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Tree-level harvest optimization for investment and institutional analysis

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ABSTRACT

This dissertation develops a feasible approach for optimizing individual-tree harvest decisions in complex forest stands and applies it to forest investment and institutional analysis problems. It explores the advantages of pairing neighborhood-scale growth modeling with tree-level optimization and considers the challenges such fine-resolution analysis presents. Applications of this approach illustrate the silvicultural sophistication required to maximize investment returns from quality-differentiated forests. Increased financial returns and ecologically complexity result.

The approach also provides visibility into the decision dynamics at play in informationally rich environments. When access to information is asymmetrically distributed between forestland buyers and sellers, or forest owners and managers, dominant silvicultural strategies can deviate from those that maximize long-term value production absent such distortions. One study in this dissertation explores the implications of imperfect information in markets for forestland and the opportunity it creates to profit from deliberate degradation. A final study models optimal contracting for silvicultural expertise when the forest owner cannot supervise the forester and anticipates opportunistic behavior. Even an optimally specified contract induces substantially less effort than a forester would exert in managing their own property.

These insights help to explain a puzzling pattern of behavior where forests capable of growing premium-quality timber are more often mismanaged than those that only produce commodity-grade products. Information-related transaction costs bear particularly heavily on the management of the eastern North American hardwood forests that are the focus of this research. The tools and concepts developed in this dissertation should prove useful to those who take on the challenge of crafting institutional responses to the confounding problem of forest degradation.

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1 INTRODUCTION

A puzzle exists: in many places, forests with the best kinds of trees suffer from the worst kind of management. Forests that yield nothing better than commodity-grade sawtimber are managed with discipline and intensity while those that could grow ultra-premium hardwoods are exploited with little oversight. Why is it that as the potential quality of timber goes up, the actual quality of silviculture often goes down?

Consider the differences in the management of loblolly pine in Mississippi versus northern hardwoods in New York—a contrast so stark that Munsell et al. (2008) could refer to "a tale of two forests" (p. 431). Landowners in Mississippi invest heavily in their forests in spite of the relatively low value of the products they yield. Historically, the best quality southern pine has not earned a consistent premium over standard sawtimber (Regmi et al., 2022) and the long-term trend is toward even less differentiation by quality. In recent decades, sawtimber prices have converged toward those of pulpwood, the lowest grade product (Parajuli et al., 2019). Nonetheless, the intensity with which these forests are managed is well documented, including costly investments in site preparation, genetic improvement, artificial regeneration, fertilization, chemical and mechanical competing vegetation control, and precommercial thinning (Allen et al., 2005; Fox et al., 2007; McKeand et al., 2021). All of these investments entail up-front, out-of-pocket expenses, borne in the expectation of higher future revenue. They require capital, foresight, and patience, yet such investments are common practice (Rogers and Munn, 2003; Arano and Munn, 2006; Zhang et al., 2012).

In contrast, New York's hardwood forests are not merely *underinvested in* but *disinvested from.* Though capable of growing some of the world's finest timber, few landowners try. Far more often, landowners and loggers "chase" what good wood is currently standing and give no apparent thought to the implications for future production (Nyland, 1992). For example, from a sample of 50 harvests in New York's Catskills Mountains, Munsell et al. (2009) observed almost uniformly unsustainable harvesting, categorizing just one job as consistent with silvicultural best practice. Similar results have been noted throughout North America's eastern hardwood region (Egan and Jones, 1993; Fajvan et al., 1998; Belair and Ducey, 2018). These patterns of "selective" cutting or high grading—removing the highest quality stems from a stand to the detriment of its long-term production potential—amount to silvicultural disinvestment. As opposed to steering capital *into* the forest in the expectation of generating a return, as observed in Mississippi, landowners in New York pay a premium to take capital *out of* their forests. In this way, high grading is the silvicultural equivalent of a payday loan: an unreasonably expensive way of converting a future-dated asset to cash.

What explains the differences between these two cases? Are New Yorkers just poorer, dumber, and more short-sighted than Mississippians? Unlikely (says a New Yorker). Are there differences in policy? Perhaps, but even within states where both forest types occur (e.g. between the piedmont and mountain regions of North Carolina) the pattern persists of capital-intensive pine management and hardwood silviculture that appears allergic to leaving value on the stump (Chizmar et al., 2021). Might the attributes of the trees or forests themselves explain the difference? How?

Though an intellectually interesting puzzle in its own right, an improved understanding of why we high grade would have practical implications. High grading is not a true tragedy in Whitehead's (1925) sense of "remorseless inevitableness" (p. 11) but is instead a remediable problem. This dissertation advances the argument that much of the degradation observed in (potentially) premium-quality hardwood forests is driven by informational constraints and the resulting transaction costs that arise when these forests are bought, managed, or sold. Eastern hardwood forests are less attentively managed than southern pine forests because the required management inputs are harder to monitor and the future gains that would result are harder to verify.

To illustrate the logic of transaction costs, consider a simple (if somewhat contrived) example. A farmer, who owns no woods, approaches the widow next door who owns a small hardwood forest. The farmer offers to give her a side of beef if he can cut some firewood from her property. The widow is happy to barter but wants to make sure that the farmer doesn't cut too many trees or the wrong kinds. She is planning to harvest her timber in 20 years and knows that she has a valuable stand because her husband had been a forester and tended it himself. The farmer agrees not to remove more than a maximum volume—say, 100 cubic meters—and to only take junky, firewood-quality trees. He will leave all of the straight, valuable trees to keep growing for timber.

As good neighbors, they trust each other. She also knows she can easily check to see that he keeps his word. Just by looking at his wood pile when the job is done, she will know that he took no more than the volume they agreed to and that the trees he cut were all poor-quality stems or low-value species. The low cost of *ex post* verification facilitates mutually beneficial trade.

Suppose, now, that a forester comes along and offers her services to the widow. She would supervise the farmer's work, designating which specific trees he cuts (to improve the growth of the very best stems rather than just getting rid of the ugliest ones) and then monitoring his felling and skidding to make sure no quality growing stock gets damaged. The forester says she can't be 100% certain of the payoff, but this work would certainly increase the odds of a valuable future harvest and she is confident this is a worthwhile investment:

> Across your 10-hectare woodlot, if the farmer cuts firewood unsupervised there is a 50% chance that your timber will be worth \$25,000 in 20 years and a 50% chance it will only be worth \$20,000. If I thoughtfully mark the job and carefully supervise his work, there is a 60% chance it will be worth \$30,000 and a 30% chance it will be worth \$25,000. Though it's still possible, there's just a 10% chance that it will only be worth \$20,000.

The widow is impressed. These estimates are right in line with the management plan her husband had written. The forester says she could do the work for \$150 per hectare. At the widow's discount rate of around 3.5%, a payoff 20 years in the future is worth half as much today, so the upfront cost of the forester's services is less (in present value terms) than the increase in the expected payoff. Figure 1.1 illustrates the structure of this silvicultural investment game.

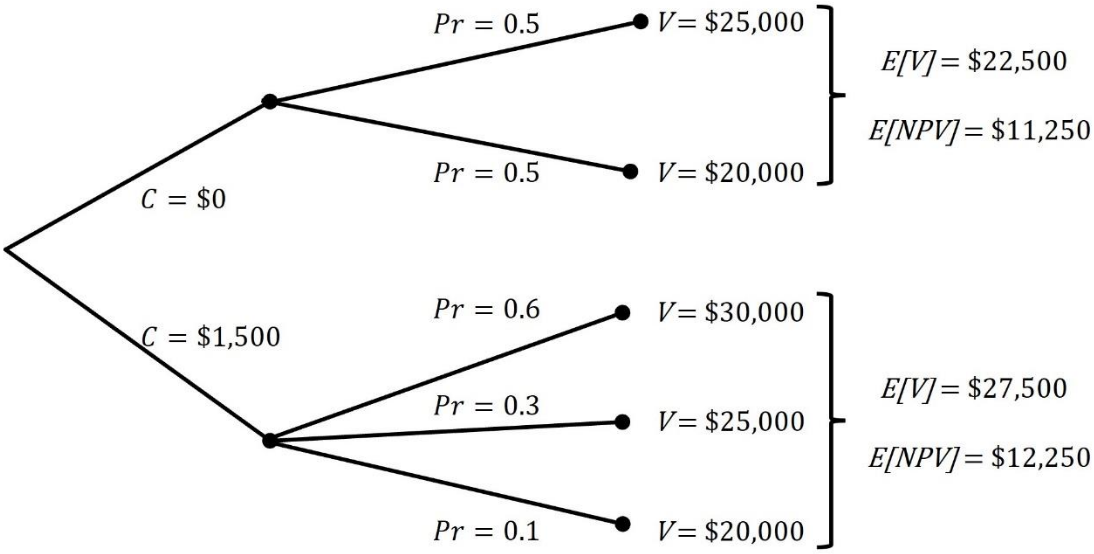

Figure 1.1: Non-strategic silvicultural investment game in extensive form

Just as the forester said, this appears to be a sound investment. And yet, the widow may have good reason to hesitate before hiring her. How would she know that the forester did the work she promised? Thoughtful silviculture requires intense focus and real cognitive strain. What would prevent the forester from skimping on that effort, pocketing the $1,500, and just letting the farmer proceed as he otherwise would have? There wouldn't be any obvious signals to look to during the job to determine how much effort the forester was putting in. From the landowner's kitchen window, a hard day of silvicultural analysis and logging supervision looks no different than an easy day with a short hike and a long picnic—in each case, she only sees the forester walking off into the woods at some point and coming back again later.

Similarly, even after the job was done the landowner would struggle to figure out if she had gotten her money's worth. On the surface, the residual stand conditions would look similar whether the forester had worked hard or not—some stumps in the woods and a pile of firewood on the landing. The adage about physicians, unlike foresters, getting to bury their mistakes (e.g. Ashton and Kelty, 2018, p. 80) belies the fact that the initial conditions of a stand—on which any assessment of the quality of the silviculture must be based—are lost to history once a job is cut.

And not even the final harvest, 20 years later, would necessarily reveal whether the forester had given it her all, 20 years earlier. High-touch silviculture would reduce, but not eliminate, the probability of the $20,000 or $25,000 outcomes. The occurrence of the best outcome *would* prove she exerted high effort, but a disappointing outcome wouldn't prove that she didn't.

Without a reliable accountability measure, the forester could avoid the cost of high-effort exertion but still collect the full payoff. Her best move is obviously to shirk. Anticipating that response, the landowner's best move is obviously to not hire the forester. Investing in silviculture, in this context, is an act of faith.

Some forests provide clearer signals than others on the quality of past management. U.S. President Lyndon Johnson purportedly said that the best fertilizer for a piece of land is the footprints of its owner, but actual fertilizer comes with receipts. Management-induced changes in structure, composition, and quality are more discernible in forests that are less complex, diverse, and differentiated. Shorter rotations return quality signals more frequently. Less trust is required to induce silvicultural investment in easy-to-read forests. Thus, Mississippians invest more in their forests than New Yorkers not because Mississippi foresters inspire more faith, but because loblolly silviculture demands less of it.

Forest owners (like everyone else) tend to economize on transaction costs, in part by avoiding the activities where those costs bite hardest. The solution to this problem lies in crafting new institutions or developing innovative contracting devices that mitigate transaction costs. What is called for is “public entrepreneurship” (Ostrom, 1990), “transaction cost engineering” (Gilson, 1984), or various other forms of “institutional work” (Lawrence et al., 2011) through which new institutions take shape. The research presented in this dissertation hopes to inform those who take up that challenge and guide the work that lies ahead.

2 LITERATURE REVIEW & RESEARCH METHODOLOGY

This section reviews the intellectual development of the main theoretical and methodological approaches incorporated into this dissertation: institutional theory, with an emphasis on transaction costs economics; the classical, rotation-based approach to the so-called "tree cutting problem" in forest economics; and the emerging individual-based approach to harvest optimization. The resulting work is a unique combination of ideas and approaches from across disciplines—Coase (transaction costs; 1937; 1960) meets Faustmann (optimal rotations; 1849) meets Gleason (individualistic concept; 1926), operationalized with heuristic optimization algorithms.

2.1 Institutions, transactions & contract

Imperfect information creates opportunities for one or both parties to a transaction to increase their own payoff at the expense of the counterparty. The owner of a used car known to be a "lemon" may try to represent it to an uninformed buyer as a "peach" in hope of selling it for more than its true value (Akerloff, 1970); an unscrupulous taxi driver may zig-zag back and forth across the city with the meter running as they take a tourist from the airport to their hotel (Dulleck and Kerschbamer, 2006); a farmer leasing land on crop share may put down less fertilizer than they would use on their own land (Cheung, 1969; Barzel, 1997; Allen and Lueck, 1998). Omniscient car buyers, taxi passengers, or farmland lessors would be unconcerned by these risks—with perfect information the car buyer offers no more than the fair price, the passenger demands the driver take the direct route, and the landowner stipulates best practices from their tenant. In the real world of imperfect information, however, such hazards are everywhere. The costs of safeguarding against these losses and the costs of the value lost when such opportunistic behavior does occur are called *transaction costs* (Allen, 1991).

Coase (1937) first drew attention to the role of transaction costs in the organization of economic activity. Coase asked why a firm would choose to do some tasks in-house and turn to the market (i.e. "the price mechanism") for others. He concluded that "main reason why it is profitable to establish a firm would seem to be that there is a cost of using the price mechanism" (Coase, 1937, p. 390). Though largely unnoticed by the rest of the economics profession, this insight re-emerged in Coase's important later work (1959; 1960) and eventually—through Cheung (1969), Alchian and Demsetz (1972), Williamson (1975), and others—was rediscovered and became the foundation of the New Institutional Economics.

"Coase's lasting contribution is the idea that when there are transaction costs—when people engage in bad behavior—then rules do matter and different institutions have different social outcomes" (Allen, 2012, p. 19). The view of institutions favored by economists, as "the humanly devised constraints that shape human interaction," or simply "the rules of the game" (North, 1990, p. 3), differs from the broader, sociological perspective of institutions as enduring social patterns (Hughes, 1936). Rules (or the absence of rules) affect "[t]he opportunities and constraints individuals face in any particular situation, the information they obtain, the benefits they obtain or are excluded from, and how they reason about the situation" (Ostrom, 2005, p. 3).

From a social perspective, the goal is to craft a set of rules (i.e. an institutional arrangement) that incentivizes wealth-maximizing cooperation, net of the parties' "search and information costs, bargaining and decision costs, policing and enforcement costs" (Dahlman, 1979, p. 148). Following Alchian's (1950) evolutionary logic, many economists in the transaction costs tradition presume extant institutional arrangements to be efficient. Cheung (1969) recognized efficiency as a "condition of market equilibrium logically deduced" (p. 159) rather than as a normative concept pregnant with welfare implications. The concept of efficiency is just an analytical constraint—a device economists employ to do economics better—rather than a tool for improving the world around us (Allen, 2018). Methodologically, "efficiency" only implies that the economist has sufficiently constrained the maximization process (Staten and Umbeck, 1989). An economist's first job is to try to understand the world around them. That job is over before it begins if they think they already know all the answers.

Williamson's (1996) "discriminating alignment hypothesis" is consistent with this positive approach to institutional analysis. This hypothesis states that "transactions, which differ in their attributes, are aligned with governance structures, which differ in their cost and competence, so as to effect a (mainly) transaction cost economizing result" (Williamson, 1996, p. 41). Williamson emphasizes asset specificity (leading to "hold-up" problems or the risk of *ex post* appropriation), uncertainty (i.e. measurement issues), and frequency (resolving measurement issues and allowing or not allowing for relational contracting). Increased asset specificity or uncertainty, or decreased frequency of transacting, lead to increased transactional hazards. In response, contracting parties adopt more costly safeguards or simply transact less.

Other scholars—particularly those adjacent to or entirely outside of economics—have approached institutional analysis from a more entrepreneurial perspective. Ostrom's initial approach to institutional analysis was to use the tools of the policy scientist to examine the role of "public entrepreneurs" in transforming incentives to overcome collective action problems (Nordman, 2021). Ostrom always thought of her research "as part of a general effort to understand institutions so as to provide a better formulation for improving their performance" (Ostrom, 2005, pp. 30-31).

In the field of Law and Economics, many scholars have emphasized the potential for value creation through innovative contract design. Gilson (1984) echoed Alchian (1950) in noting that "[j]ust as competitive conditions create incentives that encourage reduction of production costs, the market also encourages private efforts to reduce transaction costs" (p.254) but understood that evolution as an active process. Where some economists would presume that all efficiency gains have already been eked out through institutional evolution, legal scholars allow more scope for on-going innovation in contracting. Business lawyers (in Gilson's formulation) are natural "transaction cost engineers" when designing contracts and should take an active role in identifying and coping with transaction costs in complex deals (Goldberg, 2019).

Sociologists and organizational theorists also contest economists' presumption of efficiency in extant institutional arrangements. Sociologists understand institutions as the persistent social structures that shape individual and collective behavior and beliefs by providing templates for action, cognition, and emotion and by imposing costs on nonconformance (Meyer and Rowan, 1977; Scott, 2001). This conception maintains a role for rules and incentive structures but asserts a larger role for individual and social psychology and cultural influence. Earlier theorists suppressed the agency of actors embedded in institutions (DiMaggio and Powell, 1991), but contemporary scholarship recognizes that agents have the capacity to purposefully create, maintain, or disrupt institutions to serve their goals or advance a collective aim through "institutional work" (Lawrence et al., 2011). The focus of much institutional work scholarship is how, why, and when people work to shape institutions, the factors that affect the success or failure of those efforts, and the descriptive experiences of the actors engaged in that work (Hampel et al., 2017).

The theoretical approaches and analytical methods of policy analysts, Law and Economics scholars, and institutional work theorists differ from those employed directly in this dissertation, but they all share in the insight that institutions are susceptible to change and that such change can be directed with intentionality and effort by people willing to engage with them. This insight invites not just the analysis of persistent social problems, such as high grading, but encourages the active search for their solutions. Economists' framing of institutional analysis through the transaction costs lens provides a conceptual toolkit to guide that search, as well as a helpful note of caution (or plea for humility) to those whose analysis *begins* with the assumption that they are better equipped to "fix" the situation than the people living with the problem (Allen, 2018).

2.2 Optimal harvesting theory

The optimal rotation problem is the paradigm case in classical forest economics. When is the optimal time to harvest a growing stand of timber and begin the next rotation? Faustmann (1849) provided the first fully-developed solution to the problem, which Pressler (1860) later formalized and Samuelson (1976) much later popularized, sparking what became known as the Faustmann revival (Newman, 2002).

The textbook solution to the standard rotation problem has been presented in detail elsewhere (e.g. Amacher et al., 2009, pp. 12-23) and will only be reviewed briefly here to develop the supporting intuition. A stand should not be harvested until the gross returns from letting it continue to grow no longer exceed the opportunity costs. The optimal time to harvest is when the marginal costs equal the marginal benefits of delaying harvest.

The benefits of delaying harvest for an additional growing period are easy to conceptualize, even if hard to quantify precisely. They simply represent the change in the stand's harvestable value from one period to the next as its trees grow larger.

Conceptualizing the opportunity costs of delaying harvest is less straightforward. Each passing year entails a missed opportunity to earn an outside return on the reinvested harvest revenue. Just as the trees could have been growing larger on the stump, so too could the capital embedded in those trees have grown larger in the bank, had they been harvested. Moreover, delaying harvest of the current crop pushes back the eventual harvests of each subsequent crop, which in a sustainable production system are expected to extend indefinitely into the future.

Analytically[1], the optimal decision rule for harvesting the stand is given by:

$$\Delta V(T) < rV(T) + rLEV$$

where $V(T)$ is the production function that relates the stand's timber value to its age T; $\Delta V(T)$ is the stand's forward-looking value increment, such that $\Delta V(T) = V(T+1) - V(T)$; r is the discount rate (i.e. the opportunity cost of capital or the expected rate of return from similarly risky investments); and LEV is the present value of all future harvest revenue, referred to as the land expectation value (defined in more detail below). Delaying harvest by a year allows $\Delta V(T)$ of additional timber value to accrue but entails foregoing a year's worth of interest on the initial harvestable value and reduces the present value of future revenue by a factor of r as each future cashflow is discounted an additional year. The decision rule states that the stand should be harvested when the opportunity costs of delay first exceed the next year's projected value growth.

LEV is the discounted value of an infinite series of future harvests of T-aged stands, starting from bare land and occurring every T years, which is calculated by:

$$LEV = \frac{V(T)}{(1+r)^T - 1}$$

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[1] Though the standard approach in forest economics works in continuous time, the following analysis is presented in discrete time which corresponds more closely to the practical logic of harvesting decisions and thus facilitates more natural interpretation of the underlying economic ideas.

where $[(1+r)^T-1]^{-1}$ is the periodic perpetuity factor that gives the present value of a cashflow that recurs every T years over an infinite horizon (see Zhang and Pearse, 2012, p. 94, for a derivation). Importantly, the rotation age T that maximizes LEV is the same T the stepwise application of the harvest rule presented above would dictate.

Samuelson's famous treatment of the problem differed in style from the preceding analysis but illustrated the same underlying economic logic. Samuelson (1976) began his exposition by noting that "[i]f an unambiguous solution to the problem is to be definable, of course certain definite assumptions must be made. If the solution is to be simple, the assumptions must be heroic" (p. 471). Samuelson explicitly acknowledged four assumptions: (1) known and constant input costs and output prices; (2) a known and constant interest rate prevailing across perfect capital markets, i.e. a fixed rate at which a forestry investor "can both borrow and lend in indefinite amounts" (Samuelson, 1976, p. 472); (3) a known and deterministic biological growth function; (4) perfect markets for forestland.

Closer examination of Samuelson's work also reveals a set of five implicit assumptions. The first three closely follow Löfgren's (1990) notion of a linear forest: point-input/point-output production (i.e. clearcut silviculture without thinning); perfectly homogenous site conditions and stand structure; and undifferentiated quality. In such a model, one representative tree can suitably describe the dynamics of the entire stand.

A fourth implicit assumption relates to the interest rate. A known and constant intertest rate implies that both components of the market interest rate—the risk-free rate of return and the market risk premium (Sharpe, 1964)—are constant. Even so, the asset-specific discount rate for a stand of timber could evolve over time if the correlation between that stand's returns to those of the overall market varies at different stages of the rotation (Insley and Wirjanto, 2010; see Fama, 1977 for a general discussion). For example, if the variance in the price of pulpwood is low and largely uncorrelated to other asset markets, but sawtimber prices are more strongly correlated, then different discount rates would be called for at different times in the rotation, even if "the" interest rate remains constant. Samuelson's analysis implicitly assumes otherwise.

Finally, Samuelson's model assumes that the analytically optimal solution can be costlessly implemented—a world of zero transaction costs. Samuelson entertains the concern that "when what is at issue is a tree ... whose full fruits may not accrue until a century from now, the brute fact that our years are numbered ... prevents people from planting the trees that will not bear shade until after they are dead – altruism, of course, aside" (p. 476), but then waves it away:

> To argue in this way is to fail to understand the logic of competitive pricing. Even if my doctor assures me that I will die the year after next, I can confidently plant a long-lived olive tree, knowing that I can sell at a competitive profit the one-year-old sapling. Each person's longevity and degree of impatience to spend becomes immaterial in a competitive market place with a borrowing, lending, and capitalizing interest rate that encapsulates all which is relevant about society's effective time preferences (Samuelson 1976, pp. 476-477).

By this reasoning, landowners will take up costly olive planting—or, by extension, any form of silvicultural investment for which the capitalized payoff exceeds the initial cost—confident that the secondary market will fully price-in the expected future gains from that investment. But this "logic of competitive pricing" clearly overlooks the role of transaction costs. Samuelson may have arrived at a different conclusion if, rather than olive saplings, he had focused his analysis on the market for lemon trees.

Even in the simplest case of planting trees, consider the many dimensions over which a tree-planter might shirk or shade on quality if they were insulated from the resulting costs: the genetic quality of the seedlings; the care with which they were planted; the amount and quality of fertilizer, irrigation, or pest control measures supplied. Each activity is both expensive and hard to verify after the fact. Prudent buyers would have good reason to suspect that an opportunistic seller invested less in these areas than they claimed and would discount their offers accordingly. Wise sellers, anticipating that discount, would underinvest in establishing their plantation, knowing that the full cost of their efforts would not be recouped.

That many of these practices do occur in plantation forestry systems, as discussed in the introduction to this dissertation, does not suggest that transaction costs are absent, but rather that institutional "technologies" exist that at least partially mitigate their effects. Many of the investments in an intensive plantation system, such as genetically-improved seedlings or fertilization, entail physical inputs of a type that receipts from third-party vendors can easily verify. Others, such as site preparation and tree planting, are mechanized operations with less variability in outcomes (and less scope for shirking) than manual tasks. Others still, such as pruning or precommercial thinning, are relatively easily verifiable *ex post* given the uniform nature of the treatment and the stand's homogenous structure: a buyer that can see a line of stumps every third row is likely to accept the seller's claim to have done a third-row thinning.

Despite its limitations, the standard rotation model can thus accommodate analysis of simplified silvicultural systems reasonably well. Management inputs, including mid-rotation thinning, can be consolidated into a single, up-front cost component that enters into the production function and analysis can proceed without further modification (Chang, 1983). Transaction costs need not be analyzed explicitly in the context of rotation decisions but can instead be incorporated directly into the cost function prior to that analysis.

Standard rotation analysis can abstract from transaction costs because technologies exist to mitigate them. Nevertheless, there is still a margin beyond which the payoff from further transaction cost mitigation no longer exceeds the cost. Critically, that margin shifts in settings like hardwood forests where the nature of silvicultural investment is more complex and the informational environment is more opaque. A closer examination of those dynamics requires visibility into the drivers of that complexity, such as dynamic thinning responses, spatial heterogeneity, and differentiated timber quality. The classic analytical model of forest management, in which uniform timber is produced from linear forests and sold into static markets where information and institutions play no role, is ill suited for that task.

2.3 Individual-based modeling & tree-level optimization

The tools and techniques for optimizing complex silvicultural regimes were developed in parallel with the modern Faustmann-inspired literature. Though both approaches are grounded in a shared logic of economic optimization, their methods are notably different. Unlike the closed-form solutions of standard rotation problems and their extensions, analysis of more sophisticated management regimes requires numerical optimization methods. In this setting, end-of-rotation decisions represent just one dimension of a higher-dimensional tree-cutting problem.

Early silviculturalists marveled at the complexity of the thinning problem. One tree's removal benefits the growth of its neighbors, but a landowner must forfeit the future value the harvested tree could have added. Each possible combination of thinning and retention sends the stand along a distinctive new trajectory, each with its own unique payoff. Over most of the history of professional forestry, simply projecting growth responses to thinning—to say nothing of optimizing thinning decisions—was out of reach. Hence, Baker's (1934) lament that "[t]hinnings depend upon obscure and difficult laws of forest growth for their efficiency" (p. 358). But even once "the purely biological inferences of the growth process [were] clarified" (Assmann, 1970, p. 4) as the science of growth modeling advanced (Ek et al., 1988; Weiskittel et al., 2011; Mäkelä and Valentine, 2020), the challenge of integrating those models into economically rigorous decision models remained daunting.

For all but the most grossly simplified applications, the thinning problem proved too complex for economists' standard analytical tools. Even in the simplest relevant practical setting—say, an even-aged monocultural system with no quality assortments—the production function has too many feedbacks, and the decision space too many dimensions, for a closed-form approach to be practical. Only within the past decade did Coordes (2014) provide the first fully analytical, theoretically complete treatment of the thinning problem, upon which he concluded that "the viability of the scientific [i.e. economic] management of forest stands for profitable timber production is doubtful" (p.168) given the complexity of the solution.

Not by coincidence, then, the thinning problem was the subject of some of the earliest studies in forest economics and operations research that employed numerical methods, such as dynamic programming (e.g. Arimizu 1958; Näslund 1969; Schreuder, 1971). Adams and Ek (1974; 1975), Clark (1976), Haight (1987), Getz and Haight (1986; 1989), and Haight and Monserud (1990) provided important further advances in numerical optimization methods, leading to development of the ecologic-economic optimization approach now referred to as the "Helsinki School" of resource economics (e.g. Tahvonen and Salo, 1999; Hyytiäinen and Tahvonen, 2002; Hyytiäinen et al., 2004; Tahvonen et al., 2010; Tahvonen and Rämö, 2016; Parkatti et al., 2019; Assmuth et al., 2021).

At the center of the ecologic-economic optimization approach is a model coupling methodology, which pairs "abstract economic models of optimization and ecological models of population growth" (MacLeod and Nagatsu, 2016, p. 420). When these models are properly integrated, the optimization methods ensure economic rigor while the ecological sub-models provide a reasonable tether back to biological reality.

This modeling approach now employs optimization methods that are *a priori* agnostic on silvicultural strategy, such as the choice between a clearcutting regime and continuous cover forestry (e.g. Tahvonen and Rämö, 2016). Because the timing and intensity of intermediate cutting strongly influences the timing (and perhaps type) of eventual regeneration decisions, it follows that an effective modeling strategy needs to handle thinning and regeneration simultaneously. Thinning and regeneration are determined by a single vector of cutting decisions—specified via numerical optimization methods—which in turn controls the stand's evolution over time and its resulting cashflows.

