CommonRoad-CriMe: A Toolbox for Criticality Measures of Autonomous Vehicles

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Abstract—Criticality measures are essential for autonomous vehicles to capture the complexity of the surrounding environment, trigger emergency maneuvers, and verify safety. However, there is currently no publicly available toolbox that allows researchers to use or evaluate a large number of criticality measures on arbitrary traffic scenarios. To address this issue, we present CommonRoad-CriMe, an open-source toolbox for measuring the criticality of autonomous vehicles in a unified framework. Our toolbox covers a wide range of state-of-the-art criticality measures and provides visualized information to facilitate debugging and showcasing. Numerical experiments demonstrate how our toolbox facilitates the comparison of different criticality measures and the analysis of traffic conflicts. Our toolbox is available at [commonroad.in.tum.de](https://commonroad.in.tum.de).

I. INTRODUCTION

Autonomic manufacturers must ensure that autonomous vehicles can recognize and effectively handle unexpected situations. To achieve this, criticality measures (aka surrogate indicators or threat assessments) are often used to identify safety-critical scenarios and validate autonomous driving systems [1]. A literature review on criticality measures for autonomous driving can be found in [2]–[6]. For the scope of this paper, criticality refers to the level of risk for the involved vehicles with respect to the continuation of a certain traffic situation [1, Def. 1]. An example of a critical scenario is shown in Fig. 1. Despite decades of research, the authors are unaware of any open-source toolbox available for applying and comparing different criticality measures to a wide range of scenarios, as addressed by our work.

A. Related Work

Subsequently, we present related works on criticality measures for advanced driver assistant systems and autonomous driving as well as existing open-source toolboxes computing criticality measures.

a) Criticality Measures for Advanced Driver Assistant Systems and Autonomous Driving: Criticality measures are primarily developed to objectively determine the behavioral safety and threat level of autonomous driving systems. For example, criticality measures are used to validate the safety of autonomous vehicles through various methods, including generating safety-critical scenarios [7]–[11], falsifying the system under test [12], and formally verifying system properties [1]. For motion planning applications, criticality measures help to find traffic conflicts, repair unsafe trajectories, and provide fail-safe solutions [13]–[15].

Model-based and data-driven methods are frequently used to estimate the criticality of a traffic situation [2], [3]. The criticality is influenced, e.g., by the likelihood and consequence of a collision or other dangerous situations. However, due to the large number and variety of existing criticality measures in the literature, selecting the appropriate measure for a particular traffic scenario is challenging.

b) Existing Toolboxes with Criticality Measures: There are several publicly available toolkits that include safety and criticality measures. For example, the Apollo open platform [4] defines a limited number of safety and comfort metrics for grading simulated scenarios. Meanwhile, the open-source simulation platform CARLA [16] embeds the responsible-sensitive safety model [17], which identifies safety-critical situations. Although these tools emulate real-world driving environments, they only cover a few criticality measures, and the overhead required to set up the entire simulation environment in C++ makes it difficult to include new measures. Therefore, tools written in easy-to-use programming languages that cover a wide range of criticality measures are desired.

B. Contributions

To effortlessly measure and compare the criticality of an autonomous vehicle, referred to as ego vehicle in the following sections, we present the novel CommonRoad Criticality Measures (CommonRoad-CriMe) toolbox, which:

1) provides a framework in Python with unified notations, vehicle models, and coordinate systems for criticality measures;

2) adopts and supplements the categorization of criticality measures defined in [6];

1The snapshots are created with esmini, see [https://github.com/ApolloAuto/apollo](https://github.com/ApolloAuto/apollo)

Fig. 1: An exemplary critical scenario in which the other traffic participant (red) cuts into the lane of the ego vehicle (blue) and brakes hard. The snapshots show the inside view of the ego vehicle at three time steps.
3) is open-source and allows users to easily modify, add, and compare criticality measures; and
4) offers efficient and reliable computation by bridging to powerful scenario evaluation tools, such as a drivability checker [18], a scenario designer [19], a set-based predictor [20], and a reachability analyzer [21].

The remainder of this work is structured as follows: Sec. II introduces required preliminaries and definitions. Sec. III provides an overview and a list of representative measures in the toolbox. In Sec. IV, we showcase the benefits of CommonRoad-CriMe with numerical examples. Finally, Sec. V concludes the paper.

