand, the seller's gain from information exchange is more intricate. For example, when the market potential for direct sales is notably elevated (i.e., $\frac{a_O}{a_D} \to 0$), the platform charges nearly zero commission rate. As a result, the advantage of diminishing market uncertainty

Figure 4.6: Ex-ante Expected Profits of the Players
where, $a_O = 60$, $\nu = 0.5, \nu_S = 1, \nu_P = 1,$ and $c = 1$



(a) Comparison of Information Exchange and No Information Exchange $\frac{\tilde{\Pi^W}}{\tilde{\Pi^N}}$



(b) Platform's Ex-ante Expected Profits $\tilde{\Pi_P^W}$ vs. $\tilde{\Pi_P^N}$

through information exchange takes precedence over any potential negative impact from the platform's exploitation of higher commission rates. Furthermore, if the market potential in the online market is substantial (i.e., $\frac{a_O}{a_D} \to \infty$), the seller reaps the benefit of information exchange ($\frac{\tilde{\Pi}_S^W}{\tilde{\Pi}_S^N} > 1$) due to the reduced uncertainty in market randomness within the expansive online market potential. Hence, in both cases, it makes the seller better off to exchange information (Figure 4.6a). However, when the seller's market potentials are within an interim range, and the platform sets the commission rate mainly based on the signal exchanged (not based on dominant market potential), the seller's motivation to exchange information decreases ($\frac{\tilde{\Pi}_S^W}{\tilde{\Pi}_S^N} < 1$). This is because the platform absorbs a substantial portion of the surplus by imposing a higher commission rate linked to information exchange, but the market potentials from the two channels are not sufficient to compensate for the loss from a higher commission rate imposed on the online channel.

4.6 Conclusions and future research

Motivated by the emerging interest in sharing or exchanging market information between sellers and platforms, we present a stylized game theoretic model as an attempt to explore the potential value gained from bilateral information exchanges. Our analysis focuses on investigating the impact of bilateral information exchange on the optimal commission rate decision made by the platform and the selling price decision made by the seller. Our results reveal that when the platform's commission rate is exogenous (so that it is not dependent on the market signal), unilateral information sharing of the platform always benefits both players. We show a notable difference between information sharing and exchange that information exchange can benefit both players only under certain conditions. This is because, in the case of bilateral information exchange, the seller faces a trade-off between the benefits derived from obtaining a more accurate forecast of the market uncertainty and the potential risk of the platform leveraging the seller's market information to impose a higher commission rate, whilst the unilateral information sharing consistently makes the seller better off by reducing market uncertainty.

By extending our unilateral information-sharing model to the case of bilateral information exchanges, we find that a highly precise seller's signal can induce the platform to decrease its commission rate. However, although the platform's commission rate reduces if the seller exchanges precise information, the seller's ex-ante expected profit decreases; hence, only the platform benefits from the exchange. We also find that, if the seller's direct sales channel market potential is large, the platform's incentive to exchange information disappears as he is forced to reduce his commission rate. However, information exchange between a platform and a seller is mutually beneficial when the seller's market potential in the online channel is significantly larger than that of the direct sales channel. In contrast to the prevailing practice of platforms unilaterally sharing information with their sellers, this result has the following managerial implication: when the majority of the seller's revenue is generated from the platform's sales, engaging in information exchange can foster a mutual benefit by facilitating a better pricing decision for the seller and a commission decision for the platform, ensuring a win-win case.

Our stylized model has several limitations that deserve further investigation. First, our model does not consider the competition effect under different channels by assuming that the seller offers the same price to the direct and online sales channels. However, in practice, consumer utility differs in various channels, and a competition effect may exist and the price can be discriminated by the seller. Although the choice model in multi-channel retailing was beyond the scope of this study, it has gained much attention and is worth incorporating into information-sharing topics. Second, our model does not consider multiple sellers using a common online marketplace of a platform. Because the existence of other sellers intensifies competition, the analysis of a model based on Bertrand's competition is complex that we shall relegate it to future research. Moreover, for technical tractability, we adopt a linear demand function with market randomness that follows a normal distribution. It's worth extending our model by considering different forms of market randomness, as one may derive additional insights regarding the effects of information exchange and prior updates. Finally, we focus on voluntary information exchange without any financial incentives. However, the benefit that each player can obtain from mutually exchanging information differs. Hence, it is of interest to develop an incentive or a bargaining mechanism that can induce information exchange between sellers and platforms.

Appendix

Proof of Lemma 4.1 As random variables, m, x_S, and x_P are normally distributed, we can describe the covariance matrix as below:

$$\begin{bmatrix} \mathrm{Cov}(m, x_S) & \mathrm{Cov}(m, x_P) \end{bmatrix} = \begin{bmatrix} \sigma & \sigma \end{bmatrix},$$

$$\begin{bmatrix} \mathrm{Var}(x_S) & \mathrm{Cov}(x_S, x_P) \\ \mathrm{Cov}(x_S, x_P) & \mathrm{Var}(x_P) \end{bmatrix} = \begin{bmatrix} \sigma + \tau_S & \sigma \\ \sigma & \sigma + \tau_P \end{bmatrix}.$$

Therefore, by replacing the variance with the precision term $\nu_S = \frac{1}{\tau_S}$ and $\nu = \frac{1}{\sigma}$,

$$\mathbb{E}\{m \mid x_S\} = \frac{\sigma x_S}{\sigma + \tau_S} = \frac{\nu_S x_S}{\nu + \nu_S} \text{ and } \mathrm{Var}\{m \mid x_S\} = \frac{\sigma \tau_S}{\sigma + \tau_S} = \frac{1}{\nu + \nu_S}.$$

The derivation of $\mathbb{E}\{m \mid x_P\}$ and $\mathrm{Var}\{m \mid x_P\}$ are analoguous from $\mathbb{E}\{m \mid x_S\}$ and $\mathrm{Var}\{m \mid x_S\}$. When the information is exchanged,

$$\mathbb{E}\{m \mid x_S, x_P\} = \begin{bmatrix} \sigma & \sigma \end{bmatrix} \begin{bmatrix} \sigma + \tau_S & \sigma \\ \sigma & \sigma + \tau_P \end{bmatrix}^{-1} \begin{bmatrix} x_S \\ x_P \end{bmatrix}$$

$$\mathrm{Var}\{m \mid x_S, x_P\} = \sigma - \begin{bmatrix} \sigma & \sigma \end{bmatrix} \begin{bmatrix} \sigma + \tau_S & \sigma \\ \sigma & \sigma + \tau_P \end{bmatrix}^{-1} \begin{bmatrix} \sigma \\ \sigma \end{bmatrix}$$

By using $\begin{bmatrix} \sigma + \tau_S & \sigma \\ \sigma & \sigma + \tau_P \end{bmatrix}^{-1} = \frac{1}{\sigma(\tau_S + \tau_P) + \tau_S \tau_P} \begin{bmatrix} \sigma + \tau_P & -\sigma \\ -\sigma & \sigma + \tau_S \end{bmatrix}$, we can calculate the multiplication of matrices:

$$\mathbb{E}\{m \mid x_S, x_P\} = \frac{\sigma\left[\tau_P x_S + \tau_S x_P\right]}{\sigma\left(\tau_P + \tau_S\right) + \tau_P \tau_S} = \frac{\nu_S x_S + \nu_P x_P}{\nu + \nu_S + \nu_P}, \text{ and}$$

$$\mathrm{Var}\{m \mid x_S, x_P\} = \frac{\sigma \tau_S \tau_P}{\sigma(\tau_S + \tau_P) + \tau_P \tau_S} = \frac{1}{\nu + \nu_S + \nu_P}. \qquad \square$$

Proof of Proposition 4.1 From the expected profit of the seller in (4.5), the first order condition (FOC) with respect to p yields

$$\begin{aligned} \frac{\partial \mathbb{E}(\pi_S^N \mid p; x_S, r)}{\partial p} &= \left(a_O - \mathbb{E}\left[m \mid x_S\right] - 2p\right)(1 - r) + c + \left(a_D + \mathbb{E}\left[m \mid x_S\right] - 2p + c\right) = 0 \\ &\Leftrightarrow 2p(2 - r) = (1 - r)a_O + a_D + 2c + \mathbb{E}\left[m \mid x_S\right](2 - r) \\ &\Leftrightarrow p = \frac{(1 - r)a_O + a_D + 2c}{2(2 - r)} + \frac{1}{2}\mathbb{E}\left[m \mid x_S\right]. \end{aligned}$$

By using the precision term $\nu_i = \frac{1}{\tau_i}$ and $\nu = \frac{1}{\sigma}$, we can rewrite the conditional distribution in (4.3) as $\mathbb{E}[m \mid x_S] = \frac{\nu_S}{\nu+\nu_S} x_S$ and the FOC is rearranged to

$$p = \underbrace{\frac{a_O(1-r) + a_D + 2c}{2(2-r)}}_{A^N} + \underbrace{\frac{\nu_S}{2(\nu+\nu_S)}}_{A_S^N} x_S.$$

From the second order condition (SOC) follows $\frac{\partial^2 \mathbb{E}(\pi_S^N | p; x_S, r)}{\partial p^2} = -2(2-r) < 0$, we conclude that the expected profit of the seller is concave and unimodal in p, hence the FOC ($\frac{\partial \mathbb{E}(\pi_S^N | p; x_S, r)}{\partial p} = 0$) yields an optimal price p^N for the seller. □

Proof of Proposition 4.2 From the expected profit of the seller under information sharing,
$\mathbb{E}\left[\pi_S^W \mid p; x_P, x_S, r\right] = (p(1-r) - c)\left(a_O + \mathbb{E}[m \mid x_P, x_S] - p\right) + (p-c)\left(a_D + \mathbb{E}[m \mid x_P, x_S] - p\right)$

Based on the first order condition (FOC) with respect to p,

$$\frac{\partial \mathbb{E}[\pi \mid p, x_P, x_S, r]}{\partial p} = (a_O - \mathbb{E}[m \mid x_P, x_S] - 2p)(1-r) + c + (a_D + \mathbb{E}[m \mid x_P, x_S] - 2p + c) = 0$$

$$\Leftrightarrow 2p(2-r) = (1-r)a_O + a_D + 2c + \mathbb{E}[m \mid x_P, x_S](2-r)$$

$$\Leftrightarrow p = \frac{(1-r)a_O + a_D + 2c}{2(2-r)} + \frac{1}{2}\mathbb{E}[m \mid x_P, x_S]$$

By using the precision term $\nu_i = \frac{1}{\tau_i}$ and $\nu = \frac{1}{\sigma}$, we can rewrite the conditional distribution in (4.3) as

$$\mathbb{E}[m \mid x_P, x_S] = \frac{\nu_P}{\nu + \nu_P + \nu_S} x_P + \frac{\nu_S}{\nu + \nu_P + \nu_S} x_S.$$

The FOC is rearranged to

$$p = \underbrace{\frac{a_O(1-r) + a_D + 2c}{2(2-r)}}_{A^W} + \underbrace{\frac{\nu_S}{2(\nu + \nu_P + \nu_S)}}_{A_S^W} x_S + \underbrace{\frac{\nu_P}{2(\nu + \nu_P + \nu_S)}}_{A_P^W} x_P$$

Similar to Proposition 4.1, the second order condition (SOC) follows $\frac{\partial^2 \mathbb{E}\left[\pi_S^W | p; x_P, x_S, r\right]}{\partial p^2} = -2(2-r) \leq 0$, there exists an optimal price p^W that satisfies $\frac{\partial \mathbb{E}\left[\pi_S^W | p; x_P, x_S, r\right]}{\partial p} = 0$. □

Proof of Lemma 4.2 From the fact that $\mathbb{E}[x_S] = \mathbb{E}[x_P] = 0$, the expected prices of the seller under no information exchange and with exchange yield:

$$\mathbb{E}_{x_S}\left\{p^N(x_S)\right\} = \mathbb{E}_{x_S, x_P}\left\{p^W(x_S, x_P)\right\} = A^N = A^W.$$

Using $\mathbb{E}[x_S^2] = \frac{1}{\nu} + \frac{1}{\nu_S}$, $\mathbb{E}[x_P^2] = \frac{1}{\nu} + \frac{1}{\nu_P}$, $\mathbb{E}[x_S x_P] = \mathbb{E}m x_S = \mathbb{E}m x_P = \frac{1}{\nu}$, the variance of the online demand in two different cases can be described as:

$$Var\left\{q_O(p^N)\right\} = Var\left\{a_O + m - p^N\right\} = \frac{1}{\nu} - \frac{3\nu_S}{4\nu(\nu+\nu_S)},$$
$$Var\left\{q_O(p^W)\right\} = Var\left\{a_O + m - p^W\right\} = \frac{1}{\nu} - \frac{3(\nu_S+\nu_P)}{4\nu(\nu+\nu_S+\nu_P)}.$$

From the above results, we can compare $Var\left\{q_O(p^N)\right\}$ and $Var\left\{q_O(p^W)\right\}$ as

$$Var\left\{q_O(p^W)\right\} - Var\left\{q_O(p^N)\right\} = -\frac{3\nu_P}{4(\nu+\nu_S+\nu_P)(\nu+\nu_S)} < 0. \qquad \square$$

Proof of Lemma 4.3 Similar to (4.8), we first focus on the seller's ex-ante expected profit under information exchange. Using $p^W(x_S, x_P) = A^W + A_S^W x_S + A_P^W x_P$ and $\mathbb{E}[m|x_S, x_P] = \frac{\nu_S x_S + \nu_P x_P}{\nu+\nu_S+\nu_P}$,

$$\Pi_S^W = \mathbb{E}_{x_S,x_P}\left\{(A^W + A_S^W x_S + A_P^W x_P)(1-r) - c)(a_O + \frac{\nu_S x_S + \nu_P x_P}{\nu+\nu_S+\nu_P} - A^W - A_S^W x_S - A_P^W x_P)\right\}$$
$$+ \mathbb{E}_{x_S,x_P}\left\{(A^W + A_S^W x_S + A_P^W x_P - c)(a_D + \frac{\nu_S x_S + \nu_P x_P}{\nu+\nu_S+\nu_P} - A^W - A_S^W x_S - A_P^W x_P)\right\}.$$

From $\frac{\nu_S x_S}{\nu+\nu_S+\nu_P} = 2A_S^W x_S$ and $\frac{\nu_P x_P}{\nu+\nu_S+\nu_P} = 2A_P^W x_P$,

$$\Pi_S^W = \mathbb{E}_{x_S,x_P}\left\{(A^W + A_S^W x_S + A_P^W x_P)(1-r) - c)(a_O - A^W + A_S^W x_S + A_P^W x_P)\right\}$$
$$+ \mathbb{E}_{x_S,x_P}\left\{(A^W + A_S^W x_S + A_P^W x_P - c)(a_D - A^W + A_S^W x_S + A_P^W x_P)\right\}.$$

As $a_O(1-r) + a_D + 2c = 2(2-r)A^W$, and $\mathbb{E}[x_S] = \mathbb{E}[x_P] = 0$,

$$\Pi_S^W = (2-r)\left\{(A_S^W)^2 x_S^2 + (A_P^W)^2 x_P^2 + 2A_S^W A_P^W x_S x_P\right\} + (2-r)(A^W)^2 - c\left\{a_O + a_D\right\}$$

Using $\mathbb{E}[x_S^2] = \frac{1}{\nu} + \frac{1}{\nu_S}$, $\mathbb{E}[x_P^2] = \frac{1}{\nu} + \frac{1}{\nu_P}$, and $\mathbb{E}[x_S x_P] = \frac{1}{\nu}$, we can rearrange the terms to

$$\Pi_S^W = (2-r)\left\{(A_S^W)^2(\frac{1}{\nu} + \frac{1}{\nu_S}) + (A_P^W)^2(\frac{1}{\nu} + \frac{1}{\nu_P}) + A_P^W A_S^W \frac{2}{\nu}\right\} + (2-r)(A^W)^2 - c(a_O + a_D).$$

We now derive the platform's ex-ante expected revenue. We first define the expected revenue of the platform given the equilibrium price of the seller $p^W = A^W + A_S^W x_S + A_P^W x_P$ once he shares his signal x_P:

$$\begin{aligned}\Pi_P^W &= \mathbb{E}_{x_S,x_P}\left\{r \cdot \left(A^W + A_S^W x_S + A_P^W x_P\right)\left(a_O + B_P x_P - A^W - A_S^W x_S - A_P^W x_P\right)\right\} \\ &= r\left\{A^W\left(a_O - A^W\right) + A_S^W B_P \mathbb{E}x_S x_P + A_P^W B_P \mathbb{E}x_P^2\right\} \\ &\quad - r\left\{\left(A_S^W\right)^2 \mathbb{E}x_S^2 + 2A_S^W A_P^W \mathbb{E}x_S x_P + \left(A_P^W\right)^2 \mathbb{E}x_P^2\right\}\end{aligned}$$

where, $B_P = \frac{\nu_P}{\nu+\nu_P}$. Using $\mathbb{E}[x_S^2] = \frac{1}{\nu} + \frac{1}{\nu_S}$, $\mathbb{E}[x_P^2] = \frac{1}{\nu} + \frac{1}{\nu_P}$, and $\mathbb{E}[x_S x_P] = \frac{1}{\nu}$, we can rearrange the terms to

$$\begin{aligned}\Pi_P^W &= r\left\{A^W\left(a_O - A^W\right) + \frac{2B_P}{2\nu}A_S^W - \frac{A_S^W}{2\nu} + \frac{2A_P^W}{2\nu} - \frac{A_P^W}{2\nu}\right\}\\ &= r\left\{A^W\left(a_O - A^W\right) + \frac{A_S^W}{2\nu}(2B_P - 1) + \frac{A_P^W}{2\nu}\right\}\\ &= r\left\{A^W\left(a_O - A^W\right) + \frac{A_S^W}{2\nu}\left(B_P - \frac{\nu}{\nu+\nu_P}\right) + \frac{A_P^W}{2\nu}\right\}.\end{aligned}$$

Based on (4.8) and (4.12), we compare the ex-ante expected profit of the seller. As r is exogenous, we know $A^N = A^W$. Therefore,

$$\begin{aligned}&\Pi_S^W - \Pi_S^N\\ &= (2-r)\left\{\left(A_S^W\right)^2\left(\frac{1}{\nu} + \frac{1}{\nu_S}\right) + \left(A_P^W\right)^2\left(\frac{1}{\nu} + \frac{1}{\nu_P}\right) + A_P^W A_S^W \frac{2}{\nu} - \left(A_S^N\right)^2\left(\frac{1}{\nu} + \frac{1}{\nu_S}\right)\right\}\\ &= (2-r)\left(\frac{\nu_P}{2\left(\nu+\nu_S+\nu_P\right)}\right)^2\left(\frac{1}{\nu} + \frac{1}{\nu_P}\right) + \left(\frac{\nu_P}{2\left(\nu+\nu_S+\nu_P\right)}\right)\left(\frac{\nu_S}{2\left(\nu+\nu_S+\nu_P\right)}\right)\frac{2(2-r)}{\nu}\\ &- (2-r)\left(\frac{1}{\nu} + \frac{\nu}{\nu_S}\right)\left[\left(\frac{\nu_S}{2\left(\nu+\nu_S\right)}\right)^2 - \left(\frac{\nu_S}{2\left(\nu_S+\nu_P\right)}\right)^2\right]\end{aligned}$$

For notational convenience, we denote $T = 2(\nu+\nu_S+\nu_P)$. After some algebra, we obtain

$$\begin{aligned}\Pi_S^W - \Pi_S^N &= (2-r)\left\{\frac{\nu_S^2}{T^2}\left(\frac{1}{\nu} + \frac{1}{\nu_S}\right) + \frac{\nu_P^2}{T^2}\left(\frac{1}{\nu} + \frac{1}{\nu_P}\right) + \frac{\nu_P\nu_S}{T^2}\frac{2}{\nu} - \left(\frac{1}{\nu} + \frac{1}{\nu_S}\right)\left(\frac{\nu_S}{2\left(\nu+\nu_S\right)}\right)^2\right\}\\ &= (2-r)\left\{\frac{\nu_S+\nu_P}{2\nu T} - \frac{\nu_S}{4\nu(\nu+\nu_S)}\right\} = \frac{2-r}{4}\left\{\frac{\nu_P}{4(\nu+\nu_S+\nu_P)(\nu+\nu_S)}\right\} > 0.\end{aligned}$$

