Formalization of Intersection Traffic Rules in Temporal Logic

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Abstract— **Intersections are difficult to navigate for both human drivers and autonomous vehicles because several diverse traffic rules must be considered. In addition, current traffic rules are ambiguous and cannot be applied directly by autonomous vehicles. Therefore, national traffic rules must be concretized and formalized so that they are machine-interpretable. We present formalized intersection traffic rules in temporal logic and use the German traffic regulations as a concrete example. Our formalization considers different types of intersections, i.e., signalized, traffic-sign-regulated, and unregulated intersections. We also define predicates and functions that can be easily reused for other national traffic laws. We evaluate our formalized traffic rules on recorded real-world scenarios and manually-created test scenarios. Our evaluation validates the formalization from different legal sources.**

I. INTRODUCTION

Intersections are among the most complex locations for human drivers and autonomous vehicles to navigate. One of many reasons is the large variety of intersections, e.g., signalized, traffic-sign-regulated, or unregulated intersections. In contrast to other road types like interstates, all kinds of road users are present at intersections, including pedestrians and cyclists. Furthermore, traffic participants may arrive from different directions and may be occluded by obstacles. Therefore, the number of rules and their complexity are much higher than in other road categories. In some countries, it might even be the case that the right of way cannot be determined, e.g., when all traffic participants arrive at an uncontrolled intersection simultaneously.

In 2019, 18.9% of all car accidents[1] in Germany were caused by vehicles turning, performing a U-turn, reversing, entering the flow of traffic, and starting off from the side of the road; 17.6% of the accidents were caused by priority and precedence mistakes. These two categories of accidents occur mainly at intersections. Only 16.1% of accidents were caused by a too small distance and 10.8% by an inappropriate speed. As one can imagine, preventing safe distance and velocity-related accidents is rather simple as one only must maintain a safe distance or reduce the velocity in contrast to adhering to many rules in intersections.

This paper addresses the importance of rule compliance in intersections by formalizing traffic rules in a machine-interpretable way for autonomous vehicles. The formalized rules can be integrated into motion planners of automated vehicles or can be used to analyze recorded traffic data.

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[1]https://www.destatis.de/EN/Themes/Society-Environment/Traffic-Accidents/_node.html

A. Related Work

Several publications have formalized traffic rules for safe distance [1], lane changing [2], overtaking [3], two-way roads [4], interstates [5]–[7], uncontrolled intersections [8], and general intersections [9]. They formalized the rules using German traffic law, US state laws, or the Vienna Convention on Road Traffic. Only in [3] and [6], judicial decisions and feedback from lawyers were considered in the formalization process. The authors of [10]–[12] formalized the so-called Responsible-Sensitive Safety rules (RSS) [13]. However, the RSS rules have no legal basis since they are not based on national law.

Traffic rules can be formalized using different logics, such as answer set programming (ASP) [8], propositional logic [9], or ontologies [14], [15]. However, these logics do not directly integrate time relations necessitating a workaround, such as hiding temporal information within predicates or atomic propositions. Linear temporal logic (LTL) and co-safe linear temporal logic (scLTL) are used to formalize rules in [1]–[5], [16]. However, LTL and scLTL cannot model a duration, limiting their applicability to a smaller set of rules. In [6], [7], [10]–[12], the rules are formalized in metric temporal logic (MTL) or signal temporal logic (STL). Users can model duration with both logics.

Formalized traffic rules can be used in different application domains. The rules can be integrated into the control strategy of a trajectory planner [16], [17] or used to monitor trajectories as described in [6], [10], [11]. It is also possible to consider rules when computing the reachable sets of automated vehicles [18]. The rule-compliant reachable sets can therefore be directly considered by a trajectory planner. Rulebooks [19] allow for the prioritization of traffic rules which facilitate decision making when a rule must be violated to satisfy another. The authors of [7] predict rule violations of traffic participants using neural networks. This information can be integrated into the motion prediction of other traffic participants. In [9], the implemented code of motion planners is annotated and evaluated to whether it complies with formalized rules. The approach can also provide a counterexample if a rule is violated. However, this method can only process the code itself and cannot be used within a real vehicle.

None of the cited papers formalize intersection traffic rules for different types of intersections using temporal logic, several legal sources, and the feedback of lawyers.

B. Contributions

This paper is an extension of our previous work on formalizing traffic rules for interstates [6] by addressing the following aspects:

1) Defining a road network for intersections.
2) Providing formalized traffic rules based on several legal sources in temporal logic for different types of intersections.
3) Defining a set of predicates and functions in higher-order logic (HOL), which can be used to formalize different traffic rules and can be easily adapted to different national requirements.
4) Using more than 2000 recorded and simulated traffic participants to evaluate intersection rules.

This paper is structured as follows: after introducing the road network and other necessary information in Sec. II-IV, we present the required predicates and functions and formalize the intersection traffic rules in Sec. V-VI. The traffic rules are evaluated in Sec. VII and the paper is concluded in Sec. VIII.

II. Legal Information and Assumptions

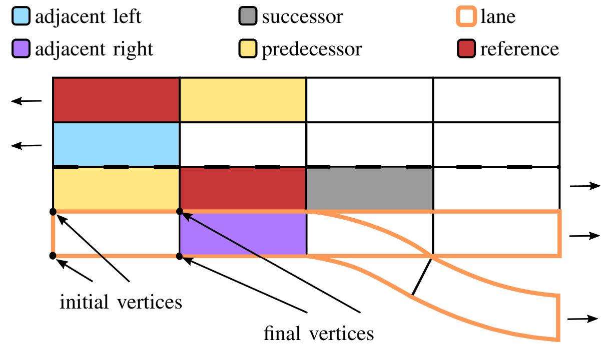
The formalized rules in this work are defined from the perspective of an autonomous vehicle, which we refer to as ego vehicle. Other traffic participants can also be autonomous vehicles or driven by humans. The legal sources for the traffic rules are the German traffic regulations (StVO), judicial decisions, comments in law literature, and feedback from lawyers. The translation of the StVO used for this paper is based on the *German Law Archive*[2]. Due to space limitations, we only considered a limited set of possible road networks and types of traffic participants. We have the following assumptions:

- We do not consider signs given by police officers.
- We assume that only one intersection is within the road network in the evaluated scenario. However, this does not limit our formalization when applying the rules in a real vehicle since we can update the evaluated scenario frequently.
- We do not consider crossings, pedestrians, cyclists, railroad vehicles, and buses.
- The ego vehicle has an unobstructed view over the intersection.
- We assume right-hand traffic. However, our formalization can be easily adapted to left-hand traffic.
- The considered rules address the central parts of intersection traffic rules. Therefore, they do not cover all possible situations at intersections, e.g., congestion, waiting within an intersection, behavior before an intersection, among others.

