R-code for Chapter 3: Bivariate copula classes, their visualization and estimation

Claudia Czado

01 March, 2019

Contents

Required R-packages

- VineCopula
- rafalib
- copula

Section 3.3 Archimedean copulas

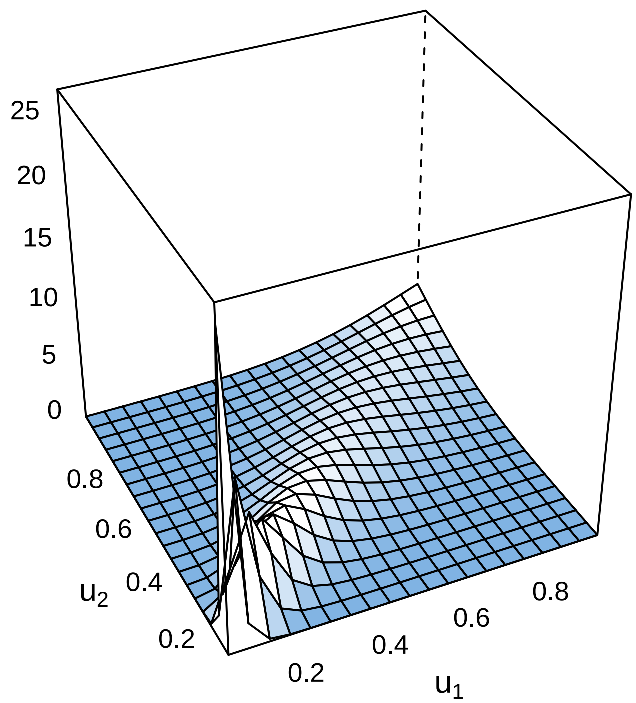
Figure 3.1: Archimeden copulas: Bivariate Archimedean copula densities with a single parameter: top left: Clayton, top right: Gumbel, bottom left: Frank, bottom right: Joe (The copula parameter is chosen in such a way that the correspond-ing Kendall's $\tau = .7$).

Clayton, Frank, Gumbel and Joe copula setup

```
par.clayton<-BiCopTau2Par(3, .7, check.taus = TRUE)
par.gumbel<-BiCopTau2Par(4, .7, check.taus = TRUE)
par.frank<-BiCopTau2Par(5, .7, check.taus = TRUE)
par.joe<-BiCopTau2Par(6, .7, check.taus = TRUE)
obj.clayton <- BiCop(family = 3, par = par.clayton)
obj.gumbel <- BiCop(family = 4, par = par.gumbel)
obj.frank <- BiCop(family = 5, par = par.frank)
obj.joe <- BiCop(family = 6, par = par.joe)
```

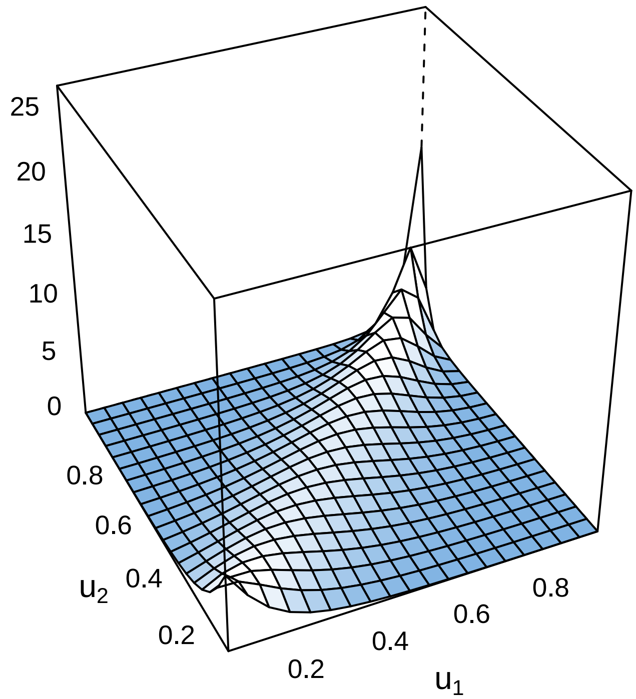
Clayton copula density surface

```r
plot(obj.clayton,zlim=c(0,27))
```



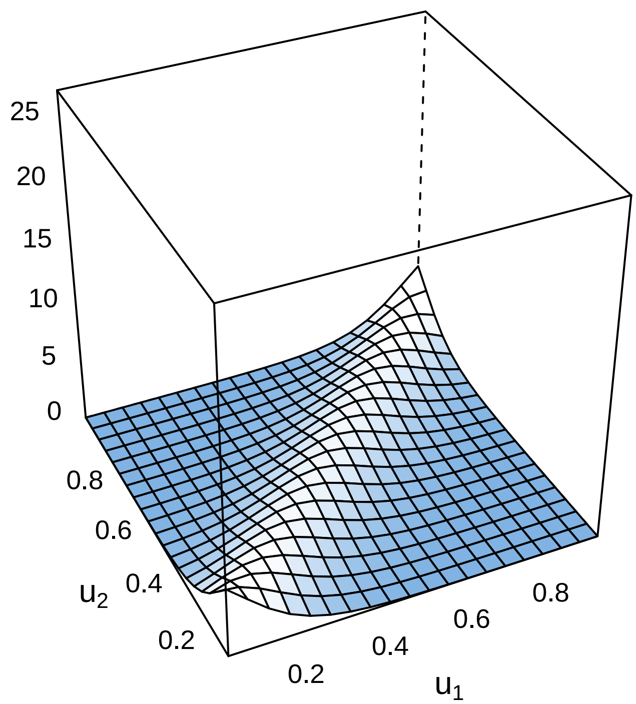
Gumbel copula density surface

```r
plot(obj.gumbel,zlim=c(0,27))
```