Secondly, the ex-ante expected revenue of the platform is:

$$\Pi_P^W - \Pi_P^N = r\left\{\left(B_P - \frac{\nu}{\nu+\nu_P}\right)\left(A_S^W - A_S^N\right) + \frac{A_P^W}{2\nu}\right\}$$

As $A_S^W - A_S^N = -\frac{\nu_S\nu_P}{2(\nu+\nu_S+\nu_P)(\nu+\nu_S)}$,

$$\begin{aligned}\Pi_P^W - \Pi_P^N &= r\left\{\frac{1}{4\nu}\left(\frac{-\nu_S\nu_P}{\left(\nu+\nu_S+\nu_P\right)\left(\nu+\nu_S\right)}\right)\left(\frac{\nu_P-\nu}{\nu+\nu_P}\right) + \frac{\nu_P}{4\nu\left(\nu+\nu_S+\nu_P\right)}\right\}\\ &= r\left\{\frac{\left(\nu_P-\nu\right)\left(-\nu_S\nu_P\right) + \nu_P\left(\nu+\nu_S\right)\left(\nu+\nu_P\right)}{4\nu\left(\nu+\nu_S+\nu_P\right)\left(\nu+\nu_P\right)\left(\nu+\nu_S\right)}\right\}\\ &= r\left\{\frac{\nu_P(\nu+2\nu_S+\nu_P)}{4(\nu+\nu_S+\nu_P)(\nu+\nu_S)(\nu+\nu_P)}\right\} > 0.\end{aligned}$$

□

Proof of Proposition 4.3 First, consider the case when $\nu_S \to \infty$. From (4.16) and (4.19), for any realized x_S and x_P, $r^N(x_P)$ solves: $\frac{(2+r)}{(2-r)^3} = (\frac{a_O + \frac{\nu_P}{(\nu+\nu_P)}x_P}{K})^2$, and $r^W(x_P, x_S)$ solves

$\frac{(2+r)}{(2-r)^3} = (\frac{a_O+x_S}{K})^2$. These conditions can be rewritten as:

$$\begin{cases} \sqrt{\frac{(2+r)}{(2-r)^3}} = \frac{a_O+\frac{\nu_P}{(\nu+\nu_P)}x_P}{K} \text{ for } r^N & \sqrt{\frac{(2+r)}{(2-r)^3}} = \frac{a_O+x_S}{K} \text{ for } r^W \text{ when } K>0 \\ \sqrt{\frac{(2+r)}{(2-r)^3}} = -\frac{a_O+\frac{\nu_P}{(\nu+\nu_P)}x_P}{K} \text{ for } r^N & \sqrt{\frac{(2+r)}{(2-r)^3}} = -\frac{a_O+x_S}{K} \text{ for } r^W \text{ when } K\leq 0 \end{cases} \tag{4.21}$$

where $K = -a_O + a_D + 2c$. While $r^N(x_P)$ is dependent on x_P and $r^W(x_S)$ is thoroughly based on x_S when $\nu_S \to \infty$. By focusing on the case when $K > 0$ and the right-hand side of the equilibrium condition in (4.21), we get $\mathbb{E}_{x_P}\left[\frac{a_O+\frac{\nu_P}{(\nu+\nu_P)}x_P}{K}\right] = \mathbb{E}_{x_S}\left[\frac{a_O+x_S}{K}\right] = \frac{a_O}{K}$. Further,

$$Var\left(\frac{a_O+\frac{\nu_P}{(\nu+\nu_P)}x_P}{K} \mid x_P\right) < Var\left(\frac{a_O+x_S}{K} \mid x_S\right) \Leftrightarrow \frac{\nu_P}{K^2\nu(\nu+\nu_P)} < \frac{1}{K^2\nu} \Leftrightarrow \frac{\nu_P}{\nu+\nu_P} < 1$$

as when $\nu_S \to \infty$, $\mathbb{E}[x_P] = 0$, $\mathbb{E}[x_S] = 0$, $Var(x_P) = \frac{1}{\nu} + \frac{1}{\nu_P}$, and $Var(x_S) = \frac{1}{\nu}$. Using these properties with the linear transformations of normal random variables (x_P and x_S), we define $z_P \equiv \frac{a_O+\frac{\nu_P}{(\nu+\nu_P)}x_P}{K}$ and $z_S \equiv \frac{a_O+x_S}{K}$, where, $z_P \sim N(\frac{a_O}{K}, \frac{\nu_P}{K^2\nu(\nu+\nu_P)})$ and $z_S \sim N(\frac{a_O}{K}, \frac{1}{K^2\nu})$. Denote the probability density functions as $\phi_{z_P}(\cdot)$ and $\phi_{z_S}(\cdot)$ and the cumulative distribution functions as $\Phi_{z_P}(\cdot)$ and $\Phi_{z_S}(\cdot)$ for two random variables z_P and z_S, respectively.

We define $f(r) \equiv \sqrt{g(r)}$ so that $r^N = f^{-1}(\frac{a_O+\frac{\nu_P}{(\nu+\nu_P)}x_P}{K})$ and $r^W = f^{-1}(\frac{a_O+x_S}{K})$ when $K > 0$. Note that the same proof applies to the case when $K < 0$ as $\mathbb{E}_{x_P}[-\frac{a_O+\frac{\nu_P}{(\nu+\nu_P)}x_P}{K}] = \mathbb{E}_{x_S}[-\frac{a_O+x_S}{K}]$ and $Var(-\frac{a_O+\frac{\nu_P}{(\nu+\nu_P)}x_P}{K} \mid x_P) < Var(-\frac{a_O+x_S}{K} \mid x_S)$. Then, by using integration by parts ($\int_{-\infty}^{\infty} f^{-1}(x)\phi(x)dx = f^{-1}(x)\Phi(x)]_{-\infty}^{\infty} - \int_{-\infty}^{\infty} f'^{-1}(x)\Phi(x)dx$), we can express $\mathbb{E}[r^N]$ and $\mathbb{E}[r^W]$ as below:

$$\mathbb{E}_{z_P}[f^{-1}(z_P)] = \int_{-\infty}^{\infty} f^{-1}(t)\phi_{z_P}(t)dt = f^{-1}(t)\Phi_{z_P}(t)]_{-\infty}^{\infty} - \int_{-\infty}^{\infty} f'^{-1}(t)\Phi_{z_P}(t)dt$$

$$\mathbb{E}_{z_S}[f^{-1}(z_S)] = \int_{-\infty}^{\infty} f^{-1}(t)\phi_{z_S}(t)dt = f^{-1}(t)\Phi_{z_S}(t)]_{-\infty}^{\infty} - \int_{-\infty}^{\infty} f'^{-1}(t)\Phi_{z_S}(t)dt.$$

$f^{-1}(t)\Phi_{z_S}(t)]_{-\infty}^{\infty} = f^{-1}(t)\Phi_{z_p}(t)]_{-\infty}^{\infty}$. Also, as $f'^{-1}(t) \geq 0$, $\mathbb{E}z_P = \mathbb{E}z_S$ and $Var(z_S) > Var(z_P)$, we can conclude $\int_{-\infty}^{\infty} f'^{-1}(t)\Phi_{z_S}(t)dt \geq \int_{-\infty}^{\infty} f'^{-1}(t)\Phi_{z_P}(t)dt$. Therefore,

$$\mathbb{E}_{z_S}[f^{-1}(z_S)] \leq \mathbb{E}_{z_P}[f^{-1}(z_P)] \Leftrightarrow \mathbb{E}_{x_S}[r^W(x_S)] \leq \mathbb{E}_{x_P}[r^N(x_P)].$$

Second, consider the case when $\nu_P \to 0$. It is easy to check from (4.16) and (4.19) that, for any realized x_S and x_P, $r^N(x_P)$ solves: $\frac{(2+r)}{(2-r)^3} = (\frac{a_O}{K})^2$, and $r^W(x_P, x_S)$ solves $\frac{(2+r)}{(2-r)^3} = (\frac{a_O+\frac{\nu_S}{\nu+\nu_S}x_S}{K})^2$, which are both independent of x_P. Now, let $g(r) \equiv \frac{(2+r)}{(2-r)^3}$ so that $r^N = g^{-1}((\frac{a_O}{K})^2)$ and $r^W(x_S) = g^{-1}((\frac{a_O+\frac{\nu_S}{\nu+\nu_S}x_S}{K})^2)$. By noting that $g(r) > 0$ and $g(r)$ is increasing and convex in r for $r \in (0,1)$, we can conclude that $g^{-1}(\cdot)$ is increasing and concave. As our assumption stated in §4.2 that a_O is large enough to ensure $a_O + x_S \geq 0$, r^N and r^W satisfy:

$$\begin{cases} \sqrt{\frac{(2+r)}{(2-r)^3}} = \frac{a_O}{K} \text{ for } r^N & \sqrt{\frac{(2+r)}{(2-r)^3}} = \frac{a_O+\frac{\nu_S}{\nu+\nu_S}x_S}{K} \text{ for } r^W \text{ when } K > 0 \\ \sqrt{\frac{(2+r)}{(2-r)^3}} = -\frac{a_O}{K} \text{ for } r^N & \sqrt{\frac{(2+r)}{(2-r)^3}} = -\frac{a_O+\frac{\nu_S}{\nu+\nu_S}x_S}{K} \text{ for } r^W \text{ when } K \leq 0 \end{cases} \tag{4.22}$$

where $K = -a_O + a_D + 2c$. Define $f(r) \equiv \sqrt{g(r)}$ so that $r^N = f^{-1}(\frac{a_O}{K})$ and $r^W(x_S) = f^{-1}(\frac{a_O+\frac{\nu_S}{\nu+\nu_S}x_S}{K})$ when $K > 0$. (Below, we present the proof for the case when $K > 0$, but the same proof applies to the case when $K < 0$.) From $f(r) = \sqrt{\frac{(2+r)}{(2-r)^3}}$, we can derive $f'(r)$ and $f''(r)$ as follows:

$$f'(r) = \frac{4+r}{(2-r)^4\sqrt{\frac{(2+r)}{(2-r)^3}}} \geq 0 \text{ and } f''(r) = \frac{2(r^2+8r+10)}{(2-r)^8(\frac{(2+r)}{(2-r)^3})^{\frac{3}{2}}} \geq 0.$$

By using that $f(r)$ is increasing and convex in $r \in (0,1)$, we can conclude that $f^{-1}(\cdot)$ is a concave and increasing function. From $r^N = \mathbb{E}[r^N] = f^{-1}(\frac{a_O}{K})$ and $\mathbb{E}[x_S] = 0$, by applying Jensen's theorem, we obtain

$$\mathbb{E}_{x_S}[f^{-1}(\frac{a_O + \frac{\nu_S}{\nu+\nu_S}x_S}{K})] \leq f^{-1}(\mathbb{E}_{x_S}[\frac{a_O + \frac{\nu_S}{\nu+\nu_S}x_S}{K}]) = f^{-1}(\frac{a_O}{K}).$$

Therefore, $\mathbb{E}_{x_S}[r^W(x_S)] \leq \mathbb{E}[r^N] = r^N$. □

Proof of Proposition 4.4 Based on (4.13), the optimal prices for the seller under information exchange and no exchange given the platform's equilibrium commission rates r^N and r^W are

$$\tilde{p}^N = \frac{a_O}{2} + \frac{K}{2}\cdot\frac{1}{2-r^N(x_P)} + \tilde{A}_S^N x_S \text{ and } \tilde{p}^W = \frac{a_O}{2} + \frac{K}{2}\cdot\frac{1}{2-r^W(x_S,x_P)} + \tilde{A}_S^W x_S + \tilde{A}_P^W x_P$$

where $K = -a_O + a_D + 2c$. Recall from Proposition 4.3 that when $\nu_P \to 0$, r^N suffices the equilibrium condition $\frac{2+r^N}{(2-r^N)^3} = (\frac{a_O}{K})^2$ that is independent on x_P. In the meantime, r^W depends on x_S under the equilibrium condition of $\frac{2+r^W}{(2-r^W)^3} = (\frac{a_O+\frac{\nu_S x_S}{\nu+\nu_S}}{K})^2$. From $f(r) = \sqrt{\frac{2+r}{(2-r)^3}}$, the equilibrium commission rates can be described as $r^N = f^{-1}(\frac{a_O}{K})$ and $r^W = f^{-1}(\frac{a_O+\frac{\nu_S x_S}{\nu+\nu_S}}{K})$. Denoting $t(y) = \frac{1}{2-f^{-1}(y)}$, the expected prices are

$$\mathbb{E}_{x_S}[\tilde{p}^N] = \tilde{p}^N = \frac{a_O}{2} + \frac{K}{2}\cdot t(\frac{a_O}{K}) \text{ and } \mathbb{E}_{x_S,x_P}[\tilde{p}^W] = \frac{a_O}{2} + \frac{K}{2}\cdot\mathbb{E}\left\{t(\frac{a_O + \frac{\nu_S x_S}{\nu+\nu_S}}{K})\right\}.$$

As $f^{-1}(y)$ is increasing and concave in y ($f^{'-1}(y) > 0$ and $f^{''-1}(y) < 0$), it is easy to find $2 - f^{-1}(y)$ is decreasing and convex, consecutively making $t(y) = \frac{1}{2-f^{-1}(y)}$ is increasing and concave in y. Based on Jensen's theorem under a concave function, $\mathbb{E}\left\{t(\frac{a_O+\frac{\nu_S x_S}{\nu+\nu_S}}{K})|x_S\right\} < t\left(\mathbb{E}\left\{\frac{a_O+\frac{\nu_S x_S}{\nu+\nu_S}}{K}|x_S\right\}\right) = t\left(\frac{a_O}{K}\right)$. Hence, when $\nu_P \to 0$, $\mathbb{E}_{x_S,x_P}[\tilde{p}^W] \leq \mathbb{E}_{x_S}[\tilde{p}^N]$ for $K \geq 0$ $(a_D + 2c \geq a_O)$ and $\mathbb{E}_{x_S,x_P}[\tilde{p}^W] > \mathbb{E}_{x_S}[\tilde{p}^N]$ for $K < 0$ $(a_D + 2c < a_O)$.

Next, for the case $\nu_S \to \infty$, define $z_P \equiv \frac{a_O+\frac{\nu_P}{(\nu+\nu_P)}x_P}{K}$ and $z_S \equiv \frac{a_O+x_S}{K}$ with the probability density functions as $\phi_{z_P}(\cdot)$ and $\phi_{z_S}(\cdot)$ and the cumulative distribution functions as $\Phi_{z_P}(\cdot)$ and $\Phi_{z_S}(\cdot)$ for two random variables z_P and z_S, where $z_P \sim N\left(\frac{a_O}{K}, \frac{\nu_P}{K^2\nu(\nu+\nu P)}\right)$ and $z_S \sim N\left(\frac{a_O}{K}, \frac{1}{K^2\nu}\right)$ respectively. By using the terms z_P and z_S, the equilibrium commission rates for the platform for $\nu_S \to \infty$ are expressed as $r^N = f^{-1}(z_P)$ and $r^W = f^{-1}(z_S)$. With the expression of $t(y) = \frac{1}{2-f^{-1}(y)}$ the seller's expected prices can be rearranged to

$$\mathbb{E}_{z_P}[\tilde{p}^N] = \frac{a_O}{2} + \frac{K}{2} \cdot \mathbb{E}\left\{t(z_P)\right\} \text{ and } \mathbb{E}_{z_S}[\tilde{p}^W] = \frac{a_O}{2} + \frac{K}{2} \cdot \mathbb{E}\left\{t(z_S)\right\}$$

where, $\mathbb{E}\left\{t(z_P)\right\}$ and $\mathbb{E}\left\{t(z_S)\right\}$ are equivalent to

$$\mathbb{E}_{z_P}\left[t\left(z_P\right)\right] = \int_{-\infty}^{\infty} t(x)\phi_{z_P}(t)dx = t(x)\Phi_{z_P}(t)\Big]_{-\infty}^{\infty} - \int_{-\infty}^{\infty} t'^{-1}(x)\Phi_{z_P}(t)dx$$

$$\mathbb{E}_{z_S}\left[t\left(z_S\right)\right] = \int_{-\infty}^{\infty} t(x)\phi_{z_S}(t)dx = t(x)\Phi_{z_S}(t)\Big]_{-\infty}^{\infty} - \int_{-\infty}^{\infty} t'^{-1}(x)\Phi_{z_S}(t)dx.$$

From $\mathbb{E}z_P = \mathbb{E}z_S$ and $Var(z_P) < Var(z_S)$, $\int_{-\infty}^{\infty}\left[\Phi_{z_S}(x) - \Phi_{z_P}(x)\right]dx \geq 0$. Moreover, since $t'^{-1}(x) = \frac{f'^{-1}(x)}{[2-f^{-1}(x)]^2} \geq 0$,

$$t(x)\Phi_{z_P}(t)]_{-\infty}^{\infty} - \int_{-\infty}^{\infty} t'(x)\Phi_{z_P}(x)dx \geq t(x)\Phi_{z_S}(t)]_{-\infty}^{\infty} - \int_{-\infty}^{\infty} t'(x)\Phi_{z_S}(x)dx$$

$$\Leftrightarrow \mathbb{E}_{z_P}\left[t\left(z_P\right)\right] \geq \mathbb{E}_{z_S}\left[t\left(z_S\right)\right].$$

Hence, $\mathbb{E}_{z_P}[\tilde{p}^N(z_P)] > \mathbb{E}_{z_S}[\tilde{p}^W(z_S)]$ for $K > 0$ $(a_D + 2c > a_O)$, equivalently $\mathbb{E}_{z_P}[\tilde{p}^N(z_P)] \leq \mathbb{E}_{z_S}[\tilde{p}^W(z_S)]$ for $K \leq 0$ $(a_D + 2c \leq a_O)$. □

Proof of Proposition 4.5 First, when $\nu_P \to 0$, the equilibrium commission rate conditions without and with information exchange follow $\frac{2+r^N}{(2-r^N)^3} = (\frac{a_O}{K})^2$ and $\frac{2+r^W}{(2-r^W)^3} = (\frac{a_O}{K} + \frac{\nu_S x_S}{K(\nu+\nu_S)})^2$, respectively. Using these expressions, r^N and $r^W(x_S)$, the ex-ante expected revenue of the platform without and with information exchange are

$$\tilde{\Pi}_P^N = \mathbb{E}_{x_S}\left\{r^N \cdot \left(\frac{a_O}{2} + \frac{K}{2\left(2-r^N\right)} + \tilde{A}_S^N x_S\right)\left(\frac{a_O}{2} - \frac{K}{2\left(2-r^N\right)} - \tilde{A}_S^N x_S\right)\right\}$$

$$= r^N\left(\frac{a_O^2}{4} - \frac{K^2}{4\left(2-r^N\right)^2} - \frac{\nu_S}{4\nu\left(\nu+\nu_S\right)}\right) = \frac{K^2}{2} \cdot \frac{\left(r^N\right)^2}{\left(2-r^N\right)^3} - \frac{\nu_S r^N}{4\nu\left(\nu+\nu_S\right)}.$$

$$
\begin{aligned}
\tilde{\Pi}_P^W &= \mathbb{E}_{x_S,x_P}\left\{\tilde{\pi}_P^W\left(r^W\left(x_S,x_P\right),\tilde{p}^W\left(x_S,x_P\right)\right)\right\}\\
&= \mathbb{E}_{x_S}\left\{r^W(x_S)\cdot\left(\frac{a_O}{2}+\frac{K}{2\left(2-r^W(x_S)\right)}+\frac{\nu_S x_S}{2\left(\nu+\nu_S\right)}\right)\left(\frac{a_O}{2}+\frac{\nu_S x_S}{2\left(\nu+\nu_S\right)}-\frac{K}{2\left(2-r^W(x_S)\right)}\right)\right\}\\
&= \frac{K^2}{2}\cdot\mathbb{E}_{x_S}\left\{\frac{(r^W(x_S))^2}{(2-r^W(x_S))^3}\right\}
\end{aligned}
$$

