Quantum Process Tomography from Time-Delayed Measurements

Irene López Gutiérrez

Technische Universität München

Faculty of Informatics

Chair of Scientific Computing

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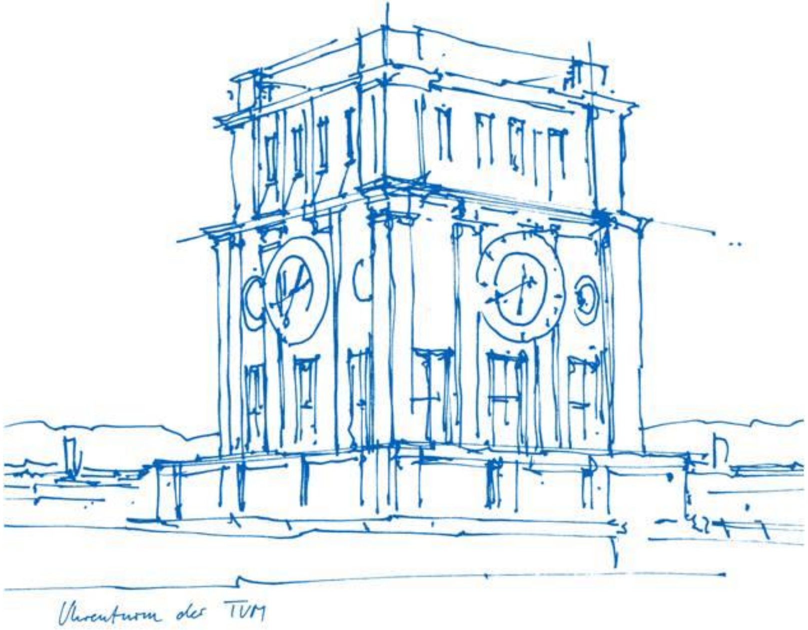

ArXiv preprint: arXiv:2112.09021

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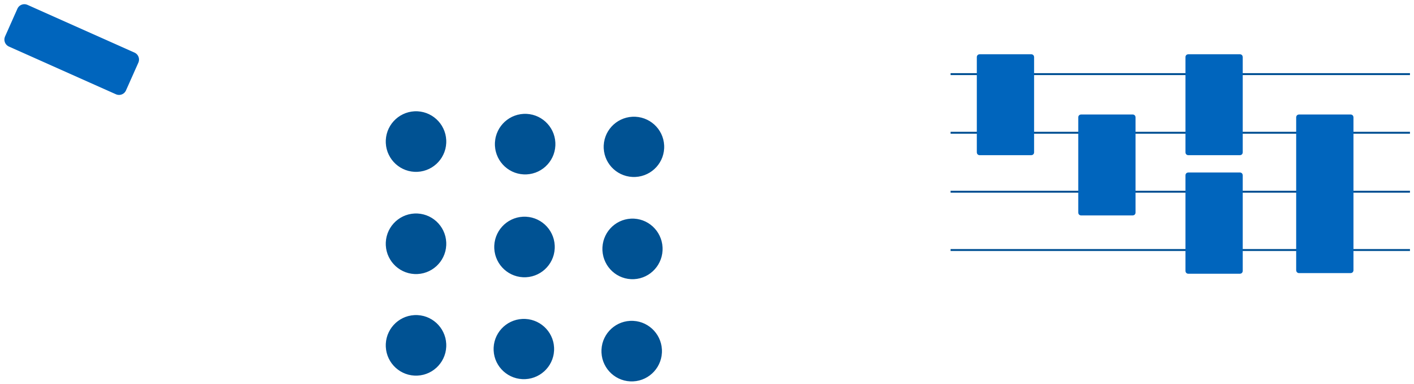
Dr. Felix Dietrich

Prof. Christian Mendl

Quantum Process Tomography

Goal: Learn unitary (or Kraus operators) describing time evolution.



Mathematical Framework: Takens Theorem

Let $\mathcal{M}$ be a d-dimensional manifold.