In forest resource economics, the main line of the Helsinki School operates primarily at the stand scale, using size-class-level harvest percentages as the control variable. This strategy is well-suited for Nordic forests, which typically have more species diversity and a broader distribution of size classes than monocultural plantations, but which are still simple enough in terms of stand structure and quality differentiation to be effectively managed with uniform silvicultural treatments. Nordic thinning strategies are thus more complex than the geometric methods employed in plantations but nevertheless aim to maintain relatively consistent composition, density, and structure across a stand. Size-class-level modeling approaches reduce the harvesting problem to only as many dimensions as there are defined size classes and management periods.

An alternative approach is emerging that operates at finer spatial scales but is more cumbersome computationally. Early ecologists pioneered a mechanistic view of plant associations and population dynamics (Gleason, 1926; Watt, 1947) which provided the conceptual foundations for individual-based modeling in forest ecology (e.g. Botkin et al., 1972; Mitchell, 1975; Shugart, 1984; Pacala et al., 1993; Bugmann, 2001; Pretzch et al., 2002; Pommerening and Grabarnik, 2019). Forest economists have also taken an interest in tree-level decision processes in recent decades (Moog, 1990; Yoshimoto et al., 1990; Moog and Karberg, 1992; Pukkala and Miina, 1998; Hof and Bevers, 2000; Hagner et al., 2001; Härtl et al., 2010). Only recently, however, have computing methods advanced sufficiently to operationalize tree-level optimization research (e.g. Meilby and Nord-Larsen, 2012; Vauhkonen and Pukkala, 2016) leading to a flourishing of scholarship among clusters of researchers

(Lohmander, 2019; Dong et al., 2020; Frannson et al., 2020; Packalen et al., 2020; Pascual, 2021a; West et al., 2021; Koster and Fuchs, 2022; Sun et al., 2022; Dong et al., 2022; Pascual and Guerra-Hernández, 2022).

Tree-level optimization problems are inherently more complex than stand- or size-class-level problems because of the exploding combinatorial nature of the problem space. A single stand might easily contain thousands of trees. Even if the solution is constrained to a discrete number of potential harvest entries (e.g. one entry every ten years), the number of individual-tree cutting combinations is inconceivably large. Consider a stand with "just" 1,000 trees, each of whose removal was limited to ten potential entries over a 100-year horizon. The different potential combinations of individual-tree cutting schedules is a number so large—$10^{1,000}$—as to be literally inconceivable. Following some of Bookstaber's (2017) examples, imagine a supercomputer that could evaluate 100 trillion potential solutions to this problem per nanosecond. If such a computer could be shrunk to a single cubic nanometer, and if the entire volume of the universe were occupied by such computers, and if those computers ran continuously for the next 100 trillion years, only 10^{168} possible solutions to this unrealistically simplistic problem would have been evaluated—much, much less than one trillionth of a trillionth of a trillionth of a trillionth of a trillionth of a trillionth of the $10^{1,000}$ potential solutions.

Researchers employ different strategies, often in combination, to overcome this challenge. Most tree-level optimization studies use heuristic optimization tools to find near-optimal solutions within the vast problem space (e.g. West et al., 2021). Heuristic optimization methods search for near-optimal solutions to complex problems, often through an iterative selection process that mimics natural processes such as genetic algorithms, particle swarm optimization, simulated annealing, or ant colony optimization (see Gilli and Winker, 2009 for a general overview).

Another tactic in tree-level optimization research is to reduce the dimensionality of the problem by isolating non-interacting (or only weakly interacting) components of the system. Stand sizes are often reduced from typical operational scale (10^1-10^2 ha) to plot scale (10^{-2}-10^0 ha) (e.g. Frannson et al., 2020; Koster and Fuchs, 2022; Sun et al., 2022). Analysis may be limited to a single, representative plot, or multiple, non-interacting plots can be aggregated into a stand, following the gap model framework from forest ecology (Botkin et al., 1972; Shugart, 1984; Pacala et al., 1993).

Finally, many tree-level optimization studies optimize the decision rules applied to individual trees, rather than specifying the optimal timing or size threshold of each individual tree's removal (e.g. Pukkala et al., 2015; Lohmander, 2019). This approach often entails weighting various decision criteria, such as current value, projected growth increment, or neighborhood competitive influence, and then defining a cutting rule based on those criteria. This approach is significantly less computationally demanding than optimizing decisions over each individual tree but produces solutions that are necessarily farther from the true optimum.

Most tree-level optimization studies optimize on financial performance, but not all. Several studies have optimized stand structure for habitat or other conservation values (e.g. Bettinger and Tang, 2015; Dong et al., 2020) and Pascual (2021b) used a mixed integer programming approach to map Pareto frontiers across financial performance, timber volume production, harvesting efficiency and a stand structural index. The third and fourth manuscripts in this dissertation appear to be the only studies in the tree-level optimization literature that employ the approach to explore institutional questions or incorporate informational dimensions of the problem. Tree-level analysis allows for a much richer representation of the informational environment than stand-level approaches, which inherently aggregate attributes such as quality or vigor that are naturally differentiated among trees. Maintaining visibility into these dimensions of stand composition and structure allows for analysis of situations where access to that information is determinative of optimal management, including those where it is asymmetrically distributed between parties.

3 MANUSCRIPT I

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This study explores the development of individual- and neighborhood-based methods in forest ecology and silviculture and argues for the benefits of incorporating similar methods into forest economics. It presents evidence of the technical advantages to modeling forest demographic processes at finer spatial resolutions. Such models often provide more accurate predictions of tree-level growth and mortality. In a practical sense, if silviculturalists increasingly work at the neighborhood scale—formulating and implementing prescriptions at that level of resolution—then economists should meet them where they are. This study explores the implications of Jensen's Inequality in an optimal control setting, where the aggregation problem causes neighborhood- and stand-level optimal control paths to diverge and can introduce substantial bias into optimization models. This study then presents a simple model of a landowner's tree-level thinning decision and operationalizes this model with a numerical application. That analysis illustrates the scale of the potential gains from shifting silvicultural decision making from stand- to neighborhood-scale resolution.

Economic analysis at the neighborhood scale: Toward a new framework for forest valuation, management and research

Abstract

Shifting analysis to individual trees and their immediate neighborhood environments represents an important development in the continued evolution of forest economics. We review the history of individual- and neighborhood-based approaches in ecology and silviculture, and find evidence from those fields that neighborhood models of tree growth and dynamics are more descriptive than coarser-scale models, particularly for the diverse, structurally complex stands that are of increasing interest and importance to forest managers and society. Moreover, we show how economic analysis of heterogeneous-quality forests is highly susceptible to aggregation problems. A thinning-and-harvest schedule optimized on the basis of stand-average stocking, structure or demographics, and applied uniformly across the stand, will generate systematically less value than the aggregate value produced from unique cutting schedules optimized for individual neighborhoods. From Jensen's inequality, the optimum of the mean is not the mean of the optima. We then outline an iterative, forward recursive solution method to optimize individual-tree thinning and retention decisions at the neighborhood scale, and illustrate this neighborhood-based approach with a numerical example. Finally, we discuss the operational advantages of this approach and briefly explore some new avenues of economic research that this framework could open up.

3.1 Introduction

Forest ecologists have increasingly turned their attention to the fine-scale interactions between individual trees and their close neighbors. Forest economists, in general, have not. In this paper we make the case for situating economic research at the neighborhood level of analysis. We provide a brief history of the development of the theory of neighborhood dynamics in ecology and its application in silviculture and forest management, and we explore some of the key implications of this theory for economic analysis. Our central proposition emerges naturally from this context: the appropriate frame for studying the economics of complex stands employs the neighborhood as the level of analysis and the individual tree as the unit of observation.

There are unique challenges in operationalizing this approach without deviating too far from the spirit of neighborhood dynamics. We introduce a discrete-population, discrete-time harvest decision model built around a neighborhood-scale, distance-independent model of individual tree growth. This deterministic setup allows for an iterative solution by a forward dynamic programming procedure, which we outline. We provide an illustrative application of the model to highlight its potential implications for forest valuation and management, and we discuss some new lines of forest economics research this approach might open up.

3.2 Neighborhood dynamics in ecology and silviculture

In his seminal paper, "Pattern and process in the plant community," Watt (1947) articulated the challenge of studying the structure and dynamics of plant communities, including forests: "The ultimate parts of the community are the individual plants, but a description of it in terms of the characters of these units and their spatial relations to each other is impracticable at the individual level" (Watt, 1947, p. 1). Much of ecological theory sidesteps this challenge by invoking the "mean-field" assumption. That assumption posits that individual organisms interact with each other in proportion to their average abundance across space (Dieckmann et al., 2000). The assumption is likeliest to hold in physically homogenous environments with populations of highly mobile organisms (either of their own accord or subject to strong exogenous mixing forces) or among organisms that interact with each other over long distance. In systems where these conditions don't hold—of which forests are a conspicuous example—the immediate neighborhood environments around individuals tend to deviate from the spatial average (Dieckmann et al., 2000, p. 4).

Watt suggested that an intermediate scale exists between the levels of the individual and the community that was not "impracticable" to study, but still rich enough in detail to offer insight into the functional processes structuring plant communities. As his own work demonstrated, detailed empirical investigations were "feasible in terms of the aggregates of individuals and species which form different kinds of patches; these patches form a mosaic and together constitute the community" (Watt, 1947, pp. 1-2).

The neighborhood approach essentially agrees with Watt (1947) that individuals are the "ultimate parts of the community" (p.1) but contends that it is now feasible to model interactions between individuals. Individual-based models—in which each individual tree is tracked through time as it is established, grows and eventually dies, all contingent on the evolving structure of its immediate neighborhood (Judson, 1994; Grimm and Railsback, 2005; Pommerening and Grabarnik, 2019)—encapsulate the theory of neighborhood dynamics (Canham and Uriarte, 2006).

Though undoubtedly more demanding, the neighborhood approach better reflects several broad principles of plant-plant interactions that field ecologists have established with confidence: "plants do interact locally; local crowding reduces plant growth, reproductive output, and probability of survival; the effect of neighbors attenuates with distance (although the nature of this attenuation is not well understood); beyond a certain distance plants have no detectable effect on each other" (Stoll and Weiner, 2000, p. 18). Resource-mediated competition is generally thought to be the primary channel through which plants interact to affect each other's growth and development (Reineke, 1933; Tilman, 1982; Bazzaz, 1990) and these interactions turn almost entirely on competition for *local* resources.

Ecologists have formulated the neighborhood concept in a variety of ways (Stoll and Weiner 2000). One class of models relies on spatial data for every individual tree. These distance-dependent individual-based models, such as SORTIE (Pacala et al., 1996) and SILVA (Pretzsch et al., 2002), construct a unique index of available resources or a zone of competition, respectively, for each tree, based on its crown dimensions and those of its neighbors. We focus here on the simpler, distance-independent approach. It essentially describes neighborhoods as areas in space that are sufficiently small that a mean-field approximation adequately describes the within-neighborhood environment.

Specifically, the approach assumes that within the area of the neighborhood, competition among individual trees is homogeneous in the horizontal dimension (though it may be spatially-explicitly modelled in the vertical dimension), so precise within-neighborhood spatial locations can be disregarded (Shugart et al., 2018). All individuals within a neighborhood can potentially interact, but no interactions occur between individuals in different neighborhoods or among neighborhoods themselves. Models that operate within this framework are often referred to as "gap models" (Shugart and West, 1980; Bugmann, 2001). Development of the JABOWA simulation model (Botkin et al., 1972) represented a particularly significant development. It is the "parent" to hundreds of subsequent models or variants (Bugmann, 2001) and its model structure serves as the point of departure for our proposed approach to neighborhood economic analysis.

As with many theoretical developments, the theory of neighborhood dynamics followed a different path in silviculture and forest management than it did in ecology. "The stand" has been called "the foundational concept in forestry" (O'Hara and Nagel, 2013, p. 335). The tension between the stand as an administrative convenience versus the stand as an ecological descriptor has always been present (Peuttmann et al. 2008), so it is unsurprising that re-evaluation of the stand concept from a purely ecological perspective has been contentious. For example, over 100 years ago Mayr (1909) noted that stand delineation often lacked an ecological basis, suggesting that mini-stands, often less than a hectare, would otherwise predominate. Schädelin (1934) emphasized the inherently local nature of appropriate-sized management units, with each unit consisting of a group of competitors anchored around a high-value tree. Smith (1962), on the other hand, warned forcefully against indulging in such an approach, which would lead to a "chaos of little stands" (p. 467).

Over most of the history of forestry as a profession, this tension was resolved in favor of the stand as an efficiently-large operational unit. The *Dictionary of Forestry* (Helms, 1998) defines a stand as "a contiguous group of trees sufficiently uniform in age-class distribution, composition, and structure and growing on a site of sufficiently uniform quality, to be a distinguishable unit" (p. 92). Taken together with Oliver's (1986) definition of silviculture as "the technical service of converting forestland to stands of particular structures, tree sizes, and species compositions for particular uses" (p. 32), these statements highlight the extent to which 20th century forestry emphasized the creation of homogenous stands.

Recent years, however, have seen a noticeable shift by researchers and practitioners away from this approach. In Europe, neighborhood-scale treatments play a prominent role in traditional *Plenter* systems (Schütz, 1999), in the increasingly common close-to-nature approaches (Bauhus et al., 2013), and in the related "freestyle silviculture" method (Boncina, 2011). The synonymous "future tree" (*Zukunftsbaum*) or "frame tree" concepts in German and British usage, respectively, focus treatments on individual trees and are now widely employed in continuous cover forestry (CCF) systems and for thinning even-aged stands (Pommerening and Grabarnik, 2019). In North America, as well, individual-tree treatments or assessments have become more common in a variety of silvicultural settings, including in even-aged crop-tree management systems (Miller et al., 2007), uniform single-tree selection systems (e.g. Webster et al., 2009) and non-uniform systems (e.g. Nolet et al., 2013). A prominent review of silviculture in the United States over the 30-year period ending around 2016 highlighted the growing prevalence of multi-aged silvicultural systems that target treatments at the sub-stand level, thus emulating the effect of partial disturbance regimes (D'Amato et al., 2018). Pommerening and Grabarnik (2019) provide a thorough review of the historical development and recent advances in individual-based forest management research.

Puettmann and colleagues' critical assessment of silviculture (2008) consolidated much of the thinking behind this shift and presented the rationale for a more ecologically-informed, neighborhood-based approach to silviculture and forest management. Beginning with the recognition that "silviculturists manage the establishment, survival, and growth of trees and all these demographic processes unfold at local neighborhood scales" (Puettmann et al., 2008, p. 103; cf. Oliver and Larson's "Silviculture is the manipulation of forest stands" formulation [1996, p.6]), they argue that effective silvicultural treatments should therefore "specifically consider the scale of the processes that are managed and they should be *applied at that scale*" (p. 138, emphasis added).

3.3 Economic rationale for neighborhood-scale analysis

The central argument of this paper—namely, that adopting a neighborhood dynamics perspective could improve applied forest economic analysis and enrich many lines of forest economics research— rests on three general points. First, as discussed above, neighborhood models of tree growth processes are more descriptive than coarser-scale models. Tree growth underpins every production function in forest economics, so the validity of those models is of obvious relevance to the field. Second, economic analysis of heterogeneous forests is highly susceptible to aggregation problems. The mean of the optima is not the optimum of the mean. In general, forest economists have not grappled with the implications of this insight. Finally, the neighborhood is a tractable scale for approaching some of the remaining unanswered questions or unexplored topics in forest economics. Description of many economically relevant phenomena requires visibility down to the individual-tree level, or would at least benefit from analytical resolution at that scale. Neighborhood analysis provides that resolution in a manageable way and thus opens up new potential lines of inquiry.

The preceding section of this paper addressed the first point, and the third point will be taken up in the sections on operationalization and applications. The remainder of this section will address the second point, on aggregation problems, in detail, followed by discussion of the existing body of forest economics literature that operates in or adjacent to the neighborhood dynamics framework.

Jensen's Inequality (Jensen, 1906; see also Dixit and Pindyck, 1994) states that if x is a random variable and $f(x)$ is a convex function of x, then $\bar{f}(x) > f(\bar{x})$, where $\bar{x}$ denotes the mean of x and $\bar{f}(x)$ denotes the mean of f over all x's. Naturally, the inequality is reversed for concave functions of x. Implications of Jensen's Inequality have been discussed in the ecology literature (e.g. Cale et al., 1983; Ruel and Ayers 1999, Duursma and Robinson, 2003) and some forest biometrics studies have examined plot aggregation as a source of bias (e.g. Moeur and Ek, 1981; Sambakhe et al., 2014; Green et al., 2019). In forest economics, however, consideration of Jensen's Inequality has generally been limited to stochastic stand- or forest-level processes, such as arrival rates for natural disturbances or timber price fluctuations (e.g. Plantinga, 1996). In this discussion, we instead examine the aggregation problem as it arises in an optimal control setting, causing neighborhood- and stand-level optimal control paths to diverge.

Suppose that all the timber-quality attributes that relate to potential value production could be reduced to an index, q, such that q represents the probability, as a percentile, of a neighborhood of a specified quality being drawn from the distribution of all neighborhoods. We contend that for most economically relevant scenarios, and particularly for high-value hardwood forests, the distribution of maximized net present value (NPV) over plots is a convex function in q.

Begin with the observation from Miller et al. (2007) that "nearly all of the economic value in hardwood stands is found in a relatively small number of trees" (p. 10). In their study of an unmanaged 53-year-old upland hardwood stand in the eastern United States, they measured and ranked 441 trees by current timber value (CTV). The top ten trees contributed 45% of the stand's CTV, followed by 18% from the next ten trees, and then 13%, 10%, 7%, 5%, and 1% from each of the next sets of ten trees up to the 70th most valuable tree in the stand; the 371 least valuable trees together contributed just 1% (Miller et al., 2007). Shifting from trees to neighborhoods, Arner et al. (1990) calculated the annualized compound rate of value change over ten years for more than 1,100 plots (0.04-0.08 ha) in New England. They found the cumulative distribution of value growth rates was also strongly convex. That such a small proportion of trees or plots attain the highest quality reflects the "Anna Karenina" principle: all premium-quality trees are alike; each low-quality tree is low-quality in its own way.

The assertion that for most stands of high-value hardwood species, optimized plot NPV is a convex function in q thus appears justified. From Jensen's Inequality, mean-field-based economic analysis of heterogeneous stands will misspecify the optimal thinning and harvest regime, leading to the systematic undervaluation of these stands. Implicitly, this is an important rationale for the observed "over-performance" of uneven-aged management systems with target diameter harvesting rules vis-à-vis uniform even-aged methods (e.g. Hanewinkel, 2001; Knoke and Plusczyk, 2001; Nord-Larsen et al., 2003). Put simply, a stand-level approach leaves bad trees standing too long and cuts good trees down too soon.

In spite of the recent developments in adjacent fields and a compelling economic rationale, neighborhood-scale analysis in forest economics research has been limited to a small number of studies. Matrix models (e.g. Buongiorno and Michie, 1980) and related age- or size-class models (e.g. Adams and Ek, 1974; Tahvonen et al., 2019) have been used to explore increasingly complex optimization problems, but at a coarser level of analysis than the individual tree and within a framework that still relies on mean-field assumptions. Many studies have examined optimal harvesting from an individual-tree perspective, in both North America (e.g. Godman and Mendel, 1978; Reed and Mroz, 1997; Webster et at., 2009; Demchik et al. 2018) and Europe (e.g. Moog and Karberg, 1992; Kuper, 1994; Hagner et al., 2001), but until Härtl et al. (2010) did not examine neighborhood competitive interactions. Separately, Pukkala and Miina (1998), and Hof and Bevers (2000) contributed important, if simplified, individual-tree optimization studies. Meilby and Nord-Larsen (2012) appear to provide the first rigorous numerical solution to an individual-tree optimal thinning-and-harvest problem, using a simulated annealing algorithm to optimize target diameters for a stand with over 500 trees. Coordes (2014) obtained a fully analytical solution to the thinning-and-harvest problem. That solution analyzes individual-tree harvest and retention decisions at the stand scale, but it extends naturally to—and, in fact, is more easily applied at—the neighborhood scale.

3.4 Operationalization

We begin with a simple model of a neighborhood comprised of only two trees: a crop tree and a competitor, harvested at times $t = T$ and $t = \tau$, respectively, where $\tau \leq T$. The objective function is given by:

$$\mathcal{N} = \max_{\tau,T}\{v(\tau) + V(T,\tau) + L(T)\} \qquad \text{Eq. 3.1}$$

where v and V denote the discounted net revenue from harvesting the competitor and crop tree, respectively, such that $v = v(t) = s(t)\delta^t = p(f;q)f(t)\delta^t$, where s is (undiscounted) stumpage value at time t, p is unit price, which is a function of volume f and quality q, and δ is the discount factor $\delta = (1+r)^{-1}$ with the real discount rate r specified exogenously. Note that q is conceptualized as a fixed attribute of a given tree. Mechanically, it specifies the parameters of a function that relates price to volume, where a higher value of q has the effect of "shifting up" in that price-volume curve. V and its component functions are analogously defined, with the noted addition of the variable τ into the biological production function $F = F(t,\tau)$, which relates the crop tree's volume at harvest to the timing of the competitor's removal.

L denotes the discounted value of LEV realized immediately after the harvest of the second tree, so that $L = L(T) = LEV\delta^T$, where LEV is the capitalized value of the productive potential of bare land. Generally, $LEV$is the maximum expected NPV of the infinite series of cash flows, starting from bare land and following a path of optimal harvesting. In analyzing structurally and compositionally heterogeneous neighborhoods, it is appropriate to treat LEV as independent in the solution of $\mathcal{N}$, in that the expected composition of a regenerated neighborhood (with particular regard to its quality distribution) is not controlled by the unique realization of the stochastic regeneration process that established the neighborhood. Analytically, as v and V vary with changes in τ and T, LEV remains constant. That is to say, increases in $\mathcal{N}$ as τ and T are optimized do not directly "feed forward" into an adjusted LEV. In a Bayesian sense, the solution to the optimization problem for an individual plot can be thought of as having a negligible effect on updating the prior distribution of $\mathcal{N}$'s over stochastic regeneration realizations. By the same logic, any costs for establishing regeneration enter directly into LEV independent of the maximization problem in Eq. (3.1) (cf. Halbritter and Deegen [2015] on the role of first-rotation regeneration costs in a stand-scale model with deterministic regeneration).

The discrete-time approximation of the first order conditions which maximize $\mathcal{N}$ are simply $\Delta\mathcal{N}/\Delta\tau = \Delta\mathcal{N}/\Delta T = 0$. The first order condition on T reduces to the familiar Faustmann rule: $\Delta\mathcal{N}/\Delta T = \Delta S/\Delta T - rS(T) - rLEV = 0$. The components of $\Delta\mathcal{N}/\Delta\tau$ are somewhat more complex to define. For convenience, we specify τ in such a way that the initial thinning decision occurs at time $t = 0$ (and acknowledging that we use the term "thinning" somewhat loosely to refer to any partial harvest not leading to regeneration, regardless of initial age-class structure). $\Delta v/\Delta \tau$ is analogous to $\Delta V/\Delta T$, i.e.:

$$\frac{\Delta v}{\Delta \tau} = \frac{\Delta s}{\Delta \tau}\delta - rs(\tau)\delta \qquad \text{Eq. 3.2}$$

The inherent challenge in analyzing thinning lies in assessing $\Delta V/\Delta \tau$. If the two trees are growing solitarily, then the problem is simple: $\Delta V/\Delta \tau = 0$ and τ is chosen to satisfy Jevon's single-rotation rule (Amacher et al., 2009). If the crop tree grows under competition from τ to $(\tau + 1)$, changes could be induced in either T or S or both, such that $T \neq T' = T + \Delta T/\Delta \tau$ and/or $S(T,\tau) \neq S(T',\tau + 1) = S(T,\tau) + \Delta S/\Delta \tau$.

$$\begin{aligned} \frac{\Delta V}{\Delta \tau} &= S(T';\tau + 1)\delta^{T'} - S(T;\tau)\delta^{T} \\ &= \left[S(T,\tau) + \frac{\Delta S}{\Delta \tau}\right]\delta^{T'} - S(T,\tau)\delta^{T'}\delta^{-\frac{\Delta T}{\Delta \tau}} \\ &= \frac{\Delta S}{\Delta \tau}\delta^{T'} - S(T,\tau)\delta^{T'}\left(\delta^{-\frac{\Delta T}{\Delta \tau}} - 1\right) \qquad \text{Eq. 3.3} \end{aligned}$$

Similar analysis shows $\Delta L/\Delta \tau = -LEV\delta^{T'}\left(\delta^{-\frac{\Delta T}{\Delta \tau}} - 1\right)$ and operates entirely through $\Delta T/\Delta \tau$.

If the volume production function is size-dependent but age-independent, the crop tree will return to its same growth path following the competitor's delayed removal, and it will be harvested at the same *size*, $S(T,\tau) = S'(T',\tau + 1)$, but at a later *time*, $T' > T$, than if thinning had not been delayed. Alternatively, if postponing thinning leads to persistently lower growth, the crop tree will be harvested at a smaller size, $S(T',\tau + 1) < S(T,\tau)$, (and correspondingly lower value), in order to satisfy the Faustmann rule (Eq. 3.2). Here, the sign of $\Delta T/\Delta \tau$ is ambiguous and will depend on the magnitude of the growth effect. In a continuous-time set up it would be exceedingly uncommon for T' to *exactly* equal T, given the multiple margins of adjustment in play. In discrete-time, however, this scenario is far more plausible; the iterative solution procedure we outline later ensures this result as it converges on the optimal solution.

For $\Delta T/\Delta\tau = 0$, the second term on last line of Eq. (3.3) and $\Delta L/\Delta\tau$ both go to zero, so the total change in $\mathcal{N}$ induced by delayed thinning reduces to:

$$\frac{\Delta\mathcal{N}}{\Delta\tau} = \left[\frac{\Delta s}{\Delta\tau} - rs(\tau)\right]\delta + \frac{\Delta S}{\Delta\tau}\delta^{T} \qquad \text{Eq. 3.4}$$

Next, we introduce Δt to explicitly represent the number of years in each discrete time step. As before, we assume that the initial thinning decision takes place at $t = 0$. If $s(\Delta t) - s(0) > rs(0) - (\Delta S/\Delta\tau)\delta^{T-\Delta t}$, then the thinning decision is deferred to the next period. Most of the elements of this thinning rule are easily obtained: $s(0)$ is given by the initial conditions and $s(\Delta t)$ can be computed by modelling just one growth period. The second term on the RHS of the inequality appears more challenging to quantify. When solved iteratively, though, the value for T can be stored from the previous run, and $\Delta S/\Delta\tau$ can be approximated with only one additional computation.

We introduce a new term, $\mathcal{Z}$, to denote competition-induced growth lag (measured in years). Graphically, $\mathcal{Z}$ can be thought of as the horizontal distance between $F(t = \Delta t, \tau = \Delta t)$ and $F(t = \Delta t, \tau = 0)$, i.e. crop tree volume at the end of the first period with and without competition, respectively.

$$\mathcal{Z} = -\left[\frac{F(\Delta t, 0) - F(\Delta t, \Delta t)}{F(\Delta t, 0)}\right]\Delta t \qquad \text{Eq. 3.5}$$

To be concrete, suppose that one-period growth without competition is normalized to 1; that competition reduces growth by 20% (i.e. $F(\Delta t, \Delta t) = 0.8$); and that $\Delta t = 10$ years. Then $\mathcal{Z} = -\left[\frac{1-0.8}{1}\right]10 = (-2)$ years. If there are no persistent growth effects following release, then the crop tree will always be two years behind where it otherwise would have been, $F(t, \tau = 10) = F(t-2, \tau = 0)$ for all t. Rather than modelling growth all the way through period $(T + \mathcal{Z})$, harvest volume (or value) can be calculated from just $\mathcal{Z}$ if the rate of growth around T is known: $\Delta S/\Delta\tau = \mathcal{Z}(\Delta S/\Delta t)$ for $\Delta S/\Delta t$ around $t = T$. The one-period thinning rule can now be completely evaluated in just two steps, allowing for step-wise forward recursive solution, with values for T and $\Delta S/\Delta t$ around T having been stored from each previous run and updated after each iteration.

The solution procedure is similar for neighborhoods with more than two trees. Eq. (3.4) is extended to include the growth effects on the crop tree and other residual competitor trees:

$$\frac{\Delta \mathcal{N}}{\Delta \tau} = \left[\frac{\Delta s}{\Delta \tau} - rs(\tau)\right]\delta + \sum \frac{\Delta s_i}{\Delta \tau}\delta^{T_i} \qquad \text{Eq. 3.6}$$

For the initial run, thinnings and the final harvest can be determined from Jevon's rule, with each $\Delta S_i/\Delta t$ and T_i stored for use in the following run. Over successive iterations, each approximated thinning date, $\hat{T}_i$, will approach the optimum T_i^*strictly from the right. Eventually, the thinning schedule converges so that no $\hat{T}_i$ changes after updating each $\Delta\hat{S}_i/\Delta t$.