II. PRELIMINARIES

A. System Description

We model the motion of vehicles as discrete-time systems:

$$x_{k+1} = f_d(x_k, u_k),$$

where $x_k \in \mathbb{R}^{n_x}$ is the state vector, $u_k \in \mathbb{R}^{n_u}$ is the input vector, and the index $k \in \mathbb{N}_0$ maps to a discrete time step according to $t_k = k\Delta t$ with $\Delta t$ being a fixed time increment. At each time step, the system is bounded by sets of admissible states $X_k \subset \mathbb{R}^{n_x}$ and admissible control inputs $U_k \subset \mathbb{R}^{n_u}$. We use the notation $\chi(k', x_k, u_{[k,k']} \}$ to represent the solution of (1) at time step $k' \geq k$, given an initial state $x_k$ and an input trajectory $u_{[k,k']} \}$ for the time interval $[k, k']$. Due to limited space, we use the shorthand $\chi(k') := \chi(k', x_k, u_{[k,k']} \}.$

The state vector of a vehicle $x^0 := (s_x, s_y, v, \theta)^T$ in a global Cartesian coordinate frame $\mathbb{G}$ typically consists at least of the position $(s_x, s_y)^T$, the velocity $v$, and the heading $\theta$ as the basic elements. Other models have either state variables that can be converted to the previous ones and/or further state variables that are irrelevant to this work. Considering the criticality measures based on structured road scenarios, we can localize the vehicle in curvilinear coordinate systems $\mathbb{G}$ [22] formulated locally with respect to a reference path $\Gamma$, e.g., the centerline of the road. As shown in Fig. 2, the vehicle is described by the longitudinal position $s_x$, the orthogonal deviation of the reference path $s_y$, and the relative heading $\theta := \theta - \theta_{\Gamma}(s_x)$ measured with respect to $\Gamma$ with orientation $\theta_{\Gamma}(s_x)$. When localizing a point-mass vehicle model in the curvilinear coordinate system, the state vector is denoted as $x^l := (s_x, s_y, v_x, v_y)^T$ and the system receives inputs $u^l := (a_x, a_y)^T$. For brevity, we omit the superscript when it is clear which coordinate system is being used based on the context.

Let $\Box$ be a variable, we denote its value associated with the ego vehicle by $\Box_{\text{ego}}$ and with the obstacle $b \in \mathbb{B}$ by $\Box_b$, where $\mathbb{B}$ is the set of all criticality-relevant obstacles. The functions $\text{front}(x_k) : \mathbb{R}^{n_x} \rightarrow \mathbb{R}$, $\text{rear}(x_k) : \mathbb{R}^{n_x} \rightarrow \mathbb{R}$, $\text{occ}(x_k) : \mathbb{R}^{n_x} \rightarrow \mathbb{P}((\mathbb{R}^{3})^2)$ and $\text{lanes}(x_k) : \mathbb{R}^{n_x} \rightarrow \mathbb{P}((\mathbb{N}_0))$ return the position of the front bumper, rear bumper, the spatial occupancy, and the indices of occupied lanes of a vehicle at time step $k$, respectively. Given the occupancy of an obstacle $b$ at time step $k$, we denote the set of forbidden state for the ego vehicle as $\mathcal{F}(x_{b,k}) := \{ x_{\text{ego},k} \in X_k | \text{occ}(x_{\text{ego},k}) \land \text{occ}(x_{b,k}) \neq \emptyset \}.$

B. Definitions

The following definitions are necessary for introducing the presented criticality measures:

Definition 1 (Maneuver $m$):

A maneuver $m$ is an element of the set that consists of braking, constant velocity, kickdown, steering, turning, overtaking, and lane change.

The control input of a vehicle with maneuver $m$ is denoted as $u^m$. In other works, e.g., [23]–[25], the terms action and behavior are used interchangeably for the same purposes as maneuver.

Definition 2 (Scene and Scenario [26, Sec. II and VI]):

A scene is a snapshot of the environment, which includes the lane network as well as states and inputs of vehicles. A scenario is a temporal sequence of maneuvers and scenes.

Definition 3 (Criticality Measure $c$ [1, Def. 12]):

A criticality measure $c : X^h_{\Gamma} \times (X^h_{\Gamma})^3 \rightarrow \mathbb{R}$ is a function that maps the vehicle states and/or inputs to the criticality of a traffic scene at time step $k$, where $h \in \mathbb{N}_0$ is the prediction horizon of the input trajectory $u_{[k,k+h]}$.