Commun. Math. Phys. 20, 167 (1971)

Generic delay embeddings: *For pairs* $(\phi, y), \phi: \mathcal{M} \to \mathcal{M}$ *a smooth diffeomorphism and* $y: \mathcal{M} \to \mathbb{R}$ *a smooth function, it is a generic property that the map* $\Phi_{(\phi,y)}: \mathcal{M} \to \mathbb{R}^{2d+1}$, *defined by*

$$\Phi_{(\phi,y)}(x) = \left(y(x), y(\phi(x)), \ldots, y(\phi \circ \phi \ldots \circ \phi(x))\right)$$

is an embedding of $\mathcal{M}$.

i.e. given 2d + 1 measurements throughout the time-evolution we can uniquely identify the process

Mathematical Framework: Applied to Quantum Unitaries

Generic delay embeddings: *For pairs* $(\phi, y), \phi: \mathcal{M} \rightarrow \mathcal{M}$ *a smooth diffeomorphism and* $y: \mathcal{M} \rightarrow \mathbb{R}$ *a smooth function, it is a generic property that the map* $\Phi_{(\phi,y)}: \mathcal{M} \rightarrow \mathbb{R}^{2d+1}$, *defined by*

$$\Phi_{(\phi,y)}(x) = \Big(y(x), y\big(\phi(x)\big), \ldots, y\big(\phi \circ \phi \ldots \circ \phi(x)\big)\Big)$$

is an embedding of M.

$$x = U = e^{-iH}$$

$$\phi(U) = \mathrm{e}^{-\mathrm{i}\gamma H} = U^{\gamma}$$

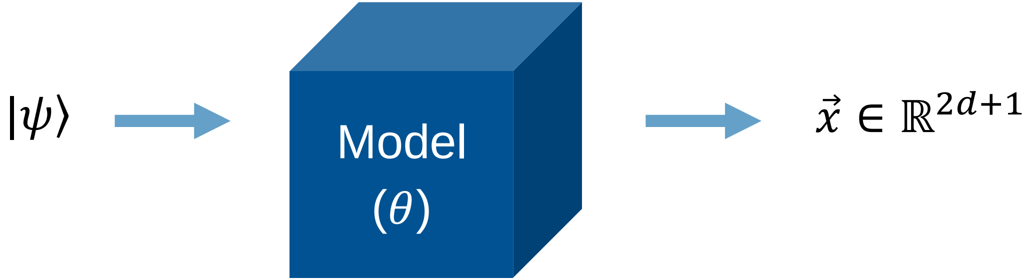
$$y(U) = \langle\psi|U^{\dagger}AU|\psi\rangle$$

$$\Phi_{(\phi,y)}(\psi) = \left(\langle\psi|U^{\dagger}AU|\psi\rangle, \cdots, \left\langle\psi\left|\left(U^{\gamma^{(2d+1)}}\right)^{\dagger} AU^{\gamma^{(2d+1)}}\right|\psi\right\rangle\right)$$

