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The Interplay Between Parental (STEM) Occupation, Parental Attitudes, and the Home Numeracy Environment and Their Impact on Children's Mathematical Competencies

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Corrigendum

The coding for participants' genders (girls and boys) had been confused. All significant gender differences in favour of girls are therefore in favour of boys. Thus, the discussion contains erroneous conclusions from incorrect parts of the analysis. However, the confusion does not affect the analytical results that this study produced.

Table of Contents

Abstract

Early numeracy skills are an essential predictor of children's later academic achievement and thus play a critical role in their lives early on. As these skills develop before children start school, their parents' influence is important. However, the home numeracy environment (HNE) young children find themselves in is greatly influenced by family characteristics such as the parents' occupation or attitudes towards learning. This suggests that parents who work in numeracy-related fields, namely STEM fields, would provide their children with numeracy activities that are of higher quality than non-STEM parents. Additionally, STEM parents should regard mathematics as more important and enjoyable than parents working in other fields, thus conveying these positive attitudes to their children. Furthermore, as a gender gap still exists in both mathematical achievement and STEM occupations, it can be assumed that parents' numeracy-related interactions with their children differ in regard to their child's gender. Data of N = 500 German preschool children and their parents were conducted via mathematical tests, a parental survey, and an observation. Using a structural equation model, this study investigated the relationship between parental STEM occupation, their attitudes toward mathematics, the quality of interaction between parents and children at a game of dice as an indicator of the HNE and the children's mathematical competencies. Unlike previous studies, the findings suggest that there are no significant differences between STEM and non-STEM parents regarding their HNE or their children's mathematical competencies. Findings revealed parental attitudes and the HNE as the most important factors being associated with children's mathematical competencies, independent of the children's gender, as well as a close relation between children's behaviour and the quality of the HNE. The findings are discussed and implications for further research are suggested.

Children's Mathematical Competencies – how and why are They Associated With Parental and Family Characteristics?

Math is everywhere: not just at school or work, but also when measuring ingredients for cooking, counting change, or calculating sales discounts. Therefore, being proficient in mathematics is not only important for people who work in science, technology, engineering, or mathematics (STEM) fields but also for everybody else participating in everyday life. Although primary school children are introduced to formal mathematics in first grade, evidence suggests that differences in children's mathematics competency already manifest themselves before the start of school (e.g., Bachman et al., 2015; Dearing et al., 2012; Niklas & Schneider, 2014) and are highly likely to grow during the course of children's school lives (Leyva et al., 2018).

These early differences stem from several influences. As children's families are their first learning environment and the one they are involved in most frequently, especially family factors are suggested to be the most influential ones on children's mathematics development: the family's socio-economic status (SES), parental attitudes towards learning (and, in this context, specifically towards mathematics), and the home numeracy environment (HNE) (e.g., Elliott & Bachman, 2018a; Harackiewicz et al., 2012; Niklas et al., 2016). As parents who work in STEM disciplines are more likely to enjoy mathematics and as both mathematics anxiety and achievement is assumed to be genetically heritable (Elliott & Bachman, 2018a), it can be suggested that early differences are also affected by parents' professions. Additionally, researchers report differences in mathematics competencies between boys and girls, suggesting unequal learning opportunities in families (Bowden et al., 2018; Lee et al., 2011; Mejía-Rodríguez et al., 2021).

These influential family factors, the SES, parental attitudes, and HNE, affect each other. Parents with lower SES tend to have more negative attitudes about mathematics such as little enjoyment or even mathematics anxiety (Elliott & Bachman, 2018a), feel less capable of supporting their children in their early mathematics development (Leyva et al., 2018), and are less likely to engage in informal mathematics activities in everyday life like measuring (Elliott & Bachman, 2018a). Furthermore, SES acts as the strongest predictor of children's academic achievement (Lee et al., 2011).

As parental attitudes influence both their children's attitudes (Harackiewicz et al., 2012) and the HNE they grow up in (Hornburg et al., 2021; Niklas et al., 2016), parental attitudes can also greatly affect children's academic performance (LeFevre, Polyzoi et al., 2010).

While there are mixed findings on the relationship between the HNE and children's mathematics achievement (Elliott & Bachman, 2018a), some researchers report that the learning environment children find themselves in can significantly predict children's cognitive development (Niklas et al., 2016). As the quality of the HNE parents provide differs depending on their SES (e.g., Jordan et al., 2007), low-SES children are more likely to find themselves in lower quality HNEs (Leyva et al., 2018). Additionally, parents who work in the STEM field are more likely to provide their children with supportive and engaging HNEs than parents who do not work in the STEM field (Mues et al., 2021).

So far, there is a dearth of research on the HNE and its influences on children's mathematics competencies (Dearing et al., 2012), and investigations of any possible impact of parents' occupations are even more scarce (Mues et al., 2021). The present study addresses this research gap by empirically investigating the interaction between SES, parental attitudes, and the HNE of STEM and non-STEM parents and their children's mathematics competencies one year before the start of school.

Theoretical Background

The following section will briefly present the considered constructs and will discuss their associations by means of empirical findings.

Socio-Economic Status

The socio-economic status (SES) is a latent construct that comprises economic and social components, generally measured by income, (maternal) education, and occupation (Baker, 2014; Tonizzi et al., 2021). Although there is no consistent operationalisation of SES across studies (see for example Lombardi & Dearing, 2020, who explicitly disaggregated SES, or Niklas & Schneider, 2014, who only used occupational prestige), most researchers define family income and maternal education as indicators for SES (Elliott & Bachman, 2018b; Rowe, 2017). Especially the latter is reported to have a strong impact on children's academic welfare (Bachman et al., 2015).

Differences in their family's SES can gravely affect children's lives: Children from low-SES families are more likely to show less self-regulation, have worse language skills and cognition, more unfavourable health, and a lower chance of high academic achievement (DeFlorio & Beliakoff, 2015; Tonizzi et al., 2021). Furthermore, when starting school, children from low-SES families are one developmental year behind their middle- and high-SES peers and this gap tends to widen throughout the school years (DeFlorio & Beliakoff, 2015; Leyva et al., 2018). Parental factors, i.e. family income and parental education, are not only positively correlated with children's academic achievement (Chi et al., 2017) but even appear to be its strongest predictor (Lee et al., 2011).

STEM-Employment as Facet of the SES

As the SES and thus a child's development depends on the parents' education and income, both of which predict occupational prestige (Powell & Jacobs, 1984), it can be assumed that parents who learnt or work in prestigious professions also attain a higher SES. Among the most prestigious, highest-earning occupations are those in the disciplines of science, technology, engineering and mathematics, commonly abbreviated as STEM (Bundesagentur für Arbeit, 2016, slide 15; Jahnke-Klein, 2014). In 2020 in Germany, more specifically in Bavaria, where the present study was conducted, STEM employees earned on average 17 per cent more money than the average employee (Bundesagentur für Arbeit, 2021a for STEM employees in Bavaria; Bundesagentur für Arbeit, 2021b for employees in general in Bavaria).

While working in STEM fields can improve an individual's SES, the SES can also influence the motivation and achievement in STEM fields: The to this date most recent PISA study reports an achievement gap between low- and high-SES students in all participating countries in reading and scientific performance (OECD, 2019), and students from lower SES families are more likely to feel less motivated to engage in mathematics and science (Murphy, 2020). Researchers observed that parental education, one of the indicators of SES, predicted course choice: Children of highly educated parents took more STEM courses in high school than children of lesser-educated parents (Harackiewicz et al., 2012).

So far, there are rather few studies operationalising parental occupation as an indicator for SES. This leaves out the properties and influences parental occupation brings with it: For instance, in their sample using Finnish register data, Erola et al. (2016) found income to have no independent effect on children's achievement. Rather, an observable effect of family income emerged only as a shared effect alongside parental education or occupation.

Omolade and Salomi (2011) report similar importance of parental occupation on students' achievements. They suggest that parental occupation was outweighed only by parental education when investigating their significance on students' mathematics outcomes.

Chi et al. (2017) also detected significant associations between students' scientific competencies and their parents' occupations. Specifically for girls, Guo and colleagues (2019) found in their analysis of PISA 2015 data that girls with at least one STEM parent indicated a significantly higher interest in science than girls with non-STEM parents.

Considering these scientific findings, parental occupation appears to be a factor that can significantly influence the interests and performance of children. By looking at parental (STEM) occupation as an indicator of the SES, the present study contributes not only to the underrepresentation of parental occupation as an influential factor in research but also to a clearer differentiation and definition of the SES construct.

SES and Parental Attitudes

While only a small number of studies used parental occupation as a stand-alone operationalisation for family SES, the global construct of SES had been applied in many studies to examine its relationship with other influential variables.

Previous studies suggest differences in SES levels on parental attitudes towards engaging in (mathematical) learning activities with their children (e.g., LeFevre, Fast et al., 2010; Niklas & Schneider, 2014). Researchers found low-SES parents to show a less positive view of mathematics when compared to higher-SES parents (Elliott & Bachman, 2018b; Tonizzi et al., 2021).

SES was positively linked with parental attitudes about mathematics and parents' expectations for their children's academic development, which ultimately predicted children's mathematics outcomes (Elliott & Bachman, 2018a; Melhuish et al., 2008). These attitudes and expectations were higher in middle- and high-SES parents (DeFlorio & Beliakoff, 2015; Elliott & Bachman, 2018a). Especially parents with lower SES felt less capable of supporting their children in their early mathematics development and rather believed this to be the teachers' responsibility (Leyva et al., 2018), which might be explained by the observation that lower SES parents reported a lack of knowledge about how to support their children's development in numeracy and literacy skills (Sonnenschein & Sun, 2017). At the same time, low-SES mothers rather believed in performance-oriented, adult-led activities to teach their children foundational mathematical concepts and were more likely to engage in formal mathematical activities while mothers with higher SES rather believed in child-centred informal activities (DeFlorio & Beliakoff, 2015; Elliott & Bachman, 2018a).

These findings suggest that low-SES parents see themselves less fit in providing their children with adequate material and social resources to support their early mathematical and language development while simultaneously believing these formal activities to be the most important means in order to foster children's early numeracy and literacy skills. This incongruent mixture of parents' attitudes, beliefs and expectations and their engagement in formal or informal learning activities leads to different mathematical and academic outcomes for children with different SES backgrounds. In fact, maternal education and family income act as the strongest predictors of both reading and mathematical skills (Sonnenschein & Sun, 2017).

Conclusion

The vast number of findings on the influence of SES on children's development point out that children from low-SES families are disadvantaged in several areas of life. Low-SES parents are more likely to display less confidence in their knowledge of children's development and academic matters and provide less favourable home learning environments for their children. Children from low-SES backgrounds are less likely to have at least one parent who holds a STEM occupation, which would positively influence their motivation to engage in mathematics and science, thus enhancing their chances of carrying out a STEM profession themselves.

Parental Attitudes

Parental attitudes are generally defined as parents' feelings, beliefs and behaviours toward something or someone, influenced by their own experiences and perceptions (Elliott & Bachman, 2018a; Hogg & Vaughan, 2005 in Chi et al., 2017). Parental attitudes and thus their role modelling are highly influential for their children's (academic) development (Niklas et al., 2020). Elliott and Bachman (2018a) suggest two perspectives of parental promotion of children's learning: what parents do and who parents are.

What parents do comprises for example their engagement in their children's development and education through the provision of adequate resources or a supportive communication style - in short, their behaviour. *Who parents are* relates more to their cognition - their beliefs about education and academic competency, which are influenced by their own school experiences. Who parents are affects what parents do. Parental attitudes rather relate to *who parents are*, and thus act as a base for parents' actions.

In their model on attitudes, Zanna and Rempel (1988) further argue that these action-shaping attitudes evaluate objects and situations against the background of cognitive, affective, and behavioural components. Applying this model to parental attitudes towards mathematics, it implies that parents assign certain values and emotions to mathematics and will be more or less likely to engage in mathematical actions with their children.

In line with these models, negative parental attitudes, as Dowker (2021) summarises in her review article, can affect children's mathematics-related experience in several ways: Parents with negative attitudes towards mathematics are less likely to engage with their children in mathematical content at all, more likely to convey their negative attitudes onto their children and more likely to be stressed or feel anxious when engaging in mathematical activities. Therefore, it is important for parents to display nurturing attitudes that can serve as a solid foundation for their actions and behaviours.

So far, the findings on the connection between parental attitudes and children's academic competencies remain mixed. Several researchers report no relationship between parental attitudes and their children's educational development (e.g., Elliott et al., 2021; Mejía-Rodríguez et al., 2021; Missall et al., 2015). Other studies were able to find significant positive connections between parental attitudes and parents' involvement in their children's development or children's academic performance (e.g., Giannelli & Rapallini, 2019; LeFevre, Polyzoi et al., 2010; Perera, 2014).

Valuing mathematics and showing positive attitudes toward STEM disciplines positively influences parents' engagement in mathematics-related activities, leads to higher quality mathematics talk between parents and children (Niklas et al., 2016) and positively impacts children's STEM and general academic performance (Elliott & Bachman, 2018a). This positive influence can even lead to a feedback loop: Children's level of attainment influences parents' stimulation of their children (Melhuish et al., 2008). Parents with positive attitudes toward mathematics are more likely to watch their children enjoy and frequently engage in mathematics which in turn will lead to more parental engagement in mathematical activities (Hornburg et al., 2021; Lane, 2017; Silinskas et al., 2020). Overall, parents inhibit highly influential roles in children's attitudes toward mathematics (Lane, 2017).

