Model selection of vine copulas with applications

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Why are vine copulas useful?

- Multivariate data has often complex dependency patterns, such as asymmetry and dependence in the extremes
- Cannot be captured by the multivariate normal distribution.
- The copula approach allows for these dependency patterns
- Current classes of multivariate copulas such as Gaussian, Student t and Archimedean copulas are too restrictive
- They require often exchangeability and that the distribution of pairs are of same kind
- Vine copulas allow for flexible modeling of (conditional) pairs

Overview

What are copulas and how it all started ...

Consider d random variables $\mathbf{X} = (X_1, ..., X_d)$ with

	density function	distribution function
marginal	$f_i(x_i),\ i = 1, ..., d$	$F_i(x_i),\ i = 1, ..., d$
joint	$f(x_1, ..., x_d)$	$F(x_1, ..., x_d)$
conditional	$f_{i\|j}(x_i\|x_j),\ i \neq j$	$F_{i\|j}(x_i\|x_j),\ i \neq j$

Copula

A d-dimensional copula C is a multivariate distribution on $[0,1]^d$ with uniformly distributed marginals.

Copula density function: $c(u_1, ..., u_d) := \frac{\partial^d}{\partial u_1 ... \partial u_d} C(u_1, ..., u_d)$

Sklar's Theorem

Theorem (Sklar 1959)

$$F(x_1, ..., x_d) = C(F_1(x_1), ..., F_d(x_d))$$
$$f(x_1, ..., x_d) = c(F_1(x_1), ..., F_d(x_d))f_1(x_1)...f_d(x_d)$$

for some d-dimensional copula C.

$d = 2:$

$$f(x_1, x_2) = c_{12}(F_1(x_1), F_2(x_2))f_1(x_1)f_2(x_2)$$
$$f_{2|1}(x_2|x_1) = c_{12}(F_1(x_1), F_2(x_2))f_2(x_2)$$

What are these vine copulas?

- Multivariate vine copulas are copulas built out of bivariate copulas.
- A pair copula construction (PCC) is possible through conditioning. Joe (1996) gave a first example.
- Many PCC's are feasible. Bedford and Cooke (2002) introduced a graphical structure to organize them.
- Gaussian vines were analyzed in Kurowicka and Cooke (2006) while ML estimation for Non Gaussian ones started with Aas et al. (2009).
- See also `vine-copula.org`

How does this work in 3 dimensions?

$$f(x_1, x_2, x_3) = f_{3|12}(x_3|x_1, x_2) f_{2|1}(x_2|x_1) f_1(x_1)$$

Using Sklar for $f(x_1, x_2), f(x_2, x_3)$ and $f_{13|2}(x_1, x_3|x_2)$ implies

$$\begin{aligned} f_{2|1}(x_2|x_1) &= c_{12}(F_1(x_1), F_2(x_2)) f_2(x_2) \\ f_{3|12}(x_3|x_1, x_2) &= c_{13;2}(F_{1|2}(x_1|x_2), F_{3|2}(x_3|x_2)) f_{3|2}(x_3|x_2) \\ &= c_{13;2}(F_{1|2}(x_1|x_2), F_{3|2}(x_3|x_2)) c_{23}(F_2(x_2), F_3(x_3)) f_3(x_3) \end{aligned}$$

$$\begin{aligned} f(x_1, x_2, x_3) &= c_{13;2}(F_{1|2}(x_1|x_2), F_{3|2}(x_3|x_2)) c_{23}(F_2(x_2), F_3(x_3)) \\ &\times c_{12}(F_1(x_1), F_2(x_2)) \\ &\times f_3(x_3) f_2(x_2) f_1(x_1) \end{aligned}$$

The copula corresponding to the distribution of (X_1, X_3) given $X_2 = x_2$ is denoted by $c_{13;2}$. Only bivariate copulas and univariate conditional cdf's are used. This can be easily generalized to d dimensions.

What bivariate copula families are available?

Elliptical: Construction by inversion

$$C(u_1, u_2) = F(F_1^{-1}(u_1), F_2^{-1}(u_2)), \quad u_1, u_2 \in (0, 1),$$

where F is elliptical.

