CONSTRAINED QAOA USING AUTOENCODERS

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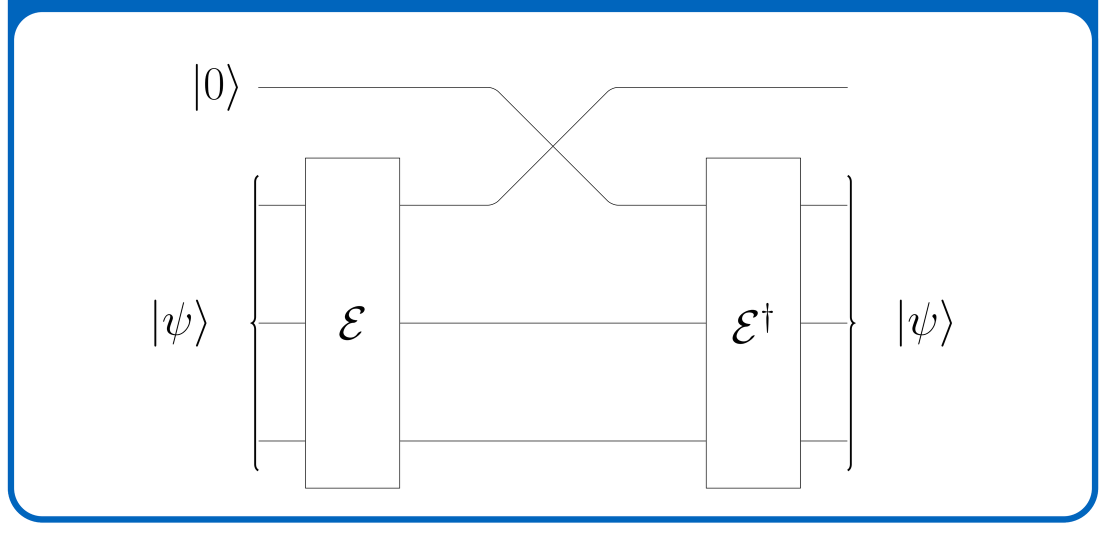
Motivations

The Quantum Approximate Optimization Algorithm (QAOA) [1] is a variational quantum algorithm used for solving combinatorial optimization problems. Usually, the solutions to such an optimization problem are provided as a binary string, therefore any computational basis state can be considered as one solution to the optimization problem. In the algorithm, the application of a problem-specific cost Hamiltonian achieves phase separation, in relation to the costs of each solution. The algorithm also includes another Hamiltonian called the mixer, defined as a Hermitian matrix that does not commute with the cost Hamiltonian. The mixer Hamiltonian tries to mix the probabilities of all possible solutions as a way of exploring the whole solution space and avoiding potential local minima.

In recent work, Hadfield [2] showed that the role of mixer Hamiltonians can be expanded when there are constraints involved in the optimization problem. Preparing according to the hard constraints of a problem, the mixer is ensured, not only to explore the whole solution space but to also restrict it according to the corresponding constraints. However, preparing a mixer can be cumbersome, since it requires distinct ansatz for different problems and constraints. **Here, we propose an alternative based on an encoding scheme that shrinks the search space into the subspace of feasible solutions.**