Frank copula density surface

```
plot(obj.frank,zlim=c(0,27))
```



Joe copula density

surface

```r
plot(obj.joe,zlim=c(0,27))
```

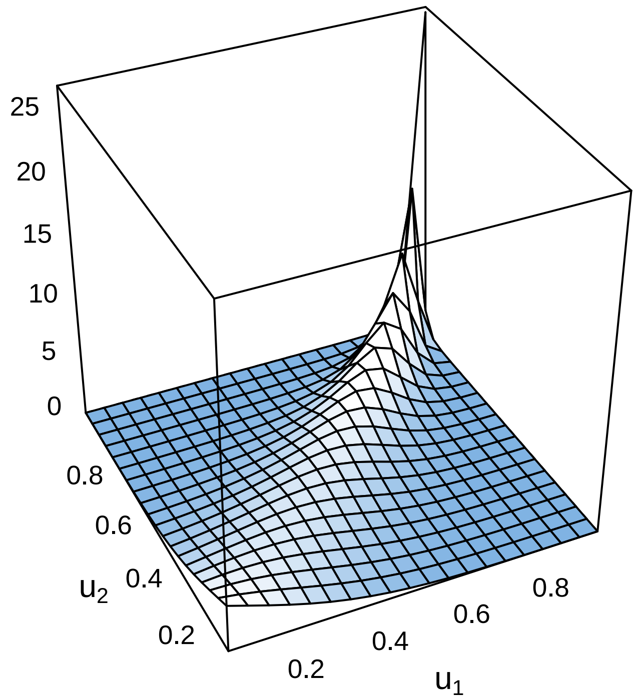


Figure 3.2: BB copula densities: top left: BB1 with $\delta = 20/12$ and $\theta = 2$, top right: BB1 with $\delta = 2$ and $\theta = 4/3$, bottom left: BB7 with $\theta = 2$ and $\delta = 4.5$, bottom right: BB7 with $\theta = 1.3$ and $\delta = 2$ (Copula parameters are chosen in such away that the corresponding Kendall's $\tau = .7$).

BB1 copula densities setup

```
tauv1.bb1<-BiCopPar2Tau(fam=7,par=2,par2=20/12)
print(tauv1.bb1,digits=2)
```

```
## [1] 0.7
```

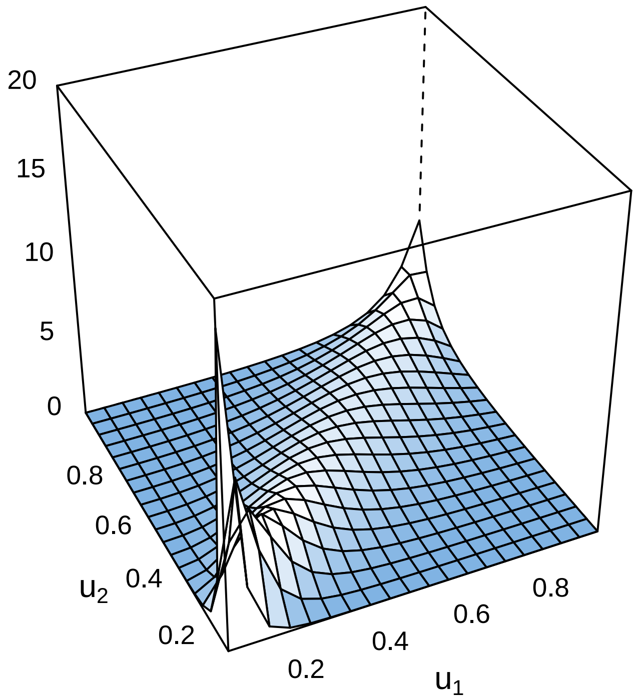
```
obj.bb1v1 <- BiCop(family = 7, par = 2,par2=20/12)

tauv2.bb1<-BiCopPar2Tau(fam=7,par=4/3,par2=2)
print(tauv2.bb1,digits=2)
```

```
## [1] 0.7
```

```
obj.bb1v2 <- BiCop(family = 7, par = 4/3,par2=2)
```

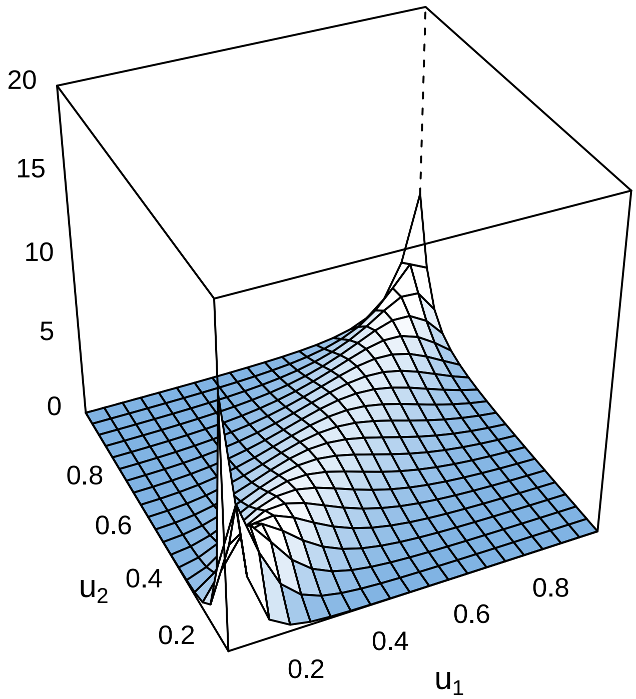
BB1 copula density surface ($\delta = 20/12$,$\theta = 2$,$\tau = .7$)

```
plot(obj.bb1v1,zlim=c(0,20))
```



BB1 copula density surface ($\delta = 2,\ \theta = 4/3$,$\tau = .7$)

```
plot(obj.bb1v2,zlim=c(0,20))
```



BB7 copula densities setup

```
tauv1.bb7<-BiCopPar2Tau(fam=9,par=2,par2=4.5)
print(tauv1.bb7,digits=2)
```

```
## [1] 0.7
```

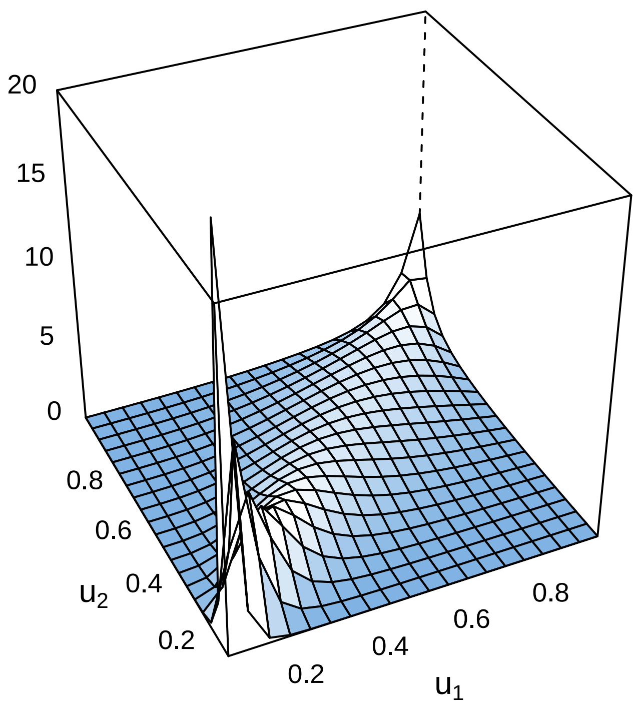
```
obj.bb7v1 <- BiCop(family = 9, par = 2,par2=4.5)

tauv2.bb7<-BiCopPar2Tau(fam=9,par=4.0,par2=2)
print(tauv2.bb7,digits=2)
```

```
## [1] 0.7
```

```
obj.bb7v2 <- BiCop(family = 9, par = 4.0,par2=2)
```