Examples: Gaussian, Student's t

Archemedean: Construction through generator φ

$$C(u_1, u_2) = \varphi^{-1}(\varphi(u_1) + \varphi(u_2)), \quad u_1, u_2 \in (0, 1).$$

(McNeil and Nešlehová 2009)

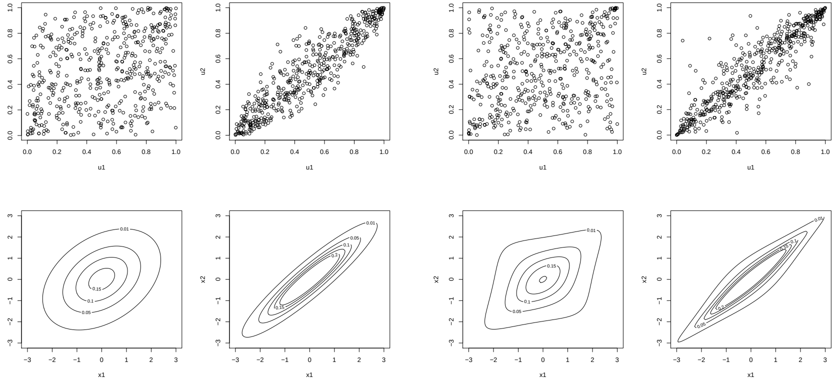
Examples: Clayton, Gumbel, Frank, Joe ...

Books: Joe (1997) and Nelsen (2006)
Extensions: Rotations by 90, 180 (survival) and 270 degree

Bivariate elliptical copula families

Gaussian copula
(left $\tau = .25$, right: $\tau = .75$)
symmetric dependence

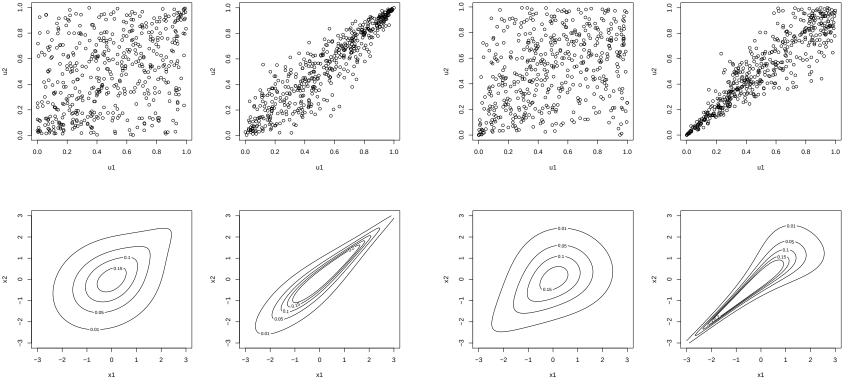
t-copula with $df = 3$
(left $\tau = .25$, right: $\tau = .75$)
symmetric dependence



Bivariate Archimedean copula families

Gumbel copula
(left $\tau = .25$, right: $\tau = .75$)
upper tail dependent

Clayton copula
(left $\tau = .25$, right: $\tau = .75$)
lower tail dependent



How do vines work in higher dimensions?

- **Which pairs of variables are needed?**
- **What are the conditioning variables?**

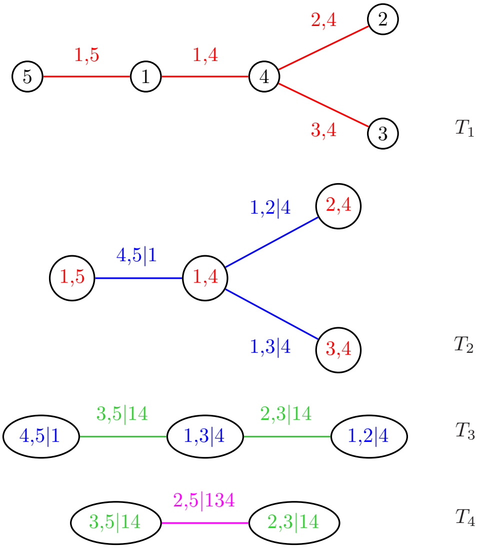
Components of a regular vine $R(\mathcal{V}, \mathcal{C}, \boldsymbol{\theta})$ distribution

1. Tree structure $\mathcal{V}$ of linked trees identifies the pairs of variables and conditioning variables.
2. Parametric bivariate copulas $\mathcal{C} = \mathcal{C}(\mathcal{V})$ for each edge in the tree structure
3. Corresponding parameter value $\boldsymbol{\theta} = \boldsymbol{\theta}(\mathcal{C}(\mathcal{V}))$

- Joe (1996) showed that conditional distribution functions can be computed recursively. For $\mathbf{v} = (v_j, \mathbf{v}_{-j})$ we have

$$F(x|\mathbf{v}) = \frac{\partial C_{xv_j;\mathbf{v}_{-j}}(F(x|\mathbf{v}_{-j}), F(v_j|\mathbf{v}_{-j}))}{\partial F(v_j|\mathbf{v}_{-j})}.$$

Can we see an example of a tree structure?