Monotonicity is a desired relationship between the criticality measure and criticality [27, Def. 1]. For instance, criticality increases as the time-to-collision (TTC) decreases [28]. If the measure input only contains information about the ego vehicle, criticality is computed as the minimum or maximum value of all pairs of the ego and other vehicles, depending on its monotonic relationship to the measure:

$$\min_{b \in \mathbb{B}} \max_{b \in \mathbb{B}} c(x_{\text{ego},k}, u_{\text{ego},[k,k+h]}).$$

Definition 4 (Reachable Set $\mathcal{R}$ [29, (1)]):

The reachable set at time step $k'$ is the set of states that can be reached from the initial set of states $X_k$ including measurement uncertainties while avoiding any forbidden states from time step $k$ to $k'$:

$$\mathcal{R}_{k'}(X_k, x_{b,k}, u_{b,[k,k']} \} := \left\{ x_k \bigg| \exists x_k \in X_k \land \forall \tau \in [k, k'), \exists u_{\tau} \in U_{\tau} : \chi(\tau, x_{b,k}, u_{b,[k,k']} \} \notin \mathcal{F}(x_{b,k}) \right\},$$

$^{3}$P($\phi$) is the power set of $\phi$.

$^{4}$Maximum is for positive monotonic relationships, whereas minimum is for negative ones. For measures that operate on sets, the infimum or supremum is used.
### TABLE I: List of state-of-the-art criticality measures

We express the positive and negative monotonic relationship between the measure and the criticality as $\oplus$ and $\ominus$, respectively.