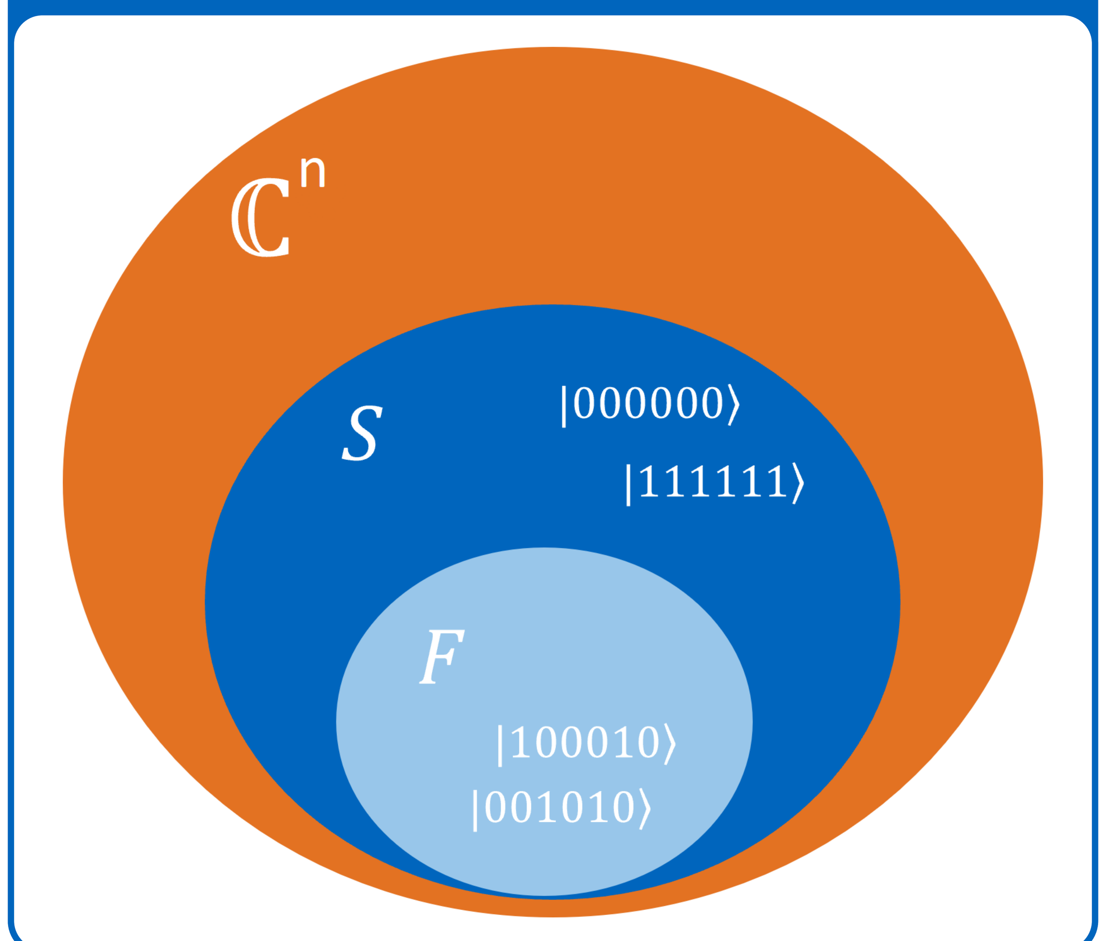
Quantum Autoencoders

General idea

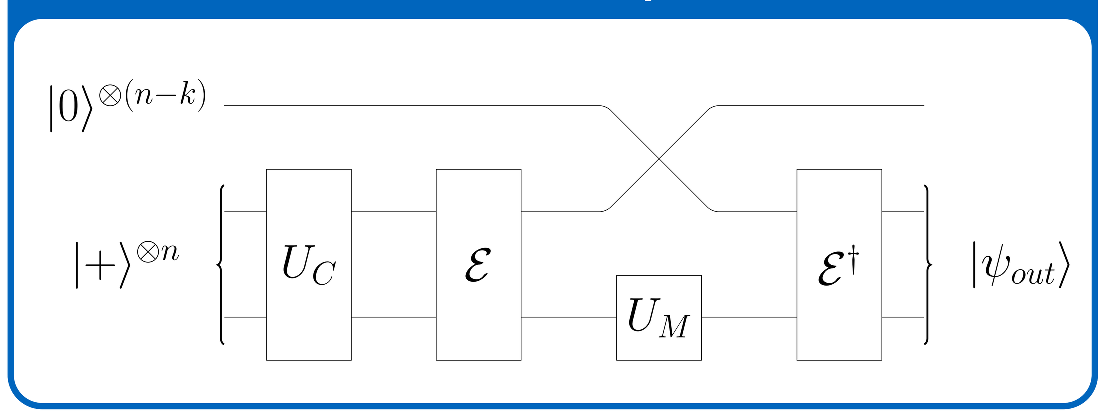


Constrained QAOA

Solution subspaces



QAOA step



Results

We test our approach on the Travelling Salesman Problem. Due to computational constraints, the experiments are run on a 3-vertex graph, with weights as shown on the right-hand side. The states that refer to the most optimal routes are encoded as: 010 100 001 and 010 001 100, that refer to the routes $1 \to 0 \to 2$ and $2 \to 0 \to 1$ respectively, which are the only possible routes that do not include the edge that connects the nodes 1 and 2.

