Parameterizations for bead-like regular shapes in node-based shape optimization

Bastian Devresse*, David Schmölz*, Armin Geiser*,

Kai-Uwe Bletzinger*

* Chair of Structural Analysis,

Technical University of Munich

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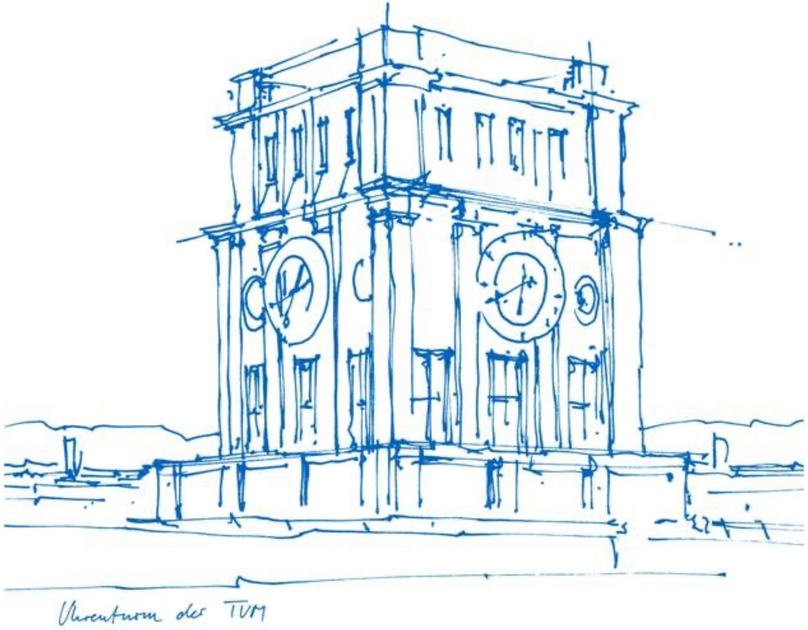
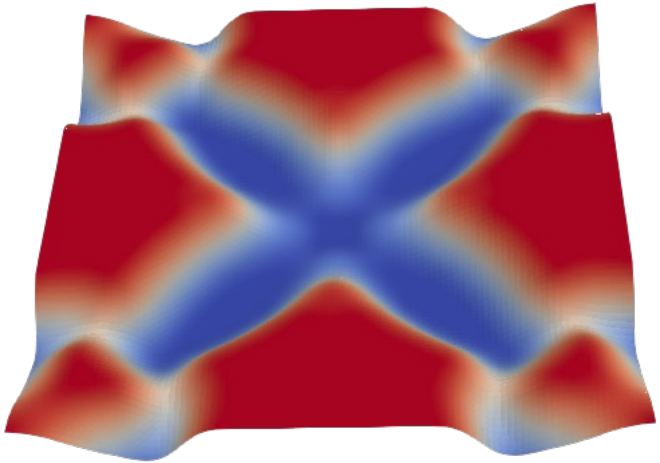


Outline

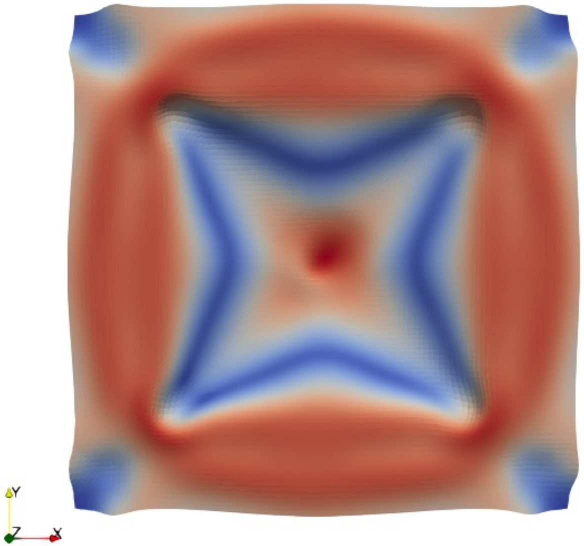
1. Introduction
2. Vertex Morphing
3. Bead Parameterization
4. Enhancement to variable bead heights
5. Enable full bead formation
6. Conclusion / Outlook



Introduction

Free-form node-based parameterization techniques:

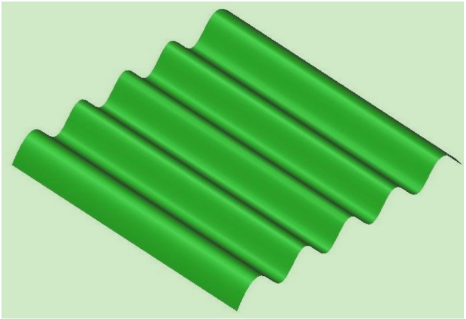
- Large freedom of optimal design
- Identification of patterns and interpretation of final shapes not straight forward



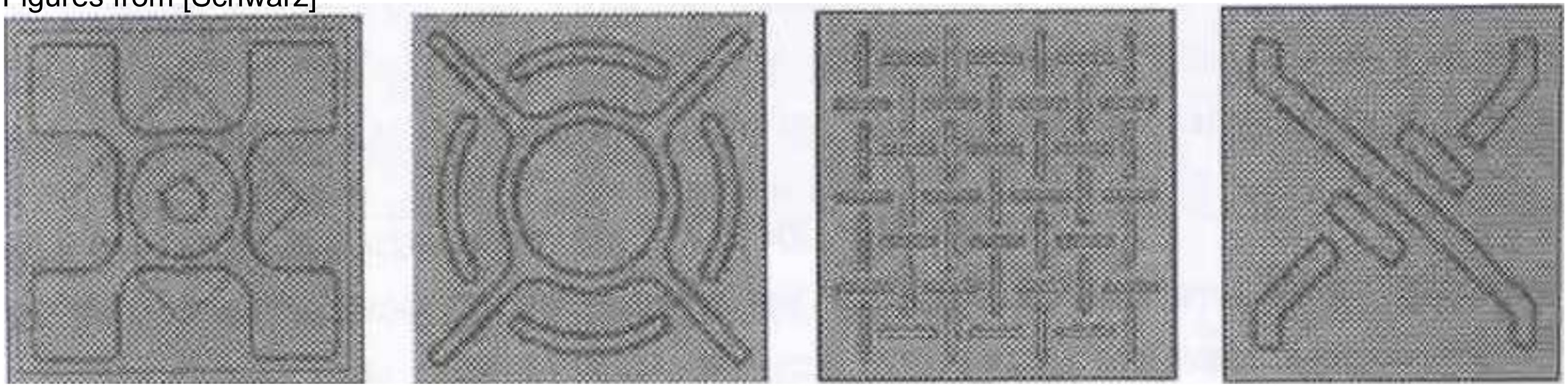
Introduction

- Bead shapes are preferred
- Patterns can be identified, interpreted

Figure from [Daoud]



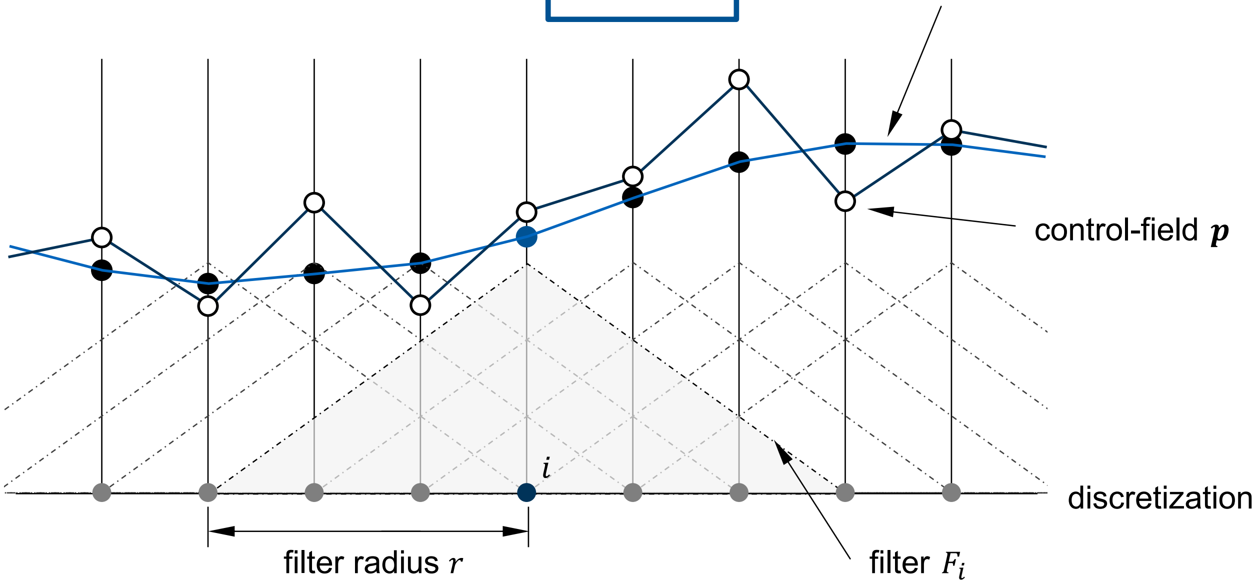
Figures from [Schwarz]