<table>
<thead>
<tr>
<th>Domain</th>
<th>Measure</th>
<th>Acronym</th>
<th>Output Range</th>
<th>Mono.</th>
<th>Unit</th>
<th>Implemented Coord. Sys.</th>
<th>Source</th>
</tr>
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<tbody>
<tr>
<td>Time</td>
<td>Time headway</td>
<td>THW</td>
<td>$\mathbb{R}_+ \cup {\infty}$</td>
<td>$\oplus$</td>
<td>s</td>
<td>$\mathbb{L}$</td>
<td>[30]</td>
</tr>
<tr>
<td></td>
<td>Encroachment time</td>
<td>ET</td>
<td>$\mathbb{R}_+ \cup {\infty}$</td>
<td>$\oplus$</td>
<td>s</td>
<td>$\mathbb{L}$</td>
<td>[31]</td>
</tr>
<tr>
<td></td>
<td>Post-encroachment time</td>
<td>PET</td>
<td>$\mathbb{R}_+ \cup {\infty}$</td>
<td>$\oplus$</td>
<td>s</td>
<td>$\mathbb{L}$</td>
<td>[31]</td>
</tr>
<tr>
<td></td>
<td>Accepted gap size</td>
<td>AGS</td>
<td>$\mathbb{R}_+$</td>
<td>$\ominus$</td>
<td>s</td>
<td>$\mathbb{L}$</td>
<td>[32]</td>
</tr>
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<td></td>
<td>Time-to-collision</td>
<td>TTC</td>
<td>$\mathbb{R}_+ \cup {\infty}$</td>
<td>$\oplus$</td>
<td>s</td>
<td>$\mathbb{L}$</td>
<td>[28]</td>
</tr>
<tr>
<td></td>
<td>Time-to-collision with given prediction</td>
<td>TTC*</td>
<td>$\mathbb{R}_+ \cup {\infty}$</td>
<td>$\oplus$</td>
<td>s</td>
<td>$\mathbb{G}$</td>
<td>[33]</td>
</tr>
<tr>
<td></td>
<td>Potential time-to-collision</td>
<td>PTTC</td>
<td>$\mathbb{R}_+ \cup {\infty}$</td>
<td>$\oplus$</td>
<td>s</td>
<td>$\mathbb{L}$</td>
<td>[34]</td>
</tr>
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<td></td>
<td>Worst-time-to-collision</td>
<td>WTTTC</td>
<td>$\mathbb{R}_+ \cup {\infty}$</td>
<td>$\ominus$</td>
<td>s</td>
<td>$\mathbb{G}$</td>
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</tr>
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<td>Time-exposed time-to-collision</td>
<td>TET</td>
<td>$\mathbb{R}_+$</td>
<td>$\ominus$</td>
<td>$s^2$</td>
<td>$\mathbb{L}$</td>
<td>[23]</td>
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<td>TIT</td>
<td>$\mathbb{R}_+$</td>
<td>$\ominus$</td>
<td>$s^2$</td>
<td>$\mathbb{L}$</td>
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<td>TTCE</td>
<td>$\mathbb{R}_+ \cup {\infty}$</td>
<td>$\ominus$</td>
<td>s</td>
<td>$\mathbb{L}$</td>
<td>[36]</td>
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<td>Time-to-zebra</td>
<td>TTZ</td>
<td>$\mathbb{R}_+ \cup {\infty}$</td>
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<td>$\mathbb{L}$</td>
<td>[37]</td>
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<td>Time-to-brake</td>
<td>TTB</td>
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<td>TTK</td>
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<td></td>
<td>Time-to-react</td>
<td>TRR</td>
<td>$\mathbb{R}_+ \cup {\infty, -\infty}$</td>
<td>$\ominus$</td>
<td>s</td>
<td>$\mathbb{L}$</td>
<td>[38], [39]</td>
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<tr>
<td></td>
<td>Worst-time-to-react</td>
<td>WTRR</td>
<td>$\mathbb{R}_+ \cup {\infty, -\infty}$</td>
<td>$\ominus$</td>
<td>s</td>
<td>$\mathbb{L}$</td>
<td>[33]</td>
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<td>Time-to-violation</td>
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<td>s</td>
<td>$\mathbb{L}$</td>
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<td>Time-to-comply</td>
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<td>s</td>
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<td>[14]</td>
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<td>Distance</td>
<td>Headway</td>
<td>HW</td>
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<td>m</td>
<td>$\mathbb{L}$</td>
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<td>Distance-of-closest-encounter</td>
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<td>$\ominus$</td>
<td>m</td>
<td>$\mathbb{L}$</td>
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<td>Acceptable minimum stopping distance</td>
<td>MSD</td>
<td>$\mathbb{R}_+$</td>
<td>$\ominus$</td>
<td>m</td>
<td>$\mathbb{L}$</td>
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<td>Proportion of stopping distance</td>
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<td>Velocity</td>
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<td>Delta-v</td>
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<td>$\ominus$</td>
<td>$m/s$</td>
<td>$\mathbb{L}$</td>