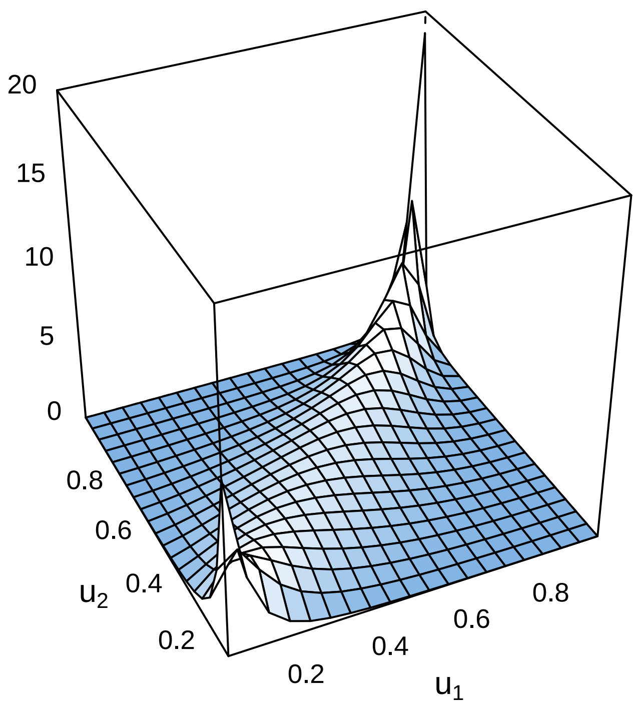
BB7 copula density surface ($\theta = 2$, $\delta = 4.5$,$\tau = .7$)

```
plot(obj.bb7v1,zlim=c(0,20))
```



BB7 copula density surface ($\theta = 4.0$, $\delta = 2$,$\tau = .7$)

```
plot(obj.bb7v2,zlim=c(0,20))
```



Section 3.5 Relationship between copula parameters and Kendalls's τ

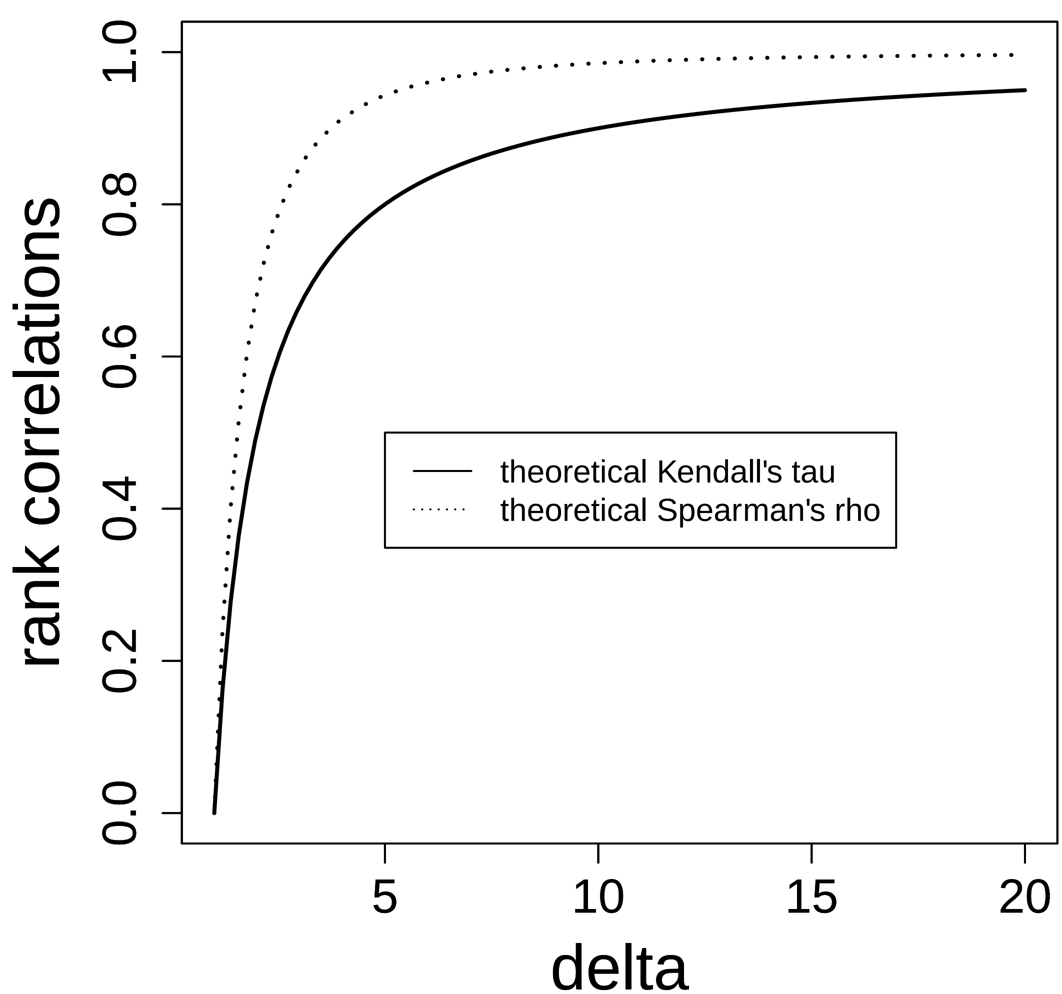
Figure 3.6: Relationship: Copula parameter and rank based dependence measures: Clayton copula (left), Gumbel copula (right).