Density

$$f = f_1 \cdot f_2 \cdot f_3 \cdot f_4 \cdot f_5$$

$\cdot\ c_{14} \cdot c_{15} \cdot c_{24} \cdot c_{34}$

$\cdot\ c_{12;4} \cdot c_{13;4} \cdot c_{45;1}$

$\cdot\ c_{23;14} \cdot c_{35;14}$

$\cdot\ c_{25;134}$

How is a regular vine tree structure defined?

An d-dimensional vine tree structure $\mathcal{V} = \{T_1, \ldots, T_{d-1}\}$ is a sequence of $d-1$ linked trees with

Vine tree structure (Bedford and Cooke (2002))

- Tree T_1 is a tree on nodes 1 to d.
- Tree T_j has $d+1-j$ nodes and $d-j$ edges.
- Edges in tree T_j become nodes in tree T_{j+1}.
- **Proximity condition:** Two nodes in tree T_{j+1} can be joined by an edge only if the corresponding edges in tree T_j share a node.

Are there special cases?

- D-vines use only path like trees
- canonical (C)-vines use only star like tree

How do these C and D-vines look like?

C-vine: each tree has a unique node connected to $d - j$ edges

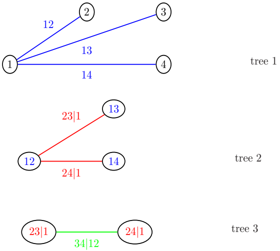
$$f_{1234} = \left[\prod_{i=1}^{4} f_i\right] \cdot c_{12} \cdot c_{13} \cdot c_{14} \cdot c_{23;1} \cdot c_{24;1} \cdot c_{34;12}$$

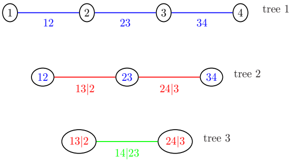
useful for ordering by importance

D-vine: no node is connected to more than 2 edges

$$f_{1234} = \left[\prod_{i=1}^{4} f_i\right] \cdot c_{12} \cdot c_{23} \cdot c_{34} \cdot c_{13;2} \cdot c_{24;3} \cdot c_{14;23}$$

useful for temporal ordering of variables





General density expressions

C-vine (Aas et al. 2009)

$$f(x_1, \ldots x_d) = \left[\prod_{k=1}^{d} f(x_k)\right] \times \left[\prod_{j=1}^{d-1} \prod_{i=1}^{d-j} c_{j,j+i;1,\ldots,j-1}\right]$$

D-vine (Aas et al. 2009)

$$f(x_1, \ldots x_d) = \left[\prod_{k=1}^{d} f(x_k)\right] \times \left[\prod_{j=1}^{d-1} \prod_{i=1}^{d-j} c_{i,i+j;i+1,\ldots,i+j-1}\right]$$

Regular vine (Dißmann et al. 2013)

$$f(x_1, \ldots, x_d) = \left[\prod_{k=1}^{d} f_k(x_k)\right] \times \left[\prod_{j=d-1}^{1} \prod_{i=d}^{j+1} c_{m_{j,j}, m_{i,j}; m_{i+1,j}, \ldots, m_{n,j}}\right]$$

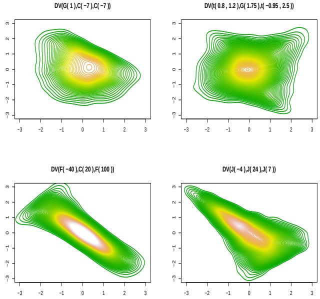
Here, $m_{i,j}$ refers to element (i, j) in the matrix representation of the R-vine.

What is the scope of the vine models?