Comparing parents' attitudes towards mathematics and their engagement in early numeracy activities to language and literacy shows that parents tend to view the latter as being more important (Elliott & Bachman, 2018a; Missall et al., 2015; Niklas et al., 2016). This leads to less engagement in early numeracy activities (Elliott & Bachman, 2018a). Missall and colleagues (2015) suggest that this preference for literacy activities stems from parents' beliefs that mathematics is less important than language and that children are less interested in numeracy activities. This relates to the abovementioned reciprocal influence of parents and children: Just like their parents, children can influence their parents' attitudes and behaviours through their interests and skills (Silinskas et al., 2020). If parents believe their children to be less interested in mathematics, they are also less likely to frequently engage in mathematical activities with them.

Parental attitudes are not only important because they underlie the quality and quantity of actions that support children's educational development, but they are also suggested to mediate certain effects of the SES (Elliott & Bachman, 2018a; Li et al., 2020). As a higher SES is associated with more positive parental attitudes towards mathematics (Elliott & Bachman, 2018a, 2018b), the achievement gap between low- and high-SES children can be narrowed by improving parental attitudes towards mathematics and learning in general (Li et al., 2020; Perera, 2014). Thus, even though the SES accounts for grave differences in children's academic achievement, targeting more easily adjustable factors such as parental attitudes can help improve disadvantages.

Parental Attitudes and Children's Gender

The awareness of gender stereotypes begins in early childhood and starts to shape children's understanding of what girls and boys can or should do and what they cannot or should not do from a young age (Picho & Schmader, 2018). As children are greatly influenced by their parents and family surroundings, family factors such as parents' gender ideologies and gender-related beliefs and attitudes play a big part in children's self-efficacy, self-confidence and overall attitudes (Bowden et al., 2018; Parsons et al., 1982).

Ambady et al. (2001) reported that elementary school girls showed worse cognitive performance once they got confronted with negative gender stereotypes. Consequently, while parents who endorse gender stereotypes can greatly impede their daughters' self-efficacy and attitudes, parents who actively reject these stereotypes can support their daughters' self-concepts (Picho & Schmader, 2018).

Regarding children's academic development, specifically their STEM development, the attitudes parents hold towards their daughters are of great importance. Generally, parents' mathematics expectations towards their daughters appear to be lower than towards their sons, which can result in lower mathematics ambitions for daughters (Bowden et al., 2018).

Conclusion

To conclude these findings on parental attitudes, it can be stated that parents' actions and the beliefs they display about their children can have a great influence on children's general and academic development. With a special focus on STEM disciplines, positive parental attitudes are essential for fostering children's STEM motivation, self-concept and achievement and can even create a feedback loop. As parents seem to rather focus on language than on mathematics and science, stressing the importance of their attitudes on their children's development is essential. Concerning the differences parents (unintentionally) display in their supportive attitudes in regard to their children's gender, it is ascertainable that although parents cannot even out all structural incidents that affect their children, they can at least actively work against them to establish a more equal environment for their children, regardless of their gender.

Home Numeracy Environment

The environment that parents provide for their children to support their upbringing is called home learning environment (HLE). This home learning environment can be further specified into home literacy and home numeracy activities, which themselves can be classified into formal and informal activities (Lehrl et al., 2019). Formal activities include teaching letters or numbers for home literacy and home numeracy activities, respectively, while informal activities comprise less directed teaching methods that cover language and numbers, such as shared reading or playing dice games (Lehrl et al., 2019). To act as a fostering, supportive environment, the HLE should provide children with stimulating learning resources as well as meaningful parental engagement (Dearing et al., 2012). Especially the latter holds high importance: Being mentored by and learning from a more knowledgeable person, in this case, the child's parents, proved to be essential for children's development. Bronfenbrenner (1979) ascribes in his ecological theory an essential impact to proximal aspects in children's surroundings, such as the interactions between children and their parents, to foster children's development. Parents can help their children by scaffolding their learning processes and by role modelling (Burghardt et al., 2020).

As the HLE is made up of learning resources, parents' engagement and behaviour towards their children as well as children's own engagement in it, it is evident that families' HLEs differ both in quantity and quality (Lehrl et al., 2019). Many researchers report a correlation between a family's HLE and SES, suggesting that children from lower SES families are provided with less favourable HLEs (e.g., Dearing et al., 2012; Melhuish et al., 2008; Niklas et al., 2016; Niklas & Schneider, 2017, but see Missall et al., 2015 for no significant differences in engagement in HLE activities between low- and high-SES families).

It is advisable to divide the HLE into its two components, home literacy and home numeracy environment, to identify specific crucial advantageous or disadvantageous activities that help shape children's development and academic achievement. As this thesis focuses particularly on children's mathematical achievement, the main focus of attention will be laid on the numeracy activities. The part of the HLE including these number-related activities is called home numeracy environment, or HNE.

In empirical research, this subtype of learning environment is not as well researched as its counterpart, the home literacy environment (Dearing et al., 2012), but by no means less important: Between early mathematical, attention and socioemotional skills, mathematical competency acts as the strongest predictor of general academic achievement in children's early school life (DeFlorio & Beliakoff, 2015). The HNE is not just related to early numeracy skills, but also to mathematical tasks, intelligence, and letter knowledge (Niklas et al., 2016).

Formal home numeracy activities can include measuring or teaching sums and are related to early symbol knowledge, for example through digit naming; informal activities, such as playing dice games, are related to non-symbolic mathematics skills, for example adding objects (Silinskas et al., 2020). Further, mathematics talk, an informal numeracy activity that is assumed to be beneficial for children's mathematics competencies (e.g., counting, identifying and comparing numbers; Elliott & Bachman, 2018b; Lehrl et al., 2019), acts as a predictor for children's mathematical competencies (e.g., Elliott et al., 2017; Leyva, 2019; Susperreguy & Davis-Kean, 2016).

Moreover, informal numeracy activities such as playing board games or puzzle play can improve children's numerical skills and arithmetic performance and can predict their later transformation skills and overall mathematical achievement in elementary school (Dearing et al., 2012; Niklas et al., 2015); as these activities offer the chance of using mathematics talk, they can indirectly improve children's mathematical understanding by giving parents the opportunity to prompt their children to exclaim or compare numbers (see Eason et al., 2021, for the importance of prompts for the support of children's mathematics skills).

Niklas and Schneider (2014) sum up the advantages of informal mathematical games: "[P]arents who play these games with their children offer differential support as a function of age and knowledge base of the child. [They] are able to provide sensitive instructions to their children when playing games in a mathematical context, taking into account the competency level of the child" (p. 338).

Meanwhile, research on the HNE shows varying findings on its actual influence on children's mathematical achievement. According to Elliott and Bachman (2018b), some studies find a significant relationship between the HNE and children's mathematical skills while others do not. Researchers explained these discordant findings with various propositions: Some authors suggest that the frequency and quality of home numeracy activities and thus its effect on children's early numeracy skills also depends on child factors, such as children's age (Elliott & Bachman, 2018b), intelligence and working memory (Kleemans et al., 2012, but see Hornung et al., 2014 and Niklas & Schneider, 2014 for counter-arguments), or general performance (Silinskas et al., 2020). For instance, Silinskas and colleagues (2020) found that fewer home numeracy activities aligned with better performance in mathematical tests: Those parents whose children performed well in mathematics at the end of kindergarten engaged less in home numeracy activities with them at the beginning of school and the end of grade 1.

Another possible reason for the mixed findings between research papers on the influence of the HNE on children's early mathematical and academic skills is the insufficiently designed and not fully theoretically embedded concept of the HNE, which often differentiates between studies and thus leads to heterogeneous observations (Elliott & Bachman, 2018a). To solve this problem, "a clearer conceptualization of formal and informal practices [...] is warranted" (Elliott & Bachman, 2018a, p. 7; Hornburg et al., 2021).

In their systematic review, Mutaf-Yıldız and colleagues (2020) suggest that one reason for these divergent results is the difference in the used measurement instruments. Among all articles the authors reviewed, the majority used questionnaires while only a few used observations to assess home numeracy. Out of those to whom the latter applies, Leyva (2019) as well as Susperreguy and Davis-Kean (2016) found in their longitudinal observational studies that parents' mathematics talk with their pre-kindergarteners was predictive of children's mathematical skills one year after recordings (Susperreguy & Davis-Kean, 2016) or at the end of kindergarten (Leyva, 2019), even when taking parental education or child factors like age or working memory into account.

Observational studies on the relationship between the HNE and children's mathematical skills mostly focused on parent-child math talk and their findings generally align with those from studies using questionnaires, such as that advanced math talk at home is positively related to children's mathematical competencies (Mutaf-Yıldız et al., 2020). This indicates that both observations and questionnaires are equally fit for investigating the HNE. While questionnaires are generally easier to apply, observations are more timely, less reliable on memory, and harbour less risk of social desirability (Amodio et al., 2007; Mutaf-Yıldız et al., 2020). As the observational approach has not been as thoroughly utilised as questionnaires, the present study contributes to extending the findings of observational studies.

SES and Home Numeracy Environment

As mentioned above, many studies find SES differences in families' HNEs. Parents with lower SES and lower education are less likely to be able to provide their children with stimulating resources like books, instruments, or art supplies, and are less likely to engage with their children in meaningful, cognitively stimulating ways, such as shared reading or teaching numbers (Dearing et al., 2012). As changes in a family's SES occur, its HLE changes as well (Elliott & Bachman, 2018a).

Typically, higher parental education is associated with more frequent and qualitatively higher parental engagement in home learning activities and greater parental school involvement (Elliott & Bachman, 2018a; Giannelli & Rapallini, 2019; Leyva et al., 2018). High-SES parents are more likely than low-SES parents to engage in math talk with their children (Bachman et al., 2015; Lombardi & Dearing, 2020; Rowe, 2017), as well as in formal mathematical games, such as dice games, board games, shared number-game plays or playing with blocks and puzzles (Elliott & Bachman, 2018a; Susperreguy et al., 2020).

All in all, many researchers found disadvantaging HNEs as the reason for SES differences in children's mathematical competencies (e.g., Elliott & Bachman, 2018a; Niklas & Schneider, 2014; Tonizzi et al., 2021; but see also Silinskas et al., 2020, who found no significant relationship between HNE and children's mathematics skills). Jordan and colleagues (2007) state that these SES differences in children's early mathematical competencies in kindergarten do not diminish in the course of the school years but rather widen.

Variations in the HNE Between STEM and non-STEM Parents

The research on HNE variations between STEM and non-STEM parents is very scarce. As most empirical research on the influence of parental occupations was conducted on adolescents, their influence on young children and the surroundings they find themselves in is still a wide field to explore.

One recent study that focused on parental occupation and young children's HNE was conducted by Mues and colleagues (2021), who differentiated between parents' learned and current STEM or non-STEM occupations. They found that parents' current occupation significantly predicted the quality of their HNE, but not children's mathematics competencies, while the opposite applied for parents' learned occupations. The authors assume "that a current STEM background is related to more activity in and discussion of numerical and scientific topics in general" (Mues et al., 2021, p. 12).

This assumption relates to other studies on parental numerical activities. For instance, Elliott and Bachman (2018b) reported that parental beliefs play an important role in the design and quality of the HNE as well as the frequency of mathematics activities and that parents who do not or rarely engage in numerical activities can diminish the influence of the HNE on children's mathematical competencies. Furthermore, Sonnenschein and colleagues (2012) found that parental beliefs and parental role-modelling were important influences on children's engagement in mathematical activities as well as parents' own engagement in these activities with their children. As already stated, parents who work in STEM fields are more likely to enjoy mathematical activities, convey this enjoyment to their children and rather include mathematical activities in their HNE.

Variations in the HNE Between Children's Gender

As children's first and most frequent surrounding environment, parents' behaviours and actions can greatly influence those of their children (Bandura, 1997; Eccles, 1993).

According to Eccles (2011), home practices that differ in regard to children's gender impact and ultimately shape differences between boys' and girls' individual self-perceptions. When looking closer into the home practices, parents' behaviours are suggested to be more influential on their children than parents' gender ideologies (Halpern & Perry-Jenkins, 2016). Thus, the activities parents provide their children with and behaviours they dispose onto them are not only seen as highly influential towards children's development but are also likely to differ depending on the children's gender.

However, research on gender differences in families' HNEs, especially with samples of young children, is still insufficient and existing findings are discordant.

In their sample of 920 German kindergarteners, Niklas and Schneider (2017) found relations between the HLE, children's SES and migrant background, but not their gender. However, the authors included both the literacy and numeracy components of the HLE and did not conduct gender differences in the HNE separately.

When asking parents about their mathematical beliefs, Mues et al. (2022) found both mothers and fathers to regard girls as less competent in mathematics than boys. These beliefs are highly likely to be reflected in not only the type and frequency of numeracy activities provided at home but also the verbal and nonverbal gendered expressions parents convey.

Dearing and colleagues (2012) reported close relations between home numeracy activities and girls' arithmetic skills, even diminishing the effects of girls' low-SES background. However, this study only included girls in the sample, making it impossible to draw conclusions about gender differences.