Here, d is the number of free parameters in U

Numerical Approach: general idea

Choose Ansatz to represent Φ

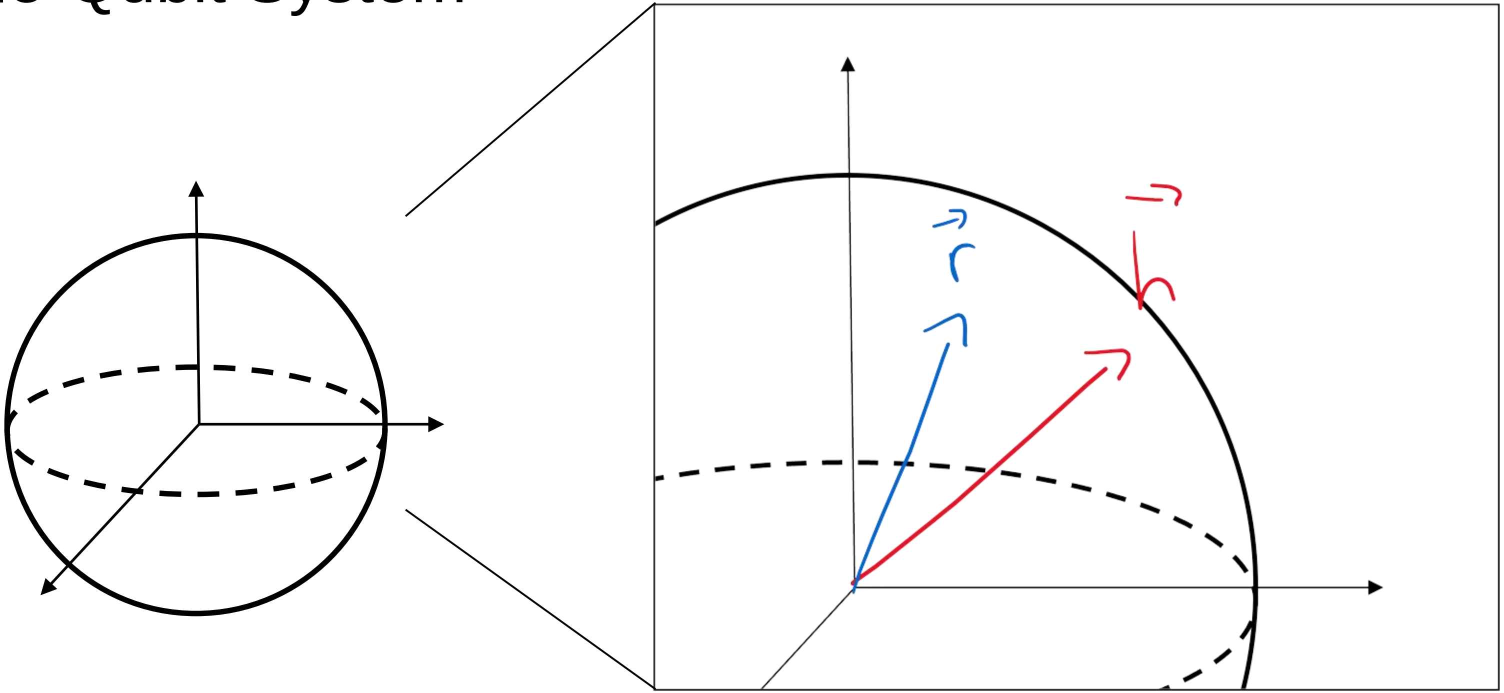


Optimize by:

$$\arg\min_{\theta} \; C(\vec{y}, \vec{x}(\theta))$$

e.g. $\|\vec{y} - \vec{x}(\theta)\|^2$

Single-Qubit System



Single-Qubit System

- Rewrite Hamiltonian vector: $\vec{h} = \frac{1}{2}\omega\vec{v}$
- Evolution described by Rodrigues' rotation formula:

$$U_{\omega,t,\vec{v}}\vec{r} = \cos(\omega t)\vec{r} + \sin(\omega t)(\vec{v} \times \vec{r}) + (1 - \cos(\omega t))(\vec{v} \cdot \vec{r}) \cdot \vec{v}$$

- Measurements are: $y_t = \langle \vec{m}, U_{\omega,t,\vec{v}}\vec{r} \rangle$

First find ω

$$\mathrm{argmin}_\omega \ \min_{a,b,c} \sum_{n=0}^{2d} |y_n - a\cos(\omega t_n - b) - c|^2$$