Vine copula classes
(Stöber et al. (2013))

- multivariate Gaussian copula (pair copulas are Gauss and parameters are partial correlations)
- multivariate t copula (pair copulas are t and df increases by ℓ for trees $\ell \geq 2$)
- multivariate Clayton copula (Takahasi (1965))

Contours of bivariate (1,3) margins with standard normal margins



(C=Clayton, G=Gumbel, t=Student, F=Frank, J=Joe)

Number of R-vine tree structures and copulas

Dimension n	#R-vine tree structure [1]	#R-vine copulas [2]
2	1	7
3	3	1,029
4	24	2,823,576
5	480	1.3559e+11
6	23,040	1.0938e+17
7	2,580,480	1.4413e+24
8	660,602,880	3.0387e+32
9	3.8051e+11	1.0090e+42
10	4.8705e+14	5.2118e+52

Efficient estimation and model selection are crucial

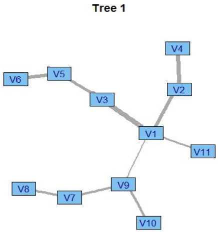
[1] see Morales-Nápoles et al. (2010) for details.

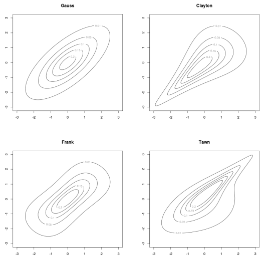
[2] This assumes 7 candidate pair copula families.

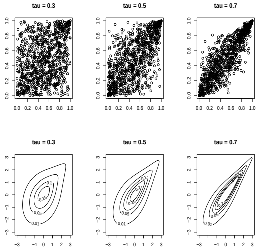
How can we estimate and select PCCs?

Three problems: (Czado et al. (2013))

1. How to estimate the pair copula parameters for a given vine tree structure and pair copula families for each edge?
2. How to choose the pair copula families and estimate the corresponding parameters for a given vine tree structure?
3. How to select and estimate all components of a regular vine?







Problem 1: Parameter estimation for given tree structure and copula families

- **Sequential estimation:**
  - Parameters are sequentially estimated starting from top tree until last (Aas et al. (2009), Czado et al. (2012)).
  - Asymptotic theory developed by Hobæk Haff (2013), Hobæk Haff (2012), however standard error estimates can only be bootstrapped.
  - starting values for maximum likelihood.
- **Maximum likelihood estimation:**
  - Asymptotically efficient and standard errors have been directly estimated in Stoeber and Schepsmeier (2013)
  - Uncertainty in value-at-risk (high quantiles) is difficult to assess.
- **Bayesian estimation:**
  - Posterior is tractable using Markov Chain Monte Carlo (Min and Czado (2011) for D-vines and Gruber (2011) for R-vines)
  - Prior beliefs can be incorporated and credible intervals allow to assess uncertainty.

How does sequential and ML estimation work?

Parameters: $\Theta = (\theta_{12}, \theta_{23}, \theta_{13;2})$

Observations: $\{(x_{1t}, x_{2t}, x_{3t}), t = 1, \cdots, T\}$

Sequential estimates:

- Estimate θ_{12} from $\{(x_{1,t}, x_{2,t}), t = 1, \cdots, T\}$
- Estimate θ_{23} from $\{(x_{2,t}, x_{3,t}), t = 1, \cdots, T\}$.
- Define pseudo observations

$$\hat{v}_{1|2t} := F(x_{1t}|x_{2t}, \hat{\theta}_{12}) \text{ and } \hat{v}_{3|2t} := F(x_{3t}|x_{2t}, \hat{\theta}_{23})$$

Finally estimate $\theta_{13;2}$ from $\{(\hat{v}_{1|2t}, \hat{v}_{3|2t}), t = 1, \cdots, T\}$.

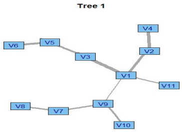
Maximum likelihood

$$\begin{aligned} L(\Theta|x) &= \sum_{t=1}^{T} [\log c_{12}(x_{1t}, x_{2t}|\theta_{12}) + \log c_{23}(x_{2t}, x_{3t}|\theta_{23}) \\ &+ \log c_{13;2}(F(x_{1t}|x_{2t}, \theta_{12}), F(x_{3t}|x_{2t}, \theta_{23})|\theta_{13;2})] \end{aligned}$$

Problem 2: Joint estimation of pair copula families and parameters

- **Classical approach:**
  - Restrict to a set of bivariate pair copula families and use AIC or Vuong test to select family
  - Check for truncation possibilities (Brechmann et al. (2012)) by using independence copulas in higher trees
- **Bayesian approach:**
  - Reversible jump (RJ) MCMC (Min and Czado (2011))
  - MCMC with model indicators (Smith et al. (2010)) choosing between an independence copula and a fixed copula family.

Only one more problem to go ...



- **sequential treewise approach** (see Dißmann et al. (2013))
- **Bayesian sequential and joint approaches** (see Gruber and Czado (2012), Gruber and Czado (2013))

Problem 3: How does this treewise selection work?

