

# Optimized 3D Placement of LiFi Access Points towards maximizing Wireless Network Performance

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**Abstract**—In the next generation of networks, the high data rate and large bandwidth offered by Light-Fidelity (LiFi) are expected to be fully exploited to satisfy the diverse Quality of Service (QoS) requirements of users. LiFi networks are expected to provide full data coverage while also satisfying illumination requirements in indoor environments. Thus, the optimized placement of LiFi Access Points (APs) is of utmost importance. Extending the commonly applied 2D placement, we include the height of the AP, which is an important factor to consider due to the short range of LiFi and limited Field of View (FoV) of the receivers. To this end, we propose a 3D LiFi Access Point placement framework that formulates the placement as a multi-objective optimization problem minimizing the number of APs and maximizing the sum rate while providing a guaranteed minimum rate and illumination level. Since the exact positions of users are unknown during network planning and change dynamically after deployment, the probability distribution of user occurrence is considered in the optimization. This results in the network performance being maximized in areas where users are likely to be present. This optimization problem is solved with the proposed multi-objective genetic algorithm. The framework is evaluated with simulations and the results show that the height of the AP greatly influences the network performance. We conclude that a free choice of the height of each AP results in an average rate that is significantly better than the rate achievable when all APs are placed at the same height.

**Index Terms**—LiFi, Placement, 3D, NSGA, Multi-Objective Optimization

## I. INTRODUCTION

The number of machine-to-machine connections is forecast to reach 14.7 billion by 2023 and these connections are expected to support indoor applications like home automation, security, and video applications like Telemedicine and Augmented Reality (AR) [1]. These applications contribute to the exponentially increasing data traffic that requires higher bandwidth and lower latency. Recently, Light-Fidelity (LiFi) has emerged as a wireless access technology operating on the visible light and infra-red spectrum [2] with great potential to serve these high bandwidth needs. Since LiFi can be operated with existing infrastructure like off-the-shelf Light Emitting Diodes (LEDs) and does not interfere with Radio Frequency (RF) technologies, it is a promising solution for indoor wireless applications.

The LiFi channel degrades rapidly with distance which restricts the communication to a short-range. Therefore, LiFi cells are typically deployed in an ultra-dense fashion. In such a network, the placement of the LiFi Access Points (APs) or LEDs

gains importance to be able to provide optimal communication coverage as well as illumination in the indoor environment. In the rest of this work, the terms LiFi AP and LED are used interchangeably. Although multiple works analyze the coverage of a LiFi cell, they do not consider optimizing the number of APs. Placing a larger number of LEDs might ensure sufficient illumination but the communication network suffers due to interference in the overlap regions of the coverage area of the cells. Furthermore, a larger number of cells increases the number of handovers. However, a smaller number of LEDs would cause outage areas and may not be able to satisfy the network requirements of the users or illuminate the entire area. While some existing works do consider optimizing the placement of LiFi APs for various objectives like minimizing the number of APs or maximizing the throughput of the network, they do not consider optimizing for both objectives simultaneously. Multi-Objective Optimization (MOO) is important since the two objectives are contradictory and need to be solved simultaneously for optimal placement.

While optimizing for the position of the APs, the height of each access point from the user plane is an important factor since this influences the range of communication as well as the Signal to Interference and Noise Ratio (SINR) at the user device. This highly influences the network performance. The expected distribution of the users in an indoor environment is another important factor that has an impact on the system performance and must be considered while optimizing the placement for network performance.

### A. Related Work

In [3], the authors consider an LED array with a fixed number of LEDs and derive the optimal  $x,y$  position of each LED to minimize power consumption under data rate, and illumination constraints. The authors in [4] consider a variable number of LEDs and minimize this number to find the optimal 2D placement along the  $x$  and  $y$  coordinates. In our work, we also consider the height of the AP as a variable since this is an important factor that influences the network performance. Reference [4] also takes the expected user distribution in a room into account. However, they do not optimize the system for network performance in terms of throughput. Reference [5] places importance on the network performance and optimizes the placement towards maximizing the average throughput. Moreover, they factor in the stationary distribution of users in a room following the Random Waypoint mobility model. This