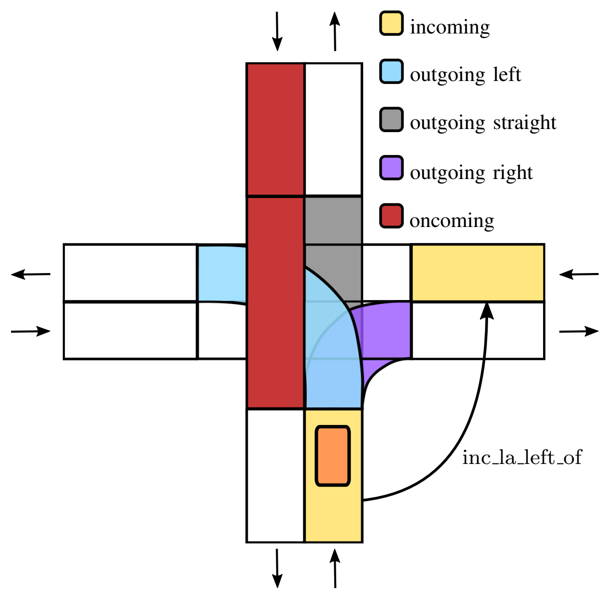
III. Road Network and Regulatory Elements

Subsequently, using the functions defined in Tab. I, we describe our model for road networks. The road network is based on the CommonRoad [20] format and is an extension of the road network described in our previous work [6]; both are based on lanelets [21].

[2]https://germanlawarchive.iuscomp.org/?p=1290



(a) A road network with opposite driving directions. The lanelet relationships are shown based on two reference lanelets. Additionally, the initial and final vertices of a lanelet and two lanes with a common part are highlighted.



(b) A schematic intersection with labels assigned from the orange vehicle's perspective.

Fig. 1: Schematic road networks.

A. Lanelet and Lane

The road network is defined by a set of lanelets $\mathcal{L}\cup\{\bot\}$, with the bottom element ($\bot$) being used if no lanelet exists. A lanelet is defined by a left and right boundary given by a polyline [21], where the direction of a lanelet is implicitly defined by its boundaries. A single lanelet is denoted by $l\in\mathcal{L}$ in the following. The functions $\mathrm{l_{ini}}(l)$ and $\mathrm{l_{fin}}(l)$ return the two initial and final vertices of a lanelet, the functions $\mathrm{l_{lb}}(l)$ and $\mathrm{l_{rb}}(l)$ return the left and right boundaries of a lanelet, and the function $\mathrm{l^{l}_{fin}}(l)$ returns the final vertex of the left boundary of a lanelet. The interconnection between lanelets can be described by their successors and predecessors as well as their adjacent left and adjacent right lanelet, where $\mathrm{suc}(l)$, $\mathrm{pre}(l)$, $\mathrm{adj_r}(l)$, and $\mathrm{adj_l}(l)$ return the successor, predecessor, right adjacent, and left adjacent lanelets of a lanelet, respectively (cf. Fig. 1a and Tab. I), where adjacent lanelets must be of the same length, which has no limitation since lanelets can be arbitrarily short.

A lane is a combination of lanelets that are connected to their predecessor and successor lanelets. To determine all lanes to which a given lanelet belongs, we first create a set of all predecessor lanes by recursively following the predecessors of the given lanelet (cf. function $\mathrm{lanes}_{\mathrm{pre}}(la, l)$ in Tab. I), where $la \subseteq \mathcal{L}$. If a lanelet with multiple successors is reached, a lane for each successor is created by appending the common predecessors. Analogously, a set of all predecessor lanes is created (cf. function $\mathrm{lanes}_{\mathrm{suc}}(la, l)$ in Tab. I). Finally, we combine every successor lane with every predecessor lane (cf. function $\mathrm{lanes}(l)$ in Tab. I) to generate the set of all lanes spanned by a lanelet. For example, if $\mathrm{lanes}_{\mathrm{pre}}(\emptyset, 1)$ and $\mathrm{lanes}_{\mathrm{suc}}(\emptyset, 1)$ return $\{\{1,2\},\{1,3\}\}$ and $\{\{1,4\},\{1,5\}\}$, respectively, $\mathrm{lanes}(1)$ returns $\{\{1,2,4\}, \{1,2,5\}, \{1,3,4\}, \{1,3,5\}\}$, where the numbers represent lanelets. To obtain the spatial occupancy of a lane, lanelet, or lanelet boundary, we define the operator $\mathrm{occ}(l)$, where l is the corresponding object. The function $\mathrm{reach}(l)$ (cf. Tab. I) returns the set of lanelets reachable from a given lanelet l using the successor and predecessor relationships, e.g., for the purple lanelet in Fig. 1a all lanelets included in the orange lanes are part of its reachable lanelets. Analogously, the sets for the reachable predecessor and successor lanelets are defined (cf. Tab. I).

B. Stop Line, Traffic Signs, and Traffic Lights

A lanelet can also reference regulatory elements, such as a stop line, traffic signs, or traffic lights. We assume that regulatory elements are always referenced by incoming lanelets. A stop line is modeled as a line segment. The function $\mathrm{stop\_line}(l)$ returns a set containing two Cartesian points $p_1, p_2 \in \mathbb{R}^2$ defining the position of a stop line for a given lanelet, where a lanelet can only reference a single stop line. This has no limitation since lanelets can be arbitrarily short.

A traffic sign is defined by its ID according to the StVO. Tab. II lists the symbols, IDs and the associated legal texts of the traffic signs used in this work. The set of traffic sign IDs is denoted as $\mathcal{S}$ and the set of traffic sign elements as $\mathcal{TS}$. The function $\mathrm{traffic\_sign}(l, ty)$ and $\mathrm{traffic\_sign\_type}(l)$ return the traffic sign $ts \in \mathcal{TS}$ of a given type $ty \in \mathcal{S}$ assigned to the lanelet l and the set of traffic sign types assigned to a lanelet. Each lanelet can only reference a single traffic sign of the same type. To infer whether a vehicle has the right of way, we must determine the priority of each lanelet occupied by a vehicle. We assign a priority value for each driving direction to a traffic sign, with a higher value indicating a higher priority. The priority values are derived from the StVO using the following rules:

- Priority roads have a higher priority than other roads.
- Roads with no traffic sign or sign 102 yield higher priority than roads with traffic signs prompting to wait (signs 205 and 206).
- Stop signs yield the same priority for all directions, lower than for all other traffic signs, except the green arrow.
- The green arrow sign has the lowest priority.

Additionally, traffic signs are assigned an evaluation index that defines the relevance of the sign in case several traffic signs are valid simultaneously for a vehicle. The signs are ordered so that supplementary signs have a lower evaluation index than general signs (e.g., 1002-10 has a lower evaluation index than 306). Tab. II shows the resulting priority values and evaluation index for each traffic sign. The functions $\mathrm{argmin\_s}(ts)$ and $\mathrm{get\_priority}(i, dir)$ return the minimum evaluation index of a set of traffic sign IDs $ts \subseteq \mathcal{S}$, and the priority for turning into the direction $dir \in \{left, straight, right\}$ (cf. Tab. IV), given the evaluation index $i \in \mathbb{N}_0$. A lanelet without a priority traffic sign in front of an intersection is interpreted as referencing sign 102. We can infer which vehicle has the right of way at an intersection by comparing the priority values of two vehicles, e.g., a vehicle with priority 5 has the right of way over a vehicle with priority 2.

A traffic light is described by a turning direction and its current state. The set of traffic lights is defined as $\mathcal{TL}$, the set of traffic light states as $\mathcal{C}$ (cf. Tab. III and Tab. IV), the set of all possible combinations of turning directions as $\mathcal{D}$, and the set of possible right turn directions as $\mathcal{RD}$, where the sets for the left and straight directions ($\mathcal{LD}$, $\mathcal{SD}$) are defined analogously. The functions $\mathrm{traffic\_light}(l)$, $\mathrm{dir}_{\mathrm{tl}}(tl)$, and $\mathrm{state}_{\mathrm{tl}}(tl)$ return the traffic light $tl \in \mathcal{TL}$ associated with a lanelet, the direction of a traffic light, and the state of a traffic light tl, respectively. Each lanelet can only reference a single traffic light. The functions $\mathrm{stop\_line}(l)$, $\mathrm{traffic\_sign}(l, ty)$, $\mathrm{traffic\_sign\_type}(l)$, and $\mathrm{traffic\_light}(l)$ return $\emptyset$ if the lanelet does not reference the corresponding regulatory element or if $l = \perp$, where $\emptyset$ indicates that the corresponding regulatory element is not referenced.