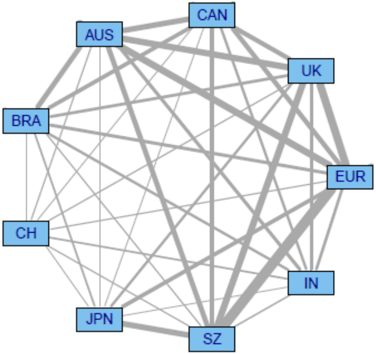
Idea: Capture strong pairwise dependencies first

For Tree $\ell = 1, \ldots, d-1$

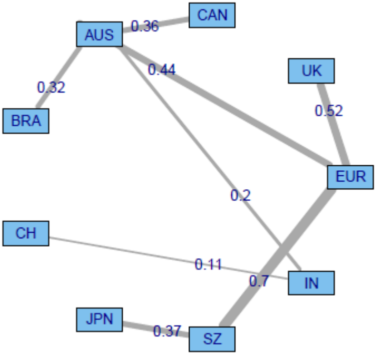
1. Calculate an empirical dependence measure $\hat{\delta}_{jk|D}$ for all variable pairs $\{jk|D\}$ ($\rightarrow$ **edge weights**: Kendall's τ, tail dependence coefficients) allowed by proximity (D is empty for Tree 1).
2. Select the tree on all nodes that maximizes the sum of absolute empirical dependencies ($\rightarrow$ **maximum spanning tree**). Choose independence copula if possible.
3. For each selected edge $\{j,k\}$ ($\{j,k\}|D$) in Tree 1 (in Tree $\ell > 1$), select copula family and estimate the corresponding parameter(s).
4. Transform to pseudo observations: $F_{j|k \cup D}(u_{ij}|\mathbf{u}_{i,k\cup D}, \hat{\theta}_{j,k;D})$ and $F_{k|j\cup D}(u_{ik}|\mathbf{u}_{i,j\cup D}, \hat{\theta}_{j,k;D})$, $i = 1, \ldots, n$.

What does this look like for Tree 1?

(1) Pairwise dependencies.



(2) Maximum dependence tree.



Czado, Jeske, and Hofmann (2013) compare sequential selection strategies

Problem 3: Three approaches to full R-vine selection

- $d_{\mathcal{V}}$ number of parameters in R-vine $(\mathcal{V}, \mathcal{B}(\mathcal{V}), \boldsymbol{\theta}(\mathcal{B}(\mathcal{V})))$
- T_ℓ tree ℓ of vine tree structure $\mathcal{V}$
- $\mathcal{B}_\ell$ set of pair copula families for all edges in T_ℓ
- d_ℓ number of parameters in Tree T_ℓ

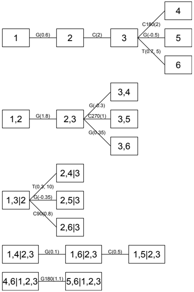
	Dissmann et. al 2013	Gruber/Czado 2012	Gruber/Czado 2013
Appr.	Frequentist level-by-level	Bayesian level-by-level	Bayesian all levels jointly
Priors		$T_\ell \sim Unif(\cdot)$ $\mathcal{B}_\ell \mid T_\ell \sim \exp(-\lambda d_\ell)$ $\boldsymbol{\theta}_\ell \mid T_\ell, \mathcal{B}_\ell \sim Unif(\cdot)$	$\mathcal{V} \sim Unif(\cdot)$ $\mathcal{B}_{\mathcal{V}} \mid \mathcal{V} \sim \exp(-\lambda d_{\mathcal{V}})$ $\boldsymbol{\theta}_{\mathcal{V}} \mid \mathcal{V}, \mathcal{B}_{\mathcal{V}} \sim Unif(\cdot)$
Method	Select MST with weights	RJ MCMC	RJ MCMC

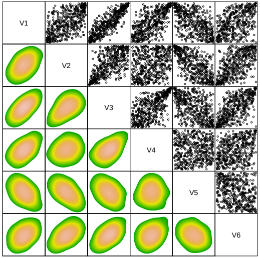
Proposal strategies (Gruber and Czado 2012; Gruber and Czado 2013)

- Use a mixture of two mutually exclusive, collectively exhaustive algorithms for the between models move:
  - FAM only updates the pair copula families;
  - TREE updates the tree structure and the pair copula families and guarantees that the current tree is not proposed.
- Draw proposal trees from a uniform distribution over all trees allowed by the proximity condition (only TREE).
- Compute the MLEs of the parameters for all candidate pair copula families.
- Draw the proposal pair copulas from a discrete distribution with weights proportional to the copulas' maximum likelihoods.
- Draw the proposal parameters from a mixture of truncated normal distributions with varying variances, centered at the MLE.