Gustafsson et al. (2011) were able to find gender differences in children's HNEs when they observed that parents engaged their boys more in numeracy activities while the girls were engaged more in literacy activities. However, in their analysis of the TIMSS data of 2015, Mejía-Rodríguez and colleagues (2021) found no overall correlation between gender and early numeracy activities in all 32 analysed countries and the same frequency of engagement with girls and boys on average. Contrary to gender stereotype beliefs (cp. Mues et al., 2022), del Río and colleagues (2017) found mothers to engage in numeracy activities more often with their daughters than with their sons.

Regarding the divergence of these findings, there is a necessity for more research on the relation between the HNE and children's gender to draw secure conclusions about possible directions.

Conclusion

Summed up, the HNE plays an important role in the educational development of a child. The HNE is both associated with the SES and with children's academic achievement and thus acts as a mediator. There is the possibility that parents who work in STEM occupations engage their children in more favourable HNEs, but this suggestion still needs more research to build upon. Likewise, it cannot yet be assumed whether parents provide their children with qualitatively and quantitatively less favourable HNEs depending on the children's gender or not.

All in all, the HNE proves to be an important influence on children's academic development and can even be assumed to be the most essential influence, "not only because it is the child's primary learning environment but also because it can be more easily manipulated than other family characteristics, such as the socioeconomic status" (Niklas et al., 2015, p. 3).

Children's Mathematical Competencies

Early mathematical competencies that children acquire before they start school form the base that is crucial for later mathematical and academic achievement (Hornburg et al., 2021; Hornung et al., 2014). Early mathematical skills such as enumeration or advanced counting act as a strong predictor of mathematics achievement in first grade and children's further academic lives (Hornung et al., 2014; Jordan et al., 2007; Kleemans et al., 2012; Lehrl et al., 2019; Susperreguy et al., 2020). Children who start school with lower levels of early mathematical competencies than their peers start with a disadvantage they will very unlikely be able to recover from (Niklas et al., 2015).

The previous sections illustrated three factors that strongly influence children's mathematical development and competencies. In this section, these three factors - a family's SES, parental attitudes and the HNE a child finds themself in - are highlighted again to clarify their influence and interplay on children's mathematical competencies. Additionally, the association between children's gender and their mathematical competencies found in previous research will be described.

Children's Mathematical Competencies, Socio-Economic Status, Parental Attitudes, the Home Numeracy Environment, and Children's Gender

The SES is suggested to be the strongest predictor of children's academic achievement (Lee et al., 2011; Sonnenschein & Sun, 2017) and can thus greatly influence children's later mobility, career chances and income (Elliott & Bachman, 2018a; Hornburg et al., 2021). As an indicator of the SES, parental STEM occupation positively correlates with children's STEM-related interests and achievement (Chi et al., 2017; Guo et al., 2019; Plasman et al., 2021). Therefore, it can be assumed that children whose parents work in STEM fields show higher mathematical competencies than children whose parents do not work in STEM fields.

At the same time, SES effects on children's mathematical competencies may also be mediated by parental attitudes (Elliott & Bachman, 2018a), as parents' attitudes are intertwined not only with their education, income, and professional occupation but also with their engagement in learning activities with their children (e.g., Dearing et al., 2012; DeFlorio & Beliakoff, 2015; Kleemans et al., 2012). Parental attitudes toward mathematics can be transmitted to their children (Hildebrand et al., 2022; Soni & Kumari, 2017), and are strongly related to children's mathematical outcomes (Makur et al., 2019).

Looking at the association between the HNE and children's mathematical competencies, mixed findings emerge (e.g., Missall et al., 2015 who found no significant connection, or Napoli & Purpura, 2018, who found significant connections). This is due to the fact that a child's development is impacted by a conglomeration of factors, many of which are hardly or not at all manipulable by parents, such as political or cultural influences (Hornburg et al., 2021).

Researchers who were able to report significant relations emphasised the role of informal numeracy activities, such as playing games, cooking, or measuring, which they argued influence children's mathematical thinking and thus mathematical competencies later on in school and could be especially important for low-SES children (Chi et al., 2017; Dearing et al., 2012; Niklas et al., 2015, 2016). Thompson and colleagues (2017) found an initial relation between basic home numeracy practices and children's numeracy skills to diminish once parental education was considered, highlighting the importance of the SES and thus parental occupation on children's mathematical development (see also DeFlorio & Beliakoff, 2015).

Elliott and Bachman (2018a) put the interplaying relation of parental attitudes, the HNE and SES in a succinct statement: "[P]arents mediate SES disparities in [children's mathematics] achievement" (p. 3). This puts parents in a role whose task it is to actively act against the disadvantages SES differences pose on children, as the HLE they engage their children in "seems to act as a mediator between [..] family characteristics [such as the SES] and child outcomes" (Niklas et al., 2016, p. 6). By adequately supporting their children's development through informal and formal numeracy activities and displaying positive, supportive attitudes toward them, parents are in the power to diminish the dire differences their SES poses on their children, at least to some extent.

Whether or not children's mathematical achievement differs due to their gender is not clear. While Levine and colleagues (2012) found differences between boys and girls in spatial tasks already in preschool, suggesting gender differences even before the start of school, Bakker et al. (2019) achieved opposite results that were able to provide substantial support for the gender equality hypothesis, indicating that their sample of preschoolers did not differ in their numerical competencies because of their gender. However, as children's gender may have an impact on the HNE they are provided with (Gustafsson et al., 2011), it should be taken into consideration as an associative factor on both parental and child characteristics.

Conclusion

Considering previous findings, children's mathematical competencies are influenced by a variety of family- and child-related factors which seem to influence one another, as a clear distinction does not seem feasible (Hornburg et al., 2021; Wachs, 2000). Furthermore, research on certain topics, such as gender's association with mathematical competencies, is still far from being extensive, especially in the area of early childhood.

However, a recognisable body of literature argues that the three introduced influencing factors (i.e. a child's SES, parental attitudes and the HNE a child finds themself in) are intertwined and influence children's mathematical development and competencies in crucial ways. Children from low-SES families are more likely to live in less favourable HNEs and show lower mathematical competencies than children from high-SES families (Niklas et al., 2016; Niklas & Schneider, 2014). Low-SES parents rather display lower qualitative parental attitudes, such as lower expectations about children's learning or more negative attitudes towards mathematics, which ultimately negatively influence their children's mathematical competencies (Elliott & Bachman, 2018b). Children's gender can play an additional role in their parents' attitudes and practices, subliminally influencing them (Eccles, 2011).

By investigating the relations between parental (STEM) occupations, parental attitudes toward mathematics, the HNE, children's mathematical competencies, and their gender, the present study contributes to a growing field of early childhood research and further intents to expand the understanding of the multifaceted constructs SES and HNE.

Research Question and Hypotheses

The dearth of research on the influence of parental occupation and the HNE on preschoolers' mathematical competencies stresses the need for studies such as the present thesis. As parents generally act as the first and closest contact members to young children and provide them with their first and most frequently surrounding numeracy environment that is shaped by parents' own attitudes and beliefs towards mathematics, one can expect that all these factors are affected by the experiences and expertise parents acquire in their (STEM) profession. Thus, this thesis will focus on the relationship between parental STEM occupation, parental attitudes towards mathematics, parent-child interactions that shape the HNE, and children's mathematical competencies.

Accordingly, the following hypotheses were tested:

1. Significant positive correlations between parental STEM vs. non-STEM occupations, parental attitudes, the HNE - more specifically the quality of the parent-child interaction during a game of dice -, and children's mathematical competencies are expected.
    a. Parental STEM vs. non-STEM occupations are significantly associated with parental attitudes towards mathematics, the HNE, and children's mathematical competencies. Here, STEM parents are expected to display more positive attitudes towards mathematics and a higher interaction quality during a game of dice than non-STEM parents.
    b. A positive association between parental attitudes, the HNE, and children's mathematical competencies is expected.

As findings on gender differences in mathematical competencies in young children are mixed, but research on adolescents confirms a persistent gender gap in mathematical competencies, favouring boys, this thesis additionally focuses on the mathematical competencies of boys and girls, in relation to their parents' STEM vs. non-STEM occupations.

Accordingly, the following additional hypothesis was tested:

2. A significant association between STEM vs. non-STEM children, their gender and their mathematical competencies can be found.
    a. Children of STEM parents are expected to display qualitatively greater mathematical competencies than children of non-STEM parents.
    b. A significant association between parental (STEM) occupations and children's mathematical competencies in regard of their gender can be found: Here, boys should show better mathematical competencies than girls, and children of STEM parents should show better mathematical competencies than children of non-STEM parents. These expectations lead to the following hypothesis: Boys of STEM parents will show the greatest mathematical competencies, followed by girls of STEM parents, followed by boys of non-STEM parents, followed by girls of non-STEM parents, who are expected to show the lowest mathematical competencies.

As parental attitudes and their design of and engagement in the HNE is influenced by their children's interests and motivation, it is important to also consider children's behaviour. Therefore, this study additionally included the quantity and quality of children's behaviour during a game of dice. This behaviour on the other hand is assumed to depend on parents' occupation, attitudes, and HNE. A further possible influencing variable may be constituted by children's gender, which is assumed to impact parents' attitudes and the HNE. As there is no existing literature on these factors and their association with children's behaviour during informal numeracy activities such as a game of dice, the following hypothesis will be of an explorative nature:

3. A significant association between parental (STEM) occupation and children's gender with parental attitudes, the HNE, children's behaviour during a game of dice, and children's mathematical competencies is expected.

Material and Methods

The following section highlights the materials that were used for the execution of this study as well as the methods applied. These include a description of the sample, the procedure, and the measured variables including their reliability.

Sample and Procedure

The data analysed in this study was taken from the first measurement point (t1) of both cohorts of the EU-funded, 5-years longitudinal study *Learning4Kids*, conducted in Bavaria, Germany. The sample comprised N = 500 children (243 girls and 257 boys) with an average age of M = 60.96 months (SD = 4.61). The assessments included a wide range of instruments measuring children's academic skills, including standardised mathematics tests to assess their numerical knowledge and skills. All assessments were conducted by trained research assistants. In addition, parents were requested to fill in surveys about their children's characteristics as well as the parents' occupations, the family background and the HNE. Out of all participants, 301 indicated speaking German as their main language at home. The parental surveys were translated into several different languages to ensure that parents who did not speak German as their main language would be able to fully understand all items.

The participating families were recruited through kindergartens, public places and through a company specialised in study recruitment. The assessments took place at the families' homes when the children were in their second to last year of kindergarten. Parental consent was given informally at their recruitment and formally at their first visit. All research activities had been approved by the European Research Council Executive Agency and the ethics committee of the Faculty of Psychology and Educational Sciences at LMU Munich.

Measures

The following section will highlight the measures that were used for the analysis with their reliability measures, as well as a description of the statistical procedure.

Parental (STEM) Occupations

In the parental survey, parents were asked to indicate both their learned and current occupations. The categorisation of their answers into STEM or non-STEM professions was based on the *Aggregate Determination of STEM Professions* ("Aggregatbestimmung MINT-Berufe"; Bundesagentur für Arbeit, 2017) and *Aggregate Determination of Engineering Professions* ("Aggregatbestimmung Ingenieurberufe"; Bundesagentur für Arbeit, 2013). Despite the suggestion by Mues and colleagues (2021) that parents' learned and current occupations pose individual influences on their own, this thesis will, due to its framework, only focus on parents' current occupations.

The categorisation of the indicated occupations was coded independently by two coders. The intercoder reliability for the mothers' current occupation was Cohen's K = 0.95. The intercoder reliability for the fathers' current occupation was Cohen's K = 0.99. If at least one parent indicated a STEM background, the occupational background was coded with 1 = STEM. If none of the parents indicated a STEM background, the occupational background was coded with 0 = non-STEM. Families were only excluded from the analysis if neither the mother nor the father[1] indicated an occupational status.

[1] The same procedure was carried out for same-sex parental couples.

Parental Attitudes

To investigate their attitudes towards mathematics, the parental survey additionally included three items (McDonald's Ω = .79) that asked parents to indicate the importance and enjoyment of mathematics in their home (e.g., "mathematics is considered important at our home"; see also Appendix B). The items were adapted from a survey used by Niklas et al. (2016). Parents were asked to rate the items on a 5-point Likert scale (*very strongly* to *not at all*). Values ranging from 4 to 0 were assigned correspondingly, with higher values suggesting parental attitudes of higher quality. The mean was applied for the analyses.

The Home Numeracy Environment

To measure the HNE, the interaction between parent and child during a game of dice was conducted. Research assistants observed these interactions using an adapted version of the "Family Rating Scale" (Kuger et al., 2005; cf. Lehrl, 2018). The scale comprised seven items rating the parental instruction and behaviour regarding counting, their nonverbal behaviour, mentioning of numbers, free language level, explanations of mathematical concepts, comparing or ordering numbers, and references to everyday life (McDonald's Ω = .87). The observations were evaluated with values ranging from 1 to 7, with higher values suggesting qualitatively greater interaction on part of the parent. The mean was used for the analyses.

As several different research assistants observed and rated the parent-child interactions, potential differences in the ratings between research assistants were computed. Results of an analysis of variance between research assistants and their estimated HNE for each participant showed significant differences ($F(1) = 16.26$, $\eta = .03$, $p < .001$). The significance of this disparity is discussed in the limitations.