C. Intersection

Fundamental elements in road networks are intersections. We define an intersection by assigning special intersection labels $\mathcal{I}$ to a lanelet that is part of an intersection. The labels describe an intersection from a vehicle perspective and are listed in Tab. IV and shown in Fig. 1b. The lanelets after an incoming lanelet until an outgoing lanelet are labeled with type *intersection* and an additional type indicating the turning direction (*leftTurn, rightTurn, goingStraight*). The function $\mathrm{type}(l)$ can be used to access the intersection labels of a lanelet and the function $\mathrm{inc\_la\_left\_of}(l)$ returns the set of counterclockwise-located lanelets of type *incoming* with respect to a lanelet with type incoming, where the function returns $\emptyset$ if $l = \perp$ or if l is no incoming lanelet. The outgoing and oncoming lanelets are specified for each incoming lanelet individually. The function $\mathrm{oncom}(l)$ returns the set of oncoming lanelets belonging to an incoming lanelet l or returns $\emptyset$ if l is no incoming lanelet.

IV. METHODOLOGY

A. Vehicle Configuration

In this subsection, we briefly introduce our vehicle configuration. For more details, we refer the reader to [6]. We use a curvilinear coordinate system with the transformation from Cartesian to curvilinear coordinates as described in [22]. We choose the centerline of lane that the traffic

TABLE I: Functions for extracting specific elements from a road network.

Name	Formalization
Predecessor/Successor lanelets	$\mathrm{pre}(l) = \{l' \in \mathcal{L} \mid \mathrm{l_{ini}}(l) = \mathrm{l_{fin}}(l')\},\ \mathrm{suc}(l) = \{l' \in \mathcal{L} \mid \mathrm{l_{fin}}(l) = \mathrm{l_{ini}}(l')\}$
Adjacent left lanelet	$\mathrm{adj_l}(l) = \begin{cases} l' & \text{if } l' \in \mathcal{L} \wedge \mathrm{occ}(\mathrm{l_{lb}}(l)) \subseteq \mathrm{occ}(l') \ \wedge \big(\mathrm{l_{ini}}(l) \cap \mathrm{l_{ini}}(l') \neq \emptyset \wedge \ \mathrm{l_{fin}}(l) \cap \mathrm{l_{fin}}(l') \neq \emptyset \\ & \quad \vee\ \mathrm{l_{ini}}(l) \cap \mathrm{l_{fin}}(l') \neq \emptyset \wedge \mathrm{l_{fin}}(l) \cap \mathrm{l_{ini}}(l') \neq \emptyset\big) \\ \bot & \text{otherwise} \end{cases}$
Adjacent right lanelet	$\mathrm{adj_r}(l)$ - Identical to $\mathrm{adj_l}(l)$, except that $\mathrm{l_{lb}}(l)$ is replaced by $\mathrm{l_{rb}}(l)$.
Lane predecessors	$\mathrm{lanes_{pre}}(la, l) = \begin{cases} \Big(\bigcup_{pre \in \mathrm{pre}(l)} \big(\mathrm{lanes_{pre}}(la \cup \{l\}, pre)\big)\Big) & \text{if } \ \mathrm{pre}(l) \neq \emptyset \wedge l \notin la \\ \{la \cup \{l\}\} & \text{otherwise} \end{cases}$
Lane successors	$\mathrm{lanes_{suc}}(la, l)$ - Identical to $\mathrm{lanes_{pre}}(la, l)$, except that $\mathrm{pre}(l)$ is replaced by $\mathrm{suc}(l)$.
Lanes	$\mathrm{lanes}(l) = \{p \cup s \mid p \in \mathrm{lanes_{pre}}(\emptyset, l) \wedge s \in \mathrm{lanes_{suc}}(\emptyset, l)\}$
Reachable lanelets	$\mathrm{reach}(l) = \bigcup \mathrm{lanes}(l),\ \mathrm{reach_{suc}}(l) = \bigcup \mathrm{lanes_{suc}}(\emptyset, l),\ \mathrm{reach_{pre}}(l) = \bigcup \mathrm{lanes_{pre}}(\emptyset, l)$

TABLE II: Overview of traffic signs considered in this work together with their left, straight, and right priority, and evaluation index. The descriptions are excerpts of the translated original legal text.

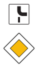
Symbol	Sign ID	Pr. Le.	Pr. Str.	Pr. Ri.	Eval. Idx.	Legal Description
	1002-10	5	4	4	1	The supplementary sign to sign 306 indicates a left turning priority road.
	1002-12	5	4	-	2	The supplementary sign to sign 306 indicates a left turning priority road.
	1002-13	5	-	4	3	The supplementary sign to sign 306 indicates a left turning priority road.
	1002-20	4	4	5	4	The supplementary sign to sign 306 indicates a right turning priority road.
	1002-22	-	4	5	5	The supplementary sign to sign 306 indicates a right turning priority road.
	1002-23	4	-	5	6	The supplementary sign to sign 306 indicates a right turning priority road.
	1002-11	2	2	2	7	The supplementary sign to sign 306 indicates the course of the priority road. No turning priority.
	1002-14	2	2	-	8	The supplementary sign to sign 306 indicates the course of the priority road. No turning priority.
	1002-21	2	2	2	9	The supplementary sign to sign 306 indicates the course of the priority road. No turning priority.
	1002-24	-	2	2	10	The supplementary sign to sign 306 indicates the course of the priority road. No turning priority.
	306	4	5	4	11	This sign indicates priority until signs 205, 206, or 307.
	301	4	5	4	12	This sign indicates priority at the next intersection.
	205	2	2	2	13	A person operating the vehicle must give way.
	206	1	1	1	14	A person operating a vehicle must stop and give way.
	102	3	3	3	15	Traffic coming from the right has priority.
	720	-	-	0	16	After stopping, traffic may turn right even if the traffic light shows red.

TABLE III: Definition of traffic light states.

State	Description
Red	Stop in front of the intersection. If the traffic light shows an arrow, this only applies to the specified direction. Traffic may turn right if sign 720 is placed at the intersection.
Yellow	Wait in front of the intersection for the next signal.
Green	Traffic may proceed only in the specified direction.
Inactive	The traffic light should be ignored.

TABLE IV: Sets defining regulatory elements and intersections.