Following the initial run, individual trees can also be ranked according to the opportunity cost of premature thinning, which we denote $\mathcal{C} = \left[V_i(\hat{T}_i) - S_i(0)\right]/G_i(0)$, where G is an index of competitive influence. If trees are ranked such that $\mathcal{C}_1 \geq \mathcal{C}_2 \geq \cdots \geq \mathcal{C}_n$, and thinning decisions are then evaluated in reverse order, $T_n \leq T_{n-1} \leq \cdots \leq T_1$, then the solution procedure is especially efficient. Rankings can be reevaluated after each iteration, but in heterogeneous stands the original ranking is usually determinative.

3.5 Numerical application

Simulation results are presented here for a 50 ha northern hardwood stand located in a private forest in northern New York State, USA. Inventory data for 0.02 ha plots describe individual-tree attributes, including: species; diameter at breast height; total height; and stem quality assessments along 2.5 m log increments. Data were collected for 591 trees over 23 plots. Individual-tree growth was modelled from the parameterized height- and diameter-increment equations in Weiskittel et al. (2016) and individual log prices were assigned according to unit price functions provided by a local forest management company (pers. comm.). Initial basal area was 20.9 m^2/ha and initial current timber value (US dollars) was $1,176/ha.

Table 3.1 presents summarized results for three simulated silvicultural strategies. The strategy labeled "Optimal" simulates the optimized, NPV-maximizing thinning-and-harvest schedule for each plot; "Irregular" simulates a system of neighborhood-scale silvicultural decision-making informed by a simple heuristic an unassisted field forester could feasibly apply; and "Even-aged" simulates management following the same heuristic decision model but under the constraint that the end-of-rotation harvest must occur in a single time period across all plots. For all strategies, thinning-and-harvest decisions were specified to maximize NPV, subject to each strategy's constraints, based on individual trees' modelled harvested value, $LEV =$ $618/ha (realized following the harvest of the last tree on each plot), and a discount rate $r = 0.035$. Cutting decisions were evaluated and growth responses were projected over ten ten-year time steps (so that any plots not regenerated by year 90 were fully harvested in year 100 by default).

Columns in Table 3.1 under the "Residual stocking" heading describe the evolution of aggregate basal area, across all plots, of the initial growing stock. These values do not include the projected basal area accrued on regenerated plots (i.e. after the last tree from the initial growing stock is harvested). "Coefficient of variation" presents the mean basal area divided by the standard deviation of basal areas between plots. Here, stocking from regeneration-cohorts is accounted for and assumed to accrue linearly at a rate of 0.42 m^2/ha/yr. Finally, "Discounted revenue" presents the schedule of projected cash flows, in present value terms, from the initial endowment of growing stock; LEV is not accounted for in these values, though it is accounted for in the analysis of cutting decisions that generated these cash flow schedules and in the NPV values reported below.

Year	Residual stocking (m^2/ha)			Coefficient of variation			Discounted revenue (USD/ha)		
	Even-aged	Irregular	Optimal	Even-aged	Irregular	Optimal	Even-aged	Irregular	Optimal
0	13.4	13.4	16.8	0.31	0.31	0.64	*$209*	*$209*	*$236*
10	15.0	15.0	22.2	0.15	0.15	0.59	*$118*	*$118*	*$500*
20	16.3	14.1	24.3	0.10	0.41	0.54	*$81*	*$525*	*$662*
30	16.7	13.7	25.3	0.08	0.37	0.51	*$188*	*$311*	*$164*
40	17.5	12.6	21.6	0.10	0.42	0.62	*$53*	*$303*	*$353*
50	0	8.3	18.2	0.00	0.71	0.54	*$1,480*	*$373*	*$198*
60	0	4.6	17.2	0.00	0.78	0.53	--	*$259*	*$152*
70	0	1.7	17.5	0.00	0.77	0.54	--	*$156*	*$107*
80	0	0	15.9	0.00	0.64	0.61	--	*$86*	*$176*
90	0	0	10.4	0.00	0.47	0.74	--	--	*$261*
100	0	0	0	0.00	0.37	1.87	--	--	*$261*
						TOTAL:	*$2,130*	*$2,340*	*$3,070*

Table 3.1: Comparison of stand structural development and financial performance between three simulated silvicultural strategies. “Optimal” = plot-by-plot optimized thinning-and-harvest schedule; “Irregular” = thinning as per conventional management guidelines, with plot-by-plot regeneration; “Even-aged” = conventional thinning leading to stand-wide regeneration.

The “Optimal” strategy implies full flexibility regarding stand-scale structure and an “omniscient forester” able to properly calculate the optimal thinning-and-harvest schedule. NPV from this strategy was $3,102/ha, with an excess value (i.e. NPV in excess of immediate liquidation [CTV + LEV]) of $1,308/ha. The strategy labeled “Irregular” retained the former assumption but relaxed the later. Here, regeneration decisions were again evaluated plot-by-plot, though a less sophisticated heuristic method was used to prescribe thinning treatments prior to the terminal harvest. Simulated removals followed the conventional “B-line” thinning method (Leak et al. 2014), applied at the plot-scale. As with the “Optimal” strategy, trees within each plot were ranked by $\mathcal{C}$ (i.e. competition-weighted opportunity cost). At each time step, individual trees were removed in ascending order until an additional removal would have reduced stocking below the prescribed target (around 15-16

m^2/ha for typical northern hardwood stand structures). Plot-by-plot regeneration decisions were specified with simple linear programming methods (Buongiorno and Gilless 2003). Here, the stand develops toward a system of patches located within a matrix of approximately uniform stocking, though we imposed no constraints on distribution of patches over time, as with an area-controlled group selection system (hence our use of the term "Irregular"). The "Irregular" strategy generated an NPV of \$2,373/ha and an excess value of \$578/ha. The "Even-aged" strategy simulated the same thinning method described above but restricted end-of-rotation harvests to a single time period across all plots (i.e. a uniform clearcut). The strategy generated an NPV of \$2,162/ha and an excess value of \$368/ha.

3.6 Discussion

As presented in the numerical example above, the gains from shifting silvicultural decision making from stand- to neighborhood-scale resolution may be substantial. Even when constrained to simple heuristics, this shift would increase the stand's excess value—or what might be termed "silvicultural alpha"—by over 50%. The gains from more sophisticated decision processes are larger still: the "Optimal" strategy would produce more than twice the excess value of the "Irregular" strategy and over 350% more than "Even-aged". Notably, this strategy generates these higher returns while simultaneously retaining higher stocking and producing more complex stand structures, as the non-financial values in Table 1 illustrate.

To obtain a true optimum, the thinning-and-harvest problem must be approached at the individual-tree level, but to approach it at that level is computationally demanding nearly to the point of intractability. Coordes (2014) goes so far as to conclude that "the viability of the scientific management of forest stands for profitable timber production is doubtful" (p. 168) given the complexity of the thinning problem. As Meilby and Nord-Larsen (2012) note, in their study of just one stand of 511 trees, the problem amounts to finding a single point in a 511-

dimensional space. Even after reducing the solution space by limiting potential occurrence of thinning to 20 discrete events, (via five-year cutting cycles over 100 years) "solving a problem with 20^{511} potential solutions is still hard" (p. 294). Consider, though, the implications of shifting to a neighborhood level of analysis: moving from a stand with 500 trees to 50 neighborhoods with ten trees each (and retaining, for illustrative purposes, the imposed limit of 20 discrete thinning opportunities) the problem reduces from 20^{500} to $50{\cdot}20^{10}$ potential solutions, a difference of 487 orders of magnitude. Even so, $50{\cdot}20^{10}$ is a number larger than 512 trillion. If the *order* in which trees are harvested can be strictly determined within each neighborhood, as outlined above, the possible solutions for the *timing* of harvests is further reduced to just over 1 billion potential solutions, five additional orders of magnitude simpler than when harvest order is unspecified.

Though ecological purists might still object to some of the assumptions underlying the "gap model" structure proposed in this study—specifically, the non-interaction between discrete neighborhoods or between boundary trees in adjacent neighborhoods, and the oversimplification of the regeneration process—we contend that the computational efficiency gained from our approach justifies the consequent loss of realism. For many applications, those losses will be relatively slight, while the gains over stand- or size-class-models will be large. Further, just as the neighborhood approach provides a tractable framework for managing the complexity of the thinning problem, it could function as a similarly useful device for integrating complex of regeneration processes into optimization problems.

In practice, the value of this approach is obvious for forest management planning and forestland valuation. Economic analysis at the neighborhood scale better describes the potential value obtainable from a well-managed forest, generates richer projections of financial and ecological outputs and attributes, and provides a practical map for implementing that strategy. This approach can also serve as a foundation for learning: offering foresters a conceptual framework to help frame their decisions in the field; producing an extensive dataset of optimized simulation results, from which 'rules-of-thumb' might be inferred; and providing a rigorous tool with which suggested cutting decisions can be evaluated in training exercises.

In research applications, a neighborhood-based approach provides the necessary analytical resolution to explore a variety of questions that the conventional "Faustmann lab" (*sensu* Deegan et al., 2011) struggles to unpack. Two areas stand out. First, within-stand spatial structure plays a critical role in the provision of many ecosystem services and amenities, particularly wildlife habitat and biodiversity (Lindenmayer et al., 2000). How changing policy instruments or market structures will affect production of these non-financial goods and services are highly relevant questions. Because the ecological dynamics driving these economic relationships often turn on within-stand spatial structure, uniform-stand models of forest management are ill suited to address these questions. Classic point-input/point-output rotation models can often describe spatial and demographic heterogeneity at the landscape scale suitably well, but these models offer no insight into finer-scale structural dynamics. Age-class models can capture compositional and demographic heterogeneity and vertical structural complexity, but still rely on assumptions of uniform horizontal stand structure. The neighborhood-based approach—simply optimizing individual plot data rather than aggregating it and then optimizing—could build on the growing line of research in what might be called the modern Hartmann tradition (e.g. Tahnoven et al., 2019) and offers a natural avenue for introducing a spatial dimension to that analysis.

Secondly, agency-theoretic optimal contracting questions have been explored in the forestry literature in the context of timber harvesting contracts (Leffler and Rucker, 1991; Leffler et al., 2000), planting and pre-commercial tending (Wang and van Kooten, 2001), and incentive programs for afforestation (Immorlica et al., 2020), but the implications of contract theory on thinning-and-harvest decisions have been less explored (see Tatoutchoup, 2015; Jensen et al., 2018; Tatoutchoup and Nijiki, 2018). Interesting contracting questions arise in settings where information is imperfect, asymmetric and/or costly to obtain. Foresters managing structurally complex, heterogeneous-quality forests have access to richer and more accurate information than the forest owner who contracts for their services, setting up a potent principal-agent dynamic. Similarly, many forest governance and institutional design questions depend on the structure and symmetry of information and can be framed similarly to contracting problems (e.g. Campbell et al., 2001; Poteete and Ostrom, 2004; Deegen, 2016). Economic analysis at the neighborhood scale provides a tractable framework for operationalizing the informational dimensions of implementing thinning-and-harvest decisions and for examining the contracting implications of these dynamics.

4 MANUSCRIPT II

Foppert, J.D. & Maker, N.F. When economically optimal is ecologically complicated: Modeling tree-by-tree cutting decisions to maximize financial returns from northern hardwood stands. *Forestry* (*in review*).

This study develops a rigorous bioeconomic model of forest growth and timber production parameterized for the forest types and product markets of the Northern Forest region of the northeastern United States. It develops computational methods suitable for optimizing individual-tree harvesting decisions in the complex, mixed-species stands that occur in the region. These methods are applied to three different case study northern hardwood stands representing a range of initial stand structures. The results contradict the conventional wisdom about the presumed conflict between investment objectives and conservation outcomes. Rather than employing heavy-handed management strategies leading to simplified stand structures—a supposedly "nearly universal outcome of timber-focused silviculture" (Palik et al., 2021, p. 295)—these results show that truly maximizing financial returns from northern hardwood forests requires silvicultural finesse and would result in ecologically complicated stands.

Authors' contributions

John Foppert developed the research question and conceptual modeling approach, conducted the formal analysis, supported the development of simulation and optimization methods, and drafted the original manuscript and produced the tables and figures. Neal Maker wrote the code, supported the development of the research question and simulation and optimization methods, and reviewed the manuscript.

When economically optimal is ecologically complicated: Modeling tree-by-tree cutting decisions to maximize financial returns from northern hardwood stands

Abstract

This study challenges a long-standing and often uncontested assertion in the forestry discourse that maximizing financial returns requires ecologically simplified stands. We developed a high-resolution simulation tool for northern hardwood stands in eastern North America and integrated advanced numerical optimization methods to model the tree-level harvest decisions that maximize financial returns. We modeled each individual tree's growth and its probability of natural mortality, conditioned on the evolving neighborhood-scale competitive environment it resides in. We developed size-, species-, and grade-specific price functions to assign potential harvest revenue values to each discrete bole section of each standing tree, and we used an evolutionary search algorithm to specify the financially optimal timing of tree-by-tree removals. We modeled three different case studies, representing a broad range of northern hardwood stand conditions, including a hypothetical young, even-aged stand and two inventoried stands in northern New York, USA. We observed consistent results across all three cases: maximizing financial returns from northern hardwood forests requires silvicultural finesse and results in ecologically complicated stands.

4.1 Introduction

It is often assumed that forests managed to maximize financial returns simply *must* be uniform, monocultural, and, in some general sense, ecologically disinteresting. Conversely, ecologically rich forests simply *must* underperform financially. Palik et al. (2021), for example, assert that simplified age- and size-class structure is a "nearly universal outcome of timber-focused silviculture" (p. 295), and Himes et al. (2022) refer to the "substantially lower financial returns" (p. 2) associated with silvicultural practices that create complex stands. The results we report here suggest otherwise. Combining high-resolution growth simulation and decision optimization tools, we find that maximizing financial returns from northern hardwood forests requires silvicultural finesse and results in ecologically complicated stands.

Fostering within-stand structural complexity is a key objective of "ecological" and "closer-to-nature" approaches to forest management (Franklin et al., 2018; Larsen et al., 2022) and numerous silvicultural strategies have been developed specifically to enhance structural complexity (Seymour et al., 2002; Schutz, 2002; Graham and Jain, 2005; Raymond et al., 2009). We take a different approach, modeling financially optimal silviculture without regard to ecological outcomes and then observing the resulting stand structure. We optimize harvest decisions on a tree-by-tree basis without limiting removal patterns to a menu of standard silvicultural treatments (cf Pauwels et al., 2007; Stout and Brose, 2014; Meek and Lussier, 2014; Labelle et al., 2018). That is, we put no constraints on the complexity of the silviculture employed, provided it increases economic returns.

We model the behavior of both trees and foresters at the neighborhood scale (Canham and Uriarte, 2006; Foppert, 2019). We model the growth of each tree individually, subject to the structure of the neighborhood it resides in and its relative position there and we evaluate harvest decisions at the tree-level. A global optimization algorithm searches over the universe of possible individual-tree cutting schedules to find the most valuable result. Intermediate tending and neighborhood-level end-of-rotation regeneration decisions are specified independently across neighborhoods. Our approach is agnostic on silvicultural strategy: neighborhood-level composition and density and stand-level demographics and spatial structure are all unconstrained. The individual-based modeling and tree-level optimization methods we use are well suited to handle compositionally diverse and heterogeneously arranged stands (Pommerening and Grabarnik, 2019), but, again, we assign no weight to ecological outcomes. We optimize strictly on financial returns.

We apply this method to three different case study northern hardwood stands representing a range of initial stand structures: (1) the relatively uniform structure typical of a young, even-aged stand (Marquis, 1967; Wang and Nyland, 1996); (2) the irregular spatial and demographic structure that results from a mixed history of natural disturbances and some regulated and unregulated cutting (Canham et al., 2013; Belair and Ducey, 2018); and (3) the distinct structure typical of a mostly unmanaged, conventionally "mature" stand (Angers et al., 2005). The results are consistent across all cases: financially optimal harvesting leads to structurally complex stands.

4.2 Background

Ashton and Kelty (2018) refer to regeneration and tending as the "two broad categories" of silvicultural treatments (p. 30). A silvicultural system results from the arrangement of such treatments within a stand, their timing across a sequence of entries, and the specific patterns of cutting they prescribe. In order to accommodate irregular management and stand structures, models of stand development must therefore account for regeneration and tending at sub-stand scales, allowing both to be carried out at different times in different areas, and accurately projecting residual growth and regeneration responses. Attempts to optimize management must quantify the payoffs that result from various combinations of treatments.

Regeneration is perhaps the more studied of the two treatment types. Contemporary silviculturalists devote considerable attention to novel regeneration strategies (D'Amato et al., 2018; Achim et al., 2022) and forest economists continue to probe rotation-ending “optimal stopping” problems (Newmann, 2002; Kant, 2013). Close examination of tending decisions has proven more challenging and has attracted less attention (Parkatti and Tahvonen, 2020). This stems from the difficulty of both projecting growth responses to partial cutting and solving multi-dimensional optimization problems.

Recently, advances in growth modeling (Assmann, 1970; Ek et al., 1988; Weiskittel et al., 2011; Mäkelä and Valentine, 2020) have allowed for the prediction of thinning responses with reasonable confidence, while advances in numerical methods have opened the door to integrating complex biometric models into rigorous economic analysis. The modern ecologic-economic optimization approach, sometimes referred to as the “Helsinki School” of resource economics (e.g. Tahvonen and Salo, 1999; Hyytiäinen and Tahvonen, 2002; Tahvonen et al., 2010; Tahvonen and Rämö, 2016; Parkatti and Tahvonen, 2020; Assmuth et al., 2021), has been successful in pairing more sophisticated “abstract economic models of optimization and ecological models of population growth” (MacLeod and Nagatsu, 2016, p. 420).

In this study, we diverge from the main line of that tradition—which in forestry operates primarily at the stand scale, using size-class-level harvest percentages as the control variable—by shifting our analytical attention to finer spatial scales. Our focus on tree-by-tree harvesting decisions follows a rich history of individual-based modeling in forest ecology (e.g. Botkin et al., 1972; Mitchell, 1975; Shugart, 1984; Pacala et al., 1993; Bugmann, 2001; Pretzch et al., 2002; Pommerening and Grabarnik, 2019), and an emerging trend in the forest management and economics literature (Härtl et al., 2010; Meilby and Nord-Larsen, 2012; Coordes, 2014; Vauhkonen and Pukkala, 2016; Lohmander, 2019; Foppert, 2019; Dong et al., 2020; Frannson et al., 2020; Packalen et al., 2020; Pascual, 2021a; West et al., 2021; Foppert, 2022; Koster and Fuchs, 2022; Sun et al., 2022; Dong et al., 2022; Pascual and Guerra-Hernández, 2022). Our study makes further contributions to tree-level optimization literature, employing more refined growth and mortality models, tracking neighborhood-level regeneration, and explicitly relating our results back to larger debates in silviculture, forest management and policy.

4.3 Methods

4.3.1 Formal model

Consider a neighborhood of some fixed area A, sufficiently small that all of the trees within it interact in a nearly homogenous competitive environment but sufficiently large that neighborhood-level summary statistics accurately describe that environment. That is, a neighborhood must not be so large that substantial environmental variation exists within it, but not so small that its environment is significantly (and unaccountably) influenced by trees outside of it. Functionally, a neighborhood should be large enough to accommodate two or three trees of end-of-rotation stature. Target crop tree densities of 150-175 tr·ha^{-1} (e.g. Miller et al, 2007) imply neighborhoods in the range of 0.01-0.02 ha, consistent with how the neighborhood concept has been operationalized in the ecology literature (Canham and Uriarte, 2006).

The growth of tree $i \in \{1, \ldots, n\}$ from time t_0 to t_1, is a function of its physiological attributes; of the initial neighborhood-level competitive environment it resides in at time t_0, as controlled by the attributes of the trees neighboring it; and of the fixed site attributes that relate to tree growth processes (and which, as fixed exogenous attributes, can be suppressed in the analytical notation that follows without any loss of generality). Let ΔDBH_{i0} denote the forward periodic diameter increment of tree i over the first modeling timestep, where $\Delta DBH_{i0} = DBH_{i1} - DBH_{i0} = \Delta DBH_{i0}(DBH_{i0}, CR_{i0}, BA_0, BAL_{i0}; SPP_i)$. SPP is a categorical variable that denotes the species of tree i; DBH denotes diameter at breast height, $DBH\colon R^{+}$; CR

denotes live crown ratio (the ratio of the length of tree's photosynthetically active crown to its total height and a proxy measure for a tree's overall vigor and productive potential), $CR \in [0,1]$; BA is the neighborhood-level basal area (a measure of density):

$$BA_0 = \sum_{i=1}^{n} \left(DBH_{i0}{}^2 \frac{\pi}{4} A^{-1} \right) \qquad \text{Eq. 4.1}$$

and BAL_{i0} is the overtopping basal area (i.e. basal area of larger trees), relative to tree i:

$$BAL_{i0} = \sum_{j=1}^{n} \left(DBH_{j0}{}^2 \frac{\pi}{4} A^{-1} I\big(DBH_{j0} > DBH_{i0}\big) \right) \qquad \text{Eq. 4.2}$$

where I is an identity function, ($I = 1$ if $DBH_{jt} > DBH_{it}$, $I = 0$ otherwise). Define ΔCR_{i0} analogously to ΔDBH_{i0}.

Given the significant role natural mortality plays in a forest's development, optimization modeling should account for it explicitly. Let ϕ_{i0} denote tree i's probability of survival over the first timestep, where $\phi(\cdot)$ is a function of the same factors that determine ΔDBH and ΔCR: $\phi_{i0} = \phi_{i0}(DBH_{i0}, CR_{i0}, BA_0, BAL_{i0}; SPP_i)$.

In a dynamic model, the states of each tree i evolve over successive timesteps, $t \in \{t_0, t_1, \dots, T\}, T \in [t_0, \infty)$. Expected growth and mortality functions for times $t > 0$ require modification to account for reduced stocking resulting from probabilistic natural mortality. Let $E[BA_{i\tau}]$ denote the expected basal area affecting the growth and mortality of tree i from an arbitrary time τ to $\tau + 1$, conditional on tree i having survived until time τ:

$$E[BA_{i\tau}] = \left(DBH_{i\tau}{}^2 + \sum_{j=1}^{n} \left(\Phi_{j\tau} DBH_{j\tau}{}^2 \right) \right) \frac{\pi}{4} A^{-1}, \forall j \neq i \qquad \text{Eq. 4.3}$$

and let $E[BAL_{i\tau}]$ denote expected overtopping basal area, similarly qualified:

$$E[BAL_{i\tau}] = \sum_{j=1}^{n} \left(\Phi_{j\tau} DBH_{j\tau}{}^2 \frac{\pi}{4} A^{-1} I\big(DBH_{j0} > DBH_{i0}\big) \right) \qquad \text{Eq.4.4}$$

where $\Phi_{j\tau}$ denotes the cumulative probability of survival up to time τ:

$$\Phi_{j\tau} = \prod_{t=0}^{\tau} \phi_{jt} \tag{Eq.4.5}$$

from discrete, individual-tree survival probabilities:

$$\phi_{it} = \phi_{it}(DBH_{it}, CR_{it}, E[BA_{it}], E[BAL_{it}]; SPP_i) \tag{Eq. 4.6}$$

where the state of tree i diameter and crown ratio in time τ is given by:

$$DBH_{i\tau} = DBH_{i0} + \sum_{t=0}^{\tau} \Delta DBH_{it} \tag{Eq. 4.7}$$

$$CR_{i\tau} = CR_{i0} + \sum_{t=0}^{\tau} \Delta CR_{it} \tag{Eq. 4.8}$$

and where general diameter increment and crown ratio change functions are similarly modified to account for expected cumulative survival:

$$\Delta DBH_{it} = \Delta DBH_{it}(DBH_{it}, CR_{it}, E[BA_{it}], E[BAL_{it}]; SPP_i) \tag{Eq. 4.9}$$

$$\Delta CR_{it} = \Delta CR_{it}(DBH_{it}, CR_{it}, E[BA_{it}], E[BAL_{it}]; SPP_i) \tag{Eq. 4.10}$$

The economic dimension of the individual-tree harvesting problem entails the choice, in each period, of which trees to harvest and which to retain, and the valuation of the resulting cash flows. Let v_{it} denote the potential gross harvest revenue of tree i at time t, such that $v_{it} = v_{it}(DBH_{it}, HT_{it}, CR_{it}; SPP_i, q_i)$ where HT_{it} denotes tree height, which evolves according to a height growth function, ΔHT, defined analogously to ΔDBH and ΔCR, and where q_i is a quality vector, $q_i = \{q_{i1}, q_{i2}, \dots, q_{ib_{\max}}\}$, where q_{ib} denotes the timber quality class of the bth 'bolt' (i.e. discrete merchantable bole section, typically 2.5 m in length) in the stem of tree i and $b_{\max}$ denotes the bole from the upper-most feasibly merchandizable stem section. Quality class q_{ib} is a fixed attribute of bolt b of tree i and specifies the coefficients for the function that relates unit price, p_{itb}, to individual-bolt volume, f_{ibt}, (for bolts of a fixed length) where $p_{itb} = p_{itb}(f_{itb}; SPP_i, q_{ib})$, and $p(\cdot)$ is an increasing, sigmoidal function in f (see Foppert, 2022, Figure 1).

For bolts of a fixed length, individual-bolt volume is determined by the tree's DBH, the bolt's position in the bole, b, and the stem taper form of tree i, which is a function of its physiological attributes, such that $f_{itb} = f_{itb}(DBH_{it}, HT_{it}, CR_{it}; SPP_i, b)$. The total potential gross harvest revenue of an individual tree is:

$$v_{it} = \sum_{b=1}^{b_{\max}} [p_{itb} \cdot f_{itb}] \qquad \text{Eq. 4.11}$$

Let c_{it} denote the potential cost of harvesting tree i at time t. Harvesting cost can be represented strictly as a function of tree size, such that $c_{it} = c_{it}(f_{it})$, where $f_{it} = \sum_{b=1}^{b_{\max}} f_{itb}$, or as a function of v_{it} if the harvesting contractor's compensation includes some component of gross revenue to incentivize efficient utilization, or as a linear combination of these functions.

Let the binary variable χ_{it} operate as the control variable, such that tree i is harvested in period t if $\chi_{it} = 1$ and is retained through the subsequent period if $\chi_{it} = 0$. An n-dimensional neighborhood-level harvest vector, h, specifies the timing of harvest for each tree i, such that $h = \{h_1, h_2, \dots, h_n\}, h_i \in \{0,1,\dots,T\}$. Define χ_{it} as $\chi_{it} = 1$ for $t = h_i$ and $\chi_{it} = 0 \forall t \neq h_i$. Realized stumpage (i.e. net revenue), s_{it}, is thus given by:

$$s_{it} = \chi_{it}(v_{it} - c_{it}) \qquad \text{Eq. 4.12}$$

Define $\Phi_{kt} = 0 \forall t > h_k$ to remove the influence of any tree k on the neighborhood competitive environment following its harvest. The expected present value in time t_0 of realized stumpage from harvesting tree i at time $t = h_i$ is given by:

$$u_i = \Phi_{ih_i} s_{ih_i} (1 + r)^{-h_i} \qquad \text{Eq. 4.13}$$

where r is the per-period discount rate and, by assumption, is specified exogenously for all forestry projects. Let U denote the aggregated neighborhood-level present value of expected future cashflows from the current growing stock, $U = \sum_{i=1}^{n} u_i$.

By assumption, a neighborhood will regenerate naturally following the harvest of the last tree from the initial cohort. Natural regeneration is a stochastic process, so the exact distribution of attributes among trees that initialized the previous cohort will not necessarily be replicated. Following the establishment of regeneration, the expected present value of the infinite series of rotations within the neighborhood's fixed area is thus independent of the harvest schedule h specified for the initial cohort (see Foppert [2019] for an extended discussion). This "land expectation value" (LEV) can be assigned exogenously or may be derived numerically if the outcome of regeneration, in expectation, is known. In either case, the solution to the optimization problem for the initial neighborhood relies on a fixed value of LEV independent of the choice of harvest schedule. Here, we outline a method for solving for LEV explicitly by representing regeneration over a single neighborhood.

Assume that the expected structure and attributes of the regenerated cohort is known. Populate a hypothetical neighborhood with trees such that the distribution of species, quality, DBH, CR, and HT within the neighborhood correspond to the expected distribution of a regenerated neighborhood at the close of the stand establishment phase. Model the development of that neighborhood forward according to Equations (3) – (11). Denote the present value of the expected cashflows from the first rotation in the representative regenerated neighborhood by $\check{U}$. Define $\check{T}$ such that $\check{T}(h) = max\{h_1, h_2, \dots, h_n\}$ for any specified harvest vector. The maximum value of an infinite series of repeated rotations (see Zhang and Pearse [2011] for a general derivation) is given by:

$$LEV = \max_{h} \left[\frac{\check{U}(h)}{1 - \frac{1}{(1+r)^{\check{T}(h)}}} \right] \qquad \text{Eq. 4.14}$$

Having defined LEV, the general silvicultural optimization problem for any initial neighborhood population can be stated as:

$$\max_{h}\left[U(h)+\frac{LEV}{(1+r)^{-T(h)}}\right] \qquad \text{Eq. 4.15}$$

Specifying harvest vectors independently across all the neighborhoods in a stand results in an unconstrained (often irregular) age class structure likely to evolve over time.