Children's Behaviour

The observations of the parent-child interactions during a game of dice for cohort 2 participants (*N* = 302; 144 girls and 158 boys; M_{age} = 59.31, *SD* = 3.93) included two additional items that were used for the third hypothesis which focused on the children's behaviour. These items rated the child's qualitative and quantitative response behaviour during the game of dice (McDonald's Ω = .92) and were based on a study of Convery (2021). The values ranged from 1 to 7, with higher values suggesting qualitatively greater response behaviour of the child. The mean was used for the analyses.

As with HNE, any possible differences in the rating between the research assistants were computed. The results of an analysis of variance between all research assistants and their ratings for children's behaviour showed significant differences ($F(1) = 18.82$, $\eta = .06$, $p < .001$). The relevance of these significant differences is discussed in the limitations of this study.

Children's Mathematical Competencies

Children's mathematical competencies were assessed with six different test sets: the "Mathematik- und Rechenkonzepte im Vorschulalter-Screening" (MARKO-S) (Ehlert et al., 2020) with 21 items comprising numbers, ordinal number bars, cardinality, number division, and inclusion and relations; the "Würzburger Vorschultest" (WVT) (Endlich et al., 2017) that was divided into three subtests, measuring number sequences forwards (Zfv, 8 items), number sequences backwards (Zfr, 6 items), number predecessors and successors (VuN, 8 items), and number symbol knowledge (Zk, 10 items); as well as the "Test mathematischer Basiskompetenzen im Kindergartenalter" (MBK-0) (Krajewski, 2018), assessing mathematical multiplication and subtraction (R, 8 items) (McDonald's Ω between .61 and .88). Each subtest was summarised into its own scale. All numerical competencies were then measured via a latent variable (MSkill) which included all summarised subtest scales. The z-standardisation of this variable was used for the analyses.

Statistical Model Analysis

All analyses were conducted using R 4.1.2 (R Core Team, 2021). As a first step, correlational analyses were conducted to check the association between STEM occupations, parental attitudes towards mathematics, the HNE, and children's mathematical competencies.

In order to further test the associations between the variables as well as children's behaviour during a game of dice and their gender, two structural equation models were developed. The first model did not include children's behaviour, as this variable was only conducted in the second cohort. Therefore, the second model, which included children's behaviour, used a reduced data set consisting only of cohort two participants (N = 302). Parental occupations and children's gender were utilised as manifest predictor variables while the HNE, parental attitudes, children's behaviour, and children's mathematical competencies were introduced as latent variables, comprising several subitems (see materials). Finally, two additional versions of the structural equation models were developed including children's age and intelligence as covariates.

The model fit was evaluated using the following goodness-of-fit criteria: The comparative fit index (CFI), the root mean square error of approximation (RMSEA) and its confidence interval, the non-normed fit index/Tucker-Lewis Index (TLI), and the relative/normed chi-square (χ^2/df), replacing the conventional chi-square due to its sensitivity in large samples (Hooper et al., 2008).

According to Hooper and colleagues (2008), the following values are suggested to indicate a good model fit between the data and developed model and were therefore considered to determine the model fit: Relative/Normed Chi-square (χ^2/df) between .03 and .05, CFI $\geq$ .95, TLI $\geq$ .95, RSMEA $\leq$ .07 and its 90 % confidence interval with a lower limit close to 0 and the upper limit ≤ .08. As Hair et al. (2006) stated that the standardised root mean squared residuals (SRMR) is rather biased upwards when using a sample of N > 250 participants and a variable count of m < 12, this fit index was not considered to assess the model fit.

To develop the models using the package *lavaan* in R, all covariances among the latent variables were set to zero (orthogonal = TRUE), the covariates were considered random and the means, variances and covariances were considered as free parameters (fixed.x = FALSE), and missing values were estimated using the maximum likelihood approach (missing = ‘ml.x’).

Modification indices for the model fit were considered as far as they seemed reasonable and in line with the underlying theory. Other modification indices were not applied but instead taken as an impulse to compute confirmatory and further exploratory factor analyses for the latent variables.

Missing Data

Most variables of interest were subject to missing values. The greatest number of missing values was seen in parental (STEM) occupation with $n = 57$ missing values. Reasons could be that the indicated occupations could not have been characterised, or that parents did not indicate any occupation at all, either because they did not want to, they did not have a profession, or because they missed the question in the survey.

For children’s mathematical competencies, $n = 15$ observations were missing. It is possible that the corresponding children could not be motivated or refused to do the tests or did not understand the tasks.

$N = 9$ observations were missing in the variable HNE. As this variable was operationalised as an observation, it might be that the parent or child did not want to participate in the game of dice or did not understand what to do. Further explanations could be that no parents or caregivers were present at the time of the assessments and no other appointment could be arranged between the family and the research assistant.

Among the data for parental attitudes, $n = 3$ values were missing, potentially due to parents missing the question in the survey, not wanting to give any indication or not understanding the questions.

Finally, while there is a total of $n = 198$ values missing in the variable child behaviour, it is necessary to repeat that this variable was only measured in cohort 2 with a sample size of $N = 302$. Thus, no data points for the observation of children's behaviour during a game of dice were missing.

While missing values were replaced by means of the maximum likelihood approach for the structural equation modelling, the procedure for the other analyses was different. The computations that were used all originated from the *stats* package in R (*cor.test* for correlations, *t.test* for t-tests, and *aov* for ANOVA and ANCOVA) and omitted any missing values by default.

Findings

The following section highlights the results the computed analyses yielded. After a descriptive overview of the variables, the findings for each tested hypothesis will be shown.

Descriptive Statistics

An overview of all investigated variables and their descriptives is given in Appendix A in Table A1.

Nearly half of all participating families (44.69 %) indicated that at least one parent had a STEM background. Compared to the average of STEM-employees in Germany (29 %, as of 2018; Statistik der Bundesagentur für Arbeit, 2019), this distribution is above average.

Between all variables, seven significant correlations were detected, as shown in Table 1. As gender and child behaviour were the only variables that were not investigated more closely in order to test the hypotheses, the latter stood out with three significant correlations with parental attitudes, HNE, and mathematical competencies. These correlations will be discussed further down.

Table 1

Bivariate correlations between all variables

	Variables	1	2	3	4	5	6
1	(STEM) Occupation	1					
2	Parental Attitudes	.13**	1				
3	HNE	.00	.17***	1			
4	MSkill	.03	.24***	.19***	1		
5	Child Behaviour[a]	.06	.22***	.57***	.28***	1	
6	Gender	.08	-.07	.06	-.07	.04	1

Note. Variables measured are (STEM) occupation, parental attitudes, HNE, and children's mathematical competencies. HNE = Home numeracy environment; MSkill = Mathematical Skill; (STEM) Occupation (0 = non-STEM, 1 = STEM).

**p < .01. ***p < .001.

[a] Child Behaviour was only conducted for cohort 2.

Associations Between (STEM) Occupations, Parental Attitudes, HNE, and Children's Mathematical Competencies

To test the first hypothesis, whether there were positive associations between STEM occupations, parental attitudes towards mathematics, the HNE, and children's mathematical competencies, bivariate correlations between all variables were conducted.

Results showed small significant correlations for the relation between parental attitudes and parental (STEM) occupation (r = .13, p < .01), the HNE (r = .17, p < .001), and children's mathematical competencies (*r* = .24, *p* < .001), as well as between the HNE and children's mathematical competencies (*r* = .19, *p* < .001). However, no statistically significant correlations between parental (STEM) occupation and the HNE (*r* = .00, *p* > .05) or children's mathematical competencies (*r* = .03, *p* > .05) were detected.

To further test whether STEM parents displayed significantly greater attitudes toward mathematics and a greater quality of behaviour during a game of dice, thus a better HNE, t-tests were computed.

Results showed that STEM and non-STEM parents significantly differed in their attitudes toward mathematics (*t*(441) = -1.61, *d* = .25, *p* = .01, 95 % CI [-0.34, -0.05]). STEM parents indicated higher attitudes (*M* = 3.03, *SD* = .77) than non-STEM parents (*M* = 2.84, *SD* = .78). However, no statistically significant differences between STEM and non-STEM parents in their HNE could be detected (*t*(435) = -0.05, *d* = .00, *p* = .96, 95 % CI [-0.21, 0.20]).

As there were only significant associations between parents' (STEM) occupation and their attitudes but not between parental (STEM) occupation and the HNE or children's mathematical competencies, as well as only significant differences between STEM and non-STEM parents in their attitudes but not their HNE, the first hypothesis must be partially rejected.

Associations Between (STEM) Occupation, Children's Gender and Their Mathematical Competencies

To test for differences between children's mathematical competencies in dependence on their parents' (STEM) occupation, a t-test was computed. Results showed a small non-significant difference between the means of non-STEM and STEM children (*t*(427) = -.83, *d* = .08, *p* > .05, 95 % CI [-0.27, 0.11]). This implies that children of STEM parents did not display significantly qualitatively greater mathematical competencies than children of non-STEM parents. Therefore, the first assumption of the second hypothesis must be rejected.

To further test whether an association between parental STEM occupations and children's mathematical competencies could be found in regard to their gender, an analysis of variance was conducted. Results indicated that children's gender was a significant predictor of their mathematics skills (η_p^2 = .01, *p* = .03), whereas neither parental STEM occupations (η_p^2 = .00, *p* = .32) nor the interaction between children's gender and their parents' STEM occupation (η_p^2 = .00, *p* = .69) acted as a predictor for children's mathematical competencies. When looking at the means in mathematical competencies between girls and boys descriptively, girls scored slightly higher (*M* = .08, *SD* = .87 versus boys *M* = -.06, *SD* = .75; Cohen's *d* = -.17). All in all, however, this small difference between the means was not sufficient to provide any evidence for the second hypothesis.

To control for possible relations between the variables and children's age and intelligence, an analysis of covariance (ANCOVA) was conducted. Assumptions for this statistical test were checked descriptively and no violations were detected. See Table 2 for an overview of the results.

Table 2

Analysis of Covariance

Source	Sum of squares	Mean Square	F(1, 421)	η_p^2
(STEM) Occupation	0.70	0.69	0.76	.00
Gender	4.90	4.94	5.44*	.02
Intelligence	14.10	14.07	15.51***	.02
Age	18.70	18.69	20.61***	.05
(STEM) Occupation:Gender	0.30	0.24	0.33	.00
(STEM) Occupation:Intelligence	0.20	0.24	0.26	.00
(STEM) Occupation:Age	0.10	0.08	0.09	.00

Note. *p < .05. ***p < .001. Variables measured are (STEM) occupation, gender, and mathematical competences with the covariates intelligence and age. Effect size stated as partial eta squared.

After controlling for intelligence and age, no interaction effect between STEM occupation and children's gender were observed ($\eta_p^2 = .00$, $p = .56$). However, 1.4 % of variance in children's mathematical skills could still be explained by children's gender ($\eta_p^2 = .01$, $p < .05$). Post-hoc contrasts revealed a small significant difference in favour of girls ($\beta = -.24$, $SE = .09$, $t(421) = 2.51$, $p = .01$).

Results of the ANCOVA further showed significant relations between children's mathematical competencies and their age ($\eta_p^2 = .05$, $p <$ 001), as well as their intelligence ($\eta_p^2 = .02$, $p < .001$).

Taking into account the ANOVA's result of gender as a significant predictor of children's mathematical competencies and the suggestion that this effect still remained after controlling for children's age and intelligence, it can be assumed that there were small significant differences between boys and girls in their mathematical skills, but these differences did not seem to depend on the (STEM) occupation of their parents. Therefore, as all conducted analyses did not support the hypothesis that there were significant associations between (STEM) children, their gender, and their mathematical competencies, the second hypothesis must be rejected.