Name and Abbreviation	Elements
Intersection labels $\mathcal{I}$	incoming, intersection, oncoming, outgoingLeft, outgoingRight, outgoingStraight leftTurn, rightTurn, goingStraight
Sign ID $\mathcal{S}$	1002-10, 1002-12, ..., 205, 206, 103, 720
Turning directions $\mathcal{D}$	left, straight, right, leftStraight, straightRight, leftRight, all
Right turning directions $\mathcal{RD}$	leftRight, straightRight, right, all
Traffic light states $\mathcal{C}$	red, yellow, green, inactive

participant's trajectory occupies for the most time steps as the reference path Γ for the curvilinear coordinate system. The six-dimensional state $x = \begin{bmatrix} s & d & v & a & \theta & \mathcal{D} \end{bmatrix}^T$ of a vehicle consists of the longitudinal position $s \in \mathbb{R}$ along the reference path, lateral distance $d \in \mathbb{R}$ to the reference path, velocity $v \in \mathbb{R}$, acceleration $a \in \mathbb{R}$, orientation $\theta \in \mathbb{R}$, and the occupied lanelets based on the driving direction of the vehicle $\mathcal{D} \subseteq \mathcal{L}$. The function $\mathrm{lanelets\_dir}(x)$ extracts $\mathcal{D}$ from a state and the operator $\mathrm{proj}_{\square}(x)$ projects x to the elements specified by the placeholder $\square$. The spatial occupancy of a vehicle shape for a given state is denoted by $\mathrm{occ}(x)$. The functions $\mathrm{front}(\Gamma, x)$ and $\mathrm{rear}(\Gamma, x)$ return the longitudinal position of the front and rear bumper of a vehicle state x with respect to a reference path. The functions $\mathrm{trans_s}(\Gamma, p)$, $\mathrm{line\_in\_front}(x, sl)$, and $\mathrm{min\_dist}(x, sl)$ compute the longitudinal position along a reference path Γ for a Cartesian point $p \in \mathbb{R}^2$, whether the line sl defined by a set of two Cartesian points $p_1, p_2 \in \mathbb{R}^2$ is in front of a vehicle, and the minimum distance from a vehicle to a line sl, respectively. We denote the variables associated with the ego vehicle by the subscript $\square_{\text{ego}}$, with other vehicles by the subscript $\square_{\text{o}}$, and with arbitrary vehicles, including the ego vehicle, either by the subscripts $\square_{\text{k}}$ or $\square_{\text{p}}$. The functions $\text{ref\_path\_lanelets}(x)$ and $\text{overappr\_braking\_pos}(x, a_{\text{min}})$ compute all lanelets belonging to the reference path and the frontmost point in the Cartesian coordinate frame of the over-approximated reachable set [23], where x is the initial state and a_{min} is the minimum acceleration. The functions $\text{turning\_left}(x)$, $\text{turning\_right}(x)$, and $\text{going\_straight}(x)$ return whether a vehicle is turning left, turning right, or going straight at an intersection.

B. Metric Temporal Logic

We use discrete-time point-based MTL [24] to include time intervals in our formalization. Given a set $\mathcal{AP}$ of atomic propositions, where each atomic proposition $\sigma_{\text{i}} \in \mathcal{AP}$ represents a Boolean statement, an MTL formula ϕ is defined as [25]:

$$\phi ::= \sigma_i \,|\, \neg\phi \,|\, \phi_1 \wedge \phi_2 \,|\, \phi_1 \vee \phi_2$$
$$\phi ::= \text{G}_{\text{I}}(\phi) \,|\, \text{X}_{\text{I}}(\phi) \,|\, \text{F}_{\text{I}}(\phi) \,|\, \text{O}_{\text{I}}(\phi) \,|\, \phi_1 \text{S}_{\text{I}} \phi_2$$

where G, X, F, O, and S are temporal operators described later. The subscript I represents an interval $[lb, ub]$, where $lb, ub \in \mathbb{N}_0$ and $lb \leq ub$, expressing time constraints relative to the current time. If an operator's interval is not specified, we assume that the interval is specified until the end of the trace, i.e., $[0, \infty)$. We also require the Boolean operators $\neg$, $\wedge$, and $\vee$ with the precedence among these operators as they are listed. The implication $a \implies b$ is defined as $\neg a \vee b$. For a detailed description of the semantics, we refer the reader to [25]. We describe the semantics of the presented temporal operators informally:

- $\text{G}_{\text{I}}(\phi)$: ϕ holds globally for all future states within the time interval I.
- $\text{X}_{\text{I}}(\phi)$: ϕ holds for the next state within the time interval I.
- $\text{F}_{\text{I}}(\phi)$: ϕ holds for some future state within the time interval I.
- $\text{O}_{\text{I}}(\phi)$: ϕ holds at least once for some previous state within the time interval I.
- $\phi_1 \text{S}_{\text{I}} \phi_2$: ϕ_1 holds at least since ϕ_2 holds within the time interval I.

V. PREDICATES

Subsequently, we introduce the essential predicates and functions for the formalization of the traffic rules in Sec. VI. We categorize them in terms of position, regulatory, velocity, braking, and temporal elements.

A. Position Elements

A vehicle is on a lanelet with a specific type $ty \in \mathcal{I}$ if any occupied lanelet has the type ty:

$$\text{on\_lanelet\_with\_type}(x_{\text{k}}, ty) \iff ty \in \bigcup_{l \in \text{lanelets}(x_{\text{k}})} \text{type}(l),$$

where the occupied lanelets of a state x_{k} are defined by the intersection of the vehicle's occupancy with the occupancy of the corresponding lanelets:

$$\text{lanelets}(x_{\text{k}}) = \big\{ l \in \mathcal{L} \,\big|\, \text{occ}(l) \cap \text{occ}(x_{\text{k}}) \neq \emptyset \big\}.$$

A vehicle is on a lanelet in driving direction with a specific type if any occupied lanelet in the vehicle's driving direction has this type:

$$\text{on\_dir\_lanelet\_with\_type}(x_{\text{k}}, ty) \iff ty \in \bigcup_{l \in \text{lanelets\_dir}(x_{\text{k}})} \text{type}(l).$$

The k[th] vehicle is approaching an intersection from the left of the p[th] vehicle if the k[th] vehicle occupies a lane that is left of the lane of the p[th] vehicle in terms of incoming lanelets:

$$\begin{aligned} &\text{on\_incoming\_left\_of}(x_{\text{k}}, x_{\text{p}}) \iff \\ &\quad \exists l_{\text{k}} \in \text{lanelets\_dir}(x_{\text{k}}) : \exists la_{\text{k}} \in \text{reach}_{\text{suc}}(l_{\text{k}}), \\ &\quad \exists l_{\text{p}} \in \text{lanelets\_dir}(x_{\text{p}}) : \exists la_{\text{p}} \in \text{reach}_{\text{suc}}(l_{\text{p}}) : \\ &\qquad la_{\text{p}} \in \text{inc\_la\_left\_of}(la_{\text{k}}). \end{aligned}$$

The k[th] vehicle occupies an oncoming lanelet of the p[th] vehicle if the following predicate evaluates as true:

$$\begin{aligned} &\text{on\_oncom\_of}(x_{\text{k}}, x_{\text{p}}) \iff \\ &\quad \exists l_{\text{k}} \in \text{lanelets\_dir}(x_{\text{k}}), \exists l_{\text{p}} \in \text{lanelets\_dir}(x_{\text{p}}) : \\ &\qquad \exists la_{\text{p}} \in \text{reach}_{\text{pre}}(l_{\text{p}}) : l_{\text{k}} \in \text{oncom}(la_{\text{p}}). \end{aligned}$$