<td>[40], [41]</td>
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<td>Conflict severity</td>
<td>CS</td>
<td>$\mathbb{R}$</td>
<td>$\ominus$</td>
<td>$m/s$</td>
<td>$\mathbb{L}$</td>
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<td>Acceleration</td>
<td>Deceleration-to-safety-time</td>
<td>DST</td>
<td>$\mathbb{R}$</td>
<td>$\ominus$</td>
<td>$m/s^2$</td>
<td>$\mathbb{L}$</td>
<td>[42]</td>
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<td></td>
<td>Required longitudinal acceleration (aka deceleration rate to avoid crash)</td>
<td>$a_{\xi, req}$ (DRAC)</td>
<td>$\mathbb{R}_-$</td>
<td>$\ominus$</td>
<td>$m/s^2$</td>
<td>$\mathbb{L}$</td>
<td>[30], [43]</td>
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<td></td>
<td>Required lateral acceleration</td>
<td>$a_{\eta, req}$</td>
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<td>$\ominus$</td>
<td>$m/s^2$</td>
<td>$\mathbb{L}$</td>
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<td>$\mathbb{R}_+$</td>
<td>$\ominus$</td>
<td>$m/s^2$</td>
<td>$\mathbb{L}$</td>
<td>[30]</td>
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<td>Jerk$^6$</td>
<td>Longitudinal jerk</td>
<td>LongJ</td>
<td>$\mathbb{R}_+$</td>
<td>$\ominus$</td>
<td>$m/s^3$</td>
<td>$\mathbb{L}$</td>
<td>[41]</td>
</tr>
<tr>
<td></td>
<td>Lateral jerk</td>
<td>LatJ</td>
<td>$\mathbb{R}_+$</td>
<td>$\ominus$</td>
<td>$m/s^3$</td>
<td>$\mathbb{L}$</td>
<td>[44], [45]</td>
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<td>Index</td>
<td>Conflict index</td>
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<td>$[0, 1]$</td>
<td>$\ominus$</td>
<td>$kg \cdot m^2/s^2$</td>
<td>$\mathbb{L}$</td>
<td>[46]</td>
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<td>Crash potential index</td>
<td>CPI</td>
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<td>$\ominus$</td>
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<td>Aggregated crash index</td>
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<td>$-$</td>
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<td>Trajectory criticality index</td>
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<td>$-$</td>
<td>$\mathbb{G}$</td>
<td>[48]</td>
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<td>PRI</td>
<td>$\mathbb{R}_+$</td>
<td>$\ominus$</td>
<td>$m^2/s^3$</td>
<td>$\mathbb{L}$</td>
<td>[49]</td>
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<td>Space occupancy index</td>
<td>SOI</td>
<td>$\mathbb{R}_+$</td>
<td>$\ominus$</td>
<td>$1/m^2$</td>
<td>$\mathbb{L}$</td>
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<td>BTN</td>
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<td>$\ominus$</td>
<td>$-$</td>
<td>$\mathbb{L}$</td>
<td>[30], [51]</td>
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<td>Steer threat number</td>
<td>STN</td>
<td>$\mathbb{R}_+$</td>
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<td>Responsibility sensitive safety-dangerous situation</td>
<td>RSS</td>
<td>$[0, 1]$</td>
<td>$\ominus$</td>
<td>$-$</td>
<td>$\mathbb{L}$</td>
<td>[17]</td>
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<td>Reachable-Set</td>
<td>Drivable area</td>
<td>DA</td>
<td>$\mathbb{R}_+$</td>
<td>$\ominus$</td>
<td>$m^2$</td>
<td>$\mathbb{L}$</td>
<td>[7]–[9]</td>
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<td>Probability</td>
<td>Collision probability via Monte Carlo simulation</td>
<td>P-MC</td>
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<td>$\ominus$</td>
<td>$-$</td>
<td>$\mathbb{L}$</td>
<td>[24], [52]</td>
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<td>Collision probability via scoring multiple hypotheses</td>
<td>P-SMH</td>
<td>$[0, 1]$</td>
<td>$\ominus$</td>
<td>$-$</td>
<td>$\mathbb{L}$</td>
<td>[53]</td>
</tr>
<tr>
<td></td>
<td>Collision probability via stochastic reachable sets</td>
<td>P-SRS</td>
<td>$[0, 1]$</td>
<td>$\ominus$</td>
<td>$-$</td>
<td>$\mathbb{L}$</td>
<td>[54]</td>
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<td>Potential functions as superposition of scoring functions</td>
<td>PF</td>
<td>$\mathbb{R}$</td>
<td>$\ominus$</td>
<td>$-$</td>
<td>$\mathbb{L}$</td>
<td>[55]</td>
</tr>
<tr>
<td></td>
<td>Safety potential</td>
<td>SP</td>
<td>$\mathbb{R}_+$</td>
<td>$\ominus$</td>
<td>$-$</td>
<td>$\mathbb{L}$</td>
<td>[56]</td>
</tr>
</tbody>
</table>