Denote $f(r^W)=\sqrt{\frac{2+r^W}{(2-r^W)^3}}$ and define $g(x_S)=\frac{[r^W(x_S)]^2}{(2-r^W(x_S))^3}$. As $r^W(x_S)=f^{-1}\left(\frac{a_O}{K}+\frac{\nu_S x_S}{K(\nu+\nu_S)}\right)$,

$$
\frac{\partial g(x_S)}{\partial x_S}=\frac{f^{-1}\left(\frac{a_O}{K}+\frac{\nu_S x_S}{K(\nu+\nu_S)}\right)\cdot f'^{-1}\left(\frac{a_O}{K}+\frac{\nu_S x_S}{K(\nu+\nu_S)}\right)\cdot\left(4+f^{-1}\left(\frac{a_O}{K}+\frac{\nu_S x_S}{K(\nu+\nu_S)}\right)\right)}{\left(2-f^{-1}\left(\frac{a_O}{K}+\frac{\nu_S x_S}{K(\nu+\nu_S)}\right)\right)^4}.
$$

From the inverse function theorem, $f'^{-1}\left(\frac{a_O}{K}+\frac{\nu_S x_S}{K(\nu+\nu_S)}\right)=\frac{1}{f'(r^W)}=\frac{(2-r^W)^4\sqrt{\frac{(2+r^W)}{(2-r^W)^3}}}{4+r^W}$ in equilibrium. Further Proposition 4.3 showed that $f(r^W)$ is an increasing and convex function in r. Accordingly, the inverse function has an increasing and concave function. Therefore,

$$
\begin{aligned}
\frac{\partial g(x_S)}{\partial x_S} &= f(r^W)\cdot f^{-1}\left(\frac{a_O}{K}+\frac{\nu_S x_S}{K\left(\nu+\nu_S\right)}\right)>0\\
\frac{\partial^2 g(x_S)}{\partial x_S^2} &= f'(r^W)\cdot f^{-1}\left(\frac{a_O}{K}+\frac{\nu_S x_S}{K\left(\nu+\nu_S\right)}\right)+f(r^W)\cdot f'^{-1}\left(\frac{a_O}{K}+\frac{\nu_S x_S}{K\left(\nu+\nu_S\right)}\right)>0.
\end{aligned}
$$

As $g(x_S)$ is an increasing and convex function in x_S, from the Jensen's theorem

$$
\tilde{\Pi}_P^W=\frac{K^2}{2}\cdot\mathbb{E}\left\{g(x_S)\right\}>\frac{K^2}{2}\cdot\left\{g(f^{-1}(\frac{a_O}{K}))\right\}=\frac{K^2}{2}\cdot\frac{(r^N)^2}{(2-r^N)^3}.
$$

As $\frac{K^2}{2}\cdot\frac{(r^N)^2}{(2-r^N)^3}>\frac{K^2}{2}\cdot\frac{(r^N)^2}{(2-r^N)^3}-\frac{\nu_S r^N}{4\nu(\nu+\nu_S)}$, we can conclude $\tilde{\Pi}_P^W>\tilde{\Pi}_P^N$. Next, the seller's ex-ante expected profit function for each information exchange case follows:

$$
\begin{aligned}
\tilde{\Pi}_S^N &= \mathbb{E}_{x_S,x_P}\left\{\tilde{\pi}_S^N\left(r^N\left(x_P\right),\tilde{p}^N\left(x_S\right)\right)\right\}\\
&= \mathbb{E}_{x_S}\left\{\left(\frac{a_O}{2}+\frac{K}{2(2-r^N)}+\frac{\nu_S x_S}{2\left(\nu+\nu_S\right)}-c\right)\left(a_D+\frac{\nu_S x_S}{\nu+\nu_S}-\frac{K}{(2-r^N)}\right)\right\}\\
&\quad - r^N\mathbb{E}_{x_S}\left\{\left(\frac{a_O}{2}+\frac{\nu_S x_S}{2(\nu+\nu_S)}\right)^2-\frac{K^2}{4(2-r^N)^2}\right\}\\
&= \left(\frac{a_O a_D}{2}-a_D c+\frac{\nu_S}{2\nu(\nu+\nu_S)}\right)+\frac{K^2}{2}\frac{1-r^N}{(2-r^N)^2}-\frac{K^2}{2}\frac{(r^N)^2}{(2-r^N)^3}+\frac{\nu_S r^N}{4\nu(\nu+\nu_S)}\\
\tilde{\Pi}_S^W &= \mathbb{E}_{x_S,x_P}\left\{\tilde{\pi}_S^W\left(r^W\left(x_S,x_P\right),\tilde{p}^W\left(x_S,x_P\right)\right)\right\}\\
&= \left(\frac{a_O a_D}{2}-a_D c+\frac{\nu_S}{2\nu(\nu+\nu_S)}\right)+\frac{K^2}{2}\mathbb{E}_{x_S}\left\{\frac{1-r^W(x_S)}{(2-r^W(x_S))^2}\right\}-\frac{K^2}{2}\mathbb{E}_{x_S}\left\{\frac{(r^W(x_S))^2}{(2-r^W(x_S))^3}\right\}.
\end{aligned}
$$

From the platform's ex-ante revenue comparison, we have shown that

$$\mathbb{E}_{x_S}\left\{\frac{(r^W(x_S))^2}{(2-r^W(x_S))^3}\right\} > \frac{(r^N)^2}{(2-r^N)^3} \Leftrightarrow -\frac{K^2}{2}\mathbb{E}_{x_S}\left\{\frac{(r^W(x_S))^2}{(2-r^W(x_S))^3}\right\} < -\frac{K^2}{2}\mathbb{E}_{x_S}\frac{(r^N)^2}{(2-r^N)^3}.$$

Defining $g(x_S) = \frac{1-r^W(x_S)}{(2-r^W(x_S))^2}$,

$$\begin{aligned}
\frac{\partial g(x_S)}{\partial x_S} &= -f(r^W)\cdot f^{-1}\left(\frac{a_O}{K}+\frac{\nu_S x_S}{K\left(\nu+\nu_S\right)}\right)\frac{2-r^W}{4+r^W} < 0\\
\frac{\partial^2 g(x_S)}{\partial x_S^2} &= -f^{'}(r^W)\cdot f^{-1}\left(\frac{a_O}{K}+\frac{\nu_S x_S}{K\left(\nu+\nu_S\right)}\right) - f(r^W)\cdot f^{'-1}\left(\frac{a_O}{K}+\frac{\nu_S x_S}{K\left(\nu+\nu_S\right)}\right)\\
&\quad - \frac{6f^{'-1}\left(\frac{a_O}{K}+\frac{\nu_S x_S}{K(\nu+\nu_S)}\right)}{(4+f^{-1}\left(\frac{a_O}{K}+\frac{\nu_S x_S}{K(\nu+\nu_S)}\right))^2}\cdot f(r^W)\cdot f^{-1}\left(\frac{a_O}{K}+\frac{\nu_S x_S}{K\left(\nu+\nu_S\right)}\right) < 0.
\end{aligned}$$

Using the fact that $g(x_S)$ is increasing and concave function in x_S, we obtain

$$\mathbb{E}_{x_S}\left\{\frac{1-r^W(x_S)}{(2-r^W(x_S))^2}\right\} < \frac{1-r^N}{(2-r^N)^2}.$$

Therefore,

$$\begin{aligned}
&\tilde{\Pi}_S^N - \tilde{\Pi}_S^W\\
&= \frac{\nu_S r^N}{4\nu(\nu+\nu_S)} + \frac{K^2}{2}\left(\frac{1-r^N}{(2-r^N)^2} - \mathbb{E}_{x_S}\left\{\frac{1-r^W(x_S)}{(2-r^W(x_S))^2}\right\}\right)\\
&\quad - \frac{K^2}{2}\left(\frac{(r^N)^2}{(2-r^N)^3} - \mathbb{E}_{x_S}\left\{\frac{(r^W(x_S))^2}{(2-r^W(x_S))^3}\right\}\right) > 0.
\end{aligned}$$

Secondly, for the case when $\nu_S \to \infty$, the seller's ex-ante expected profit function for each information exchange case follows:

$$\begin{aligned}
\tilde{\Pi}_S^N &= \mathbb{E}_{x_S,x_P}\left\{\tilde{\pi}_S^N\left(r^N\left(x_P\right),\tilde{p}^N\left(x_S\right)\right)\right\}\\
&= \mathbb{E}_{x_S}\left\{\left(\frac{a_O}{2}+\frac{K}{2(2-r^N(x_P))}+\frac{x_S}{2}-c\right)\left(a_D+x_S-\frac{K}{(2-r^N(x_P))}\right)\right\}\\
&\quad - r^N\mathbb{E}_{x_S}\left\{\left(\frac{a_O}{2}+\frac{x_S}{2}\right)^2 - \frac{K^2}{4(2-r^N(x_P))^2}\right\}\\
&= \left(\frac{a_O a_D}{2} - a_D c + \frac{1}{2\nu}\right) + \frac{K^2}{2}\mathbb{E}_{x_P}\left\{\frac{1-r^N(x_P)}{(2-r^N(x_P))^2}\right\}\\
\tilde{\Pi}_S^W &= \mathbb{E}_{x_S,x_P}\left\{\tilde{\pi}_S^W\left(r^W\left(x_S,x_P\right),\tilde{p}^W\left(x_S,x_P\right)\right)\right\} = \mathbb{E}_{x_S}\left\{\tilde{\pi}_S^W\left(r^W\left(x_S\right),\tilde{p}^W\left(x_S\right)\right)\right\}\\
&= \left(\frac{a_O a_D}{2} - a_D c + \frac{1}{2\nu}\right) + \frac{K^2}{2}\mathbb{E}_{x_S}\left\{\frac{1-r^N(x_S)}{(2-r^N(x_S))^2}\right\}
\end{aligned}$$

Define $g(f^{-1}(x)) = \frac{1-f^{-1}(x)}{(2-f^{-1}(x))^2}$ where $r^N(x_P) = f^{-1}(\frac{a_O}{K} + \frac{\nu_P x_P}{K(\nu+\nu_P)})$ and $r^W(x_S) = f^{-1}(\frac{a_O}{K} + \frac{x_S}{K})$ from Proposition 4.3. Because

$$\frac{\partial g(f^{-1}(x))}{\partial x} = \frac{f'^{-1}(x) f^{-1}(x)}{(2 - f^{-1}(x))^3} > 0,$$

by rearranging $z_P \equiv \frac{a_O}{K} + \frac{\nu_P x_P}{K(\nu+\nu_P)}$ and $z_S \equiv \frac{a_O}{K} + \frac{x_S}{K}$ where, $z_P \sim N\left(\frac{a_O}{K}, \frac{\nu_P}{K^2\nu(\nu+\nu P)}\right)$ and $z_S \sim N\left(\frac{a_O}{K}, \frac{1}{K^2\nu}\right)$ we obtain

$$\mathbb{E}_{z_P}\left[g\left(z_P\right)\right] = \int_{-\infty}^{\infty} g(x)\phi_{z_P}(t)dx = g(x)\Phi_{z_P}(t)\Big]_{-\infty}^{\infty} - \int_{-\infty}^{\infty} g'^{-1}(x)\Phi_{z_P}(t)dx$$
$$\mathbb{E}_{z_S}\left[g\left(z_S\right)\right] = \int_{-\infty}^{\infty} g(x)\phi_{z_S}(t)dx = g(x)\Phi_{z_S}(t)\Big]_{-\infty}^{\infty} - \int_{-\infty}^{\infty} g'^{-1}(x)\Phi_{z_S}(t)dx.$$

From $\mathbb{E}z_P = \mathbb{E}z_S$ and $Var(z_P) < Var(z_S)$, $\int_{-\infty}^{\infty}\left[\Phi_{z_S}(x) - \Phi_{z_P}(x)\right]dx \geq 0$, and $g'^{-1}(x) > 0$,

$$\mathbb{E}_{z_P}\left[g\left(z_P\right)\right] \geq \mathbb{E}_{z_S}\left[g\left(z_S\right)\right] \Leftrightarrow \tilde{\Pi}_S^N > \tilde{\Pi}_S^W.$$

Finally, the platform's ex-ante expected revenue follows:

$$\begin{aligned}
\Pi_p^N &= \mathbb{E}_{x_S,x_P}\left\{r^N\left(x_P\right)\left(\frac{a_O}{2} + \frac{K}{2\left(2 - r^N\left(x_P\right)\right)} + \frac{x_S}{2}\right)\left(\frac{a_O}{2} + \frac{\nu_P}{\nu+\nu_P}x_P - \frac{x_S}{2} - \frac{K}{2\left(2 + r^N(x_P)\right)}\right)\right\} \\
&= \mathbb{E}_{x_S,x_P}\left\{r^N\left(x_P\right)\left(\left(\frac{a_O}{2} + \frac{x_S}{2}\right)^2 - \frac{K^2}{4\left(2 - r^N\left(x_P\right)\right)^2}\right)\right\} \\
&+ \mathbb{E}_{x_S,x_P}\left\{\left(\frac{\nu_P x_P}{\nu+\nu_P} - x_S\right) r^N\left(x_P\right)\left(\frac{a_O}{2} + \frac{K}{2\left(2 - r^N\left(x_P\right)\right)} + \frac{x_S}{2}\right)\right\}
\end{aligned}$$

$$\begin{aligned}
\Pi_p^W &= \mathbb{E}_{x_S}\left\{r^W\left(x_S\right)\left(\frac{a_O}{2} + \frac{K}{2\left(2 - r^W\left(x_S\right)\right)} + \frac{x_S}{2}\right)\left(\frac{a_O}{2} + \frac{x_S}{2} - \frac{K}{2\left(2 - r^W\left(x_S\right)\right)}\right)\right\} \\
&= \mathbb{E}_{x_S}\left\{r^W\left(x_S\right)\left(\left(\frac{a_O}{2} + \frac{x_S}{2}\right)^2 - \frac{K^2}{4\left(2 - r^W\left(x_S\right)\right)^2}\right)\right\}.
\end{aligned}$$

Although when $\nu_S \to \infty$, $\mathbb{E}[\frac{\nu_P x_P}{\nu+\nu_P}x_S] < \mathbb{E}[x_P x_S] = \mathbb{E}[x_S x_S] = \frac{1}{\nu}$, it is difficult to obtain an analytical expression for the platform's no information exchange ex-ante expected revenue $\tilde{\Pi}_P^N$ because they involve the expression of a non-linear function of $r^N(x_S)$ and $r^W(x_P)$ with the multiplication of two random variables leading to the intractable expected value (i.e., $\mathbb{E}_{x_S,x_P}\left\{\frac{r^N(x_P)x_S^2}{2}\right\}$). However, we have also conducted extensive numerical experiments and the experiments exhibit that the result ($\Pi_p^W > \Pi_p^N$) is robust against the choice of parameters. □

Proof of Lemma 4.4 1) When $K \to -\infty$ ($a_O \to \infty$), the platform's equilibrium commission rate condition follows:

$$\frac{r+2}{(2-r)^3} \to 1.$$

$$\tilde{\Pi}_P^N = r^* \left\{ \tilde{A}^N (a_O - \tilde{A}^N) - \frac{\nu_S(2\nu + \nu_P)}{2\nu(\nu + \nu_P)(\nu + \nu_S)} \right\}, \text{ and}$$
$$\tilde{\Pi}_P^W = r^* \left\{ \tilde{A}^W (a_O - \tilde{A}^W) + \frac{(\nu_S + \nu_P)}{\nu(\nu + \nu_S + \nu_P)} \right\}.$$

From the above expressions,

$$\tilde{\Pi}_P^W - \tilde{\Pi}_P^N = r^* \left\{ \frac{(\nu_S + \nu_P)}{\nu(\nu + \nu_S + \nu_P)} + \frac{\nu_S(2\nu + \nu_P)}{2\nu(\nu + \nu_P)(\nu + \nu_S)} \right\} > 0.$$

Further, the seller's ex-ante expected profits follow:

$$\tilde{\Pi}_S^N = (\tilde{A}^N - c)(a_O + a_D - 2\tilde{A}^N) - r^* \tilde{A}^N (a_O - \tilde{A}^N) + (2 - r^*) \frac{\nu_S}{\nu(\nu + \nu_S)}, \text{ and}$$

$$\tilde{\Pi}_S^W = (\tilde{A}^N - c)(a_O + a_D - 2\tilde{A}^N) - r^* \tilde{A}^N (a_O - \tilde{A}^N) + (2 - r^*) \frac{\nu_S + \nu_P}{\nu(\nu + \nu_S + \nu_P)}.$$

As $\tilde{\Pi}_S^W - \tilde{\Pi}_S^N = (2 - r^*) \frac{\nu_P}{(\nu+\nu_S)(\nu+\nu_S+\nu_P)} > 0$, when $K \to -\infty$, $\tilde{\Pi}_S^W > \tilde{\Pi}_S^N$ and $\tilde{\Pi}_P^W > \tilde{\Pi}_P^N$.

2) Secondly, suppose $K \to \infty$, equivalent to $a_D \to \infty$. The platform's equilibrium commission rate decisions for both information exchange and no exchange are:

$$\frac{r + 2}{(2 - r)^3} \to 0.$$

Therefore, for any given x_S and x_P, we obtain $r^* = r^N(x_P) = r^W(x_S, x_P) \to 0$ which is the opposite case of $K \to 0$. As the platform's revenue function is $\pi_P = rpq_O$, the platform gains no revenue ($\pi_P = 0$) under this market condition ($K \to \infty$). Hence, $\tilde{\Pi}_P^N = \tilde{\Pi}_P^W = 0$. On the other hand, using that the seller's optimal prices given that the platform's commission rate is zero are $\tilde{p}^N = \frac{a_O+a_D+2c}{4} + \tilde{A}_S^N x_S$ and $\tilde{p}^W = \frac{a_O+a_D+2c}{4} + \tilde{A}_S^W x_S + \tilde{A}_P^W x_P$ from Proposition 4.1 and 4.2, the ex-ante expected profits are

$$\tilde{\Pi}_S^N = \left(\frac{a_O + a_D - 2c}{4} + \tilde{A}_S^N x_S \right) \left(\frac{a_O + a_D + 2c}{2} + 2\tilde{A}_S^N x_S \right) = \frac{(a_O + a_D - 2c)^2}{8} + \frac{2\nu_S}{\nu\,(\nu + \nu_S)},$$
$$\begin{aligned}\tilde{\Pi}_S^W &= \left(\frac{a_O + a_D - 2c}{4} + \tilde{A}_S^W x_S + \tilde{A}_P^W x_P \right) \left(\frac{a_O + a_D - 2c}{2} + 2\tilde{A}_S^W x_S + 2\tilde{A}_P^W x_P \right) \\ &= \frac{(a_O + a_D - 2c)^2}{8} + 2(\tilde{A}_S^W)^2 x_S^2 + 4\tilde{A}_S^W \tilde{A}_P^W x_S x_P + 2(\tilde{A}_P^W)^2 x_P^2 \\ &= \frac{(a_0 + a_D - 2c)^2}{8} + \frac{2(\nu_S + \nu_P)}{\nu(\nu + \nu_S + \nu_P)}.\end{aligned}$$

Because $\tilde{\Pi}_S^W - \tilde{\Pi}_S^N = \frac{2\nu_P}{(\nu+\nu_S)(\nu+\nu_S+\nu_P)}$, for $K \to \infty$, $\tilde{\Pi}_P^N = \tilde{\Pi}_P^W = 0$ and $\tilde{\Pi}_S^W - \tilde{\Pi}_S^N > 0$. $\square$

Table 4.2: Summary of Notation

Symbol	Description
Superscripts and Subscripts	
P, S	Platform, seller, respectively
O, D	Online channel, direct sales channel, respectively
N, W	No information exchange, and with information exchange, respectively
Parameters	
a_i	Market potential in channel $i \in (O, D)$
m	Random variable, representing market randomness where, $m \sim N(0, \sigma)$ with market uncertainty σ
ν	Intrinsic market uncertainty, where $\nu = \frac{1}{\sigma}$
x_j	Noisy private signal that player $j \in (P, S)$ possesses on market uncertainty,
	where $x_j = m + \varepsilon_j$, and $\varepsilon_j \sim N(0, \tau_j)$ with signal noise τ_j
ν_j	The precision of private signal of player $j \in (P, S)$, where $\nu_j = \frac{1}{\tau_j}$
r	Exogenous commission rate (%) the platform charges to the seller in the sharing model
c	Unit cost of the sellers per product
Decision Variables	
r^t	Endogenous commission rate (%) of the platform for information policy $t \in (N, W)$
p^t, $\tilde{p}^t$	Seller's market price when r is exogenous
	and r^t is endogenous for information policy $t \in (N, W)$, respectively
Functions	
π_j^t	Profit of player $j \in (P, S)$ for information policy $t \in (N, W)$ in the sharing model
Π_j^t	Ex-ante expected profit of $j \in (P, S)$ for information policy $t \in (N, W)$ in the sharing model
$\tilde{\pi}_j^t$	Profit of player $j \in (P, S)$ for information policy $t \in (N, W)$ in the exchange model
$\tilde{\Pi}_j^t$	Ex-ante expected profit of $j \in (P, S)$ for information policy $t \in (N, W)$ in the exchange model

Chapter 5.

