Backstepping Control of a DC-DC Boost Converters Under Unknown Disturbances

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Abstract—This paper presents a novel control scheme for DC-DC boost converter, maintaining the desirable voltage regulation performance under high load variation and large change of voltage reference. The model of converter is reformulated, in which the unknown equivalent load, input voltage, model uncertainties and unmodeled dynamics are lumped as external disturbance. The control strategy is designed with backstepping control technology, similar to the cascade control method in which the intermediate variable is introduced to fast respond to the control demand, effectively dealing with the nonlinearities of the boost converter dynamics. The disturbance observers are established to estimate the lumped disturbances, rejecting disturbances and removing steady-state errors to improve the closed-loop performance. The simulation results demonstrate that the proposed control strategy, backstepping control combined with disturbance observer, provides lots of advantages superior to the conventional PI control such as faster dynamic response and less output voltage drop.

Index Terms—DC-DC Boost Converters; Backstepping control; Disturbance Observer.

I. INTRODUCTION

The role of dc/dc boost converter is to step up an input voltage to a higher output voltage. During the past decades, the dc/dc boost converters have been widely applied in various industrial applications, such as battery power systems, hybrid electric vehicle systems, new energy resources systems (photovoltaic and wind energy system) and dc motor drivers systems, etc [1]–[4]. Particularly, for the application of new energy resources systems involving many uncertainties and external disturbances, it requires that the devices must turn new energy resources to electrical energy efficiently and economically while minimizing the impact on the power grids. Therefore, the main control objectives are to operate dc/dc boost converters with small steady-state error, fast dynamical response, low overshoot, low noise susceptibility and high efficiency even in presence of uncertainties and disturbances.

However, from the control perspective, it is more difficult to control the boost converter than the buck converter since the dynamic model of the boost converter is an highly nonlinear system and exhibits nonminimum-phase behavior. In order to control the dc/dc boost converter effectively, a great number of control approaches have been proposed in literature [5]–[8]. In [9], a modified version of cascade control algorithm has been established for boost converter to deal with its nonminimum-phase behavior, which consists of internal-loop current control and external-loop voltage control. Based on the equivalent control approach, two sliding mode controllers are developed for the boost converter in [10], where current and voltage controls are reported. It has demonstrated that since the boost converter is nonminimum-phase system, direct voltage control for it can lead to zero dynamics [10]. Combined a cascade control with nested reduced-order proportional-integral observers, the authors in [11] have studied how to regulate the output voltage of a dc/dc boost converter under parametric uncertainty and input voltage change. According to the multivariable method, a proportional-type controller has been proposed for the boost converter in [12], where the nonlinear observer is designed to approximate the disturbances caused by model mismatches.

On the other hand, as an effective control scheme for nonlinear systems, backstepping control has been widely applied into various systems [13]–[15]. In the [16], a backstepping controller is designed for the distributed hybrid photovoltaic power system. Experimental results have shown that the controller could achieve maximum power transfer to the grid and deal with grid nonlinearity and uncertainties effectively. The backstepping control was applied to control for a multisource vehicle with fuel cell and supercapacitors in [17]. An adaptive backstepping controller has been proposed for a pump-controlled electrohydraulic actuator to achieve position control in [18]. In addition, the backstepping controller is implemented for modular multilevel converter in [19]. Compared with the linear PI control, the backstepping controller shows lots of advantages.

In this paper, a novel control strategy, backstepping control combined with disturbance observer, is proposed for dc/dc boost converter. First, the model of converter is reformulated, in which the unknown equivalent load, input voltage, model uncertainties and unmodeled dynamics are lumped as exter-
nal disturbance. The disturbance observer is constructed to estimate the lumped disturbance, rejecting disturbance and removing steady-state errors to improve the closed-loop performance. Then the backstepping controller plus disturbance observer is applied to established mathematical model and the stability of the closed-loop system is guaranteed based on the Lyapunov stability theory.