Association Between Parental (STEM) Occupation and Children's Gender With Parental Attitudes, the HNE, Children's Behaviour, and Children's Mathematical Competencies

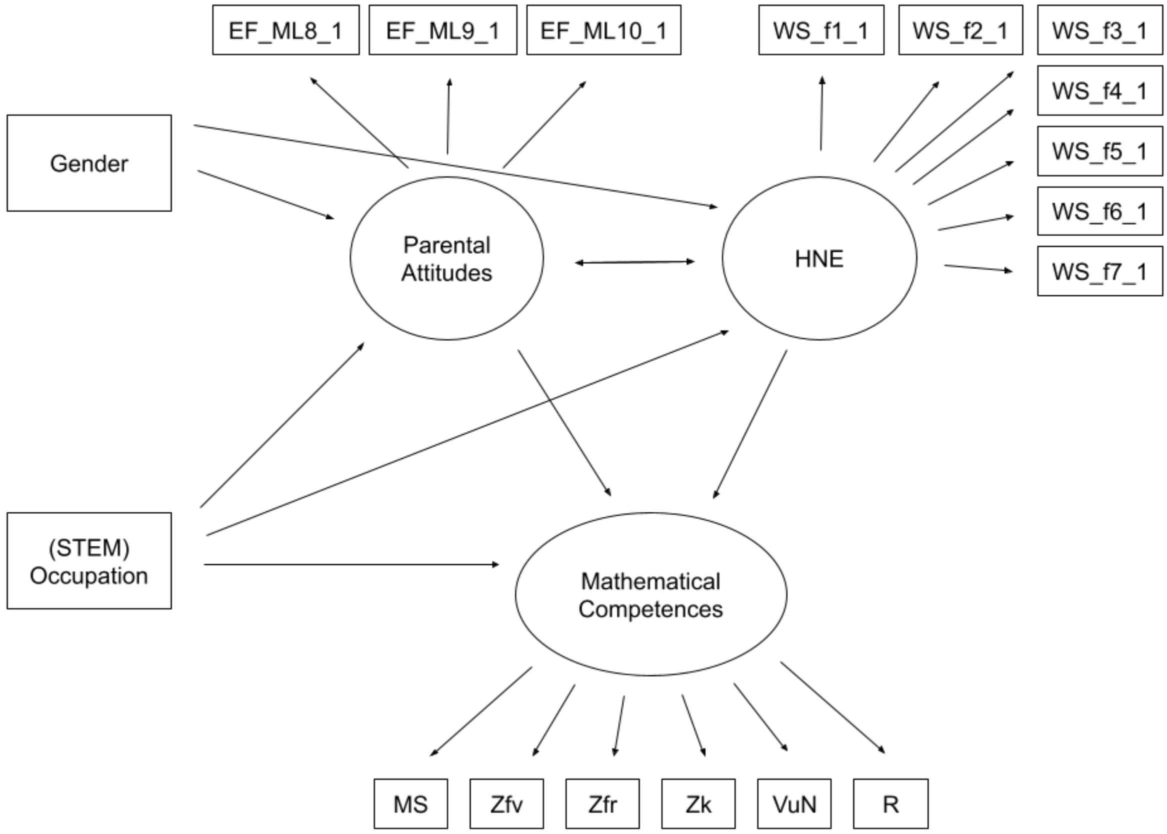
The third hypothesis was checked by developing structural equation models (SEM). The procedure, model fits and results are described in the following.

Model Fit

To test the third hypothesis, whether there is an association between parental (STEM) occupation and children's gender with parental attitudes, the HNE, children's behaviour during a game of dice, and children's mathematical competencies, two statistical models were developed. The first model, shown in Figure 1, comprised all data of N = 500 participants not including the variable child behaviour, while the second model, shown in Figure 2, included child behaviour and therefore comprised the data of cohort two only, resulting in N = 302 participants. The models included the two manifest variables (STEM) occupation (in the following: STEM) and child gender (gender), as well as three (model 2: four) latent variables. The latent variable parental attitudes was measured with three manifest items; the home numeracy environment (HNE) through seven items; mathematical competencies (mathematics) with six subtests comprising 61 items in total; and child behaviour, which was added in model 2, was measured with two items. Table A2 shows the initial model fit of both models.

Figure 1

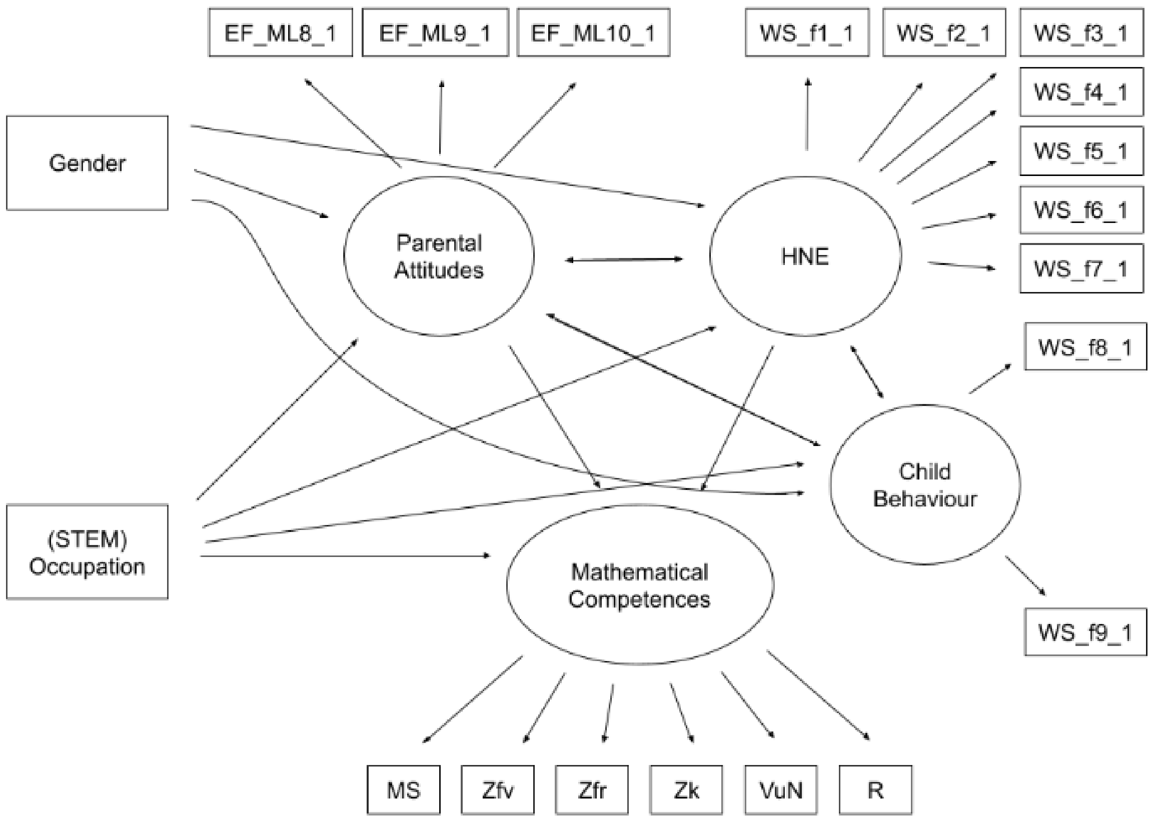
Structural equation modelling of pathways between variables of interest – Model 1



Note. HNE = Home Numeracy Environment. Rectangles indicate manifest variables; circles indicate latent variables.

Figure 2

Structural equation modelling of pathways between variables of interest – Model 2



Note. HNE = Home Numeracy Environment. Rectangles indicate manifest variables; circles indicate latent variables.

Modification indices from the *lavaan* package in R suggested adding correlations between the items of each variable, more specifically: For mathematics, the modification indices suggested correlating MARKO-S with R and Zfr; for the HNE it was recommended to correlate the first and sixth with the seventh, and the second, fourth and fifth with the sixth item; for parental attitudes, R's modification indices suggested to correlate the first with the third item; and between items, it was suggested to regress the second item of parental attitudes on mathematics as well as Zfv, one subtest of mathematics, on the HNE. As most of these suggestions referred to intra-variable correlations and all three variables covered various facets of the same construct, the suggestions regarding intra-variable correlations were accepted and the correlations were added to the first and second model. Modification indices suggesting additional correlations between mathematics and other variables were rejected.

The second model included another latent variable, children's behaviour during a game of dice (child behaviour), measured via two manifest observational items. As this variable was only measured in cohort two, the model also included only the data from cohort two, resulting in $N = 302$ participants. Modification indices suggested primarily intra-variable correlations, such as correlations between Marko-S and R, Marko-S and Zfr, as well as various combinations of correlations between the sub-items of the construct HNE. Further, assigning one of the sub-items comprised in parental attitudes (EF_ML9_1, "My child is very interested in learning arithmetic and counting and is really looking forward to it") to the latent construct Math Skill was suggested. See Table A3 for an overview of the implemented recommendations for models 1 and 2.

Although modification indices regarding the assignment of an item related to parental attitudes to the construct that measured children's mathematical competencies are in line with findings from Giannelli and Rapallini (2019), including these suggestions showed too large differences compared to the hypothesis and were therefore not included. The only suggestions accepted and included in model 2 were those between MARKO-S, R and Zfr as well as the intra-item correlations between items comprised in the construct measuring HNE and the construct measuring parental attitudes. These inclusions were due to the fact that all items substituting these two variables measured the constructs in a quite faceted manner.

In the final statistical models, regressions from STEM on parental attitudes, STEM on the HNE, STEM on mathematics, parental attitudes on mathematics, the HNE on mathematics, gender on parental attitudes, and gender on the HNE, as well as a correlation between parental attitudes and the HNE were formed.

In model 2, additional correlations between parental attitudes and child behaviour as well as the HNE and child behaviour were included. Positive associations between all variables were expected. Table A4 shows the model fit for both models as well as for models 3 and 4 which included the control variables age and intelligence.

It can be seen that model 2 did in fact not meet the requirements the fit criteria proposed, not even after considering the modification indices. Similarly, after adding the control variables intelligence and age into the models, the goodness-of-fit criteria were not met by either of the models, resulting in the rejection of both of them.

Consequently, model 2 and thus also the variable child behaviour was rejected respectively excluded, and only model 1 was used to test the hypothesis.

Exploratory Factor Analysis

Considering the suggestions the modification indices put out for both models, it seemed as though there were inconsistencies within the variables themselves, regarding the recommendations to add intra-variable correlations into the model. This suggested that the items used to measure the latent variables did not measure the same singular construct. This seemed especially the case with the latent variable mathematics which was composed of six sub-variables.

To address this curiosity, a confirmatory factor analysis of mathematics and its variables MS, Zfv, Zfr, Zk, VuN, and R was conducted. Its fit indices affirmed a bad model fit ($\chi^2 = 88.96$, df = 9, CFI = .95, TLI = .92, RMSEA = .13, 90 % CI [.11, .16]). Consequently, an exploratory factor analysis for mathematics was conducted to determine its factor count.

Results suggested a two factors solution: one that rather focuses on number knowledge and counting (with loadings of Zfv, Zfr, Zk, VuN, and R) and one that focuses more on the general understanding of mathematics (with loadings of MARKO-S and R). However, this distinction was not added in the present study. The suggestions that the subitems comprising the latent variables describe more than one factor posed a limitation to this study and will be discussed further down. An overview of the results of the exploratory factor analysis is given in Table A5.

Statistical Model of Parental (STEM) Occupation, Children's Gender, Parental Attitudes, HNE, and Children's Mathematical Competencies

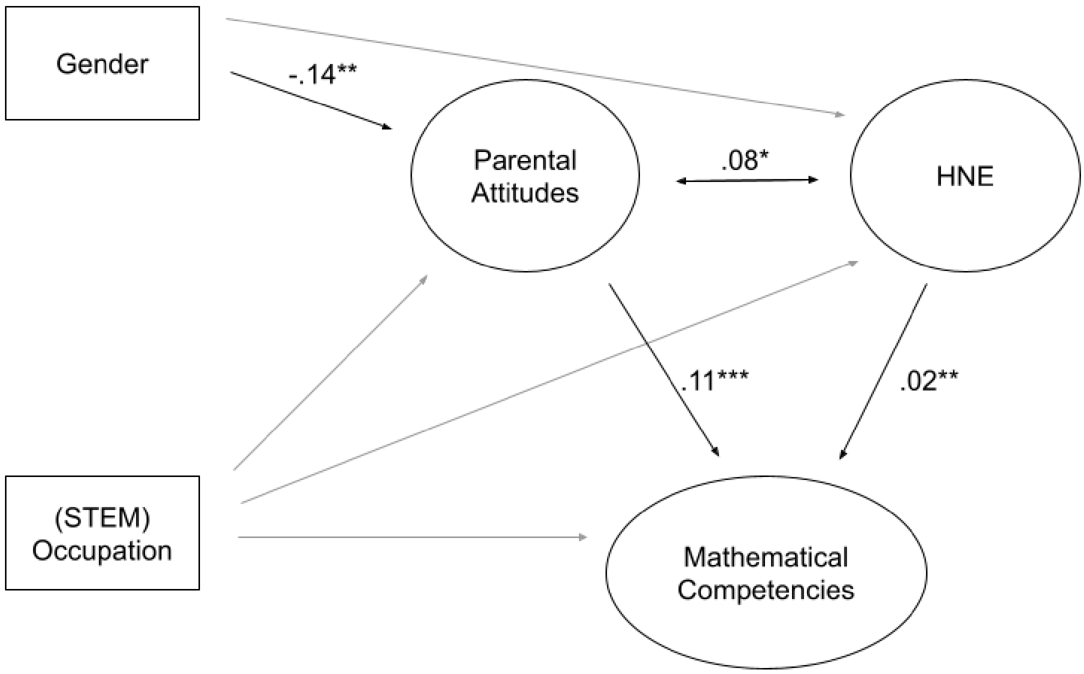
Out of all four developed models, only model 1 met the fit criteria. Therefore, only the results of model 1 will be presented hereafter (see Table A6 for a total overview of the results). For the results and pathway structures of models 2, 3, and 4, see Tables A7, A8, and A9 as well as Figure A1 and A2.

To investigate the relations between all variables of interest except for children's behaviour during a game of dice – i.e., parental (STEM) occupation, parental attitudes towards mathematics, the HNE, and children's mathematical competencies, model 1 was developed and its results will be presented in the following.

Figure 3 illustrates the significant pathways of model 1. In line with the hypothesis, the correlation between HNE and parental attitudes was significant ($r = .08$, $p = .01$). Regressions from STEM to parental attitudes, the HNE, and mathematical competencies showed no significant results, meaning that parents' occupation was not a significant predictor for their attitudes, the quality of their interaction with their child during a game of dice, or their children's mathematical competencies. While children's gender had no significant association with HNE, it acted as a significant predictor for parental attitudes ($\beta = -.14$, $p < .01$), indicating higher parental attitudes towards mathematics for parents from girls than from boys. Furthermore, for mathematics, both parental attitudes ($\beta = .11$, $p < .001$) and HNE ($\beta = .02$, $p < .01$) acted as significant predictors.