model can, however, be unrealistic in many indoor scenarios. We, on the other hand, analyze our model for multiple indoor scenarios with varying user distributions. The authors also assume that users in a certain region would be served by a dedicated AP. This raises issues in regions of interference and is oblivious to the device orientation of the users. The orientation of user devices is important in LiFi communication since each receiver has a Field of View (FoV) outside of which no signal is received. All works mentioned so far do not optimize the number of APs and the position simultaneously, thus differing from our work. They also do not consider the height to be a variable. Works like [6] and [7] propose a solution for the 3D placement of Unmanned Aerial Vehicles (UAVs) towards the goal of wireless communication resource allocation. For their application, they assume the position of users is known. This is an invalid assumption for our problem since the optimization of LED placement should happen in the network planning stage before deployment in an environment with users.

### B. Contribution

We examine the network planning of a LiFi communication and illumination network in an indoor environment and frame an optimization problem with multiple objectives of minimizing the number of APs, thus minimizing the cost, and maximizing the sum rate while constraining the minimum guaranteed achievable rate and minimum illumination level. Furthermore, we factor in the expected user distribution while calculating expected rates. Among the optimization variables, is the height of each AP which we allow to be either freely placed (3D free-height) or constrained such that all APs have the same height which is then freely selected (3D fixed-height). We consider both options to emphasize the impact of the height of an AP on the network performance. To solve this 3D placement problem, we propose a solution method employing a genetic MOO algorithm. Finally, we evaluate the solutions found for varying system parameters and scenarios and establish the validity of our optimization framework.

The rest of the paper is organized as follows. The system model considered is first introduced in Section II. The following Section III, details the 3D placement optimization problem formulation. Section IV describes the solution to this problem using the proposed algorithm and the results of this solution are analyzed in Section V. The paper is then concluded in Section VI which summarizes the important contributions of this work.

## II. SYSTEM MODEL

### A. Network Architecture

This work considers an indoor LiFi network with a maximum of  $M^L$  APs. The LiFi APs are LEDs operating on the visible light spectrum that are mounted above the user plane facing downwards as represented in Fig. 1. Thus, they enable illumination and data transmission simultaneously. The three-dimensional coordinates of each AP is denoted by  $(x, y, z)_l$ . The maximum height of the APs is denoted by  $z_{maxdim}^L$  and this is the height of the ceiling in the indoor environment. The

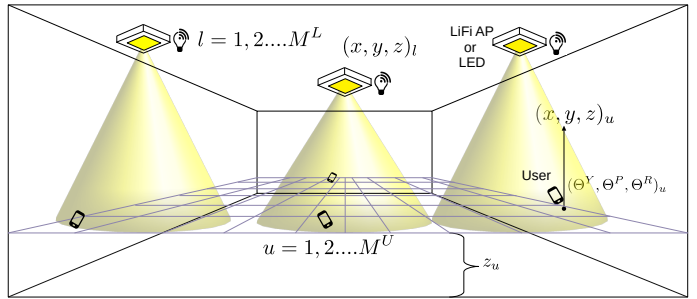


Fig. 1. Architecture of a LiFi communication and illumination network

minimum height is denoted by  $z_{mindim}^L$  and this is assumed to be at least one meter above the user plane to avoid saturating the receivers. Since all the APs operate at the same frequency, this results in co-channel interference in the areas of overlap of the coverage of the LiFi cells. The region where we expect to find users is called the user plane and this plane is quantized into grids with a spacing of 0.25 m. Each position on this grid is represented with the coordinates  $(x, y, z)_u$  and there are  $M^U$  such positions. We expect each user in such a position to be equipped with a LiFi photodiode receiver for downlink traffic. The orientation of these receivers is denoted by  $(\Theta^Y, \Theta^P, \Theta^R)_u$  which represents the Yaw, Pitch, and Roll angles of the device. If this value is  $(0, 0, 0)$  then the user device is parallel to the floor facing the the LED. Since the exact positions of the users are not known and dynamically changing, each user grid position  $\in M^U$  is associated with a weight  $P_u$  that is proportional to the expected probability of occurrence of a user in that position. The user is expected to be able to connect to the AP with the highest offered SINR. This adds another degree of freedom to our model since there are no dedicated APs for the users based on distance. With this information, the 3D placement of the LiFi APs is then optimized in the network planning phase and then the LiFi-enabled LEDs are deployed in the indoor environment.

