Proceedings of the IASS 2022 Symposium affiliated with APCS 2022 conference
Innovation·Sustainability·Legacy
19 – 22 September 2022, Beijing, China

Exploring the potential of equilibrium-based methods in additive manufacturing: the Digital Bamboo pavilion

Davide TANADINI*, Patrick Ole OHLBROCK, Marirena KLADEFTIRA[a], Matthias LESCHOK[a], Eleni SKEVAKI[a], Benjamin DILLENBURGER[a], Pierluigi D'ACUNTO[b]

*ETH Zurich, Institute for Technology in Architecture ITA, Chair of Structural Design
Stefano-Franscini-Platz 1, HIB E11, 8093, Zurich, Switzerland
tanadini@arch.ethz.ch

[a] ETH Zurich, Institute for Technology in Architecture ITA, Chair of Digital Building Technologies
[b] Technical University of Munich, School of Engineering and Design, Professorship of Structural Design

Abstract
Additive manufacturing allows using naturally grown structural elements in combination with 3D printed components, aiming to foster a more sustainable building culture. However, from a structural perspective, the combined use of natural and artificial materials introduces new uncertainties and challenges in relation to the design and analysis of the global load-bearing behavior of structures, as well as their connection systems. In this context, equilibrium-based methods derived from the lower bound theorem of the theory of plasticity are particularly effective since they are not restricted to a specific structural typology, material, or scale. This paper illustrates the application of equilibrium-based methods such as strut-and-tie modeling and graphic statics to the structural design and analysis of a full-scale demonstrator, the Digital Bamboo pavilion. The load-bearing system of the pavilion is a bamboo space frame reinforced by post-tensioned cables. The bamboo elements are connected via bespoke 3D printed nylon and steel connections. Thanks to the combination of natural and artificial materials, the Digital Bamboo exhibits a filigree high-performance geometry that goes beyond the ordinary space frame architecture.

Keywords: Space frame, equilibrium-based methods, bamboo, bespoke connections, additive manufacturing

1. Introduction
Bamboo is an excellent sustainable building material because of its rapid growth and the low weight-to-strength ratio [1]. However, bamboo canes are by nature not perfectly straight and are of variable diameter and thickness. This natural variability of bamboo can inhibit its use in space frame structures where several truss elements are often connected via complex three-dimensional nodes that require the transfer of tension and compression forces. Developing an appropriate connection system for bamboo elements is therefore challenging as it requires bespoke solutions [2]. Additive manufacturing (AM) technologies allow the manufacturing of custom connections that can accommodate irregularities of natural materials. However, adopting these hybrid solutions poses challenges, as the structural behavior of the bespoke components in combination with naturally grown elements is difficult to simulate. Classical structural analysis methods based on the Finite Element Method (FEM) and elasticity theory present shortcomings when exploring new AM construction technologies, as they are computationally intensive and require a high degree of precision in defining the input material properties. Furthermore, they appear more oriented toward the analysis rather than the design, producing opaque results where

the influence of single design parameters is not always intuitive and easy to interpret. On the contrary, equilibrium-based methods such as strut-and-tie modeling and graphic statics can support the design of non-standard structures, as they are not bound to any specific structural typology, material, or scale. Furthermore, the graphical representation of the state of internal forces allows understanding the behavior of the structural systems in an intuitive way.

This paper illustrates the use of equilibrium-based design and analysis methods, such as strut-and-tie modeling, stress fields, and graphic statics, for the development of the Digital Bamboo (Figure 1), a bespoke pavilion composed of bamboo canes and customized 3D printed connections. With a covered area of more than 40 square meters, a height of 4.6 meters, and a mass of only 200 kg, the pavilion demonstrates how the novel combination of naturally grown materials such as bamboo with additive manufacturing makes it possible to transcend the standardized structural typologies.

Figure 1: The Digital Bamboo at the *Biennale Architettura: Time, Space, Existence* exhibition in Venice in 2021.

2. Equilibrium-based methods

2.1. The lower bound theorem of the theory of plasticity

The lower bound theorem of the theory of plasticity is one possible basis for developing equilibrium-based solutions [3]. Methods grounded in plastic theory have been developed to determine the collapse load of structures at their ultimate limit state [4]. They comprise the static solution, based on the lower bound theorem, and the dual kinematic solution, based on the upper bound theorem. When the two solutions coincide, the exact solution is achieved. The collapse load of a structure calculated according to the lower bound theorem is always smaller or at most equal to the actual collapse load. Therefore, applying the lower bound theorem leads at most to an underestimation of the structure's load-bearing capacity.

For a lower bound solution to be valid, three conditions must be fulfilled: a rigid-plastic behavior, an admissible state of equilibrium, and compliance with the yield conditions [5]. In a rigid-plastic model, the plastic strains are considerably larger than the elastic strains, so the latter can be neglected. Furthermore, the assumption of a rigid-plastic behavior allows the presence of static discontinuities, which facilitate the development of the equilibrium state. In fact, given external forces and a geometric boundary, the designer is free to define the path of the internal forces, provided that the boundary conditions and yield conditions are fulfilled.

It is worth noting that singularities such as the inherent fragile behavior or anisotropy of a material do not represent limits for applying the plastic theory. Furthermore, using a rigid-plastic model does not necessarily imply the presence of rigid plastic materials, but rather that the overall behavior of the structural system can be assumed to be rigid plastic. To this end, the structural system must guarantee the necessary deformation capacity to redistribute the internal forces plastically. This can be achieved through low stiffness and controlled brittle and ductile failures. As a result, the lower bound theorem can be applied to determine the collapse load in steel [5], reinforced concrete [6], masonry [7], or timber [8] structures.