Gumbel copula

```
delta.vec=seq(1,20,length=100)
tau.vec=delta.vec
rho.vec=delta.vec
for(i in 1:100){
  tau.vec[i]=tau(gumbelCopula(delta.vec[i]))
  rho.vec[i]=rho(gumbelCopula(delta.vec[i]))
}
```

```
## parameter at boundary ==> returning indepCopula()
## parameter at boundary ==> returning indepCopula()
```

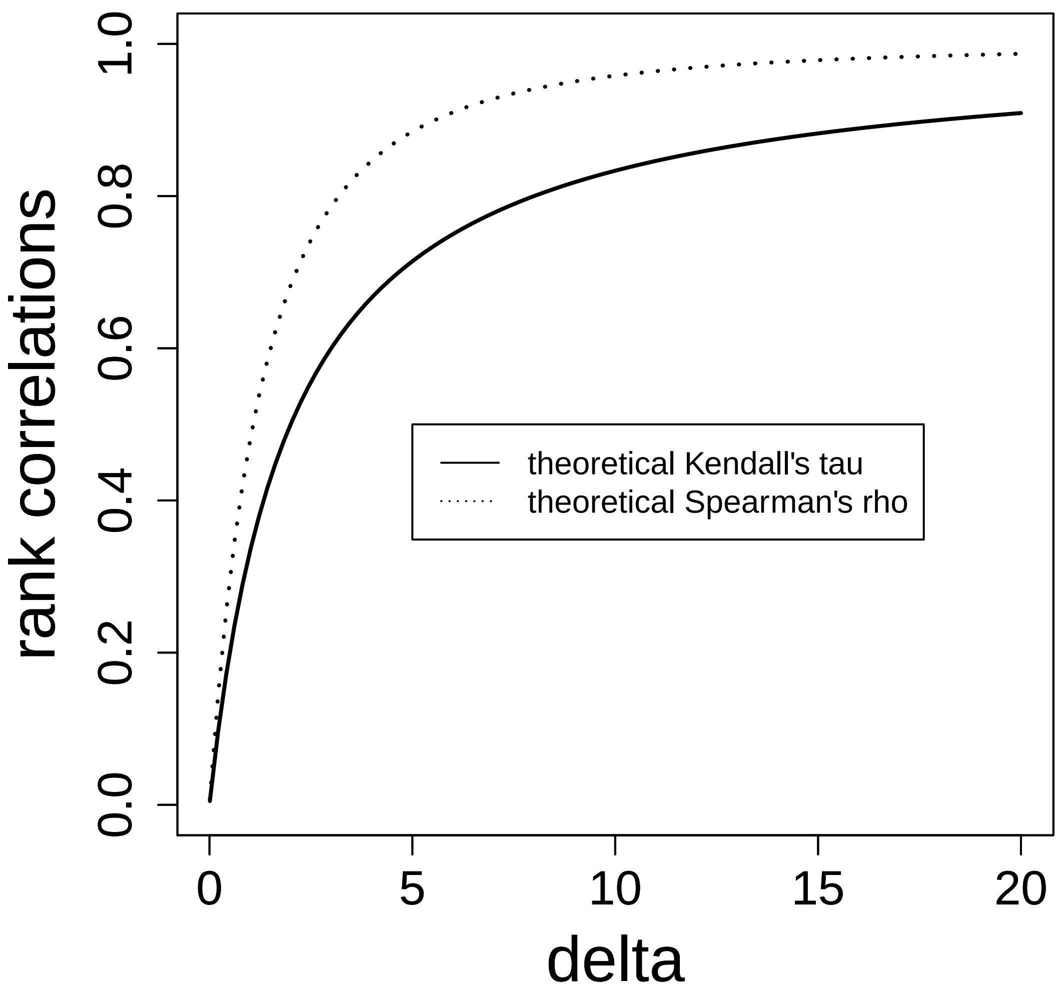
```r
bigpar(1,1)
plot(delta.vec,tau.vec,type="l",lty=1,lwd=2,
     xlab="delta",ylab="rank correlations",ylim=c(0,1))
par(new=T)
plot(delta.vec,rho.vec,type="l",lty=3,lwd=2,
     xlab="delta",ylab="rank correlations",ylim=c(0,1))
legend(5,.5,legend=c("theoretical Kendall's tau","theoretical Spearman's rho"),lty=c(1,3))
```



Clayton copula

```r
delta.vec=seq(0.01,20,length=100)
tau.vec=delta.vec
rho.vec=delta.vec
for(i in 1:100){
  tau.vec[i]=tau(claytonCopula(delta.vec[i]))
```

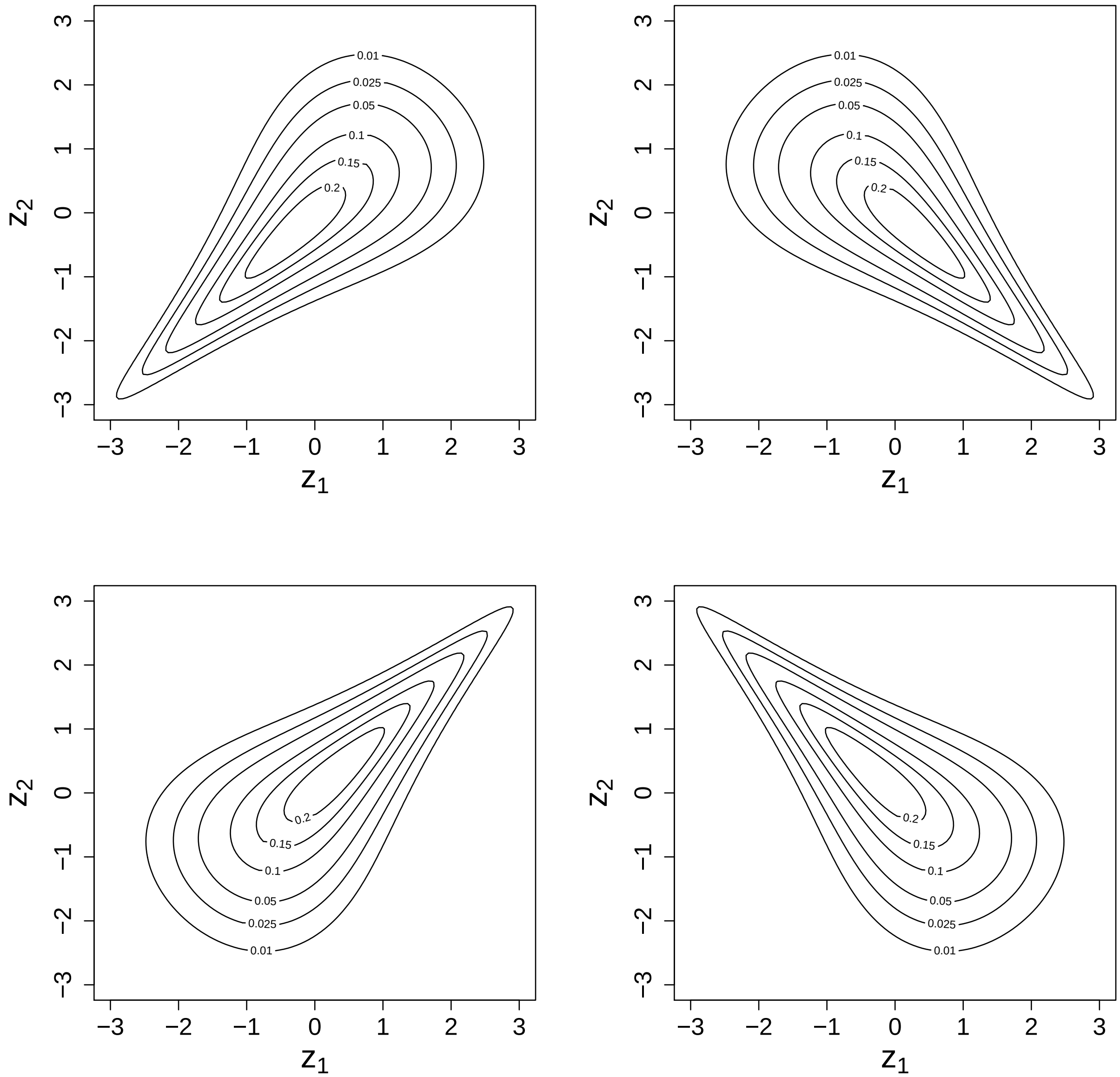
```
  rho.vec[i]=rho(claytonCopula(delta.vec[i]))
}
bigpar(1,1)
plot(delta.vec,tau.vec,type="l",lty=1,lwd=2,
     xlab="delta",ylab="rank correlations",ylim=c(0,1))
par(new=T)
plot(delta.vec,rho.vec,type="l",lty=3,lwd=2,
     xlab="delta",ylab="rank correlations",ylim=c(0,1))
legend(5,.5,legend=c("theoretical Kendall's tau","theoretical Spearman's rho"),lty=c(1,3))
```