4.3.2 Numerical simulation methods

To illustrate the theoretical approach described above, we modeled optimal cutting schedules for three different northern hardwood stands, one constructed synthetically and two real stands inventoried in the field. For all cases, individual-tree data are represented over 7.32 m radius fixed-area plots, following the design of US Forest Service Forest Inventory and Analysis (FIA) subplots (Woudenberg et al., 2010), treating each plot as a discrete neighborhood. For the empirical case studies a systematic sample of plots was taken and species, DBH (measured in inches, converted to centimeters), CR (estimated to the nearest 10%-class), and quality `assessments for each potentially-merchantable 2.5 m bole section were recorded for every tree within each plot. Bole sections were evaluated for *potential* log grade, irrespective of current diameter, consistent with the methods described in Demchik et al. (2018). For the hypothetical stand, all of the above individual tree attributes were synthesized to produce a plot with the targeted structure and composition, as described in sub-section 4.1. All analysis was done in the R programming language (R Core Team, 2022).

Initial tree heights were projected from a nonparametric model developed following the methods described in Maker and Foppert (*In press*) and trained to the same FIA dataset[2] of remeasured plots in the Northern Forest region, restricted to data from the current national standard plot design (see Woudenberg et al., 2010). Diameter-inside-bark (dib) was calculated at the top of each bole section from Westfall and Scott's (2010) tree taper equations.

Prices were estimated for each bole section of each tree following the methods described in Foppert (2022). A relative price factor (RPF) relates the projected unit price of the subject log section to the price of a reference grade, such as a #2 sawlog. For a given species group and quality class, dib prescribes RPF according to the sigmoidal function:

$$RPF = \frac{\beta_0}{1 + exp\left[\frac{-\beta_1}{\beta_0}(dib - \beta_2)\right]} + \beta_3 \qquad \text{Eq. 4.16}$$

Note that RPF is a unitless factor.

RPF curves were fitted for three hardwood species groups, with groupings based on historic price performance: high-value species, including sugar maple (*Acer saccharum*), black cherry (*Prunus serotina*), and yellow birch (*Betula allegeniensis*); mid-value species, including red maple (*Acer rubrum*), white ash (*Fraxinus americana*), and paper birch (*Betula papyrifera*); and low-value species including aspen (*Populus* spp.) and American beech (*Fagus grandifolia*). RPF curves were estimated from log price data from the Indiana Forest Products Price Report[3] from 1957-2019. Log prices across grades were referenced to #2 sawlog prices and plotted along the grade-limiting dib values for potential veneer (4 clear faces [cf]), sawtimber

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[2] https://apps.fs.usda.gov/fia/datamart/datamart.html, downloaded December 6, 2020

[3] Indiana imports a substantial volume of hardwood sawlogs from across the northeast and upper Midwest and the Indiana Forest Products Price Reports consists of the longest and most complete record of delivered sawlog prices; recent reports are available at https://www.in.gov/dnr/forestry/forestry-publications-and-presentations/ and historic reports at https://docs.lib.purdue.edu/timber/.

(1-3 cf), and pallet/tie (0 cf) quality classes. For each species group and quality class, RPF-function coefficients were fitted to minimize residual squared errors.

Table 4.1 presents RPF-function coefficients for each species group and quality class.

Species group	**Veneer class**				**Sawtimber class**				**Pallet/tie class**			
	β_0	β_1	β_2	β_3	β_0	β_1	β_2	β_3	β_0	β_1	β_2	β_3
High-value	3.746	1.788	14.495	0.563	1.485	1.032	12.311	0.563	0.203	0.144	6.261	0.501
Mid-value	–	–	–	–	0.884	0.629	12.379	0.759	0.041	0.064	4.583	0.759
Low-value	–	–	–	–	0.366	0.293	12.945	0.936	0.059	0.012	13.145	0.936

Table 4.1: RPF-function coefficients, by species group and quality class

Roadside prices were calculated for individual trees in their specified harvest year from quality assessments, dib, and RPF values of all merchantable log sections. Reference prices (#2 sawlog) were assigned to each individual species from an average of advertised local sawmill price sheets (Table 4.2).

Species	**Reference price**
Ash	\$400
Aspen	\$170
Beech	\$190
Black cherry	\$450
Paper birch	\$320
Red maple	\$400
Sugar maple	\$550
Yellow birch	\$450

Table 4.2: Reference prices (#2 sawlog), by species

Prices were reported in units of U.S. dollars per thousand board feet[4] (\$/MBF) and were assumed to remain constant, in real terms, over the simulation horizon. Harvesting costs for log products (i.e. veneer and sawtimber) were calculated per cord were modeled as:

$$c_{it} = 0.4v_{it} + 36 \qquad \text{Eq. 4.17}$$

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[4] Measures of volume were converted between units at 1 MBF = 2 cords; for reference, 1 cord is approximate equal to approximately 2.5 m^3.

A fixed trucking rate of \$37.50/cord was assigned for all log products and a fixed roadside price of \$15/cord was assigned for all harvested pulp-quality volume other than aspen, which was assigned a price of \$5/cord. A precommercial thinning cost of \$1.50/tree was assigned to all harvested trees less than 15 cm in diameter. Future-period net revenues were discounted back to present value terms at a 3.5% real discount rate.

For all cases, we simulated individual-tree development within each plot over five-year timesteps using a refined version of the nonparametric diameter increment model from Maker and Foppert (*In press*) and similarly developed mortality, height increment, and crown ratio change models. These are all random forest models (Breiman, 2001) developed using the ranger (Wright and Ziegler, 2017), Rborist (Seligman, 2022) and caret (Kuhn, 2008) software packages.

Harvest vectors were specified for all plots, restricting harvests to alternate simulation timesteps (10-year intervals) beginning in year 0. To search for near-optimal harvest vectors, we employed the genoud evolutionary algorithm using Mebane and Sekhon's (2011) rgenoud package. Genoud is a metaheuristic, global optimizer that can accommodate complex problems with multiple local maxima and integer solutions (Mebane and Sehon, 2011). The algorithm takes a given population of potential solutions and evolves them over multiple generations with reproductive strategies that favor higher fitness solutions, iteratively approaching a near-optimal solution. For each plot, we initialized the algorithm with a population of 50 potential solutions (harvest vectors) and allowed it to evolve for up to 70 generations.

4.5 Case study descriptions

We modeled three different stands representing a variety of structural conditions observed across northern hardwood forests. Two of the case studies analyzed real stands located in the Adirondack region of northern New York, USA. The other case study was constructed synthetically to represent the expected composition and structure of a stand following regeneration on a representative site in the region. The subsections below provide detailed descriptions of each of the three case study stands. Information on the empirical stands was obtained from communication with the current owners and recent forest managers.

4.5.1 Case Study 1 – Young, even-aged

Population attributes of the hypothetical even-aged stand were taken from Marquis's (1967) study of clearcutting in the White Mountains of New Hampshire. The granitic soils on that study site are similar to those in the Adirondacks. Marquis's (1967) observed species distributions (30% American beech, 29% sugar maple, 12% yellow birch by stem count at age 30) and stand density of 6,541 tr·ha^{-1} are also consistent with local experience of regeneration on productive hardwood sites. DBH values were drawn from species-specific lognormal distributions fit to Marquis's (1967) reported mean values and randomly assigned to individual trees. Quality scores for individual bolt sections were randomly assigned from the quality score distributions, by species group, observed from empirical assessments in Adirondack hardwood stands (Table 4.3). CR and HT values were randomly assigned from species-specific normal distributions derived from analysis of filtered FIA data.

	Bolt	Veneer	Sawtimber	Pallet	Pulp
Maple	1st	28%	51%	10%	11%
	2nd	11%	65%	14%	10%
	3rd	1%	40%	30%	30%
Birch	1st	48%	33%	7%	13%
	2nd	27%	40%	14%	19%
	3rd	1%	40%	19%	41%
Beech	1st	0%	5%	48%	47%
	2nd	0%	3%	42%	55%
	3rd	0%	0%	24%	76%

Table 4.3: Quality score distributions of key species for hypothetical even-aged northern hardwood stand

A representative neighborhood was populated and grown forward in five-year timesteps from age 30 to age 50, following the growth simulation methods described above, excluding any harvesting. At age 50, the neighborhood population was reduced to account for stochastic natural mortality by drawing random values $\in [0,1]$ for each tree and removing any tree for which the drawn value exceeded its year-50 cumulative survival probability. Values were redrawn over 1,000 realizations. The best fitting year-50 tree list was retained for subsequent optimization, with fit defined as the least absolute variance between the realized and predicted proportions of quality score #1 and #2 stems over the first two bolts of sugar maple and yellow birch, restricted to realizations with neighborhood-level stocking within 5% of mean basal area over all stochastic realizations. From the selected year-50 tree list, the harvest schedule was optimized to maximize LEV, following Equation (4.14), for the market and financial parameters specified above.

4.5.2 Case Study 2 – Managed, uneven-aged

The first empirical case study stand is located on land owned by Lincoln Brook Timber Company in the Town of St. Armand, Essex County, New York. Stand 8 of the Pigeon Roost Lot is 24 ha in a mid-slope position on a moderately productive, northwest-facing granitic till hillside formed of bouldery, fine sandy loams (Beckett, Skerry, and Adirondack soil series). Elevations range from approximately 550-580 m.a.s.l. The stand was likely cleared for charcoal production in the late 19th century and regenerated as a primarily even-aged hardwood stand around that time. It was cut intermittently in the 20th century, including harvests in the 1950s and 1980s that led to the establishment of distinct cohorts. Most of the stand only suffered moderate damage in the 1998 ice storm, though the paper birch component (which was then 110-120 years old) was more severely damaged. The stand was lightly cut in 2000 to salvage storm damaged trees and harvest sawtimber deemed mature by the forester administering the timber sale. Lincoln Brook Timber

Company acquired the tract containing this stand in 2003 and in the winter of 2016/2017 implemented a single-tree selection harvest under the planning and supervision of an experienced forester. Data from nine sample plots provide the basis for simulation and were collected in November 2020.

4.5.3 Case Study 3 – Mature, unmanaged

The second empirical case study stand is located in the Lot 57 Operational Unit of the Paul Smith's College Forest, in the Town of Brighton, Franklin County, New York. The 30-ha stand occupies a similar site as Case Study 2: mid-slope on a granitic till hillside (Beckett and Beckett-Tumbridge complex soils), with elevations from 530-570 m.a.s.l. Unlike the Lincoln Brook Timber tract, the Lot 57 Unit of the College Forest does not have well-developed road access. This stand has been unmanaged since at least the mid-1900's. Some earlier logging may have occurred but could only have been light, opportunistic cutting. Many trees in the stand predate the area's settlement in the late 1800's. Trees of different ages are present, but the majority of trees are over 100 years old and the canopy is mostly closed. Composition is typical for an older northern hardwood stand. Sugar maple accounts for 57% of stand basal area, and yellow birch, red maple, and American beech each account for around 12%. Timber quality is variable, but high-quality stems are present across size classes. The stand is represented over 16 sample plots inventoried in April 2021.

4.6 Results

4.6.1 Case Study 1 – Young, even-aged

The optimal harvest schedule for the simulated even-aged stand was projected to generate an LEV of $562/ha over a 170-year rotation. Figure 4.1 illustrates the evolution of projected neighborhood-level stocking and associated cashflows.

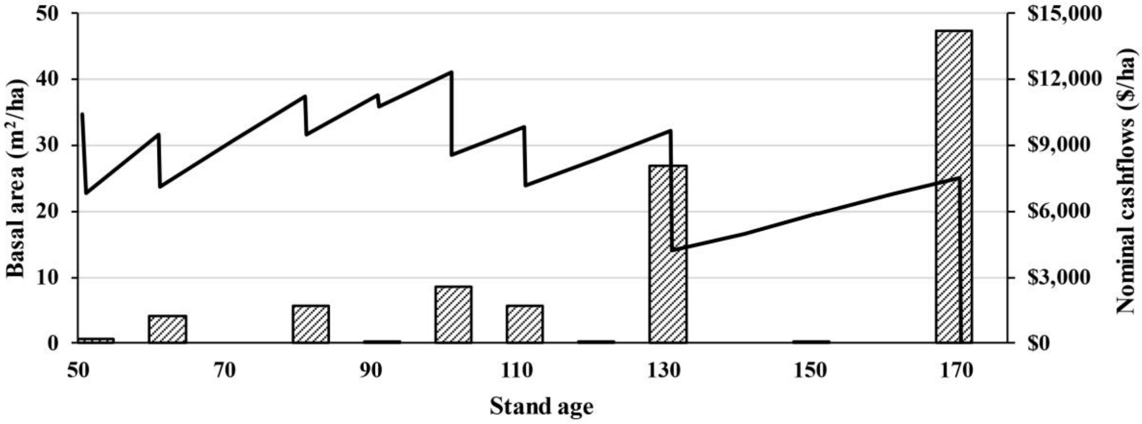

Figure 4.1: Simulated optimal stocking path (solid line) and undiscounted cashflow schedule (shaded bars, secondary axis) for hypothetical even-aged northern hardwood stand

Tending began in year 50, at the specified start of the optimization period. The first entry brought stand basal area (stems $\geq$ 14 cm) from $35\text{m}^2\text{ha}^{-1}$ to $23\text{m}^2\text{ha}^{-1}$. Removals in the first entry were concentrated among poorer quality stems (Table 4.4) distributed across intermediate size classes (Figure 4.2, left box). Subsequent tending maintained expected basal area within a range of approximately $25\text{-}40\text{m}^2\text{ha}^{-1}$.

Stand age	Harvested		Retained	
	Mean	*Max*	*Mean*	*Max*
50	$0.52	$0.90	$0.63	$11.00
60	$6.85	$20.08	$0.62	$7.97
70	$0.00	$0.00	$1.20	$15.95
80	$14.37	$28.67	$1.40	$15.39
90	$0.99	$0.99	$2.71	$26.20
100	$21.70	$39.75	$4.00	$20.05
110	$14.46	$28.75	$8.19	$37.86
120	--	--	$15.72	$65.83
130	$67.73	$93.50	$4.92	$10.92
140	--	--	$9.60	$23.78
150	--	--	$18.65	$49.37
160	--	--	$32.39	$94.00
170	$59.71	$143.97	--	--

Table 4.4: Mean and maximum expected values per tree harvested or retained following the optimal thinning schedule for hypothetical even-aged northern hardwood stand

Thinnings between ages 50 and 130 employed a combination of methods. Crown-thinning-type treatments removed poorer-quality stems from the upper half of the diameter distributions, but a strategy consistent with dominant thinning (a.k.a.

selection thinning) was observed at times. Beginning in year 80, high-quality soft maple stems were removed from overtopping positions as they reached diameters around 40-50 cm, as were good-quality (but not premium) yellow birch at around 30-40 cm (Figure 4.2, center box). At age 130, a final thinning removed good-quality soft maple and yellow birch (40-50 cm) overtopping or competing with mostly premium-quality hard maple and yellow birch. The residual stand was then free to grow, with an expected residual basal area of 14 m^2ha^{-1}. No removals occurred from years 130-170 when the remaining growing stock reached diameters around 50 cm and a final harvest brought the rotation to a close.

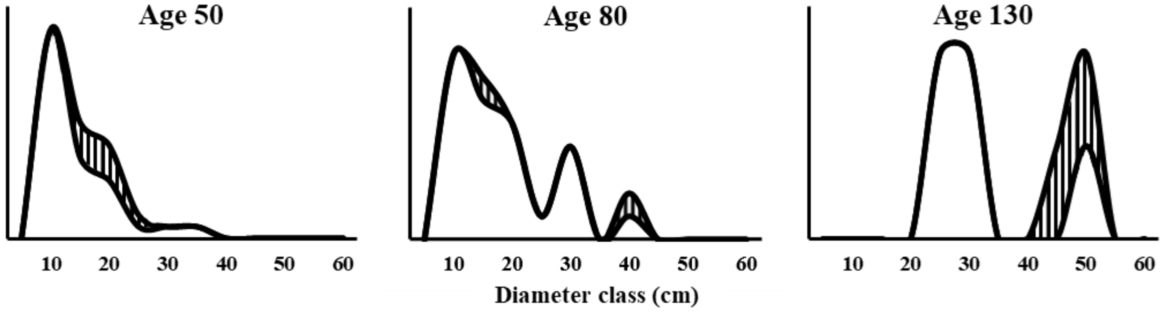

Figure 4.2: Simulated diameter distributions at selected mid-rotation entries along the optimal thinning schedule for the hypothetical even-aged northern hardwood stand; y-axis: relative size-class abundance (smoothed); hashed areas represent removals

4.6.2 Case Study 2 – Managed, uneven-aged stand

The optimal harvest schedule for the Lincoln Brook stand was projected to generate discounted cashflows, inclusive of discounted plot-level *LEV* realizations, of $2,761/ha. Subtracting from this value the initial liquidation value (i.e. initial standing timber value plus *LEV*) results in excess returns of $573/ha, a 26.2% premium over liquidation. Stand level cashflows are shown in Figure 4.3.

Regeneration began in the first period and proceeded irregularly over 130 years. No regeneration occurred in years 10, 30, 50, 70, 80, or 100-120; harvests in years 0, 20, 40, and 60 regenerated around 11% of the total stand area during each entry; one-third of the stand area was regenerated in year 90; and the remainder of the stand was regenerated in year 130.

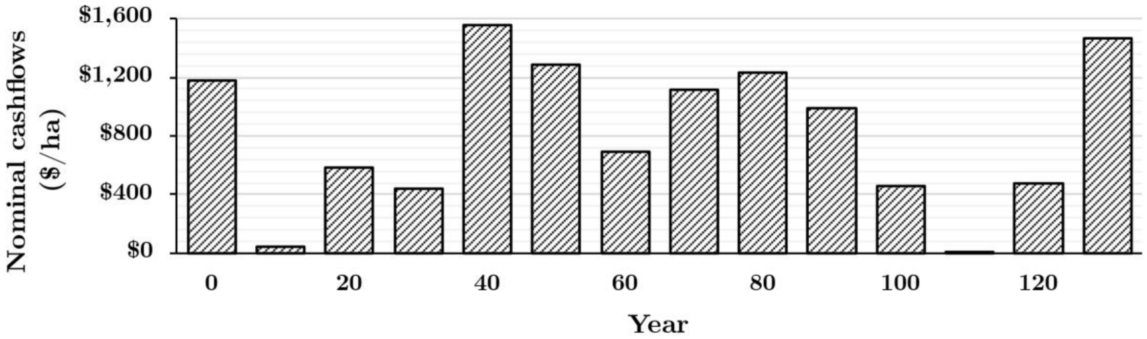

Figure 4.3: Optimized cashflow schedule, Lincoln Brook Timber Company, Pigeon Roost Lot, Stand 8

Over the full simulation period, basal area of unregenerated plots averaged 14.5 m^2ha^{-1}, ranging at the stand level from a high of 19.7 m^2ha^{-1} in year 40 to a low of 8.7 m^2ha^{-1} following the entry in year 80. Naturally, stocking varied more widely at the individual-plot level, ranging from 3.8-33.4 m^2ha^{-1}, with a standard deviation of 6.5 m^2ha^{-1} around the mean of 14.5 m^2ha^{-1}.

As with Case Study 1, a variety of patterns of within-plot removals were observed. Though these removals do not represent specific silvicultural thinning methods, *per se*, they corresponded closely to the patterns of removal prescribed by crown and dominant thinning methods (Ashton and Kelty, 2018).

4.6.3 Case Study 3 – Mature, unmanaged

The present value of revenue from the projected optimal harvest schedule (inclusive of LEV realizations) for the Paul Smith's Lot 57 stand was $5,258/ha. Compared to a strategy of uniform liquidation and regeneration, this harvest schedule generated excess returns of $228/ha. A variety of tending patterns were observed and, as with Case Studies 1 and 2, target diameters were generally around 30-40 cm for mid-quality stems and 50 cm for the highest quality sugar maple and yellow birch stems.

Plot-level end-of-rotation harvest timing varied across the stand. 50% of plots were regenerated in the first entry. Regeneration of the remaining plots unfolded in an irregular pattern over the following 60 years; at no point did the stand approach a balanced age-class distribution (Figure 4.4).

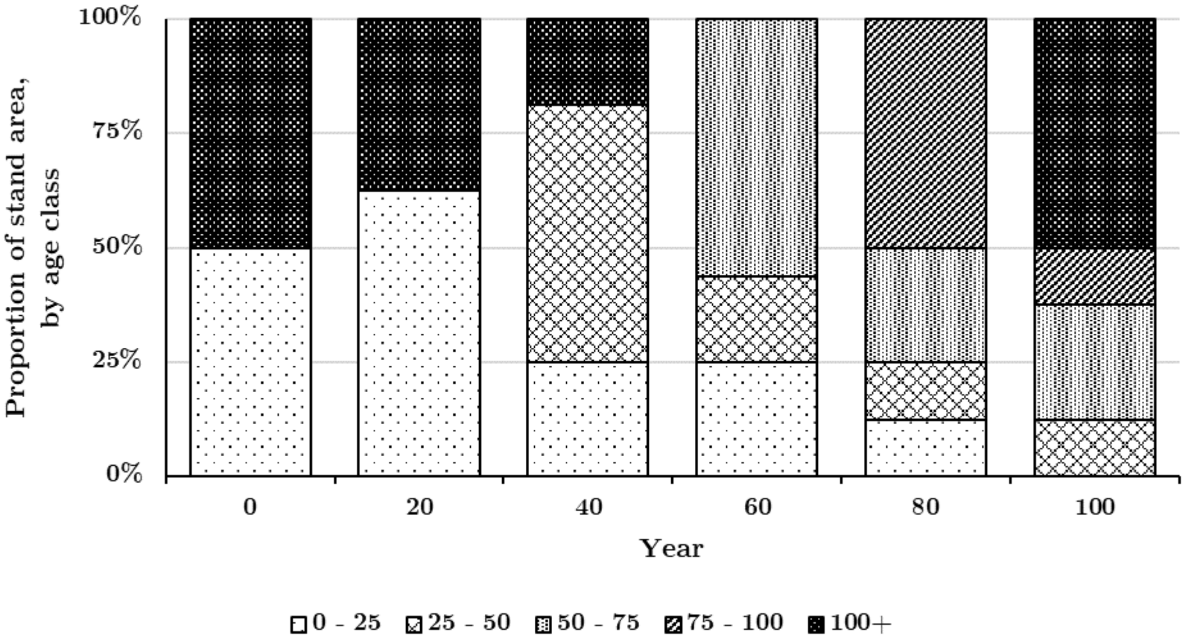

Figure 4.4: Evolution of projected (post-harvest) stand age-class structure, Paul Smith's College Forest, Lot 57; all initial growing stock assigned to 100+ age class, by assumption

4.7 Discussion

4.7.1 The gap model framework in silviculture and economics

Silviculturalists have long recognized the importance of thinning. Fernow (1911) gave credit for "the first good statement of the theory of thinnings" (p.67) to Berlepsch in 1761. Its basic logic has two parts. First, as Toumey and Korstian (1937) observed, "When a dense stand is opened by thinning, the remaining trees grow faster than previously" (p. 321). Second, economic output "can be enhanced by simply favoring the trees of best potential quality and discriminating against the poor ones. This effect of thinning on wood quality is vastly more important than any other" (Ashton and Kelty, 2018, p. 400).

Consolidating these informal observations into a rigorous analytical framework has proved challenging, despite forest economists' progress fleshing out the formal logic of the thinning question (Härtl et al., 2010; Coordes 2014; Koster and Fuchs, 2022). Foppert (2019) argued that for both theoretical and numerical problems, the stand-level thinning problem can be broken up into smaller, neighborhood-scale problems and advocated for economists to borrow the gap model framework from forest ecology.

The gap model framework (e.g. Botkin et al., 1972; Shugart and West, 1980; Bugmann, 2001) represents a stand as an aggregation of independent "gaps" or neighborhoods in which all individual trees interact, but where no interactions occur between individuals in different neighborhoods or among neighborhoods themselves. The approach can reduce the complexity of a stand-scale, individual-tree thinning optimization problem by hundreds of orders of magnitude (Foppert, 2019). It is also a practical approach, retaining the advantages of individual-based modeling without requiring spatially-explicit tree maps.

Beyond the thinning problem, the gap model framework is useful for quantifying non-uniform regeneration strategies. Similar to the original applications of gap models in forest ecology, in quantitative silviculture and economic analysis they offer visibility into the spatial dimension of stand dynamics. Our approach follows earlier work developing irregular silvicultural strategies that apply multiple treatments within heterogeneous stands (Meek and Lussier, 2014; Labelle et al., 2018), but where Labelle et al.'s (2018) multi-treatment planning tool was explicitly "not meant to optimize or produce a heuristic solution based on financial returns" (p. 485), we show that the gap model framework offers a feasible (if still challenging) method for integrating economic optimization into ecological modeling. As it relates to both thinning and regeneration, this framework can help bring rigor to the irregular silviculture deemed "seemingly less demanding and more informal as a silvicultural system" (Nyland et al., 2016, p. 518).

4.7.2 Limitations and future direction

Our model could benefit from refinement in several areas. Four opportunities stand out: mortality, harvesting costs and damage, price dynamics, and regeneration.

We addressed mortality by discounting each tree's competitive influence and its eventual harvest value by its cumulative survival probability in each period. This kept the model deterministic and the optimization problem manageable while still accounting for mortality in evaluating cutting decisions and valuating the results. We consider this an improvement over other tree-level economic analyses which simply assumed away mortality, either explicitly (e.g. Meilby and Nord-Larsen, 2012; Koster and Fuchs, 2022) or implicitly (e.g. Lohmander, 2019; Foppert, 2022). Nonetheless, the approach is problematic. It assumes that a tree with a 50% survival probability will exert an influence on its neighborhood competitive environment equivalent to half its basal area. "Expected basal area" perhaps makes sense in a tree-list model comprised of representative trees, but it challenges the logic of an individual-based model such as ours. A tree will either have survived and exert its full competitive influence or it will have died and exert none. To split the difference is to model an impossible scenario. Future work could incorporate stochastic mortality into the adaptive control function optimization procedure Lohmander (2019) described, though this would require a nearly complete overhaul of the modeling strategy we used. For now, we consider our current approach a reasonable compromise between the high computational cost of a fully stochastic treatment of mortality and the bias of neglecting it altogether.

Harvest cost functions could also be more dynamic than the simple formula we employed. Ideally, they would account for job-level fixed costs and the stand-level attributes that affect variable operating costs (e.g. Germain et al., 2019; George et al., 2022). Unfortunately, such changes would substantially complicate our modeling approach, requiring a shift from independent neighborhood-level optimization to a significantly less efficient bi-level optimization approach similar to Tahvonen and

Rämö (2016) or Sun et al. (2022). There may be some other, more modest improvements to our cost functions that could more easily be incorporated into the existing model structure. Future work should explore these, but we note that both the form and the values of the stumpage formula we used track with observed local timber sale contracts.

As with harvesting costs, residual stand damage and grade degradation can be estimated as a function of stand and harvest attributes (Wiedenbeck and Smith, 2019; Kizha et al., 2021). In the future, we could adjust quality scores to account for expected harvest damage, following the same mechanics as our current deterministic treatment of mortality.

Stochastic prices are also well known to affect optimal harvesting strategies (e.g. Brazee and Mendelsohn, 1988; Platinga, 1998; Gong and Löfgren, 2007; Manley and Niquidet, 2017) but have been ignored in this study. Insulating decision making from price fluctuations results in under-valuation of forest assets. It also likely leads to sub-optimal model recommendations for the initial treatment. As with mortality, future studies may have to abandon the convenience of modeling prices as deterministic (in this case, static) processes if they are to offer a fully formed solution to the problem at hand.

Our treatment of regeneration is also oversimplified but may be remediable without incorporating fully stochastic behavior into the model. The primary purposes of the tools we developed in this paper are to improve long-term valuation and to support near-term silvicultural decision making. In both cases, accounting for and planning around the *expected* outcomes of regeneration processes should not lead to distortions relative to an approach that evaluates the full distribution of potential outcomes. An improved model could treat regeneration as a deterministic dynamic process. Rather than using a simple binary process, in which a neighborhood regenerates always and only after the last tree is cut, different patterns of neighborhood-level residual growing stock (including, perhaps, time-lagged effects of structure over preceding timesteps) could result in the establishment of different cohorts that vary in their density and composition. Crucially, modeling need not represent a new cohort as individually identified trees, so long as the effects of current neighborhood structure on future composition and density—and the resulting expected value of that regeneration—are quantified.

4.7.3 Management and policy implications

This study rebuts the claim that investment-oriented silviculture inevitably results in uniform, ecologically disinteresting stands. We offer three case studies, representing a wide variety of initial stand conditions, in which maximizing investment returns is the only consideration shaping management decisions. In each case, the financially optimal strategies are silviculturally sophisticated and lead to ecologically complex stands. Managers who insist on uniform silviculture leave money on the table.

Optimization in the even-aged stand produced a rotation age of 170 years and target diameters in excess of 50 cm. Cutting strategies varied over the rotation, tailored to species-specific growth patterns and differentiated individual-tree vigor and quality. Mixed-species composition was maintained through the final entry.