$^5$The categorization is based on [6], of which the up-to-date descriptions can be found at [https://criticality-metrics.readthedocs.io](https://criticality-metrics.readthedocs.io).

$^6$We take the absolute jerk since the sign is insignificant for the criticality and affects the monotonic relationship.

### III. CommonRoad-CriMe

#### A. Overview

The architecture of CommonRoad-CriMe is depicted in Fig. 3 as a unified modeling language (UML) class diagram.
We provide the user interface for setting configurations, selecting criticality measures, and loading traffic scenarios in the class CriMeInterface. The class Scenario [57] contains the information of a series of continuous traffic scenes (class Scene). The criticality measures are defined in the core module Measure, of which the categorization is based on their output domains adapted from [6] (cf. Tab. I). It should be noted that there are numerous criticality measures in the literature and new ones are continually being developed, making it impossible to cover them all. However, in the following sections, we provide a representative list and aim to use the most prominent definitions for a comprehensive analysis. Please note that our calculation does not take into account discretization errors or vehicle reaction time. We reset time for each evaluation without loss of generality.

Fig. 3: UML class diagram of CommonRoad-CriMe.

### B. Headway and Time Headway

The headway (HW) is the distance from the ego vehicle to the leading vehicle. Since the distance along the lane centerline is more representative than the straight-line distance [58], the HW is defined in the curvilinear coordinate system [30, Sec. 5.3.1]:

\[ HW(x_{ego,k}, x_{b,k}) = \begin{cases} \text{rear}(x_{b,k}) - \text{front}(x_{ego,k}) & \text{if } \text{lanes}(x_{ego,k}) \cap \text{lanes}(x_{b,k}) \neq \emptyset \\
\infty & \text{and } \text{rear}(x_{b,k}) \geq \text{front}(x_{ego,k}), \\
\infty & \text{otherwise.} \end{cases} \]

Similarly, the time headway (THW) is the time until the ego vehicle reaches the position of the leading vehicle [30, Sec. 5.3.1]. Assuming the ego vehicle drives at a constant velocity, the THW is [30, (5.23)]:

\[ THW(x_{ego,k}, x_{b,k}) = \frac{HW(x_{ego,k}, x_{b,k})}{v_{ego,k}}. \]

### C. Time-To-X

The most used criticality measure is time-to-x (TTX), where x denotes a relevant event on the path toward a potential collision.

1) **Time-To-Collision**: TTC is a measure of the time remaining until a collision occurs [2]. When calculating the TTC, we often assumed that the relative acceleration between vehicles is zero [23] or constant [30, Sec. 5.3.2]. Without loss of generality, we consider the latter case and compute the TTC based on the HW, the relative velocity \( \Delta v_\xi := v_{c,b} - v_{ego} \), and the relative acceleration \( \Delta a_\xi := a_{c,b} - a_{ego} \) in \( \xi \)-direction [30, (5.26)]:

\[
\text{TTC}(x_{ego,k}, x_{b,k}) = \begin{cases} \frac{HW(\ldots)}{\Delta v_\xi} & \text{if } \Delta v_{\xi,k} < 0 \text{ and } \Delta a_\xi = 0, \\
\infty & \text{if } \Delta v_{\xi,k} = -2HW(\ldots)\Delta a_\xi < 0, \\
\frac{\Delta v_{\xi,k} - 2HW(\ldots)\Delta a_\xi}{\Delta a_\xi} & \text{if } \Delta v_{\xi,k} < 0 \text{ and } \Delta a_\xi \neq 0, \\
\infty & \text{if } \Delta v_{\xi,k} \geq 0 \text{ and } \Delta a_\xi < 0, \\
\infty & \text{otherwise.} \end{cases}
\]

A more accurate alternative is to compute the TTC with an intended trajectory of the ego vehicle and the given prediction of other vehicles [13], [33], which we call TTC*. Given a most-likely or set-based prediction [59] of a vehicle \( b \), we define that the intended trajectory of the ego vehicle can be executed without collisions through the predicate 

\[
\text{noCollision}(x_{ego,k}, x_{b,k}, u_{ego,[k+k+h]}, u_{b,[k+k+h]}) \iff \forall k' \in [k, k+h] : \text{ego}_{k'} \notin F(x_{b,k'}).
\]

TTC* is then computed by [33, Def. 1]:

\[
\text{TTC}^*(x_{ego,k}, x_{b,k}, u_{ego,[k+k+h]}, u_{b,[k+k+h]}) = \begin{cases} \infty & \text{if noCollision(\cdots) holds,} \\
\min \{ t_{k'-k} | k' \in [k, k+h], \text{ego}_{k'} \notin F(x_{b,k'}) \} & \text{otherwise.} \end{cases}
\]

2) **Time-To-Collision Variants**: Based on the TTC, the time-exposed time-to-collision (TET) and time-integrated time-to-collision assess the criticality considering future vehicle trajectories over space and time [23]. Since the effectiveness of TTC decreases significantly in the lateral direction of the ego vehicle, the encroachment time (ET) and post-encroachment time (PET) are proposed to measure traffic conflicts in intersections [31]. A similar approach is employed for the time-to-closest-encounter (TTCE) and distance-to-closest-encounter (DCE), which generalizes the TTC to non-collision cases [36]. To obtain a worst-case approximation, the TTC is extended to the worst-time-to-collision (WTTTC) in [35], which takes lateral traffic into account and tends to overestimate the criticality of traffic scenarios.