How power structure and markup schemes impact channel efficiency under price-dependent stochastic demand

Although considerable attention has been separately given to factors such as power structures, price-dependent demand, and markup pricing schemes, there has been limited exploration of the combined effects of these factors on supply chain efficiency and the leader's advantage. We propose a game theoretic model in which a manufacturer sells a single product to a newsvendor retailer who sets both optimal order quantity and selling price under uncertain price-dependent demand. Furthermore, we examine a supply network wherein a single retailer fulfills orders using a global manufacturer for regular orders and a local manufacturer to clear any shortages.

Through numerical analysis, we show that the retailer always prefers to charge a percentage markup. In a two-player game, channel efficiency is higher when the retailer is the leader under linear demand; however, under iso-elastic demand, the manufacturer being a leader brings a higher channel efficiency. When a local manufacturer is involved as a second manufacturer, channel efficiency is higher when the retailer remains a follower, as this induces more fierce wholesale price competition between the two manufacturers. Additionally, when demand uncertainty is high in the two-player game with linear demand, a follower can achieve higher profits, whilst high uncertainty under iso-elastic demand decreases both players' profits. Moreover, it becomes advantageous for the retailer to have a local manufacturer as demand uncertainty increases, even when the local manufacturer announces the wholesale price first.

5.1 Introduction

Game theory finds extensive application in supply chain contract design. When the game parameters are common knowledge, a decision-maker can make optimal decisions while considering the reaction of other participants (Eisenhardt 1989). The results under deterministic information summarized by Lau and Lau (2003) show that when a supplier sets a wholesale price and a retailer decides on a selling price, the deterministic two-echelon Stackelberg game leads to channel efficiency (CE) of 75%, and the leader's advantage (LA) brings two times more profits than the follower's under linear demand. Even though these results no longer hold for iso-elastic demand, CE and LA remain constant in the elasticity factor of the demand function (i.e., b from iso-elastic demand curve $y(p) = ap^{-b}$) as stated by Lau et al. (2008).

In most cases, however, the decision-maker faces demand uncertainty when making operational decisions, such as determining order quantities or pricing. Despite the extensive literature on contract design in the context of stochastic demand information, most of these studies resort to numerical methods due to challenges in achieving tractability (Lau and Lau 2005). Additionally, the outcomes of a game-theoretic model are greatly influenced by the specific forms of price-sensitive demand functions (Petruzzi and Dada 1999, Lau and Lau 2003, Chiu et al. 2011, Shi et al. 2013). For example, iso-elastic demand makes the dominant player lose the first-mover advantage. Therefore, if the market has iso-elastic demand, the leader of the game would search for a way to stay as a follower (Lau et al. 2008). However, if the retailer sets the price with a percentage markup whenever she becomes the leader of the game, she can easily redeem her leader's advantage (Wang et al. 2013).

Moreover, in stochastic settings, results on channel efficiency and leader's advantages under deterministic settings do not hold (Lau et al. 2008, Shi et al. 2013, Wang et al. 2016, 2019a). Optimal decisions and economic benefits, in a certain game, change significantly by multiple factors such as randomness of demand, price-dependent demand function, markup schemes, and power structure. However, how these factors influence supply chain performance and whether there exists some consistent results that decision-makers can learn from is not well understood.

In this context, our focus revolves around the question of how supply chain performance is affected by decision sequences within a supply chain and pricing markup strategies in the face of uncertain demand. As Matsui (2021) mentioned, the power structure in supply chains denotes the sequence of decisions in which supply chain members set their respective margins (i.e., wholesale and selling prices). Based on the definition, in our study, we refer to a dominant player as the one who demands its margin earlier than a dominated player as a leader. Initially, we delve into the optimal pricing decision within an integrated market, aiming to identify the maximum channel efficiency. Subsequently, we shift our attention to a supply chain consisting of a single manufacturer deciding on a wholesale price and a newsvendor retailer determining a market price and order quantity. Within this setup, we explore two distinct power structures: 1) where the manufacturer wields dominant bargaining power (referred to as Domi-manu) and 2) where the retailer possesses dominant bargaining power (referred to as Domi-reta).

In economic literature, linear, iso-elastic, exponential, and logit demand functions are most commonly used (Huang et al. 2013). Although the logit function has its own benefit of capturing more precise consumer willingness-to-pay (WTP) distributions in a global range of price settings (i.e., the price-demand relationship is more sensitive in the middle range of price), the linear and iso-elastic demand functions are still found to be useful and most widely applied to derive analytical implications within a reasonable price variation (Duan and Ventura 2021). Therefore, for tractability, we consider linear and iso-elastic demand functions. Further, the retailer can set a selling price with either an absolute markup or a percentage markup on the manufacturer's wholesale price. Especially, when the retailer is a leader, the manufacturer's wholesale price is set differently depending on which markup scheme the retailer charges.

Although a wholesale price-only contract is known to be unable to coordinate the supply chain because of double marginalization (Katok et al. 2014), it is yet the most commonly used and preferred contract mechanism in various industry sectors (i.e., semiconductor and agriculture) due to its simplicity (Hwang et al. 2018). Further, conducting laboratory experiments, Ho and Zhang (2008) show that more elaborate mechanisms, such as two-part tariff and quantity discount, do not necessarily improve channel efficiency compared to the wholesale price-only contract. To this end, we also apply the wholesale price-only contract in this study based on its extensive practicality. We evaluate the supply chain performance of each setting based on two aspects: 1) channel efficiency, which quantifies the relative profit achieved in a decentralized game compared to that obtained by an integrated (centralized) system, and 2) leader's advantage, which shows the relative profit gained by the leader of a game.

While most existing literature predominantly focuses on a retailer and a manufacturer game, recent years have experienced the significance of supply-driven markets. This surge in importance can be attributed to global geopolitical and economic uncertainties, exemplified by incidents like shortages in gas and batteries and the global pandemic. Drawing inspiration from this observation, we encompass a supply network characterized by a single retailer and two manufacturers, where the retailer has a local backup manufacturer. Within this network, one manufacturer assumes the role of a global player responsible for fulfilling the retailer's order requests. At the same time, the other operates as a local (backup) manufacturer, charging a higher wholesale price but offering the advantage of immediate response to the retailer's anticipated shortages. With the retailer's option to source from the local manufacturer, we examine how the sequence of decisions (power structure) influences channel efficiency in the supply chain and the leader's advantages for each player in this complex dynamic. Also, we investigate how the decision sequence of the players impacts the two competing manufacturers' wholesale price decisions.

In a two-player game, Domi-manu always leads to the highest channel efficiency under iso-elastic demand, while under linear demand, Domi-reta, charging a percentage markup, has the highest channel efficiency. Further, Domi-reta charging a percentage markup obtains the highest leader's advantages, regardless of the demand function. As demand uncertainty increases, the follower attains higher profits under linear demand, while under iso-elastic demand, an increasing demand uncertainty makes all players' expected profits decrease. Specifically, when the manufacturer is the leader under linear demand, he substantially lowers the wholesale price to encourage the retailer to place larger orders when faced with high demand uncertainty. Consequently, in this scenario, the retailer, acting as the follower, benefits from reduced double marginalization effects. Conversely, when the retailer is the leader, she increases the selling price in response to market demand uncertainty, resulting in a subsequent increase in order quantity. In this situation, the manufacturer can achieve higher profits as a follower by gaining from the retailer's increased order quantity without substantially reducing his wholesale price.

The remainder of this chapter is structured as follows: In Section 5.2, we outline assumptions and describe the model. Section 5.3 presents a two-player Stackelberg game and provides a numerical analysis of channel efficiency and the leader's advantages. Section 5.4 delves into a three-player Stackelberg game, offering insights from our numerical experiments. Finally, in Section 5.5, we conclude with a summary and propose potential avenues for future research.

5.2 Model formulation

We consider a Stackelberg game with one manufacturer (He) who sets the wholesale price w and one retailer (She) who decides the order quantity q and markup u. The power structure of the game consists of two cases: 1) the manufacturer plays as the leader (Domi-manu), and 2) the retailer plays as the leader (Domi-reta). Under the Domi-manu game, the sequence of decisions is: 1) wholesale price w, 2) markup u and order quantity q, while under the Domi-reta game: 1) markup u, 2) wholesale price w, and 3) order quantity q. The retailer has two options to set her markup: absolute markup u ($p = u + w$), or percentage markup $u\%$ ($p = (1 + u) \cdot w$). The manufacturing cost c, and retailer's salvage cost s are exogenously given. All cost parameters, s

and c, are common knowledge to the manufacturer and retailer where $s \leq c$. The decisions are made before uncertain demand is realized over a single selling season. The market has a price-sensitive demand function $D(p)$. We apply the most commonly used price-sensitive demand functions from price theory, linear and iso-elastic demands in the following form (see Simon et al. 1989):

1. Linear demand curve: $y(p) = a - bp$, where $a > 0$ and $b > 0$
2. Iso-elastic demand curve: $y(p) = ap^{-b}$, where $a > 0$ and $b > 2$.

Price-dependent demand is subject to uncertainty. The random variable $\widetilde{x}$ has a probability density $f(\cdot)$ and a cumulative distribution $F(\cdot)$. Newsvendor-type problems typically assume that the random demand distribution has an increasing generalized failure rate (IGFR) (Ziya et al. 2004). By definition, the generalized failure rate is $x \cdot f(x)/\left[1 - F(x)\right]$, an adjusted form of the failure rate, $f(x)/\left[1 - F(x)\right]$. The IGFR assumption is a relatively mild restriction compared to the other two assumptions, and IGFR distributions contain the most frequently employed distributions such as normal, exponential, and uniform distributions (Kocabıyıkoğlu and Popescu 2011).

Conventionally, the randomness of demand ($\widetilde{x}$) is applied additively (location) or multiplicatively (scale) to the demand function (Mills 1959, Karlin and Carr 1962). If demand uncertainty is formed in an additive way, the demand function is $\widetilde{D}(p, \widetilde{x}) = y(p) + \widetilde{x}$. For multiplicative randomness, demand follows $\widetilde{D}(p, \widetilde{x}) = y(p)\widetilde{x}$. Depending on the form of uncertainty applied to the demand function, the moments of uncertain demand vary and lead to different properties (Petruzzi and Dada 1999). For example, under additive randomness, the price decision contributes to the coefficient of variation (CV) of demand from the demand function $y(p)$ because the expected mean demand is $\mathbb{E}\left[\widetilde{D}(p, \widetilde{x})\right] = y(p) + \mu$, and the standard deviation equals $\sigma\left[\widetilde{D}(p, \widetilde{x})\right] = \sigma$; hence $CV = \frac{\sigma\left[\widetilde{D}(p,\widetilde{x})\right]}{\mathbb{E}\left[\widetilde{D}(p,\widetilde{x})\right]} = \frac{\sigma}{y(p)+\mu}$. Under multiplicative randomness, the CV is independent of price as $CV = \frac{\sigma\left[\widetilde{D}(p,\widetilde{x})\right]}{\mathbb{E}\left[\widetilde{D}(p,\widetilde{x})\right]} = \frac{\sigma y(p)}{\mu y(p)} = \frac{\sigma}{\mu}$. To avoid the endogenous pricing decision impacts on the CV through the price-dependent demand function $y(p)$, we consider multiplicative randomness in our study (where the CV is purely defined by the moments of distribution such as μ and σ) and investigate the effect of demand stochasticity represented as CV on each supply chain member's profit.

Based on Gal-Or (1985a), we consider two types of leaders' advantages. Type-I leader's advantage represents that the player with dominant power always prefers to move first rather than second. Type-II leader's advantage refers to the leader gaining more than the follower in a game.

To investigate the supply chain performance, this study covers the individual player's profit and analyzes channel efficiency and these two types of leader's advantages.

We denote the player's expected profits as π_j^i, where superscript $i \in \{I, M, R\}$ represents a game setting (I integrated market, M Domi-manu, and R Domi-reta). Subscript $j \in \{m, r\}$ stands for the respective player. For instance, π_r^M represents the retailer's profit under the Domi-manu game. As Petruzzi and Dada (1999) and Lariviere and Porteus (2001), we use the stocking factor z and modified price elasticity $\varepsilon(z)$ for the analysis tractability and assume $\varepsilon(z) > 1$. The players are risk-neutral and maximize their expected profits.

5.3 Two-player Stackelberg game

5.3.1 Integrated decision

We first analyze the integrated decision where an integrated company needs to decide the price p and the order quantity q. The objective function is to maximize expected profit.

$$\pi^I(p, q) = (p - c) \cdot q - (p - s) \cdot \mathbb{E}\left[q - y(p) \cdot \widetilde{x}\right]^+ \tag{5.1}$$

Using the stocking factor expression $z = \frac{q}{y(p)}$, the profit function can be rearranged to

$$\pi^I(p, z) = \begin{cases} \pi_+^I(p, z) = p \cdot y(p) \cdot \widetilde{x} - c \cdot y(p) \cdot z + s \cdot y(p)[z - \widetilde{x}]^+ & \widetilde{x} < z \\ \pi_-^I(p, z) = (p - c) \cdot y(p) \cdot z & \widetilde{x} \geq z. \end{cases} \tag{5.2}$$

Hence, the expected profit of the integrated market is:

$$\mathbb{E}\left[\pi^I(p, z)\right] = (p - c)y(p)\mu - y(p)[(c - s)\Lambda(z) + (p - c)\Theta(z)] \tag{5.3}$$

where $\Lambda(z) = \int_A^z (z - \tilde{x}) f(\tilde{x}) d\tilde{x}$ and $\Theta(z) = \int_z^B (\tilde{x} - z) f(\tilde{x}) d\tilde{x}$. Under the integrated market, the two decision variables (p and z) are set simultaneously before demand realization. The optimal solutions can be found by substituting z to p (Whitin 1955) or p to z (Zabel 1970). The first- and second-order conditions with respect to the stocking factor z from the expected profit are

$$\begin{aligned} \frac{\partial \mathbb{E}\left[\pi^I(p, z)\right]}{\partial z} &= y(p)(c - s) - y(p)(p - s)(1 - F(z)) \\ \frac{\partial^2 \mathbb{E}\left[\pi^I(p, z)\right]}{\partial z^2} &= -(p - s) f(z) < 0 \end{aligned} \tag{5.4}$$

From (5.4), the shape of z is concave for a given p, hence $F(z^*) = \frac{p-c}{p-s}$. The equilibrium function of price p with respect to z is

$$\frac{\partial \mathbb{E}\left[\pi^I(p, z)\right]}{\partial p} = (\mu - \Theta(z)) \cdot \left\{y(p) + y'(p) \cdot (p - c)\right\} - y'(p) \cdot (c - s)\Lambda(z).$$

In an integrated market, for a given stocking factor z, the optimal price is

$$p(z) = p_0 + \frac{\Lambda(z)(c-s)}{2(\mu - \Theta(z))} \quad where, \quad p_0 = \frac{a+bc}{2b} \text{ for a linear demand function}$$
$$p(z) = p_0 + \frac{b}{b-1}\left[\frac{(c-s)\Lambda(z)}{\mu - \Theta(z)}\right] \quad where, \quad p_0 = \frac{bc}{b-1} \text{ for an iso-elastic demand function.} \tag{5.5}$$

p_0 is the price that maximizes the riskless profit, $(p-c)y(p)\mu$. By replacing the price decision variable p with the stocking factor z, we have one variable equation to obtain an optimal solution. Given the relationship between $p(z)$ and $z(p)$, the first-order condition in (5.4) is

$$\frac{\partial \mathbb{E}\left[\pi^I(p(z), z)\right]}{\partial z} = y(p(z)) \cdot [1 - F(z)]\, R(z) \quad \text{where, } R(z) = p(z) - s - \frac{c-s}{1-F(z)}. \tag{5.6}$$

The multiplication term $y(p(z))$ and $[1-F(z)]$ is strictly positive. Therefore, focusing on the last term, $R(z) = \left[p(z) - s - \frac{c-s}{1-F(z)}\right]$, the optimal stocking factor z^* makes the first-order condition (FOC) equal to zero, $R(z) = 0$. The optimal z^* and $p^*(z)$ suffice $F(z^*) = \frac{p-c}{p-s}$ and $p^*(z) = s + \frac{c-s}{1-F(z)}$. It is noteworthy that even if the optimal stocking factor, z, and price p are the same under two demand functions (i.e., linear and iso-elastic demands), it does not imply that the optimal quantity and price of both functions are equal as the stocking factor $z = q/y(p)$ is a relative indicator of order quantity q depending on different demand functions $y(p)$.

5.3.2 Decentralized decision

Power Structure with Domi-manu. The retailer's profit is analogous to the integrated one since both the retailer's price and quantity decisions need to be made before the demand has materialized. Therefore, u and q are simultaneous decisions. However, now the cost is the wholesale price w, instead of the manufacturing cost c. We can proceed with the same approach for the retailer's optimal solution. Another structural difference in the retailer's profit is that now the price-dependent demand depends on the retailer's markup decision u and the manufacturer's wholesale price decision w.

The retailer's expected profit functions for absolute markup and percentage markup are:

$$\mathbb{E}\left[\pi_r^M(u, z; w)\right] = \begin{cases} u \cdot y(u+w)\mu - y(u+w)[(w-s)\Lambda(z) + u \cdot \Theta(z)] & \text{for } p = u + w \\ u \cdot w \cdot y(w \cdot (1+u))\mu - y(w \cdot (1+u))[(w-s)\Lambda(z) + u \cdot w \cdot \Theta(z)] & \text{for } p = w(1+u). \end{cases} \tag{5.7}$$

As the retailer being a follower sets u and z simultaneously, the markup under the two schemes remains the same ($u_+ = u_\%$) as the retailer essentially sets the price p that maximizes her expected profit, jointly with the stocking factor z after the manufacturer's wholesale price w announcement. This means that even though the retailer decides to apply the percentage markup, under the Domi-manu game, as the retailer's margin is chosen after the manufacturer announces his wholesale price w, the retailer essentially seeks the optimal markup price of p that maximizes her profit. Therefore, as a follower of the game, the retailer focuses on the optimal market price p given the wholesale price w, regardless of the markup schemes. For this reason, we focus on optimizing z paired with the selling price p. Based on the partial derivatives of the retailer's expected profit with respect to z, the retailer's optimal stocking factor is $z^*(u, w) = F^{-1}\left(\frac{p-w}{p-s}\right)$. Further, the equilibrium price is

$$\begin{aligned} p^*(w, z) &= u_0 + \frac{\Lambda(z)(w-s)}{2(\mu - \Theta(z))} + w \text{ where, } u_0 = \frac{a - bw}{2b} \text{under a linear demand,} \\ p^*(w, z) &= u_0 + \frac{b}{b-1}\left[\frac{(w-s)\Lambda(z)}{\mu - \Theta(z)}\right] + w \text{ where, } u_0 = \frac{w}{b-1} \text{under an iso-elastic demand.} \end{aligned} \tag{5.8}$$

u_0 denotes the riskless markup, which differs from the riskless price of p_0 under the integrated market. It is worth mentioning that the integrated one merely needs to consider the stocking factor z and corresponding underage/overage for its price decision since all the other factors are exogenously given parameters. However, the price under the decentralized market is influenced by the wholesale price w decision of the manufacturer and the stocking factor z of the retailer. Hence, the manufacturer observes the reaction function of the retailer from (5.8) and optimizes his decision w.