Figure 3

Significant pathways in structural equation model 1



Note. HNE = Home Numeracy Environment. Rectangles indicate manifest variables; circles indicate latent variables. Grey lines indicate non-significant pathways, black lines indicate significant pathways between variables.

*p < .05. **p < .01. ***p < .001.

Discussion

The growing research on the association between family characteristics and children's early mathematical competencies gives insight into their interplay and how children's skills and abilities can best be supported (e.g., Chi et al., 2017; Dearing et al., 2012; Niklas & Schneider, 2017). However, previous research shows mixed findings (cp. Elliott & Bachmann, 2018a), and the relation between parental occupation as a standalone component of the SES and children's numeracy development has not been sufficiently considered yet (cp.

Mues et al., 2021). Further, while indications of children's own influence on the support they receive and the growth they experience has been reported (e.g., Elliott & Bachman, 2018a), the role of child characteristics like interest and behaviour was hardly investigated.

In the present study, parental occupations as indicators of families' SES, parental attitudes toward mathematics, and the interaction between parents and their children during a game of dice as an indicator of the HNE were investigated in relation to children's mathematical achievement.

Associations Between (STEM) Occupations, Parental Attitudes, HNE, and Children's Mathematical Competencies

In line with previous research (e.g., DeFlorio & Beliakoff, 2015; Niklas & Schneider, 2017; Sonnenschein & Sun, 2017), the family characteristics HNE and parental attitudes correlated with children's mathematical competencies. While STEM parents showed significantly greater attitudes toward mathematics than non-STEM parents, there were no significant findings between (STEM) occupation and the HNE or children's mathematical competencies, in contrast to the findings of Mues and colleagues (2021), who investigated a similar sample.

These disparities might be due to the differences in the operationalisation of both the HNE and parental occupation in the present study and that by Mues et al. (2021): The present study operationalised the HNE via observation while Mues and colleagues used items from a questionnaire measuring both formal and informal HNE. Further, the authors differentiated between parents' learned and current occupations in their analysis while the present study is limited to parents' current occupations. Additionally, although the sample was similar, there were important differences between the sample Mues and others used and the sample that was used in the present study. In the latter, the sample comprised both cohorts (N = 500) of the Learning4Kids study (Niklas et al., 2020) including only the first measurement point (M_{age} =

60.96 months, *SD* = 4.61), while Mues et al. analysed a sample of only the first cohort (*N* = 190; M_{age} = 63.58 months, *SD* = 4.41) including the first two measurement points.

Several factors could be decisive for the incongruent findings in this study and the one by Mues et al.: The greater age in Mues and colleagues' sample, the distinction between learned and current parental occupation, the smaller sample size, or the longitudinal aspect of the study may all impact the findings and lead to differing results. An investigation of the same sample on the same variables in both a cross-section and longitudinal design would be interesting to check for possible remaining differences.

Correlational analyses further revealed a non-significant association between parental (STEM) occupation and children's mathematical competencies. These findings suggest that while STEM employment does play a significant role in adolescents' achievements (e.g., Chi et al., 2017), it does not seem to be as influential in a younger age. Differences between young children's mathematical competencies are more likely to occur due to parental, more specifically, maternal education (Bachman et al., 2015; Erola et al., 2016; Omolade & Salomi, 2011).

This suggestion is in line with findings by Mues and colleagues (2021), who found parents' *learned* occupations to be significantly related to children's numeracy achievement, but not their *current* occupations. It is possible that parents who did not indicate to currently work in the STEM field do have a STEM background. Cech and Blair-Loy (2019) found new parents to have a high chance of leaving full-time STEM occupations and often switching to other full-time professions, mothers far more often than fathers. Thus, separating parental employment into learned and current occupations is a vital addition to research on family characteristic influences.

Interestingly, while Mues and colleagues (2021) found significant associations between parents' current occupations and the HNE, the present study did not. Again, this difference may have occurred due to the different operationalisation and thus different types of items. As Mutaf Yildiz and colleagues (2018) put it: Both questionnaires and observations "tap into different aspects of home numeracy and can be an important factor explaining inconsistencies in literature" (p. 2). While parental questionnaires are easier subject to social desirability bias, observations are able to display behaviour that is closer to parents' and children's actual behaviour outside of the study context.

Further, while the construct HNE was measured through an observation of parents interacting with their children during a game of dice, Mues et al.'s (2021) construct HNE was measured using items comprising informal and formal facets of the HNE, such as the frequency parents engage their children in number counting or the frequency their children see them handling numbers and calculating. It makes sense that parents who currently work in STEM fields would more likely be seen dealing with numbers or engage their children in counting, as they are confronted with mathematical content on an everyday basis. When preparing for work, working from home, or talking about work, they are more likely to (in)formally and subliminally procure a learning environment with numerical contents.

Playing a game of dice, however, constitutes an inclusive form of HNE. Parents do not need to handle numbers daily or be exceptionally proficient in STEM fields to be able to engage and interact with their children in a meaningful way. They are more likely to engage in mathematics talk during a game of dice than in their everyday lives, as the situation demands it in a subtle, playful way. Prompts can be made more naturally and interacting with the child in a numeracy-related environment seems more casual.

In fact, the insignificant findings between parents' occupation and the HNE are good news: Games like a joined game of dice, which can greatly support children's mathematical development (see e.g., Burghardt et al., 2020; Dearing et al., 2012; Niklas et al., 2015), are accessible to people from all educational backgrounds, regardless of if they are employed in the STEM field or not, and seem to be equally supportive for children's competencies throughout all parents.

Considering the differences that possibly occur due to the operationalisation of the HNE (when comparing these present results with the ones from Mues et al. [2021]), it is important to look more closely at the relationship between observational measures and questionnaires in the future in order to better understand the construct of the HNE (cp. Mutaf Yildiz et al., 2018).

Despite Mues and colleagues' (2021) assumption "that parental learned occupations (but not their current occupations) are connected to parents' [..] attitudes" (p. 11), the present study did find significant associations between parents' current occupations and parental attitudes, indicating that STEM parents have greater attitudes toward mathematics than non-STEM parents. As attitudes toward STEM professions influence people's motivation to engage with them (Guo et al., 2019), the same can be assumed as vice-versa: Engaging with STEM professions presumably influences people's attitudes toward them. Thus, parents currently working in the STEM field are more likely to display more positive attitudes toward mathematics and science and pass these positive attitudes on to their children (cp. also Elliott & Bachman, 2018a, 2018b).

The attitudes parents displayed towards mathematics were further significantly associated with children's mathematical competencies. As parental attitudes are assumed to be transmitted to their children (Hildebrand et al., 2022; Soni & Kumari, 2017), children with parents with more positive attitudes toward mathematics are also more likely to display greater mathematical competencies. This outcome is in line with that by Makur and colleagues (2019) and contributes to the manageable series of findings on parental attitudes toward mathematics and their children's mathematical achievement.

Associations Between (STEM) Occupation, Children's Gender and Their Mathematical Competencies

Previous research showed mixed findings regarding the relationship between gender and children's mathematical competencies but suggested rather small significant differences (e.g., Levine et al., 2012; Mejía-Rodríguez et al., 2021). Consistent with this suggestion, the present study found gender to be a significant predictor of children's mathematical competencies. This effect even remained after controlling for children's age and intelligence. While the gender gap in adolescents is obvious and growing, consistently favouring boys (Lee et al., 2011), gender differences in mathematics skills between younger children are not as apparent and often favour girls, as is the case in the present study. Unlike Bowden and colleagues (2018), who found differences between girls and boys and their parents' occupations, always favouring those children whose parents were employed in STEM professions, this study did neither find parental (STEM) occupation nor the interaction between children's gender and their parental occupation to act as a predictor to children's mathematics skills. Further, comparing children from STEM and non-STEM parents showed no significant results, implying that children do not significantly differ in their mathematical competencies subject to their parents' occupation.

Regarding the above-mentioned results on the HNE and STEM occupation, it can be assumed that the HNE children find themselves in does not differ in relation to their parents' occupations. This suggests that both STEM and non-STEM parents are likely to treat their children in a similar way and promote their (mathematical) development in a similar way.

Considering prior research on family characteristics and children's mathematical achievements, parental occupation seems more likely to play a bigger role later on: STEM occupation was observed to predict (STEM) course choice in adolescents, and to have an impact on adolescents' enjoyment, motivation and self-concept in mathematics (Harackiewicz et al., 2012; Lane, 2017). Here, age seems to be a driving force that enables various modes of action. Results of the analysis of covariance also depicted the significance of children's age on their mathematical competencies. But getting older can impact many skills and characteristics of the self, not just mathematical skills. Self-concept in particular, a construct that constitutes an influential factor for children's and adolescents' academic achievement, is not only subject to parental and family characteristics (Harackiewicz et al., 2012; Lane, 2017), but is also dependent on children's characteristics, such as their age.

According to Marsh (1986), students' self-concepts are formed both by social and dimensional processes, ascribing schools to an influential and transforming place. Thus, fostering a strong and positive mathematics self-concept is more important at younger ages than just focusing on mathematical knowledge, so that children can enter school with a healthy self-concept.

Mathematics self-concept is assumed to be able to mediate between mathematics anxiety and children's mathematics outcome (Justicia-Galiano et al., 2017) and predict later academic achievement (Susperreguy et al., 2018). While several researchers (Cvencek et al., 2011; Lindberg et al., 2013; Mejía-Rodriguez et al., 2021) detected a significant gender gap in children's mathematics self-concept in elementary school kids, favouring boys - with no gender differences in actual mathematical outcomes detected -, Arens et al. (2016) did not find one in their sample of preschoolers. In an older sample consisting of fifth and eighth graders, Onetti et al. (2019) found that older students showed lower self-concepts.

Thus, the possibility that significant differences in girls' and boys' mathematics self-concepts and abilities start once they enter school where they are exposed to comparisons with their peers is highly likely (see also Niklas & Schneider, 2012). Future research should include children's mathematics self-concept in addition to their competencies to further investigate this matter.

Association Between Parental (STEM) Occupation and Children's Gender With Parental Attitudes, the HNE, Children's Behaviour, and Children's Mathematical Competencies

Pathways from the structural equation model 1 showed a significant covariance between parental attitudes and the HNE and further revealed these variables as predictors of children's mathematical competencies.

This covariance confirms Elliott and Bachman's (2018a) model of the two perspectives of parental promotion of children's learning: *Who parents are* affects *what parents do.* Parental attitudes toward mathematics affect their interaction with their children during a game of dice, i.e., their HNE, and thus it makes sense that both of these constructs act as predictors of children's numerical competencies, as one influences the other. Also, the model by Zanna and Rempel (1988) gains further confirmation from these results, implying that the attitudes parents display toward mathematics shape the quantity and quality of their HNE (see also Dowker, 2021).

Further, parental attitudes presumably predict home numeracy practices and children's mathematical achievement as well as mediate SES (in this case STEM occupation) effects (Elliott & Bachman, 2018a; Kleemans et al., 2012). This exemplifies the close connection between parental attitudes and the HNE and their covariation.

Another significant pathway the structural equation modelling revealed was the significant association between children's gender and parental attitudes. Contrary to Hildebrand et al.'s (2022) recent findings on parental attitudes towards mathematics in relation to their children's gender, the present study was able to report parental attitudes in favour of girls. Parents of daughters had more positive attitudes towards mathematics than parents of sons. Despite the small effect size, this outcome contributes to the mixed findings on parental attitudes towards their sons and daughters.

As the variable investigating parental attitudes consisted of only three items (see Appendix B for an overview), with just one of them focusing on the parents' attitudes towards their children and their relation to mathematics, it could be interesting to more closely investigate this variable in particular. According to several researchers (e.g., Marsh et al., 2019; Picho & Schmader, 2018; Tomasetto et al., 2011), mothers who explicitly rejected the stereotype threat girls are faced with were more likely able to prevent their daughters from its negative effect on their susceptibility towards it. As mostly mothers (72.34 %) filled in the survey and thus indicated these positive attitudes towards mathematics, it could be that especially the mothers of daughters try to actively support their daughters' attitudes and self-esteems towards mathematics in order to fight the stereotype threat.

However, this significant association between gender and parental attitudes only occurred in the SEM that included recommended modification indices. Here, a correlation between the first and third item of parental attitudes was added. Regarding this fact and the scarce theory on children's gender and their relation to their parents' attitudes on mathematics in general (instead of parents' attitudes on their children's mathematical competencies), these findings need to be interpreted cautiously and require further replication.

Even though prior research agrees on the dire influences of SES on nearly every family characteristic and child outcome (e.g., DeFlorio & Beliakoff, 2015; Niklas et al., 2016; Sonnenschein & Sun, 2017; Tonizzi et al., 2021), no associations between parental (STEM) occupation as an indicator of SES and other variables could be detected in the present study. This suggests that parental occupation is unable to act as a standalone component describing the SES. Simultaneously, it shows that other parts of a family's SES have to be the crucial component(s) influencing children's academic development and outcomes. There is still a need for a more detailed investigation of the many facets of the SES and their influence on family characteristics and child outcomes in order to crystallise its decisive factors (Elliott & Bachman, 2018a; Mues et al., 2021).