3) **Time-To-Maneuver and Time-To-React**: The TTC and its variants do not provide enough information for collision avoidance, as they do not include possible evasive maneuvers [33]. To address this limitation, the time-to-maneuver (TTM) is proposed as the latest possible time before the TTC*, at which an evasive maneuver still exists [39, (8)]:

\[
\text{TTM}(x_{ego,k}, x_{b,k}, u_{ego,[k+k+h]}, u_{b,[k+k+h]}) = \begin{cases} \infty & \text{if noCollision(\cdots) holds,} \\
\max \left( \left\{ \infty \cup \left\{ t_{k'-k} | k' \in [k, k+\text{TTC}^*(\cdots)] \right\} \right\} | u_{ego,[k+k+h]} \subset U: \right. \\
\left. \text{noCollision}(\cdots | u_{ego,[k+k]}, u_{ego,[k+k+h]} \ldots) \right) & \text{otherwise.} \end{cases}
\]
For emergency braking, evasive steering, and kickdown, the corresponding TTM is denoted as the time-to-brake (TTB), time-to-steer (TTS), and time-to-kickdown (TTK), respectively. In [38], the authors propose the time-to-react (TTR) as the maximum TTM of all possible maneuvers. Since computing the exact TTR is computationally intractable [33], the TTR is underapproximated using a set of selected evasive maneuvers [39, (10)]:

$$\text{TTR}(x_{\text{ego}}, k, \mathbf{u}_{\text{ego}}, [k, k+h], \mathbf{u}_b[k, k+h]) = \max\{\text{TTB}(\ldots), \text{TTS}(\ldots), \text{TTK}(\ldots)\}.$$  

In contrast, the TTR is tightly overapproximated by iteratively checking the existence of collision-free reachable sets of the ego vehicle, which is denoted as the worst-time-to-react (WTTR) [33, Prop. 1]:

$$\text{WTTR}(x_{\text{ego}}, k, \mathbf{u}_{\text{ego}}, [k, k+h], \mathbf{u}_b[k, k+h]) = \begin{cases} \infty & \text{if } \text{noCollision}(\ldots) \text{ holds}, \\ \max \{-\infty\} \cup \{t_{\text{req},k} | k' \in [k, k + \text{TTC}'(\ldots)]\}, \\ \mathcal{R}_{k+h} (x_{\text{ego}}, k', \chi_{b,k'}, \mathbf{u}_b[k', k+h]) \neq \emptyset \} & \text{otherwise}. \end{cases}$$

**E. Brake and Steer Threat Number**

Delta-v measures the change in velocity a vehicle experiences as a result of a collision, which is widely used for measuring crash severity [40]. Assuming an inelastic collision between vehicles, we compute Delta-v taking their masses $M$ into account [41, (2)]:

$$\text{Delta-v}(x_{\text{ego}}, k, \mathbf{u}_{\text{ego}}, k) = \frac{M_k v_{\text{ego},k} + v_{\text{ego},k} \cos(\theta_{\text{ego},k} - \theta_{b,k})}{M_{\text{ego}} + M_k}.$$  

**F. Drivable Area**

The drivable area is the configuration space in which the ego vehicle can operate safely without colliding [8]. By projecting the reachable set (cf. Def. 3) to the position domain in the Euclidean space, the drivable area at time step $k'$ is obtained by [60, Def. 4]:

$$\mathcal{D}_{k'}(x_{\text{ego}}, k, \mathbf{u}_{\text{ego}}, k, \mathbf{u}_b[k, k+h]) = \text{proj}_{\mathcal{R}_{k'}}(\mathcal{R}_{k'}(x_{\text{ego}}, k, \mathbf{u}_{\text{ego}}, k, \mathbf{u}_b[k, k+h])).$$

7 CommonRoad ID: DEU_Gar-1_1_T-1
IV. NUMERICAL EXAMPLES

We evaluate the criticality of scenes and scenarios from the CommonRoad benchmark suite [57] using CommonRoad-CriMe with exemplary measures and show the benefits of using our toolbox. Scenario I, with $h = 20$ and $\Delta t = 0.1s$, depicts a rural environment in which the intended trajectory of the ego vehicle and the most likely trajectories of the other traffic participants are given (cf. Fig. 4a). In the urban scenario II, we present a set-based prediction of the other vehicles for $h = 30$ time steps using $\Delta t = 0.25s$ as shown in Fig. 5a. We refer to scenes I and II as the initial scenes of scenarios I and II, respectively. We employ the CommonRoad vehicle models [57] and use the point-mass model to demonstrate the results. The parameters for computing the measures are obtained from the original papers (cf. Tab. II).