Single-Qubit System

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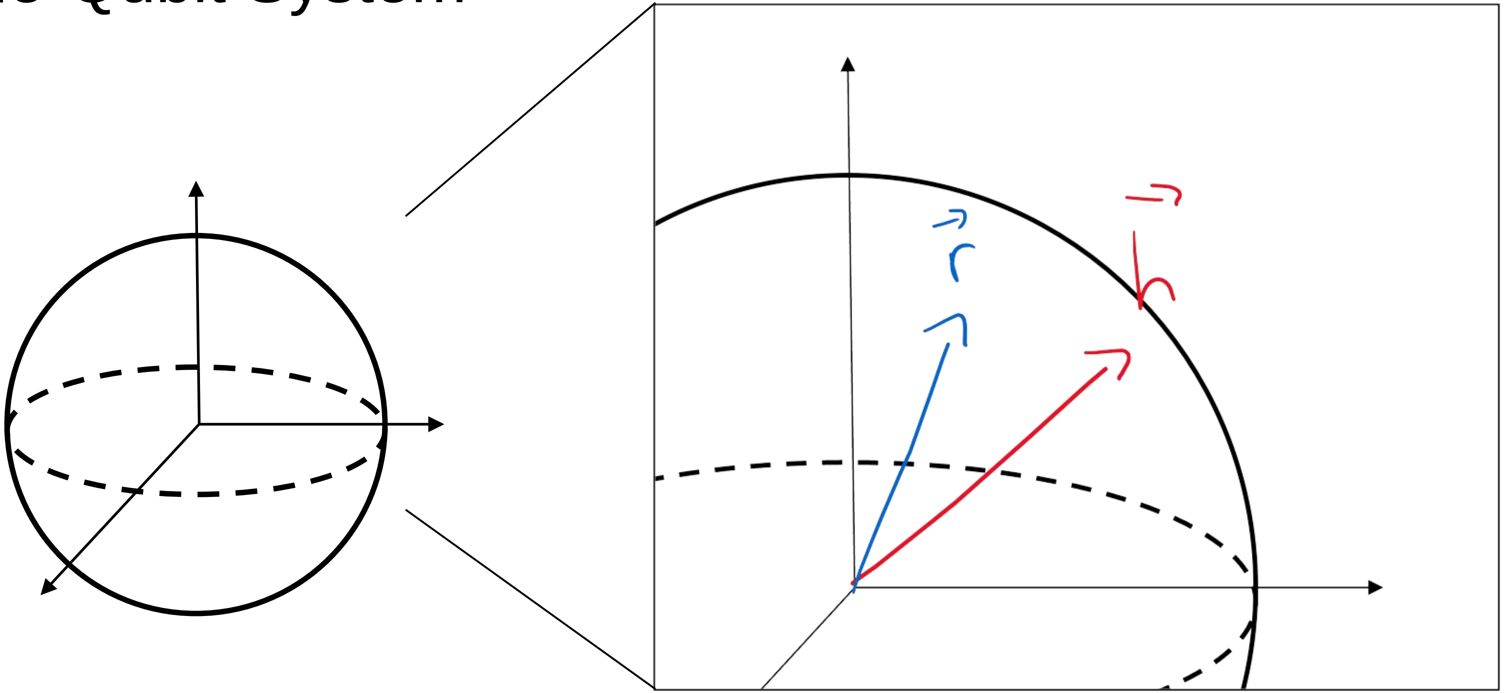
- Measurements are: $y_t = \langle \vec{m}, U_{\omega,t,\vec{v}}\vec{r} \rangle$

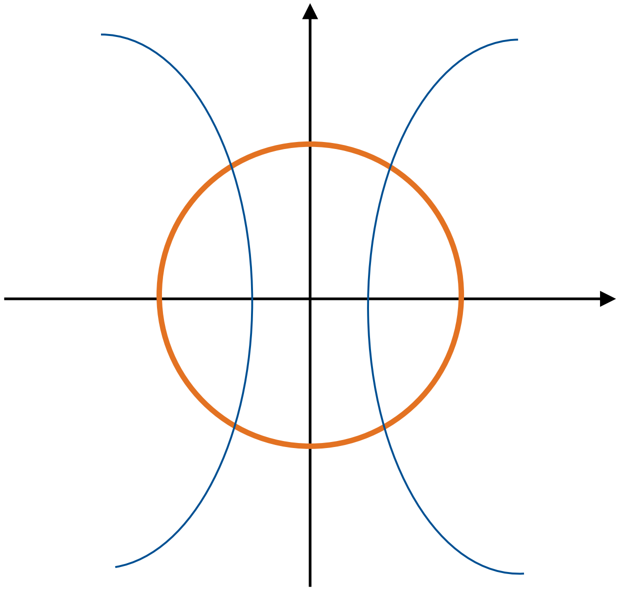
Next find $\alpha = \vec{m} \cdot (\vec{v} \times \vec{r})$ and $\kappa = (\vec{v} \cdot \vec{r}) \cdot (\vec{m} \cdot \vec{v})$

$$\arg\min_{\alpha,\kappa} \sum_{n=0}^{2d} |\tilde{y}_n - \sin(\omega t_n)\alpha - (1 - \cos(\omega t_n))\kappa|^2$$

Single-Qubit System



Single-Qubit System



- 4 solutions
- Optimization let’s us find one of them
- Then, decide between 4 with one extra measurement in different basis

Single-Qubit System with Noise

Recall: $y(U) = \langle\psi|U^\dagger A U|\psi\rangle$

Not fully known?