The second case study examined a stand that began with high quality growing stock well-distributed across size classes and a spatial arrangement indicative of a professionally managed uneven-aged stand on path toward a desirably complex structure. Optimal management neither steered the stand away from that path nor rigidly bound it to a target structure. Within-neighborhood patterns of removal varied over time and across neighborhoods. At no point did the stand approach a steady distribution of size classes, age classes, or neighborhood- or stand-level stocking.

The third case study can be thought of as an exit strategy scenario. Given a mature stand, what would be the most profitable way to end the current rotation and establish the next? Here, we observed a 60-year process of distributed establishment cuts with attentive tending in the unregenerated matrix, resulting again in large target diameters, mixed species composition, and irregular spatial and demographic structure that never converged to a static equilibrium.

The management implications of this study are clear: owners of quality hardwood forests stand to profit by calling on skilled foresters to implement thoughtful, silviculturally sophisticated treatments. High-value hardwood production is a dynamic process, a key ingredient of which is silvicultural skill (and the time required to deploy it). Foresters would do well to acquire such skills and to advocate forcefully for the value they can create.

The policy implications of this study are significant but perhaps less clear. First, forest policy aimed at restraining landowners from acting in their economic self-interest, based on the premise that financially optimal management is ecologically degrading, may be misplaced. Value-destructive behavior such as high grading should be understood as perverse, resulting from factors that distort landowner behavior away from what is efficient and enriching. Rather than *restrain*, policy should *empower* landowners to act more effectively in their own best interests. Good policy should aim to break down distortionary institutional factors and build up those that bring financial and conservation outcomes back into alignment.

Second, this study illustrates one side of the U-shaped relationship between financial performance and structural complexity. The best- and worst-managed stands each tend toward higher structural complexity than silviculturally middling stands. High grading requires no skill on the forester’s part, while unconstrained optimal silviculture “obviously requires much skill,” as Baker (1934, p. 373) put it with regard to high-touch thinning. But both approaches lead to more variable stands than uniform thinning methods or stand-scale regeneration treatments.

Regulatory or third-party certification schemes that are administratively biased toward conventional, uniform silvicultural treatments have a safeguarding effect by precluding the worst practices. The homogenizing tendency of early European "scientific forestry" methods (Puettmann, et al., 2009) likely stemmed from the perceived need to protect against the hazards of messier silvicultural systems, like what Vaselow (1963) referred to as *Ungeordnetes Plentern* ('disorderly' selection cutting). From the start, "scientific forestry" may have been less about maximizing production than about curtailing degradation. Yet, while mandating uniformity may establish a floor that protects against the worst cutting practices, it also puts a ceiling in place that precludes the best. This study provides further encouragement to reevaluate regulations and certification standards that inhibit foresters from pursuing the flexible, creative silviculture that ultimately leads to more valuable and ecologically richer stands.

5 MANUSCRIPT III

Foppert, J.D., 2022. Worse off on purpose: An economic analysis of deliberate forest degradation. *Forest Ecology and Management*, *504*, 119771.

This study turns explicitly to the question of high grading. It briefly reviews earlier commentators' explanations of the phenomenon (e.g. Nyland, 1992) which typically blame the practice on some combination of landowners' greed, short-termism, or ignorance. This study offers an alternative explanation centered on the role of imperfect information in markets for forestland. Forestland buyers are uncertain of the true value of a prospective property—even a thorough forest inventory will result in some sampling error around the estimated mean value. Rational buyers combine the information revealed from an inventory with their prior knowledge of the distribution of values across forests in the region. They shade their estimated valuation of the prospective property up or down toward the average region-wide value, reflecting their prior beliefs that especial high- or low-value properties are statistically uncommon. High grading is a rational strategy if buyers systematically undervalue well-managed forests and overvalue degraded ones. A formal theoretical model is developed to illustrate these "statistical discounting" dynamics and their implications for landowner harvesting decisions. The theoretical results are supported by a numerical simulation employing tree-level optimization methods and incorporating the effects of statistical discounting.

Worse off on purpose: An economic analysis of deliberate forest degradation

Abstract

High grading is, by definition, wealth-destructive and yet the practice is widespread. Explanations of high grading typically assume landowners are greedy, impatient, or ignorant. None of these explanations are robust. This paper develops an analytical model of landowner decision making that provides a richer explanation for this counterproductive practice. The model centers on buyers' behavior in markets for heterogenous-quality forestland. Facing imperfect information, it is rational for buyers to shade their estimated valuation of a prospective property up or down toward the region-wide average. Because high-quality forestland thus sells at a discount, and low-quality forestland for a premium, the so-called strip-and-flip strategy can outperform good, long-term silviculture. A simulation case study for a northern hardwood forest in the Adirondack region of New York illustrates this theoretical model. The simulation incorporates empirical growth models, continuous quality-specific price functions, and integer programming methods to specify the tree-by-tree harvest schedule that maximizes long-term net present value. An alternative simulation conditions cutting decisions on the expected sale price at the end of a ten-year investment period, resulting in a systematic—or, perhaps, "selective"—shift in harvesting patterns favoring removal of high-value trees. In presenting an improved theory of high grading, this study helps direct policy makers' attention away from dismissive characterizations of landowners as dumb, greedy, or both, and toward closer analysis of the institutional factors that drive deliberate forest degradation.

5.1 Introduction

High grading is a perplexing phenomenon. Its defining feature is that it destroys wealth and yet smart, disciplined investors profitably employ the practice. This paper presents a new theory that explains the rationale behind the purposeful use of this seemingly irrational practice. In doing so, it identifies the features of the institutional environment that enable deliberate high grading and suggests potential responses to discourage it.

What makes high grading high grading is how it's done and what it does. A three-part definition of high grading specifies that it is (1) the selective removal of individual trees (2) in roughly descending order of their current value (3) resulting in reduced capitalized value of the residual forest asset, inclusive of its reinvested cash component. Starting from the most valuable tree, some but not all individual stems are selected for removal. If the immediate harvest revenue and the residual stand's future yields (discounted to present values) add up to less than the net present value (NPV) of the initial stand's expected cashflows under proper silviculture, then that pattern of cutting constitutes high grading. In this way, high grading is, by definition, wealth destructive.

Why, then, would anyone high grade? "The greed factor" (Nyland, 1992, p. 36) is often cited but is too simplistic. After all, flushing money down the toilet is not such a greedy thing to do. Short-termism is another common explanation (e.g. Seymour et al., 1986) but does not hold up, either. If landowners are hyper-motivated to generate immediate revenue, why do they leave so much money on the table by merely high grading rather than (commercial) clearcutting?

If not greed or impatience, then most analysis just blames ignorance. Undoubtedly, there are cases where information and cognition interact in complex ways to perpetuate mismanagement, though simply chalking this up to "ignorance" may be too dismissive. In other cases, though, high grading is the work of sophisticated forest investors. Far from ignorant, these landowners and managers seem to know exactly what they are doing.

The remainder of this paper investigates the economic logic behind deliberate high grading and its implications. The basic theory turns on the role of imperfect information in markets for forestland and runs as follows: forestland buyers are uncertain of the true value of a prospective property and shade their estimated valuation up or down toward the average; anticipating this discount on high value properties and over-payment for low value ones, savvy landowners purposefully deplete their forests to exploit buyers' informational disadvantages.

5.2 Formal analysis

Consider a pure stand comprised of n trees. Denote the volume of tree i at time t by $f_i = f_i(t)$, $t \in \{t_0, T\}$, such that t_0 represents the first period in the two-period model, T represents the second period, and the time between t_0 and T is arbitrary and fixed. Stand structure is homogenous, so that $f_1 = f_2 = \cdots = f_n = f$ for all i. In general, price per unit volume is a function of a tree's size and quality. Denote the unit price of tree i in time t as $p_i = p_i(f(t); q_i)$, where q_i denotes individual-tree quality and is a fixed attribute of tree i. Assume timber quality varies across individual trees in the stand. Let q_i assign the parameters of the price curve mapping f onto p_i, where p is an increasing, sigmoidal function in f, as illustrated in Figure 5.1.

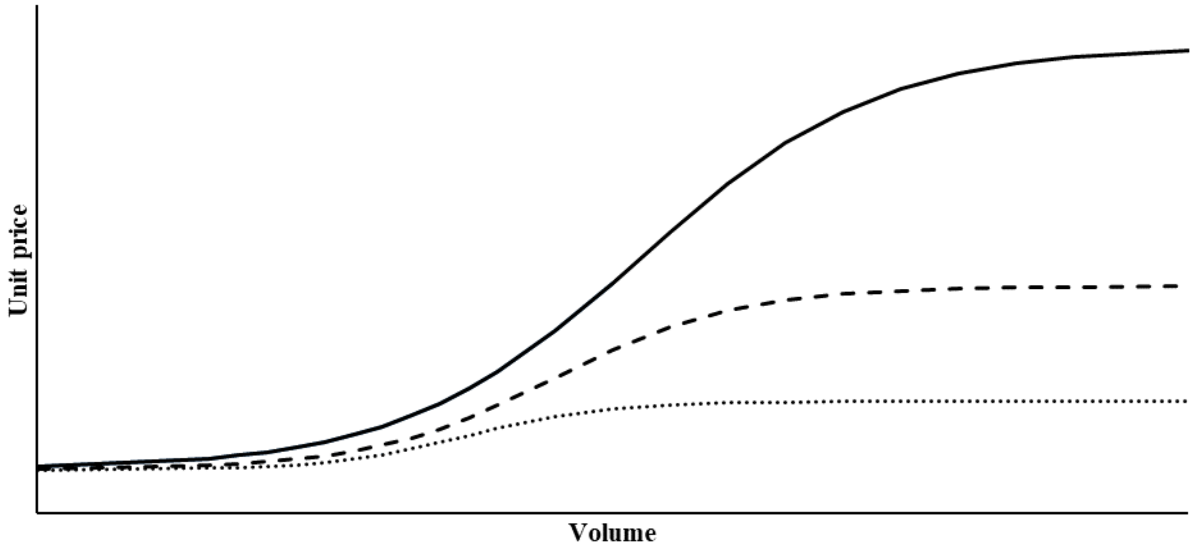

Figure 5.1: Stylized unit price curves, by quality: solid – high-quality; dashed – medium quality; dotted – low-quality.

Let $v_i(t)$ denote the stumpage value (i.e. the landowner's net revenue from the sale of standing timber) of tree i in period t, such that $v_i(t) = p_i(f(t); q_i)f(t)$. The present value of a tree harvested in the second period is thus $v_i(T)/(1+r)$ where r is the discount rate used to compare the value of cashflows across time periods and, by assumption, is specified exogenously and fixed for all forestry investments.

In the first period, the landowner chooses to either harvest tree i immediately or to delay harvest until the second period, described by the decision variable $\kappa_i \in \{0,1\}$, where $\kappa_i = 0$ denotes the decision to cut tree i in the first period and $\kappa_i = 1$ denotes the decision to keep it until the second period (at which point all residual trees are harvested). Let $\mathbf{K}$ denote the vector of harvest-or-retention decisions for each individual tree, such that $\mathbf{K} = \{\kappa_1, \kappa_2, \dots, \kappa_n\}, \kappa_i \in \{0,1\}$. The capitalized value of the stand is a function of the landowner's harvesting decisions and is given by:

$$NPV(\mathbf{K}) = \sum_{i=1}^{n} (1-\kappa_i)v_i(t_0) + \kappa_i \left(\frac{v_i(T)}{1+r}\right) \qquad \text{Eq. 5.1}$$

Let Κ denote an index of stand stocking, $\mathrm{K} = \mathrm{K}(\mathbf{K}) = n^{-1}\sum_{i=1}^{n} \kappa_i$. Assume biological growth is conditioned on a tree's initial volume and stand-level stocking, such that $f(T) = f\big(T|\mathrm{K}; f(t_0)\big)$. Let $\dot{f}$ denote the change in volume between periods, such that $\dot{f} = \dot{f}\big(\mathrm{K}, f(t_0)\big) = f\big(T|\mathrm{K}; f(t_0)\big) - f(t_0)$. Assume that $\dot{f}$ is a decreasing concave function in K, (i.e., $\dot{f}_{\mathrm{K}} < 0$, $\dot{f}_{\mathrm{KK}} < 0$) consistent with nonlinear growth responses to resource-mediated competition. Changes in unit price are also responsive to stand stocking, though indirectly. Let $\dot{p}_i$ denote the change in unit price of tree i, such that $\dot{p}_i = \dot{p}_i\big(K; q_i, f(t_0)\big) = p_i\Big(f\big(T|\mathrm{K}; f(t_0)\big); q_i\Big) - p_i\big(f(t_0); q_i\big)$.

Define a new term to consolidate the terms developed above. Let g_i denote the percentage change in stumpage value, $g_i = g_i(\mathbf{K}) = v_i(T)/v_i(t_0) - 1$

$$g_i(\mathbf{K}) = \frac{\Big(p_i\big(f(t); q_i\big) + \dot{p}_i\big(\mathbf{K}; q_i, f(t)\big)\Big) \cdot \Big(f(t) + \dot{f}\big(\mathbf{K}, f(t)\big)\Big)}{p_i\big(f(t); q_i\big)\ \cdot f(t)} - 1$$

$$= \frac{\dot{p}_i\big(\mathbf{K}; q_i, f(t)\big)}{p_i\big(f(t); q_i\big)} + \frac{\dot{f}\big(\mathbf{K}, f(t)\big)}{f(t)} + \frac{\dot{p}_i \dot{f}}{v_i(t)} \qquad \text{Eq. 5.2}$$

Rewrite Eq. 5.1 as:

$$NPV(\mathbf{K}) = \sum_{i=1}^{n} (1 - \kappa_i) v_i(t_0) + \kappa_i v_i(t_0) \left(\frac{1 + g_i(\mathbf{K})}{1 + r}\right) \qquad \text{Eq. 5.3}$$

An additional term helps to further clarify the landowner's objective function. When an investment "beats the market," finance practitioners use the term *alpha* to refer to the excess returns, (i.e. the market-beating portion of the investment return). In a silvicultural context, excess returns are the difference between the future value of a tree (or stand or forest) and its current timber value (CTV) compounded forward at the risk-adjusted market rate of return (corresponding to the discount rate, r). Excess returns quantify how much more valuable a tree is for having grown "on the stump" rather than "in the bank," given by $v(T) - v(t_0)(1 + r)$ in the single-tree version of the two-period model presented above[5]. Consistent with the informal usage

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[5] A fully developed, infinite-horizon version of the model would need to account for opportunity cost of the land in the form of the land expectation value and the definition of excess returns would include a $-rLEV$ term.

of *alpha*, let α_i denote the present value of future excess returns, proportional to the initial asset value:

$$\alpha_i(\mathbf{K}) = \frac{v_i(T|\mathbf{K}) - v_i(t_0)(1+r)}{1+r} v_i(t_0)^{-1}$$

$$= \frac{v_i(t_0)\big(1+g_i(\mathbf{K})\big) - v_i(t_0)(1+r)}{1+r} v_i(t_0)^{-1}$$

$$= \frac{1+g_i(\mathbf{K})}{1+r} - 1 \qquad \text{Eq. 5.4}$$

which (omitting some notation for convenience) reduces to $\alpha = (g-r)/(1+r)$. This expression provides for the most intuitive interpretation of the term α: future growth in excess of the cost of capital, discounted. Rewrite Eq. 5.3 as:

$$NPV(\mathbf{K}) = \sum_{i=1}^{n} (1-\kappa_i)v_i(t_0) + \kappa_i v_i(t_0)\left(1 + \left[\frac{1+g_i(\mathbf{K})}{1+r} - 1\right]\right) \qquad \text{Eq. 5.5}$$

and note that the term in the square brackets in Eq. 5.5 is equivalent to the definition of α_i in Eq. 5.4, such that:

$$NPV(\mathbf{K}) = \sum_{i=1}^{n} (1-\kappa_i)v_i(t_0) + \kappa_i v_i(t_0)\big(1+\alpha_i(\mathbf{K})\big) \qquad \text{Eq. 5.6}$$

Simplify Eq. 5.6 and write the landowner's objective function as:

$$\max_{\mathbf{K}} NPV = \sum_{i=1}^{n} \{v_i(t_0) + \kappa_i v_i(t_0)\alpha_i(\mathbf{K})\} \qquad \text{Eq. 5.7}$$

which states that the present value of a forest in the two-period model is value of its initial growing stock and maximum *alpha* obtainable from the residual stand. The solution to the maximization problem in Eq. 5.7 must satisfy the discrete approximation of the equi-marginal principle, such that:

$$\mathbf{K}^* = \underset{\mathbf{K}}{\operatorname{argmin}} \sum_{i=1}^{n} \left\{ \frac{\Delta\kappa_i}{\Delta\mathbf{K}} v_i(t_0)\alpha_i(\mathbf{K}^*) + \sum_{j=1}^{n} \kappa_j v_j(t_0) \frac{\Delta\alpha_j}{\Delta\kappa_i} \right\} \qquad \text{Eq. 5.8}$$

where each $\Delta\kappa_i/\Delta\mathbf{K}$ is evaluated at $\mathbf{K} = \mathbf{K}^*$ and where $\Delta\kappa_i/\Delta\mathbf{K} \in \{-1,1\}$, depending on the value of κ_i in the decision vector $\mathbf{K}^*$, such that $\Delta\kappa_i/\Delta\mathbf{K} = 1$ for $\kappa_i|_{\mathbf{K}^*} = 0$ and $\Delta\kappa_i/\Delta\mathbf{K} = -1$ for $\kappa_i|_{\mathbf{K}^*} = 1$. The first term inside the curly brackets in Eq. 5.8 thus represents either *foregone* or *recovered* excess returns, depending on the value of $\Delta\kappa_i/\Delta\mathbf{K}$. The inside summation in Eq. 5.8 quantifies the aggregate response to marginal thinning or retention, in terms of excess returns, by every other individual tree in the residual stand. Crucially, cutting a low-quality rather than high-quality tree (denoted by subscripts L and H, respectively) entails lower foregone excess returns ($\alpha_L < \alpha_H$), while, if retained, the good tree adds more value in response to thinning than its poor-quality neighbor ($\Delta\alpha_H/\Delta\kappa_i > \Delta\alpha_L/\Delta\kappa_i$). In the uniformly-structured, heterogeneous-quality stand examined here, the economically optimal thinning schedule unambiguously favors high-quality trees over low-quality trees for retention (see Appendix).

The analysis above assumes the landowner captures the full value of the second-period stumpage. Suppose, instead, that the landowner sells their forest just prior to the second-period harvest. NPV at the start of the first period is given by:

$$NPV(\mathbf{K}) = \sum_{i=1}^{n}[(1-\kappa_i)v_i(t_0)] + \frac{E[\Upsilon(T)]}{1+r} \qquad \text{Eq. 5.9}$$

where $\Upsilon(T)$ denotes the realized sale price at the start of the second period. Abstracting from any search or trading costs, assume the landowner can sell their forest in a competitive market for the present value of its expected future cashflows. With perfect information $E[\Upsilon(T)] = V = PF$, where $F = \sum_{i=1}^{n} f_i(T)$ denotes total volume at time T and $P = \sum_{i=1}^{n} \kappa_i p_i(T) \,/\, \sum_{i=1}^{n} \kappa_i$ denotes the average price per unit volume. The expected selling price under perfect information corresponds perfectly with the second-period initial CTV and there is no distortion in $\mathbf{K}^*$.

Next, consider the buyer's valuation process in an imperfect-information environment. The buyer conducts an inventory to estimate the liquidation value of the asset, $\bar{V}$. Assume that F is known with certainty. All variance in the estimate of

V is therefore attributable to variance in individual-tree quality, which leads to variance in estimates of P. Let $\bar{P}$ denote the buyer's estimate of P, which is generated from a sample drawn from the population of individual-tree unit prices, $\{p_i(T)\}, i = 1, .., n$. From the Central Limit Theorem, the buyer's post-inventory beliefs about V are normally distributed, $V \sim N(\bar{V}, s^2)$, where s denotes the inventory's standard error of the mean. For an unbiased sample, $E[\bar{V}] = V = E[\Upsilon(T)]$. Absent any additional information, the expected selling price still equals second-period CTV and NPV and $\mathbf{K}^*$ are both independent of the landowner's ownership horizon.

Finally, consider how prior knowledge of the asset universe affects a buyer's valuation process and its implications for harvesting strategy. Suppose a buyer holds beliefs about the distribution asset values. Let $\mathcal{M}$ denote the common-knowledge beliefs about the distribution of standing timber value across all properties that comprise the relevant market, where $\mathcal{M} \sim N(\mu, \sigma^2)$, such that μ is the average standing timber value (per unit area) across the market. When the subject property enters the market, the buyer considers it a random draw from the distribution $\mathcal{M}$, just as they consider $\bar{V}$ a random draw from the sampling distribution around $\bar{V}$ (where values of $\bar{V}$ are now also expressed on a per-unit-area basis, for consistency with the definition of $\mathcal{M}$). The probability that a property with a true value of $\check{V}$ would be simultaneously drawn from the population distribution $\mathcal{M}$ and the sampling distribution around $\bar{V}$ is given by the joint probability distribution:

$$\Pr\{V^*\} = \frac{1}{2\pi\bar{s}\sigma}\exp\left[-\frac{1}{2}\left(\left(\frac{\check{V}-\bar{V}}{\bar{s}}\right)^2 + \left(\frac{\check{V}-\mu}{\sigma}\right)^2\right)\right] \qquad \text{Eq. 5.10}$$

which is itself Gaussian. The expected true value of the subject property (and also, therefore, $E[\Upsilon(T)]$) is given by the first moment of the joint probability distribution:

$$E[\check{V}] = \frac{\bar{V}\sigma^2 + \mu\bar{s}^2}{\sigma^2 + \bar{s}^2} \qquad \text{Eq. 5.11}$$

Eq. 5.11 has important implications. For $\bar{V} > \mu$ and $\bar{s}^2 > 0$ (i.e. an above-average-value property inventoried with a non-zero sampling error), $E[Y(T)] =$

$E[\check{V}] < \bar{V}$. Rational, well-informed buyers' best estimate of the property's true value is less than their own (unbiased) estimate of V derived from direct measurement. The discount they apply reflects their prior beliefs that high-value properties are statistically uncommon. For a sample estimate with some range of uncertainty around it, it is more probable that the true value lies to the left of $\bar{V}$, where $\mathcal{M}$ is denser. This 'statistical discounting' is functionally similar to the process posited by statistical discrimination theory in economics (Phelps, 1972). The higher the sample variance or the lower the variance in the asset universe, the larger the magnitude of the statistical discount. Conversely, for $\bar{V} < \mu$, a rational buyer's willingness to pay includes a premium on top of their sample estimate $\bar{V}$, weighted proportional to the relative variances of the sample and the prior (see Figure 5.2).

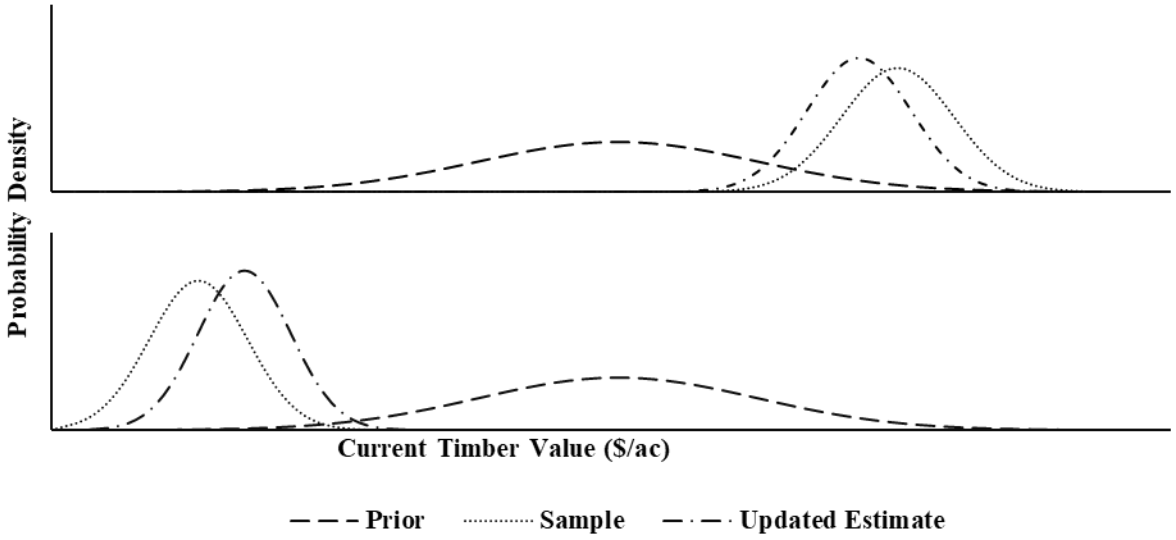

Figure 5.2: Statistical discount on above-average-value asset (top) and statistical premium on below-average-value asset (bottom)

Let δ denote the statistical discount or premium factor, such that:

$$\delta(\mathbf{K}) = \frac{E[\check{V}]}{V} = \frac{\sigma^2 + \frac{\mu}{V(\mathbf{K})}\bar{s}^2(\mathbf{K})}{\sigma^2 + \bar{s}^2(\mathbf{K})} \qquad \text{Eq. 5.12}$$

The forest's present value is therefore given by:

$$NPV(\mathbf{K}) = \sum_{i=1}^{n} \left[(1-\kappa_i) v_i(t_0) + \delta(\boldsymbol{K}) \kappa_i \frac{v_i(T)}{1+r} \right] \qquad \text{Eq. 5.13}$$

which can be re-expressed as the landowner's objective function:

$$\max_{\mathbf{K}} NPV(\mathbf{K}) = \sum_{i=1}^{n} v_i(t_0) - \big(1 - \delta(\mathbf{K})\big)\kappa_i v_i(t_0) + \delta(\mathbf{K})\alpha_i(\mathbf{K})\kappa_i v_i(t_0) \qquad \text{Eq. 5.14}$$

The decision vector $\mathbf{K}^*$ that solves Eq. 5.14 is analytically cumbersome to derive but the key implication is that the optimal harvesting strategy with respect to quality is ambiguous. Retaining a high-quality tree provides for additional *alpha* production, but at the expense of an increased statistical discount applied to that *alpha*, to the initial value of that tree, and to the terminal value of all the other trees in the residual stand. Particularly for cases where σ^2 is low and s^2 is high, retaining an above-average-quality tree may contribute less to NPV than retaining a below-average-quality tree. Deliberate high grading can thus emerge as a dominant strategy.

5.3 Numerical methods

The assumptions of homogenous stand structure and uniform growth dynamics in the theory presented above are unrealistic and potentially confounding. Variations in individual-tree health and vigor are key dimensions of silviculture, generally, and of the high-grading problem in particular. Especially for structurally-complex, mixed-species forests, numerical simulation provides the only tractable approach to examining these dynamics.

Data that provide the basis for simulation were collected from a 56 ha (138 ac) maple-beech-birch stand in the Adirondack region of northern New York. The stand is unbalanced, multi-aged with variable timber quality. High quality stems are distributed throughout the stand but are more prevalent in intermediate size classes.

Individual-tree data are represented over 24 fixed area plots (0.02 ha [0.05 ac]) and include species, diameter at breast height (dbh), live crown ratio (CR), and quality assessments for each potentially-merchantable 2.5 m (8.3 ft) bole section.

Diameter-inside-bark (dib) was calculated at the top of each log-section from Westfall and Scott's (2010) tree taper equations.

Prices were estimated for each log section of each tree. For a given species and quality class, a relative price factor (RPF) relates the expected unit price of the subject log section to the price of a reference grade, such as a #2 sawlog. An RPF of 2.0, for example, implies a unit price double that of a #2 sawlog of the same species. For a given species group and quality class, dib prescribes RPF according to the sigmoidal function:

$$RPF = \frac{\beta_0}{1 + \exp\left[-\frac{\beta_1}{\beta_0}(dib - \beta_2)\right]} + \beta_3 \qquad \text{Eq. 5.15}$$

where β coefficients can be fit from empirical price data.

Species common to the Northern Forest were grouped into five species groups: high-value hardwoods (e.g. hard maple, black cherry, red oak, yellow birch); mid-value hardwoods (e.g. soft maple, white ash); low-value hardwoods (e.g. aspen, basswood, beech); pine and hemlock; and other softwoods. High-value hardwoods were categorized into three quality classes: (1) veneer potential (4 clear faces [cf]), (2) sawtimber potential (1-3 cf), and (3) pallet/tie potential (0 cf). Log price data from the Indiana Forest Products Price Report[6] from 1957-2019 were referenced to #2 sawlog prices, by species, and plotted along the grade-limiting dib values for each quality class. For each quality class, RPF-function coefficients were fitted to minimize residual squared errors. For all species groups other than high-value hardwoods, RPF-functions were fitted for quality classes (2) and (3). Functions for mid- and low-value hardwood species groups were fitted from the Indiana Forest Products Price Reports dataset and for softwood species groups from a collection of price sheets from

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[6] Indiana sawmills are major importers of roundwood from across the northeast and Midwest, and the Indiana Forest Products Price Reports consists of the longest and most complete record of delivered sawlog prices; recent reports are available at https://www.in.gov/dnr/forestry/forestry-publications-and-presentations/ and historic reports at https://docs.lib.purdue.edu/timber/.

sawmills in the local market (Northern Forest states and Quebec) compiled since 2013. RPF-function coefficients for each species group and quality class are presented in Table 5.1.