Section 3.6: Rotated and reflected copulas

Figure 3.7: Rotations: Normalized contour plots of Clayton rotations: top left: 0 degree rotation ($\tau = .5$), top right: 90 degree rotation ($\tau = -.5$), bottom left: 180 degree rotation ($\tau = .5$), bottom right: 270 degree rotation ($\tau = -.5$).

```
par.clayton0<-BiCopTau2Par(3, .5, check.taus = TRUE)
par.clayton90<-BiCopTau2Par(23, -.5, check.taus = TRUE)
par.clayton180<-BiCopTau2Par(13, .5, check.taus = TRUE)
par.clayton270<-BiCopTau2Par(33, -.5, check.taus = TRUE)
obj.clayton0 <- BiCop(family = 3, par = par.clayton0)
obj.clayton90 <- BiCop(family = 23, par = par.clayton90)
obj.clayton180 <- BiCop(family =13, par = par.clayton180)
obj.clayton270 <- BiCop(family = 33, par = par.clayton270)
bigpar(2,2)
contour(obj.clayton0)
contour(obj.clayton90)
contour(obj.clayton180)
contour(obj.clayton270)
```


0.01
0.025
0.05
0.1
0.15
0.2
−3 −2 −1 0 1 2 3
z_1
z_2

Section 3.8: Exploratory visualization

Figure 3.8: Bivariate elliptical copulas: first column: Gauss with $\tau = .7$, second column: Gauss with $\tau = -.2$, third column: Student t???:2, third column: Student t with $\nu = 4, \tau = .7$, forth column: Student t with $\nu = 4; \tau = -.2$ (top row: normalized bivariate copula contours of $g(\cdot;\cdot)$ bottom row: pairs plot of a random sample $(u_{i1}; u_{i2})$ on the copula scale).

Setup of plot objects and simulated copula data

```
par.gaussh<-BiCopTau2Par(1, .7, check.taus = TRUE)
par.sth<-par.gaussh
par.gaussl<-BiCopTau2Par(1, -.2, check.taus = TRUE)
par.stl<-par.gaussl
par.claytonh<-BiCopTau2Par(3, .7, check.taus = TRUE)
par.gumbelh<-BiCopTau2Par(4, .7, check.taus = TRUE)
par.frankh<-BiCopTau2Par(5, .7, check.taus = TRUE)
par.joeh<-BiCopTau2Par(6, .7, check.taus = TRUE)
par.claytonl<-BiCopTau2Par(33, -.2, check.taus = TRUE)
par.gumbell<-BiCopTau2Par(34, -.2, check.taus = TRUE)
par.frankl<-BiCopTau2Par(5, -.2, check.taus = TRUE)
par.joel<-BiCopTau2Par(36, -.2, check.taus = TRUE)
obj.claytonh <- BiCop(family = 3, par = par.claytonh)
obj.gumbelh <- BiCop(family = 4, par = par.gumbelh)
obj.frankh <- BiCop(family = 5, par = par.frankh)
obj.joeh <- BiCop(family = 6, par = par.joeh)
obj.claytonl <- BiCop(family = 33, par = par.claytonl)
obj.gumbell <- BiCop(family = 34, par = par.gumbell)
obj.frankl <- BiCop(family = 5, par = par.frankl)
obj.joel <- BiCop(family = 36, par = par.joel)
obj.gaussh <- BiCop(family = 1, par = par.gaussh)
obj.gaussl <- BiCop(family = 1, par = par.gaussl)
obj.sth <- BiCop(family = 2, par = par.sth,par2=4)
obj.stl <- BiCop(family = 2, par = par.stl,par2=4)
sim.gaussh<-BiCopSim(500,obj.gaussh)
sim.gaussl<-BiCopSim(500,obj.gaussl)
sim.sth<-BiCopSim(500,obj.sth)
sim.stl<-BiCopSim(500,obj.stl)
sim.claytonh<-BiCopSim(500,obj.claytonh)
sim.gumbelh<-BiCopSim(500,obj.gumbelh)
sim.frankh<-BiCopSim(500,obj.frankh)
sim.joeh<-BiCopSim(500,obj.joeh)
sim.claytonl<-BiCopSim(500,obj.claytonl)
sim.gumbell<-BiCopSim(500,obj.gumbell)
sim.frankl<-BiCopSim(500,obj.frankl)
sim.joel<-BiCopSim(500,obj.joel)
```

Figure 3.8: Bivariate elliptical copulas: first column: Gauss with $\tau = .7$, second column: Gauss with $\tau = -.2$, third column: Student t???:2, third column: Student t with $\nu = 4, \tau = .7$, forth column: Student t with $\nu = 4; \tau = -.2$ (top row: normalized bivariate copula contours of $g(\cdot;\cdot)$ bottom row: pairs plot of a random sample $(u_{i1}; u_{i2})$ on the copula scale).

```
bigpar(2,4)
contour(obj.gaussh)
contour(obj.gaussl)
contour(obj.sth)
contour(obj.stl)
plot(sim.gaussh,xlab=expression("u"[1]),ylab=expression("u"[2]),xlim=c(0,1),ylim=c(0,1))
plot(sim.gaussl,xlab=expression("u"[1]),ylab=expression("u"[2]),xlim=c(0,1),ylim=c(0,1))
plot(sim.sth,xlab=expression("u"[1]),ylab=expression("u"[2]),xlim=c(0,1),ylim=c(0,1))
plot(sim.stl,xlab=expression("u"[1]),ylab=expression("u"[2]),xlim=c(0,1),ylim=c(0,1))
```