2.2. Equilibrium-based analysis and design

The lower bound theorem of the theory of plasticity allows the visualization of the equilibrium state in the form of strut-and-ties (discrete case) or stress field (discrete and continuous case). The illustrative representation of the stress state provides the basis for proper development of the load-bearing structure both globally and locally (Figure 2), as in the case of bespoke connections for space frames. Unlike other approaches in which the initial prerequisite of defining the geometry a priori limits the structural design process, equilibrium-based methods allow an interdependent design with forces, facilitating the understanding of the load-bearing structure.

The possibility to model internal forces combined with a basic understanding of the material behavior allows the identification of possible critical situations. Consequently, it improves the structure's efficiency by manipulating the geometry or the flow of internal forces. Furthermore, the explicit force-geometry relationship permits the integration of the structural performance with other requirements, such as those provided by manufacturing methods.

2.3. Graphic statics

Graphic statics provides significant support for developing equilibrium-based solutions that rely on the lower bound theorem of the theory of plasticity. In particular, graphic statics enables the analysis and design of structures in static equilibrium based on reciprocal form and force diagrams. The form diagram shows the geometry of the load-bearing structure and the external forces acting on it while the force diagram provides a representation of the equilibrium of the forces acting at each node of the form diagram [9].

The geometric dependency between the two diagrams underlines the form-force relationship and provides unique and intuitive tools for developing equilibrium-based solutions. It allows the transformation of the equilibrium state in the form diagram (e.g. strut-and-tie model) while keeping the magnitude of the forces under desirable limits in the force diagram (e.g. by respecting the yield conditions).

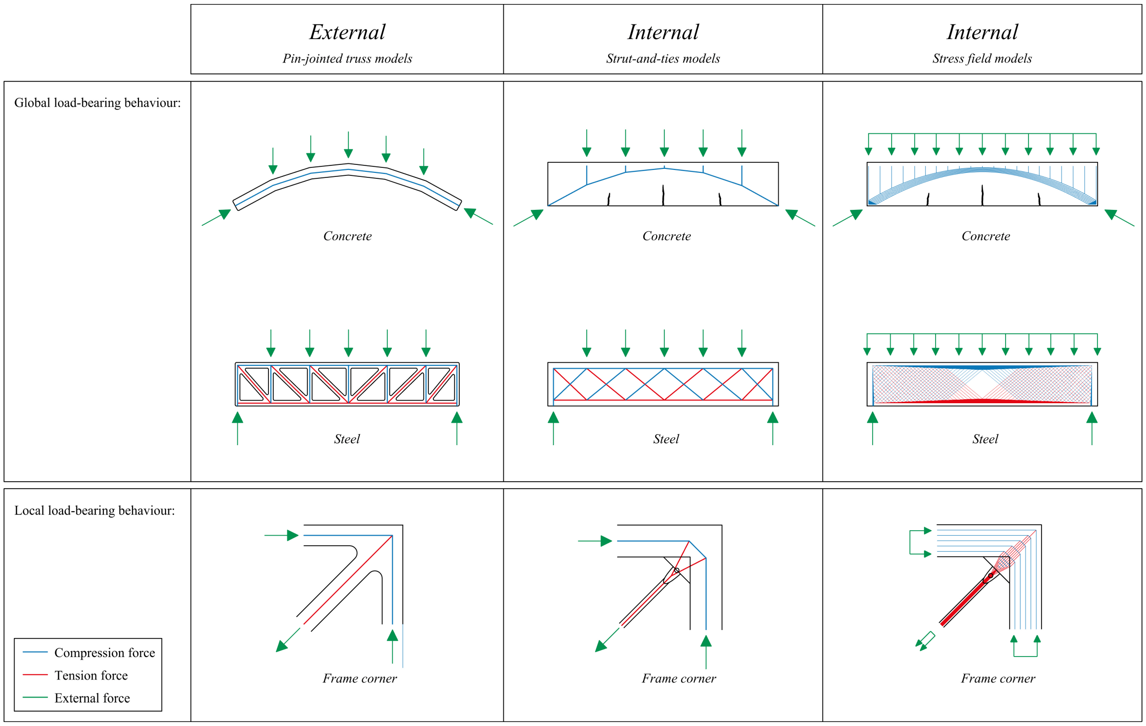


Figure 2: Equilibrium-based design allows dealing with different structural typology, materials or scales by respecting the three basic conditions of the lower bound theorem of plasticity theory.

3. Digital Bamboo

Equilibrium-based methods were applied in the structural design and analysis of the Digital Bamboo pavilion. The pavilion is a complex space frame with three cantilevering wings supported by three columns (Figure 3). The structure comprises more than 900 bamboo canes, 6 post-tensioning cables, and 381 bespoke 3D-printed connections. All connections are made of nylon, apart from one central connection, which is manufactured in stainless steel. Fabric panels that provide shade cover the upper part of the structure. These elements are manufactured by an add-on 3D printing process on a perforated lycra fabric. The structure's foundation consists of a thick, rigid metal plate with a mass of around 1400 kg that connects the three concrete footings positioned at the base of the columns. These footings are manufactured through a 3D printed formwork technique. For more information on the computational methods and fabrication processes employed in the pavilion, the reader is referred to [2].