Species group	Class 1 (Veneer potential)				Class 2 (Sawlog potential)				Class 3 (Pallet/tie potential)			
	β_0	β_1	β_2	β_3	β_0	β_1	β_2	β_3	β_0	β_1	β_2	β_3
High-value hardwoods	3.746	1.788	14.495	0.563	1.485	1.032	12.311	0.563	0.203	0.144	6.261	0.501
Mid-value hardwoods	–	–	–	–	0.884	0.629	12.379	0.759	0.041	0.064	4.583	0.759
Low-value hardwoods	–	–	–	–	0.366	0.293	12.945	0.936	0.059	0.012	13.145	0.936
Pine & hemlock	–	–	–	–	1.141	1.571	7.644	0.192	0.701	6.533	8.031	0.200
Other softwoods	–	–	–	–	0.813	3.798	4.885	0.187	0.500	6.514	7.969	0.199

Table 5.1: RPF-function coefficients, by species group and quality class

Following the approach described in Foppert (2019), a dynamic model was developed to simulate individual-tree growth subject to neighborhood-level competitive interactions. Measures of resource competition (i.e. total basal area and basal area of larger trees) were aggregated at the plot-level. Each plot was grown forward over ten ten-year timesteps using Weiskittel et al.'s (2016) Adirondack growth equations.

Roadside prices were calculated for individual trees in their specified harvest year from quality assessments, dib, and RPF values of all merchantable log sections. Stumpage was calculated as 50% of roadside log prices.

Reference prices (#2 sawlog) were assigned to each individual species from an average of advertised local sawmill price sheets and were assumed to remain constant, in real terms, over the simulation horizon; fixed unit prices were assigned for pulp-quality log sections (see Table 5.2). A fixed trucking rate per unit volume was assumed for all products. Future-period revenues were discounted back to present value terms with a 3.5% real discount rate.

As in Foppert (2019), stochastic regeneration with mixed species and variable quality actually simplifies the treatment of land expectation value (LEV). LEV represents the present value of expected future cashflows from optimal management following the establishment of regeneration after the last tree in a neighborhood is cut. Because LEV affects harvesting decisions, it must be estimated with iterative

Monte Carlo simulation, but once established its value is fixed and cutting decisions for the current stand do not “feed forward” as in the conventional analysis. A value of \$617/ha was assigned for LEV for all plots.

Species	Reference price (\$/MBF)	Pulp price (\$/cord)
Ash	\$400	\$15
Aspen	\$170	\$5
Basswood	\$190	\$5
Beech	\$190	\$15
Black cherry	\$450	\$15
Hard maple	\$550	\$15
Hemlock	\$300	\$10
Other hardwood	\$150	\$15
Other softwood	\$150	\$5
Paper birch	\$320	\$15
Soft maple	\$400	\$15
Spruce	\$340	\$5
Yellow birch	\$450	\$15

Table 5.2: Reference prices (#2 sawlog) and fixed roadside pulp prices, by species

A harvest vector, subject to optimization, specified the tree-by-tree cutting schedules under two contrasting ownership-horizon scenarios. Under long-term ownership, the landowner’s objective function is to maximize the present value of all cashflows over a 100-year horizon, including harvest revenues and LEV realizations. Values for the optimal harvest vector were generated using standard integer programming methods (Buongiorno and Gilless, 2001).

An early-exit scenario involves a simpler control function but a more complex objective function. Harvesting decisions were only evaluated for the first period, after which the residual plots were grown forward one time period before the property was “sold.” Sale prices were specified following Hutchinson et al.’s (2015) empirical sales price model for timberlands in New York, after adjusting the CTV parameter subject to statistical discounting according to Eq. 5.11 with $\mu =$ \$2,046/ha and $\sigma^2 =$ \$884/ha, as inferred from Hutchinson et al. (2015) and inflated to 2020 values.

5.4 Simulation results

The simulation results present starkly divergent harvesting strategies under long-term ownership and a planned early exit. Figure 5.3 illustrates the simulated optimal long-term strategy. The solid line in Figure 5.3depicts the evolution of the initial growing stock over the 100-year simulation horizon. It does not account for the contribution of newly established growing stock to stand basal area (around 30% of plots were regenerated prior to the year-100 harvest). Similarly, the projected cashflow schedule only accounts for timber harvest revenue and does not include LEV realizations following regeneration.

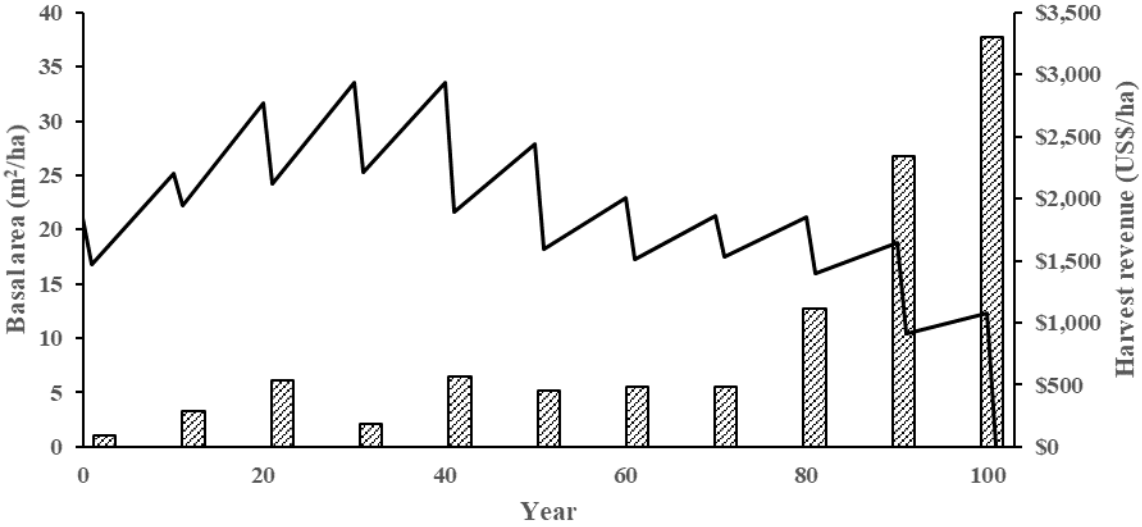

Figure 5.3: Simulated stand stocking trajectory (solid line) and timer revenue cashflow schedule (shaded bars, secondary axis) for indefinite ownership horizon optimal harvesting; cashflows exclude LEV realizations

Figure 5.3 tells a story of consistent value accrual and quality improvement through tending. Some individual trees are harvested via thinning from above, but the principal action is to reallocate growing space to high quality stems. The initial harvest removed 21.3% of total basal area (initial: 20.6 m^2/ha; residual: 15.8 m^2/ha). The harvest removed nearly the same proportion of CTV, reducing the value of the standing timber from $1,176/ha to $905/ha. The harvest removed 33 of the 222 trees with 4cf-quality butt logs (14.5% by count; 21.7% by basal area), none of which were high-value species. Most notably, the harvest had a significant impact on composition. The percentage (by basal area) of high-value hardwood species among sawtimber-size trees (dbh > 22 cm) increased from 54% initially to 82% post-harvest (see Figure 5.4).

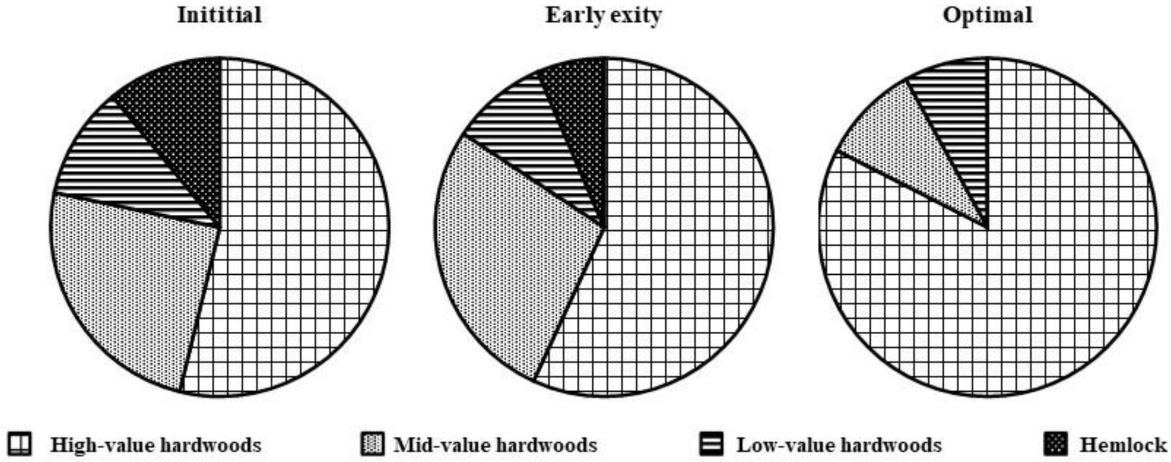

Figure 5.43: Proportional basal area of sawtimber trees (dbh > 22 cm) by species group in the initial stand (left) and the residual stand following simulated removals under the early exit scenario (middle) and optimal management over an indefinite horizon (right)

Shortening the ownership horizon and anticipating statistical discounting dynamics significantly distorted the direction and intensity of harvesting decisions. The simulated harvest in the early exit scenario reduced CTV by 52.1% and removed 136 of 222 trees with 4cf-quality butt logs. The early exit scenario removed 60.2% (by basal area) of sawtimber-sized trees of high-value hardwood species, while 100% of these trees were retained in the long-term optimal scenario. Similarly, 52.1% of the basal area of 4cf-quality-butt-log trees was removed in the early exit scenario. High grading clearly emerged as the dominant strategy in the early exit scenario.

5.5 Discussion

The theory developed above points to a dispiriting conclusion: high grading often makes sense. Worse still is to recognize its implications in a dynamic setting. The more prevalent high grading becomes, the farther buyers' distribution of prior beliefs skews to the left; the farther the distribution skews, the higher the discount on above-average value properties, further disincentivizing good silviculture. And this dynamic is frustratingly asymmetric: if good silviculture were to become widespread, shifting buyers' prior distributions to the right, the payoff from the "strip-and-flip" strategy would only increase. Heads, high grading wins; tails, silviculture loses.

Nonetheless, the mechanics of the dilemma also point to potential responses. High grading is individually profitable while still being wealth destructive only because its costs are shifted to unwitting buyers. Forestland buyers are ripe for such exploitation because information about quality and value is so uncertain to begin with and the horizon over which that uncertainty is resolved (as the initial growing stock is eventually harvested and its true quality is revealed) is so long. Still, buyers eventually bear the cost of the degradation the inherit, so institutional work (Lawrence et al., 2009) aimed at convincing buyers to "shop smarter" will likely be more productive than scolding would-be high graders.

Three opportunities stand out for savvier buyers to increase their own wealth and discourage high grading. They could cruise harder, cruise slower, and dig deeper. The models in this paper assume sampling intensity is exogenous, but smart buyers could invest in more rigorous sampling design to reduce the sampling error around their estimates. Similarly, if buyers were to cruise slower—paying more attention to product grades and grade improvement potential—they would eliminate one of the unmeasured margins high grading exploits. Both of these tactics would narrow the distribution around the sample mean, diminishing the effect of statistical discounting. Of course, cruising harder and cruising slower both imply cruising more expensively, but improvements in data collection and analysis technologies should shift norms toward more and better inventorying.

The third opportunity for smarter buying aims to narrow the prior distribution rather than the sampling distribution. Digging deeper refers to buyers becoming more adept at "reading" a property and interpreting the qualitative signals that indicate either good silviculture or past mismanagement. Buyers would form priors specific to subsets of similarly managed properties, rather than one aggregated prior for all properties across the market. A seller's gains from high grading vanish if buyers can recognize what has been done and price-in that degradation.

Each of these approaches discourage high grading by reducing its payoff. At best, markets would force high graders to fully own the costs of the wealth they destroy and would fully reward stewardship-minded landowners for the wealth their good silviculture creates. Realistically, however, there will always be unpriced margins sellers can exploit and deadweight welfare losses from the efforts expended to safeguard against exploitation (Barzel, 1997). A concerted effort to discourage high grading must confront the problem on multiple fronts. As much as the opportunities to strengthen the institutions of forestland acquisition can help mitigate high grading, so too can strengthening the institutions of forest ownership and management.

An expectation of longer tenures and less frequent ownership turnover would shift the calculus around high grading. Long-horizon investors evaluate the payoff of a high-grade-and-sell strategy against manage-well-and-hold. The gains from high grading may sometimes exceed the fundamental returns from sustainable management, but it is at least a fair fight between high grading and good silviculture. If, however, a landowner expects to sell their property soon, the contest is high-grade-and-sell versus manage-well-and-sell, which tilts systematically toward high grading. Longer ownership horizons tip the marginal cases of high grading back toward good silviculture.

While there are public policies that could help foster smarter buying practices and longer ownership horizons, the forestry profession has a larger role to play. High grading depends on unsophisticated buyers propping up the market for cut-over land. Normalizing foresters' place in the due diligence and acquisition process would knock the bottom out of that market. Foresters must advocate more persuasively for the value they can add at these stages. Similarly, foresters need to have a seat at the table when timberland investment strategies are conceived. The profession needs to argue forcefully for the competitive advantages of planning for long ownership horizons.

Finally, it is hard to overstate the importance of individual foresters and the profession as a whole drawing a line in the sand and calling out deliberate high grading for what it is: *unethical*. The profession stands to benefit from a clearer definition of high grading and more targeted identification of its motivators. Suggesting that it is simply a matter of greed creates the impression that high grading may actually serve landowners' financial interests. It does not. High grading hurts landowners, unless they plan to sell the property and expect to dupe some future buyer. One of two things happens when foresters coordinate, or even just condone, high grading: they either make the current landowner poorer or enrich them by making a future landowner even poorer still. Neither outcome is consistent with the mission the forestry profession purports to serve.

5.6 Conclusion

High grading is pernicious. Not only does it destroy value but in so doing it perpetuates the conditions that incentivize deliberate high grading in the first place. Addressing the problem requires a clear model of the structures, dynamics, and incentives that drive it. This paper explores one important facet of that problem space: purposeful high grading as a strategy to exploit imperfect information in forestland markets. This paper provided a formal model of optimal harvesting in heterogeneous-quality stands and showed that high grading is strictly sub-optimal if a landowner internalizes the payoff of their management decisions. High grading can be optimal, though, if a property will be sold and the statistical discounting effect is anticipated. A numerical simulation illustrated these dynamics in a more ecologically complex setting and supported the premise of the stylized theory. Understanding this driver of high grading should help guide the development of effective responses to the problem.

5.A Appendix

Proof, by contradiction of the sub-optimality of high grading in the two-period, perfect-information model: for some arbitrary harvest vector $\breve{\mathbf{K}}$, a high grading strategy would dominate an improvement cutting strategy if $\Delta NPV/\Delta\kappa_L >$ $\Delta NPV/\Delta\kappa_H$, i.e. NPV following the decision to modify $\breve{\mathbf{K}}$ by retaining a low-value tree L is greater than NPV following the decision to modify $\breve{\mathbf{K}}$ by retaining a high-value tree H.

From the terms inside the summation in Eq. 5.8:

$$\frac{\Delta NPV}{\Delta\kappa_H} = \frac{\Delta\kappa_H}{\Delta\breve{\mathbf{K}}} v_H(t_0)\alpha_H + \left(\kappa_L v_L(t_0)\frac{\Delta g_L}{\Delta\kappa_H} + \sum_{j\neq\{H,L\}}^{n} \kappa_j v_j(t_0)\frac{\Delta g_j}{\Delta\kappa_H}\right)(1+r)^{-1} \qquad \text{Eq. 5.A1}$$

Note that for the marginal retention decision evaluated here, $\Delta\kappa_H = 1$, $\kappa_L = 0$, and $\Delta\kappa_L = 0$, by construction, so the equation above reduces to:

$$\frac{\Delta NPV}{\Delta\kappa_H} = \frac{\Delta\kappa_H}{\Delta\breve{\mathbf{K}}} v_H(t_0)\alpha_H + \left(\sum_{j\neq\{H,L\}}^{n} \kappa_j v_j(t_0)\frac{\Delta g_j}{\Delta\kappa_H}\right)(1+r)^{-1} \qquad \text{Eq. 5.A2}$$

and $\Delta NPV/\Delta\kappa_L$ is analogously defined. Note also that summation inside the parentheses in preceding equation is of strictly equal value for both the high-quality and low-quality scenarios (from the assumption of uniform volume and stand structure). The inequality condition thus reduces to $v_L(t_0)\alpha_L > v_H(t_0)\alpha_H$, which holds if and only if:

$$v_L(t_0)\left(\frac{1+g_L}{1+r} - 1\right) > v_H(t_0)\left(\frac{1+g_H}{1+r} - 1\right) \qquad \text{Eq. 5.A3}$$

$$v_L(t_0)(g_L - r) > v_H(t_0)(g_H - r) \qquad \text{Eq. 5.A4}$$

From Eq. 5.2 and the definition of v, express the inequality condition as:

$$p_L f\left(\frac{\dot{p}_L}{p_L} + \frac{\dot{f}}{f} + \frac{\dot{p}_L\dot{f}}{p_L f} - r\right) > p_H f\left(\frac{\dot{p}_H}{p_H} + \frac{\dot{f}}{f} + \frac{\dot{p}_H\dot{f}}{p_H f} - r\right) \qquad \text{Eq. 5.A5}$$

$$\dot{p}_L f + p_L \dot{f} + \dot{p}_L \dot{f} - p_L f r > \dot{p}_H f + p_H \dot{f} + \dot{p}_H \dot{f} - p_H f r \tag{Eq. 5.A6}$$

Consolidating terms and rearranging yields:

$$\frac{(\dot{p}_L - \dot{p}_H) f + (p_L - p_H) \dot{f} + (\dot{p}_L - \dot{p}_H) \dot{f}}{(p_L - p_H) f} > r \tag{Eq. 5.A7}$$

$$\frac{(\dot{p}_L - \dot{p}_H)}{(p_L - p_H)} + \frac{\dot{f}}{f} + \frac{(\dot{p}_L - \dot{p}_H)}{(p_L - p_H)} \frac{\dot{f}}{f} > r \tag{Eq. 5.A8}$$

The analysis above is only interesting for cases where $r \leq g_L$ (otherwise $\alpha_L < 0$ and it is never rational to retain tree L). As a boundary condition, let $r = g_L$. For $\Delta NPV / \Delta \kappa_L > \Delta NPV / \Delta \kappa_H$ to hold, it must be that:

$$\frac{(\dot{p}_L - \dot{p}_H)}{(p_L - p_H)} + \frac{\dot{f}}{f} + \frac{(\dot{p}_L - \dot{p}_H) \dot{f}}{(p_L - p_H) f} > \frac{\dot{p}_L}{p_L} + \frac{\dot{f}}{f} + \frac{\dot{p}_L \dot{f}}{p_L f} \tag{Eq. 5.A9}$$

which, by rearrangement, implies:

$$\frac{(\dot{p}_L - \dot{p}_H)}{(p_L - p_H)} \left(1 + \frac{\dot{f}}{f}\right) + \frac{\dot{f}}{f} > \frac{\dot{p}_L}{p_L} \left(1 + \frac{\dot{f}}{f}\right) + \frac{\dot{f}}{f} \tag{Eq. 5.A10}$$

$$\frac{(\dot{p}_L - \dot{p}_H)}{(p_L - p_H)} > \frac{\dot{p}_L}{p_L} \tag{Eq. 5.A11}$$

which reduces to:

$$\frac{\dot{p}_H}{p_H} < \frac{\dot{p}_L}{p_L} \tag{Eq. 5.A12}$$

The Chapman-Richards' growth equation is commonly used in plant science, ecology, and other fields, and it provides a convenient functional form for the general specification of volume-to-price curves:

$$\frac{\partial p}{\partial f} = a \cdot p(f)\left(1 - \left(\frac{p(f)}{\mathcal{P}_{MAX}}\right)^{b}\right) \qquad \text{Eq. 5.A13}$$

where a defines the relative growth rate, $\mathcal{P}_{MAX}$ defines the upper limit of p, and b is a constant that allows the shape of the sigmoid curve to be varied asymmetrically. For economically relevant values of a and b, the integrated form of the differential equation is:

$$p(f) = \mathcal{P}_{MAX}(1 + Qe^{-abf})^{-\frac{1}{b}} \qquad \text{Eq. 5.A14}$$

where Q shifts the curve along the horizontal axis by indirectly defining the value for f at which $p = \mathcal{P}_{MAX}/2$. Let q assign specific values to the price curve parameters, such that $a = a(q)$, $b = b(q)$, $Q = Q(q)$, and $\mathcal{P}_{MAX} = \mathcal{P}_{MAX}(q)$. Naturally, the maximum unit price increases with increasing quality: $\mathcal{P}_{MAX}(q^H) > \mathcal{P}_{MAX}(q^M) > \mathcal{P}_{MAX}(q^L)$. Similarly, assume $Q(q^H) > Q(q^M) > Q(q^L)$, in that the high-quality trees sustain grade improvement into larger size classes than lower-quality trees. Assume all other price-curve parameters are constant across quality classes; for convenience, normalize a and b to 1 for all q.

Normalize f to approximate $\dot{p}$ as $\partial p/\partial f$ and substitute into the inequality condition:

$$\frac{p_H\left(1 - \frac{p_H}{\mathcal{P}_{MAX_H}}\right)}{p_H} < \frac{p_L\left(1 - \frac{p_L}{\mathcal{P}_{MAX_L}}\right)}{p_L} \qquad \text{Eq. 5.A15}$$

Simplify and substitute the definition of p from the integrated form of the differential equation:

$$1 - \frac{\mathcal{P}_{MAX_H}(1 + Q_H e^{-f})^{-1}}{\mathcal{P}_{MAX_H}} < 1 - \frac{\mathcal{P}_{MAX_L}(1 + Q_L e^{-f})^{-1}}{\mathcal{P}_{MAX_L}} \qquad \text{Eq. 5.A16}$$

Which further reduces to:

$$1 - \frac{1}{1 + Q_H e^{-f}} < 1 - \frac{1}{1 + Q_L e^{-f}} \qquad \text{Eq. 5.A17}$$

The inequality $Q_H < Q_L$ therefore must hold for $\Delta NPV/\Delta\kappa_L > \Delta NPV/\Delta\kappa_H$ to hold, the condition under which the high-quality tree should be removed before the low-quality tree. But quality was defined in part by the relation that the parameter Q is larger for high-quality trees than low-quality trees. This proves, by contradiction, that in a stand with uniform structure but heterogeneous quality, high-grading—cutting a high-quality tree while retaining a low-quality tree—is unambiguously sub-optimal for the objective of NPV production.

6 MANUSCRIPT IV

Foppert, J.D. & Maker, N.F. The best forestry money can buy: Efficient contracting for silvicultural expertise (and the limits thereof). *Journal of Forest Economics* (*in review*).

This study further explores the informational dimensions of harvesting decisions. This work examines the implications of information asymmetry between a forest owner and the forester contracted to provide expert services. Quality hardwood forests are responsive to silvicultural effort, though in this context effort takes the form of an expert forester's costly deliberation rather than the costly physical inputs associated with intensive plantation management. But the time a forester takes and the cognitive effort they exert in making individual-tree harvest and retention decisions are unobservable to a landowner in the moment and unverifiable after the fact. Is there an incentive contract a landowner can offer, based only on observable output—such as revenue generated, or volume or acreage harvested—that will induce the forester to exert an efficient level of silvicultural effort? In short, no. Employing bi-level optimization methods, this analysis models optimal contracting and demonstrates that the most efficient feasibly contractable outcome results in a significant deadweight loss relative to the first-best management strategy.

Authors' contributions

John Foppert developed the research question and conceptual modeling approach, supported the development of simulation and optimization methods, and drafted the original manuscript and produced the tables and figures. Neal Maker wrote the code, supported the development of the research question and simulation and optimization methods, and reviewed the manuscript.

The best forestry money can buy: Efficient contracting for silvicultural expertise (and the limits thereof)

Abstract

From the typical forest owner's kitchen window, thoughtful silviculture and lazy silviculture don't look very different. Thus, most landowners cannot directly compensate a forester based on their evaluation of the quality of the work performed. Is there an incentive contract, reliant only on observable outcomes, that could induce optimal effort from a skillful forester? We frame this contracting question as a bi-level optimization problem in which the forester (agent) solves an integer programming problem to choose the cutting schedule that will maximize her payoff, net of the cost of her effort, given the contract parameters specified by the forest owner (principal). The forest owner optimizes his choice of those parameters so as to maximize the value of returns generated from the resulting cutting schedule, net of the forester's compensation. We employ an optimization approach that combines a genetic algorithm with a derivative-based method well suited to handle the roughness and richness of this problem's solution space. We apply this approach to empirical data collected from a hardwood forest in northern Vermont, USA. Harvest schedules differ noticeably between a naïve, costless baseline scenario, a scenario in which only management costs (but not contracting distortions) are accounted for, and the bi-level optimal contracting model. We observe not just a transfer of wealth between the landowner and forester, but a deadweight loss as the maximum feasibly contractable gross value production is less than the first-best level of output.

6.1 Introduction

Choosing the optimal schedule for harvesting individual trees is no trivial problem. Consider 20 trees growing together in what forest ecologists would refer to as a neighborhood—an area sufficiently small that all the individuals present interact with each other in competition for limited resources. Suppose these are all healthy, high-quality trees. If left to grow, each would increase in value at a rate that exceeds the expected returns obtainable from harvesting the tree and reinvesting the revenue. That is, each tree is poised to add more value “on the stump” than it could “in the bank”. And yet, harvesting a tree would boost its neighbors’ growth. Does the improved growth of the residual trees justify forfeiting the excess returns the harvested tree would have generated? Exactly which trees should be retained and which should be removed to maximize value production? A forester approaching this neighborhood must choose from over a million different combinations of harvest and retention just in the present period. But the optimal choice at present depends on the harvest choices in subsequent periods. Given tree lifespans (and commercially relevant production horizons) that extend well over a century, billions of trillions of unique harvest schedules exist for this single neighborhood.

Recent advances in forest management and economics are beginning to provide tools and concepts to help confront this challenge (e.g. Foppert, 2019; Koster and Fuchs, 2022, Pascual and Guerra-Hernández, 2022). In many cases, numerical optimization tools, such as simulated annealing and evolutionary search algorithms, have proven extremely useful for searching the complex solution spaces of individual-tree cutting problems (e.g. Meilby and Nord-Larsen, 2012; West et al, 2021; Foppert and Maker, *in review).* However, behind the technical problem of tree-level optimization lies a second-order dilemma: how can a forest owner incentivize their forest manager to fully exert themselves in making those individual-tree selection decisions when neither the gains from that effort nor the effort itself are directly observable?

As with the optimization approaches introduced in the forest management literature, this paper utilizes simulation methods and an evolutionary search algorithm to evaluate a numerical contracting problem too complex for closed-form analysis. We frame the contracting question as a bi-level optimization problem in which the forester optimizes tree-level harvest decisions to maximize her payoff, given the contract parameters specified by the forest owner. Finding the vector of contract parameters that will maximize the forest owner's payoff—net of the cost of the forester's compensation, given her expected response to the contract offered—is the top-level optimization problem. This paper develops a framework for exploring otherwise inaccessible contractual problem spaces *in silico*, rendering them susceptible to analysis while retaining the richness and roughness that tractable, stylized models abstract away.

6.2 Background

6.2.1 The tree-cutting problem

The tree-cutting problem has been the object of serious analytical attention since at least the 17th century (Viitala, 2013). The problem has traditionally been formulated at the stand scale within the context of identical, infinitely-repeated rotations (Amacher et al., 2009). Given a known biological production function that relates merchantable volume to age, what is the optimal time to end one rotation and begin the next?

Though not a simple problem, the standard solution is now well known. Faustmann (1849) is credited as the first to correctly valuate a series of infinite rotations. Samuelson's (1976) modern treatment of the problem ushered in the so-called "Faustmann revival," inspiring a still-growing body of related literature (Newmann, 2002; Kant et al., 2013).

The standard Faustmann approach relies on several simplifying assumptions, including point-input, point-output production in which no intermediate treatments occur between the establishment and harvest of a stand (Amacher et al., 2009). In practice, however, intermediate treatments such as thinning are common in many silvicultural systems (see Matthews, 1989). Accounting for thinning significantly complicates the tree-cutting problem. Thinning crowds some revenue forward but forfeits the future excess returns the harvested growing stock could have earned. Most confoundingly, thinning affects the growth dynamics of the residual growing stock (see Oliver and Larson, 1996, pp. 228-234 for a detailed discussion), potentially altering both the value and the timing of the final harvest. While some studies have approached optimal thinning analytically (e.g. Coordes, 2014; Halbritter and Deegen, 2015), the complexity of the problem typically requires either strongly simplifying assumptions or numerical solution methods. In many ways, the development of modern numerical methods marked an advance in the field as significant as Faustmann's original contribution.

