Preconditioning of a FETI-solver for a nonlinear asynchronous time-integrator applied to structural dynamics

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In this work we construct a Dirichlet-type preconditioner for a FETI-solver in a nonlinear version of the BGC-macro asynchronous time-integrator. This preconditioner is then tested on a simple beam-model with six substructures and an impact load.

1 Introduction

When applying FETI-solvers, as originally developed by Farhat and Roux [4], to nonlinear structural dynamics problems with local nonlinearities, e.g. damage, efficiency might drop. The portion of computations that corresponds to communication increases, while some local processes might be idling. An approach to reduce global computations is based on subcycling time-steps on a substructure, as originally presented in the GC-method [8] and modified to the PH- [9] and BGC-macro method [3]. Both enforce compatibility only at the coarse synchronization time-step. In this work, we use this state-of-the art asynchronous or multirate time-integrator and solve the interface-problem with an iterative FETI-solver. To unlock the full potential of a FETI-solver, a proper preconditioner is required. In this work we construct a new taylored Dirichlet-type preconditioner.

2 Asynchronous time-integration and preconditioning

2.1 Nonlinear BGC-macro method

We consider an undamped dynamical system with nonlinear internal forces $\vec{f}_{int}$, depending of the displacements $\vec{q}^{(s)}$. The system's inertia is described by the mass-matrix $\mathbf{M}$ and acceleration $\ddot{\vec{q}}$, and time-dependent external forces are written as $\vec{f}_{ext}(t)$. Between neighboring subdomains, Lagrange-Multipliers $\vec{\lambda}$ are applied on the interfaces via the signed Boolean-matrix $\mathbf{B}$, which maps local degrees of freedom (dof) onto interface-dofs. These Lagrange-Multipliers are determined by the FETI-solver iteratively such that the compatibility-condition is fulfilled [4]. In case of dynamic problems, it is recommended to enforce compatibility on velocities $\dot{\vec{q}}$ or accelerations, due to weak instabilities otherwise [5]. A very common scheme for the time-integration is the Newmark-β scheme with parameters $\beta \in [0, 1/4]$ and $\gamma \in [0, 1/2]$. The BGC-macro as well as GC- and PH-method are based on a linear interpolation of $\vec{\lambda}$. In case of the BGC-macro method, these interpolated $\vec{\lambda}$ are directly derived from the final macro time-step $\vec{\lambda}_{n+1}$. This leads to the residuals of a subdomain s

$$\tilde{\vec{r}}^{(s)}_{m+1}(\tilde{\vec{q}}^{(s)}_m, \tilde{\vec{q}}^{(s)}_{m+1}, \vec{\lambda}_{n+1}) = \begin{bmatrix} \mathbf{M}^{(s)}\ddot{\vec{q}}^{(s)}_{m+1} + \vec{f}^{(s)}_{int}(\vec{q}^{(s)}_{m+1}) + \mathbf{B}^{(s)^T}\left((1-\frac{m+1}{N_m})\vec{\lambda}_n + \frac{m+1}{N_m}\vec{\lambda}_{n+1}\right) - \vec{f}^{(s)}_{ext}(t_{m+1}) \\ \dot{\vec{q}}^{(s)}_{m+1} - \dot{\vec{q}}^{(s)}_m - (1-\gamma)\Delta t^{(s)}\ddot{\vec{q}}^{(s)}_m - \gamma\Delta t^{(s)}\ddot{\vec{q}}^{(s)}_{m+1} \\ \vec{q}^{(s)}_{m+1} - \vec{q}^{(s)}_m - \Delta t^{(s)}\dot{\vec{q}}^{(s)}_m - (1/2-\beta)\,\Delta t^{(s)^2}\ddot{\vec{q}}^{(s)}_m - \beta\Delta t^{(s)^2}\ddot{\vec{q}}^{(s)}_{m+1} \end{bmatrix} \tag{1}$$

$$\tilde{\vec{r}}^{I}(\tilde{\vec{q}}^{(s)}_{N_m}) = \sum_{s=1}^{N_s}\mathbf{B}^{(s)}\dot{\vec{q}}^{(s)}_{N_m} = \vec{0} \tag{2}$$