Following a hierarchical design approach [10], the structural system of the Digital Bamboo pavilion is composed of three distinct parts: the primary, secondary, and tertiary sub-structures (Figure 4). The primary sub-structure is responsible for transferring the vertical load (i.e. self-weight) to the ground. It includes the central part of the wings, the columns, and the post-tensioning cables. The design of the primary sub-structure was developed through a constraint form-finding process using Combinatorial Equilibrium Modelling (CEM) [11]. The secondary sub-structure is responsible for transferring forces to the adjacent primary structure. It also provides the bracing as well as rotational resistance. The secondary sub-structure is composed of a three-dimensional truss, which is entirely triangulated to form tetrahedrons. Ensuring tetrahedral cells in the spatial truss limits the bending stresses in the bamboo canes. This is in line with the characteristics of bamboo since it can withstand high axial forces but displays a weak bending behavior. Thus, in the present design, the members are mainly subject to axial compression and tension forces. The final geometry is achieved through the tertiary sub-structure, which consists of non-load-bearing planar elements.

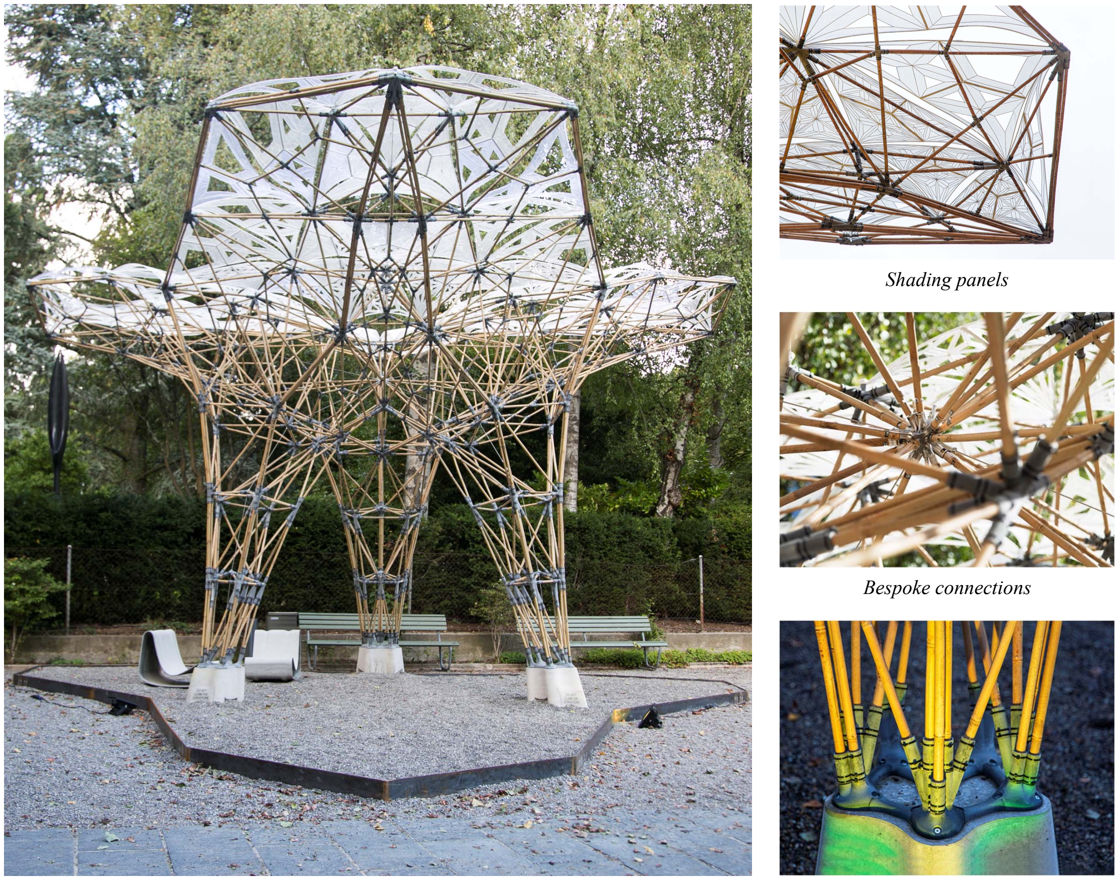


Figure 3: The Digital Bamboo at the *Bellerive Zentrum Architektur* in Zurich in 2020. [Photo: A. Jipa]

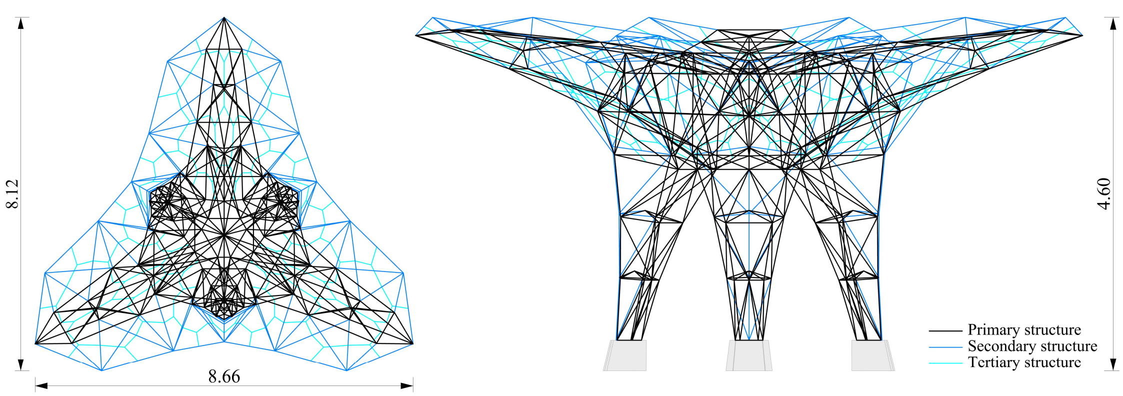


Figure 4: The hierarchical space frame structure of Digital Bamboo is composed of primary, secondary and tertiary sub-structures.