To derive the optimal decision of the manufacturer, we define the manufacturer's expected profit function and apply the modified price elasticity term of $\varepsilon(z)$ based on Lariviere and Porteus (2001). Given the retailer's decisions, the manufacturer's profit is deterministic,

$$\pi_m^M(w; p, z) = (w - c) \cdot y(p) \cdot z(w). \tag{5.9}$$

The response function of z from the retailer concerning the wholesale price w, the manufacturer uses this relationship between w and z to maximize his profit, $w(p, z) = p(1-F(z))+sF(z)$ from (5.8). By definition, the modified price elasticity represents the ratio of proportionate change in stocking factor z caused by a given proportional change in wholesale price w. We derive the optimal decision of z from the manufacturer's profit function by deriving the FOC in z and use the price elasticity term $\varepsilon(z)$ to obtain an optimal decision on $w(z)$ for the manufacturer.

$$\frac{\partial \pi_m^M(w, p, z)}{\partial z} = w(p, z) \cdot y(p) + w'(p, z) \cdot y(p) \cdot z - c \cdot y(p) = y(p)\left(w(p, z)\left[1 - \frac{1}{\varepsilon(z)}\right] - c\right) \tag{5.10}$$

The optimal solution for the manufacturer is

$$w^*(z) = \frac{\varepsilon(z^*)c}{\varepsilon(z^*) - 1}. \tag{5.11}$$

Even though our study addresses the optimal wholesale price w based on the retailer's stocking factor z, the optimal result $w^*(z)$ can be driven in the same way when the manufacturer optimizes his wholesale price by using the reaction function of the retailer's price p since the retailer's p and z are set simultaneously in Domi-manu game. Further, the impact of the demand function $y(p)$ is negligible to derive the equilibrium wholesale price w ($\frac{\partial \pi_m^R(u,w,z)}{\partial z} = 0$) when the manufacturer optimizes his decision concerning the stocking factor z as shown in (5.11). Hence, the optimal solution for the manufacturer under iso-elastic demand has the same structure as for linear demand.

Power Structure with Domi-reta. Under the Domi-reta setting, the retailer announces her markup u before the manufacturer sets his wholesale price w. As the retailer knows that the manufacturer's optimal wholesale price of w is derived based on the reaction function of z, up to the second stage (Stage 1: z and Stage 2: w), we employ the same procedure to the Domi-manu game. By doing so, the stocking factor and the wholesale price decisions suffice, $F(z^*) = \frac{p-w}{p-s}$ and $w^*(z) = \frac{\varepsilon(z^*)c}{\varepsilon(z^*)-1}$ under both linear and iso-elastic demand functions. The retailer, as a leader, anticipates the manufacturer's optimal wholesale decision for a given u as $w(u)$. Hence, by substituting $w(z)$ into the FOC function $\mathbb{E}\left[\pi_r^R(u(z), w(z), z)\right]$, an optimal absolute markup u_+^* and a percentage markup $u_\%^*$ are the solution to

$$u_+^*(w(z), z) = (w(z) - s)\frac{F(z)}{1 - F(z)} \quad \text{and} \quad u_\%^*(w(z), z) = \frac{w(z) - s}{w(z)} \frac{F(z)}{1 - F(z)}. \tag{5.12}$$

Note that the equilibrium function in (5.12) has one variable z. Based on the implicit function theorem, $\frac{\partial u}{\partial w} = -\frac{\partial F}{\partial w} / \frac{\partial F}{\partial u}$ where, $F = u_+ - (w - s)\frac{F(z)}{1-F(z)}$ (or $F = u_\% - \frac{w-s}{w}\frac{F(z)}{1-F(z)}$). As $\frac{\partial F}{\partial w} < 0$ and $\frac{\partial F}{\partial u} > 0$, a higher wholesale price leads to a higher markup price from the retailer; hence, a higher p ($\frac{\partial u}{\partial w} > 0$). However, as an increasing wholesale price reduces the order quantity, the retailer as a leader may balance between a higher price and a lower quantity. Based on the optimal decision of each player, u^*, w^*, and z^*, hereafter, we conduct numerical experiments to better understand the impact of various factors on the players' profits, channel efficiency, and leader's advantages.

5.3.3 Numerical results

We set the following parameter values: $c = 1.5$, $s = 0.5$, $a = 80$ and $b = 3$ for $y(p) = a - bp$ under the linear demand, and $y(p) = ap^{-b}$ under the iso-elastic demand for numerical studies suggested by Shi et al. (2013). We also present the sensitivity analysis on the market size a and the price sensitivity b for the range $a \in (60, 100)$ and $b \in (2, 4)$.

5.3.3.1 Supply chain performance

The impact of sequence on channel efficiency. Table 5.1 shows that under iso-elastic demand, channel efficiency does not show significant differences among different power structures. However, under linear demand, when the retailer is the leader charging a percentage markup, channel efficiency is notably higher than under the other power structures, making the supply chain achieve close to the integrated market profit.

Such a high efficiency occurs because when the retailer imposes a percentage markup, the stocking factor z^* is $F^{-1}(\frac{u \cdot w}{p-s})$ while under an absolute markup $z^* = F^{-1}(\frac{w}{p-s})$. These optimal stocking factors show that by announcing a high percentage markup u as a leader, the retailer induces the manufacturer to lower his wholesale price to ensure sufficient demand $y(p)$ where $p = w(1+u)$ as indicated in the numerator. Since the retailer exploits more control over the wholesale price by announcing a percentage markup beforehand, the double marginalization effect is mitigated; hence, the supply chain achieves a high channel efficiency.

Table 5.1: Channel Efficiency under Different Power Structures

	Iso-elastic			Linear		
CV	Domi-M	Domi-R(%)	Domi-R(+)	Domi-M	Domi-R(%)	Domi-R(+)
0.1	0.743	0.743	0.738	0.752	**0.943**	0.763
0.2	0.741	0.741	0.730	0.729	**0.927**	0.750
0.3	0.741	0.741	0.730	0.718	**0.912**	0.736
0.4	0.741	0.741	0.738	0.741	**0.930**	0.745

Other factors that may influence channel efficiency are the market size a and the price sensitivity b in the demand function. Table 5.2 shows that under iso-elastic demand, channel efficiency is relatively robust to different parameters a and b. Especially, when the retailer is the leader, channel efficiency almost remains constant over an increasing market size a. However, as price sensitivity b increases, channel efficiency decreases under all power structures. Further, under the iso-elastic demand function, channel efficiency from the Domi-manu case is always the highest in any combination of demand parameters.

Under linear demand, channel efficiency is more sensitive to both market size a and price sensitivity b. One notable observation is that the retailer charging a percentage markup shows the opposite results to the other two cases (i.e., Domi-manu and Domi-reta with absolute markup). More specifically, channel efficiency increases in market size a and decreases in price sensitivity b under the Domi-reta with a percentage markup. In contrast, when the manufacturer is the leader (Domi-M), or the retailer charges an absolute markup (Domi-R(+)), the channel

Table 5.2: Sensitivity Analysis in Market Size a and Price Sensitivity b on Channel Efficiency

Domi-M		Iso-elastic Demand					Linear Demand				
		b					b				
		2	2.5	3	3.5	4	2	2.5	3	3.5	4
	60	0.783	0.759	0.747	0.757	0.739	0.775	0.777	0.778	0.779	0.778
a	80	0.783	0.760	0.747	0.740	0.738	0.771	0.774	0.776	0.777	0.779
	100	0.781	0.758	0.745	0.757	0.740	0.769	0.771	0.774	0.775	0.778
Domi-R(+)		b					b				
		2	2.5	3	3.5	4	2	2.5	3	3.5	4
	60	0.772	0.755	0.743	0.738	0.738	0.760	0.762	0.764	0.765	0.765
a	80	0.772	0.755	0.743	0.738	0.738	0.756	0.759	0.761	0.763	0.764
	100	0.772	0.755	0.743	0.738	0.738	0.753	0.756	0.759	0.761	0.762
Domi-R(%)		b					b				
		2	2.5	3	3.5	4	2	2.5	3	3.5	4
	60	0.774	0.754	0.743	0.737	0.737	0.968	0.958	0.949	0.940	0.931
a	80	0.774	0.754	0.743	0.737	0.737	0.979	0.971	0.963	0.956	0.949
	100	0.774	0.754	0.743	0.737	0.737	0.986	0.979	0.973	0.966	0.960

efficiency decreases in the market size a while an increasing price sensitivity b leads to higher channel efficiency.

The impact of sequence on leader's advantages. Although Table 5.1 implies that depending on the demand function, the power structure combined with markup schemes leads to different channel efficiency, if the leader's advantage is not ensured, such a sequence cannot be realized as the leader would try to be a follower so that a higher profit can be obtained. In this regard, we present the Type 1 leader's advantage in Table 5.3. By definition, Type 1 leader's advantage is a relative gain that the leader can obtain as being a leader of a particular game ($LA_1^m = \frac{\pi_m^M}{\pi_m^R}$ and $LA_1^r = \frac{\pi_r^R}{\pi_r^M}$). Hence, in case the Type 1 leader's advantage is below 1, $LA_1^m < 1$ or $LA_1^r < 1$, the leader tries to gain a higher profit by enforcing to be a follower.

Similar to the deterministic iso-elastic Stackelberg model, where the leader's advantage equals $\frac{b-1}{b} < 1$ (Wang et al. 2016), Table 5.3 shows that when demand is iso-elastic, none of the players can benefit from being a leader and, hence, are better off by remaining a follower. However, a clear benefit exists to being the leader under a linear demand function. In particular, the retailer charging a percentage markup can profit significantly more by being a leader under the linear demand function. Moreover, under the linear demand function, as demand uncertainty increases, the leader's advantage under all power structures decreases. However, when demand follows an iso-elastic function, although demand uncertainty is high, the leader's advantage does not always reduce. Under iso-elastic demand, all players' profits decrease as the demand uncertainty increases. However, when demand is linear, a high uncertainty makes the follower obtain a higher profit.

Table 5.3: Type 1 Leader's Advantage under Different Power Structures

	Iso-elastic			Linear		
CV	Domi-M	Domi-R(%)	Domi-R(+)	Domi-M	Domi-R(%)	Domi-R(+)
0.1	0.68	1.00	0.68	1.91	3.23	1.94
0.2	0.70	1.00	0.69	1.78	3.06	1.83
0.3	0.72	0.99	0.72	1.61	2.75	1.61
0.4	0.72	0.99	0.73	1.51	2.55	1.46

Type 2 leader's advantage denotes that a leader in a game yields a higher profit than a follower. Given a specific channel efficiency, the leader's proportional gain is higher than that of the follower ($LA_2^m = \frac{\pi_m^M}{\pi_r^M}$ and $LA_2^r = \frac{\pi_r^R}{\pi_m^R}$). Similar to the observation from Table 5.3, when demand is linear, the Type 2 leader's advantage is evident, as depicted in Table 5.4. This implies that under a linear demand function, being the leader lets the player obtain a higher profit than being a follower and secure a higher proportion of gain compared to the follower.

Under the iso-elastic function, generally, being the leader does not lead to a higher gain than that of being the follower. However, when the retailer charges a percentage markup, even if her profit is not as high as being a follower of the game, she can at least extract a higher profit than the manufacturer by being a leader due to the increased control over the manufacturer's wholesale price. Such results that the retailer having a dominant market power imposes a percentage markup to the supplier lead to the highest leaders' advantage explain that most consumer goods retailers to opt for percentage markups as stated by Wang et al. (2013). Lastly, under the linear demand function, the Type 2 leader's advantage monotonically decreases as demand uncertainty increases, whilst, under the iso-elastic demand function, no systematic result is observed.

Table 5.4: Type 2 Leader's Advantage under Different Power Structures

	Iso-elastic			Linear		
CV	Domi-M	Domi-R(%)	Domi-R(+)	Domi-M	Domi-R(%)	Domi-R(+)
0.1	0.67	1.50	0.69	1.89	8.44	1.97
0.2	0.67	1.50	0.73	1.71	8.01	1.91
0.3	0.66	1.49	0.78	1.46	7.38	1.78
0.4	0.67	1.47	0.79	1.33	6.87	1.65

To summarize the results from channel efficiency and leader's advantage analysis, Table 5.5 shows that under iso-elastic demand, Domi-manu always leads to higher channel efficiency, while under linear demand, Domi-reta charging a percentage markup has the highest channel efficiency.

Further, Domi-reta charging a percentage markup obtains the highest leader's advantages, regardless of the demand function.

Table 5.5: Comparison of Supply Chain Performance

	Iso-elastic Demand	Linear Demand
Channel Efficiency	$\{CE^{R(+)}, CE^{R(\%)}\} < CE^{M}$	$\{CE^{R(+)}, CE^{M}\} < CE^{R(\%)}$
Leader's Advantage (Type I)	$\left\{LA_1^{R(+)}, LA_1^{M}\right\} < LA_1^{R(\%)}$	$\left\{LA_1^{R(+)}, LA_1^{M}\right\} < LA_1^{R(\%)}$
Leader's Advantage (Type II)	$LA_2^{M} < LA_2^{R(+)} < LA_2^{R(\%)}$	$LA_2^{M} < LA_2^{R(+)} < LA_2^{R(\%)}$

5.3.3.2 Equilbrium decisions and expected profits

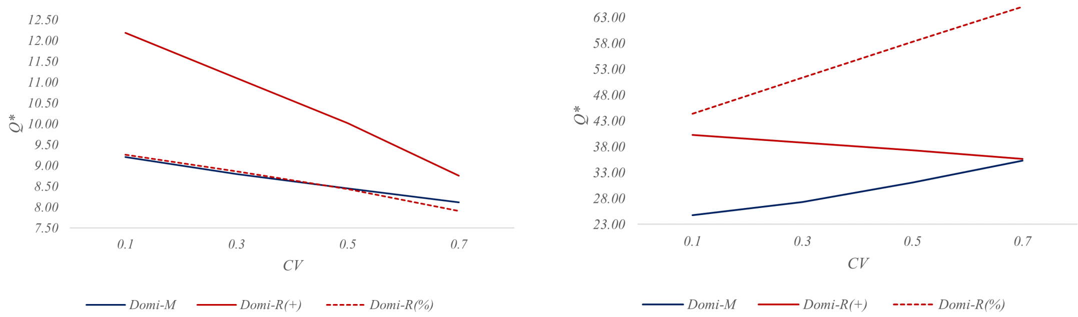
The impact of sequence on equilibrium quantity. Under iso-elastic demand (Figure 5.1a), high demand uncertainty makes the retailer reduce her order quantity under any power structure. Compared to the order quantity of the retailer setting an absolute markup, the order quantity set by the retailer charging a percentage markup is similar to the quantity when the manufacturer is a leader.

When demand is linear, the retailer charging a percentage markup orders the most while the manufacturer, being a leader, makes the retailer reduce the optimal order quantity compared to the other cases (Figure 5.1b). In contrast to the iso-elastic demand function, as demand uncertainty increases under linear demand, the order quantity increases under all power structures.

The observation that the Domi-manu case does not lead to the largest order quantity is intriguing as one may conjecture that the manufacturer, being a leader, might force the retailer to order more by reducing the selling price p (equivalently, increasing the price-dependent demand $y(p)$) so that he can increase profit. We delve into this question by investigating the optimal wholesale price and selling price decisions of the players.

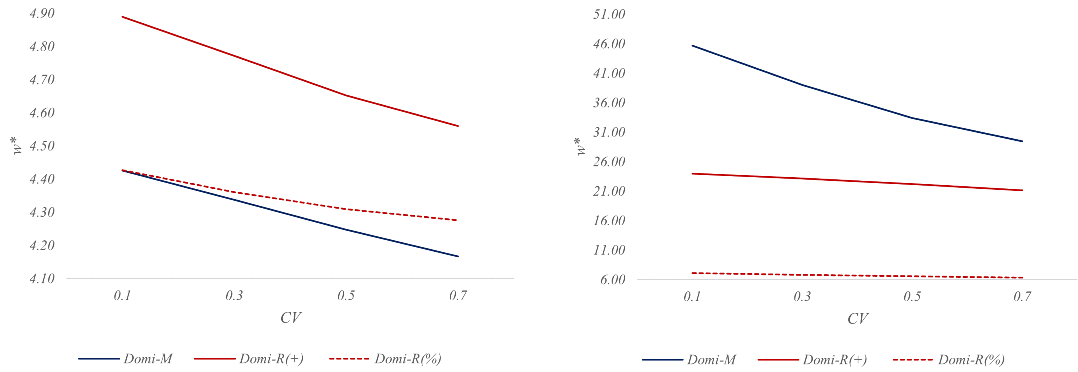
The impact of sequence on equilibrium wholesale price. When demand is iso-elastic, although the manufacturer is the leader of the game, he does not charge a higher wholesale price, as depicted in Figure 5.2a. Aligned with the finding that the iso-elastic demand function makes the leader's advantage disappear (Lau et al. 2008), this observation partly explains the reason that the manufacturer, even being a leader, cannot yield a higher profit than being a follower; hence, it is better to let the retailer be the leader instead. When demand is linear, however, the manufacturer imposes the highest wholesale price when he is the leader, as shown in Figure 5.2b.

Figure 5.1: Equilibrium Quantity q^*



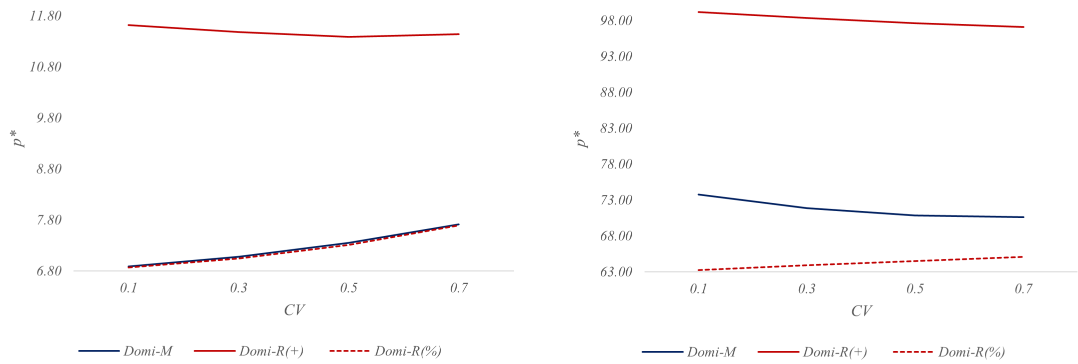
(a) Iso-elastic Demand: $y = ap^{-b}$ **(b)** Linear Demand: $y(p) = a - bp$

Figure 5.2: Equilibrium Wholesale Price w^*



(a) Iso-elastic Demand: $y = ap^{-b}$ **(b)** Linear Demand: $y(p) = a - bp$

Figure 5.3: Equilibrium Selling Price p^*



(a) Iso-elastic Demand: $y = ap^{-b}$ **(b)** Linear Demand: $y(p) = a - bp$

The impact of sequence on equilibrium selling price. Regarding the retailer's optimal selling price (Figure 5.3), one observation is that when the retailer is the leader charging an absolute markup, the optimal price is the lowest under the iso-elastic demand, while under the linear demand, the retailer charges the highest price. Under the iso-elastic demand function, as demand uncertainty increases, the retailer imposes a higher price, as shown in Figure 5.3a.