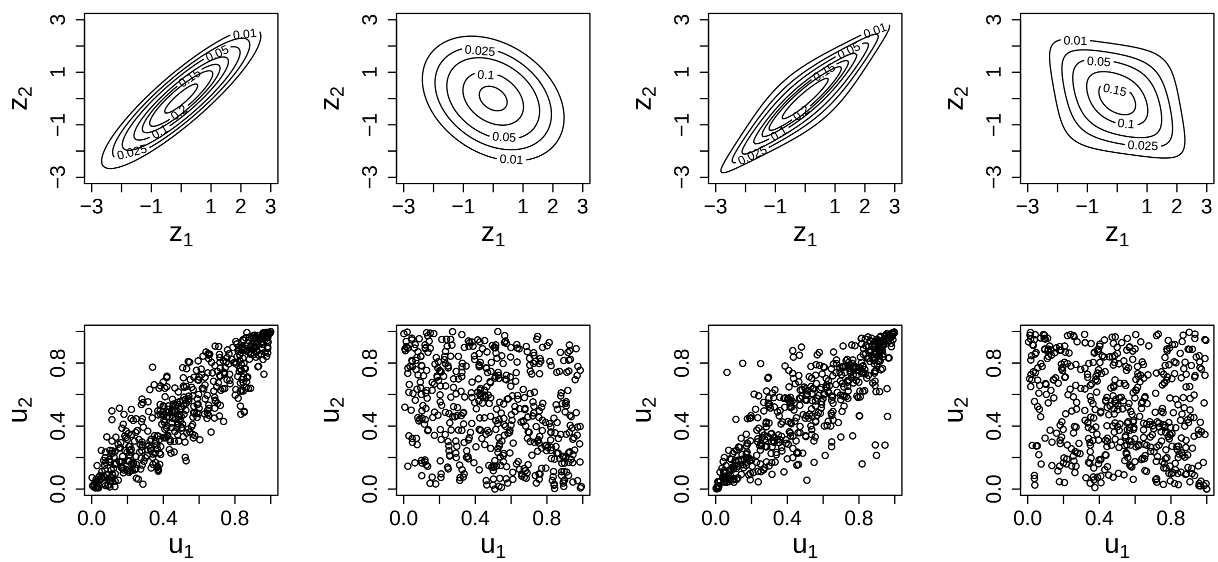
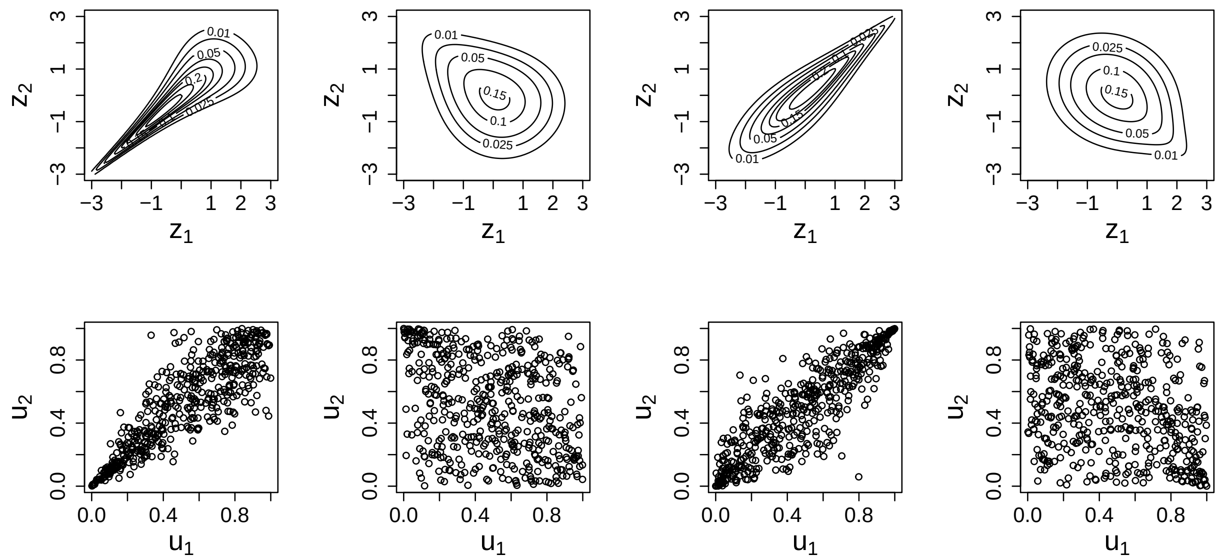


Figure 3.9: Bivariate Archimedean copulas: first column: Clayton with $\tau = .7$, second column: Clayton with $\tau = -.2$, third column: Gumbel with $\tau = .7$, forth column: Gumbel with $\tau = -.2$ (top row: normalized bivariate copula contours of $g(\cdot;\cdot)$ bottom row: pairs plot of a random sample $(u_{i1}; u_{i2})$ on the copula scale).

```
bigpar(2,4)
contour(obj.claytonh)
contour(obj.claytonl)
contour(obj.gumbelh)
contour(obj.gumbell)
plot(sim.claytonh,xlab=expression("u"[1]),ylab=expression("u"[2]),xlim=c(0,1),ylim=c(0,1))
plot(sim.claytonl,xlab=expression("u"[1]),ylab=expression("u"[2]),xlim=c(0,1),ylim=c(0,1))
plot(sim.gumbelh,xlab=expression("u"[1]),ylab=expression("u"[2]),xlim=c(0,1),ylim=c(0,1))
plot(sim.gumbell,xlab=expression("u"[1]),ylab=expression("u"[2]),xlim=c(0,1),ylim=c(0,1))
```