3.1. Conceptual structural design

In the first phase of the design process of the Digital Bamboo, the overall structural concept was developed with hand sketches (Figure 5). The use of equilibrium-based methods proved to be crucial in this exploratory and conceptual phase of the design process. In fact, it allowed the understanding of the influence of the individual geometric design parameters on the structural performance without overlooking architectural aspects. An accurate initial design based on simple and easily representable concepts avoids contradictions between design goals and mitigates the risk of potential complications in the advanced design phase.

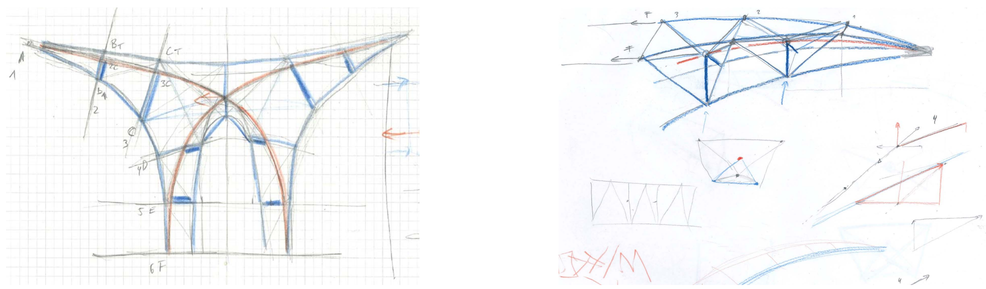


Figure 5: A sketch of the global structural system of the Digital Bamboo pavilion (left) and a conceptual study of the effect of post-tensioning on the space frame (right); red-tension, blue-compression.

3.1.1. Constraint form-finding

The initial sketch was then developed into a form-finding model using the Combinatorial Equilibrium Modelling (CEM) [11], a computational form-finding tool based on vector-based graphic statics [12]. Starting from the topology of the structure, the tool allowed exploring different equilibrium solutions for the global geometry of the pavilion by keeping the equilibrium condition. In particular, in developing the Digital Bamboo, CEM was used in the form-finding of the primary sub-structure (Figure 6). Since the structure has several planes of symmetry, it was possible to define the shape of only one-sixth of the primary sub-structure. Once this sub-system is specified, the primary sub-structure is obtained by mirroring and rotating the sub-system. During the process, local supports were introduced in the intersection points between the sub-system and the symmetry planes to interface the sub-system with the rest of the structure. This resulted in a geometrically constrained form-finding, where the elements connected to the local supports must be perpendicular to the symmetry planes.

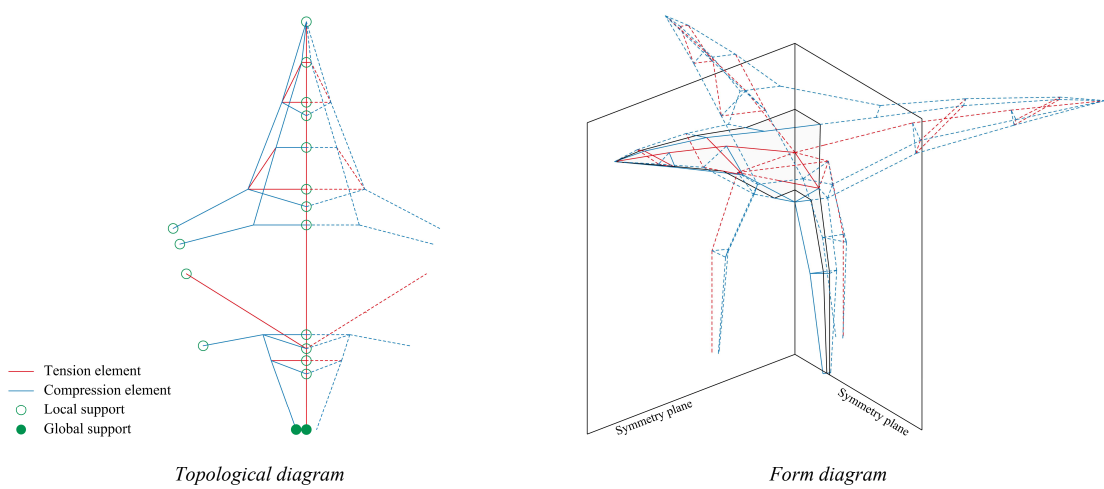
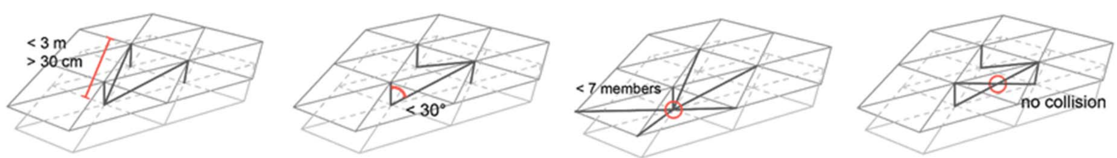


Figure 6: Only one-sixth of the primary sub-structure (continuous line) was generated using the Combinatorial Equilibrium Modelling (CEM) [11] and then mirrored using symmetry planes.