Note that the difference between absolute markup price and percentage markup price under the iso-elastic demand function increases as demand uncertainty increases. On the other hand, the price difference between the two schemes decreases under the linear demand function as the demand uncertainty increases. Under the Domi-manu, the retailer reduces her optimal price as demand uncertainty increases (Figure 5.3b). While the wholesale prices reduce and the quantities increase regardless of power structure under linear demand, the decreased optimal price under the Domi-manu explains a slow drop in the leader's advantage in high demand uncertainty. Table 5.6 summarizes the comparison among equilibrium decisions in different power structures and demand functions.

Table 5.6: Comparison of Equilibrium Decisions

	Iso-elastic Demand	Linear Demand
Stocking Factor (z^*)	$z^{R(+)} < \left\{z^{R(\%)}, z^M\right\}$	$z^M < z^{R(+)} < z^{R(\%)}$
Wholesale Price (w^*)	$\left\{w^{R(\%)}, w^M\right\} < w^{R(+)}$	$w^{R(\%)} < w^{R(+)} < w^M$
Selling Price (p^*)	$p^{R(+)} < \left\{p^{R(\%)}, p^M\right\}$	$p^{R(\%)} < p^M < p^{R(+)}$

The impact of sequence on players' profits. Under the iso-elastic demand function, demand uncertainty decreases all players' profits for any given power structure, as shown in Table 5.7. More interestingly, under the linear demand function, a high demand uncertainty does not always harm the players' profits. Ridder et al. (1998) state that an increasing demand variability does not always reduce the expected profit under newsvendor settings. Similarly, our results show that as demand uncertainty increases, being a follower under the linear demand function achieves a higher profit (i.e., π_r^M, $\pi_m^{R(\%)}$, and $\pi_m^{R(+)}$).

The interpretation of these counter-intuitive follower's benefits is as follows. First, when the manufacturer is the leader under a linear demand function, as demand uncertainty increases, the order quantity also increases while both the wholesale price and the retailer price decrease from Figure 5.1b, 5.2b, and 5.3b. Mainly, to induce the retailer to make a large order, the manufacturer reduces his wholesale price more significantly when he is a leader compared to the Domi-reta case. However, the retailer's price is moderate, although uncertainty increases, consecutively generating a market demand. Hence, under the Domi-manu, the retailer as a follower can benefit from a considerable reduction of the wholesale price while keeping her selling price relatively the same in increasing demand uncertainty.

In addition, when the retailer is the leader, similar to the Domi-manu case, the retailer's order quantity increases substantially as demand uncertainty increases (Figure 5.1b). However, as the manufacturer is not the leader, he does not have an incentive to decrease his wholesale price significantly to induce a higher retailer order quantity (Figure 5.2b). On the other hand, even if the retailer increases her selling price as uncertainty increases, the retailer's optimal price increase is not as significant as the quantity increase (Figure 5.3b). Therefore, the manufacturer in such a case can achieve a higher profit as a follower, benefiting from the retailer's increased order quantity and his relatively similar wholesale price, although the demand uncertainty rises.

Table 5.7: Expected Profit of Each Player under Different Power Structures

CV	Int	Domi-M		Domi-R(%)		Domi-R(+)	
		π_m^M	π_r^M	π_m^R	π_r^R	π_m^R	π_r^R
Iso-elastic Demand Function: $y = ap^{-b}$							
0.1	4.95	1.47	2.21	1.47	2.21	2.16	1.49
0.2	4.53	1.34	2.02	1.34	2.02	1.91	1.39
0.3	3.84	1.14	1.71	1.14	1.71	1.58	1.23
0.4	3.22	0.95	1.43	0.97	1.42	1.33	1.05
Linear Demand Function: $y(p) = a - bp$							
0.1	461.39	226.65	**120.20**	**46.05**	388.81	**118.57**	233.46
0.2	461.39	212.24	**124.09**	**47.51**	380.32	**119.11**	227.00
0.3	456.66	194.56	**133.32**	**49.68**	366.77	**120.94**	215.20
0.4	436.06	184.57	**138.69**	**51.54**	353.91	**122.61**	202.32

Table 5.8 summarizes the relationships among each player's expected profit in different settings. Similar to Table 5.3, the leader of the game obtains a higher profit than the follower under linear demand. However, under iso-elastic, the follower has a higher profit than the leader, unless the retailer is the leader, charging a percentage markup.

Table 5.8: Comparison of Expected Profits

	Iso-elastic Demand	Linear Demand
Domi-manu	$\pi_m^M < \pi_r^M$	$\pi_m^M > \pi_r^M$
Domi-reta (%)	$\pi_m^{R(\%)} < \pi_r^{R(\%)}$	$\pi_m^{R(\%)} < \pi_r^{R(\%)}$
Domi-reta (+)	$\pi_m^{R(+)} > \pi_r^{R(+)}$	$\pi_m^{R(+)} < \pi_r^{R(+)}$

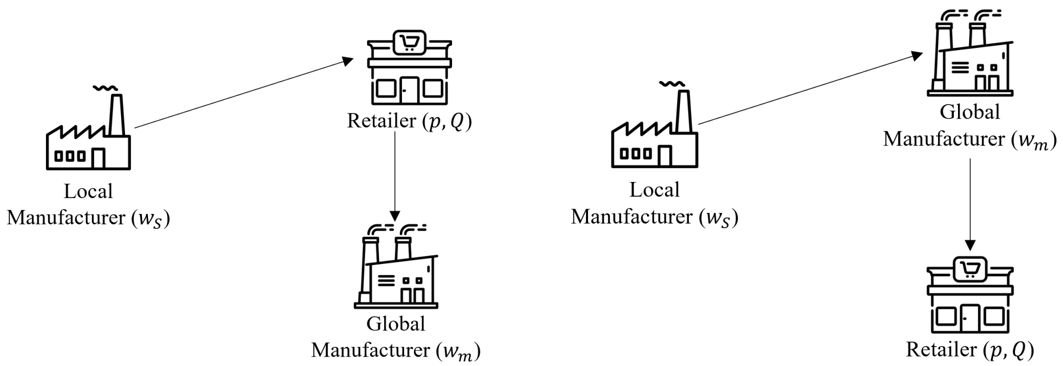
5.4 Three-player Stackelberg game

5.4.1 Decentralized decision

Now consider that a retailer sources from both a global and a local manufacturer offering identical products. The retailer decides its order quantity q from the global manufacturer, charging a wholesale price w_m before demand is realized. For any shortage, the local manufacturer can instantaneously deliver additional products to the retailer with another wholesale price of w_s. Such a local manufacturer charges a premium wholesale price for its dedicated fulfillment service. Due to the local manufacturer's premium service, the wholesale price set by the local manufacturer is higher than that of the global manufacturer $w_s \geq w_m$. Both manufacturers decide their wholesale prices, w_m and w_s, for the retailer before the selling season. Our focus is to investigate how the power structure between the retailer and the global manufacturer impacts the leader's advantage and channel efficiency with the existence of the local manufacturer.

Note that when the local manufacturer is a follower (M-S-R, R-S-M, M-R-S, or R-M-S), the optimal solution leads to an extreme case, where w_s^* is either $w_s^* = p^*$ for the sequence R-S-M, or $w_s^* = w_m^*$ for the sequence M-S-R, M-R-S, or R-M-S. Hence, we consider the case where the local manufacturer is a leader ($w_m^* \leq w_s^* \leq p^*$) to capture non-trivial results as shown in Figure 5.4. One sequence is where the retailer announces an optimal selling price p after the local manufacturer's wholesale price decision w_s. Upon the retailer's announcement of a selling price p, and the global manufacturer sets his wholesale price w_m to charge to the retailer's upcoming order quantity q. Based on p, w_m, and w_s, finally, the retailer sets the order quantity q, and the shortage is defined accordingly (Figure 5.4a). We refer to this sequence as Domi-reta under the involvement of a local manufacturer (S-Domi-reta). In the other sequence (Figure 5.4b), the local manufacturer announces his wholesale price w_s first, and the global manufacturer sets the wholesale price w_m based on the local manufacturer's price w_s. Finally, the retailer sets both of selling price p and order quantity q simultaneously. Hereafter, this sequence of the game is denoted as Domi-manu under the involvement of a local manufacturer (S-Domi-manu).

Although the local manufacturer always determines the wholesale price w_s for shortage first, the optimal decision of the local manufacturer results in a trade-off in the remaining players' decisions. For instance, if the local manufacturer imposes a high wholesale price w_s, the retailer's increasing order quantity q may induce the retailer to increase her selling price p while the global manufacturer lowers his wholesale price w_m to further encourage the retailer to increase the order quantity. It is of interest how the sequence between the followers (the retailer and the global manufacturer) influences the local manufacturer's wholesale price.



(a) S-Domi-reta **(b)** S-Domi-manu

Figure 5.4: Supply Network Sequence Structure

Retailer's profit function. The retailer's expected profit for given w_m and w_s is

$$\mathbb{E}\left[\pi_r(p, z; w_m, w_s)\right] = p \cdot y(p) \cdot \mu - w_m \cdot y(p) \cdot z + y(p)[s \cdot \Lambda(z) - w_s \cdot \Theta(z)]. \tag{5.13}$$

For the retailer, the involvement of the local manufacturer is considered to be an emergency purchasing cost. Using $\Lambda(z) = z - \mu + \Theta(z)$, the retailer's expected profit is rearranged to:

$$\mathbb{E}\left[\pi_r(p, z; w_m, w_s)\right] = y(p) \left\{ (p - s)\mu - (w_s - s)z - (w_s - s) \int_z \overline{F}(\widetilde{x}) d\widetilde{x} \right\}. \tag{5.14}$$

The first-order derivative with respect to z yields:

$$\frac{\partial \mathbb{E}\left[\pi_r(p, z; w_m, w_s)\right]}{\partial z} = y(p) \left\{ -w_m + s + \overline{F}(z)(w_s - s) \right\}.$$

Therefore, the optimal stocking factor and the order quantity of the retailer are

$$z^*(w_s, w_m) = F^{-1}\left(\frac{w_s - w_m}{w_s - s}\right) \text{ and } q^*(p, w_s, w_m) = y(p) \cdot F^{-1}\left(\frac{w_s - w_m}{w_s - s}\right) \tag{5.15}$$

The optimal quantity decision of the retailer shows that when the local manufacturer imposes a high wholesale price w_s, she increases the order quantity from the global manufacturer. In contrast, a high wholesale price w_m from the global manufacturer reduces the order quantity. Similar to the optimal price in (5.8) under the two-player setting, the first term of the retailer's price is interpreted as a riskless price that the retailer, without demand uncertainty, would charge. Notably, as the retailer's shortage cost increases by w_s, an additional term of a risk premium for the expected underage $\Theta(z)$ is imposed in her optimal price as the last term. Note that the optimal stocking factor z^* no longer depends on the retailer's selling price p as the underage and the overage costs are defined by w_s, w_m, and s. Further, with the involvement of the local manufacturer, the retailer no longer charges a certain markup (+ or %) to the global manufacturer but sets a single selling price p to the market.

Manufacturers' profit functions. As both manufacturers' profits are generated by the retailer's order quantity and subsequent expected shortages, we present the optimization problems of the manufacturers below.

$$\mathbb{E}\left[\pi_m(w_m; p, z, w_s)\right] = (w_m - c_m) \cdot y(p) \cdot z \text{ and } \mathbb{E}\left[\pi_s(w_s; p, z, w_m)\right] = (w_s - c_s) \cdot y(p) \cdot \Theta(z) \quad (5.16)$$

As the global manufacturer's w_m decision is set after the local manufacturer's wholesale price w_s, the global manufacturer's optimal decision is analogous to the selling to a newsvendor retailer problem as presented in 5.11, sufficing $w_m^*(z) = \frac{\varepsilon(z^*)c_m}{\varepsilon(z^*)-1}$. For the local manufacturer, the wholesale price suffices the optimal condition:

$$w_s^*(z) = -\frac{\Theta(z^*)}{\partial\Theta(z^*)/\partial w_s} + c_s. \quad (5.17)$$

The structure of the profit function is similar to that of the global manufacturer, as the profit is defined by the profit margin factored by the underage quantity. From the optimal stocking factor z^* in (5.15), an increasing w_m reduces the retailer's order quantity q^* while the local manufacturer's increasing w_s reversely increases the retailer's order quantity to the global manufacturer. Based on the profit functions of three players in (5.13) and (5.16), we investigate how the power structure (i.e., S-Domi-reta and S-Domi-manu) can affect channel efficiency and leader's advantages when an additional manufacturer is involved.

5.4.2 Numerical results

We now observe how the equilibrium decisions of the manufacturers and the retailer change compared to the two-player settings.

5.4.2.1 Supply chain performance

The impact of sequence on channel efficiency. Table 5.9 shows that channel efficiency is higher when the global manufacturer is the second mover, following directly the local manufacturer (S-Domi-manu) than when the retailer is the second mover. Especially, compared to the two-player game where channel efficiency is higher in Domi-reta under linear demand (Table 5.1), the global manufacturer setting his wholesale price w_m, before the retailer's price decision p leads to a higher channel efficiency due to the local manufacturer and the early announcement of w_s. This is because when the global manufacturer's decision directly follows the local manufacturer's wholesale price decision, they both can influence the retailer's price decision p and, hence, have better control over the optimal order quantity q^*. Such a sequence (S-Domi-manu) leads to more fierce wholesale price competition between the two manufacturers, resulting in lower w_m and w_s. However, in the case of S-Domi-reta, the retailer indirectly serves as a moderator in the wholesale price competition by announcing p before the wholesale price w_m which essentially increases both wholesale prices w_m and w_s and decreases channel efficiency.

Table 5.9: Channel Efficiency under Different Power Structures

	Iso-elastic Demand		Linear Demand	
CV	S-Domi-M	S-Domi-R	S-Domi-M	S-Domi-R
0.1	0.77	0.53	0.78	0.76
0.2	0.77	0.56	0.90	0.76
0.3	0.86	0.67	0.93	0.78
0.4	0.97	0.85	0.97	0.82

The impact of sequence on leader's advantages. With the existence of a local manufacturer being the first mover of the game, the sequence between the global manufacturer and the retailer leads to different leader's advantages. Note that here, the leader's advantage stands for the advantage to be a "first mover" between the retailer and the global manufacturer in the game who succeeds after the decision of w_s from the local manufacturer (as the local manufacturer always precedes the other players). The result in Table 5.10 shows interesting cases against the common knowledge that the leader under iso-elastic demand loses her advantage to be a leader, whilst under linear demand, being a leader ensures a higher profit than being a follower.

Table 5.10: Type 1 Leader's Advantages under Different Power Structures

	Iso-elastic Demand		Linear Demand	
CV	S-Domi-M	S-Domi-R	S-Domi-M	S-Domi-R
0.1	1.20	0.63	0.91	0.57
0.2	1.25	0.71	0.93	0.54
0.3	1.10	0.82	0.91	0.54
0.4	1.04	0.87	0.90	0.55

Under iso-elastic demand, with the existence of a local manufacturer, it is more beneficial for the global manufacturer to directly decide his wholesale price w_m after the local manufacturer's announcement of w_s (S-Domi-manu) than wait until the retailer decides on p (S-Domi-reta) although wholesale price competition is higher than S-Domi-reta. Specifically, direct competition between the global and the local manufacturer makes the wholesale prices w_s and w_m reduce while the order quantity q increases as the retailer can afford to lower the selling price p. Therefore, the global manufacturer experiences a profit increase due to the increased order quantity from the retailer under S-Domi-manu.

Under linear demand, direct competition between the global and the local manufacturer harms the global manufacturer as the benefit obtained from the retailer's price reduction does not compensate for the two manufacturers' lower wholesale prices w_s and w_m. Such competition still benefits the retailer under S-Domi-manu, as illustrated in Table 5.10. To summarize supply network performance under the three-player game, in general, the retailer being a follower leads to both higher channel efficiency and leader's advantages, as shown in Table 5.11.

Table 5.11: Comparison of Supply Network Performance

	Iso-elastic Demand	Linear Demand
Channel Efficiency	$CE^{S-R} < CE^{S-M}$	$CE^{S-R} < CE^{S-M}$
Leader's Advantage (Type I)	$LA_1^{S-R} < LA_1^{S-M}$	$LA_1^{S-R} < LA_1^{S-M}$

5.4.2.2 Equilbrium decisions and expected profits

The impact of sequence on equilibrium quantity. We investigate how the existence of the local manufacturer changes the optimal decision of the manufacturers and the retailer. The quantity decision in Table 5.12 shows that the involvement of the local manufacturer reduces the retailer's order quantity compared to the two-player game under both demand functions, in general. Especially, in the three-player game, if the retailer's price decision follows the local manufacturer's wholesale decision w_s (S-Domi-reta), the order quantity is significantly smaller than the case where she jointly decides on price and quantity after the global manufacturer's wholesale announcement w_m (S-Domi-manu).

The impact of sequence on equilibrium wholesale prices. Comparing the cases where the global manufacturer's wholesale decision w_m precedes the retailer's price decision p under two-player and three-player games (i.e., Domi-manu and S-Domi-manu), the existence of the local manufacturer makes the wholesale price of the global manufacturer w_m reduce as they compete over the wholesale prices to achieve more favorable order quantity from the retailer (Table 5.13).

Surprisingly, when the retailer's price decision p is made earlier than the global manufacturer's wholesale price w_m (i.e., Domi-reta and S-Domi-reta), the wholesale price increases with three players compared to two players. Because given an announced selling price p of the retailer, under the two-player game, the stocking factor $z^* = F^{-1}(\frac{p-w}{p-s})$ depends on both selling price

Table 5.12: Optimal Order Quantity Decision q^*

		Two-player			Three-player	
CV	Int Market	Domi-M	Domi-R(%)	Domi-R(+)	S-Domi-M	S-Domi-R
Iso-elastic Demand Function: $y(p) = ap^{-b}$						
0.1	6.11	2.24	1.82	1.78	1.61	0.66
0.2	5.35	2.22	1.58	1.57	1.36	0.55
0.3	4.55	2.20	1.36	1.41	1.20	0.68
0.4	3.79	2.18	1.15	1.25	1.07	0.71
Linear Demand Function: $y(p) = a - bp$						
0.1	50.97	18.79	29.08	19.51	18.39	17.58
0.2	50.97	19.03	31.55	20.57	18.44	13.88
0.3	50.53	20.59	35.65	22.51	21.84	13.72
0.4	50.03	23.05	39.71	24.47	25.02	16.03

and the wholesale price. Hence, the retailer as the leader can induce the manufacturer to lower his wholesale price w_m by imposing a high selling price, p. However, under the three-player game, the stocking factor $z^* = F^{-1}(\frac{w_s - w_m}{w_s - s})$ is no longer based on the retailer's selling price p but the wholesale prices between two manufacturers, w_s and w_m. Therefore, the retailer as a leader cannot impact the global manufacturer's wholesale price through the optimal stocking factor decision z^*, and the global manufacturer sets w_m in relation to w_s.

If the wholesale price w_m follows directly the local manufacturer's decision on w_s (S-Domi-manu), the local manufacturer imposes a lower w_s compared to the S-Domi-reta case. While a higher demand uncertainty makes the global wholesale price w_m decrease, regardless of demand functions, the local manufacturer's wholesale price w_s increases as the demand uncertainty increases when the demand follows a linear function.