The paper is organized as follows. The model of dc/dc boost converter is reformulated in Section II. Section III shows the detail of the proposed control strategy based on the backstepping controller plus disturbance observer for boost converter. The simulation results comparing the performance of the proposed control approach with the classical cascade PI control are given and analyzed in Section IV. Finally, Section V concludes this paper.

II. DC-DC BOOST CONVERTER TOPOLOGY AND MODELING

The basic topology of the boost converter is shown in Fig. 1, which comprises an input DC voltage source $v_{in}$, a switch device VT, a diode VD, an output capacitor $C$, a filter inductor $L$, and the equivalent load $R_L$ considering as the unknown load in this paper. $v_o$ and $i_L$ represent the output voltage and inductor current, respectively. Here it should be pointed that we only study the converter operating in continuous conduction mode in this paper. Denoting the output voltage $v_o$ and inductor current $i_L$ as the state variables, when the switch is ON, the converter dynamics can be written as,

\[
\begin{align*}
L \frac{dv_o}{dt} &= v_{in}, \\
C \frac{di_L}{dt} &= -\frac{v_o}{R_L}.
\end{align*}
\]

and when the switch is OFF, the converter dynamics can be written as,

\[
\begin{align*}
L \frac{dv_o}{dt} &= -v_o + v_{in}, \\
C \frac{di_L}{dt} &= i_L - \frac{v_o}{R_L}.
\end{align*}
\]

Then, the average dynamic equations of the dc/dc boost converter can be expressed as [10],

\[
\begin{align*}
L \frac{di_L}{dt} &= -(1 - D)v_o + v_{in}, \\
C \frac{dv_o}{dt} &= -(1 - D)i_L - \frac{v_o}{R_L},
\end{align*}
\]

where $D \in [0, 1]$ is the duty ratio.

One can observe that the dynamic model of boost converter (3) and (4) is nonminimum phase system, mainly because duty ratio $D$ which can be viewed as control input appears in both the voltage dynamic (3) and current dynamic (4). In fact this system is a highly nonlinear system that is difficult to control the output voltage with a desired reference. On the other hand, the above system (3) and (4) are ideal models, however in practice the values of inductor and capacitor may vary in different operating conditions and the above dynamics exist unmodeled dynamics such as diode forward resistance, switch device on-resistance, and diode threshold voltage, etc. Therefore, the dynamics of dc/dc boost converter can be rewritten as,

\[
\begin{align*}
\dot{x}_1 &= \frac{1}{C}x_2 + f_1(x_1, x_2), \\
\dot{x}_2 &= \frac{1}{L}(x_1 + a)u + f_2(x_1, x_2), \\
y &= x_1,
\end{align*}
\]

where $x_1$ and $x_2$ represent output voltage, inductor current, respectively, $u$ and $y$ are the control input and system output, respectively,

\[
\begin{align*}
f_1(x_1, x_2) &= -\frac{1}{L}x_2u - \frac{x_1}{RC} + \omega_1, \\
f_2(x_1, x_2) &= -\frac{1}{L}x_1 + \frac{2x_2}{L} - au + \omega_2,
\end{align*}
\]

and $\omega_1$ and $\omega_2$ stand model uncertainties and unmodeled dynamics in voltage and current equations, respectively, and $a$ is the positive constant which can be designed to improve the system performance. It should be pointed that in the dynamics of (5) and (6) the unknown equivalent load, input voltage, model uncertainties and unmodeled dynamics are lumped as external disturbance and the control signal only appears in the current equations. This can simplify the dynamics of (3) and (4) and even does not need to know the information of input voltage. However, the disturbance appears in both the voltage and current dynamics, i.e., the disturbances appears in two channels which also brings some difficulties for control power converter, for instance the classical extended state observer can not been used to estimate the external disturbance efficiently.

The control objective in this paper is to regulate the output voltage tracking its desired value $v_o^*$ in the presence of external disturbance. In fact, for the system (5)-(7), the control objective is transformed to that design an anti-disturbance controller $u$ such that the output of system $y$ can track a given signal $y^*$.