These nonlinear residuals are solved by a Newton-Raphson procedure and a FETI-method solves the linearized system for the Langrange-Multipliers. Linearization provides for the case of two subdomains A and B

$$\tilde{\mathbf{M}}^{(s)} = \begin{bmatrix} \mathbf{M}^{(s)} & \mathbf{0} & \mathbf{K}^{(s)}_m \\ -\gamma\Delta t^{(s)}\mathbf{I} & \mathbf{I} & \mathbf{0} \\ -\beta\Delta t^{(s)^2}\mathbf{I} & \mathbf{0} & \mathbf{I} \end{bmatrix} \mathbf{N}^{(s)} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -\Delta t^{(s)}(1-\gamma)\mathbf{I} & -\mathbf{I} & \mathbf{0} \\ -\Delta t^{(s)}(1/2-\beta)\mathbf{I} & -\Delta t^{(s)}\mathbf{I} & -\mathbf{I} \end{bmatrix} \quad \tilde{\mathbf{B}}^{(s)^T} = \begin{bmatrix} \mathbf{0} \\ \mathbf{B}^{(s)^T} \\ \mathbf{0} \end{bmatrix} \tilde{\mathbf{C}}^{(s)} = \begin{bmatrix} \mathbf{B}^{(s)^T} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}$$

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$$\mathbb{A}^{(s)} = \begin{bmatrix} \tilde{\mathbf{M}}_1^{(s)} & & \\ \ddots & \ddots & \\ & \mathbf{N}^{(s)} & \tilde{\mathbf{M}}_{N_m}^{(s)} \end{bmatrix}, \quad \mathbb{C}^{(s)} = \begin{bmatrix} \frac{1}{N_m}\tilde{\mathbf{C}}^{(s)} \\ \vdots \\ \frac{N_m}{N_m}\tilde{\mathbf{C}}^{(s)} \end{bmatrix}, \mathbb{B}^{(s)} = \begin{bmatrix} \mathbf{0} & \dots & \dots & \tilde{\mathbf{B}}^{(s)} \end{bmatrix}, \quad \mathbb{R}^{(s)} = \begin{bmatrix} \tilde{\vec{r}}_1^{(s)T} & \dots & \tilde{\vec{r}}_{N_m}^{(s)T} \end{bmatrix}^T$$

$$\begin{bmatrix} \mathbb{A}^{(B)} & & \mathbb{C}^{(B)} \\ & \mathbb{A}^{(A)} & \mathbb{C}^{(A)} \\ \mathbb{B}^{(B)} & \mathbb{B}^{(A)} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \Delta\vec{z}^{(B)} \\ \Delta\vec{z}^{(A)} \\ \Delta\vec{\lambda}_{n+1} \end{bmatrix} = \begin{bmatrix} -\mathbb{R}^{(B)} \\ -\mathbb{R}^{(A)} \\ -\tilde{\vec{r}}^{I}(\tilde{\vec{q}}_{n+1}^{(A)}, \tilde{\vec{q}}_{N_m}^{(B)}) \end{bmatrix}, \quad \Delta\vec{z}^{(s)} = \begin{bmatrix} \Delta\ddot{\vec{q}}_1^{(s)T} & \Delta\dot{\vec{q}}_1^{(s)T} & \Delta\vec{q}_1^{(s)T} \dots & \Delta\vec{q}_{N_m}^{(s)T} \end{bmatrix}^T$$