### B. LiFi Channel Model

The channel model for LiFi described in [8] forms the basis for the model in our work as well. The LiFi channel highly depends on the LED and user positions in three dimensions and the orientation of the user device. The distribution of the LiFi SINR on the user plane at a height of 1.4 m from the floor, in a simple room where 4 LEDs are placed in a lattice grid format on the ceiling at 3 m is depicted in Fig. 2. The link data rate between a user  $u$  and an AP  $l$  is calculated using the modified Shannon formula [9] as

$$R_u^L = \frac{B_L}{2} \log_2 \left( 1 + \frac{e}{2\pi} SINR_u \right), \quad (1)$$

where  $B_L$  is the LiFi modulation bandwidth of an LED and  $SINR_u$  is the LiFi SINR at user  $u$ . The maximum rate that can be offered by a LiFi AP is assumed to be 250 Mbps.

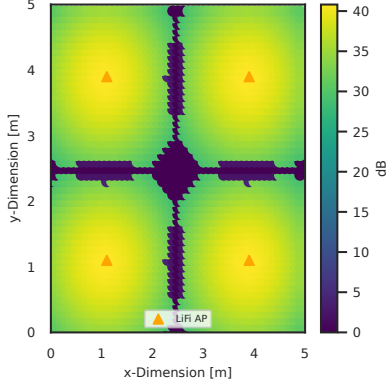


Fig. 2. LiFi SINR in a regular room with 4 LEDs placed in a lattice grid

### C. Illumination Model

The LiFi LEDs, operating on the visible light spectrum, are also responsible for the illumination in an environment. The illuminance grid need not necessarily be the same as the user grid and this grid position is denoted by  $(x, y, z)_v$  with a total of  $M^V$  grid positions. The illuminance at a grid position  $v$  provided by a single AP  $l$  is given by

$$I_{v,l} = I_0(z_l - z_v)^{m+1} (\|(x, y, z)_v - (x, y, z)_l\|_2^2)^{-\frac{m+3}{2}}, \quad (2)$$

where  $I_0$  in lux is the illuminance at the center of the LED,  $m$  is the Lambertian order of the LED, and  $\|(x, y, z)_v - (x, y, z)_l\|_2^2$  is the Euclidean distance between the AP and user position in three dimensions. The total illuminance at a grid position  $v$  provided by all APs is the sum of the individual values given by

$$I_v^L = \sum_l I_{v,l}. \quad (3)$$

### D. Scenarios

For the evaluation of our optimization model, we consider common application areas for LiFi networks which are multiple indoor environments with varying sizes and user distributions as depicted in Fig. 3.

## III. PROBLEM FORMULATION

This section describes the 3D placement optimization problem formulation. To this end, the optimization variables are the quantity and three-dimensional positions of APs. The goal of our optimization problem is to find the 3D placement of LEDs that minimizes the cost or number of APs and maximizes the sum rate on the user plane weighted by the user occurrence probability  $P_u$ . The optimization problem is constrained by the minimum guaranteed rate requirement  $R_{thresh}^L$  at all user grid positions. This ensures coverage reliability everywhere on the user plane. Since, at the planning stage, the exact number of users is unknown, we do not consider the wireless resource sharing. In contrast, we target the performance on the entire plane. We assume that a user will be associated to and served by the AP that offers the highest SINR. The other APs