3.1.2. Geometric rules

Based on the global geometry generated using the CEM, the detailed development of the three-dimensional truss of the secondary sub-structure was supported by geometric rules (Figure 7) to facilitate the manufacturing of the 3D printed connections and the assembly of the structure:

1. The length of the bamboo elements is limited to the range from 0.3 to 3.0 meters. Elements that are too short are to be avoided as any connection system requires a minimum length to effectively establish a connecting interface. Elements that are too long are difficult to handle and are more susceptible to compression stability-related issues.
2. The angle between two elements is at least 30° since narrow angles require larger connections.
3. To limit the size and complexity of the connection, the number of elements connected to a single node is limited to 7.
4. To prevent a possible collision between different elements, a minimum spatial distance between them is guaranteed. This allows the not perfectly linear shape of the bamboo canes as well as fabrication and assembly tolerances to be considered.



1. 2. 3. 4.

Figure 7: Overview of the rules adopted in the design of the space frame of the Digital Bamboo. Rules 1 and 2 constitute metric constraints, while 3 and 4 represent topological constraints.

3.2. Post-tensioning

Although the bamboo canes can withstand high compression and tension forces, the connection system shows weak resistance to tension forces, thus representing the critical point of the structural system. Hence, six post-tensioning cables were introduced (Figure 8) to reduce the number of critical members in tension (Figure 11). Three cables run in the center of their respective wings, while other three through the three columns. The post-tensioning cables are connected to the primary structure through specific connections, and they all meet in the central node at the intersection of all axes of symmetry. This central node is 3D printed in stainless steel as it is expected to withstand higher stresses than the other nodes.

3.3. Bundling

To facilitate the connection of the bamboo canes to the bespoke 3D printed joints, only bamboo canes with a diameter of 20±3 mm were used in the space frame. Whenever the capacity of the individual bamboo canes was exceeded, bundled bamboo canes were used to withstand the higher internal forces (Figure 8). All bundled elements consist of three bamboo canes, which triples the load-bearing capacity of the individual elements. About 10% of the bamboo elements in the structure are bundled.

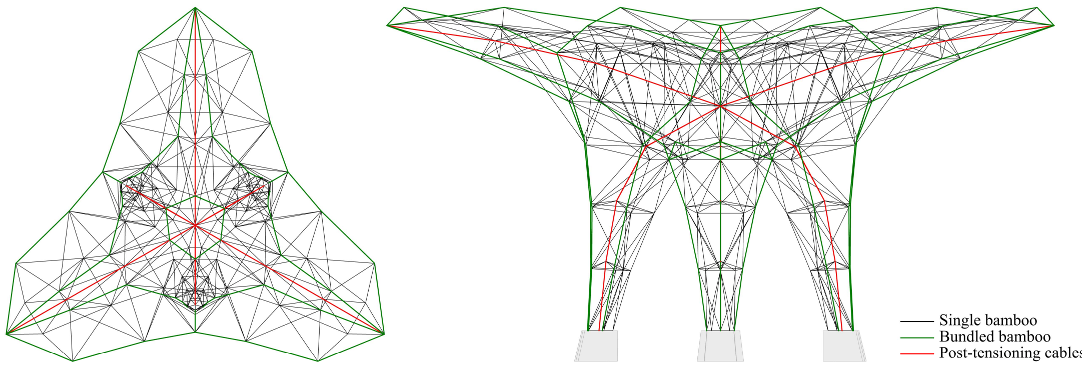


Figure 8: Schematic representation of the bundled elements and the six post-tensioning cables.

3.4. Design of the connections

The three-dimensional 3D-printed connections were designed to transfer forces from one bamboo element to the others. The connection system evolved during the design process, looking for a compromise between structural performance and manufacturing constraints in terms of cost and printing volume. Ultimately, an interlocking connection was chosen, with force transfer via contact surfaces under compression. Other connection typologies, such as those requiring chemical binders, were deliberately avoided in favor of a simple mechanical connection that guarantees quick assembly on site and the possibility of dismantling and replacing damaged bamboo canes.

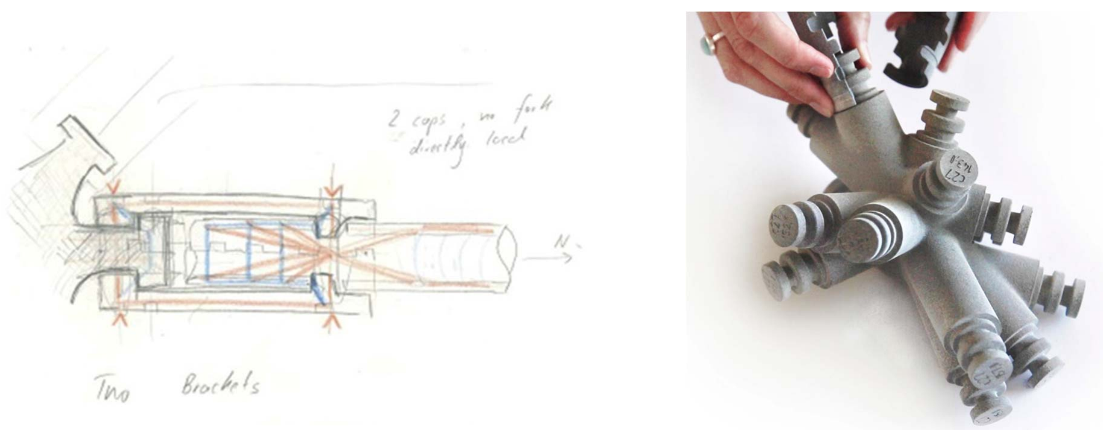


Figure 9: Equilibrium-based methods provided the basis for developing a connection that fulfilled the mechanical requirements and ensures the required hinge behavior and assembly tolerance. [Photo: A. Bargetzi]