A. Evaluation on Scenes

The evaluation results of scenes I and II with selected criticality measures are listed in Tab. II. The TTC is computed as $3.70s$ for scene I (see Fig. 4a), indicating a low criticality. It does, however, rule out the possibility that the leading vehicle fully brakes, implying a high risk for the ego vehicle. This false-negative indication is the same for scene II (see Fig. 5a) when the other participant drifts to the lane of the ego vehicle. Therefore, the TTC is less effective in assessing collision risks in the lateral driving direction. In contrast to TTC, the TTC* allows one to consider a more sophisticated prediction of vehicles, especially the set-based prediction. In Fig. 5a, we can observe a possible collision at TTC* if all the possible behaviors of the other traffic participant are considered. WTTC also attempts to address the inefficiency of using TTC, as shown in Fig. 5b. However, its threshold value for triggering warnings is difficult to define since worst-case scenarios are infrequent in real traffic.

In contrast to the TTC and its variants, TTM and TTR provide evasive trajectories. Fig. 5a shows that the ego vehicle needs to fully brake from the state at TTR, i.e., TTB, to ensure safety. As shown in Fig. 5c, there exist only small drivable areas if the ego vehicle executes evasive maneuvers from the state at time step $\text{TTR/}\Delta t - 1$.

B. Evaluation on Scenarios

Fig. 6 depicts the evaluation results for the entire scenario I. From the profiles of the TTC and $\alpha_{\xi,\text{req}}$, we can see that the ego vehicle is getting closer to the leading vehicle, but the level of criticality with respect to the other traffic participants is unknown. The curves of the BTN, STN, and P-MC indicate that a collision is unlikely to occur in this scenario, as their values are significantly lower than the collision threshold of $1.0$ [52], [62]. The DA curve confirms this observation by showing that there are ample collision-free, reachable positions for the ego vehicle in the near future. Instead, the PF reaches its maximum potential starting from time step 10, indicating that the ego vehicle does not maintain a safe distance when the preceding vehicle fully brakes, i.e., safety is not ensured. In conclusion, we can infer that relying on a single criticality measure may not provide enough information, and that multiple measures are required to accurately assess the criticality of a scenario.

We further measure the TTC of 1,000 randomly selected scenarios from the CommonRoad benchmark suite to demonstrate the usefulness of our toolbox. The CommonRoad collection includes a mix of recorded and handcrafted scenarios, such as those on highways, rural roads, and in urban settings. All calculations were carried out on a laptop equipped with an Intel Core i7-1165G7 2.8 GHz processor. There exist 21,713 scenes in total in the selected scenarios, and

<table>
<thead>
<tr>
<th>Measure</th>
<th>Scene I</th>
<th>Scene II</th>
<th>Measure</th>
<th>Scene I</th>
<th>Scene II</th>
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<td>PET</td>
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</tr>
<tr>
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<td>LatJ</td>
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<td>PF</td>
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<td>-36.70</td>
</tr>
</tbody>
</table>

TABLE II: Criticality measures of scenes I and II.
the average computation time of each scene was 17.90 ms (1.80 ms of each vehicle pair). Fig. 7 shows the histogram of the TTC for all scenes in the collection. By analyzing the distribution, we can identify safety-critical situations in the longitudinal driving direction, e.g., using a threshold of \( \text{TTC} = 1.0 \) s [28]. Therefore, CommonRoad-CriMe simplifies the process of selecting scenarios of interest from a large database.

![Figure 6: Criticality profiles of scenario I with exemplary measures.](image)

![Figure 7: Histogram of the TTC over 21,713 scenes from the CommonRoad collection.](image)

**V. Conclusions**

This paper presents the first toolbox for measuring the criticality of autonomous vehicles that is open-source, easy-to-use, and contains state-of-the-art measures. In CommonRoad-CriMe, we provide a unified evaluation framework and support a wide range of criticality measures.

We hope that CommonRoad-CriMe will make it easier for intelligent transportation researchers to evaluate their autonomous driving functions with various criticality measures and traffic scenarios. Contributions to further improve and expand the capabilities of CommonRoad-CriMe are welcomed. Future work includes a more thorough comparison of measures and user studies in order to create a reference for human criticality estimation.

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**References**