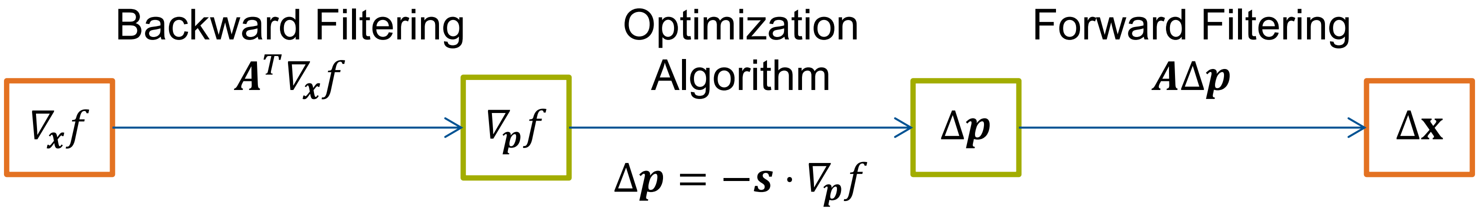
Vertex Morphing

$$x_i = \int F(\xi, \xi_i, r) p \, d\Gamma = A_{ij} \cdot p_j$$

$$\boldsymbol{x} = \boldsymbol{A}\boldsymbol{p}$$



Vertex Morphing

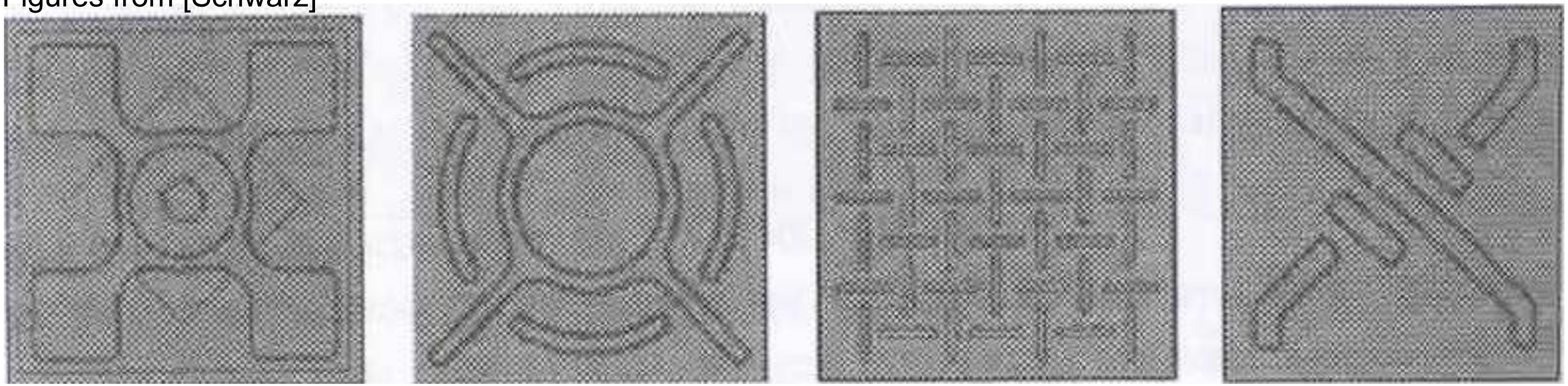


Detailed study: [Hojjat] [Bletzinger]

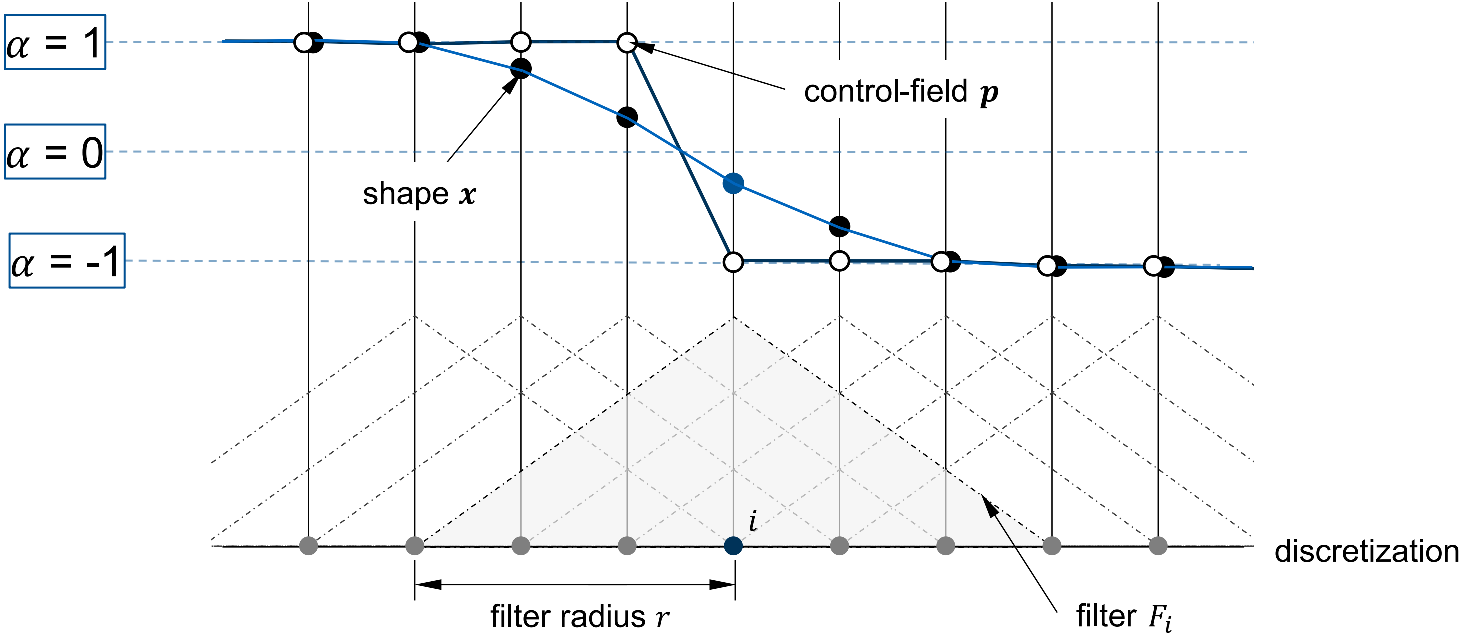
Bead Parameterization

- Start from initial flat plate
- Updates only in defined „bead direction“
- Bead parameter α goes from -1 (lower bound) to 1 (upper bound)