In the adopted connection system, compression forces are transferred directly to the core of the joint. In contrast, tension forces are redirected and transferred through notches in the bamboo and 3D printed caps. The notch system ensures a hinge behavior and complies with the tolerance required by both the variable nature of bamboo canes and the assembly process. The three parts that compose the connection are tightened together with zip ties. Thanks to the equilibrium-based method adopted in the design of the connections, it was possible to adapt the connection system to the non-standard structural behavior of bamboo. Due to its microstructure consisting of fibers oriented in a predominant direction, bamboo shows a strongly anisotropic behavior. Once the governing failure mode is identified, the graphic representation of the force flow makes it possible to modify the joint's geometry and improve its efficiency. In this specific case, the governing failure is represented by a sliding of the fibers close to the notches. With the aid of graphic statics, it can be easily demonstrated how a greater distance between the notch and the end of the bamboo cane has a beneficial effect on the capacity of the connection system.

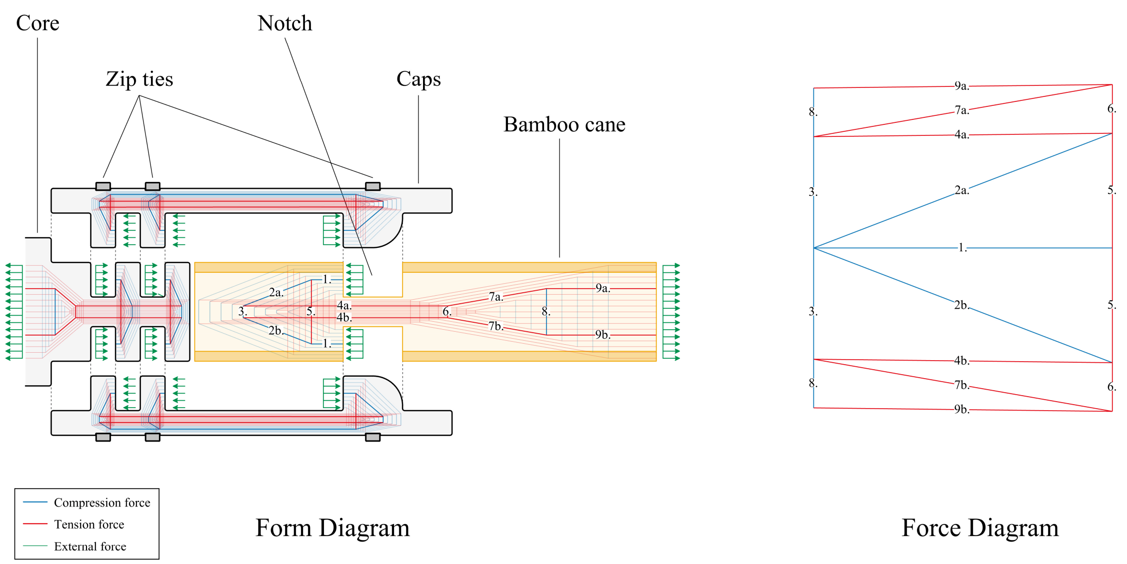


Figure 10: Illustration of the force flow in a connection under tension in the form of strut-and-ties and stress fields. Once the critical forces causing the failure in the bamboo have been identified (here 2. and 5.), it is possible to optimize the connection by adjusting its geometry, thus reducing the magnitude of the critical forces.

3.5. Structural analysis

3.5.1. Load cases

The analysis of the structure of the Digital Bamboo (Figure 11) was performed with a Finite Element Analysis (FEA) software [13], considering the linear elastic response (i.e. first-order theory). The FE model considers stiff bars and pin-jointed connections. The characteristic load-cases considered in the analysis are:

- Self-weight of bamboo canes and nylon/steel connections (*g*)
- Post-tensioning of the steel cables (*p*)
- Wind acting on the perforated fabric patches (*w*)

Based on the Swiss code SIA 261 [14], a wind load of 1.15 kN/m^2 is considered (recurrence of 50 years, sum of c_{pi} and c_{pe} = 1.0). The wind load case is applied on the fabric patches, which redistribute the loads to the corresponding adjacent connections. Given that the fabric has a perforation ratio of around 30%, the wind pressure is reduced by 30%. The direction of the wind is varied in steps of 10 degrees. Due to the rotational symmetry, 12 load case combinations were analyzed. In the FEA, no safety factors were applied on the side of the applied loads. Hence, the load case combination investigated is: 1.0 *g* + 1.0 *p* + 1.0 *w*. The analysis showed that the highest axial load in a truss element for the worst-case scenario corresponds to ±1.2 kN.