The k[th] vehicle is in the conflict area of the p[th] vehicle if it is in the lane of the p[th] vehicle but has another driving direction and the vehicles approach the intersection from different incomings:

$$\begin{aligned} &\text{in\_intersection\_conflict\_area}(x_{\text{k}}, x_{\text{p}}) \iff \\ &\quad \exists l_{\text{k}} \in \text{lanelets}(x_{\text{k}}), \exists la_{\text{p}} \in \text{ref\_path\_lanelets}(x_{\text{p}}) : \\ &\quad \mathit{intersection} \in \text{type}(l_{\text{k}}) \wedge \\ &\qquad l_{\text{k}} = la_{\text{p}} \wedge l_{\text{k}} \notin \text{lanelets\_dir}(x_{\text{k}}) \wedge \\ &\qquad \nexists la_{\text{k}} \in \text{lanelets\_dir}(x_{\text{k}}) : \text{same\_incom}(la_{\text{k}}, la_{\text{p}}), \end{aligned}$$

where $\text{same\_incom}(l_{\text{k}}, l_{\text{p}})$ evaluates whether two lanelets are part of the same intersection incoming:

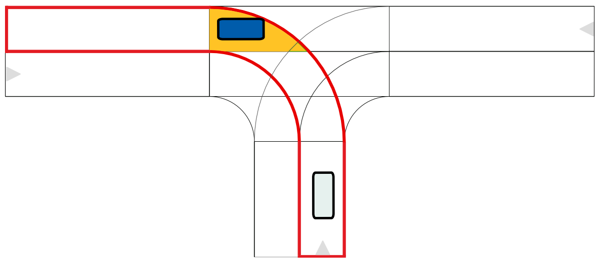
$$\begin{aligned} &\text{same\_incom}(l_{\text{k}}, l_{\text{p}}) \iff \\ &\quad \exists la_{\text{k}} \in \text{reach}_{\text{pre}}(l_{\text{k}}) : \mathit{incoming} \in \text{type}(la_{\text{k}}) \\ &\qquad \wedge \text{adj\_lanelets}(la_{\text{k}}) \cap \text{reach}_{\text{pre}}(l_{\text{p}}) \neq \emptyset, \end{aligned}$$

where $l_{\text{k}}, l_{\text{p}} \in \mathcal{L}$ and $\text{adj\_lanelets}(l)$ returns all adjacent lanelets for a given lanelet and is defined as in our previous work [6]. Fig. 2 shows the conflict area between two vehicles.

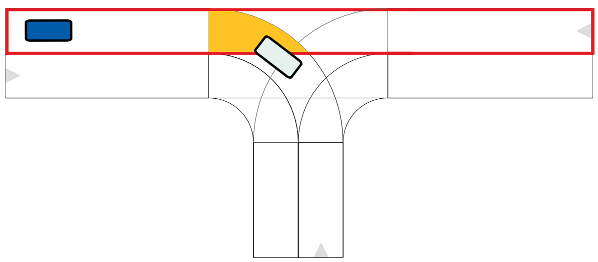
B. Priority Elements

A vehicle has to consider a specific traffic sign if any occupied lanelet references a sign of the specified type:

$$\begin{aligned} &\text{at\_traffic\_sign}(x_{\text{k}}, \mathit{signType}) \iff \\ &\quad \exists l \in \text{lanelets\_dir}(x_{\text{k}}) : \text{traffic\_sign}(l, \mathit{signType}) \neq \emptyset, \end{aligned}$$



(a) The gray vehicle approaches the intersection. The blue vehicle is in the conflict area with respect to the gray vehicle.



(b) The blue vehicle leaves the intersection. The gray vehicle is in the conflict area with respect to the blue vehicle.

Fig. 2: Intersection conflict area (yellow area) of the blue and gray vehicle for two situations. The red border indicates the lanelets belonging to the reference path of the gray (a) and blue (b) vehicle.

where $signType \in \mathcal{S}$. The k^{th} vehicle has priority (right of way) over the p^{th} vehicle if its occupied lanelets have a higher priority value:

$$\begin{aligned}&\mathrm{has\_priority}(x_\mathrm{k}, x_\mathrm{p}, dir_\mathrm{k}, dir_\mathrm{p}) \iff\\&\exists l_\mathrm{k} \in \mathrm{lanelets\_dir}(x_\mathrm{k}), \forall l_\mathrm{p} \in \mathrm{lanelets\_dir}(x_\mathrm{p}) :\\&\quad \mathrm{get\_priority}\Big(\mathrm{argmin\_s}\big(\mathrm{traffic\_sign\_type}(l_\mathrm{k})\big), dir_\mathrm{k}\Big)\\&\quad > \mathrm{get\_priority}\Big(\mathrm{argmin\_s}\big(\mathrm{traffic\_sign\_type}(l_\mathrm{p})\big), dir_\mathrm{p}\Big),\end{aligned}$$

where dir_k, $dir_\mathrm{p} \in \{left, straight, right\}$ indicate the direction which should be assumed for the k^{th} and p^{th} vehicle. Two vehicles have the same priority if

$$\begin{aligned}&\mathrm{same\_priority}(x_\mathrm{k}, x_\mathrm{p}, dir_\mathrm{k}, dir_\mathrm{p}) \iff\\&\quad \neg\mathrm{has\_priority}(x_\mathrm{k}, x_\mathrm{p}, dir_\mathrm{k}, dir_\mathrm{p})\\&\quad \wedge \neg\mathrm{has\_priority}(x_\mathrm{p}, x_\mathrm{k}, dir_\mathrm{p}, dir_\mathrm{k}),\end{aligned}$$

evaluates as true. To determine whether an upcoming intersection is regulated by traffic lights, the following predicate is introduced:

$$\begin{aligned}&\mathrm{relevant\_traffic\_light}(x_\mathrm{k}) \iff \exists l \in \mathrm{lanelets\_dir}(x_\mathrm{k}) :\\&\quad \exists succ \in \mathrm{reach}_\mathrm{suc}(l) : \mathrm{active\_tls\_by\_lanelet}(succ) \neq \emptyset,\end{aligned}$$

where $\mathrm{active\_tls\_by\_lanelet}(l)$ returns only active traffic lights referenced by a lanelet:

$$\begin{aligned}&\mathrm{active\_tls\_by\_lanelet}(l) =\\&\quad \{tl \in \mathrm{traffic\_light}(l) \mid \mathrm{state}_\mathrm{tl}(tl) \neq inactive\}.\end{aligned}$$

A stop line is in front of a vehicle if any occupied lanelet references a stop line within a distance d_sl:

$$\begin{aligned}&\mathrm{stop\_line\_in\_front}(x_\mathrm{k}) \iff\\&\quad \exists l \in \mathrm{lanelets\_dir}(x_\mathrm{k}) : \mathrm{min\_dist}\big(x_\mathrm{k}, \mathrm{stop\_line}(l)\big) < \mathrm{d}_\mathrm{sl}\\&\qquad \wedge \mathrm{line\_in\_front}\big(x_\mathrm{k}, \mathrm{stop\_line}(l)\big).\end{aligned}$$

A vehicle is at a red traffic light with a specific turning direction $dir \in \{left, straight, right\}$ and $color \in \mathcal{C}$ if the following predicate is evaluated as true:

$$\begin{aligned}&\mathrm{at\_traffic\_light}(x_\mathrm{k}, dir, color) \iff\\&\quad \exists tl \in \mathrm{active\_tl}(x_\mathrm{k}) : \mathrm{state}_\mathrm{tl}(tl) = color \wedge\\&\qquad \big(\mathrm{dir}_\mathrm{tl}(tl) \in \mathcal{RD} \wedge dir = right\\&\qquad \vee \mathrm{dir}_\mathrm{tl}(tl) \in \mathcal{LD} \wedge dir = left\\&\qquad \vee \mathrm{dir}_\mathrm{tl}(tl) \in \mathcal{SD} \wedge dir = straight\big).\end{aligned}$$