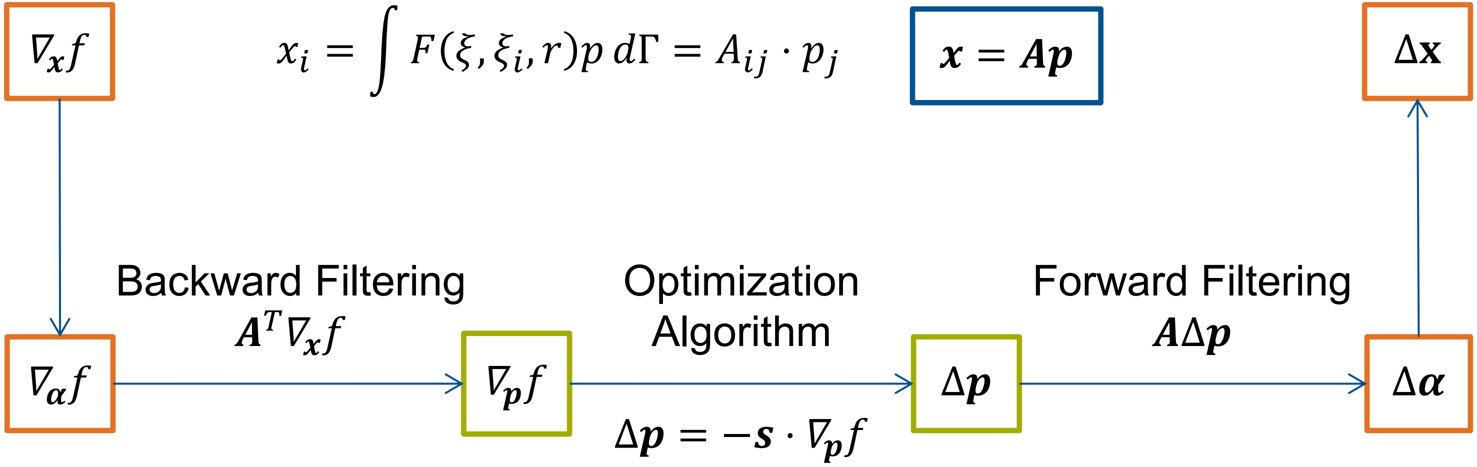
Figures from [Schwarz]



Bead Parameterization



Bead Parameterization

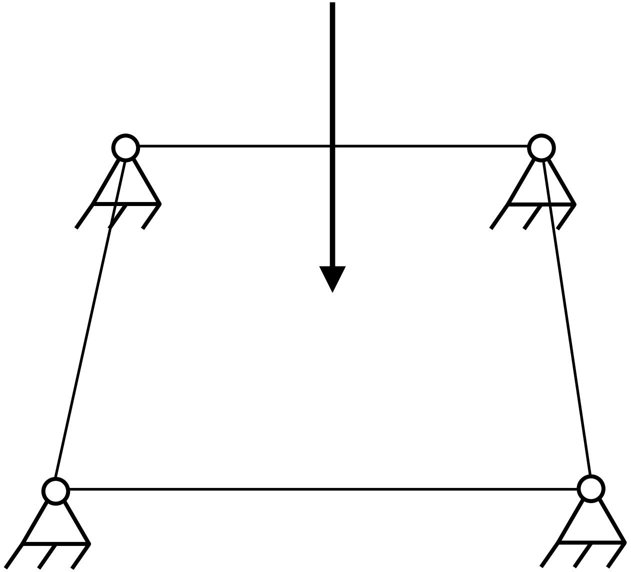


Penalty term forces control values to go to either +1 or -1:

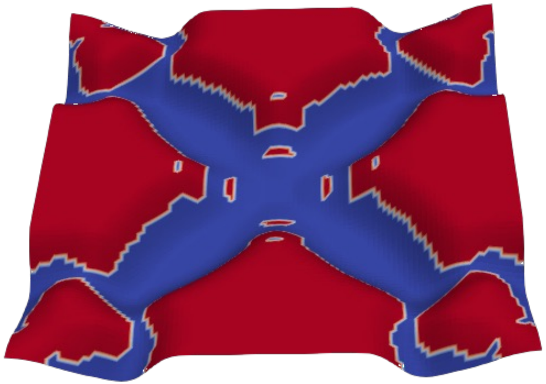
$\sigma = -p^2 + 1$

Bead Parameterization

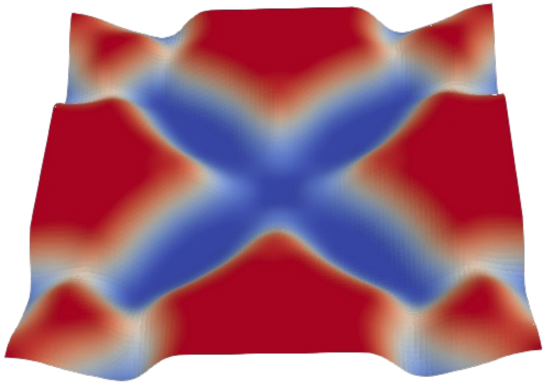
Initially flat plate (α=0, p=0)



Discrete field in control space (p=±1)

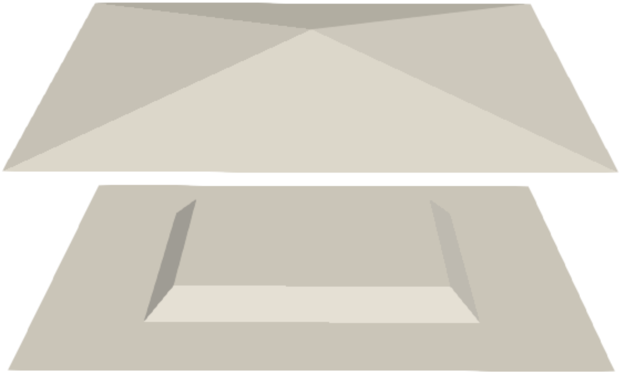


Filtered field (α)



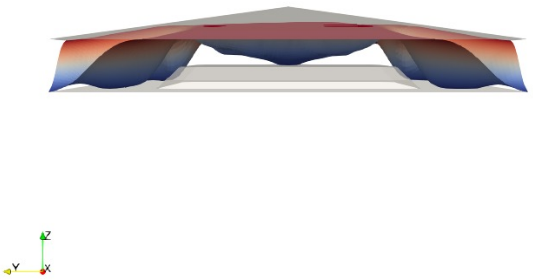
Variable Bead Height

- Starting from initial flat plate
- Using bounding geometries

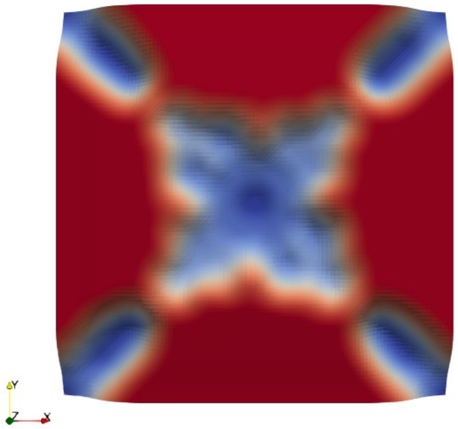


Variable Bead Height

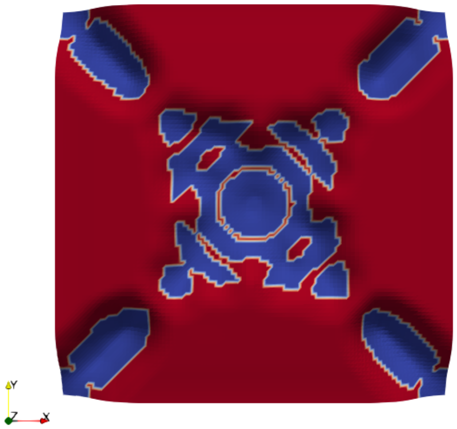
Filtered alpha field side view



Filtered field top view (α)

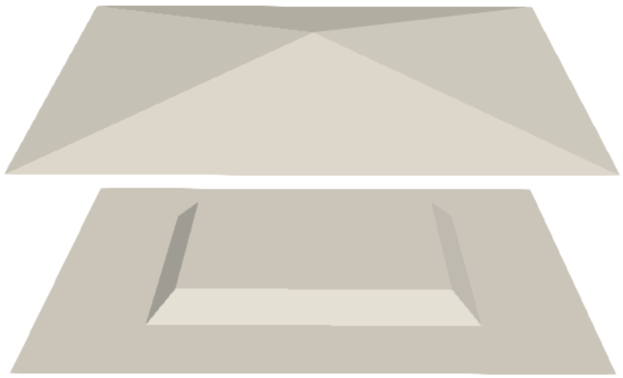


Discrete field in control space (p=±1)

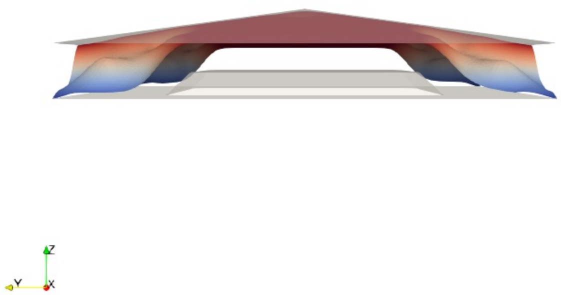


Variable Bead Height

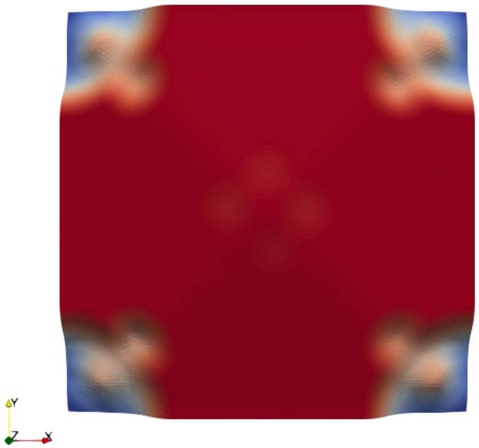
Starting from an initially non-flat geometry