Section 3.9: Simulation of bivariate copula data

Figure 3.11: Bivariate elliptical copulas:first column: Gauss with $\tau = .7$, second column: Gauss with $\tau = -.2$, third column: Student t with $\nu = 4, \tau = .7$, forth column: Student t with $\nu = 4, \tau = -.2$(top row: normalized bivariate copula contours of $g(\cdot;\cdot)$ bottom row: empirical normalized bivariate copula contours based on a sample of size n = 500.

```
bigpar(2,4)
contour(obj.gaussh)
contour(obj.gaussl)
contour(obj.sth)
contour(obj.stl)
kde.fit.gaussh<-kdecop(sim.gaussh)
kde.fit.gaussl<-kdecop(sim.gaussl)
kde.fit.sth<-kdecop(sim.sth)
kde.fit.stl<-kdecop(sim.stl)
contour(kde.fit.gaussh)
contour(kde.fit.gaussl)
contour(kde.fit.sth)
contour(kde.fit.stl)
```

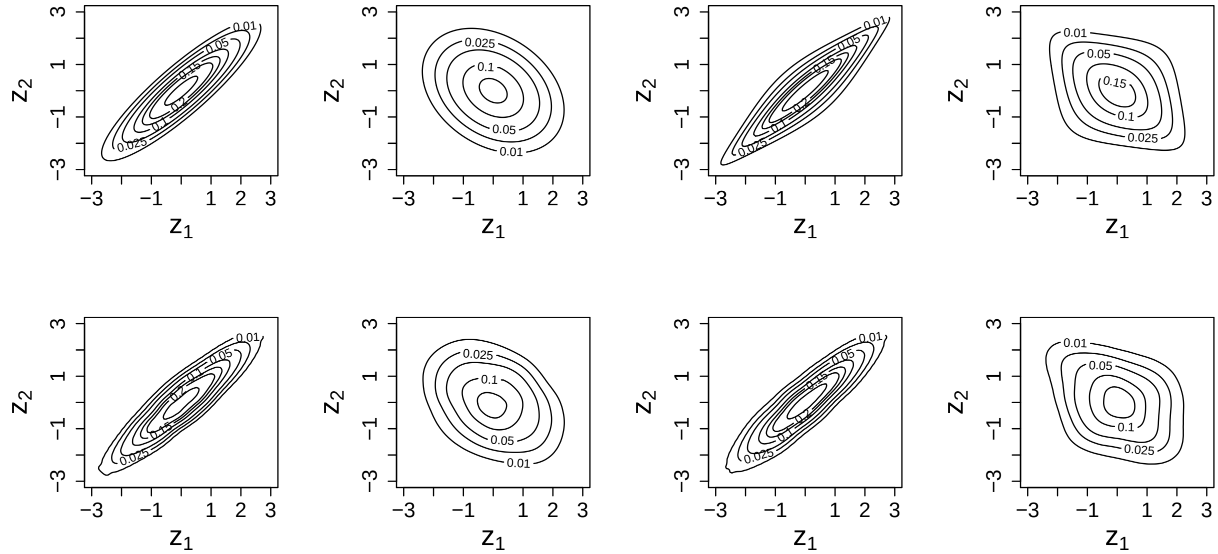
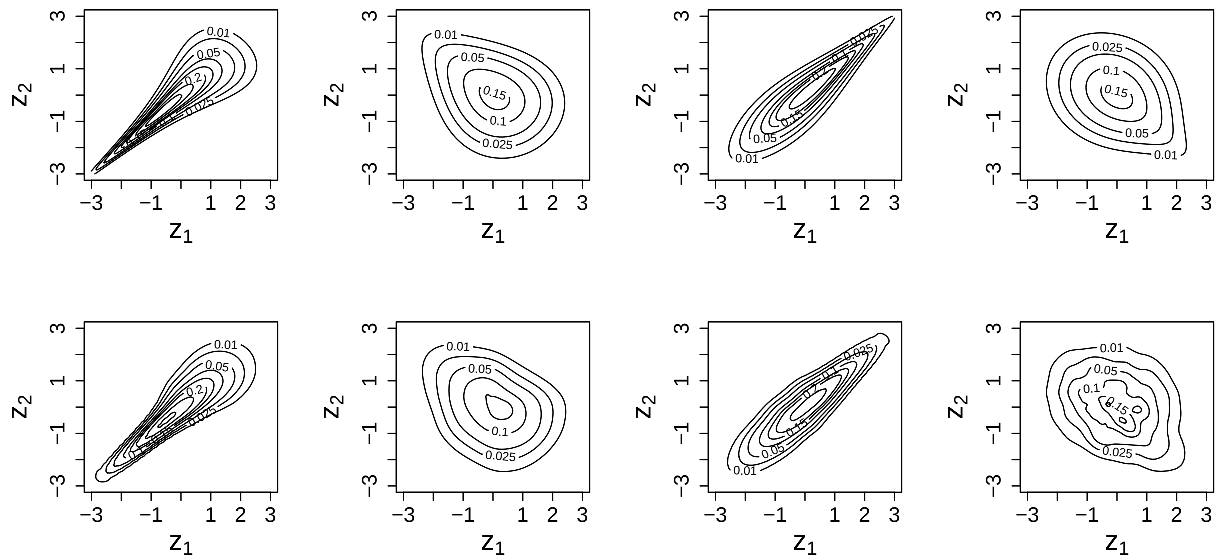


Figure 3.12: Bivariate Archimedean copulas: first column: Clayton with $\tau = .7$, second column: Clayton with $\tau = -.2$, third column: Gumbel with $\tau = .7$, forth column: Gumbel with $\tau = -.2$ (top row: normalized bivariate copula contours of $g(\cdot;\cdot)$ bottom row: empirical normalized bivariate copula contours based on a sample of size n = 500.

```
bigpar(2,4)
contour(obj.claytonh)
contour(obj.claytonl)
contour(obj.gumbelh)
contour(obj.gumbell)
kde.fit.claytonh<-kdecop(sim.claytonh)
kde.fit.claytonl<-kdecop(sim.claytonl)
kde.fit.gumbelh<-kdecop(sim.gumbelh)
kde.fit.gumbell<-kdecop(sim.gumbell)
contour(kde.fit.claytonh)
contour(kde.fit.claytonl)
contour(kde.fit.gumbelh)
contour(kde.fit.gumbell)
```