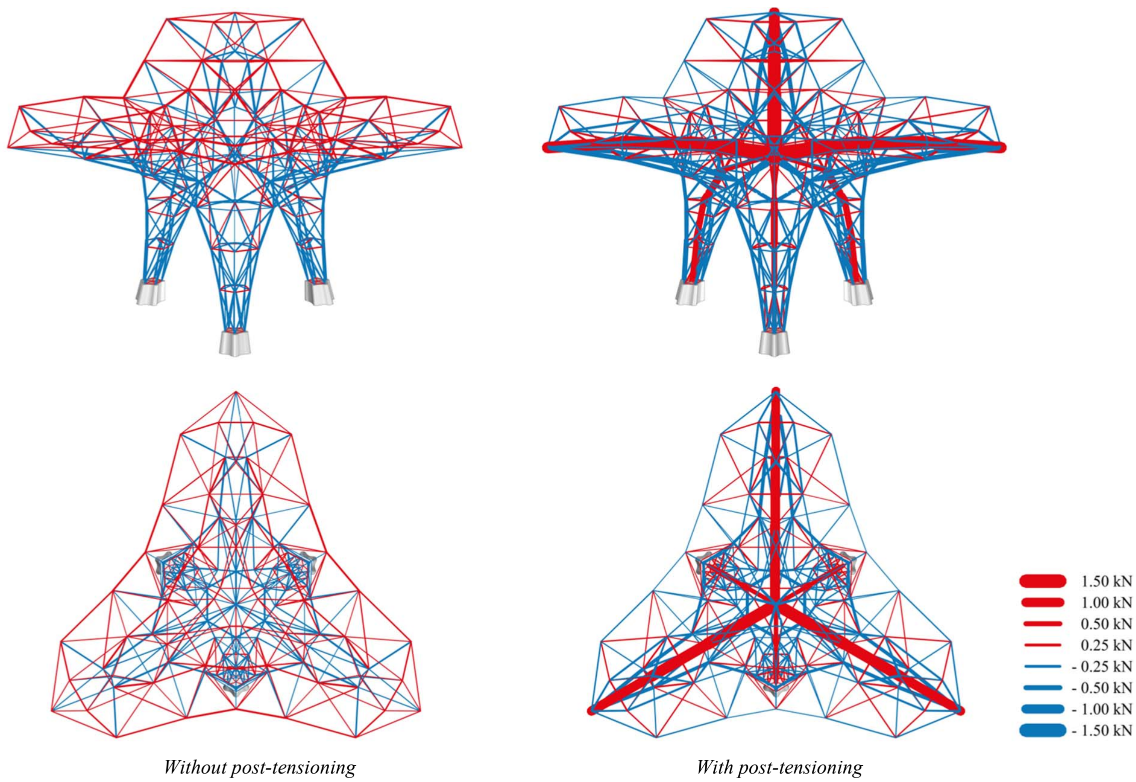


Figure 11: Structural analysis (axial forces) of the Digital Bamboo without and with post-tensioning cables. Using post-tensioning allows significantly reducing the number of bamboo canes under tension.

3.5.2. Structural capacity of the truss element

The truss element capacity refers to the resistance of the system composed of bamboo canes and 3D printed connections. The capacity is assumed to be equal for each element of the spatial truss.

Mechanical tests were carried out to determine the axial resistance of the bamboo canes. Due to the geometry of the connection, the compression forces are transmitted directly to the node's core, while the tensile forces are transferred through notches in the caps and consecutively to the node's core (Figure 10). Therefore, the transfer of tension forces from the bamboo to the connection is the most critical in the structural system. In fact, in this case, a sliding failure of bamboo fibers occurs. The results of uniaxial mechanical tension tests showed that the resistance of bamboo canes tends to increase with the increase in diameter and thickness of the sections. However, due to the inhomogeneous nature of bamboo, the exact resistance is difficult to predict accurately. Based on ca. 100 tests, the minimum diameter of the bamboo canes was set to 20 mm. In this case, the 5% percentile was set to 1.8 kN.

The reference strength of the 3D printed nylon connections was determined through mechanical tests as well as FE simulations (Figure 12). Tests indicated that the strength of the printed material corresponds to around 38 MPa. The connections were designed not to be the weaker part of the structural system, meaning the bamboo canes would fail first.

An overall safety factor of 1.5 was determined by dividing the capacity of a single truss element by the internal force magnitude of the most loaded element.

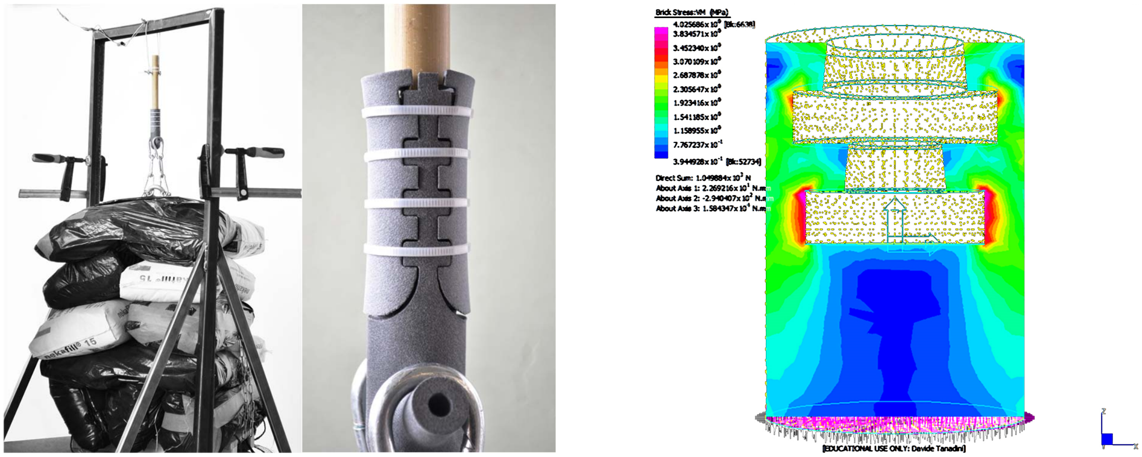


Figure 12: To validate the equilibrium-based design and analysis of the connection system, mechanical tests (left) and FE simulations (right) were performed. In the mechanical tests, cement bags (20 kg) are added incrementally. For the FE simulations, the software Strand7 [13] was used.