Simulation study for R-vine copula selection

- 4 R-vine copula models in 6 dimensions were chosen.
- Model 1 and 2 have the same R-vine tree structure and pair copula families but different parameter values, stronger dependencies in Model 2
- Model 3 is a C-vine and Model 4 is Gauss copula
- **Model 1:**





Average percentage of true likelihood recovered

Procedure	Model 1	Model 2	Model 3	Model 4	Mean
Gruber/Czado 2013	**98.4**	**98.6**	**96.5**	**99.2**	**98.2**
Gruber/Czado 2012	88.6	80.9	84.9	99.9	88.6
Dißmann et.al. 2013	86.6	75.1	78.9	99.7	85.1

- Results are from Gruber and Czado (2013) based on 10 data sets of size 500
- The joint Bayesian procedure performs best
- Model 4 is the multivariate Gaussian copula, which can be expressed as any Gaussian vine
- Because the selection of the regular vine tree structure does not matter for Model 4, all model selection procedures perform uniformly well.

How does copula based estimation work?

- **Original scale**: $\mathbf{x}_i = (x_{i1}, \ldots, x_{id}) \in \mathbb{R}^d$ i.i.d. sample
- **Copula scale**:
  - Known margins: $\mathbf{u}_i := (F_1(x_{i1}), \ldots, F_d(x_{id})) \in [0,1]^d$ (Probability integral transform)
  - Unknown margins: Estimate margins F_j either parametrically or non-parametrically and then transform (two step procedure)
- **Marginal structures**: If each margin has time series or regression structure, then a copula model will be applied to the fitted standardized residuals.

Application: Euro Stoxx 50:

- 50 large Eurozone companies.
- Major market indicator for the Eurozone.
- Brechmann and Czado (2013) consider 46 members from 5 countries (Germany, France, Italy, Spain and the Netherlands) together with their national indices.
- Daily log returns: May 2006 to April 2010 (985 obs.)

Questions

- **How do stock returns depend on the European and the national indices? Is dependence on the national index dominant?**
- **Which dependencies are most important? Are they asymmetric and/or heavy-tailed?**

Copula based models for Euro Stoxx 50 returns:

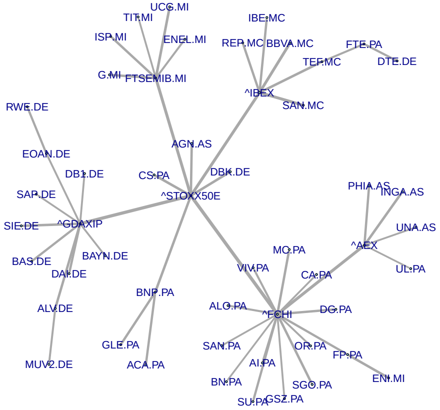
- Fit appropriate (ARMA-)GARCH models for each return time series.
- Fit copula model such as R- and C-vine copulas as well as multivariate Student-t copula for comparison to copula data based on standardized residuals

Results

Copula	Log likelihood	No. of param.	BIC
R-vine	30879.60	596	-57651.19
C-vine	30839.68	685	-56957.90
Student-t	30691.36	1327	-52236.18

R-vine $>$ C-vine $>$ Student-t

First tree of R-vine and C-vine order



Order	Root nodes
1^{st}	^STOXX50E
2^{nd}	GLE.PA
3^{rd}	^FCHI
4^{th}	^GDAXIP
5^{th}	^IBEX
6^{th}	INGA.AS
7^{th}	FTSEMIB.MI
⋮	⋮
13^{th}	^AEX
⋮	⋮

Application: Stock and volatility indices (Beil 2013)

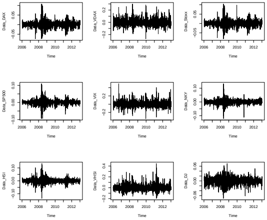
- DAX: German stock index
- VDAX: volatility index to the DAX
- STOXX: Dow Jones Euro Stoxx 50
- VSTOXX: volatility index to the STOXX
- SP500: Standard and Poor's 500 index
- VIX: volatility index to SP500
- DJ: Dow Jones UBS commodity index ex-agriculture and live stock
- NKY: Nikkei -225 stock average
- VNKY: volatility index to NKY
- HSI: Hong Kong Hang Seng index
- VHSI: volatility index to HSI