C. Velocity and Braking Elements

If the distance between the frontmost point of the p^{th} vehicle and the rearmost point of the k^{th} vehicle along the reference lane of the former is smaller than a threshold $d_\mathrm{br} \geq 0$ and the acceleration of the p^{th} vehicle is lower or equal to a threshold $a_\mathrm{br} < 0$, the k^{th} vehicle causes the braking of the p^{th} vehicle:

$$\begin{aligned}&\mathrm{causes\_braking\_intersection}(x_\mathrm{k}, x_\mathrm{p}) \iff\\&\quad 0 \leq \big(\mathrm{rear}(\Gamma_\mathrm{p}, x_\mathrm{k}) - \mathrm{front}(\Gamma_\mathrm{p}, x_\mathrm{p})\big) \leq d_\mathrm{br}\\&\qquad \wedge \mathrm{proj}_\mathrm{a}(x_\mathrm{p}) \leq a_\mathrm{br}.\end{aligned}$$

A vehicle can come to a standstill before entering an intersection without falling below an acceleration threshold if the travelled distance until the standstill is smaller than the distance to the beginning of the intersection:

$$\begin{aligned}&\mathrm{braking\_intersection\_possible}(x_\mathrm{k}) \iff\\&\quad \forall l \in \mathrm{lanelets\_dir}(x_\mathrm{k}) : \forall l_\mathrm{la} \in \mathrm{reach}_\mathrm{suc}(l) :\\&\qquad incoming \in \mathrm{type}(l_\mathrm{la}) \wedge \Big(\mathrm{trans}_\mathrm{s}\big(\Gamma_\mathrm{k}, \mathrm{l}^\mathrm{l}_\mathrm{fin}(l_\mathrm{la})\big)\\&\qquad > \mathrm{trans}_\mathrm{s}\big(\Gamma_\mathrm{k}, \mathrm{overappr\_braking\_pos}(x_\mathrm{k}, a_\mathrm{pos})\big)\Big),\end{aligned}$$

where $a_\mathrm{pos} < 0$ is a threshold for the minimum acceleration. A vehicle is at a standstill if its velocity is close to zero, where $v_\mathrm{err} \geq 0$ captures measurement uncertainties:

$$\mathrm{in\_standstill}(x_\mathrm{k}) \iff -v_\mathrm{err} \leq \mathrm{proj}_\mathrm{v}(x_\mathrm{k}) \leq v_\mathrm{err}.$$

D. Temporal Elements

Subsequently, we introduce so-called meta predicates. They combine the previously-defined predicates and temporal operators and are used to make the traffic rule formalization more compact. The definitions of the meta-predicates are listed in Tab. V.

TABLE V: Overview of the meta predicates.

Rule	MTL formula
META-1	$\mathrm{not\_endanger\_intersection}(x_\mathrm{k}, x_\mathrm{p}) ::= \Big(\mathrm{in\_intersection\_conflict\_area}(x_\mathrm{k}, x_\mathrm{p}) \implies \big(\neg\mathrm{causes\_braking\_intersection}(x_\mathrm{k}, x_\mathrm{p}) \land \neg\mathrm{F}_{[0,\mathrm{t_{ib}}]}\big(\mathrm{in\_intersection\_conflict\_area}(x_\mathrm{p}, x_\mathrm{k})\big)\big)\Big) \land \Big(\mathrm{in\_intersection\_conflict\_area}(x_\mathrm{p}, x_\mathrm{k}) \implies \neg\mathrm{F}_{[0,\mathrm{t_{ia}}]}\big(\mathrm{in\_intersection\_conflict\_area}(x_\mathrm{k}, x_\mathrm{p})\big)\Big)$
META-2	$\mathrm{passing\_stop\_line}(x_\mathrm{k}) ::= \mathrm{stop\_line\_in\_front}(x_\mathrm{k}) \land \mathrm{X}\big(\neg\mathrm{stop\_line\_in\_front}(x_\mathrm{k})\big)$

1) Not Endanger Other Vehicles - META-1: The k^{th} vehicle located in the conflict area (the intersection area of the lanes the vehicles are following) does not endanger the p^{th} vehicle if all of the following criteria are fulfilled: 1) the k^{th} vehicle does not cause the p^{th} vehicle to brake and the k^{th} vehicle is not within the conflict area if the p^{th} vehicle will be there within $\mathrm{t_{ib}}$; 2) the p^{th} vehicle is not within the conflict area if the k^{th} vehicle will be there within $\mathrm{t_{ia}}$ (the predicate is based on § 1(2), 8(2) StVO; [26] §8 StVO recital 36; OLG Schleswig VRS 80, 5).

The time parameters $\mathrm{t_{ib}}$ and $\mathrm{t_{ia}}$ specify the minimum time to pass before the p^{th} vehicle enters the conflict area and the time the k^{th} vehicle has to wait before entering the conflict area after the p^{th} vehicle has left. Note that the parameter order for the predicate $\mathrm{in\_intersection\_conflict\_area}$ is relevant (cf. Sec. V-A).

2) Passing Stop Line - META-2: The k^{th} vehicle passes a stop line if the stop line is in front of the vehicle and is not in front of it at the next time step.

VI. Traffic Rule Formalization

As introduced in our previous work [6], several steps are necessary to formalize traffic rules in temporal logic: 1) extracting rules from legal sources; 2) concretizing extracted rules; 3) extracting functions, predicates, and propositions; and 4) creating temporal logic formulas. Unfortunately, we cannot provide formalizations for all necessary intersection traffic rules and cannot consider every possible traffic situation due to the limited amount of available space. However, we provide rules for different intersection categories and our predicates and functions are modularly defined so that they can be used for new rules. We list the most relevant law sources in brackets after each rule concretization. As previously mentioned, we formalize traffic rules in MTL; further formalized intersection traffic rules are available at commonroad.in.tum.de.

A. Stop Sign - R-IN1

The ego vehicle has to stop with respect to a stop sign (sign 206) before it enters the intersection at least for a duration of t_{slw} in front of the associated stop line. [§ 8(1), § 41(1), Annex 2 sign 206, sign 294 StVO] The parameter $\mathrm{t_{slw}}$ indicates the duration of how long a vehicle has to wait. This rule does not consider stop lines referencing signs other than sign 206 and traffic lights. Those are covered by the rules defined in Sec. VI-B and Sec. VI-D.

B. Waiting At Traffic Light - R-IN2

The ego vehicle is not allowed to cross a red traffic light. If the traffic light is yellow and the ego vehicle can come to a standstill in front of the intersection without falling below an acceleration threshold a_{pos}, the ego vehicle is not allowed to cross a yellow traffic light. [§ 37(1) StVO; [26] §37 StVO recital 11, 14; BayObLG VRS 70, 384 mwN; OLG Hamm NZV 1992, 409; BGH NJW 2005, 1940; NZV 2012, 217]

The relevant traffic light depends on the direction the vehicle is turning into. Therefore, lanes controlled by two traffic lights, such as one for straight and one for right turning are considered. When turning right and when the traffic light is accompanied by a green arrow sign (sign 720), this rule does not apply. Instead, this case is handled by the rule specified in Sec. VI-D. Furthermore, this rule does not invalidate the left turn rule (cf. Sec. VI-E).