4. Conclusion

As AM technologies are expected to become more accessible in the near future, their use in the construction industry is estimated to increase steadily over the next years [15]. Leveraging the opportunities AM offers, bespoke structural elements such as 3D printed connections can facilitate the design and construction of high-performance structures. This paper presented the application of equilibrium-based methods to the design and analysis of the Digital Bamboo, a novel spatial structure in which AM technologies are used to address the non-standard character of naturally grown materials. In particular, the paper showed how equilibrium-based methods could be effectively used to tackle those structural challenges arising from the use of unprocessed natural and 3D printed materials, which are therefore subject to a large number of uncertainties. As demonstrated by the Digital Bamboo, equilibrium-based methods are particularly suited to explore novel hybrid, non-conventional structures, as they are not tied to a structural typology, scale, or material.

Acknowledgements

The authors acknowledge the contribution of the students of the MAS DFAB 2019 - 2020 to the initial concept studies of the project: F.S. Amiri, M.P. Assaf, J. Baddad, F. Brisson, Y. Chiu, R. Clémente, M. Goto, I. Georgiou, A.T. Johansson, L. Mangliár, D. Martínez Schulte, E. Sallin, I. Santosa, E. Sounigo, C. Techathuvanun, T.P. Hsun, E. Morales Zúñiga. Special thanks go to Prof. Dr. Joseph Schwartz for his support in all phases of the project; Yael Ifrah for her support and coordination between all parties.

This research project is supported by the National Centre of Competence in Research in Digital Fabrication funded by the Swiss National Science Foundation (NCCR DFAB Agreement #51NF40-141853). The realization of the structure was possible thanks to our research collaboration partner on additive manufacturing HS HI-TECH (South Korea), and our sponsors Hewlett Packard, SGSolution AG, AF Fercher AG, Lafarge Holcim, Abuma AG, and ZAZ Bellerive.

The Bamboo pavilion has been exhibited at *Bellerive Zentrum Architektur Zurich* in Zurich (2020) and at *Biennale Architettura: Time, Space, Existence* exhibition in Venice (2021).

References

[1] M. A. Ogunbiyi, S. O. Olawale, O. E. Tudjegbe and S. R. Akinola, "Comparative analysis of the tensile strength of bamboo and reinforcement steel bars as structural member in building construction," *International Journal of Scientific & Technology Research*, vol. 4 (11), Nov. 2015.

[2] M. Kladeftira, M. Leschok, E. Skevani, D. Tanadini, P. O. Ohlbrock, P. D'Acunto and B. Dillenburger, "Digital Bamboo. Reconciling bamboo and 3D printing technologies for reversible ultralight structures," in *Hybrids & Haecceities - Proceedings of the 41st Annual Conference of the ACADIA 2022*, Philadelphia, 2022.

[3] J. Schwartz, "Graphic statics and their potential for digital design and fabrication with concrete," *Cement and concrete research,* vol. 112, pp. 122-135, Oct. 2018.

[4] H. J. Greenberg and W. Prager, *Design of beams and frames,* Transactions ASCE 117, pp. 447–484, 1952.

[5] P. Marti, *Theory of structures: fundamentals, framed structures, plates and shells.* (1st ed.). Berlin: Wiley, 2013.

[6] A. Muttoni, J. Schwartz and B. Thürlimann, *Design of concrete structures with stress fields.* (1st ed.). Basel: Birkhäuser, 1997.

[7] B. Zimmerli, J. Schwartz and G. Schwegler, *Mauerwerk - Bemessung und Konstruktion.* (1st ed.). Basel: Springer, 1999.

[8] D. Tanadini and J. Schwartz, "Analysis and design of timber-to-timber connections based on the lower bound theorem of the theory of plasticity," in *Proceedings of WCTE 2020*, Santiago, 2021.

[9] M. Konstantatou, P. D'Acunto and A. McRobie: "Polarities in structural analysis and design: n-dimensional graphic statics and structural transformations," in *Int. J. Solids Struct.*, vol. 152-153, pp. 272-293, 2018.

[10] P.O. Ohlbrock, P. D'Acunto and J.-P. Jasienki. "Hierarchical form-finding with Combinatorial Equilibrium Modelling," in *Creativity in structural design - Proceedings of the IASS Annual Symposium 2018*, Boston, 2018.

[11] P.O. Ohlbrock and P. D'Acunto, "A computer-aided approach to equilibrium design based on graphic statics and combinatorial variations," in *Computer-Aided Design*, vol. 121, 2020.

[12] P. D'Acunto, J. P. Jasienski, P. O. Ohlbrock, C. Fivet, J. Schwartz, and D. Zastavni, "Vector-based 3D graphic statics: A framework for the design of spatial structures based on the relation between form and forces," in *Int. J. Solids Struct.*, vol. 167, pp. 58–70, Aug. 2019.

[13] Strand 7 – Finite Element Analysis, Student Version, Available online: https://www.strand7.com/.

[14] Swiss Society of Engineers and Architects, *SIA 261: Actions on structures*, 2020.

[15] A. Paolini, S. Kollmannsberger and E. Rank, "Additive Manufacturing in Construction: A Review on Processes, Applications, and Digital Planning Methods," *Additive Manufacturing*, vol. 30, Dec. 2019.