Daily values from June 2006 until June 2013 considered (1786 obs)

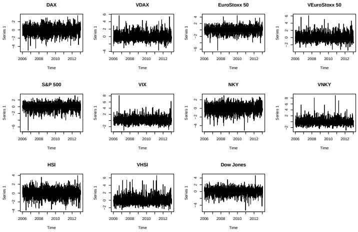
Marginal models

- Value of volatility index is the implied volatility of a 30 day option on its underlying asset by the Black Scholes model
- ARMA-(e)GARCH models with generalized hyperbolic innovation distribution are to each time series

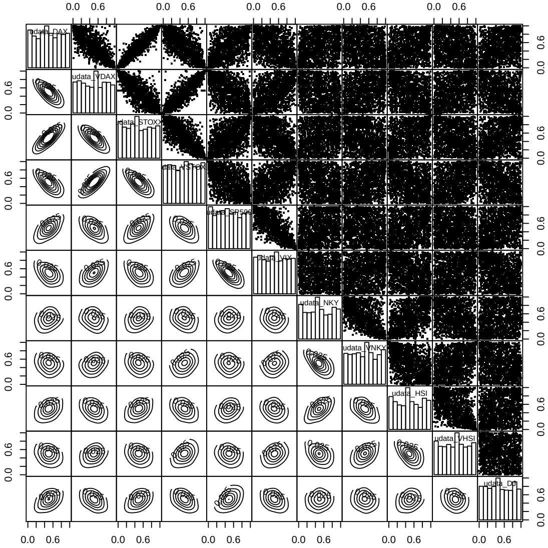
original time series



residual time series



Copula data and normalized contour plots



Using AIC/BIC to compare models not reduced by independence tests

	loglik	par	AIC	BIC
R-vine	8663.53	71	-17185.05	-16795.42
D-vine	8652.68	70	-17165.35	-16781.21
C-vine	8541.96	73	-16937.91	-16537.31
Gauss-copula	8033.01	55	-15956.03	-15654.20
T-copula	8308.15	56	-15755.92	-15448.60
T-vine	8525.14	80	-16890.28	-16451.26

If $df > 30$ for pair in T-vine then Gauss copula is used

- R-vines are selected using the Dissmann algorithm with Gauss, T, Gumbel, Clayton, Frank, Joe pair copulas and rotations are allowed
- For T-vine only pair t-copulas are allowed
- R-vine performs best compared to Gauss and T-copula

Using AIC/BIC to compare models reduced by independence tests comparison

More parsimonious models can be achieved by independence tests using asymptotic theory of $\hat{\tau}$.

	loglik	par	AIC	BIC
R-vine-ind	8606.27	51	-17110.54	-16830.67
D-vine-ind	8692.77	71	-17235.54	-16823.96
C-vine-ind	8524.54	58	-16933.07	-16614.78
Gauss-vine-ind	8013.50	41	-15945.00	-15720.01
T-vine-ind	8454.55	68	-16773.10	-16399.93
T-vine-fixed-ind	8470.37	59	-16822.74	-16498.96

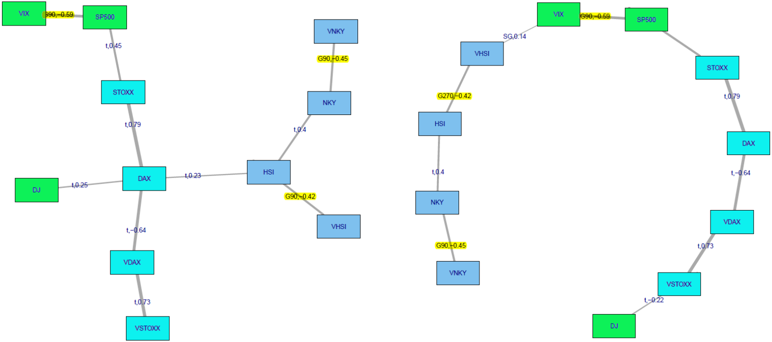
- T-vine-fixed-ind has the same tree structure as R-vine-ind but only T-pair copulas are allowed
- T-vine-ind might not have the same tree structure as R-vine-ind and only T- pair copulas are allowed