Variable Bead Height



Filtered field top view (α)



Discrete field in control space (p=±1)

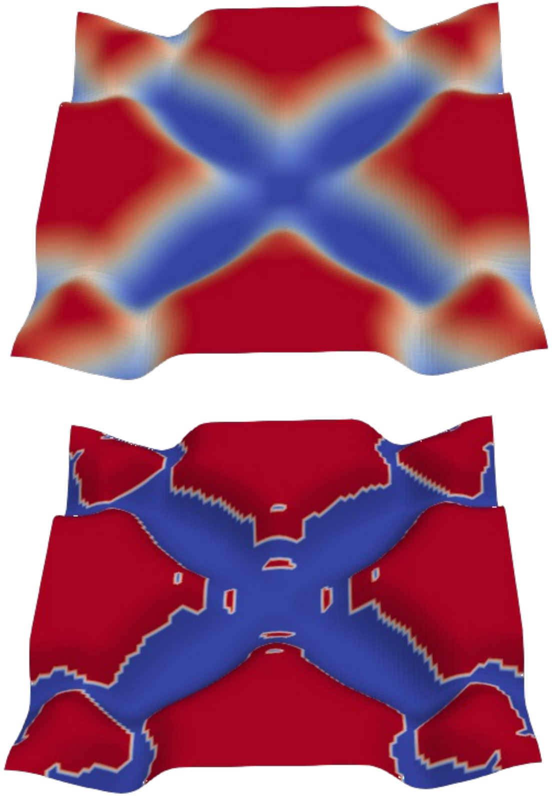
First Results

Bead patterns have been realized with

- Vertex Morphing
- Parameterization with bead parameter α
- Penalty
- Variable bead heights
- Initially curved geometry

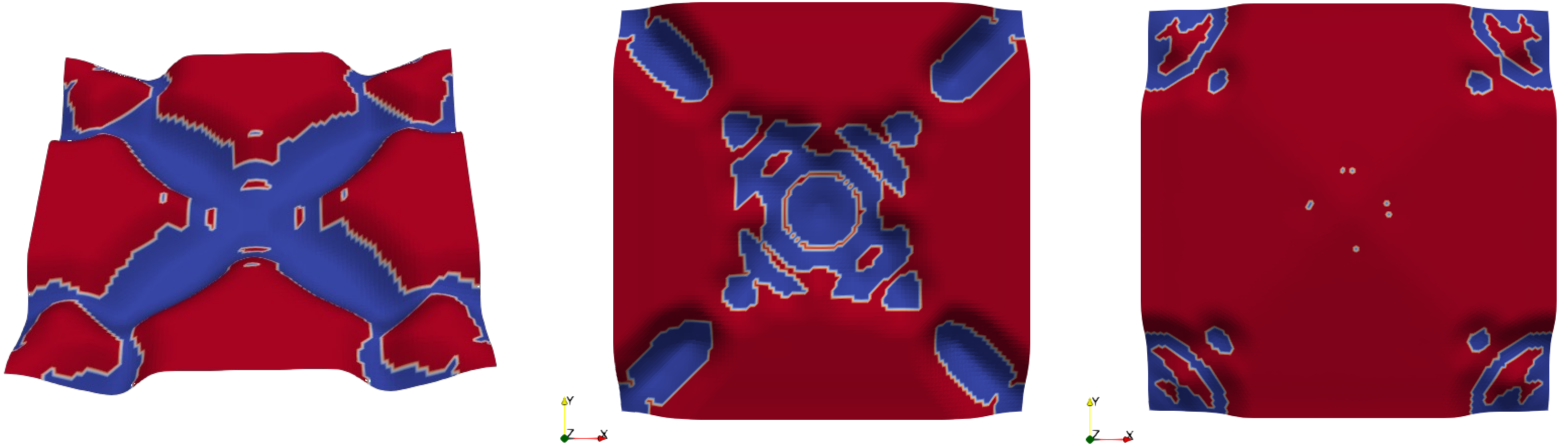
Problems:

- Avoid small bead “islands“
- Enable fully formed beads



Minimum Bead Member Size

Goal: avoid small islands in the discrete field and enable full beads

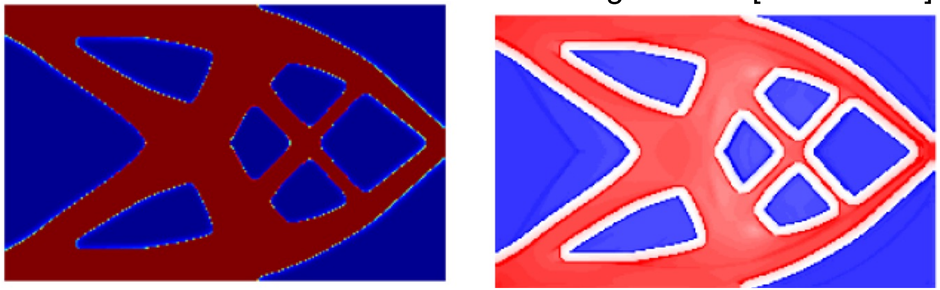


Minimum Bead Member Size – Literature

Approaches by [Guest] and [Carstensen] for minimum length scale control in topology optimization.

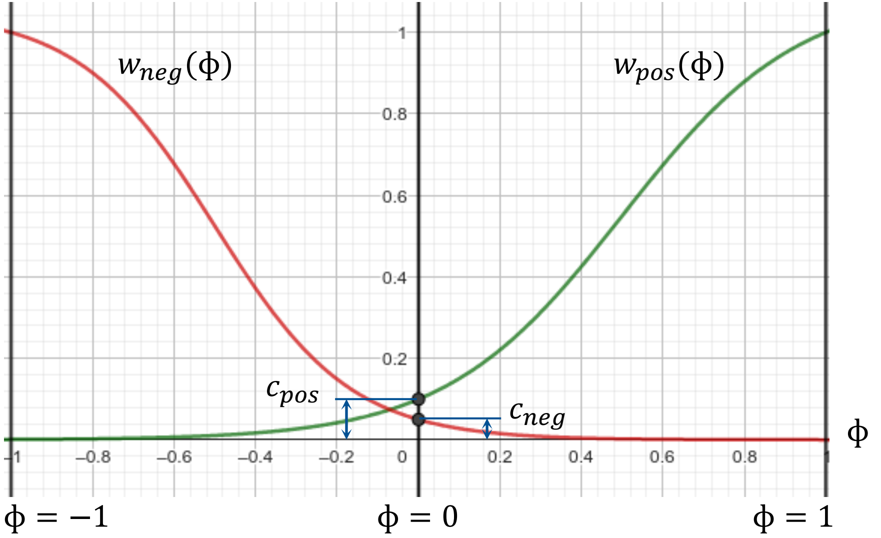
- Split up design variables in two phases: solid and void
- Apply (non-)linear weighting in every node's neighbourhood
- Push weighted values to ±1 with regularized heaviside function
- Bring phases back together for final result

Figure from [Carstensen]



Minimum Bead Member Size

- New design variables ϕ
- Filtered and split up in two phases:
  - *Positive* phase for upper bound $\mu_{pos}(\phi)$
  - *Negative* phase for lower bound $\mu_{neg}(\phi)$
  - Parameter: value c at $\phi = 0$ for hyperbolic tangent function $(c = c_{pos} = c_{neg})$
- Bring together by $\mu(\phi) = \mu_{pos}(\phi) - \mu_{neg}(\phi)$



$$\mu_p^e = \frac{\sum_{i \in N_p^e} w(\mathbf{x}_i - \bar{\mathbf{x}}^e) \cdot w_p(\phi_i)}{\sum_{i \in N_p^e} w(\mathbf{x}_i - \bar{\mathbf{x}}^e)}$$

Formula from [Carstensen]