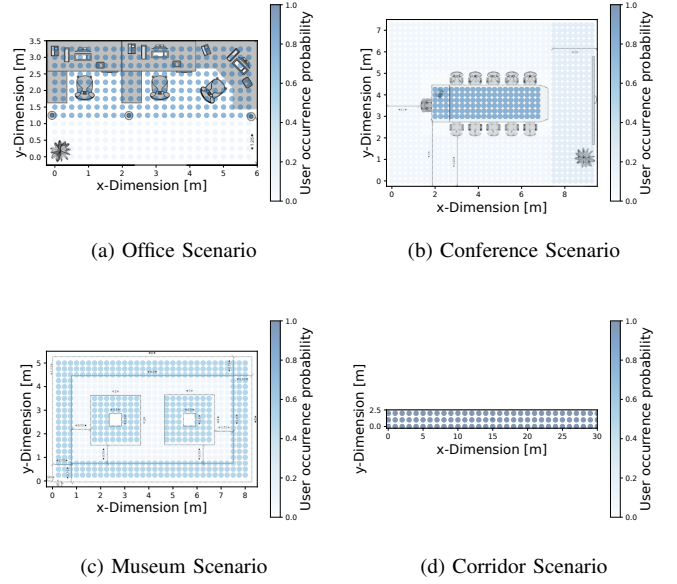


Fig. 3. Floor plans of and user distributions in indoor scenarios

act as interferers. To make the model generalizable to future generations of technology with potentially higher capacities, we include constraints on the rate in terms of a ratio of the maximum supported rate rather than using absolute rates. So the normalized rate ratio at a user  $u$  is given by

$$\tilde{R}_u^L = \frac{R_u^L}{R_{max}^L}, \quad (4)$$

where  $R_{max}^L$  is the maximum offered capacity, in bits per second, of the generation of LiFi technology considered. Another role of the LEDs is to provide the required level of illumination in the room  $I_{thresh}^L$ . Therefore, we also constrain the minimum illumination level along a plane. This could be at the floor level or on the level of desks in a work area. Hence, the illumination grid is separated from the user plane.

Given a certain maximum number of available LiFi APs  $M^L$  for placing, the optimization problem can be formulated as follows,

$$f1 : \min_{(x,y,z)_l, \alpha_l} \sum_{l=1}^{M^L} \alpha_l, \quad (5)$$

$$f2 : \max_{(x,y,z)_l, \alpha_l} \sum_{u=1}^{M^U} \tilde{R}_u^L P_u, \quad (6)$$

$$C1 : \tilde{R}_u^L \geq \tilde{R}_{thresh}^L \quad \forall u = 1, \dots, M^U, \quad (7)$$

$$C2 : I_v^L \geq I_{thresh}^L \quad \forall v = 1, \dots, M^V, \quad (8)$$

$$\alpha_l \in \{0, 1\}, \quad (9)$$

where  $u$  denotes the user grid and  $v$  the illumination grid. The objective function described in (5) corresponds to the cost minimization objective and the one in (6) corresponds to the

sum rate maximization objective of the 3D placement problem. The constraint in (7) places a requirement on the minimum guaranteed rate at every position on the user grid as a ratio of the maximum supported rate. The constraint in (8) places a requirement on the minimum illumination level applicable at every position on the illumination grid. The optimization variable  $\alpha_l$  is the AP existence which is a binary variable with a value of 1 if that AP is placed and 0 if not. In order to emphasize the importance of the height of the AP, we consider two different height models in the optimization.

#### A. 3D free-height

This model allows the height of each of the APs to be a real number freely selected from within the bounds. So the AP position variable is,

$$\begin{aligned} x_l \in [0, x_{dim}^L] \quad y_l \in [0, y_{dim}^L] \quad \forall l = 1, \dots, M^L, \\ z_l \in [z_{mindim}^L, z_{maxdim}^L] \quad \forall l = 1, \dots, M^L. \end{aligned}$$

#### B. 3D fixed-height

This model constrains all the heights of the APs to be the same but this value is a real number freely selected from within the bounds. So the AP position variable is,

$$\begin{aligned} x_l \in [0, x_{dim}^L], \quad y_l \in [0, y_{dim}^L] \quad \forall l = 1, \dots, M^L, \\ z_l = z^L \quad \forall l = 1, \dots, M^L, \\ z^L \in [z_{mindim}^L, z_{maxdim}^L]. \end{aligned}$$

### IV. 3D PLACEMENT ALGORITHM

The optimization problem described in Section III is a MOO problem. Such problems typically involve more than one objective function that conflict and there exists no one unique result but the optimal result is a set of solutions that provide the best trade-off between the objective functions called the Pareto-optimal [10] solutions. In single-objective optimization problems, the superiority of a candidate solution over others is computed by comparing the value of the objective functions evaluated with the candidate solutions. But in MOO problems, the superiority of a solution is decided by its dominance. All solutions in the feasible space that are non-dominated belong to the Pareto-optimal front. Our optimization problem is an Mixed Integer Nonlinear Programming (MINLP) problem since it has both integer  $\alpha_l$  and real  $(x, y, z)_l$  variables. Typically, MINLP problems are mathematically intractable.

