Solvable states for ternary-unitary gates

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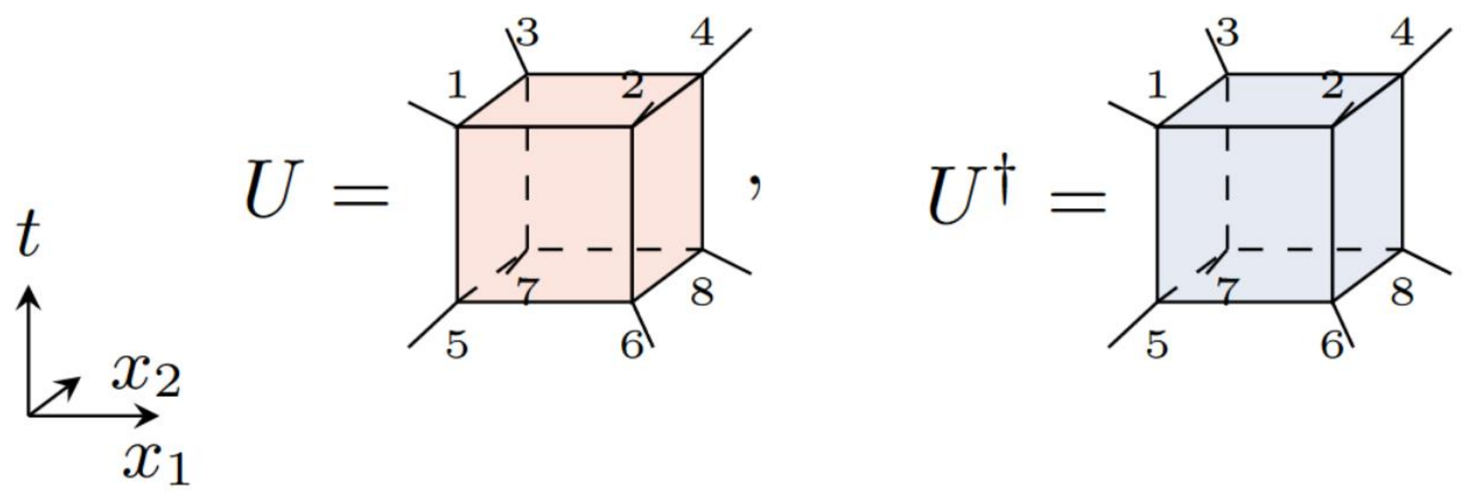
Abstract

Recently we introduced the so-called ternary unitary quantum gates. These are four-particle gates acting in 2+1-dimensions and are unitary in time and both spatial dimensions. Now we generalise the concept of solvable matrix product state[1] to two spatial dimensions under cylindrical boundary conditions. We show that such *solvable PEPS* can be identified with matrix product unitaries. In the resulting tensor network for evaluating equal-time correlation functions, the bulk ternary unitary gates cancel out, and we delineate and implement a numerical algorithm for computing such correlations.