Section 3.10: Parameter estimation in bivariate copula models

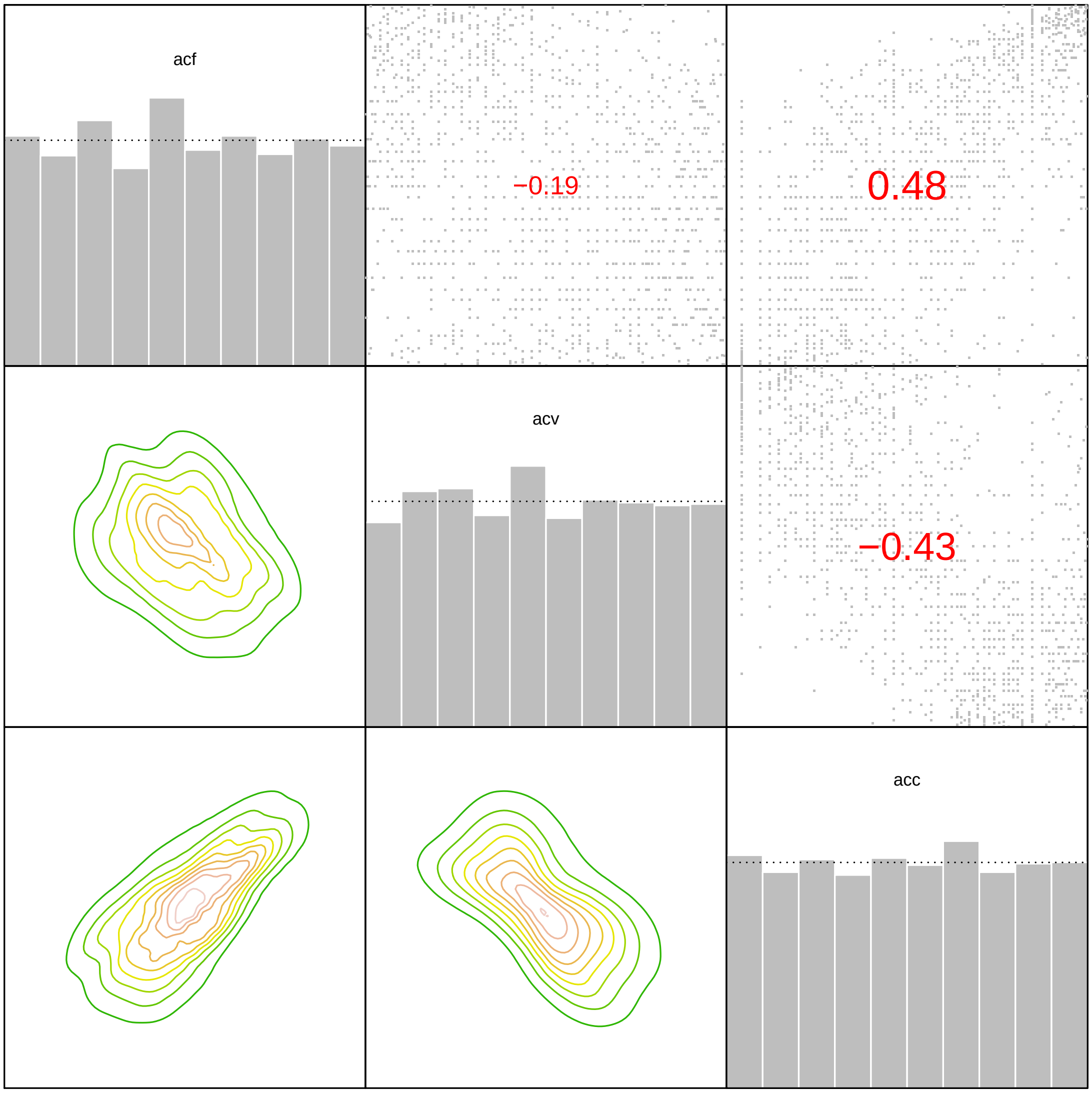
Example 3.4: WINE3: Empirical and fitted normalized contour plots

Read in data and set column names, create copula data

```
reddata<-read.csv(file="winequality-red.csv",sep=";")
n<-length(reddata[,1])
colnames(reddata)<-c("acf","acv","acc","sugar","clor","sf","st",
                     "den","ph","sp","alc","quality")
acf<-reddata[,1]
acv<-reddata[,2]
acc<-reddata[,3]
udata<-reddata
for(i in 1:12){
udata[,i]<-rank(reddata[,i])/(n+1)
}
```

Figure 3.13: WINE3: upper triangle: pairs plots of copula data, diagonal: Marginal histograms of copula data, lower triangle: empirical contour plots of normalized copula data.

```
udata<-as.copuladata(udata)
pairs(udata[,1:3])
```



From this we suggest to use for the bivariate copulas * rotated Gumbel (family=34) for the pair (acf,acv) * Gumbel (family=4) for the pair (acf,acc) * Frank (family=5) for the pair (acv,acc)

Table 3.4: WINE3: Estimated Kendall's τ , chosen copula family and through inversion estimated copula parameter for all pairs of variables

- acf,acv: 270 degree Gumbel (34)
- acf,acc: Gumbel (4)
- acv,acc: Frank (5)

```
options(digits=4)
theta.acfacv<-BiCopTau2Par(34,cor(acf,acv,method="kendall"))
theta.acfacc<-BiCopTau2Par(4,cor(acf,acc,method="kendall"))
```

```r
theta.acvacc<-BiCopTau2Par(5,cor(acv,acc,method="kendall"))
options(digits=2)
lab<-c("(acf,acv)","(acf,acc)","(acv,acc)")
fam<-c("Gumbel 270","Gumbel","Frank")
theta<-c(theta.acfacv,theta.acfacc,theta.acvacc)
corvec<-c(cor(acf,acv),
cor(acf,acc),
cor(acv,acc))
tauvec<-c(cor(acf,acv,method="kendall"),
cor(acf,acc,method="kendall"),
cor(acv,acc,method="kendall"))
table<-data.frame(lab,corvec,tauvec,fam,theta)
colnames(table)<-c("pair","cor","Kendall's tau","family","copula parameter")
print(table)
```

```
##        pair   cor Kendall's tau     family copula parameter
## 1 (acf,acv) -0.26         -0.19 Gumbel 270             -1.2
## 2 (acf,acc)  0.67          0.48     Gumbel              1.9
## 3 (acv,acc) -0.55         -0.43      Frank             -4.6
```

Figure 3.14: WINE3: Fitted normalized contour plots for the chosen bivariate copula family with parameter determined by the empirical Kendall's τ estimate (left: acf versus acv, middle: acf versus acc, right: acv versus acc).

```r
bigpar(1,3)
obj<-BiCop(family=34,par = theta.acfacv)
contour(obj, main=paste("Gumbel270: tau=",round(cor(acf,acv,method="kendall"),digits=2)))

obj<-BiCop(family=4,par = theta.acfacc)
contour(obj, main=paste("Gumbel:tau=",round(cor(acf,acc,method="kendall"),digits=2)))

obj<-BiCop(family=5,par=theta.acvacc)
contour(obj, main=paste("Frank:tau=",round(cor(acv,acc,method="kendall"),digits=2)))
```