Considering prior research suggested maternal education as being a strong influence on children's academic outcomes (e.g., Bachman et al., 2015; Lombardi & Dearing, 2020; Slusser et al., 2019), focusing on mother's learned occupations (instead of their current occupations, as mothers are still more likely to leave or change their occupation after childbirth; see Cech & Blair-Loy, 2019) and their children's outcomes could give more insight into the impact maternal *education* instead of *occupation* can have on children's development.

Outlook: The Relationship Between Child Behaviour, Parental Attitudes, the Home Numeracy Environment, and Children's Mathematical Competencies – Exploratory Considerations

The initial correlation between all variables of interest revealed significant relations between the additional variable child behaviour and parental attitudes, the HNE, and children's mathematical competencies. While of exploratory nature, these significant associations caught attention. Especially the moderate correlation between child behaviour and the HNE with r = .57 stood out. These correlations between children's behaviours and their parents' attitudes and behaviours point out the reciprocal relationship between children's and parents' actions (see e.g., Cheung et al., 2018; Lukie et al., 2014; Silinskas et al., 2020). The more interest children show in mathematical content, the more their parents will provide them with and engage them in mathematical activities. The more parents show interest towards mathematical activities and provide them their children, the more the children's interest in those contents will grow.

Further, the correlation between the HNE and children's behaviour can be attributed to the item composition of both variables: Both constructs were measured observational during a game of dice. Parents' actions during the game of dice constituted the HNE while children's behaviour during the game was measured as the additional variable. Therefore, a high correlation is not surprising. Yet, researchers suggest similar results in other settings as well: Simpkins and colleagues (2005), who investigated an older sample of 2nd and 5th graders, also reported a strong positive relation between parental and child behaviour in regard to mathematics. Thus, it is likely that children's behaviour and parental attitudes also correlate in settings that focus less on the cooperation of child and parent.

Taking into account the model of learning promotion by Elliot and Bachman (2018a), it makes sense that a significant correlation with the HNE would go hand in hand with a significant correlation with parental attitudes and vice versa.

Children’s quantitative and qualitative behaviour during a game of dice was also significantly correlated with their mathematical competencies. While the construct of child behaviour could not be investigated closer due to a bad model fit of the structural equation model 2, the significant correlation indicates a connection between children’s behaviour and mathematical competencies. These results are in line with the findings by Convery (2021), who suggests that child behaviour may be even more important for their mathematical development than parental factors. These suggestions open up a broad, exciting field of investigation that is still virtually unexplored. Future research is needed to further expand the data and replicate the (exploratory) findings of this study and Convery's.

Limitations and Implications for Further Research

This study was subject to some limitations. As the collected data were cross-sectionally, no influences of variables on each other could be determined. Analysing the data longitudinally would ascertain variables’ impacts, but it needs to be mentioned that even in a longitudinal format, causality cannot be assumed.

Although children’s age and intelligence have been added as control variables in the analyses, there are other additional variables such as ethnicity, working memory, a more global SES measure, or even literacy skills that could have a relation with the investigated variables (see e.g., Hornung et al., 2014; Kleemans et al., 2012; Niklas & Schneider, 2017).

Further, as mentioned above, adding parents’ learned occupation as an additional variable could lead to a deeper insight into the relationship between parental occupation and children’s mathematical outcomes (see Mues et al., 2021). To investigate the properties of parental (STEM) occupation more thoroughly, differing between parents’ gender is also advisable. According to prior research, especially mothers working in STEM are suggested to have a strong impact on both family characteristics and children’s outcomes (e.g., Bowden et al., 2018; Dearing et al., 2012; Guo et al., 2019). Thus, separating parental occupation into learned and current occupations as well as into mother’s and father’s occupations can bring new and deeper insights into the influence parents can have on their children's academic development.

Even though observational operationalisation holds many advantages and needs to be used more often in order to enable an adequate comparison with other methods such as surveys, measuring the HNE via observation also posed limitations. First, adding an additional quantitative measure of the HNE to be able to measure the construct more broadly can be useful, allowing differentiation between informal and formal aspects of the HNE. Secondly, as the observations were conducted by different research assistants, significant differences between the research assistants' ratings occurred. However, these statistical differences do not inform the actual rating differences between research assistants. It is not evident whether certain research assistants actually rated higher or lower than the average because of "good" or "bad" will or if their ratings more or less objectively reflected actual observations. As research assistants visited families primarily in certain districts of Munich county, it is possible that researchers who, for instance, collected data in high-SES districts were more likely to encounter families with an HNE of greater quality and rated correspondingly, which could lead to significantly differing ratings compared to research assistants visiting primarily families in low-SES districts. Each research assistant received the same instructions on the observational rating. Nevertheless, whether the differences between ratings occurred randomly or due to bias between research assistants is not apparent.

Further limitations are posed by the choice of analysis: Two structural equation models were developed, but both did not show satisfying fits. While model 1 had a fit acceptable enough to be executed once recommended modifications were applied, model 2 did not even get into consideration as its fit was insufficient, even though insight into the association between child behaviour and other variables would have been interesting (cp. Elliott & Bachman, 2018a). Including control variables into these models developed models with insufficient fits, even after imputation of recommendations.

After conducting an exploratory factor analysis for mathematics, as modification indices recommended covariances between its items, two factors concerning mathematical skills were found: one focusing on number knowledge and counting and one focusing on general mathematical understanding. This suggests that the latent variable mathematics did not measure children's mathematical competencies in a valid way. Interestingly, Mues and colleagues (2021), who worked with the same items to measure children's mathematical competencies, computed a good fit of their CFI of mathematics while the present study computed a bad fit that had to be rejected, leading to the exploratory factor analysis. Again, the researchers did not use the same sample but instead conducted a longitudinal study over two measurement points with a smaller sample of $N = 190$ children. The longitudinal aspect of their study could be decisive for the deviating results in their and the present study.

However, modification indices in the study by Mues et al. (2021) recommended similar correlations between mathematics items, reinforcing the assumption that mathematics comprises more than one factor. Distinguishing between certain skills that are part of mathematical development and competencies has been highlighted in prior research (e.g., Dearing et al., 2012; Elliott & Bachman, 2018a; Levine et al., 2012) and should be considered for future research in order to determine facets that can be easily tackled by parents.

Conclusion

This study contributes to the growing body of research on young children's mathematical competencies and their association with family characteristics such as parental occupation, attitudes, and the home numeracy environment in Germany. It further enriches research on the HNE by operationalising the variable via observations, expanding the understanding of the concept itself.

The findings of this study suggest that especially parental attitudes and the HNE they provide for their children are associated with children's mathematical competencies. Parents' current occupations seem to be less important for children's mathematics skills in this study. This leads to the assumption that children's mathematical achievement is less dependent on fixed factors but can rather be fostered by factors that are flexible and low-threshold, allowing parents of all backgrounds to manipulate them and thus support their children's mathematical skills. Making parents aware of the importance of the HNE they provide, their own attitudes and the role-modelling they display can help their children to build a strong foundation of mathematical competencies early on. Thereby, children will be able to start primary school confidently and on a more balanced level with their peers.

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Appendix A: Figures and Tables for Descriptives and Analyses

Table A1

Overview of investigated variables, their scales, and descriptives (sample sizes, means, standard deviations, minima, maxima)

Variables	N	M	SD	Min	Max	Measurement Instrument	Item Count	Example Item
Parental Attitudes	497	2.89	0.80	0.00	4.00	Survey	3	Mathematics is considered important at our home.
HNE	484	4.03	1.13	1.00	6.57	Observation	7	
Child Behaviour[a]	302	4.25	1.58	1.00	7.00	Observation	2	
MSkill	485	0.38	0.20	0.00	1.00	Test	61	
(STEM) Occupation	443	0.45	0.50	0.00	1.00	Survey		

Note. HNE = Home numeracy environment; MSkill = Mathematical Skill; (STEM) Occupation (0 = non-STEM, 1 = STEM).

[a]Child Behaviour was only conducted for cohort 2.

Table A2

Unmodified fit of the structural equation models using confirmatory factor analysis

	χ^2	df	χ^2/df	CFI	TLI	RMSEA	90 % CI
Model 1 (*N* = 500)	477.17	128	3.73	.91	.89	.07	[.07, .08]
Model 2 (*N* = 302; including Child Behaviour)	452.30	159	3.84	.89	.87	.08	[.07, .09]
Model 3 (model 1, including control variables)	590.21	162	3.64	.89	.87	.07	[.07, .08]
Model 4 (model 2, including control variables)	678.97	200	3.39	.83	.81	.90	[.08, .10]

Note. Models 1 and 3 include the variables (STEM) occupation, child gender, home numeracy environment, parental attitudes, and mathematical competences. Models 2 and 4 further include the additional variable child behaviour. The fit criteria for the statistical models were relative/normed Chi-square (χ^2/df) between .03 and .05, CFI ≥ .95, TLI ≥ .95, RSMEA ≤ .07 and its 90 % confidence interval with a lower limit close to 0 and the upper limit ≤ .08.

Table A3

Modification indices recommended by R

Model	Parameter	Modification index	Expected parameter change
1	MS[a] ~~ R[a]	68.55	0.01
	WS_f6_1[b] ~~ WS_f7_1[b]	32.63	0.44
	EF_ML8_1[c] ~~ EF_ML10_1[c]	32.43	1.09
	WS_f5_1[b] ~~ WS_f6_1[b]	29.33	0.48
	WS_f1_1[b] ~~ WS_f7_1[b]	19.76	-0.25
	MS[a] ~~ Zfr[a]	19.22	-0.00
	WS_f2_1[b] ~~ WS_f6_1[b]	16.80	-0.23
	WS_f4_1[b] ~~ WS_f6_1[b]	16.66	-0.28
2	MS[a] ~~ R[a]	55.98	0.01
	MS[a] ~~ Zfr[a]	22.21	-0.01
	WS_f5_1[b] ~~ WS_f6_1[b]	22.06	0.51
	WS_f6_1[b] ~~ WS_f7_1[b]	20.25	0.44
	WS_f4_1[b] ~~ WS_f6_1[b]	13.93	0.31
	EF_ML8_1[c] ~~ EF_ML10_1[c]	13.80	0.79
	WS_f2_1[b] ~~ WS_f6_1[b]	13.20	0.26

Note. Modification indices conducted using the *lavaan* packages (code: modindices()). Recommendations were picked by good reason and largeness of modification indices. Only those modification indices that were used in the structural equation modelling are listed. ~~ indicating covariances.

[a] Sub-item of latent construct Math Skill.

[b] Sub-item of latent construct Home Numeracy Environment.

[c] Sub-item of latent construct Parental Attitudes.

Table A4

Modified fit of the structural equation models using confirmatory factor analysis

	χ^2	df	χ^2/df	CFI	TLI	RMSEA	90 % CI
Model 1 (*N* = 500)	264.04	120	2.20	.96	.95	.05	[.04, .06]
Model 2 (*N* = 302; including Child Behaviour)	300.69	154	1.95	.95	.94	.06	[.05, .07]
Model 3 (model 1, including control variables)	418.81	157	2.67	.93	.92	.06	[.05, .06]
Model 4 (model 2, including control variables)	388.13	193	2.01	.93	.92	.06	[.05, .07]

Note. Models 1 and 3 include the variables (STEM) occupation, child gender, home numeracy environment, parental attitudes, and mathematical competences. Models 2 and 4 further include the additional variable child behaviour. The fit criteria for the statistical models were : relative/normed Chi-square (χ^2/df) between .03 and .05, CFI ≥ .95, TLI ≥ .95, RSMEA ≤ .07 and its 90 % confidence interval with a lower limit close to 0 and the upper limit ≤ .08. All fit measures listed here are based on the model fit conducted after including modification indices.

Table A5

Exploratory factor analysis of latent construct Math Skills

Item	Loading		Communality	Uniqueness	Complexity
	Factor 1	Factor 2			
Zfr	0.91		.67	.33	1.04
VuN	0.88		.75	.25	1.00
Zfv	0.72		.62	.38	1.02
Zk	0.71		.58	.42	1.02
MS		1.06	.99	.01	1.01
R	0.33	0.43	.50	.50	1.87

Note. Latent construct Math Skills comprises 61 items in total. Zfr = number sequences backwards (6 items); VuN = number predecessors and successors (8 items); Zfv = number sequences forwards (8 items); Zk = number symbol knowledge (10 items); MS = numbers, ordinal number bars, cardinality, number division, inclusion and relations (21 items); R = multiplication and subtraction (8 items). Loadings below a value of 0.15 were cut off.