To solve this MINLP, we propose a genetic algorithm. Genetic Algorithms (GAs) or Evolutionary algorithms are meta-heuristic algorithms that operate on a set of candidate solutions and select the fittest candidates from each generation and these are reproduced to create the candidates for the next generation. A common and powerful MOO algorithm based on the genetic algorithm is the Non-dominated Sorting Genetic Algorithm (NSGA-II) [11] which classifies the solutions into multiple non-dominated sets. The proposed solution method consists of the following components.

1) Population: The population is composed of all possible solutions. The algorithm starts with an initial population

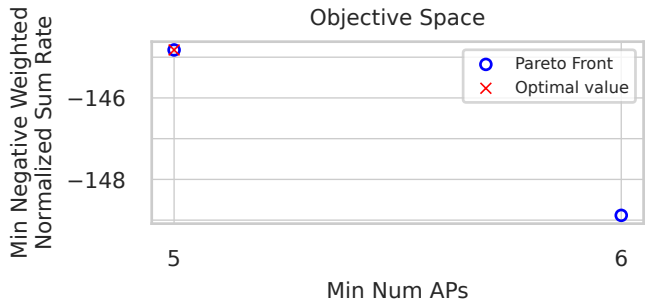


Fig. 4. An exemplary Pareto-front for the 3D Placement Optimization

that is randomly sampled from integer and real values within the bounds described by the optimization problem. We set the initial population size to 100 candidate solutions.

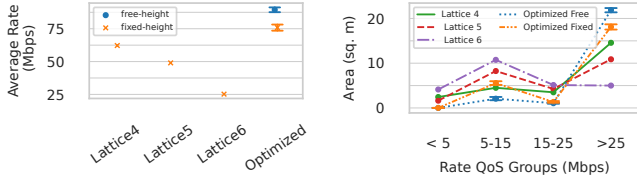
- 2) Selection: The individuals that will be carried to the next generation are selected by their fronts. The individuals are grouped into fronts by their fitness values which are decided by evaluating the objective functions with these individuals. The fronts are then ranked according to their level of non-dominance. The best individuals are then selected by comparing their rank and a second metric called the crowding distance which is decided by the density of solutions around each candidate individual.
- 3) Crossover: The selected individuals are then combined using Simulated Binary Crossover (SBX) [12] operator to produce offsprings that will form a part of the population for the next generation. The offsprings respect the bounds for the variables set in Section III.

Generally, in metaheuristics, constraints can be added to the objective function as a penalty. The individuals are made undesirable if they violate constraints since the value of the objective function, and hence the fitness of the individual, is penalized. The algorithm converges when the constraints are satisfied and the solutions belong to the Pareto-optimal front. The optimization terminates when the algorithm converges.

An exemplary run with two solutions in the Pareto-front are shown in Fig. 4. The objective space is defined by the two objective functions which are minimizing the number of APs and maximizing the weighted normalized sum rate which can also be formulated as a minimization of the negative of the sum rate. In order to select a unique solution from the Pareto front, a Multi-criteria decision-making method is employed. The two objectives are given weights according to their importance. We set these weights to be 0.8 for the cost minimization objective and 0.2 for the rate maximization objective since we want to place more importance on the number of access points used. Then each solution in the Pareto front is assigned a pseudo weight corresponding to its normalized distance to the worst solution of each objective function. The solution that has pseudo weights closest to the objective weights is considered to be the optimal solution.

TABLE I  
SIMULATION PARAMETERS

Parameter	Abbreviation	Value
Power of a LiFi LED	$P_L$	5 W
Half power beam width	$\theta_{1/2}$	60°
Physical area of the receiver	$A_p$	$10^{-4} \text{ m}^2$
FoV of the receiver	FoV	90°
Noise spectral density	$N_L$	$10^{-21} \text{ A}^2 \text{ Hz}^{-1}$
Modulation bandwidth of LED	$B_L$	20 MHz
Maximum supported Rate	$R_{max}^L$	250 Mbps
Illumination constant	$I_0$	$0.73 \times \theta_{1/2} \times \theta_{1/2}$