III. CONTROL STRATEGIES

In this section, based on the above system (5)-(7), a novel control strategy will be proposed for the dc/dc boost converter to achieve the control objective. We will use the backstepping control method combined with disturbance observer to control the system (5)-(7), in which the disturbance observer is employed to estimate the lumped disturbance to improve system performance. Next, the detailed design procedure will be presented.
First, for the system (5), a disturbance observer is constructed to estimate the disturbance \( f_1(x_1, x_2) \). The observer is defined as,
\[
\hat{f}_1 = v_1 + l_1 x_1, \quad (8)
\]
\[
\hat{v}_1 = -\dot{\hat{f}}_2 = -l_1 \left( \frac{1}{C} x_2 + \hat{f}_1 \right), \quad (9)
\]
where \( \hat{f}_1 \) is the estimation of \( f_1(x_1, x_2) \), \( v_1 \) is the state variable of the observer and \( l_1 \) is observer gain which determines the convergence rate of the observer.

Define the observer error \( \hat{f}_1 = f_1(x_1, x_2) - \hat{f}_1 \), and computing the derivative of the observer error \( \hat{f}_1 \) and using (5), (7) and (8) yield,
\[
\dot{\hat{f}}_1 = \dot{f}_1(x_1, x_2) - \dot{\hat{f}}_1
\]
\[
= -l_1 \left( \frac{1}{C} x_2 + \hat{f}_1 \right) - l_1 \dot{x}_1
\]
\[
= -l_1 \hat{f}_1 + \hat{f}_1(x_1, x_2). \quad (10)
\]

One can observer that for the system (5)-(7), the disturbances of \( f_1(x_1, x_2) \) and \( f_2(x_1, x_2) \) for the boost converter are bounded but may be unknown. Without loss of generality, assume that the \( \| f_1(x_1, x_2) \| \leq \varepsilon_1 \) and \( \| f_2(x_1, x_2) \| \leq \varepsilon_2 \). It is derived from (10),
\[
\dot{\hat{f}}_1 = e^{-l_1 t} \hat{f}_1(0) + \int_0^t e^{-l_1 \tau} \hat{f}_1(\tau) d\tau. \quad (11)
\]
Then it follows from (11) that,
\[
\| \dot{\hat{f}}_1 \| = e^{-l_1 t} \| \hat{f}_1(0) \| - \frac{\varepsilon_1}{l_1} (1 - e^{-l_1 t}). \quad (12)
\]

Note that to make the observer error system (10) stable, the observer gain has to be designed as positive scalar. The larger \( l_1 \) one selects, the higher convergence rate observer has. However, the high gain for observer can lead to high overshoot and massive consumption of control power. Thus, one should choose the observer gain \( l_1 \) considering these factors together. Next, on the basis of the above disturbance observer, we will design a composite anti-disturbance controller via backstepping control technology.

Step 1. Define \( z_1 = x_1 - y^* \), \( z_2 = x_2 - \sigma_1 \), where \( \sigma_1 \) is the virtual controller to be determined. Construct the following Lyapunov function as,
\[
V_1(t) = \frac{1}{2} z_1^2. \quad (13)
\]
Then the time-derivative of \( V_1(t) \) along the system (5) can be written as,
\[
\dot{V}_1(t) = z_1 \left( \frac{1}{C} x_2 + f_1(x_1, x_2) - \dot{y}^* \right)
\]
\[
= z_1 \left( \frac{1}{C} x_2 + \frac{1}{C} \sigma_1 + f_1(x_1, x_2) - \dot{y}^* \right). \quad (14)
\]

One can design an appropriate virtual controller \( \sigma_1 \) to make the tracking error stable,
\[
\sigma_1 = -C(\lambda_1 z_1 + \hat{f}_1 - \dot{y}^*), \quad (15)
\]
where \( \lambda_1 > 0 \). Notice that the estimation of disturbance \( \hat{f}_1 \) has been integrated in the virtual controller \( \sigma_1 \) to reject the disturbance.