C. Right Before Left - R-IN3

If the ego vehicle is left of another vehicle (in terms of incoming lanelets) and their paths are crossing, the ego vehicle is only allowed to enter the intersection if it does not endanger the other vehicle. Right before left does not apply if traffic ligths regulate the intersection. [§ 8(1) StVO; BGH VRS 27, 74; OLG Hamburg VRS 29, 126; OLG Saarbrücken SVR 2018, 255; OLG Stuttgart NZV 1994, 440]

This rule must be evaluated with respect to every other vehicle at an intersection. If the ego vehicle and the other vehicle have the same priority, right before left applies. Therefore, this formalization is also valid when both vehicles have the same priority even though traffic signs regulate the intersection (e.g., at sign 1002-11 when both vehicles approach from a non-priority road).

D. Priority - R-IN4

The ego vehicle is not allowed to enter an intersection if there is another vehicle with the right of way that will be endangered by the ego vehicle. [§ 39(2), Annex 2, Annex 3 StVO; [26] §8 StVO recital 16, 32]

This rule must be evaluated with respect to every other vehicle at an intersection and excludes cases where the ego vehicle turns left and another vehicle is either oncoming or turning right into the same street as the ego vehicle. These cases are handled by the rule introduced in Sec. VI-E. The priority of the vehicles is inferred based on the traffic signs at the intersection.

TABLE VI: Overview of the formalized intersection traffic rules.

Rule	MTL formula
R-IN1	$\mathrm{G}\Big(\big(\text{passing\_stop\_line}(x_{\text{ego}}) \land \text{at\_traffic\_sign}(x_{\text{ego}}, \mathit{stop}) \land \neg\text{relevant\_traffic\_light}(x_{\text{ego}})\big) \implies \big(\mathrm{O}(\mathrm{G}_{[0,\mathrm{t_{slw}}]}(\text{stop\_line\_in\_front}(x_{\text{ego}}) \land \text{in\_standstill}(x_{\text{ego}})))\big)\Big)$
R-IN2	$\mathrm{G}\Big(\big(\text{turning\_left}(x_{\text{ego}}) \land (\text{at\_traffic\_light}(x_{\text{ego}}, \mathit{left}, \mathit{red}) \lor \text{at\_traffic\_light}(x_{\text{ego}}, \mathit{left}, \mathit{yellow}) \land (\text{braking\_intersection\_possible}(x_{\text{ego}}) \ \mathrm{S} \ \neg\text{at\_traffic\_light}(x_{\text{ego}}, \mathit{left}, \mathit{yellow}))) \lor \text{going\_straight}(x_{\text{ego}}) \land (\text{at\_traffic\_light}(x_{\text{ego}}, \mathit{straight}, \mathit{red}) \lor \text{at\_traffic\_light}(x_{\text{ego}}, \mathit{straight}, \mathit{yellow}) \land (\text{braking\_intersection\_possible}(x_{\text{ego}}) \ \mathrm{S} \ \neg\text{at\_traffic\_light}(x_{\text{ego}}, \mathit{straight}, \mathit{yellow}))) \lor \text{turning\_right}(x_{\text{ego}}) \land (\text{at\_traffic\_light}(x_{\text{ego}}, \mathit{right}, \mathit{red}) \lor \text{at\_traffic\_light}(x_{\text{ego}}, \mathit{right}, \mathit{yellow}) \land (\text{braking\_intersection\_possible}(x_{\text{ego}}) \ \mathrm{S} \ \neg\text{at\_traffic\_light}(x_{\text{ego}}, \mathit{right}, \mathit{yellow}))) \land \neg\text{at\_traffic\_sign}(x_{\text{ego}}, \mathit{greenArrow})\big) \implies (\neg\text{on\_lanelet\_with\_type}(x_{\text{ego}}, \mathit{intersection}) \land \neg\text{passing\_stop\_line}(x_{\text{ego}}))\Big)$
R-IN3	$\mathrm{G}\Big(\big(\text{on\_incoming\_left\_of}(x_{\text{ego}}, x_{\text{o}}) \land \neg\text{relevant\_traffic\_light}(x_{\text{ego}}) \land (\text{turning\_right}(x_{\text{ego}}) \land \text{turning\_right}(x_{\text{o}}) \land \text{same\_priority}(x_{\text{ego}}, x_{\text{o}}, \mathit{right}, \mathit{right})$ *- we skip some combinations of turning directions here -* $\lor \text{turning\_left}(x_{\text{ego}}) \land \text{going\_straight}(x_{\text{o}}) \land \text{same\_priority}(x_{\text{ego}}, x_{\text{o}}, \mathit{left}, \mathit{straight}))\big) \implies \big(\mathrm{G}(\text{not\_endanger\_intersection}(x_{\text{ego}}, x_{\text{o}})) \lor \neg\text{on\_lanelet\_with\_type}(x_{\text{ego}}, \mathit{intersection})\big)\Big)$
R-IN4	$\mathrm{G}\Big(\big(\text{turning\_right}(x_{\text{o}}) \land \text{turning\_right}(x_{\text{ego}}) \land \text{has\_priority}(x_{\text{o}}, x_{\text{ego}}, \mathit{right}, \mathit{right})$ *- we skip some combinations of turning directions here -* $\lor \text{turning\_right}(x_{\text{o}}) \land \text{turning\_left}(x_{\text{ego}}) \land \text{has\_priority}(x_{\text{o}}, x_{\text{ego}}, \mathit{rigth}, \mathit{left}) \land \neg\text{on\_oncom\_of}(x_{\text{o}}, x_{\text{ego}}) \lor \text{going\_straight}(x_{\text{o}}) \land \text{turning\_left}(x_{\text{ego}}) \land \text{has\_priority}(x_{\text{o}}, x_{\text{ego}}, \mathit{straight}, \mathit{left}) \land \neg\text{on\_oncom\_of}(x_{\text{o}}, x_{\text{ego}})\big) \implies \big(\mathrm{G}(\text{not\_endanger\_intersection}(x_{\text{ego}}, x_{\text{o}})) \lor \neg\text{on\_lanelet\_with\_type}(x_{\text{ego}}, \mathit{intersection})\big)\Big)$
R-IN5	$\mathrm{G}\Big(\big(\text{turning\_left}(x_{\text{ego}}) \land (\text{going\_straight}(x_{\text{o}}) \land \neg\text{has\_priority}(x_{\text{ego}}, x_{\text{o}}, \mathit{left}, \mathit{straight}) \land \text{on\_oncom\_of}(x_{\text{o}}, x_{\text{ego}})) \lor (\text{turning\_right}(x_{\text{o}}) \land \neg\text{has\_priority}(x_{\text{ego}}, x_{\text{o}}, \mathit{left}, \mathit{right}) \land \text{on\_oncom\_of}(x_{\text{o}}, x_{\text{ego}}))\big) \implies \big(\mathrm{G}(\text{not\_endanger\_intersection}(x_{\text{ego}}, x_{\text{o}})) \lor \neg\text{on\_lanelet\_with\_type}(x_{\text{ego}}, \mathit{intersection})\big)\Big)$

E. Turning Left - R-IN5

The left turning ego vehicle that has no priority (given by traffic signs) over an oncoming vehicle may only drive onto the oncoming lane if the ego vehicle does not endanger the other vehicle. The same applies if another vehicle turns right into the same road as the ego vehicle. [§ 9(3,4) StVO; [26] §9 StVO recital 28]

This rule must be evaluated with respect to every other vehicle at an intersection. We determine whether another vehicle is oncoming by its future position. If it is going straight or right and on the oncoming lane of the ego vehicle, we can infer that the other vehicle is oncoming.