Minimum Bead Member Size

Hyperbolic tangent function

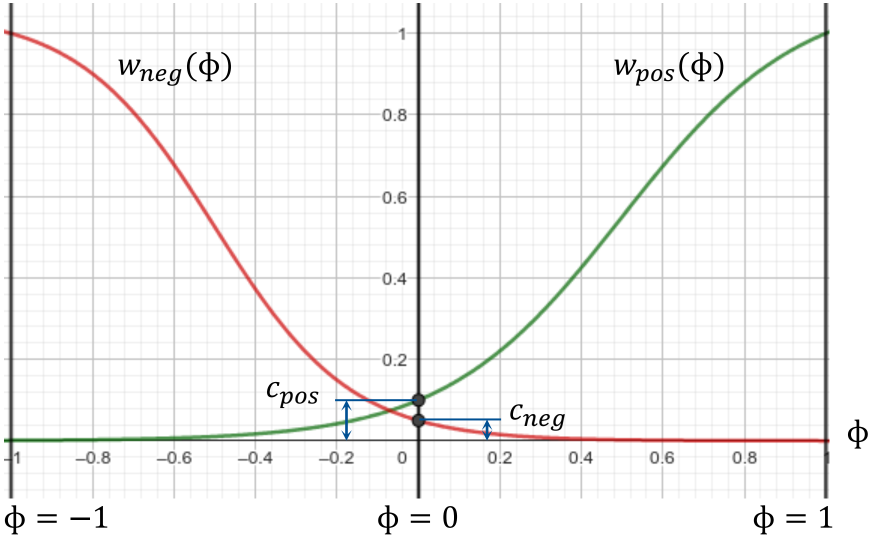
$$w_s(\phi) = \frac{1+\alpha_s}{1+\alpha_s \cdot e^{2n_s(\phi_{max}-\phi)}},$$

$$w_v(\phi) = \frac{1+\alpha_v}{1+\alpha_v \cdot e^{2n_v(\phi-\phi_{min})}},$$

where $\phi_{range} = \phi_{max} - \phi_{min}$ is used to evaluate n_p:

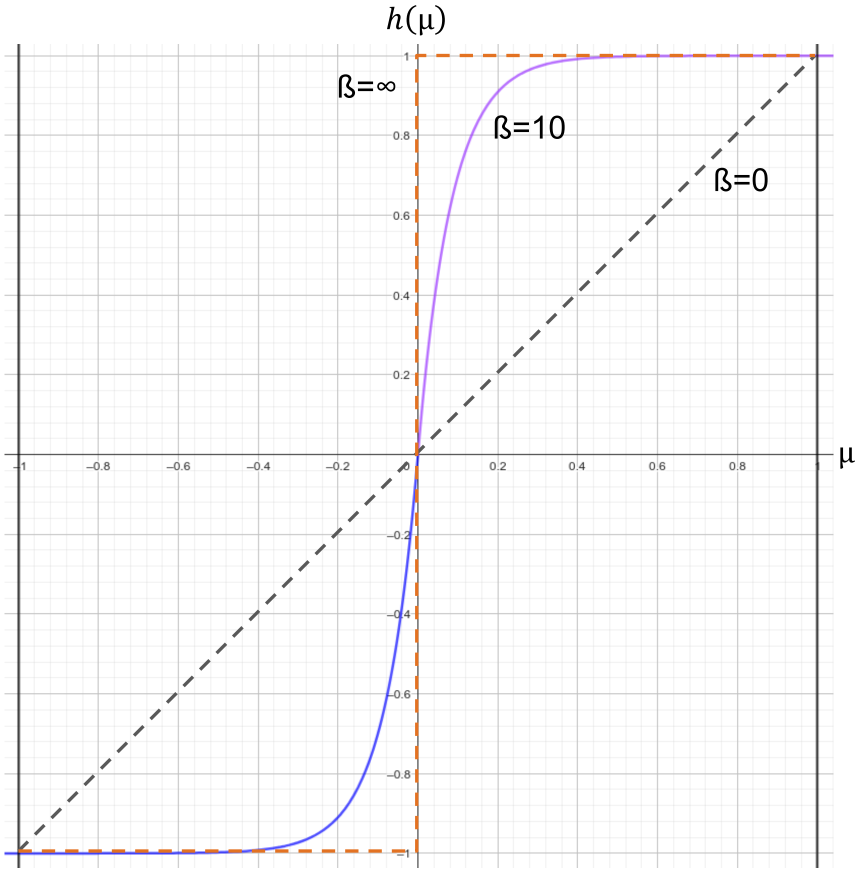
$$n_p = -\frac{2\ln(\alpha_p)}{\phi_{range}}.$$

Formulae from [Carstensen]



Minimum Bead Member Size

- New design variables ϕ
- Filtered and split up in two phases:
  - *Positive* phase for upper bound $\mu_{pos}(\phi)$
  - *Negative* phase for lower bound $\mu_{neg}(\phi)$
  - Parameter: value c at $\phi = 0$ for hyperbolic tangent function $(c = c_{pos} = c_{neg})$
- Bring together by $\mu(\phi) = \mu_{pos}(\phi) - \mu_{neg}(\phi)$
- Pushed to ± 1 by regularized heaviside function: $h(\mu) = 1 - (e^{-ß\mu} - \mu e^{-ß})$
  - Parameter ß is the regularizor for $h(\mu)$
  - ß is increased similarly to penalty
- $p = h(\mu)$



Minimum Bead Member Size

$$\mu_p^e = \frac{\sum_{i \in N_p^e} w(\mathbf{x}_i - \bar{\mathbf{x}}^e) \cdot w_p(\phi_i)}{\sum_{i \in N_p^e} w(\mathbf{x}_i - \bar{\mathbf{x}}^e)},$$

$$w(\mathbf{x}_i - \bar{\mathbf{x}}^e) = \begin{cases} \frac{r_{min,p} - ||\mathbf{x}_i - \bar{\mathbf{x}}^e||}{r_{min,p}} & \text{if } \mathbf{x}_i \in N_p^e \\ 0 & \text{otherwise,} \end{cases}$$

$$w_s(\phi) = \frac{1 + \alpha_s}{1 + \alpha_s \cdot e^{2n_s(\phi_{max} - \phi)}},$$

$$w_v(\phi) = \frac{1 + \alpha_v}{1 + \alpha_v \cdot e^{2n_v(\phi - \phi_{min})}},$$

where $\phi_{range} = \phi_{max} - \phi_{min}$ is used to evaluate n_p:

$$n_p = -\frac{2\ln(\alpha_p)}{\phi_{range}}.$$

Minimum Bead Member Size

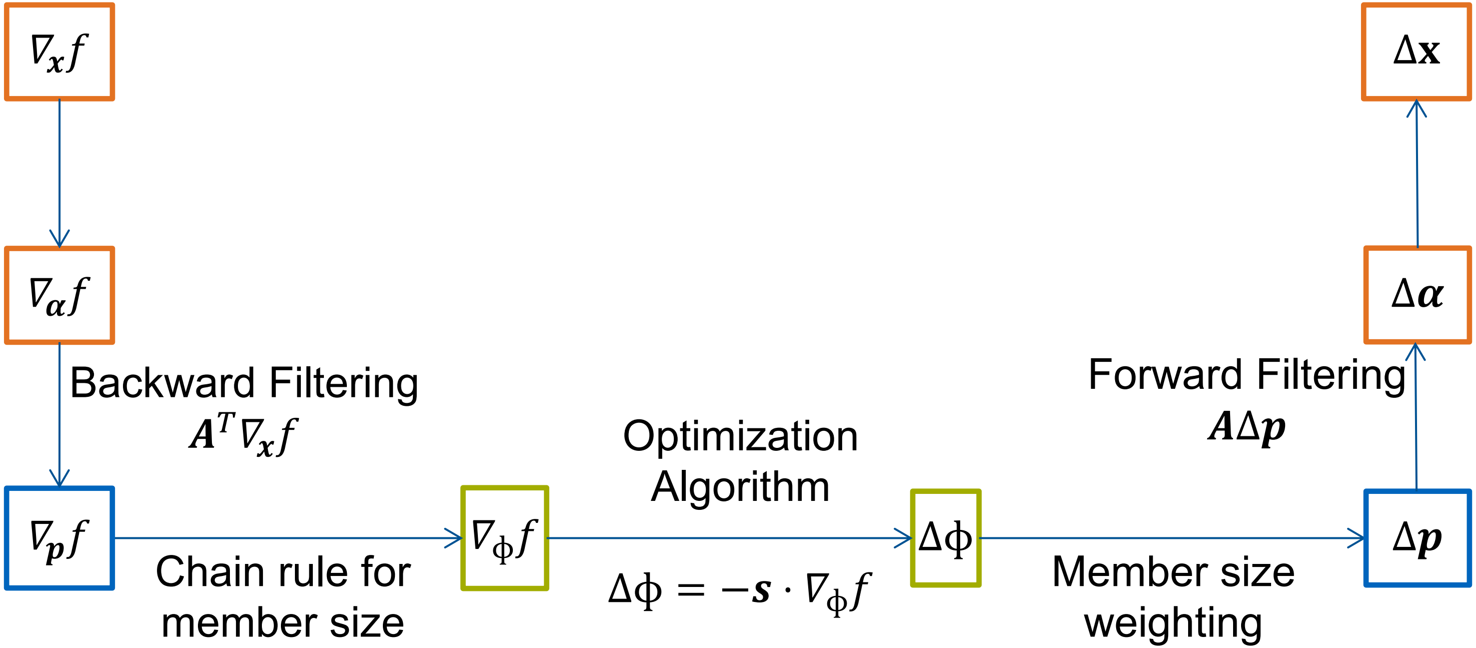
- Chain rule for sensitivities:
  - $$\frac{\partial f}{\partial \phi_i} = \sum_{e \in N} \underbrace{\frac{\partial f}{\partial p}}_{\text{known}} \frac{\partial p}{\partial \phi_i} = \sum_{e \in N} \frac{\partial f}{\partial p} \underbrace{\frac{\partial p}{\partial \mu} \frac{\partial \mu}{\partial \phi_i}}_{\text{new contribution}}$$
  - $$\frac{\partial p}{\partial \mu} = \frac{\partial h}{\partial \mu} = ßе^{-ß\mu} + e^{-ß}$$
  - $$\frac{\partial \mu}{\partial \phi} = \frac{\partial w_{pos}}{\partial \phi} - \frac{\partial w_{neg}}{\partial \phi}$$

$$\frac{\partial w_{pos}}{\partial \phi_i} = \frac{1 + \alpha_s}{(1 + \alpha_s \cdot e^{2n_s(\phi_{max} - \phi_i)})^2} \cdot 2\, \alpha_s n_s e^{2n_s(\phi_{max} - \phi_i)}$$

$$\frac{\partial w_{neg}}{\partial \phi_i} = \frac{1 + \alpha_v}{(1 + \alpha_v \cdot e^{2n_v(\phi_i - \phi_{min})})^2} \cdot (-2)\alpha_v n_v e^{2n_s(\phi_i - \phi_{min})}$$

Formulae from [Carstensen]

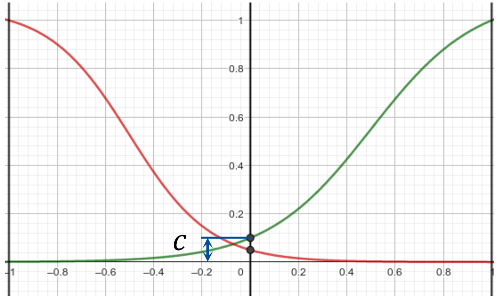
Minimum Bead Member Size



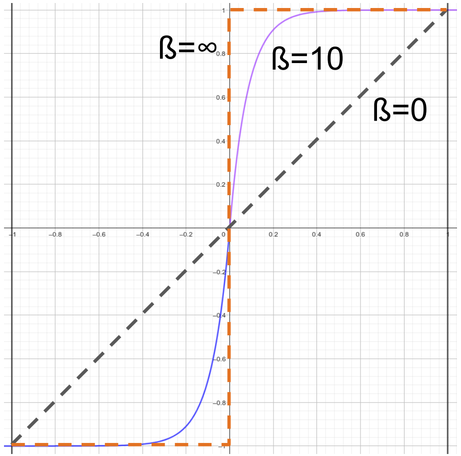
Minimum Bead Member Size

Parameters of the problem:

- Nonlinear weighting in μ is defined by parameter c

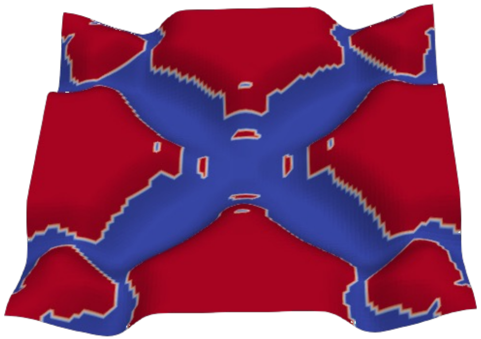


- Regularizor for the heaviside function ß
  (increases similar to the penalty previously)

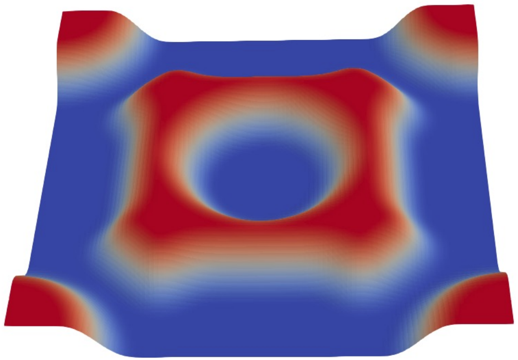


Minimum Bead Member Size – Results

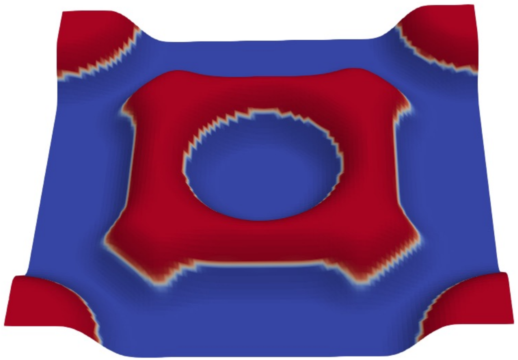
- $c = 0.01$
- Increase of ß : ß $*= 1.1$



Filtered field top view (α)

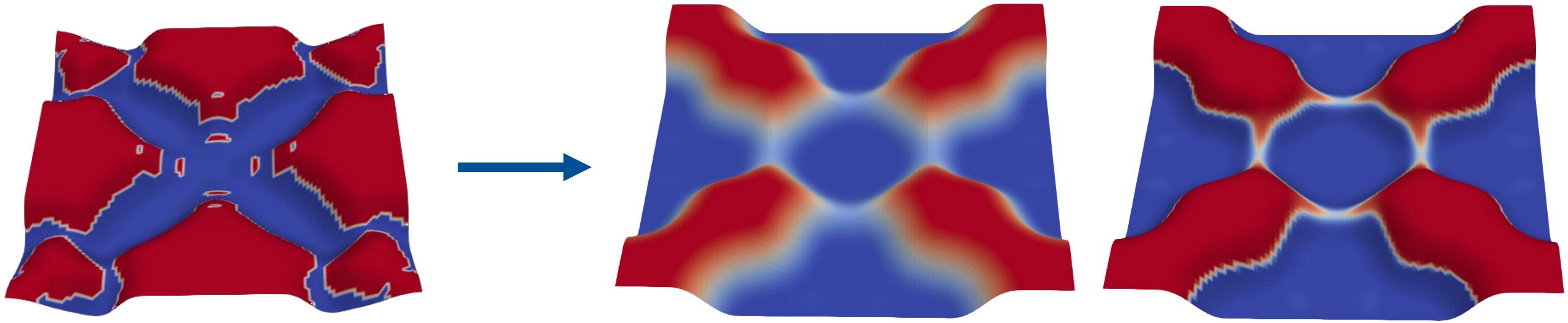


Discrete field in control space (p=±1)