(a) Average rate achieved (b) Coverage area per rate level  
Fig. 5. Results for varying placement models in a Regular room

## V. EVALUATIONS AND DISCUSSIONS

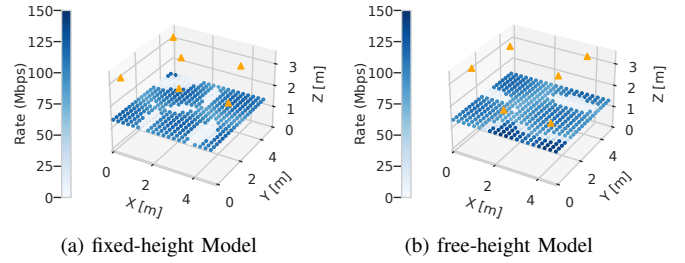
The 3D placement optimization problem is solved using the NSGA-II algorithm described in Section IV and the extensive evaluations performed are described in this section. The simulation parameters are detailed in Table I.

To validate the need for the optimization we consider a few baseline scenarios where the LEDs are placed in a deterministic pattern on the ceiling. One such pattern is the lattice grid pattern with 4 LEDs as shown in Fig. 2. We also consider the same lattice grid pattern with 5 and 6 LEDs for comparison. The placement is optimized for the regular 5 m x 5 m room scenario with a rate ratio requirement of 0.01 and an illumination requirement of 300 lux. The average rate in Mbps in the room after placement is shown in Fig. 5a along with the values for the deterministic models. The average rate is plotted with error bars representing the 95% confidence intervals. Increasing the number of access points in the deterministic model only leads to a lowering of the rate due to higher interference. This result clearly shows the need for optimization as the average rate achievable is much higher as compared to the deterministic models. Looking into this result in more detail, we split the results obtained into 4 rate groups and plot the area of the room covered with the rate range of each group. This result for the different strategies is shown in Fig. 5b. The deterministic scenarios result in areas where the rate achieved is lower than 5 Mbps while the optimized scenarios do not result in such low values of rate. The optimized placement also results in much larger areas for data rates higher than 25 Mbps.

The scenarios considered for the rest of the evaluations are summarized in Table II with two different illumination requirements at work desks and elsewhere. The pitch angle  $\Theta^P$  of the user is either 0° or 28° for sitting and standing users

TABLE II  
SCENARIO PARAMETERS

Scenario	Size	$\bar{R}_{thresh}^L$	$I_{thresh}^L$	$(z_{min}^L, z_{max}^L)$
Regular	5 m x 5 m	0.01	300 lux	(2.5 m, 3.5 m)
Office	6 m x 3.5 m	0.01	500, 200 lux	(2.5 m, 3.5 m)
Museum	8.5 m x 5.5 m	0.01	200 lux	(2.5 m, 3.5 m)
Conference	9.5 m x 7.5 m	0.01	500, 200 lux	(2.5 m, 3.5 m)
Corridor	30 m x 2.5 m	0.01	300 lux	(2.5 m, 3.5 m)



(a) fixed-height Model (b) free-height Model  
Fig. 6. 3D Positions of APs and Rate coverage in a regular room

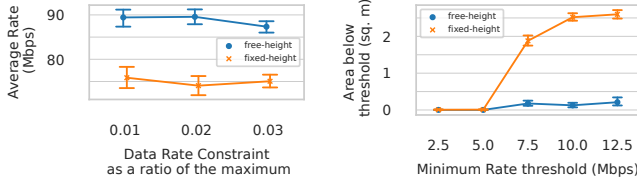
respectively. The pitch angle of 28° is the typical measured value for standing users [13].

Looking further into the results comparing free and fixed height models for the regular scenario in Fig. 6, we see that the fixed-height model has placed all APs very close to the ceiling in an attempt to maximize the rate. However, the free-height model was able to leverage the height difference among the APs to minimize the interference regions and achieve a higher rate.

Fig. 7a shows the average rate achieved in Mbps for various minimum rate requirements in a regular room. In all cases the free-height model performs better than the fixed-height model. This behavior is also confirmed with t-tests, the results of which are in Table III. The ratio of 0.03 corresponds to an absolute rate of 7.5 Mbps. In Fig. 7b, it is clear that by increasing the minimum rate requirement further, the free-height model is still able to provide this coverage resulting in a negligible outage area. Whereas, the fixed-height model struggles with this. Even with a minimum requirement of 12.5 Mbps, the free-height system is still able to provide almost full coverage with the average being much higher at 87 Mbps. This demonstrates the role the height of the AP has to play in the network performance.