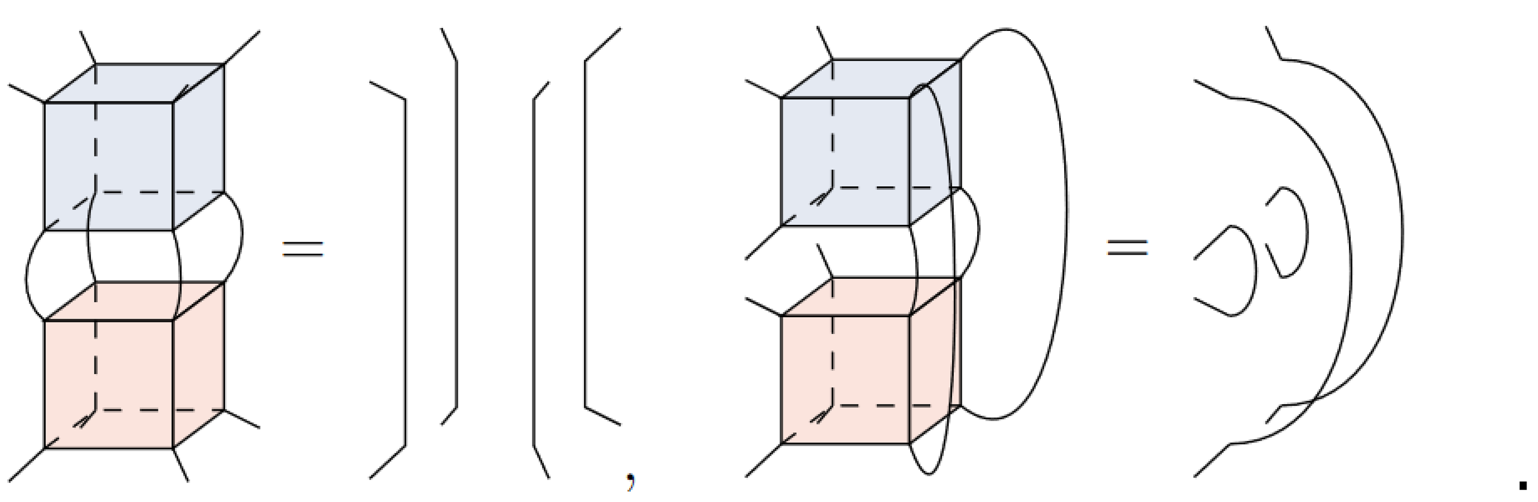
Introduction

In [2] the class of dual-unitary operators was introduced. They are unitary two-particle gates, i.e. 4-tensors, that are still unitary under a specific permutation of their leg indices. Their special structure admits exact solutions for certain quantities such as the dynamical correlation function. However, it is not generally possible to determine the dynamics of arbitrary states under the evolution of dual-unitary operators. Therefore, the concept of solvable matrix product states (sMPS) was introduced in [1]. For states in this class different dynamical properties can be determined analytically for example the expectation values of two-site operators.

We extended the concept of dual-unitary operators to two spatial dimensions by considering four-particle gates:



Such a gate U is ternary unitary if it is unitary in time t and both spatial directions x_1 and x_2. This condition is more easily seen graphically as:



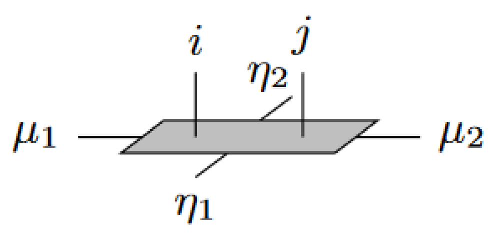
t-direction $\qquad$ x_1-direction

The condition for the x_2-direction can be obtained by rotating the x_1-condition by 90° around the t-axis.

Solvable States

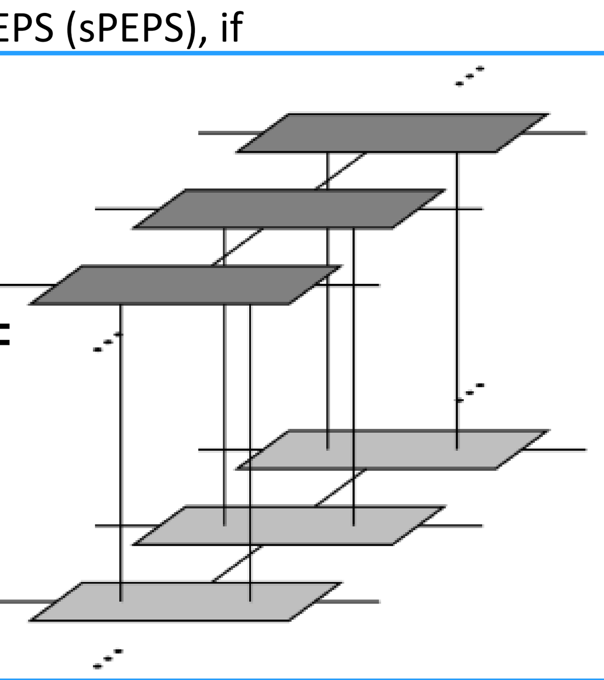
We can build a time evolution from ternary-unitary gates by letting layers of them act on a 2D-square grid. A state on such a grid can be described by projected entangled pair states (PEPS). We will use a PEPS, where each local tensor Λ describes two physical sites, i.e.:

$$\Lambda^{ij}_{\mu_1\mu_2\eta_1\eta_2} =$$



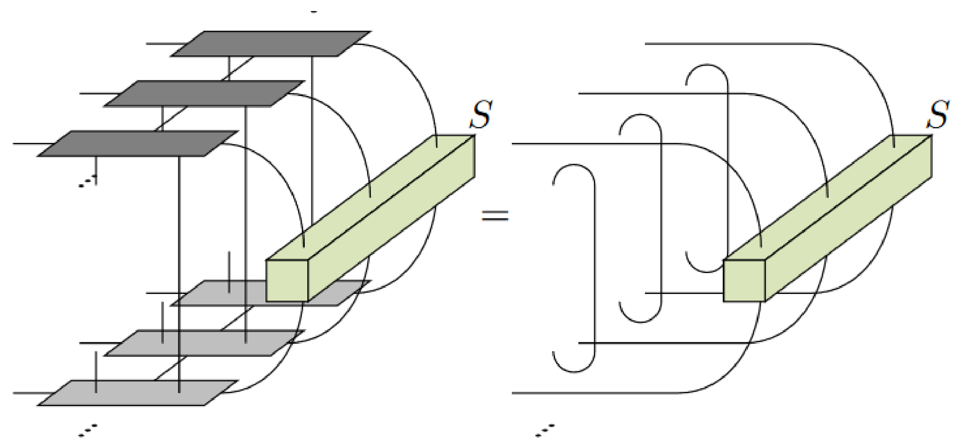
By rearranging the legs, we can make Λ a matrix product operator. Using these we create a PEPS of size $L_1 \times L_2$ with periodic boundary conditions in x_2-direction. We call such PEPS a solvable PEPS (sPEPS), if

1. $\tilde{\Lambda}^{L_2} =$



is the transfer operator and has a unique largest eigenvalue 1 with an algebraic multiplicity 1. The darker tensors denote Λ^*.

2. There exists a non-zero tensor S such that:



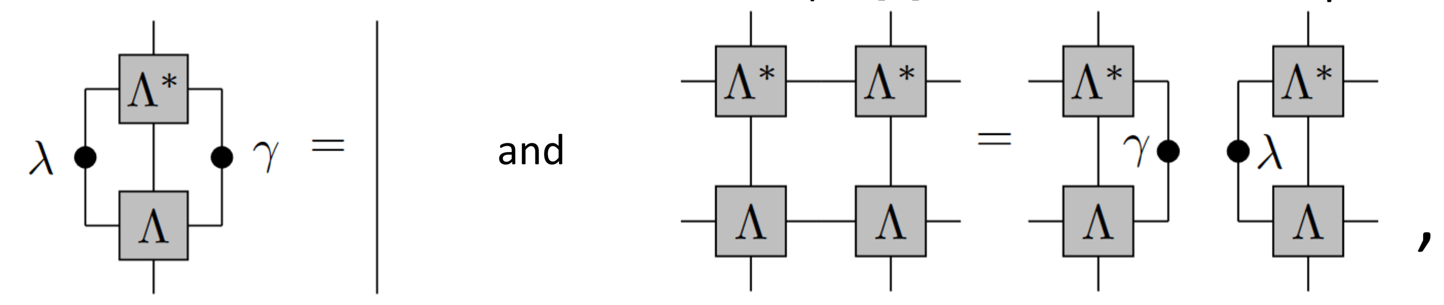
Theorem: On a square lattice of size $L_1 \times L_2$ with cylindrical boundary conditions a solvable PEPS $|\Psi_{L_1L_2}[\tilde{\Lambda}]\rangle$ as defined above is equivalent in the thermodynamic limit to some shift-invariant PEPS $|\Psi_{L_1L_2}[\Lambda]\rangle$ such that the matrix product operator associated to Λ is a matrix product unitary up to a scalar factor, i.e. $S \propto \mathbb{1}$.

Expectation Value Dynamics

For solvable states the expectation value dynamics of a two-site operator $\Omega_{\{x,y\}}$ can be determined in the thermodynamic limit where the time-evolution $\mathbb{U}$ composed of ternary-unitaries:

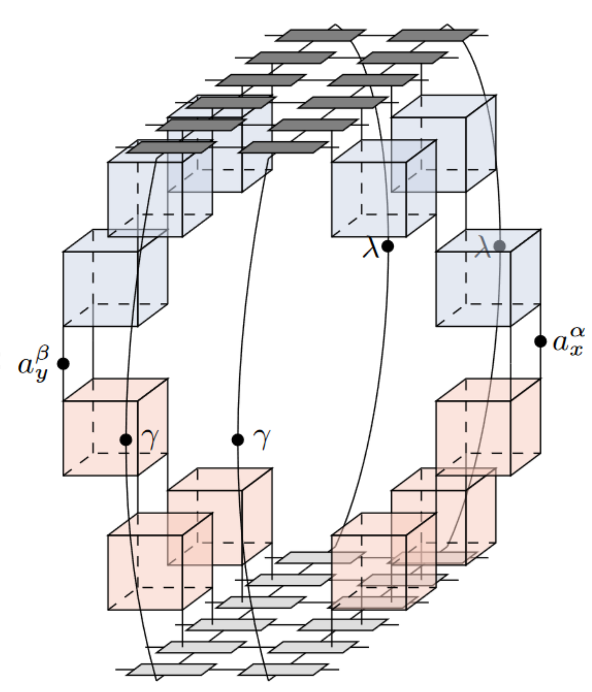
$$E\left(\Omega_{\{x,y\}}\right) = \lim_{L_2\to\infty}\lim_{L_1\to\infty}\left\langle\Psi_{L_1L_2}\middle|\mathbb{U}^{-t}\Omega_{\{x,y\}}\mathbb{U}^{t}\middle|\Psi_{L_1L_2}\right\rangle$$

If we assume the MPU associated to Λ to be simple [4], i.e. there exist λ and γ s.t.