Substituting the (15) into (14), the derivative becomes,
\[
\dot{V}_1(t) = -\lambda_1 z_1^2 + z_1 \hat{f}_1 + \frac{1}{C} z_1 \omega_1,
\]
\[
\leq -\lambda_1 z_1^2 + \frac{1}{C} z_1 \omega_1 + z_1 \omega_1, \quad (16)
\]

Step 2. Similar to the system (6), a disturbance observer is constructed to estimate the disturbance \( f_2(x_1, x_2) \). The observer is defined as,
\[
\hat{f}_2 = v_2 + l_2 x_2, \quad (17)
\]
\[
\hat{v}_2 = -l_2 \left( \frac{1}{L} (x_1 + a) u \right), \quad (18)
\]

where \( \hat{f}_2 \) is the estimation of \( f_2(x_1, x_2) \), \( v_2 \) is the state variable of the observer and \( l_2 \) is observer gain which determines the convergence rate of the observer.

We can conclude that the observer error \( \hat{f}_2 = f_2(x_1, x_2) - \hat{f}_2 \) is boundedness with \( \omega_2 \), i.e., \( \| \hat{f}_2 \| \leq \omega_2 \), whose proof process is similar to (10)-(12).

Then choose the Lyapunov function candidate as,
\[
V_2(t) = V_1(t) + \frac{1}{2} z_2^2. \quad (19)
\]

The time-derivative of \( V_2 \) is given by,
\[
\dot{V}_2(t) = \dot{V}_1(t) + z_2 \dot{z}_2
\]
\[
= \dot{V}_1(t) + z_2 (\dot{x}_2 - \dot{\sigma}_1)
\]
\[
= \dot{V}_1(t) + z_2 \left( \frac{1}{L} (x_1 + a) u + f_2(x_1, x_2) - \dot{\sigma}_1 \right)
\]
\[
\leq z_2 \left( \frac{1}{L} (x_1 + a) u + f_2(x_1, x_2) - \dot{\sigma}_1 \right) - \lambda_2 z_2^2 + \frac{1}{C} z_1 z_2 + z_1 \omega_1. \quad (20)
\]

Design the controller as,
\[
u = -\frac{L}{(x_1 + a)} (\lambda_2 z_2 + \dot{f}_2 + \frac{1}{C} z_1 - \dot{\sigma}). \quad (21)
\]

Using the controller (21) into the (20), one can obtains the following derivative,
\[
\dot{V}_2(t) \leq -\lambda_1 z_1^2 - \lambda_2 z_2^2 + z_1 \omega_1 + z_2 \omega_2
\]
\[
\leq -\lambda_1 z_1^2 - \lambda_2 z_2^2 + z_1^2 + z_2^2 + \omega_1^2 + \omega_2^2. \quad (22)
\]

Choose \( \lambda_1 = c_1 + 1 \) and \( \lambda_2 = c_2 + 1 \) with \( c_1 \) and \( c_2 \) being two positive constants.

Then, the time-derivative of \( V_2 \) becomes
\[
\dot{V}_2(t) \leq -c_1 z_1^2 - c_2 z_2^2 + \frac{\omega_1^2}{4} + \frac{\omega_2^2}{4}
\]
\[
\leq -\eta V_2(t) + \zeta, \quad (23)
\]
where }\eta = \min\{2c_1, 2c_2\}, \zeta = \frac{\omega \psi}{4} + \frac{\omega \psi}{2}^2\text{. Based on the comparison principle, it follows that }
\dot{V}_2(t) \leq (V_2(0) - \frac{\zeta}{\eta})e^{-\eta t} + \frac{\zeta}{\eta}, \quad (24)
i.e., the steady-state error of }z_1\text{ is bounded with }\lim_{t \to \infty} \|z_1\| \leq \sqrt{\frac{2\zeta}{\eta}}\text{ by computing (24).}

A schematic block diagram of the control strategy is shown in Fig. 2.