Clark (1976) brought mathematical programming approaches to prominence in the forest economics literature. Getz and Haight (1989), Tahvonen and Salo (1999), and many others built on this approach. Most numerical thinning studies (e.g. Parkatti and Tahvonen, 2020) work within a size-class model framework. They describe stand structure by the number of trees per unit area within successive size classes (see Weiskittel et al., 2011, pp. 61-67) and the proportion of growing stock removed from each size class serves as the decision variable.

An alternative approach to analyzing the tree-cutting problem has emerged somewhat recently, shifting analytical attention from the stand or size-class level to the individual-tree level. Early work by Härtl et al. (2010), Meilby and Nord-Larsen (2012), and Pukkala et al. (2015) set the stage for a string of recent studies (Lohmander, 2019; Foppert, 2019; Frannson et al., 2020; Pascual, 2021a, 2021b; West et al., 2021; Koster and Fuchs, 2022; Foppert, 2022; Pascual and Guerra-Hernández, 2022; Foppert and Maker, *in review*). While this literature is still early in its development and many of the authors above have approached the problem differently, all of these studies take up the challenging of evaluating harvesting decisions on a tree-by-tree basis. Tree-level optimization results in exponentially more complex problems than stand- or size-class-level approaches, but the added computational costs may be justifiable. Tree-level approaches are useful for analyzing complex stands (e.g. Lohmander, 2019; Foppert and Maker, *in review*) and essential for studies that incorporate within-stand spatial structure into their objective functions (e.g. Bettinger and Tang, 2015; Pascual, 2021a, 2021b; Dong et al., 2022). Similarly, in heterogenous-quality stands, where value production is unevenly distributed (as when it is concentrated in a small number of premium-quality stems), tree-level optimization mitigates the aggregation errors that result from modeling decisions at a coarser level of analysis (Foppert, 2019). Finally, information plays a critical role in many institutional settings (see Section 2.2). The tree-level approach allows the informational dimensions of thinning-and-harvest decisions to enter into analysis of settings susceptible to adverse selection (Foppert, 2022) or moral hazard, as we explore in this study.

6.2.2 The moral hazard problem

The moral hazard problem belongs to a general class of problems involving asymmetric information. Classical economic models assume that all parties to a transaction enjoy perfect information about the goods being exchanged, but this assumption is clearly unrealistic in many real-world situations. Very often, one party has more or better information than the other. Bargaining and exchange under these circumstances often leads to different outcomes than perfect-information price theory would predict (Akerlof, 1970).

The archetypal moral hazard (or *hidden action*) problem is set in an insurance context (e.g. Ross, 1973), but the framework has obvious relevance in more general management applications. Consider an unmonitored Agent facing the choice between working hard at a task for which a Principal stands to benefit, or withholding that effort. Hard work increases the probability of large payout but requires costly effort, while shirking reduces the probability of a large payout but requires no effort. The contractual options available to the Principal to induce efficient effort are limited and none are perfect.

If the Principal simply offers a fixed wage, it would be in the Agent's self-interest to shirk, collecting the full wage without having to exert costly effort. Alternatively, the Principal could offer an incentive contract, under which the Agent's payoff depends entirely on the observed outcome. But this provides, at best, only a partial solution. Agents often act on different financial parameters than the Principal (e.g. a shorter planning horizon, higher risk aversion, constrained access to credit), which would tilt an Agent's decisions away from those that best serve the Principal. And in any case, the Agent bears the full cost of her effort but only takes home a portion of the output. The management decisions (including, but not limited to, the choice of how much effort to exert) that are best for the Agent again deviate from those best for the Principal.

Either the fixed wage or the incentive contract could be augmented with increased monitoring by the Principal. He could invest in monitoring the Agent's work effort, rewarding (or punishing) the Agent based on the actions she takes, but only at a cost. As with measuring output, there is a direct cost to monitoring input (i.e. work effort). This monitoring is inherently subject to error, which can have a distortionary effect on incentives (see Ostrom, 1990, pp. 10-11 for a resource governance example).

All of these costs can be considered *transaction costs.* Transaction costs include (1) the resources spent by one party in an economic transaction to capture value from the other; (2) the resources spent to safeguard against such unauthorized appropriation; and (3) the deadweight loss resulting from the actions related to those attempts at capture or protection (Allen, 1991).

Economists' standard treatment of the moral hazard problem approaches it in a contracting setting (e.g. Salanié, 2005, pp. 119-160). The bargaining process is structured as a Stackelberg (or leader-follower) game, where the Principal makes a take-it-or-leave-it contract offer that the Agent accepts or rejects. The Principal chooses the contract terms that maximize his net payoff, subject to a pair of constraints. First, the contract must offer the Agent a payoff that induces her to freely choose the efficient level of effort (the *incentive compatibility constraint*). Second, the expected value of that payoff must exceed the outside option available to her if she were to seek employment elsewhere (the *individual rationality constraint*). As the comedian George Carlin quipped, "most people work just hard enough not to get fired and get paid just enough money not to quit."

6.2.3 Efficient contracting in forestry

Several studies have examined contracting under asymmetric information in forestry settings, though relatively few explicitly relate these ideas back to the optimal rotation problem. Leffler and Rucker (1991) and later Leffler et al. (2000) apply the property rights approach to an analysis of timber harvesting contracts and Wang and van Kooten (2001) analyze planting, weeding and cleaning, and other precommercial silvicultural interventions. Vedel et al. (2006) examine government consulting contracts for the provision of forest advisory services. Mason and Plantinga (2013) explore optimal contracting for forest carbon offsets and Fenichel et al. (2019), Immorlica et al. (2020) and Li et al. (2022) examine efficient contract design in various payment for ecosystem services or afforestation schemes.

Optimal forestry contracting studies that integrate thinning, rotation, or tree-cutting decisions are limited. Tatoutchoup (2015) develops an adverse selection model to characterize efficient royalty contracts when the harvesting firm has exclusive knowledge of their harvesting costs. Tatoutchoup's (2015) analysis accounts for adjustments in harvesting decisions by the license holder as the contract parameters change. Tatoutchoup and Njiki (2018) extend this analysis to a more complex setting with interdependent harvesting costs. Though not a contracting problem, *per se*, Deegen (2016) models public versus private forest ownership as a public choice problem under asymmetric information in which the forester's private cost of work effort plays a decisive role. Jensen et al. (2022) develop a single-rotation model to explore a situation where a regulator offers a landowner with different amenity preferences and asymmetric information a contract scheme to increase rotation age.

Our study extends this literature by more explicitly modeling forester and landowner choice environments and bringing more complex simulation methods to bear on the problem. This approach allows us to analyze tree-level harvesting decisions in heterogenous stands where a landowner's information disadvantage is especially acute and where the payoff from costly silvicultural expertise is especially high.

6.3 Methods

6.3.1 Benchmark harvest optimization

The benchmark optimal harvesting scenario can be conceptualized as a "pure" application of silviculture, abstracting away from any implementation costs or principal-agent distortions. What is the exact, tree-by-tree schedule of harvesting that maximizes the present value of gross timber sale revenue? Which is as if to ask: how would an expert hobby-forester—who owns her own woodlot and for whom time spent marking trees for harvest entails no cost—choose which trees to cut and which to keep?

We have argued elsewhere (Foppert, 2019; Foppert and Maker, *in review*) that this problem is best approached at the neighborhood level of analysis (Canham and Uriarte, 2006) and that the gap model framework from forest ecology (e.g. Botkin et al., 1972; Shugart and West, 1980; Bugmann, 2001) is especially well suited for optimization applications such as these. This framework models individual-tree growth and mortality subject to within-neighborhood competitive interactions but abstracts from any between-neighborhood dynamics. The gap model framework does not require spatially explicit tree location data but nonetheless accounts for the influence of structural heterogeneity within stands. For economic applications, neighborhoods can be modeled independently, substantially reducing the dimensionality of tree-level optimization problems without unduly forfeiting ecological realism.

Consider a neighborhood comprised of trees $i \in \{1, \dots, n\}$ at time t_0. In any period t each tree can be described by an index of physiological attributes, z_{it}, such as species, height, diameter at breast height, crown length and width, etc. Let $f_{it}(z_{it})$ denote merchantable volume of tree i as a function of those attributes.

Let k_{it} denote the competitive environment tree i resides in from period t to $t+1$, where $k_{it} = k_{it}(z_{it}, G_t)$ is a function of the attributes of tree i and of neighborhood-level composition and structure, $G_t = G_t(z_{1t}, \dots, z_{nt})$. The attributes of tree i evolve from period t to $t+1$ according to $\dot{z}_{it}(k_{it})$, such that $\dot{f}_{it} = f_{i,t+1} - f_{it}$. The equations above thus describe a dynamical system, in which the evolution of the system is determined by processes controlled by the initial state of the system.

Let v_{it} denote the potential gross harvest revenue of tree i at time t, such that $v_{it} = v_{it}(f_{it}; Q_i)$ where Q_i denotes timber quality class of tree i and specifies the coefficients for the function that relates unit price, p_{it}, to tree volume, where $p_{it} = p_{it}(f_{it}; Q_i)$. Note that Q_i is a time-invariant attribute of tree i. The total potential gross harvest revenue of an individual tree is therefore given by $v_{it} = p_{it} \cdot f_{it}$.

Let c_{it} denote the potential cost of harvesting tree i at time t. Assume harvesting cost is strictly a function of tree size, such that $c_{it} = c_{it}(f_{it})$. Stumpage value (i.e. net revenue) is the difference between v_{it} and c_{it}, provided tree i is actually harvested in period t.

The economic dimension of the individual-tree harvesting problem entails the choice, in each period $t \in \{t_0, t_1, \dots, T\}, T \in [t_0, \infty)$, of which trees to harvest and which to retain, and the valuation of the resulting cash flows. Let the binary variable

χ_{it} denote that tree i is harvested in period t when $\chi_{it} = 1$. An n-dimensional neighborhood-level harvest vector, $\boldsymbol{h}$, specifies the timing of harvest for each tree i such that $\boldsymbol{h} = \{h_1, h_2, \dots, h_n\}, h_i \in \{0, 1, \dots, T\}$. Define χ_{it} as $\chi_{it} = 1$ for $t = h_i$ and $\chi_{it} = 0 \; \forall \; t \neq h_i$ and modify G_t to account for changes in neighborhood structure resulting from harvesting, $G_t = G_t(z_{1t}, \dots, z_{nt}; \boldsymbol{h})$.

Realized stumpage, s_{it}, is given by $s_{it}(\boldsymbol{h}) = \chi_{it}(v_{it} - c_{it})$. Note the harvest vector, $\boldsymbol{h}$, controls χ_{it} directly and determines gross value, $v_{i\tau}$, and harvesting cost, $c_{i\tau}$, indirectly through their dependence on tree volume, $f_{i\tau}$, as controlled by $\dot{f}_{it}$ for all periods $t < \tau$, which is determined by the competitive environment, k_{it}, resulting ultimately from neighborhood structure contingent on harvest and retention decisions, $G_t(\boldsymbol{h})$.

The expected present value in time t_0 of realized stumpage from harvesting tree i at time $t = h_i$ is given by $u_i = s_{it}(1 + r)^{-t}$, where r is the exogenously specified per-period discount rate. Let U denote the aggregated neighborhood-level present value, $U = \sum_{i=1}^{n} u_i$.

By assumption, the neighborhood regenerates naturally following the harvest of the last mature tree. Natural regeneration is a stochastic process, so the structure and composition of the regenerated cohort will not necessarily replicate that of the initial neighborhood. Here, to restrict attention to the setup and solution of the basic optimal harvesting problem, we assume the capitalized value of future cashflows from a regenerated area (LEV, or land expectation value) is a fixed value specified exogenously. The silvicultural optimization problem can therefore be stated as an integer programming problem:

$$\max_{\boldsymbol{h}} \left[U(\boldsymbol{h}) + \frac{LEV}{(1 + r)^{T(\boldsymbol{h})}} \right]$$

s.t.

$$\boldsymbol{h} \in \{0, 1, 2 \dots\}$$

$$T(\boldsymbol{h}) = max\{h_1, h_2, \dots, h_n\}$$

A closed-form analytical solution to the objective function above would require strong concavity assumptions on the production function. Even then, problems involving more than a few trees and a few timesteps quickly become unmanageable. Numerical simulation and heuristic optimization methods provide a feasible alternative. Independently optimizing neighborhood-scale harvest schedules across the multiple neighborhoods that comprise a stand—regardless of the resulting spatial or demographic structure—represents a model of optimized irregular silviculture.

6.3.2 Optimal harvesting with costly management effort

Costly management effort adds an important dimension to the baseline harvesting problem. Management effort has traditionally been incorporated into a multivariate production function alongside rotation age (Chang, 1983; Amacher et al., 2009). In the standard formulation, the partial and cross-partial derivatives across age and management effort are "well behaved" (i.e. marginal physical products are positive and decreasing over the relevant ranges of age and effort). Management effort is typically conceptualized as silvicultural intensity and operationalized through production variables such as initial planting density or fertilization. The further one moves from the simplified production context of plantation silviculture, however, the less useful such approaches become. In a complex, natural forest, in which multiple species are present and individual trees vary widely in their quality and vigor, management effort is less a matter of silvicultural *intensity*, measured in terms of costly physical inputs, and more a matter of silvicultural *sophistication*, measured by the extent of an expert forester's costly deliberation.

Operationalizing this conception of management intensity presents distinct challenges. To motivate our modeling approach, we begin with the example of a forest owner capable of expertly managing her own property (thus sidestepping, for the time, the principal-agent issues we take up in the next section), but who *is* sensitive to management costs (unlike the hobby-forester invoked in the benchmark case). The forester-owner is capable of evaluating the exact structure of each neighborhood she comes to and of specifying the optimal selection of harvest and retention among the trees initially present. All of which, of course, requires time and effort. Alternatively, she could prescribe silvicultural treatments (i.e. *patterns* of cutting) delegatable to less expert silviculturalists. The easiest of these is to simply prescribe no action, retaining every tree and allowing the neighborhood's natural development to continue. Clearcutting decisions are also easily delegated, in that the intended action is unambiguous and is *ex post* verifiable at a glance. This treatment nevertheless requires more effort than taking no action at all, in that the harvesting contractor's actions must still be monitored and managed along non-silvicultural margins (e.g. operational considerations such as protecting water quality).

Between clearcutting and fully-optimized individual-tree silviculture lies a set of easier-to-implement heuristic treatments that would require less effort from the expert forester or that she could subcontract to a less-expert forester—what might be thought of as "sending out an intern". Much of the effort required in designating individual trees can be avoided if the treatment is incentive compatible with the logging contractor's own cutting preference. High grading, or "creaming," is when trees are removed in descending order of their current value without regard to the impact on long-term value production (Foppert, 2022). From the logger's perspective, high grading is the efficient choice of cutting pattern for timber purchased via a lump sum contract without silvicultural restrictions (Leffler and Rucker, 1991).

A forester can moderate the intensity of high grading by establishing a target level of stocking for the residual (post-harvest) stand or neighborhood. Verbal instructions to the logger can be as simple as a target level of basal area and permission to reach that target by selecting trees at their discretion. If a forester were to mark a high-grade harvest, the technical and analytical effort required would be minimal: simply identifying the most valuable trees and assessing whether stocking is within its target range. Monitoring *ex post* compliance is straightforward as well because the relevant attribute (stocking) is easily observable.

With additional effort, the high grading approach can be modified into a more sophisticated strategy, more productive of gross value. A forester again specifies a target level (or range) of residual stocking but, unlike high grading, that target is reached by removing trees in *ascending* order of *potential* value (corresponding, in the previous section's notation, to Q_i). Such "worst-first" harvesting is no longer incentive compatible for the logger, who—if not by contract, then at least by instinct—would prefer to cut high-quality trees instead of low-quality ones. Thus, the forester must actively select individual trees for harvest and then bear higher *ex post* monitoring costs to ensure that only designated trees are removed. Worst-first harvesting is, however, what we will refer to as "intern-implementable," in that its decision criteria depend only on observable current conditions, rather than requiring accurate projections of complex and conditional future growth processes. Even if the task is not literally delegated to an intern, the expert forester could implement it in less time and with less cognitive effort than the fully-optimized treatment would require. Because the worst-first and fully-optimized treatment strategies both explicitly designate which trees to harvest and which to retain, *ex post* monitoring is similarly costly for both approaches.

To summarize, a hierarchy of silvicultural effort exists, as illustrated in Table 6.1. The costs of *ex ante* and *ex post* effort vary among treatment strategies and are strictly ordered, though the gains from implementing increasingly sophisticated treatments is not always similarly ordered. Sometimes, no action is the best action. Similarly, it can be the case that the fully-optimized prescription overlaps perfectly or nearly perfectly with either clearcutting or the worst-first heuristic. In such cases, there is no payoff from the added expense of sending in the expert forester to mark every tree, versus telling the intern how to mark it or just pointing the logger in the right direction. And even where the cutting decisions from these more cheaply implemented strategies diverge from those that maximize gross value production, the savings from lower management costs may outweigh the losses from cruder silviculture.

Treatment	*Ex ante* effort	*Ex post* effort
No action	0	0
Clearcut	0	+
High grade	+	++
Worst-first	++	+++
Optimized	+++	+++

Table 6.1: Comparing timing and cost of effort across silvicultural treatments

Conceptually, the simplest approach to operationalizing this effort-cost schema would be to impose a uniform-treatment constraint. That would be to say, the forester chooses which implementation strategy to follow and that choice binds on the tree-selection decision for every neighborhood in the stand. This would reflect the case where the forester literally delegates the task of choosing trees to a third party (i.e. the logger or an intern). Such a scenario could be set up as a bi-level optimization problem and solved along similar lines as Tahvonen and Rämö (2016).

There are three shortcomings to this approach. First, the principal-agent problem motivating this paper depends on the principal's inability to monitor the agent. Uniform, stand-wide treatments, such as no action, clearcutting, or even to some extent the high grading and worst-first treatments, would provide an easily discernable signal to the landowner that the forester was not exerting a high level of effort.

Second, the question of optimal contracting over a rugged solution space is interesting at the margins, but the implementation of uniform, categorical treatment strategies would overwhelm most marginal variations. Thus, even if *delegatability* proves useful for conceptualizing management effort in this context, here we operationalize the cost of management effort as largely a function of time and attention. We envision the expert forester actually heading out into the woods, deciding neighborhood by neighborhood whether to work or to shirk, rather than making a call from the office whether or not to go out into the woods at all. The cost structure presented in Table 6.1, and much of the narrative logic developed to justify that cost structure, holds just as well when decisions are made within stands and the forester directly bears the costs of her deliberativeness as when decisions are made about stands and the forester (possibly) delegates the field work to a subordinate.

Finally, from a practical perspective, adding a third level to the optimization problem significantly complicates the search process without adding insight into the contracting dynamics. The computational costs might be justifiable if the phenomenon of interest mostly operated at a spatial level higher than the stand scale (i.e. ownership-scale, landscape-scale, etc.). For example, it might be informative to evaluate silvicultural implementation strategies in conjunction with ownership-level harvest scheduling models that accounted for fixed costs, capacity constraints, or fiber-supply commitments (e.g. Paradis et al., 2018). These are not the questions we choose to examine. The model we develop here operates at a different (and, we contend, more interesting) level where more is gained by freeing the agent to vary effort on the fly.

The objective function from the frictionless baseline model can be modified to account for the forester's effort without restructuring the overall model. Let $w_t = w_t(\boldsymbol{h})$ denote the forester's internal (and unobservable) cost of management effort at a given time t as a function of the specified harvest vector, $\boldsymbol{h}$, where $w_t \in \{w^0,\ w^{\forall}, w^+, w^-, w^*\}$ and the elements of this set are defined below. Let W denote the capitalized cost of management effort, where

$$W = W(\boldsymbol{h}) = \sum_{t=0}^{T} \frac{w_t(\boldsymbol{h})}{(1+r)^t}$$

The forester's effort-inclusive objective function can thus be written as

$$\max_{\boldsymbol{h}} \left[U(\boldsymbol{h}) + \frac{LEV}{(1+r)^{-T(\boldsymbol{h})}} - W(\boldsymbol{h}) \right]$$

s.t.

$$\boldsymbol{h} \in \{0, 1, 2 \dots\}$$
$$T(\boldsymbol{h}) = max\{h_1, h_2, \dots, h_n\}$$

The elements in the management effort cost set correspond to the five different silvicultural treatment strategies outlined above. The no action strategy, w^0, entails retaining every tree in the initial neighborhood in time t, such that $w_t = w^0$ if $\chi_{it} = 0 \; \forall \, i$. Inversely, the clearcut strategy, $w^{\forall}$, entails removing every tree standing at the start of time t, such that $w_t = w^{\forall}$ if $\max \, h_i = t$.

The high grading strategy, w^+, entails harvesting in such a way that minimizes the difference between the residual plot-level basal area and a specified target level with the condition that the current value of the lowest-valued harvested tree must exceed that of the highest-valued retained tree: $w_t = w^+$ if $\min v_{h_i=t} > \max v_{h_i \neq t}$ and $|BA_t - BA^*| < |BA_t - ba_{i+1,t} - BA^*|$ and $|BA_t - BA^*| < |BA_t - ba_{i-1,t} - BA^*|$, where BA_t denotes the stocking (basal area) at time t, BA^* denotes target stocking, $ba_{i,t}$ denotes individual-tree basal area, and trees are ordered by current value, such that $v_1 > v_2 > \cdots > v_{i-1} > v_i > v_{i+1} \ldots > v_n$ in time t.

The cost of implementing a worst-first harvest, w^-, is structured similarly. Here, $w_t = w^-$ if $\min Q_{h_i=t} > \max Q_{h_i>t}$, $|BA_t - BA^*| < |BA_t - ba_{i+1,t} - BA^*|$, and $|BA_t - BA^*| < |BA_t - ba_{i-1,t} - BA^*|$ when trees are ordered by $Q_1 < Q_2 < \cdots < Q_{i-1} < Q_i < Q_{i+1} < \cdots < Q_n$.

Optimizing the harvest vector, $\boldsymbol{h}$, in the costly management effort scenario proceeds exactly as in the baseline scenario. Aside from adjusting the forester's objective function, as described above, no further modifications to the model or the optimization procedure are required.

6.3.3 Efficient contracting

Introducing contract choice to the optimal harvesting problem changes the structure of the problem substantially. The forester is no longer the full residual claimant, so her payoff function depends on the parameters of the management contract. Consider the following five contractual devices, the first four of which issue regular (per management period) compensation, and the last of which provides exit compensation at the close of the contract. Let θ denote a fixed payment per unit area managed, independent of cutting decisions (i.e. a salary); let γ denote a contingent payment per unit volume harvested; let ρ denote a contingent payment per unit area harvested; let λ denote a contingent payment as a percent of timber sale revenue generated (i.e. a sale commission); and let ϕ denote a contingent payment as a percent of standing timber value (or "current timber value," CTV) at the end of the last management period within the contract horizon. The forester's objective function thus becomes

$$\max_{\boldsymbol{h}} \pi^A = \sum_{t=0}^{T} \frac{\left[\sum_{i=1}^{n} \chi_{it}(\gamma f_i + \lambda v_i)\right] + \theta + \rho X_t - w_t}{(1 + r^A)^t} + \frac{\phi \sum_{i=1}^{n} \prod_{t=1}^{T} (1 - \chi_{it})\,(v_{iT} - c_{iT})}{(1 + r^A)^T}$$

where X_t denotes the occurrence of harvest in period t, $\mathrm{X}_t = \begin{Bmatrix} 1 & if & \sum \chi_{it} \geq 1 \\ 0 & if & \sum \chi_{it} = 0 \end{Bmatrix}$, and r^A denotes the forester's (i.e. agent's) discount rate.

The forest owner's objective function is given by

$$\begin{aligned} \max_{\theta\,\rho\,\lambda\,\gamma\,\phi} \pi^P = \sum_{t=0}^{T} & \frac{\sum_{i=1}^{n} \chi_{it}\big((1 - \lambda) v_i - \gamma f_i\big) - (\theta + \rho X_t)}{(1 + r^P)^t} \\ & + \frac{(1 - \phi) \sum_{i=1}^{n} \prod_{t=1}^{T} (1 - \chi_{it})\,(v_{iT} - c_{iT}) + LEV}{(1 + r^P)^T} \end{aligned}$$

s.t.

$$\chi_{it} = \chi_{it}(\boldsymbol{h}^*)\ , \qquad \boldsymbol{h}^* = \operatorname{argmax} \pi^A$$

$$\pi^A(\boldsymbol{h}^*) \geq \omega$$

where ω denotes the forester's reservation payoff, such that the constraints on the forest owner's objective function correspond to the standard incentive compatibility (IC) and individual rationality (IR) constraints. Note that the landowner's discount rate, r^P, differs from the forester's and, by assumption, $r^P < r^A$, reflecting not divergent risk preferences but the principal's presumed diversified portfolio position relative to the agent's presumed overexposure to the idiosyncratic (i.e. diversifiable) risks of the specific forest asset.

6.3.4 Numerical solution methods

We modeled a stand represented by four plots drawn from a system of permanent monitoring plots on a private forest in central Vermont. The 7.32 m radius fixed-area plots (0.02 ha) follow the design of US Forest Service Forest Inventory and Analysis (FIA) subplots (Woudenberg et al., 2010). Plots were inventoried in the summer of 2022, with the following attributes recorded for each tree larger than 14 cm: species, DBH (measured in inches, converted to centimeters), height (measured in feet, converted to meters), CR (estimated to the nearest 10%-class), and quality assessments for each potentially-merchantable 2.5 m bole section, evaluated irrespective of current diameter, consistent with the methods described in Demchik et al. (2018).

We simulated growth and mortality over 5-year timesteps using the nonlinear least squares models of mortality and of DBH-, CR-, and height-increment, described in Maker and Foppert (*In press*). Opportunities to harvest occurred at for four discrete times, $t =$ 0, 10, 20, and 30. Following the Year 30 harvest, any residual growing stock was grown forward until time $t =$ 40, at which point the residual inventory was valuated (on the basis of CTV), any exit compensation was awarded, and the contract was closed. Stumpage prices were assigned according to the relative price functions, references price list, and harvest and transport cost functions described in Foppert and Maker (*in review*). Volume- or area-based prices, costs, and contract payoffs used local measurement units, such as cords (roughly equivalent to 2.5 m^3) and acres (2.471 ha).

We assigned a cost (US$) per 0.02-ha plot of $w^0 = 0$ for no removal and $w^\forall = 1.75$ for clearcutting. The per-plot cost of implementing high grading varied as a function of the number of trees harvested: marking a high grade is not hard, but the more trees there are, the longer it takes. We specified the implementation cost function for high grading as $w_t^+ = 0.70 + 1 \cdot \sum \chi_t$. We generalized target stocking for our northern hardwoods application from Leak et al., 2014 as $BA^* = 15$ m^2/ha.

To operationalize the quality ranking in the worst-first algorithm, we defined $Q_i = q_{i1} + 0.25q_{i2} + SG_i$, where q_{i1} and q_{i2} denoted the individual-bolt quality score of the first and second bolt (2.5-m log section) of a given tree, $q_{ib} \in \{1, 2, 3, 4\}$, corresponding to pulp-, pallet-, sawlog-, and veneer-potential logs, respectively; SG is a constant assigned by species group, corresponding to the relative preference for some species groups over others, $SG \in [0,2]$ with high SG values corresponding to high species desirability. Q_i is thus weighted to reflect the disproportionate importance of the first bolt and to discriminate against undesirable species. We specified $w_t^- = 0.70 + 1.5 \cdot \sum \chi_t$, so that implementation of the worst-first strategy entails the same per-plot fixed cost as high grading but higher per-tree variable costs.

Finally, the fully-optimized treatment strategy is the default management effort cost function: $w_t^* = 3.5 + 2 \cdot \sum \chi_t$. In the simulation environment, unless the pattern of tree selection for a given plot and a given timestep conformed to one of the strategies described above, the forester was assumed to have exerted maximum effort and w_t^* was applied. We specified the forester's discount rate, r^A, as 6% and the landowner's discount rate, r^P, as 3%. We assigned a value of 0 for the forester's reservation payoff, ω, reflecting a competitive market for forestry services.

We first modeled optimal thinning-and-harvest decisions under the benchmark and costly-management scenarios. We then modeled the moral hazard scenario, in which trees were selected for removal by an unmonitored agent, subject to the contract the principal offers. This scenario was evaluated as a bi-level optimization problem, with the specification of contract parameters as the top-level problem and the choice of harvest vector, $\boldsymbol{h}$, as the bottom-level problem.

For all scenarios, we used the `rgenoud` package for R (Mebane and Sekhon, 2011; R Core Team, 2022) to optimize $\boldsymbol{h}$. The `rgenoud` software package employs a genetic algorithm and, where appropriate, combines it with a derivative-based (quasi-Newton) method (Mebane and Sekhon, 2011). The genetic algorithm is an evolutionary search algorithm that accommodates local maxima and discontinuities in the solution space and can thus handle problems characterized by complex production functions and integer decision variables. However, the genetic algorithm is relatively ineffective at hill climbing and is computationally inefficient. The derivative-based method solves local hill climbing tasks effectively and is relatively parsimonious, but it is poorly suited for handling irregularities and may fail to discover a global optimum.

Combining these approaches, `rgenoud` assumes that the solution space is globally irregular but locally regular and uses evolutionary approaches generally with a derivative-based approach applied locally to the best solution in each generation (Mebane and Sekhon, 2011). Both methods are employed in solving the top-level (continuous variable) problem.