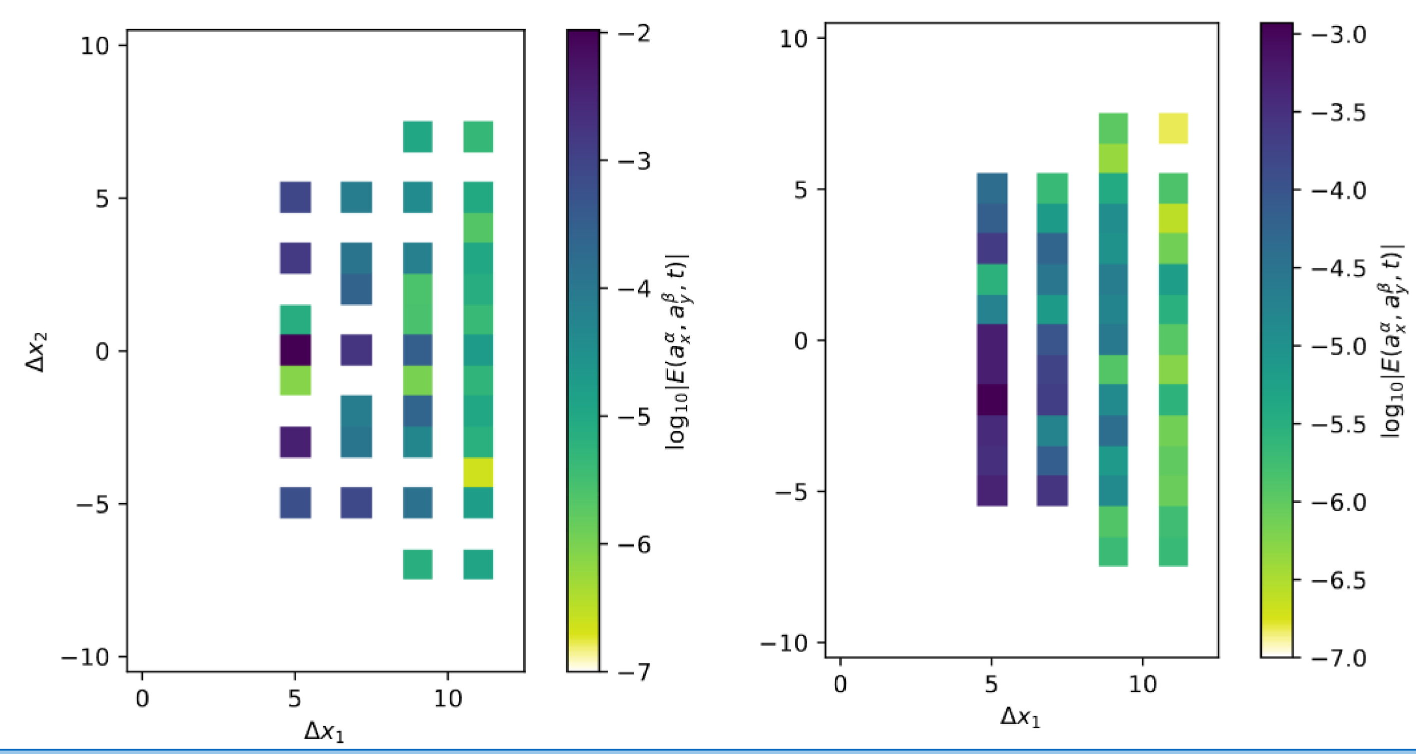


we can simplify E to a finite tensor network. In the case E is non-zero only a cylinder of tensors remains. For the example $x_1 - y_1 = 7$, $x_2 - y_2 = -1$ and $t = 1$ we obtain

$$E\left(\Omega_{\{x,y\}} = (a_x^\alpha, a_y^\beta), t = 1\right) =$$



The final result can be contracted numerically. Notably, the final result is also true if the gates are only unitary in t and x_1-direction. However, for ternary-unitaries (left) there are still more parameter values for which E is zero compared to the dual-unitary case (right):



Acknowledgements

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