Minimum Bead Member Size – Results

- $c = 0.001$
- Increase of ß : ß $*= 1.1$



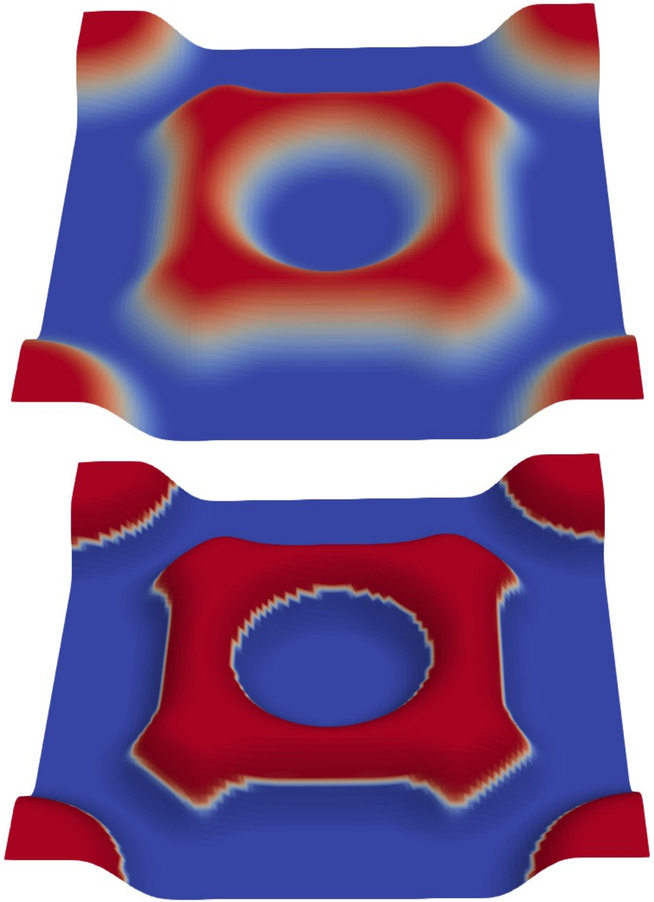
Conclusions / Outlook

Bead patterns have been realized with

- Vertex Morphing
- Parameterization with bead parameter α
- Penalty
- Variable bead heights
- Initially curved geometry
- Fully formed beads ✓

Outlook:

- Study influence of member size parameters
- Create feature based beads



References

1. M. Hojjat, E. Stavropoulou, K.-U. Bletzinger: *The Vertex Morphing method for node-based shape optimization*, Computer Methods in Applied Mechanics and Engineering, 268:494-513, 2014.
2. K.-U. Bletzinger, Kai-Uwe: *A consistent frame for sensitivity filtering and the vertex assigned morphing of optimal shape*, Structural and Multidisciplinary Optimization, 49:873-895, 2014.
3. F. Daoud: *Formoptimierung von Freiformschalen: Mathematische Algorithmen und Filtertechniken*. Shaker, 2005.
4. D. Schwarz: *Gestaltung optimierter Sickenbilder für flächige Strukturen unter Einsatz numerischer Optimierungsverfahren*. Doktorarbeit, Institut, für Kraftfahrwesen Aachen, 2003.
5. Guest, James K. "Topology optimization with multiple phase projection." *Computer Methods in Applied Mechanics and Engineering* 199, no. 1-4 (2009): 123-135.
6. Carstensen, J.V., Guest, J.K. Projection-based two-phase minimum and maximum length scale control in topology optimization. *Struct Multidisc Optim* **58**, 1845–1860 (2018).

Thank you for your attention!

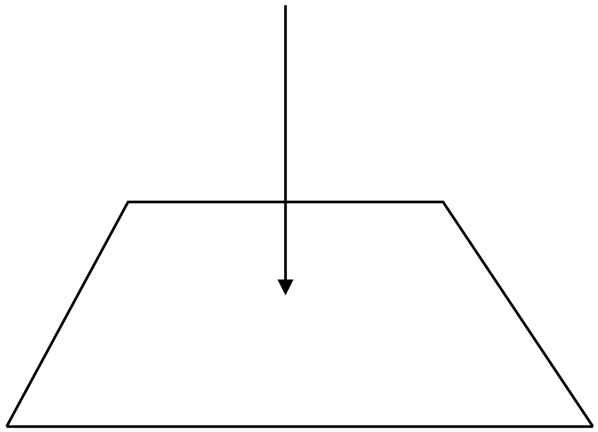
Bead Parameterization Approach

Example:

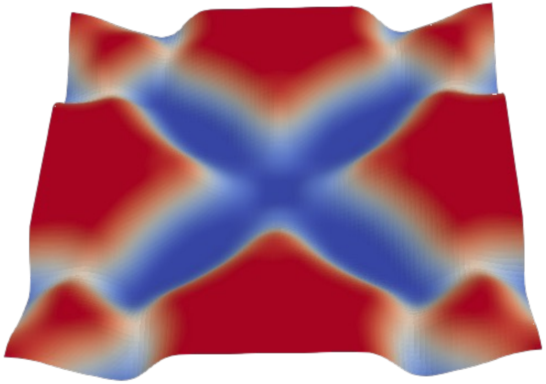
Initial design: flat plate

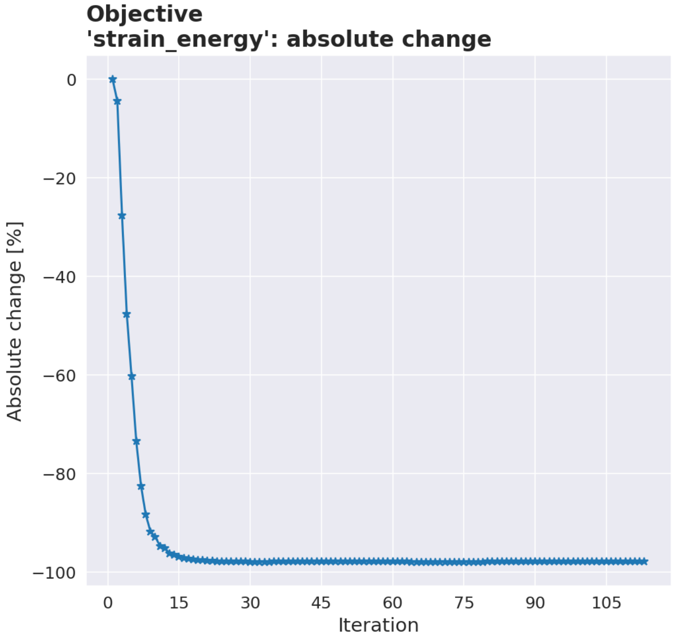
- Dimensions: 100 x 100
- Bead height: 5
- Filter radius: 7.5
- Thickness: 1
- Bead direction: vertical

Minimize compliance (no constraint)

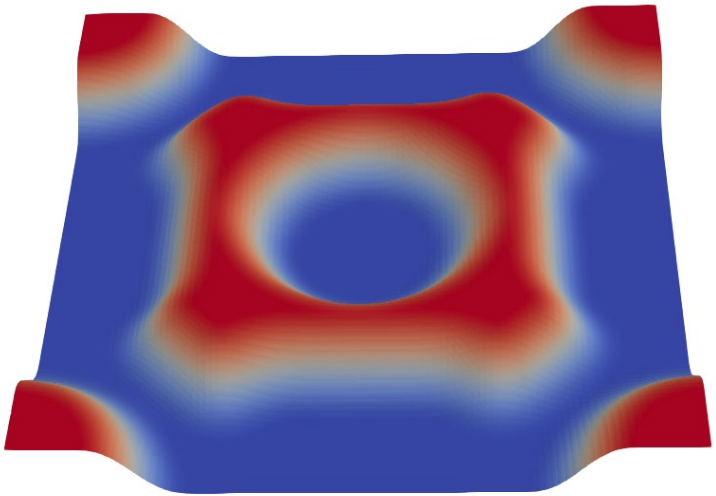


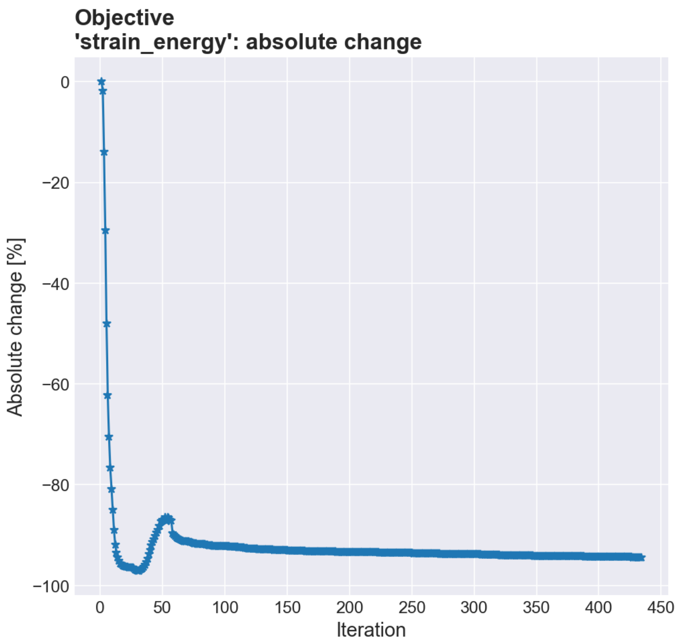
Performance original bead parameterization