Structural Equation Models

Table A6

Statistical results of structural equation model 1

Regression						
	Parameter	Coefficient	SD	95 % CI	*z*	*p*
	Attitudes ~ STEM	0.11	.06	[0.00, 0.22]	1.94	.05
	HNE ~ STEM	-0.01	.11	[-0.22, 0.20]	-0.09	.93
	Math Skill ~ STEM	0.00	.01	[-0.03, 0.02]	-0.23	.82
	Math Skill ~ Attitudes	0.11	.02	[0.07, 0.15]	6.02	< .001
	Math Skill ~ HNE	0.02	.01	[0.01, 0.03]	2.78	.01
	Attitudes ~ Gender	-0.14	.05	[-0.24, -0.04]	-2.68	.01
	HNE ~ Gender	0.09	.10	[-0.09, 0.28]	0.97	.33
Correlation						
	Parameter	Coefficient	SD	95 % CI	*z*	*p*
	Attitudes ~~ HNE	0.08	.03	[0.02, 0.15]	2.44	.01
	STEM ~~ Gender	0.02	.01	[0.00, 0.04]	1.74	.08

Note. Regressions and correlations between parental (STEM) occupation, children's gender, parental attitudes, the home numeracy environment, and children's mathematical competencies. *N* = 500. Results after adding recommended indices. Attitudes = parental attitudes; HNE = home numeracy environment; Math Skill = mathematical skill; STEM = parental (STEM) occupation.

Table A7

Statistical results of structural equation model 2

	Parameter	Coefficient	SD	95 % CI	*z*	*p*
Regression						
	Attitudes ~ STEM	0.11	.07	[-0.02, 0.24]	1.63	.10
	HNE ~ STEM	-0.04	.11	[-0.25, 0.18]	-0.33	.74
	Math Skill ~ STEM	-0.02	.02	[-0.05, 0.01]	-1.14	.25
	Math Skill ~ Attitudes	0.13	.03	[0.07, 0.18]	4.52	< .001
	Math Skill ~ HNE	0.01	.01	[-0.01, 0.03]	0.90	.37
	Attitudes ~ Gender	-0.11	.06	[-0.22, 0.00]	-2.01	.04
	HNE ~ Gender	0.02	.10	[-0.17, 0.21]	0.21	.83
Correlation						
	Parameter	Coefficient	SD	95 % CI	*z*	*p*
	Attitudes ~~ HNE	0.10	.04	[0.02, 0.17]	2.50	.01
	HNE ~~ CB	0.92	.12	[0.69, 1.14]	7.87	< .001
	Attitudes ~~ CB	0.17	.07	[0.04, 0.30]	2.50	.01
	STEM ~~ Gender	0.02	.02	[-0.01, 0.05]	1.37	.17

Note. Regressions and correlations between parental (STEM) occupation, children's gender, parental attitudes, the home numeracy environment, children's mathematical competencies, and children's behaviour during a game of dice. *N* = 302. Results after adding recommended indices. Attitudes = parental attitudes; HNE = home numeracy environment; Math Skill = mathematical skill; STEM = parental (STEM) occupation; CB = Child Behaviour.

Table A8

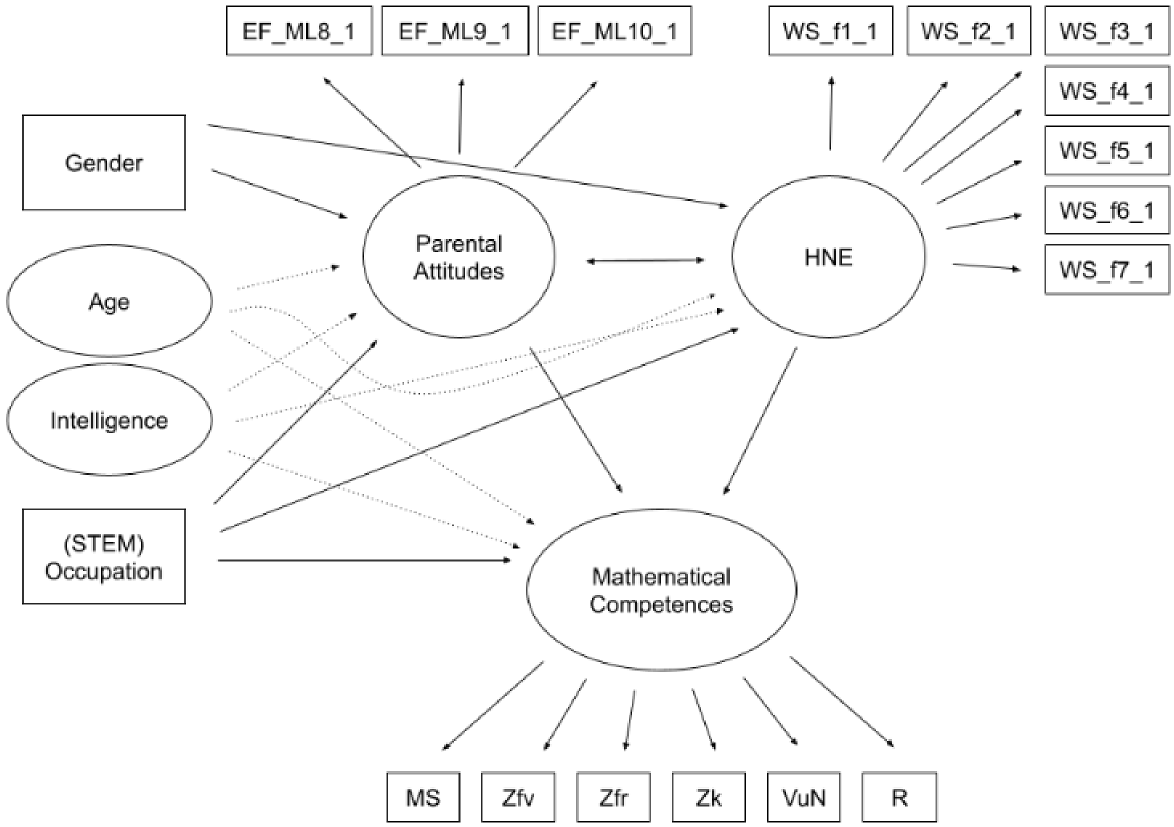
Statistical results of structural equation model 3

Regression						
	Parameter	Coefficient	SD	95 % CI	*z*	*p*
	Attitudes ~ STEM	0.24	.09	[0.06, 0.42]	2.66	.01
	HNE ~ STEM	0.02	.11	[-0.19, 0.22]	.15	.89
	Math Skill ~ STEM	0.01	.01	[-0.02, 0.03]	0.40	.69
	Math Skill ~ Attitudes	0.03	.01	[0.01, 0.05]	3.51	< .001
	Math Skill ~ HNE	0.03	.01	[0.01, 0.04]	3.90	< .001
	Attitudes ~ Gender	-0.12	.09	[-0.28, 0.05]	-1.37	.17
	HNE ~ Gender	0.09	.09	[-0.09, 0.27]	0.98	.33
	Intelligence ~ Math Skill	0.00	.00	[0.00, 0.00]	3.57	< .001
	Age ~ Math Skill	0.01	.00	[0.01, 0.01]	6.43	< .001
	Intelligence ~ STEM	2.29		[0.24, 4.34]	2.19	.03
Correlation						
	Parameter	Coefficient	SD	95 % CI	*z*	*p*
	Attitudes ~~ HNE	0.18	.05	[0.09, 0.27]	3.94	< .001
	STEM ~~ Gender	0.02	.01	[0.00, 0.04]	1.79	.08

Note. Regressions and correlations between parental (STEM) occupation, children's gender, parental attitudes, the home numeracy environment, and children's mathematical competencies including the control variables child age and intelligence. *N* = 500. Attitudes = parental attitudes; HNE = home numeracy environment; Math Skill = mathematical skill; STEM = parental (STEM) occupation.

Figure A1

Structural equation model 3



Note. Pathways between parental (STEM) occupation, child gender (Gender), parental attitudes towards mathematics (Parental Attitudes), home numeracy environment (HNE), and children's mathematical competencies as well as the control variables age and intelligence. Rectangles indicate manifest variables; circles indicate latent variables. Pathways from control variables are dotted. Direct pathways are bold.

Table A9

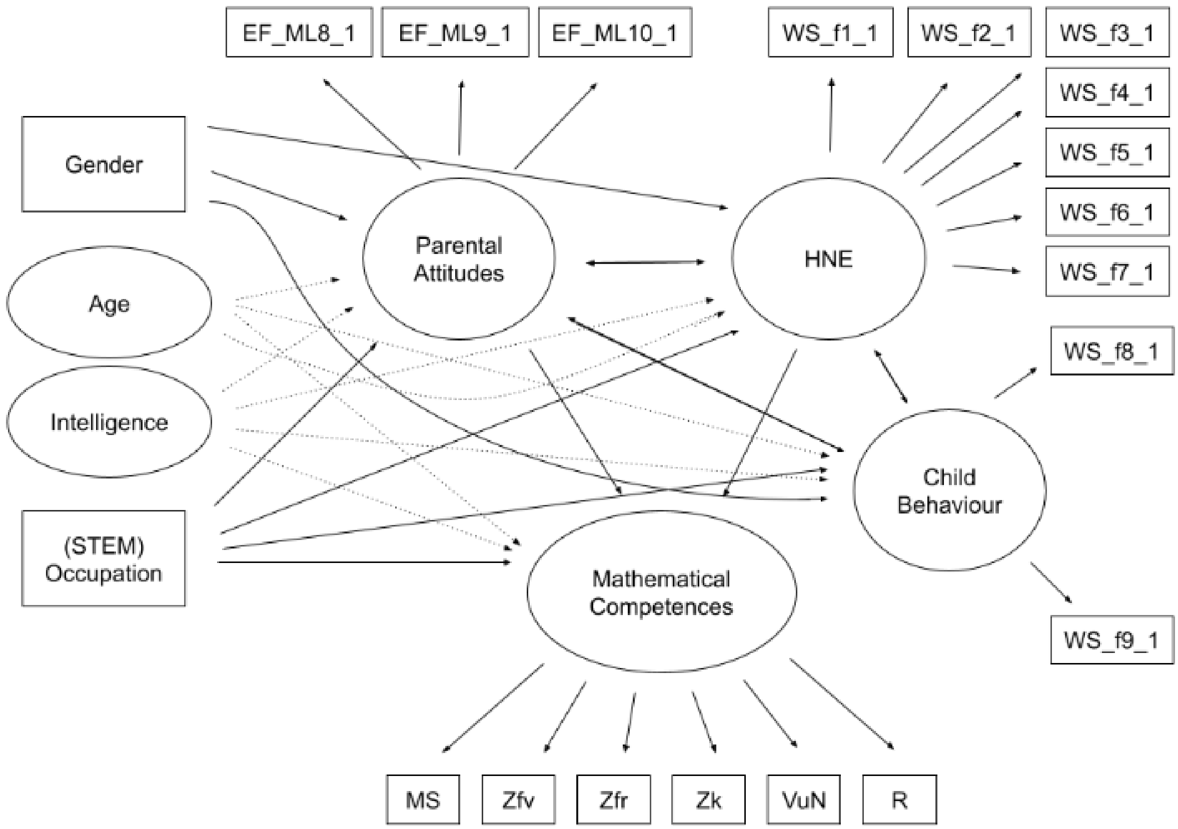
Statistical results of structural equation model 4

Regression						
	Parameter	Coefficient	SD	95 % CI	*z*	*p*
	Attitudes ~ STEM	0.34	.11	[0.13, 0.55]	3.16	.00
	HNE ~ STEM	0.09	.13	[-0.17, 0.35]	0.66	.51
	Math Skill ~ STEM	< 0.00	.02	[-0.03, 0.03]	-0.06	.95
	Math Skill ~ Attitudes	0.02	.01	[-0.01, 0.04]	1.45	.15
	Math Skill ~ HNE	0.02	.01	[0.00, 0.04]	2.42	.02
	Attitudes ~ Gender	-0.02	.10	[-0.22, 0.18]	-0.22	.83
	HNE ~ Gender	0.06	.12	[-0.17, 0.29]	0.50	.62
	Math Skill ~ Intelligence	< .001	.00	[0.00, 0.00]	1.48	.14
	Math Skill ~ Age	0.01	.00	[0.01, 0.02]	6.56	< .001
Correlation						
	Parameter	Coefficient	SD	95 % CI	*z*	*p*
	Attitudes ~~ HNE	0.17	.05	[0.06, 0.27]	3.17	.00
	STEM ~~ Gender	0.02	.02	[-0.01, 0.05]	1.45	.15
	CB ~ Age	0.42	.37	[-0.30, 1.15]	1.14	.25

Note. Regressions and correlations between parental (STEM) occupation, children's gender, parental attitudes, the home numeracy environment, children's behaviour during a game of dice, and children's mathematical competencies including the control variables child age and intelligence. *N* = 302; Attitudes = parental attitudes; HNE = home numeracy environment; Math Skill = mathematical skill; STEM = parental (STEM) occupation; CB = Child Behaviour.

Figure A2

Structural equation model 4



Note. Pathways between parental (STEM) occupation, child gender (Gender), parental attitudes towards mathematics (Parental Attitudes), home numeracy environment (HNE), children's behaviour during a game of dice (Child Behaviour) and children's mathematical competencies as well as the control variables age and intelligence. Rectangles indicate manifest variables; circles indicate latent variables. Pathways from control variables are dotted. Direct pathways are bold.

Appendix B: Descriptions of Items

Appendix B

Item description of parental attitudes

Items	Variable names	Item description
Item 1	EF_ML8_1	Being able to do arithmetic is considered important at home
Item 2	EF_ML9_1	My child is very interested in learning arithmetic and counting and is really looking forward to it
Item 3	EF_ML10_1	Mathematics is considered important at home

Note. Items were rated on a 4-point Likert scale (4 = "very strongly agree"; 0 = "do not agree at all").