Model comparison using the Vuong test

statistic	p.value	decision	alpha
0.16	0.87	R-vine equivalent D-vine	0.10
4.54	< 0.01	R-vine better C-vine	0.10
4.05	< 0.01	D-vine better C-vine	0.10
7.79	< 0.01	R-vine better T-vine	0.10
7.50	< 0.01	R-vine better T-vine-fixed	0.10

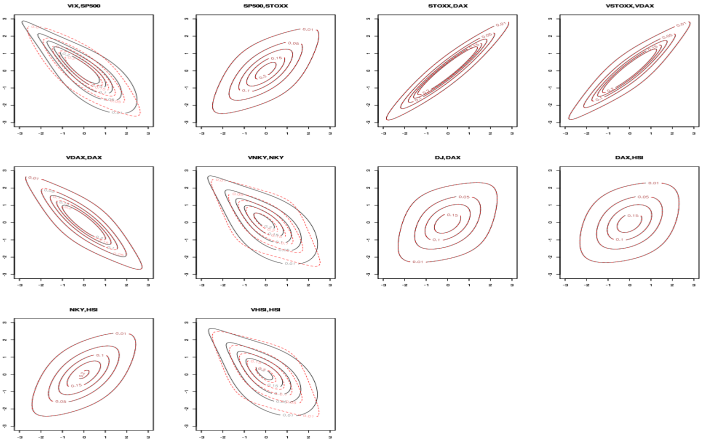
- Test of Vuong (1989) is suited for comparing non nested models
- BIC correction is used to obtain parsimony
- R-vine and T-vine-fixed have same tree structure, while this is not true for R-vine and T-vine
- Everywhere models are reduced by independence tests

First tree of R-vine and D-vine model



- tree structures are quite different
- asymmetry between volatility and asset indices
- geographic clusters

Unconditional fitted contours of R-vine and T-vine with same R-vine structure



T-vine versus R-vine: Different fit especially for dependencies between VIX-SP500, VNKY-NKY and VHSI-HSI

Recent advances for vines

- Simplified and non simplified vines: Acar et al. (2012)
- Time varying/regime switching regular vines: Almeida et al. (2012), Stöber and Czado (2013)
- Discrete and discrete/continuous vines: Panagiotelis et al. (2012), Stöber (2013)
- Non Gaussian DAG's using pair copula constructions: Hanea et al. (2006), Bauer and Czado (2012)
- Vines with non parametric pair copulas: Kauermann and Schellhase (2013), Lopez-Paz et al. (2013), Hobaek Haff and Segers (2012)
- Acceleration of MCMC algorithms: Schmidl et al. (2013)

Selected Applications

- **Financial risk management:**
  - Systemic risk simulation: Brechmann, Hendrich, and Czado (2013)
  - Operational risk: Brechmann, Czado, and Paterlini (2013)
  - Multivariate options: Bernard and Czado (2013)
  - Realized volatility: Vaz de Melo Mendes and Accioly (2013)
  - Insurance: Erhardt and Czado (2012)
  - Portfolio management: Low, Alcock, Faff, and Brailsford (2013)
- **Hydrology:** Gräler, van den Berg, Vandenberghe, Petroselli, Grimaldi, De Baets, and Verhoest (2013)
- **Machine learning:** Lopez-Paz, Hernández-Lobato, and Ghahramani (2013),
- **Health:** comorbidity Stöber, Czado, Hong, and Ghosh (2012)
- **Environmental Science**: Gräler and Pebesma (2011), Pachali (2011), Erhardt (2013)

What have we learned?

- Standard multivariate copulas are less flexible, while PCCs such as C-, D- and R-vines are much more flexible.
- Sequential and MLE parameter estimation of C-, D- and R-vines are available in **R packages** `CDVine` and `VineCopula`.
- The vine tree and the pair copula familes matter in the selection of good fitting PCCs.
- The catalog of possible pair copula families should also include nonsymmetric pair copulas such as the Tawn copula (Eschenburg (2013))
- Sequential and full Bayesian estimation and Bayesian model selection of vine trees and copula families for regular vines available, but need further testing and development

What needs to be done?

- non parametric pair copulas, spatial vines, large non Gaussian Bayesian belief networks, vines for data mining, high dimensional GoF for vines (Schepsmeier 2013)
- more applications in finance, insurance, health, genetics ...

Vine resource page: `vine-copula.org`
Vine workshop book: Kurowicka and Joe (2011)

Spatial Copula Workshop

Host: Institute for Geoinformatics, Münster, Germany
Dates: 22nd and 23rd September 2014
Organizers: C. Czado and B. Gräler

Thanks to my collaborators





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