Performance original bead parameterization





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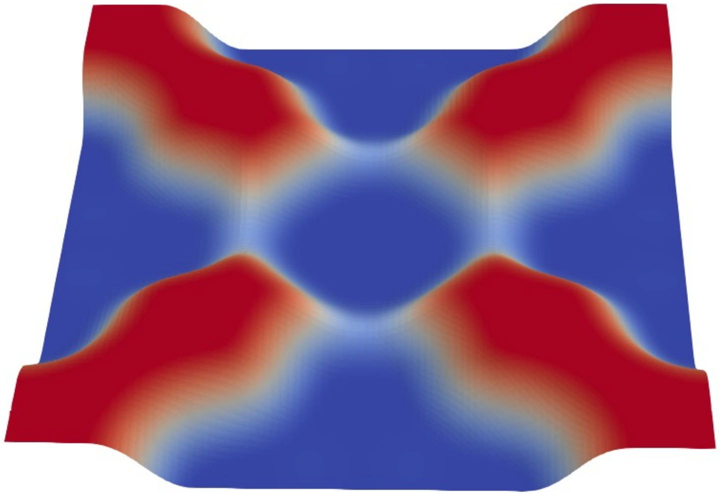
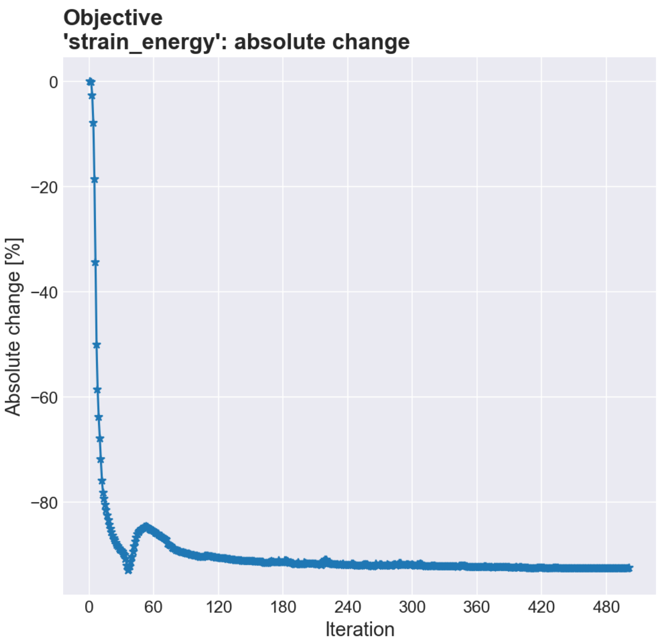
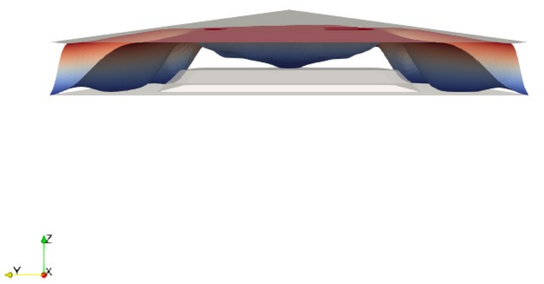
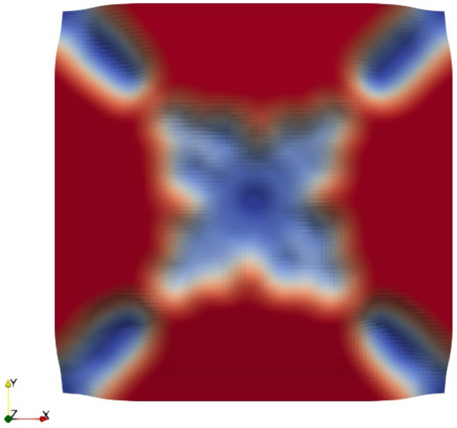
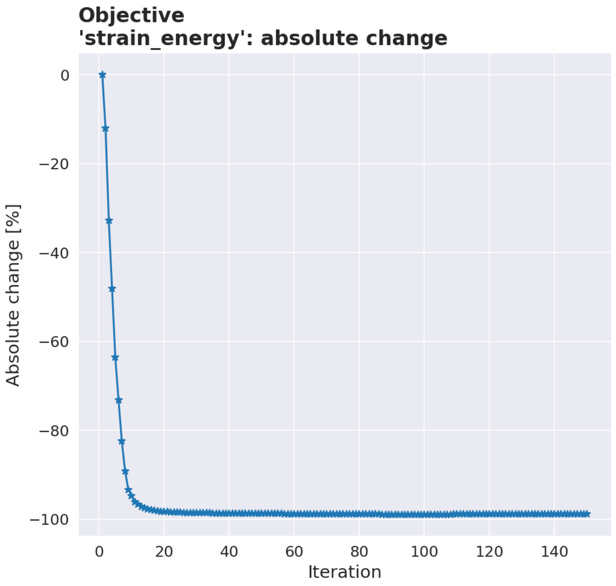




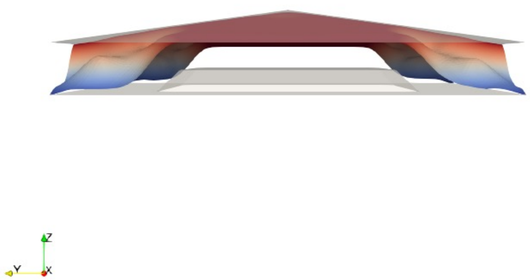
Performance variable bead height parameterization

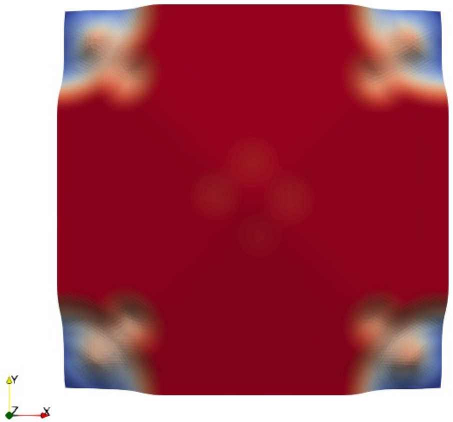


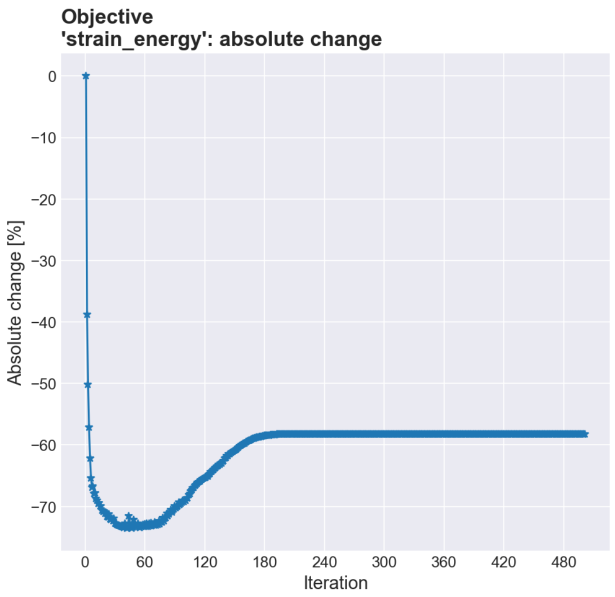




Performance initially curved geometry







Vertex Morphing

Filtering technique
Image with standard/simple optimization workflow (raw sensitivities – opt algo – design update)
Maps the sensitivities from design space to „control space“ and maps the „control updates“ back to design space (figure)

Use control design variables p that describe the actual geometry x (shape parameterization) related by: x = A(p) with A the transformation (or scaling) matrix from the design (control) field s to the actual geometry x.

Vertex Morphing

Notes from Majids Dissertation:

- The essence of the method is the filtering of the sensitivity field as well as the shape update vector by help of a suitable parametrization.
- The filtering (regularization) operations are derived consistently from the chain rule of differentiation
- Elaborate variable transformation enhanced with a suitable dimensional reduction for mesh quality regularization.
- explicit