We then move on to analyzing the effect of the user occurrence probabilities in an indoor environment. For this purpose, the regular room is adapted such that half the user plane has a low probability of 0.2 and the other half with a higher weight of 0.8. To avoid any unwanted artifacts in the results we also consider another scenario where the user plane is flipped, starting with a probability of 0.8 and ending with 0.2. Then, the average rate achieved in each of these probability groups is displayed in Fig. 8. We see that our proposed 3D Placement framework always maximizes the rate in the areas where the users are more likely to be present. This illustrates the





(a) Average rate achieved (b) Outage Area in sq.m

Fig. 7. Results for varying minimum rate requirements in a regular room

TABLE III

RESULTS OF TWO-SIDED T-TEST FOR A SIGNIFICANCE LEVEL OF 0.05 TO DEMONSTRATE THAT THE AVERAGE RATE ACHIEVABLE BY THE FREE HEIGHT MODEL IS SIGNIFICANTLY GREATER THAT THAT OF FIXED HEIGHT

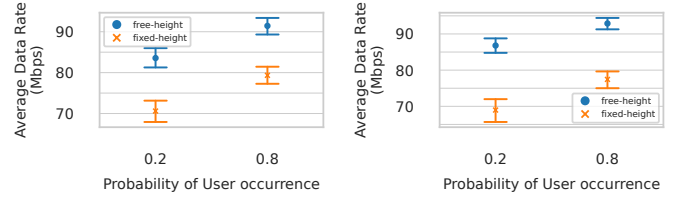
Rate Req.	t-value	p-value
0.01	8.812	6.270e-16
0.02	10.821	9.746e-22
0.03	12.490	8.827e-27

importance of considering user occurrence probabilities while optimizing the LED placement.

To see the planning that would be needed for various indoor scenarios we optimize the 3D placement for all considered scenarios and plot the number of APs needed and the average rate achievable in such a room. The parameters of the scenarios are given in Table II and the results are shown in Fig. 9. These results show that the number of APs placed is very similar for the free-height and fixed-height model but the free-height model still achieves a higher average rate. This clearly demonstrates the importance of the height of an AP and the need to consider this while optimizing the placement. Thus, the Optimized 3D placement framework proposed can be used to plan the LiFi network for any indoor scenario with various user distribution patterns.

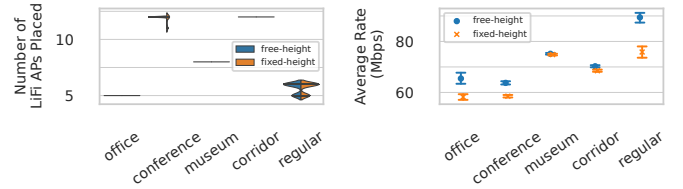
## VI. CONCLUSION

This work analyses the LiFi AP 3D Placement problem with the objectives of minimizing the number of APs placed as well as maximizing the user occurrence probability-weighted sum rate. The analysis is performed for both the free-height as well as fixed-height models. The placement problem is formulated as a multi-objective optimization problem with constraints on the minimum achievable data rate of each user position and minimum illumination level. This is then solved using the proposed NSGA-II-based algorithm. Simultaneously solving for the conflicting objectives is shown to be a necessity for optimized network performance. The proposed optimization framework is extensively evaluated using simulations and the results show that the height of each AP, greatly influences the network performance. Furthermore, we conclude that a free selection of the height of each AP provides an average rate that is significantly more than when all the APs are placed at the same height. Finally, we also see clearly that, considering the expected user distribution while evaluating the sum rate



(a) User Occurrence (0.2, 0.8) (b) User Occurrence (0.8, 0.2)

Fig. 8. Effect of user distribution in regular rooms with the first half of the user plane weighted with the first probability value in the tuple and the second half with the second value of the tuple



(a) Number of APs (b) Average Rate achievable

Fig. 9. Comparison of scenarios

objective, maximizes the network performance in areas where the user is likely to be.

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