The impact of sequence on equilibrium selling price. Table 5.14 shows that involving a local manufacturer and the retailer being a follower (S-Domi-manu) leads to a lower price than having only a global manufacturer (Domi-manu). This result is counter-intuitive as one anticipates the retailer may increase its price due to the double marginalization effect of having two manufacturers as leaders. However, when the two manufacturers compete over the wholesale prices before the retailer sets the price, the global manufacturer's optimal wholesale price w_m (S-Domi-manu) becomes smaller than without the local manufacturer (Domi-manu). This consecutively enables the retailer to offer a lower selling price p.

Table 5.13: Optimal Manufacturers' Wholesale Price Decisions w_m^* and w_s^*

	Two-player			Three-player			
CV	Domi-M	Domi-R(%)	Domi-R(+)	S-Domi-M	S-Domi-R	S-Domi-M	S-Domi-R
		w_m^*		w_m^*		w_s^*	
Iso-elastic Demand Function: $y(p) = ap^{-b}$							
0.1	2.24	2.24	2.57	2.23	2.95	3.10	3.10
0.2	2.22	2.22	2.51	2.16	2.82	2.90	2.92
0.3	2.20	2.21	2.44	2.06	2.40	2.82	2.90
0.4	2.18	2.20	2.41	1.98	2.19	2.79	2.88
Linear Demand Function: $y(p) = a - bp$							
0.1	12.65	3.01	7.29	10.28	14.33	10.65	14.73
0.2	10.95	2.89	6.87	8.00	12.60	11.80	15.97
0.3	9.51	2.80	6.51	6.95	10.81	12.69	17.31
0.4	8.46	2.72	6.19	6.20	9.49	12.81	17.49

Table 5.14: Optimal Selling Price Decision p^*

		Two-player			Three-player	
CV	Int Market	Domi-M	Domi-R(%)	Domi-R(+)	S-Domi-M	S-Domi-R
Iso-elastic Demand Function: $y(p) = ap^{-b}$						
0.1	2.33	3.58	3.48	3.43	3.57	4.50
0.2	2.46	3.64	3.64	3.48	3.62	4.30
0.3	2.62	3.89	3.88	3.57	3.67	4.05
0.4	2.80	4.22	4.22	3.76	3.76	4.02
Linear Demand Function: $y(p) = a - bp$						
0.1	14.21	19.89	17.73	20.31	18.47	20.50
0.2	14.32	19.43	17.95	20.30	17.96	20.29
0.3	14.46	19.18	18.15	20.38	17.95	20.23
0.4	14.56	19.16	18.35	20.55	17.96	20.20

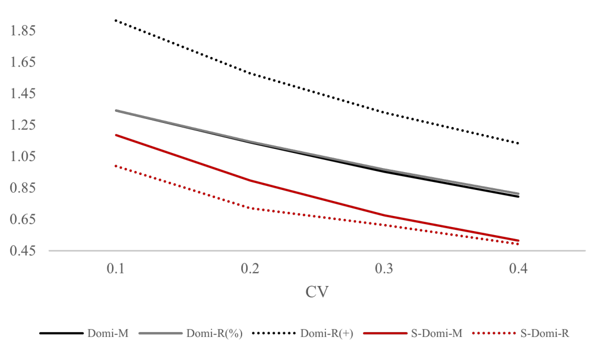
Table 5.15 summarizes the relationships among the optimal stocking factors, global and local wholesale prices, and selling prices in different settings. Aligned with the observation that S-Domi-manu leads to higher channel efficiency under both demand functions, the equilibrium prices of three players are lower under S-Domi-manu, while the order quantity of the retailer is higher.

Table 5.15: Comparison of Equilibrium Decisions

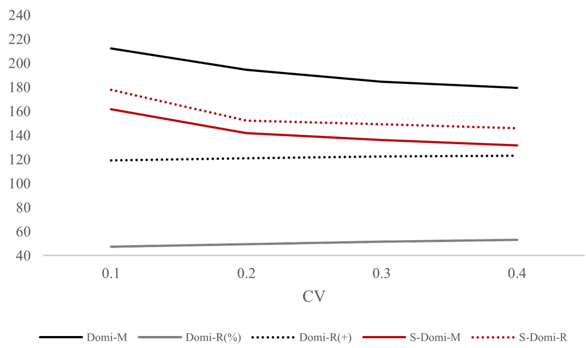
	Iso-elastic Demand	Linear Demand
Stocking Factor (z^*)	$z^{S-R} < z^{S-M}$	$z^{S-R} < z^{S-M}$
Local Wholesale Price (w_s^*)	$w_s^{S-R} > w_s^{S-M}$	$w_s^{S-R} > w_s^{S-M}$
Global Wholesale Price (w_m^*)	$w_m^{S-R} > w_m^{S-M}$	$w_m^{S-R} > w_m^{S-M}$
Selling Price (p^*)	$p^{S-R} > p^{S-M}$	$p^{S-R} > p^{S-M}$

Comparison of expected profits between two-player and three-player games. Under iso-elastic demand in Figure 5.5a, competition between the global and the local manufacturers harms the profit of the global manufacturer in general. However, with the involvement of the local manufacturer, the sequence of S-Domi-manu brings higher profits than S-Domi-reta for both the global manufacturer and the retailer, while the local manufacturer obtains a higher profit under S-Domi-reta by avoiding wholesale price competition with the global manufacturer as illustrated in Figure 5.5a, 5.5c, and 5.5e. Especially, the retailer being a follower under the three-player game implies that there exists a trade-off between being a leader against the global manufacturer and exploiting the upstream manufacturers' wholesale price competition. Further, as the demand uncertainty increases, it is beneficial for the retailer to involve the local manufacturer even though the local manufacturer announces the wholesale price first (Figure 5.5c).

Figure 5.5: Expected Profits of the Players under Iso-elastic and Linear Demand Functions



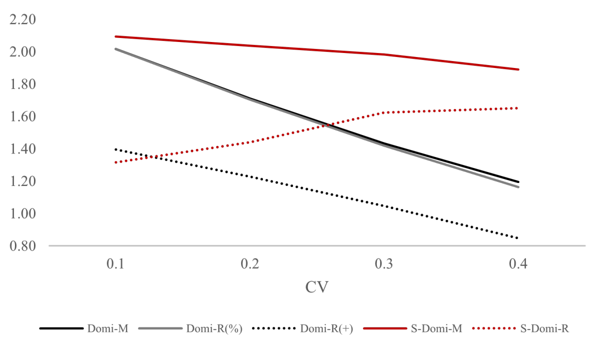
(a) Iso-elastic Demand: π_m



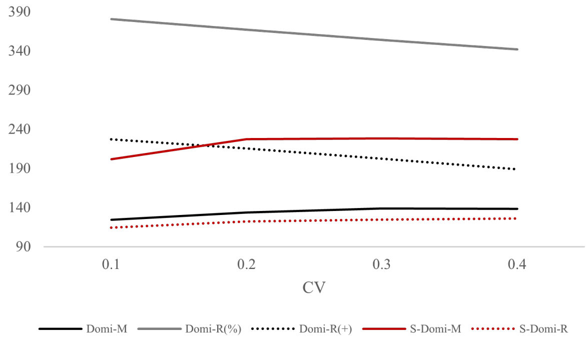
(b) Linear Demand: π_m

Under linear demand, although the global manufacturer's profit in S-Domi-manu is worse than without having the local manufacturer (Domi-manu), Figure 5.5b shows that if the retailer's price decision precedes the global manufacturer's wholesale price decision (i.e., Domi-reta, S-Domi-reta), it is better for him to involve the second manufacturer as the existence of the local

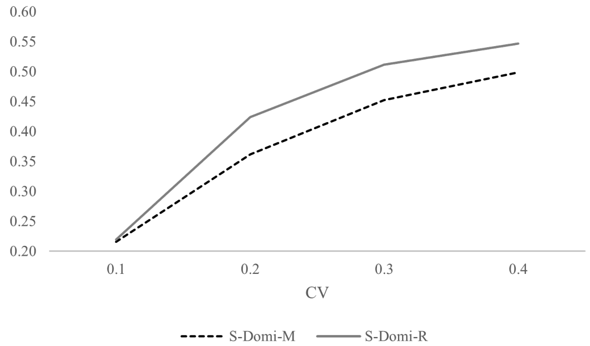
Figure 5.5: Expected Profits of the Players under Iso-elastic and Linear Demand Functions (cont.)



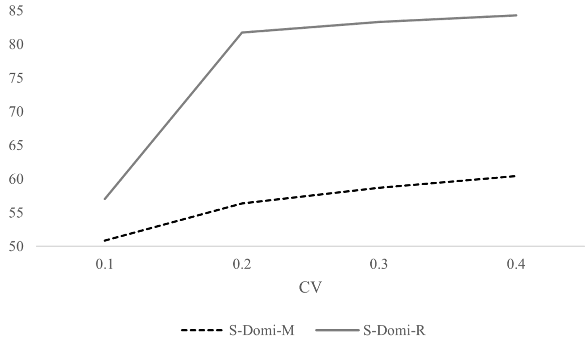
(c) Iso-elastic Demand: π_r



(d) Linear Demand: π_r



(e) Iso-elastic Demand: π_s



(f) Linear Demand: π_s

manufacturer prevents the retailer from extracting significant benefit from the global manufacturer, while the wholesale price competition is not as fierce as S-Domi-manu. Similar to the iso-elastic demand, the retailer benefits from being a follower under linear demand when the second manufacturer exists (S-Domi-manu). In particular, compared to the two-player game (Domi-manu), considerable profit improvement from being a follower (S-Domi-manu) can be acheived by the retailer. Further, under linear demand, the involvement of a second manufacturer makes the retailer's profit increase as the demand uncertainty increases, as shown in Figure 5.5d. Lastly, regardless of demand functions, the local manufacturer prefers the retailer to be to leader upon his wholesale price decision (S-Domi-reta) to avoid direct competition with the global manufacturer, and his profit increases in a high demand uncertainty as depicted in Figure 5.5e and 5.5f.

Lastly, the relationships among the players' expected profits in different settings show that upstream competition brings the retailer the highest profit. However, when demand is linear, the retailer being a leader benefits the global manufacturer. This can be explained by the fact

Table 5.16: Comparison of Expected Profits

	Iso-elastic Demand	Linear Demand
S-Domi-manu	$\pi_s^{S-M} < \pi_m^{S-M} < \pi_r^{S-M}$	$\pi_s^{S-M} < \pi_m^{S-M} < \pi_r^{S-M}$
S-Domi-reta	$\pi_s^{S-R} < \pi_m^{S-R} < \pi_r^{S-R}$	$\pi_s^{S-R} < \pi_r^{S-R} < \pi_m^{S-R}$

that the reduced wholesale price competition under S-Domi-reta induces the global manufacturer to increase the wholesale price significantly, as shown in Table 5.16.

Comparison of consumer surplus under different power structures. Consumer surplus considerably decreases as double marginalization occurs between manufacturer and retailer. Further, under the three-player game, the retailer being a follower brings higher consumer surplus as the wholesale price competition mitigates the double marginalization effect, as shown in Table 5.17. Similar to channel efficiency, under the two-player game, the consumer surplus is higher when demand follows an iso-elastic function, and the manufacturer is a leader. In contrast, under linear demand, Domi-reta brings a higher consumer surplus.

Table 5.17: Consumer Surplus under Different Power Structures

		Two-player			Three-player	
CV	Int Market	Domi-M	Domi-R(%)	Domi-R(+)	S-Domi-M	S-Domi-R
Iso-elastic Demand Function: $y(p) = ap^{-b}$						
0.1	168.94	59.32	48.28	47.30	42.65	16.79
0.2	147.45	58.51	41.68	41.63	35.98	14.04
0.3	124.47	57.48	35.52	37.40	31.66	17.58
0.4	103.17	56.13	29.77	32.75	28.03	18.57
Linear Demand Function: $y(p) = a - bp$						
0.1	805.12	192.33	386.93	199.31	212.60	131.81
0.2	792.17	217.74	429.67	218.35	262.85	133.19
0.3	777.67	249.32	470.49	235.49	301.52	156.59
0.4	766.63	279.99	509.18	248.41	337.79	179.10

5.5 Conclusion

We study the effect of power structure, demand function, and markup scheme on supply chain performance, such as channel efficiency and leader's advantages. Beyond a single retailer and a manufacturer problem, we introduced a supply network where two manufacturers offer an identical product to a retailer, but a global manufacturer is used for a regular order and a local manufacturer for shortages under a newsvendor setting.

Our numerical results in the two-player game show that the retailer being a leader leads to higher channel efficiency under linear demand while the manufacturer being a leader brings higher channel efficiency under iso-elastic demand. Especially, under the Domi-reta game, the retailer as a leader always charges a percentage markup (%) in both linear and iso-elastic demand functions as it leads to higher leader's advantages than an absolute markup (+). Further, when demand uncertainty increases, the second mover can achieve higher profit in the two-player game. This is because when the manufacturer is the leader, he reduces the wholesale price significantly to induce the retailer to order more under a high demand uncertainty. Hence, the retailer as the follower benefits from the mitigated double marginalization effect. In contrast, when the retailer is the leader, the retailer increases the selling price due to market demand uncertainty, which consecutively increases the order quantity, too. In this scenario, the manufacturer can achieve a higher profit as a follower by benefiting from the retailer's increased order quantity without significantly reducing his wholesale price in an increasing demand uncertainty.

In the three-player game, channel efficiency is higher when the retailer is the follower, regardless of demand functions. This is because when the global manufacturer's wholesale price decision directly follows the local manufacturer's wholesale price decision, both can influence the retailer's price decision. As such, the two manufacturers experience a more fierce wholesale price competition, resulting in lower wholesale prices. However, when the global manufacturer is the follower, the retailer indirectly serves as a moderator in the wholesale price competition by announcing the selling price before the wholesale price, which essentially increases both wholesale prices and decreases channel efficiency. As demand uncertainty increases, the local manufacturer, as a first-mover of the game, gains a higher profit. At the same time, it is beneficial for the retailer to involve the local manufacturer even though the local manufacturer announces the wholesale price first.

Our study is limited to showing channel efficiency under a simple price-only contact. This motivates the consideration of other contract mechanisms, such as revenue sharing and a contract menu. In our study, the local manufacturer fulfills the retailer's shortages. However, a dual-sourcing option for the retailer in which both manufacturers simultaneously decide on their wholesale prices can derive results. Furthermore, we assume that information is symmetric and commonly known to the players. Therefore, one natural extension is introducing asymmetric demand information from the retailer and investigating the impact of asymmetric information on the manufacturers' decisions and channel efficiency.

Another direction for future research is to compare the current non-cooperative equilibrium solutions to Nash bargaining solutions. For instance, as observed in the numerical analysis, channel efficiency under a particular setting can exceed the follower's disadvantages. Further, the retailer being a follower under the three-player game benefits both the global manufacturer and the retailer, while the local manufacturer prefers the global manufacturer to be the follower to avoid direct wholesale price competition. Based on these observations, investigating the players' negotiation over switching the sequence with the agreement to share the greater surplus generated from the market is worth exploring.

Appendix

Derivation of (5.6)**:** From the equilibrium functions (5.4), we can reformulate the FOC of profit function by substituting p to $p(z)$ such as below.

$$\frac{\partial \mathbb{E}\left[\pi^I(p(z),z)\right]}{\partial z} = y(p(z)) \cdot [1-F(z)] \cdot \left[p(z) - s - \frac{c-s}{1-F(z)}\right] = y(p(z)) \cdot [1-F(z)]\, R(z)$$

$where, R(z) = \left[p(z) - s - \frac{c-s}{1-F(z)}\right]$. It is obvious that $y(p(z)) > 0$ and $[1-F(z)] > 0$ by definition. Then, it needs to be proved that there is at least one z that satisfies $R(z) = 0$, given $z \in [A, B]$. The optimality proof logic of this work follows the analysis introduced by Petruzzi and Dada (1999).

$$R(A) = \left[p(A) - s - \frac{c-s}{1}\right] = p(A) - c \quad where, p > c$$

$$R(B) = \left[p(B) - s - \frac{c-s}{\approx 0}\right] \approx -\infty \quad where, p > c$$

From these upper and lower bound R(z) values, it is clear that there is at least one root exists in the range between A and B. To figure out the general shape of the function and corresponding optimalities, we examine the second-order condition of $R(z)$. For analytical tractability, we define $r(\cdot) = \frac{f(\cdot)}{1-F(\cdot)}$ as hazard rate for probability distribution function (Barlow and Proschan 1975). Then, we can obtain

$$\frac{\partial R(z)}{\partial z} = \frac{\partial p(z)}{\partial z} - \frac{(c-s)r(z)}{1-F(z)} \text{ and } \frac{\partial^2 R(z)}{\partial z^2} = \frac{\partial^2 p(z)}{\partial^2 z} - (c-s)\left[\frac{dr(z)/dz}{1-F(z)} + \frac{r^2(z)}{1-F(z)}\right].$$

To achieve FOC and SOC of $R(z)$, we firstly need to derive FOC and SOC of $p(z)$. From Proposition 5.5, we can obtain FOC and SOC with respect to z.

$$\begin{aligned}\frac{\partial p(z)}{\partial z} &= \frac{(c-s)(-F(z))}{2(\mu-\Theta(z))} - \frac{\Lambda(z)(c-s)(1-F(z))}{2(\mu-\Theta(z))^2} \\ &= \frac{(c-s)\left[F(z)\Theta(z) - F(z)\mu + \Lambda(z) - F(z)\Lambda(z)\right]}{2(\mu-\Theta(z))^2} = \frac{(c-s)\left[F(z)z - \Lambda(z)\right]}{2(\mu-\Theta(z))^2}.\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 p(z)}{\partial z^2} &= \frac{(c-s)zf(z)}{2(\mu-\Theta(z))^2} - \frac{(c-s)\left[\Lambda(z) - zF(z)\right]\left[-1+F(z)\right]}{(\mu-\Theta(z))^3} \\ &= f(z)\left[\frac{(c-s)z}{(\mu-\Theta(z))^2}\right] - \frac{2\left[1-F(z)\right]}{(\mu-\Theta(z))^3} \cdot \frac{dp(z)}{dz} \\ &= r(z)\left[\frac{(c-s)}{(\mu-\Theta(z))}\right] - \left[\frac{2\left[1-F(z)\right]}{(\mu-\Theta(z))} + r(z)\right] \cdot \frac{dp(z)}{dz}.\end{aligned}$$

Lastly, we can insert $\frac{\partial^2 p(z)}{\partial z^2}$ from the above analysis to $\frac{\partial^2 R(z)}{\partial z^2} = \frac{\partial^2 p(z)}{\partial z^2} - (c-s)\left[\frac{dr(z)/dz}{1-F(z)} + \frac{r^2(z)}{1-F(z)}\right]$ and get the following expression:

$$\begin{aligned}\frac{\partial^2 R(z)}{\partial z^2} &= \frac{\partial^2 p(z)}{\partial z^2} - (c-s)\left[\frac{dr(z)/dz}{1-F(z)} + \frac{r^2(z)}{1-F(z)}\right] \\ &= r(z)\left[\frac{(c-s)}{(\mu-\Theta(z))}\right] - \left[\frac{2\left[1-F(z)\right]}{(\mu-\Theta(z))} + r(z)\right]\cdot\frac{dp(z)}{dz} - (c-s)\left[\frac{dr(z)/dz}{1-F(z)} + \frac{r^2(z)}{1-F(z)}\right] \\ &= -(c-s)\left[\frac{dr(z)/dz + 2r^2(z)}{1-F(z)} + \frac{r(z)}{\mu-\Theta(z)}\right] - \left[\frac{2(1-F(z))}{\mu-\Theta(z)} + r(z)\right]\cdot\frac{dR(z)}{dz}.\end{aligned}$$

The second multiplication term, $\left[\frac{2(1-F(z))}{\mu-\Theta(z)} + r(z)\right]\cdot\frac{dR(z)}{dz}$, becomes zero since $\frac{dR(z)}{dz} = 0$. Therefore, if $dr(z)/dz + 2r^2(z) > 0$ then $\frac{\partial^2 R(z)}{\partial z^2}|_{\frac{dR(z)}{dz}=0} < 0$ because of $(c-s) > 0$ and $\frac{r(z)}{\mu-\Theta(z)} > 0$. This means $R(z)$ is concave, containing maximum two roots in the range of z. Note that $R(A) > 0$ and $R(B) < 0$ from the previous analysis. Therefore, we can conclude that $R(z)$ is monotone in $z \in [A, B]$ and has a unique z^* which is the optimal point for the profit function, $\mathbb{E}\left[\pi^I(p(z), z)\right]$. This optimal decision of z^* should suffice $\frac{\partial E\left[\pi^I(p(z^*), z^*)\right]}{\partial z^*} = y(p(z^*))\cdot\left[1-F(z^*)\right]R(z^*) = 0$.