Noisy measurement?

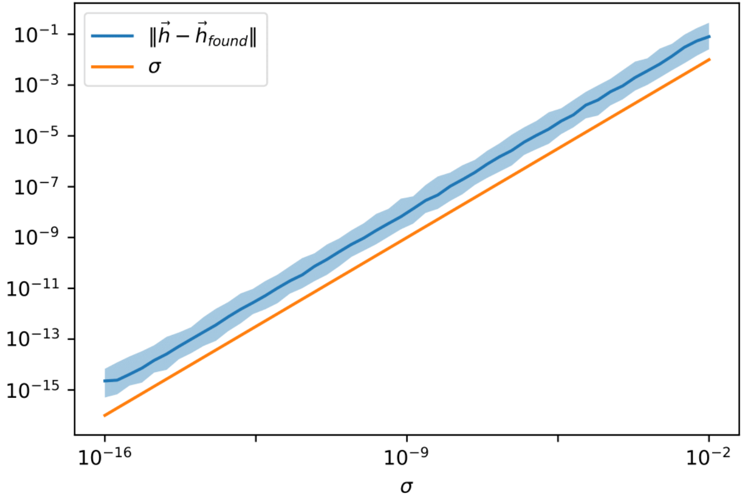
Single-Qubit System with Noise

Recall: $y(U) = \langle\psi|U^\dagger A U|\psi\rangle$

Not fully known?

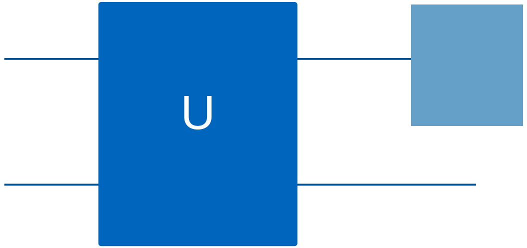
Noisy measurement?

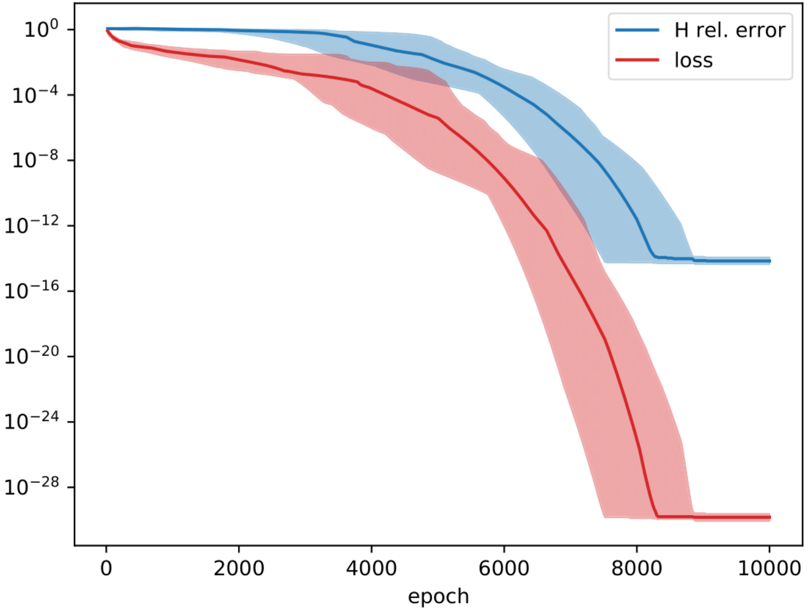
Model as: $y(U) = \langle\psi|U^\dagger A U|\psi\rangle + \mathcal{N}(0, \sigma^2)$