6.4 Results

The benchmark harvest optimization scenario generated NPV of \$5,085/ha at the 3% discount rate. The observed cutting pattern illustrates the complexity of precision hardwood silviculture (Table 6.2). In general, the highest quality, most vigorous stems were favored for retention. Four out of five veneer-quality stems were retained through exit. The one veneer tree selected for removal had low-vigor (CR=20%) and was competing with two other veneer-quality trees. Many but not all low-value species and poor-quality stems were removed in the initial entry. For trees with pallet- or pulp-quality butt logs, removal rates in the first entry were 60% and 75%, respectively. All remaining pallet trees and all but one pulp tree were removed in the second or third entry. Retention and removal decisions on sawtimber-quality trees were more varied but generally reflected a pattern of retention up to approximately 40 cm DBH.

Species	DBH (cm)	CR (%)	Quality score	CTV, by year				
				0	10	20	30	40
BF	25	30	4-3-4	**$0.13**	-	-	-	-
SM	38	30	2-2-2	$37.55	**$60.68**	-	-	-
SM	25	30	1-1-2	$3.93	$7.16	$14.59	$28.19	**$50.19**
SM	14	20	1-1-3	**$0.16**	-	-	-	-
SM	22	40	1-2-3	$2.22	$4.02	$7.99	$14.65	**$24.93**
PB	32	30	2-2-2	$4.22	$7.71	**$13.09**	-	-
SM	17	40	2-2-3	$0.43	$1.00	$1.97	$4.00	**$7.61**
SM	53	40	3-4-3	**$34.02**	-	-	-	-
SM	16	50	4-2-3	$0.30	$0.76	$1.48	$2.84	**$4.86**
SM	52	40	2-2-3	**$109.16**	-	-	-	-
SM	21	30	2-2-3	$1.40	$2.90	$5.52	$11.27	**$21.13**
YB	33	30	3-4-3	**$2.46**	-	-	-	-
SM	29	50	3-4-3	$6.47	**$9.16**	-	-	-
WA	15	50	2-2-3	$0.67	$1.61	$3.14	$5.26	**$7.98**
BE	19	90	2-4-4	$1.20	$2.30	$3.88	$5.90	**$8.51**
WA	34	30	1-2-2	$22.46	$39.31	$64.48	$98.43	**$138.35**
BE	18	60	4-4-4	**$0.25**	-	-	-	-
SM	30	30	1-2-2	$12.04	$21.79	$36.65	$64.74	**$99.95**
SM	26	20	3-2-4	**$3.52**	-	-	-	-
SM	24	10	4-4-4	**$1.06**	-	-	-	-
SM	35	20	3-2-2	$13.56	$23.55	**$37.21**	-	-

Table 6.2: Initial plot composition (thick lines between rows indicate plot breaks) and current timber value (CTV) by year; bold CTV values indicate realized value from harvest or exit valuation; species codes: BE = American beech, BF = balsam fir, PB = paper birch, SM = sugar maple, YB = yellow birch, WA = white ash; quality score indicates the assessed maximum potential product class of each of the first three 2.5 m log sections: 1 = veneer (4 clear faces [cf]), 2 = sawtimber (1-3 cf), 3 = pallet (0 cf, straight, sound), 4 = pulp (defective)

Residual stocking was generally lower than conventional management recommendations. Stand-level residual basal area ranged from 8.9 to 12.5 m^2/ha, compared to “B-line” stocking of around 15 m^2/ha for northern hardwoods (Leak et al., 2014). Plot-level residual stoking was naturally more variable, ranging from 6.2 to 16.2m^2/ha. Lower stocking also resulted in less gross production than a fully stocked stand could produce: annual basal area increment varied from 0.21 to 0.25 m^2/ha/yr, approximately half of the production potential expected locally on a good hardwood site. Thus, the baseline optimization revealed a clear strategy of favoring the individual growth of the best stems even at the cost of lower stand-level production.

Effort and agency costs significantly affected the patterns of cutting (Table 6.3). Accounting for silvicultural effort changed the harvest decision on nine out of 21 trees in the simulation, in each case harvesting later or retaining a tree through exit.

Species	DBH (cm)	CR (%)	Quality score	Harvest year		
				Benchmark	**Effort cost**	**Agency cost**
BF	25	30	4-3-4	0	--	20^{-}
SM	38	30	2-2-2	10	20^{+}	--
SM	25	30	1-1-2	--	--	--
SM	14	20	1-1-3	0	--	--
SM	22	40	1-2-3	--	--	--
PB	32	30	2-2-2	20	--	*0*
SM	17	40	2-2-3	--	--	*0*
SM	53	40	3-4-3	0	0^{+}	*0*
SM	16	50	4-2-3	--	--	*0*
SM	52	40	2-2-3	0	0^{+}	*0*
SM	21	30	2-2-3	--	--	*0*
YB	33	30	3-4-3	0	10^{*}	*0*
SM	29	50	3-4-3	10	20^{-}	--
WA	15	50	2-2-3	--	--	--
BE	19	90	2-4-4	--	--	--
WA	34	30	1-2-2	--	--	--
BE	18	60	4-4-4	0	--	20^{-}
SM	30	30	1-2-2	--	--	--
SM	26	20	3-2-4	0	--	--
SM	24	10	4-4-4	0	--	30^{-}
SM	35	20	3-2-2	20	20^{+}	10^{+}

Table 6.3: Harvest year comparison, by scenario, with treatment types indicated next to harvest years for effort cost and agency cost scenarios: "+" = high-grade, "–" = worst-first, "*" = optimal harvesting; see Table 6.2 for species codes and quality score criteria.

Of sixteen opportunities for plot-level treatments, five treatments were implemented under costly effort, compared to seven in the benchmark scenario. Three of these treatments were high grade harvests, one was a worst-first harvest, and one was optimized. The gross capitalized value of harvest revenues and terminal inventory decreased to \$4,883/ha and the owner-forester bore \$135/ha in capitalized costs of effort (also discounted at 3%; \$188/ha undiscounted), for a net present value of \$4,748/ha.

For the efficient contracting scenario, optimal contract parameters were specified as follows: $\gamma = 0.988114$; $\lambda = 0.05122812$; $\rho = 42.01945$; $\theta = 0$; $\phi =$

0.1669698. That is to say, the optimal incentive contract offered the forester approximately \$0.99 per cord harvested, 5.1% of all harvest revenue, \$42 per acre (\$103/ha) harvested, and 16.7% of CTV at exit. Plot-level treatments consisted of one clearcut, one high grade, and three worst-first harvests. Table 6.4 summarizes the resulting cashflows.

Year	Gross revenue	Net revenue	Transfer	Effort cost
0	\$2,260	\$2,085	\$175	\$22
10	\$333	\$285	\$48	\$23
20	\$19	-\$37	\$56	\$51
30	\$41	\$10	\$31	\$29
40	\$6,786	\$5,653	\$1,133	\$0
NPV (r = 3%):	*\$4,616*	*\$4,013*	*\$603*	*\$80*
		NPV (r = 6%):	*\$335*	*\$56*

Table 6.4: Modeled harvest revenue (or Year 40 CTV), incentive compensation transfers, and silvicultural effort cost schedules under efficient contracting

The gross value of harvest revenue and terminal inventory, discounted at the landowner's rate of 3%, was \$4,616/ha. Of that capitalized value, the landowner transferred 13% (\$603/ha) to the forester, leaving a net value for the landowner of \$4,013/ha. Applying the forester's discount rate of 6% to those transfer payments, the present value of the forester's compensation was \$335/ha and she bore \$56/ha in capitalized costs of silvicultural effort. The undiscounted cost of effort was \$126/ha, 33% less total effort than when agency costs were unaccounted for.

6.5 Discussion

Ashton and Kelty (2018) observe that "[t]he most profitable forest type is not necessarily the one with the greatest potential for growth or the one that can be used or harvested at lowest cost. One must also consider the silvicultural costs of growing the crop" (p. 13). Silviculturalists (and economists!) typically limit their consideration of these "silvicultural costs" to the direct costs of production, such as site preparation, planting, pruning, precommercial thinning, and harvesting costs, if they consider them at all. Rarely do they take account of the variable costs of implementation. And never, to our knowledge, has the concept of "silvicultural costs" been extended to include the information-induced transaction costs associated with different silvicultural treatments.

Consider first the direct implementation costs of sophisticated silvicultural treatments. The creative prescriptions contemporary silviculturalists often envision demand correspondingly more time and skill from the forester on the ground to implement. Beyond acknowledging the potentially "higher treatment costs due to [their] more complex prescriptions," (Palik et al., 2021, p. 17) silviculturalists rarely evaluate (much less optimize) the costs and benefits of such complexity. For their part, forest economists have generally been unhelpful in this setting, as well. The practical examples of management intensity economists analyze are almost always physical inputs or interventions such as increased planting density, improved genetics, or fertilization (e.g. Chang, 1983; Nautiyal and Williams, 1990; Amacher et al., 1991; Halbritter and Deegen, 2015). In the accompanying abstract models, management effort enters into the production function. It is taken for granted that the forester can then costlessly optimize production decisions. This approach maps awkwardly onto the notion of costly silvicultural sophistication pertinent to quality hardwood management.

In our model, effort does not enter directly into the production function but operates instead through the forester's "decision technology". We represented management effort as a discrete choice between different modes of silvicultural decision making, from crude, uniform treatments to relatively cheaply implementable heuristics to complex, optimized tree-by-tree selection. Optimization is feasible, but not free. Only in the most general sense can landowners or foresters "undertake (costly) actions that improve the growth conditions of their stands," as Amacher et al. (2009, p. 34) describe management effort. In our setup, "growth conditions" were fixed; only the decision technology was responsive to investment.

We employed reasonable, though admittedly arbitrary, cost parameters for this study. Our aim was not to precisely quantify the costs of implementing sophisticated silviculture, but merely to introduce such cost considerations into a more general framework for the economic analysis of complex stands. Our results showed clearly that these costs affected the forester's decision calculus. Overall, less work was carried out and the intensity of that work clearly decreased. Over 16 potential plot-level treatment opportunities, optimal tending was implemented just once.

Though often overlooked, the economic logic of costly management effort is nonetheless uncomplicated: intensive management is only efficient if the resulting gains exceed their added costs. Apparently, in many cases they don't. The logic of transaction costs, however, is less obvious. It unfolds in three steps. First, deadweight losses are unavoidable under imperfect information—the maximum *feasibly-contractable* level of value production will always be lower than the technically efficient level. Second, the informationally-advantaged contracting party (here, the forester) will extract further rents from the "blind" party if a single contract must span across a heterogeneous decision space. And yet, third, in a competitive environment, the informationally-advantaged party ultimately pays for that privilege by an amount greater than its value.

It is tempting to assume it would be in the best interest of all parties to manage the resources they were contracting over to maximize value production, net of the cost of parties' contributions. The crux of the contracting problem, then, would be in haggling over the division of that surplus value. In fact, such outcomes are unobtainable. Given asymmetric information, the most efficient *feasible* contract, consistent with the incentive compatibility and individual rational constraints described in Section 6.2.2, cannot induce first-best resource management. The Principal is willing to forfeit some gross production for a contracting arrangement that lets him retain a larger residual. The forest grows less but the forest owner keeps more.

The landowner's scope for losses increases as the contracting space gets more complex. Early, exploratory runs of the contracting problem optimized the contract parameters for single plots at a time. In these scenarios, the landowner consistently drove the forester's payoff (net of her private cost of effort) down toward the reservation wage, for example, \$1.736e-19/ha, which is to say, minimally in excess of zero. However, in the multi-plot scenario we modeled here, the contract terms could not be tailored as finely. At the plot level, the forester's net payoff (capitalized at 6%) varied from \$156 to \$615/ha. A contract forced to span multiple plots in a heterogenous stand necessarily cedes rents to the forester.

The operative mechanism here is the individual rationality constraint, which binds at the plot level. The forester's option to do nothing establishes a lower bound of zero on the payoff of any action she chooses. The landowner cannot recoup excessively generous compensation in one period through negative transfers back from the forester in another. She would simply walk away, in the moment, from any given plot rather than take a loss on it. And given the unobservability of her actions, the landowner cannot condition future payoffs on present performance. A premium payoff for the forester results.

Before foresters get too excited, though, they should recall Barzel's (1997) assertion that "those able to shirk, to cheat, or to enjoy 'free' perks must, under competition, pay for the privilege by an amount that *exceeds* in expected terms the value of the privilege" (p. 42). In a dynamic setting, excess rents are unlikely to prove sustainable. The opportunity costs of a forester's time are embedded in the cost of her silvicultural effort. The reason thoughtful silviculture costs more than the shoddier variety is because the former takes so much longer to implement. But if a forester could capture additional rents by curtailing her effort on one job and adding additional rent-ceding clients elsewhere, she would price those opportunities into her private cost calculations, disproportionately increasing the cost of time-intensive silvicultural strategies (i.e. worst-first and optimal tending) relative to the easier-to-implement options (clearcutting and high-grading). At the job level, she would exert less effort on the margin and less gross value would be produced; across the market, landowners would adjust the contract terms offered to reflect foresters' lower, opportunity-cost-adjusted level of effort. Modeling the resulting equilibrium would require dynamic silvicultural cost functions, as well as a richer representation of the attributes of the other forestland properties, landowners, and foresters in the market, all of which is well beyond the scope of this paper.

Nonetheless, this points to the limits of a purely contractarian approach to governing the relationship between landowners and foresters. The most efficient contract design leaves on the table 15% of the forest asset's potential capitalized value. That uncaptured value represents what could be thought of as the landowner's "governance budget". Rather than relying exclusively on contractual mechanisms of governance, savvy or creative landowners could invest in non-contractual arrangements that more effectively economize on transaction costs or even cultivate intrinsic motivation (Ryan and Deci, 2000; Grant and Shin, 2012). Aggressive incentive contracting—premised on greed and untrustworthiness—can have the opposite effect. It signals that self-interested behavior is expected, that other social norms should be ignored, and that those who feel otherwise need not apply. Thus, the real limit of efficient contracting is not in the rents it cedes or the value it leaves uncaptured, but the risk that it "crowds out" exactly the type of unmonitored effort and far-sighted behavior it was intended to induce (Frey and Jegen, 2001).

7 SUMMARY & DISCUSSION

This section summarizes the key findings and contributions from the preceding manuscripts, discusses the limitations of these studies, and considers their implications for policy, practice, and future research. This dissertation was motivated by the observation that transaction costs bear particularly heavily on the management of the structurally complex, potentially high-value hardwood forests of eastern North America. These costs—and the informational dynamics that give rise to them—are underappreciated by the scholars, policy analysts, and advocates trying to understand and (perhaps eventually) alter the puzzling, often problematic behavior of forest landowners in the region.

One aim of this dissertation was to contribute to the development of a new set of analytical tools that could better accommodate the ecologically and informationally complex forests of interest here. The first and second manuscripts of this dissertation provide the conceptual rationale and the technical specifications of the modeling approach developed to serve that end and they apply it in straightforward investment analysis applications. The third and fourth manuscripts then deploy those tools in institutionally richer settings where asymmetric information creates misaligned incentives and scope for opportunistic behavior.

The first manuscript in this dissertation argues that the shift to tree- and neighborhood-level analyses is noteworthy in the continued evolution of forest economics. Fine-scale economic analysis is about more than flattering the biometricians who build tree-level growth models; fundamental economic considerations are at play. Modeling the production dynamics of structurally-complex, quality-differentiated stands entails substantial risk of aggregation errors. The value produced by applying a uniform treatment, optimized for the average stand condition, is not equivalent to the value produced by applying a set of treatments optimized to the conditions in each neighborhood. The mean of the optima is not the optimum of the mean. The aggregate-then-optimize approach mis-specifies the optimal thinning and harvest regime, leading to the systematic undervaluation vis-à-vis an optimize-then-aggregate approach. This phenomenon is known generally as Jensen's Inequality (Jensen, 1906), a concept that has received some attention from ecologists (e.g. Cale et al., 1983; Ruel and Ayers 1999, Duursma and Robinson, 2003) and forest biometricians (e.g. Moeur and Ek, 1981; Sambakhe et al., 2014; Green et al., 2019) but much less from forest economists. This manuscript appears to be the first time Jensen's Inequality has been considered in the context of harvest optimization.

The first and second manuscripts of this dissertation both address the challenges of tree-level optimization directly. Each argues for the adoption of the gap model framework (Botkin et al., 1972) for forest economic analysis. Computationally, the gap model framework is significantly more efficient than fully distance-dependent modeling and optimization from census inventory data while only imposing small or negligible modeling distortions. This approach evaluates tree cutting decisions at the neighborhood scale (i.e. 0.01-0.1 ha), contingent only on the evolving local stand composition and structure. When the final tree in a neighborhood is removed, the area is assumed to regenerate and LEV is realized.

Economic gap modeling accurately depicts tree-tree interactions and a forester's optimal thinning response at the neighborhood scale, while illustrating the patch-wise evolution of structure at the stand scale (Pickett and White, 1985). In conventional silvicultural terms, this approach depicts an irregular group selection system with matrix free thinning. In that way, it mimics the form and logic of the high-touch silviculture many of the most thoughtful foresters implement on the ground (e.g. Susse et al., 2011). As an analytical approach, economic gap modeling feasibly delivers several useful outputs. It generates high-resolution financial and ecological projections that provide a basis for planning and valuation. At the stand level, it paints a picture of the optimal *pattern* of cutting (though stopping short of prescribing each and every removal or retention decision). It provides field foresters with a roadmap to guide their implementation of the strategy. And at the finest scale, it can be used to calibrate training tools such as marteloscopes (Soucy et al., 2016).

The concept of excess value or "silvicultural alpha"—introduced in first manuscript, employed as a reporting metric in the second, and formally incorporated it into an analytical model in the third—provides a conceptual framing for the thinning problem that could further support improved practice on the ground. Excess value is defined as the difference between a forest's long-term capitalized asset value and its immediate liquidation value. Long-term capitalized value consists of NPV, inclusive of discounted revenue from the harvest of current growing stock and discounted LEV, realized following a regenerative harvest. Liquidation value consists of current timber value (CTV) plus LEV. Alpha measures the gains from silviculture above the cost of capital, or the extent to which well-managed capital allocated to growing stock "beats the market" (i.e. exceeds the market rate of return).

The concept of silvicultural alpha deserves attention both as an analytical device and, especially, as a conceptual teaching tool. Analytically, it concentrates attention on the response of those values or attributes that are sensitive to silvicultural intervention. Given the particularly capital-intensive nature of forest production (Binkley, 1993), CTV often comprises a large fraction of total forest asset value. By setting aside this "sunk value" that is nonresponsive to intervention, the alpha metric offers a more easily interpretable basis for comparison across treatments or strategies (as in the numerical application in the first manuscript). In financial forest management problems, the silvicultural objective function is often defined as maximizing LEV—an economically valid but otherwise confusing and unintuitive concept. Maximizing silvicultural alpha is an equally valid objective function but has proven to be much easier for forestry students and practitioners to understand. It frames the thinning problem as tasking the forester with choosing which trees to cut and which trees to leave so as to maximize the amount by which capital value "on the stump" outperforms what it could earn "in the bank".

The second manuscript builds on the ideas introduced in the first and advances them within a more robust numerical modeling methodology. It uses machine learning models of mortality and of diameter, height, and crown ratio increment developed in a separate study, adjacent to this dissertation (Maker and Foppert, *in press*). It provides more complete descriptions of harvest cost, product price, and log merchandizing sub-models. And, to optimize this bioeconomic system, it employs a genetic search algorithm more flexible than the forward dynamic programming procedure described in the first manuscript.

The second manuscript also considers in more detail the technical limitations of the tree-level modeling approach employed across this dissertation. The main limitations relate to the treatment of mortality, harvesting costs and damage, price dynamics, and regeneration. A few of these could be addressed with *relatively* minor technical improvements, but those that entail a switch from deterministic to stochastic processes (i.e. the treatment of individual-tree mortality and market-level product-specific price dynamics) will require wholesale retooling of the modeling strategy. Yet these processes are fundamental components of forest production systems and are tremendously consequential to timberland investment performance over forest-relevant timescales. Future work should confront those challenges.

Emerging approaches, such as reinforcement learning (Malo et al., 2021; Tahvonen et al., 2022) offer considerable promise.

Though developed principally as a methods paper, the second manuscript's most significant contribution may be its conclusion that, at least for the northern hardwood forests it examines, financially optimized silviculture requires finesse and results in stand-structural complexity. This finding corroborates results in the first manuscript and cuts sharply against the conventional wisdom that investment-oriented forest management only and always results in ecologically simplified stands, or that ecologically interesting silviculture is necessarily less revenue productive.

The second manuscript concludes with a discussion of policy implications. In its discussion of the U-shaped relationship between financial performance and structural complexity, and of "scientific" forestry's historical aversion to "disorderly" silviculture (Vaselow, 1963; Puettmann et al., 2008), it hints at the informational dimensions of forest management and administration that are central to this dissertation. As with the first manuscript, though, it goes no further, leaving exploration of those topics for subsequent studies.

The third and fourth manuscripts are explicitly set in informationally rich environments. The third manuscript addresses the high grading question directly. It defines high grading and then develops a formal model proving that the practice is value destructive. The study then provides a model of forest buyers' behavior in markets for heterogeneous-quality forestland with imperfect information, and of forest owners' harvesting behavior in anticipation of those dynamics. Here, high grading can be wealth-maximizing. Strategic high grading before disposition—akin to the strategy private equity critics refer to as "strip-and-flip" (Stringham and Vogel, 2018)—sometimes outperforms long-term ownership and sound silviculture.

A concrete illustration of this stylistic model requires tools of the sort developed in the first and second manuscripts. Species diversity, differentiated quality, variable individual-tree health and vigor, spatial heterogeneity: high grading operates across all these dimensions. Simulating “selective” harvesting requires analytical visibility into the tree-level attributes individual stems are selected on. The simulation results in the third manuscript show a stark divergence in harvesting strategies between long-term optimal management and a limited investment horizon. To maximize returns under a shortened ownership horizon, over 60% (by basal area) of sawtimber-sized, high-value-species stems were harvested, while all of those trees were retained in the first entry in the long-term optimal scenario. Strategic high grading can be a smart strategy when asymmetric information is exploitable.

The fourth manuscript moves from information asymmetry between timberland buyers and sellers to forest owners and managers. It asks whether there could exist an incentive contract conditioned only on observable outcomes that would induce efficient effort from an (unsupervised) expert forester. This study nests the complexity of the tree-level optimization problem inside a higher-level optimal contracting problem. It presents a novel simulation-based approach for solving *in silico* an otherwise intractable contracting problem.

To motivate the moral hazard dilemma, the forester must bear a private cost for her silvicultural effort. To that end, the manuscript introduces a new conception of management effort into harvest decision analysis. Before even getting to transaction costs, the study provides a more thorough consideration of the dynamic costs of silvicultural implementation than appears elsewhere in the literature. The second scenario in the study incorporates a management cost function that varies by the silvicultural complexity of the prescribed treatment. The overall outcome was clear: putting a price on silvicultural complexity meaningfully affected the forester’s decision calculus. The forester did less work and the intensity of that work decreased. Only once did the gains from optimal tending justify the cost.

To a silviculturalist, it is frustrating to hear that thoughtful tree marking might create less value than it costs—that the juice often isn’t worth the squeeze. (Thousands of gallons of marking paint add up to a lot of juice over a forester’s lifetime.) This assertion needs to be put in perspective. Its basis is in *ad hoc* cost functions intended to be illustrative rather than authoritative, employed in a single case study with just 21 trees. Nevertheless, its general implications hold regardless of the specifics. Forests can be overmanaged. When the high implementation costs of better silviculture are properly accounted for, mild high grading or other forms of apparent silvicultural underinvestment can sometimes be efficient. The modeling in this study structured the forester's management effort decision as a choice from a discrete set. It is unclear if modeling effort as a continuous variable would change the picture. Nor is it clear if feedbacks exist, such that good silviculture in the beginning increases the payoff on more good silviculture in the future, or *vice versa*. This component of the study begs for follow up research.

The main focus of the fourth manuscript was the contracting problem. Here, again, the results represent no more than the first take on a complex research question that demands further investigation. Nevertheless, they are significant. Even the optimally specified incentive contract has a substantial distortionary effect on the forester's decision making. Compared to the benchmark scenario, agency costs reduced the gross value of production by 9% and the landowner's net payoff by 16%. The forester exerted one-third less effort as a contractor than she would have if she owned the stand herself.

From a purely contractarian perspective (e.g. Bolton and Dewatripont, 2004), these losses are unavoidable. Trying to recover some of the money left on the table would only cede more informational rents to the forester. The efficient contract fully economizes on the tradeoffs between rent cession, wealth dissipation, and incentive intensity. The best forestry money can buy turns out not to be so good.

This result relates back, with frustrating consequences, to a logic embedded in the analytical model in the third manuscript. Though not formally developed, the model implies that in a dynamic setting high grading is self-reinforcing while sound silviculture is self-discouraging. As high grading becomes more common, rational buyers discount above-average value properties more, further disincentivizing good silviculture. But if good silviculture were to become the norm, the payoff from high grading would only increase. As noted in the manuscript, the situation modeled here is one of "heads, high grading wins; tails, silviculture loses."

It is in this context that the economist's toolkit feels particularly bare. For a scholar (and proud forester) reluctant to accept that high grading represents some sort of efficient institutional outcome—as economists working from the transaction costs approach might assert (e.g. Allen, 2018)—the tools and insights from adjacent fields such as institutional work theory (Hampel et al., 2017) or Ostrom's (2005) Institutional Analysis and Development framework offer a heartening contrast to economists' grouchy insistence that "[t]here is no way a student of economics can use an economic model to 'save the world'" (Allen, 2022, p. 30).

The third manuscript identifies specific opportunities to discourage high grading that relate to the institutional structure of forestland acquisition and the institutions of forest ownership and management. It stops short, however, of proposing fully-formed institutional solutions. Devices such as signaling, screening, bonding, and warranting are all well developed in the institutional economics literature (see Ménard and Shirley, 2005) and should be incorporated into not just an improved theory of high grading but potential responses to it.

As the fourth manuscript points out in its conclusion, the purely contractarian perspective does not capture all the contours of work motivation (Grant and Shin, 2012), particularly in a field like forestry. Non-contractual governance devices may prove more effective at cultivating intrinsic motivation and inducing optimal effort (Ostrom, 2005). In fact, there is a real risk that an overly transactional employment relationship may actively crowd out the intrinsic motivation that would otherwise drive foresters' actions. Purely economic incentives are a signal that narrowly self-interested behavior is acceptable and even expected, and that other social norms should be ignored (Frey and Jegen, 2002). Clearly, norms play a substantial role in regulating foresters' and landowners' behavior and structuring their decision processes (Scott, 2001; Ostrom, 2005; Primmer and Karppinen, 2010).

A full assessment of the high grading problem will require a clearer picture of all its dimensions, not just those directly explored in the third and fourth manuscripts. Before narrowing the scope of the study to deliberate degradation, the third manuscript hints at a different, non-strategic category of high grading. It notes that some cases of high grading are driven by complex interactions of information and cognition. The anecdote about the widow and the farmer in this dissertation's introduction illustrates some aspects of this. The full logic of that behavior (especially when foresters' spouses aren't involved) likely turns on some elements of the Volvo Theorem (Brazee, 2003), a bounded-rationality-driven cost-minimization model of forest owner behavior (Simon, 1957; Wagner 2020), and the informational dimensions of markets for credence goods (Dulleck and Kerschbamer, 2006). More fully developing these ideas represents an important, if challenging, step forward for this research agenda.

Across the four manuscripts that comprise this dissertation, an innovative approach to modeling the optimal management of complex forest stands was developed and applied to investment and institutional analysis problems. This approach integrates tools from forest economics, ecology, and computer science. It takes a problem that is incomprehensively hard to solve and reduces it to one that is merely very, very hard. The approach takes its cues from the thought patterns and work habits of the world's best field foresters. At the same time, it provides a tool that might help them refine, enrich, and further advance their practice. To landowners and investment managers, it puts numbers behind the claim that there is money to be made by operating with a finer touch. And it draws attention to one of forestry's cleanest little secrets: that conservation and investment goals are often surprisingly well aligned, if not for the distortionary institutional factors that act as a wedge between them.

The final two manuscripts in this dissertation address these institutional questions squarely, though not in their entirety. They identify—directly or in passing—four drivers of high grading: the opportunity to pass off a “lemon” as a “peach” in the forestland market, leading to deliberate degradation; a combination of imperfect capital markets, cost-minimizing rather than wealth-maximizing landowner behavior, and a mistrust of foresters leading landowners to back in to unsupervised, non-strategic high grading; agency-induced underinvestment in silvicultural effort at the margin of contracting efficiency; and efficient high grading, when the costs of careful silvicultural analysis and implementation exceed the gains. This typology has not been thoroughly developed nor even preliminarily tested. Nevertheless, it offers a more constructive starting point for analyzing and addressing landowners’ puzzling behavior than simply dismissing them as greedy, impatient, or dumb. Much work remains to be done. It is my sincere hope that the tools and concepts developed in this dissertation prove useful to those who take up that challenge.

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