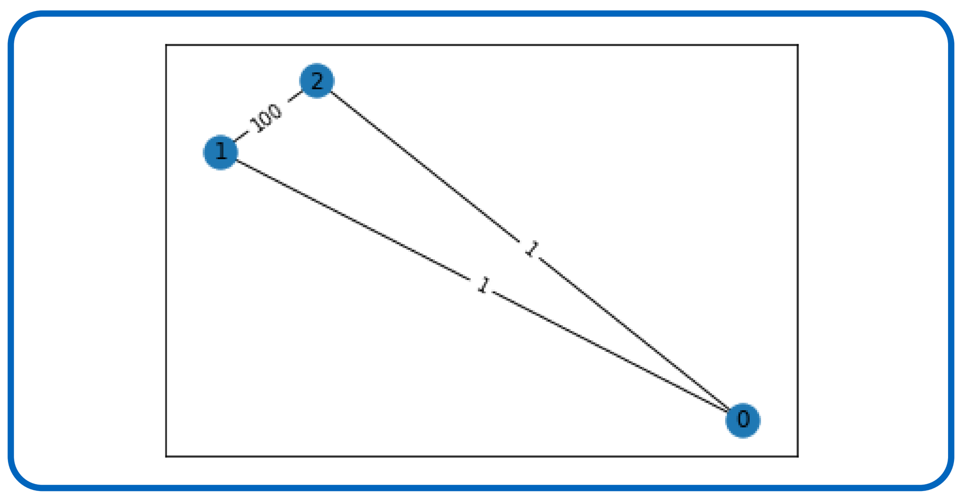


Fig. 1

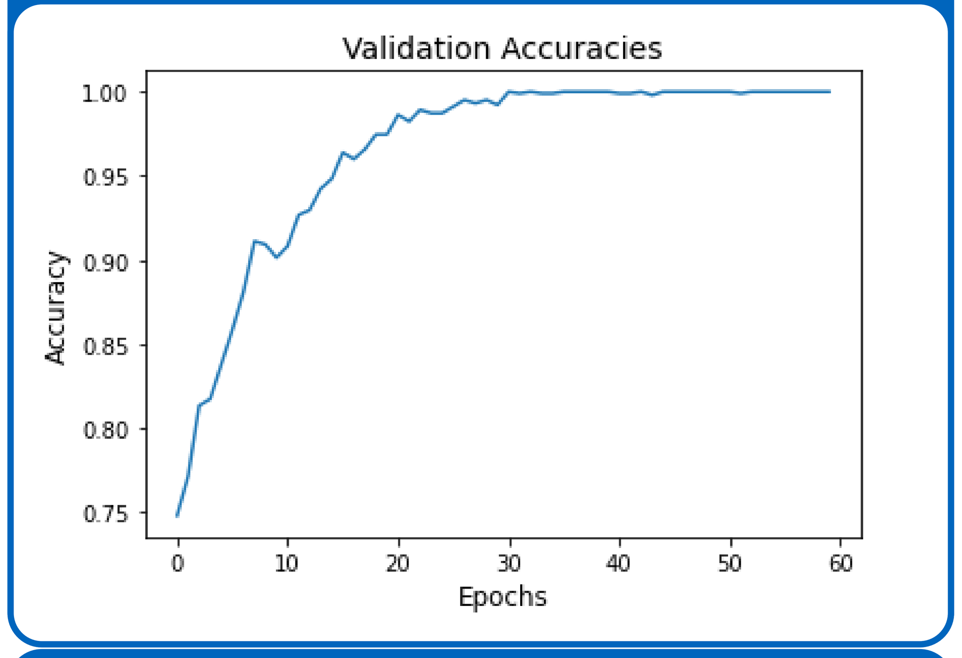
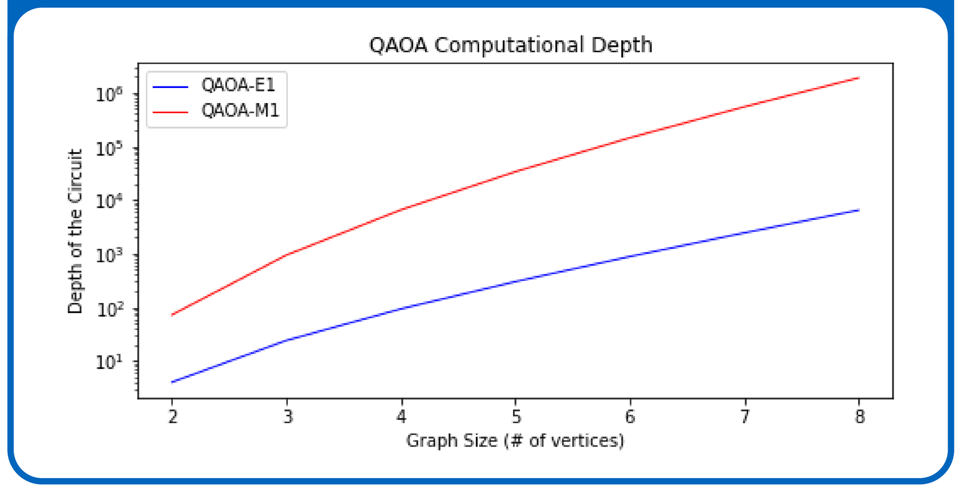