VII. Numerical experiments

We use a modified version of the MTL runtime verification monitor Hydra [25] to monitor the formalized rules. We test the rules on recorded real-world scenarios from two sign-regulated intersections of the inD dataset [27] (modified Bendplatz and Frankenburg scenarios) and on artificially-generated CommonRoad scenarios[3]. Tab. VII shows the parameters used in the evaluation.

For both intersections from the inD dataset, almost all vehicles consider the priority rule (R-IN4) and give way to oncoming traffic when turning left (R-IN5). About 90% of the vehicles adhere to the stop line rule (R-IN1). This is comparably low considering that most vehicles do not pass the stop sign in the recorded scenarios. The reason for this is that the vehicles do not come to a complete standstill at the stop line, but rather pass through at a low speed. Additionally, some vehicles stop on or shortly after the stop line rather than before it.

We have created traffic light scenarios using the traffic simulator SUMO [28] and the interface introduced in [29] with randomly behaving traffic participants and traffic light cycles (CommonRoad scenario IDs: ZAM_Trafficlight-1_1 - ZAM_Trafficlight-1_5). Almost all vehicles behave in accordance with the left turn rule (R-IN5). About 18% of the vehicles violate the traffic light rule (R-IN2). This can be mostly attributed to vehicles stopping slightly too late.

Furthermore, we created hand-crafted test scenarios covering different traffic situations for all formalized rules because the recorded and simulated scenarios may not consider certain situations. Fig. 3 and Fig. 4 show two examples of these hand-crafted scenarios. We created the scenarios using the CommonRoad Scenario Designer [30]. In Fig. 3, the intersection is regulated by priority signs. The vehicle approaching the intersection from the right does not give way to the vehicle coming from the left and therefore violates rule R-IN4. In contrast, a vehicle from the left stops in

[3]https://commonroad.in.tum.de/scenarios/

TABLE VII: User-defined parameters for the traffic rule evaluation.

Param.	Value	Param.	Value	Param.	Value
$\mathrm{t_{ib}}$	$1.0\,\mathrm{s}$	$\mathrm{t_{ia}}$	$0.5\,\mathrm{s}$	$\mathrm{t_{slw}}$	$3.0\,\mathrm{s}$
$\mathrm{d_{sl}}$	$1.0\,\mathrm{m}$	d_{br}	$15.0\,\mathrm{m}$	v_{err}	$0.1\,\mathrm{m/s}$
a_{br}	$-1.0\,\mathrm{m/s^2}$	a_{pos}	$-4.0\,\mathrm{m/s^2}$		

front of the intersection and obeys the rule. Fig. 4 shows a traffic-light-controlled scenario. The vehicle approaching the intersection from the top does not stop at the red traffic light and therefore violates rule R-IN2. The other vehicles obey this rule.

VIII. Conclusions

We have presented a formalization of several intersection traffic rules based on German traffic regulations. Our defined rules, predicates, and functions can easily be adapted by other national traffic laws. The rules cover different intersection types and can be the basis for further intersection traffic rules. Autonomous vehicles probably cannot be certified for usage alongside human-driven vehicles without these rules. The evaluation of the inD dataset and manually-created test scenarios shows that the formalization can be used for realistic traffic scenarios. Future work includes the formalization of additional intersection and urban traffic rules, such as the behavior before reaching an intersection and within an intersection as well as consideration of pedestrians and cyclists. Additionally, the traffic rules will be tested in our autonomous test vehicle within real traffic.

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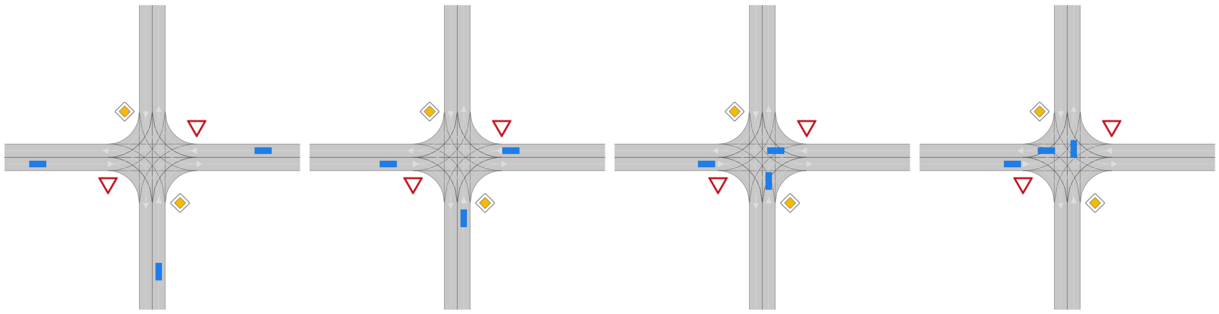
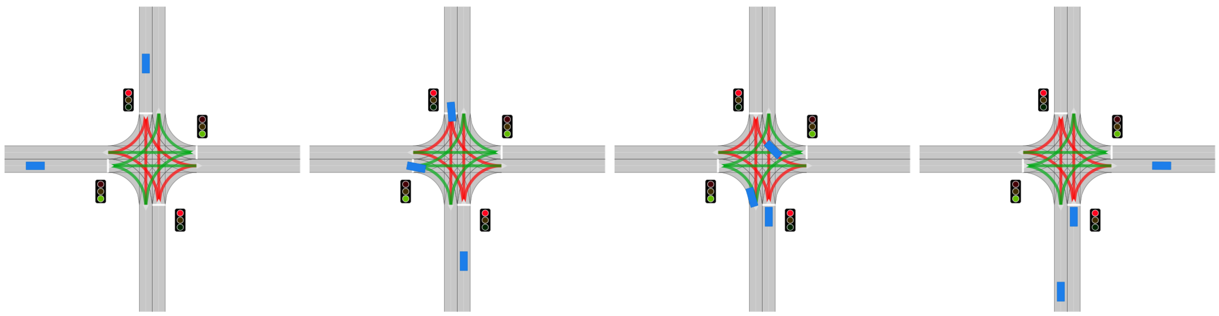


(a) $t = 0.0s$ (b) $t = 1.0s$ (c) $t = 1.7s$ (d) $t = 2.3s$

Fig. 3: A hand-crafted four-way intersection in which the vehicle coming from the right violates the priority rule R-IN4.



(a) $t = 0.0s$ (b) $t = 2.0s$ (c) $t = 3.5s$ (d) $t = 6.0s$

Fig. 4: A hand-crafted four-way intersection in which the vehicle coming from the top violates the traffic light rule R-IN2.

[29] M. Klischat, O. Dragoi, M. Eissa, and M. Althoff, “Coupling SUMO with a motion planning framework for automated vehicles,” in *SUMO: Simulating Connected Urban Mobility*, 2019.

[30] S. Maierhofer, M. Klischat, and M. Althoff, “CommonRoad Scenario Designer: An open-source toolbox for map conversion and scenario creation for autonomous vehicles,” in *Proc. of the IEEE Int. Conf. on Intelligent Transportation Systems*, 2021, pp. 3176–3182.