2.2 A Dirichlet-Preconditioner based on the local time-integration

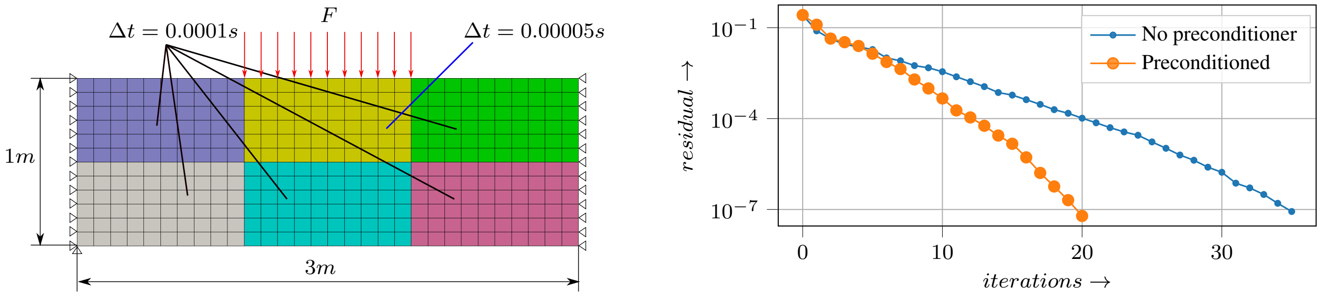
For the construction of the new Dirichlet-type preconditioner, we view the local time-stepping as a linear system with the Lagrange-Multipliers as input and the last boundary-velocities as output. With the preconditioner, we invert that. Hence, we write the local problem for internal quantities i and quantities on the boundary b on each interface between two substructures

$$\begin{bmatrix} \mathbb{A}_{ii}^{(s)} & \mathbb{A}_{ib}^{(s)} \\ \mathbb{A}_{bi}^{(s)} & \mathbb{A}_{bb}^{(s)} \end{bmatrix} \begin{bmatrix} \Delta\vec{z}_i^{(s)} \\ \Delta\vec{z}_b^{(s)} \end{bmatrix} = -\begin{bmatrix} \mathbb{C}_i^{(s)} \\ \mathbb{C}_b^{(s)} \end{bmatrix} \Delta\vec{\lambda}_{n+1}, \quad -\left(\mathbb{C}_b^{(s)} - \mathbb{A}_{bi}^{(s)} \mathbb{A}_{ii}^{(s)^{-1}} \mathbb{C}_i^{(s)}\right)^{-1} \underbrace{\left(\mathbb{A}_{bb}^{(s)} - \mathbb{A}_{bi}^{(s)} \mathbb{A}_{ii}^{(s)^{-1}} \mathbb{A}_{ib}^{(s)}\right)}_{\mathbb{S}^{(s)}} \Delta\vec{z}_b^{(s)} = \Delta\vec{\lambda}_{n+1} \quad (3)$$

where any external forces apart from the Lagrange-Multipliers are omitted. Our boundary-dofs are here the coupling dofs in $\mathbb{B}^{(s)}$. By solving for the internal solution and reordering we get the local contributions to the preconditioner. Finally the local preconditioners are assembled to the global preconditioner for each interface

$$\tilde{\mathbf{F}}_I^{-1} = \sum_{s=1}^{N_s} \beta^{(s)} \begin{bmatrix} \mathbf{0} & -\left(\mathbb{C}_b^{(s)} - \mathbb{A}_{bi}^{(s)} \mathbb{A}_{ii}^{(s)^{-1}} \mathbb{C}_i^{(s)}\right)^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbb{S}^{(s)} \end{bmatrix} \mathbb{B}^{(s)^T} \beta^{(s)} \quad (4)$$

where $\beta^{(s)}$ are scaling coefficients [7]. Throughout this work Multiplicity-scaling is used, which averages the solutions on the interfaces.



(a) Impact plate with triangular loading (St.Venant-Kirchhoff material, $E = 6.0 \cdot 10^5 N/m^2$, $\rho = 2.7 \cdot 10^{-3} kg/m^3$, $\nu = 0.34$, thickness= $0.5m$, $F_{max} = 10^4 N$)

(b) Residuals of GMRES-solver in first time-step and second Newton-iteration with and without preconditioning

Fig. 1: Geometrically nonlinear impact experiment solved with GMRES for the linearized interface-problem with and without preconditioner (Implemented in AMfe [1] and AMfeti [2]).

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