Partial (subsystem) measurements

- No restrictions on measurement
- We can choose: $I \otimes \sigma^j$





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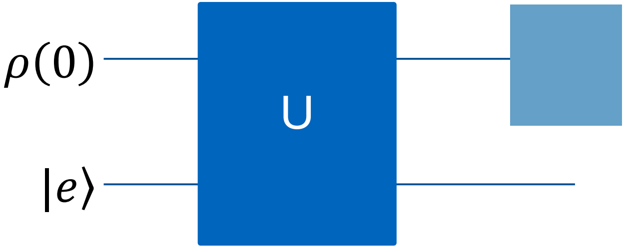


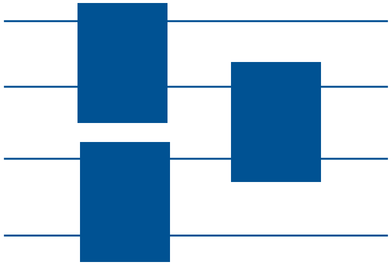
Can model a noisy process on a single qubit

$$\rho(t) = \sum_k E_k \, \rho(0) E_k^\dagger$$

$$\rho(t) = \mathrm{tr}_{\mathrm{env}}\big(U(\rho(0) \otimes |e\rangle\langle e|) U^\dagger\big)$$

Partial Knowledge of Hamiltonian

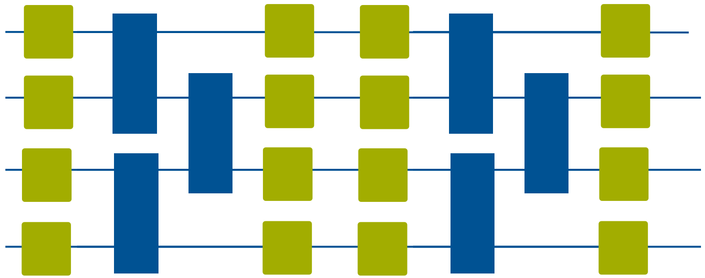
- E.g. local interactions
- Model can use less parameters
- And therefore we need less time points

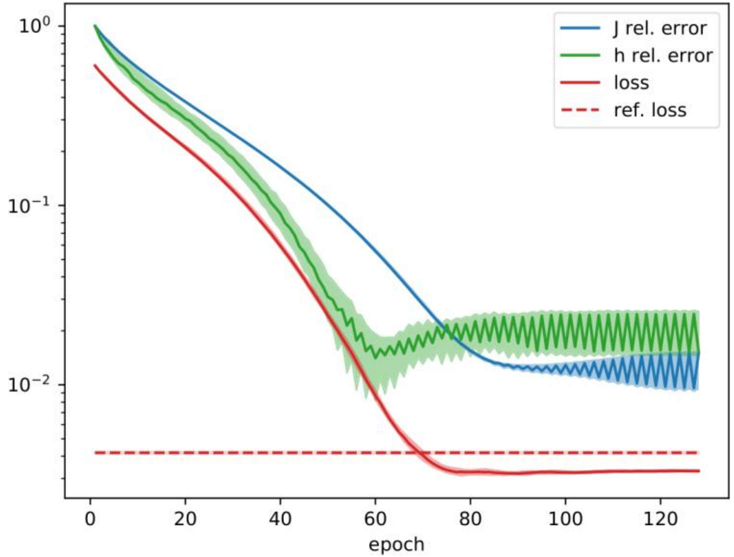


Partial Knowledge of Hamiltonian

Uniform interaction (3 × 4 lattice)

$$H = -J \sum_{\langle j,k \rangle} \sigma_j^z \sigma_k^z - \sum_j \vec{h} \cdot \vec{\sigma}_j$$





Summary

- Takens' theorem provides a mathematical framework for the reconstruction
- Number of expectation values needed: 2d + 1 (where d is the number of free parameters in U)
- Limited to one or two-qubit systems or systems with some underlying structure
- One must be careful with symmetries

Future work

- Experimental tests in collaboration with Robert Stárek and Michal Micuda from
  Department of Optics, Palacký University Olomouc (Chzech Republic)