Derivation of (5.11)**:** Based on Lariviere and Porteus (2001), we first show the relationship between price elasticity and IGFR. By definition $\varepsilon(z)$ equals $-\frac{w(z)}{z}\frac{dz}{dw(z)}$ and $g(z)$ equals $\frac{zf(z)}{1-F(z)}$. Under the absolute markup scheme market price p is simply the summation of wholesale price and markup ($p = u + w$). Applying this information, we can rearrange the optimal stocking factor condition as $F(z) = \frac{p-w}{p-w}$ and obtain $w(z) = p(1-F(z)) + sF(z)$. According to the calculus below, we achieve the relationship of $1/\varepsilon(z) + K = g(z)$.

$$1/\varepsilon(z) = -\frac{z}{w(z)}\frac{dw(z)}{dz} \quad \textit{and} \quad \frac{dw(z)}{dz} = -f(z)(p-s)$$

Therefore,

$$1/\varepsilon(z) = \frac{z}{w(z)}\frac{dw(z)}{dz} = \frac{f(z)(p-s)z}{w(z)} \quad \textit{and} \quad g(z) = \frac{zf(z)}{1-F(z)} = \frac{zf(z)(p-s)}{w(z)-s}.$$

As the numerators are the same for both terms we divide them with $zf(z)(p-s)$. The denominator for the reversed price elasticity has $w(z)$ and the IGFR has $w(z)-s$. The deviation between $\frac{1}{w(z)}$ and $\frac{1}{w(z)-s}$ is $\frac{s}{w(w-s)}$ which is referred to K. Therefore, by adding the term K to the left hand side of the equation, we conclude that $1/\varepsilon(z) + K = g(z)$ holds. The explanatory variable for IGFR as well as price elasticity, is z, hence, K is interpreted as a constant employing no effect on FOC and SOC with respect to z. This relationship finds to be useful for the following derivation. To assure the optimal solution of z under the manufacturer's profit function, we need to examine the second-order condition of the manufacturer's profit function with respect to z.

$$\frac{\partial^2 \pi_m^M(w,p,z)}{\partial z^2} = y(p)\left(w'(p,z)\left[1-\frac{1}{\varepsilon(z)}\right] + \frac{\varepsilon'(z)w(p,z)}{\varepsilon(z)^2}\right) < 0 \tag{5.18}$$

It is worthwhile to recognize the relationship, $1/\varepsilon(z) + w'(p,z)\cdot\frac{s}{w(w-s)} = g(z)$, between the modified price elasticity $\varepsilon(z)$ and the generalized failure rate $g(z)$. The mild but versatile property of IGFR greatly facilitates the analysis procedure because this reversed relationship between $\varepsilon(z)$ and $g(z)$ makes $\varepsilon(z)$ a decreasing function in z. The SOC equation shows that the unimodality of profit function of the manufacturer relies on the term $w'(p,z)\left[1-\frac{1}{\varepsilon(z)}\right]$. We can induce $w'(p,z) = -f(z)(p-s) - pF(z)$ from eq. (5.8) and see that $w'(p,z)$ is strictly negative. Based on the assumption $\varepsilon(z) > 1$, the whole term of SOC becomes negative, and the concavity is guaranteed. Therefore, the FOC condition leads to an optimal stocking factor decision for the manufacturer as below:

$$\frac{\partial \pi_m^M(w,p,z)}{\partial z} \equiv w(z)\left[1-\frac{1}{\varepsilon(z)}\right] - c = 0 \Longleftrightarrow w(z^*) = \frac{\varepsilon(z^*)c}{\varepsilon(z^*)-1}.$$

Derivation of (5.12)**:** Based on (5.6) and (5.11), while the retailer's expected profit is jointly concave in z and p (u), there exists an optimal w^* for a given z for the manufacturer. Hence, using the optimal stocking factor condition $F(z) = \frac{p-w}{p-s}$, we obtain the following expressions for the additive and multiplicative markups, respectively:

$$\frac{u}{u+w-s} = F(z) \Leftrightarrow u(z) = (w(z)-s)\frac{F(z)}{1-F(z)} \text{ and}$$

$$\frac{u\cdot w(z)}{w(z)(1+u)-s} = F(z) \Leftrightarrow u(z) = \frac{w(z)-s}{w(z)}\frac{F(z)}{1-F(z)}.$$

Derivation of (5.17)**:** The first-order condition of the local manufacturer's expected profit is:

$$\frac{\partial \mathbb{E}\left[\pi_s\right]}{\partial w_s} = y(p)\left\{\Theta\left(z\left(w_s\right)\right) + \left(w_s - c_s\right)\cdot\frac{\partial\Theta\left(z\left(w_s\right)\right)}{\partial w_s}\right\}.$$

By definition,

$$\Theta\left(z\right) = \int_z^\infty (x-z)f(x)dx.$$

Using the integral parts, as $\Theta\left(z\right) = \int_{-\infty}^{z} F(x)dx - z$

$$\frac{\partial\Theta\left(z\left(w_s\right)\right)}{\partial w_s} = \frac{-\partial z\left(w_s\right)}{\partial w_s}\left\{1-F\left(z\left(w_s\right)\right)\right\}.$$

Further, based on the inverse function theorem, $f(z) = \frac{1}{F'^{-1}(\frac{w_s - w_m}{w_s - s})}$. Accordingly, the first-order condition is rearranged to

$$\frac{\partial \mathbb{E}\left[\pi_s\right]}{\partial w_s} = y(p)\left\{\Theta\left(z\left(w_s\right)\right) - \left(w_s - c_s\right) \cdot \frac{1 - F(z)}{f(z)}\right\} = y(p)\left\{\Theta\left(z\left(w_s\right)\right) - \left(w_s - c_s\right) \cdot k(z)\right\}.$$

where, $k(z) = \frac{1-F(z)}{f(z)}$. From the assumption of IGFR where the failure rate is defined as $r(z) = \frac{f(z)}{1-F(z)}$, we observe $\frac{1}{r(z)} = \frac{1-F(z)}{f(z)} = k(z)$. As $r'(z) > 0$ and $k'(z) > 0$, we obtain the second-order condition as

$$\frac{\partial^2 \mathbb{E}\left[\pi_s\right]}{\partial w_s^2} = y(p)\left\{\Theta^{'}\left(z\left(w_s\right)\right) - \left(w_s - c_s\right) \cdot k^{'}(z)\right\}.$$

Since $\Theta^{'}\left(z\left(w_s\right)\right) < 0$ and $k^{'}(z) > 0$, $\frac{\partial^2 \mathbb{E}[\pi_s]}{\partial w_s^2} < 0$. Hence, the optimal wholesale price that maximizes the local manufacturer's profit suffices the first-order condition equals to 0 as follows:

$$w_s^*(z) = -\frac{\Theta(z^*)}{\partial \Theta(z^*)/\partial w_s} + c.$$

Table 5.18: Summary of Notation

Notation	Description
Superscripts and Subscripts	
I, M, R	Integrated, dominant manufacturer and retailer game, respectively
m, s, r	Global manufacturer, local manufacturer, retailer, respectively
$+$, $\%$	Absolute market and percentage market of retailer, respectively
Parameters	
s	Unit salvage cost
c, c_m, c_s	Unit production cost, global manufacturer's cost, local manufacturer's cost in three-player game, respectively
$\widetilde{x}$	Random variable $\widetilde{x} \in [A, B]$
$f(\widetilde{x}), F(\widetilde{x}), \overline{F}(\widetilde{x})$	Probability density, cumulative, and complementary distribution functions of $\widetilde{x}$
$y(p)$	Price dependent demand function where, $y(p) = a - bp$ or $y(p) = ap^{-b}$
$D(p, \widetilde{x})$	Stochastic demand $D(p, \widetilde{x}) = y(p) \cdot \widetilde{x}$
Decision Variables	
u_t	Retailer's markup under scheme t where, $t \in \{+, \%\}$
w, w_m, w_s	Manufacturer price in a two-player game, global manufacturer's price, local manufacturer's price in three-player game, respectively
p	Market price where, $p = w + u$ or $p = w \cdot (1 + u)$
q	Order quantity of retailer
Functions	
π_j^i	Profit function of j in setting i where $i \in \{I, M, R\}$ and $j \in \{m, s, r\}$
Definitions	
z	Stocking factor $z = q/y(p)$
$\varepsilon(z)$	Price elasticity $\varepsilon(z) = -w(z,p)/\left[z \partial w(z,p)/\partial z\right]$
$g(x)$	Generalized increasing failure rate $g(x) = x \cdot f(x)/[1 - F(x)]$
CE^i	Channel Efficiency in setting i $CE^i = \pi_r^i + \pi_m^i/\pi^I$ where $i \in \{I, M, R\}$
LA_1^i	Type 1 leader's advantage in setting i $LA_1^i = \pi_r^R/\pi_r^M$ or π_m^M/π_m^R
LA_2^i	Type 2 leader's advantage in setting i $LA_2^i = \pi_r^R/\pi_m^R$ or π_m^M/π_r^M

Chapter 6.
Conclusions

6.1 Summary

This thesis studies different aspects of information sharing and pricing decisions within the framework of game theory. In the first two chapters (Chapter 3 and 4), we explore the intrinsic incentive of supply chain members to voluntarily share their private information, employing both analytical and numerical methods. Moving on to Chapter 5, where we assume symmetrical information, our attention shifts to examining how the power structure within the supply chain and markup scheme influence channel efficiency and confers advantages to the leader under demand uncertainty.

In Chapter 3, we propose a Bayesian Nash equilibrium game where each player makes information sharing decision in anticipation of the other player's sharing decision. We extend a conventional unilateral asymmetric information game to a bilateral information asymmetry game under signal-based Bayesian updating.

RQ 1.1) If a self-interested supplier and a retailer decide whether to reveal their capacity and demand information, respectively, what is an optimal revelation decision knowing that the other also decides whether to share or not?
The demand and capacity information revelation rules in our analysis are as follows. Under the benchmark case for joint sharing decisions, the players reveal their information to alert anticipated mismatches in demand or capacity. Hence, private information is revealed if the locally observed value is significantly low or high. In the decentralized setting, a supplier exhibits a reversed information-sharing rule; he shares capacity information if the capacity level is intermediate. This is because, under decentralized sharing, the supplier with a high capacity knows that the retailer receiving the capacity information is less likely to share demand reciprocally, which

leads to a high overage cost. On the other hand, if the supplier has a small capacity, equivalent to a high capacity investment cost, a relatively low profit margin does not compensate for the information-sharing cost. Therefore, in both cases, the supplier withholds capacity information. Further, a retailer in the decentralized setting discloses demand information when it is above a certain threshold.

RQ 1.2) How does a player's risk aversion impact the information sharing decisions of supply chain members?
If a retailer is risk-averse, she is more likely to share demand information. In contrast, if a supplier is risk-averse, he can be more reluctant to share capacity information. The main reason is that the risk-averse retailer's demand sharing eliminates information asymmetry (i.e., demand and capacity) in the supply chain regardless of the supplier's sharing decision. However, the risk-averse supplier's capacity information sharing cannot eliminate the demand uncertainty the supplier faces. Hence, when the supplier has a low expected utility, the marginal utility of incurring the information sharing cost is greater than the expected utility gain from the retailer's reciprocal sharing.

In this chapter, we further showed the impact of variability of demand and capacity prior distributions on the players' sharing decisions. As the variability of the prior demand distribution increases, a company is more likely to explore both demand and capacity information. Furthermore, a high demand variability makes the locally informed retailer and supplier more likely to reveal their information in a decentralized setting. Hence, companies are more likely to exchange private demand and capacity information in a highly volatile market. On the other hand, with high variability in capacity information, the company is more likely to explore only capacity information. However, in a decentralized supply chain where a retailer has a highly variable prior belief in the supplier's cost, the supplier is less likely to reveal capacity information, while the retailer is more likely to reveal demand information.

In Chapter 4, we study the mutual information exchange problem between a platform and a seller. Under a sequential game with asymmetric demand information, the platform sets an optimal commission rate, and the seller decides on a market price.

RQ 2.1) When the commission rate is exogenously fixed, would sharing the platform's private signal with the seller unilaterally be mutually beneficial?
If the platform predefines the commission rate, it is beneficial for the platform to share private signals on market uncertainty with the seller. However, when the intrinsic market uncertainty is high, revealing a precise signal to the seller reduces the platform's gain from information sharing as the seller's price decision becomes too restrictive to the platform's signal.

RQ 2.2) When the commission rate is endogenously determined, would exchanging private signals bilaterally between the platform and the seller be mutually beneficial?
When the platform's information precision is relatively lower than that of the seller, it is more advantageous for the platform to share information, whereas the seller does not reap any benefits from this information exchange. This is because the platform benefits by being able to set a more advantageous commission rate, thanks to the enhanced accuracy in demand forecasting facilitated by the information shared by the seller. Simultaneously, the platform gains the ability to observe the seller's exact price reaction function, eliminating information asymmetry.

In addition, we explore the influence of demand sizes in the two channels, which are tied to the seller's market power, on the decisions regarding information exchange. Notably, when online demand is sufficiently large, mutual information exchange proves advantageous for both players. Further, if the demands of the seller's two sales channels (online and direct sales) substantially diverge, the seller always benefits from exchanging information with the platform. This is due to the fact that when one market demand prevails over the other, the platform's commission rate is primarily determined by the base demands from both channels. As a result, it becomes less sensitive to the signals exchanged. Hence, the seller's concern about the platform exploiting her information to charge a higher commission rate under information exchange reduces while this exchange of information contributes to a reduction in market uncertainty.

Lastly, in Chapter 5, our primary focus was on examining the pricing decisions made by both a manufacturer and a retailer, and we subsequently expanded the model to encompass a network consisting of two manufacturers and a retailer. Within this framework, our investigation delved into the influence of various power structures within a supply chain or network on channel efficiency. We also explored how these dynamics impact the advantages of the leader, considering different demand functions and markup schemes.

RQ 3.1) Does a sequence of the game (power structure) lead to different channel efficiencies and leader's advantages under stochastic price-dependent demand functions?
When a supply chain consists of a retailer and a manufacturer, the retailer being a leader leads to a higher channel efficiency than the manufacturer being a leader under linear demand. In contrast, under the iso-elastic demand, the manufacturer being a leader brings a higher channel efficiency. More interestingly, with the involvement of a local manufacturer as a first-mover (three-player) in both demand functions, the retailer being a follower shows the highest channel efficiency as letting the two manufacturers directly compete over wholesale prices brings higher efficiency to the supply network. Further, regardless of the demand function, the retailer always prefers to charge a percentage markup to an absolute markup.

RQ 3.2) Does a high demand uncertainty always reduce the players' expected profits in a supply chain or supply network?
When the demand uncertainty is high, it generally harms the players' profits. However, under a linear demand function in the two-player game, a follower observes profit increase as the market uncertainty increases. This stems from the fact that when the manufacturer is the leader, he reduces the wholesale price significantly to induce the retailer to order more under a high demand uncertainty. Hence, the retailer as the follower benefits from the mitigated double marginalization effect. In contrast, when the retailer is the leader, the retailer increases the selling price due to market demand uncertainty, which consecutively increases the order quantity, too. In this scenario, the manufacturer can achieve a higher profit as a follower by benefiting from the retailer's increased order quantity without significantly reducing his wholesale price in an increasing demand uncertainty.

Further, in the three-player game, elevated demand uncertainty amplifies the local manufacturer's profitability as the first-mover. If the demand follows an iso-elastic function, and the supply network faces substantial demand uncertainty, the retailer lowers the order quantity. Consequently, the shortage that the local manufacturer fulfills increases. In contrast, when the demand is linear, even though the order quantity increases in demand uncertainty, the local manufacturer raises its wholesale price as the demand uncertainty surges. In both cases, therefore, demand uncertainty leads to a higher profit for the local manufacturer.

6.2 Limitations and future research

The stylized analytical models presented in this dissertation possess several limitations that warrant further exploration in the future. Regarding the information sharing problems in Chapter 3 and Chapter 4, we assume truthful information sharing whenever the players decide to share their information. While our research primarily concentrates on voluntary information sharing among players, it could be worthwhile in the future to develop sharing mechanisms that do not rely on the presumption of truthful information revelation. Such mechanisms may optimize the transfer payments for information-sharing costs or distribution of the surplus generated by information sharing to incentivize the disclosing party to provide accurate information.

In this context, the integration of the bilateral information exchange problem into the framework of signaling or screening games under mechanism design (see e.g., Cachon and Lariviere 2001, Özer and Wei 2006) offers a promising avenue for future research. Especially, in Chapter 3, we assume that each player possesses perfect information on demand and capacity. Therefore, future research could investigate sharing strategies for imperfect and noisy information on demand and capacity, considering the information as private signals.

Additionally, all of the game theoretic models developed in our work consider a single-period decision. Incorporating multi-period games and implementing Bayesian learning across these periods could capture the dynamics of decision-making more comprehensively and potentially yield more nuanced insights into information-sharing choices. Also, our study primarily focuses on pricing decisions within the context of a simple wholesale price-only contract. Given the existence of various elaborate contract mechanisms, an interesting line for future research is to investigate whether the information sharing decision differs on different contract schemes and power structures in supply chains (see e.g., Lau et al. 2008, Raju and Zhang 2005).

In Chapter 4, we examined a seller who operates both a direct sales channel and an online channel. Within the scope of our research, to thoroughly focus on the information exchange decisions, our model did not account for competitive effects across different channels, assuming that the seller, assuming that the seller offers the same price in both the direct and online sales channels. Nevertheless, in practice, consumer preferences and satisfaction can vary across channels, potentially giving rise to competitive effect (see e.g., Shen et al. 2019b, Berbeglia et al. 2022, Feldman et al. 2022). While the investigation of choice models in the context of multi-channel retailing was not within the scope of our study, it is an area of growing interest and merits consideration when exploring topics related to information sharing.

Last, in Chapter 5, as incorporating uncertainty into a multi-player Stackelberg game adds complexity to the analysis, obtaining closed-form solutions becomes a challenging task. Consequently, we conducted an extensive numerical analysis to gain insights into the implications of power structure within the game. Nevertheless, delving into the analytical properties of the Stackelberg game under uncertainty holds promise for future research in supply networks (see e.g., Majumder and Srinivasan 2008). Another intriguing avenue for future investigation is to compare the current equilibrium solutions, which are non-cooperative, with the Nash bargaining solution (see e.g., Feng et al. 2022). As our numerical analysis revealed, in specific conditions, the increase in channel efficiency can surpass the disadvantages faced by the follower. In such instances, the players may engage in negotiations to alter the sequence, with an agreement to share the greater surplus generated from the market.

Despite the aforementioned limitations, the managerial implications derived from the conceptual game theoretic models presented in this thesis offer valuable insights for the development of decision frameworks for information sharing. Also, the Stackelberg model under different settings enhances our comprehension of the influence of power structure and demand functions on supply chain performance. The majority of information sharing problems are, as of yet, mainly tackling unilateral sharing, while in practice, companies often give little focus to quantifying the benefit of information revelation to their partners. Finally, there are numerous unexplored applications within the framework of game theory that hold great potential for future research endeavors, in particular, to assist decision-makers in making strategic decisions while navigating the complexities of both vertical and horizontal competitions.

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