Fig. 2

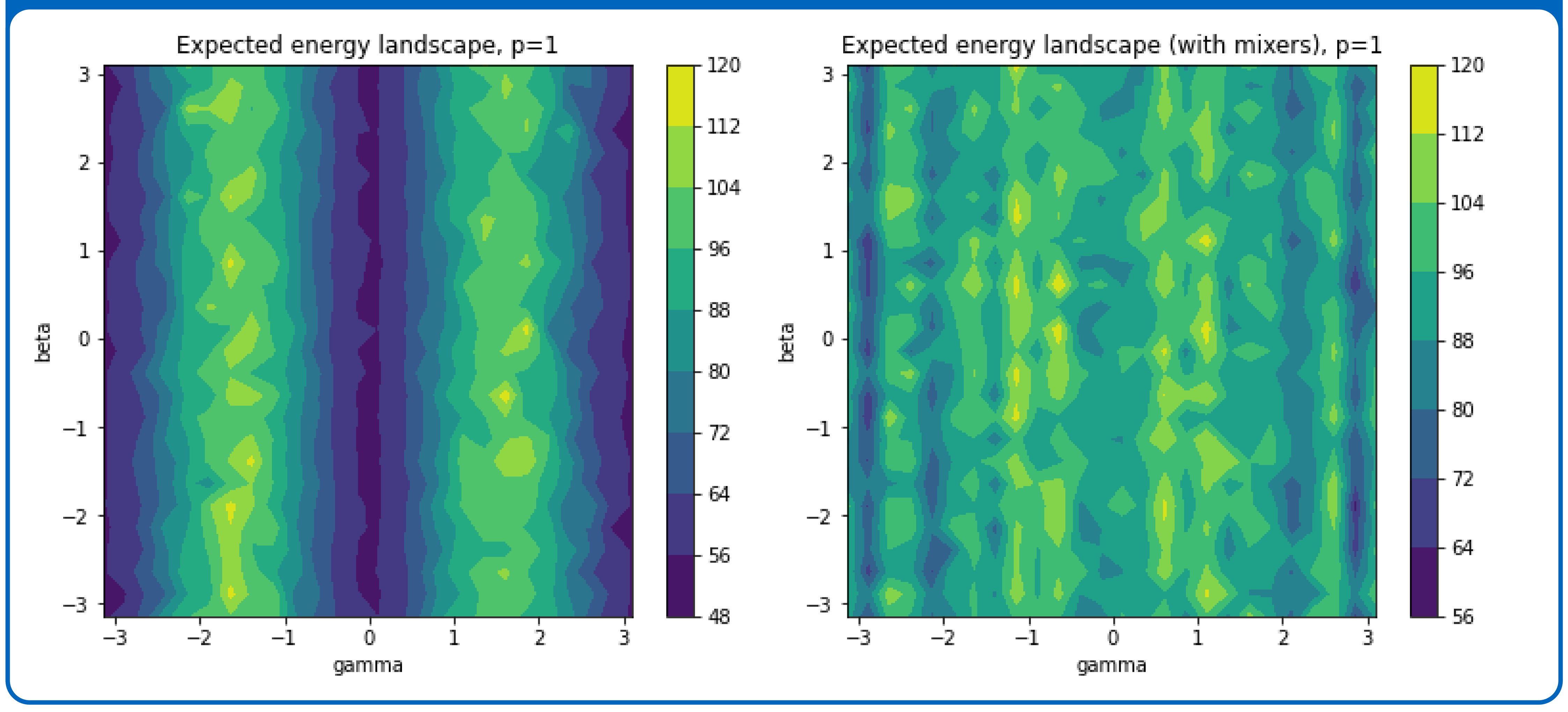


The QAOA ansatz proposed in this work, which uses encoders, is abbreviated in this section as "QAOA-E", whereas the method proposed in [2] is referred to as "QAOA-M". The final sample is selected using the majority vote approach. Fig. 1 shows the accuracy of the solutions obtained with QAOA-E on our toy-model problem.

Fig. 2 shows the expected circuit depth for a single layer using both approaches.

As it can be seen on Fig. 3, in QAOA-E, the parameters of the mixer unitaries are not decisive factors in the expectation value of the cost Hamiltonian. Instead, only the parameters of the cost unitaries are critical in finding the optimal configuration. Because the projection of the states onto the feasible subspace is done by the non-variational encoder sub-circuit, the mixers only have a limited role, that is creating an exploration mechanism over the states that are already in the feasible subspace.

Fig. 3



Conclusion

QAOA-E is a general-purpose method, which is shown to work efficiently on small-scale examples. When compared to QAOA-M, our method shows a smoother optimization landscape. Furthermore, the circuit required for QAOA-E is significantly shallower, which could avoid the problem of barren plateaus in the optimization, and makes the circuit less vulnerable to noise.

However, one disadvantage of QAOA-E is that it utilizes ancillary qubits while forming the latent space representation. QAOA-M does not use any ancillary qubits, which is generally preferred due to the limitations of real quantum hardware.

References

[1] E. Fahri et al. "A quantum approximate optimization algorithm". In: *arXiv preprint* 1411.4028 (2014).

[2] S. Hadfield et al. "Quantum approximate optimization with hard and soft constraints". In: *Proceedings of the Second International Workshop on Post Moores Era Supercomputing* (2017), pp. 15–21.

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