![Schematic block diagram of the control strategy](image)

**TABLE I**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>f</td>
<td>20 × 10^3</td>
<td>Switching Rate (Hz)</td>
</tr>
<tr>
<td>R_L</td>
<td>80 → 40</td>
<td>Load Resistance (Ω)</td>
</tr>
<tr>
<td>C</td>
<td>470 × 10^{-6}</td>
<td>Capacitance (F)</td>
</tr>
<tr>
<td>L</td>
<td>220 × 10^{-6}</td>
<td>Inductor (H)</td>
</tr>
<tr>
<td>v_{in}</td>
<td>25</td>
<td>Input Voltage (V)</td>
</tr>
<tr>
<td>v_{ref}</td>
<td>50</td>
<td>Output Voltage Reference (V)</td>
</tr>
</tbody>
</table>

**IV. SIMULATION ANALYSIS**

In this section, a simulation model of the boost dc/dc converter, whose parameters and variables are shown in the Table I, is established by using MATLAB/SIMULINK to validate the effectiveness of the proposed strategy. The classical cascade PI control is also implemented for purpose of showing the superiority of proposed control strategy. On the other hand, we will perform the simulations to regulate the output voltage under two conditions: reference voltage variation and unknown equivalent load variation.

**TABLE II**

<table>
<thead>
<tr>
<th>Control Strategy</th>
<th>Control Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>PI control</td>
<td>K_p = 5 × 10^{-4}, K_i = 2.5</td>
</tr>
<tr>
<td>Internal loop</td>
<td>K_p = 0.1, K_i = 2.5 × 10^{-4}</td>
</tr>
<tr>
<td>The Proposed control strategy</td>
<td>c_1 = 1, c_2 = 3, l_1 = 5 × 10^4, l_2 = 1 × 10^3, a = 120</td>
</tr>
</tbody>
</table>

**A. Reference voltage variation**

The control parameters of the classical PI control and the proposed control scheme are summarized in the Table II. The responses of the output voltage of the boost converter are depicted in Figs. 3 and 4 performed by PI and our proposed control strategy, respectively. It can be observed that both control strategies can regulate output voltage to 50 V perfectly. However, in comparison with classical PI control, the proposed control strategy has better steady-state performance. Then, with the same control parameters, both control strategies are utilized to regulate output voltage for the step voltage references.

In Fig. 5, one can see that when the output voltage is increased form 40 V to 50 V, it takes less than 200 ms for the proposed control scheme to recover to the steady state, while for the PI control the settling time is 500 ms. Additionally, when the voltage reference is stepped form 50 V to 30 V, the PI control has large voltage fluctuation with high voltage overshoot (the max overshot is 16 V) and drop (the max voltage drop is 5 V) and needs more than 1500 ms to recover the desired value. However, for the proposed control strategy it only needs 700 ms to finish the transient state and there is only voltage drop (the max drop is 4 V) in the transient process. The voltage errors between desired value and output voltage of the proposed control strategy and PI control are shown in the Fig. 6, in which the proposed control has better performance than the PI control. The dynamic responses of disturbance observer are pictured in Fig. 7, in which the observer can change smoothly based on the voltage variation.

Therefore, we can conclude that both control strategies can regulate output voltage perfectly with constant voltage reference. Of course, the proposed control scheme shows a slightly better performance in the transient and steady state response. However, for the stepped voltage reference, the proposed control strategy exhibit more excellent performance than the classical cascade PI control. The proposed control strategy can reduce more than half of settling time under the voltage reference increase and drop conditions. For the voltage reference increase case, the proposed control scheme has smaller voltage drop and for the voltage reference drop case, the proposed control scheme has no voltage overshoot and smaller voltage drop. This implies that the proposed control strategy is more robust against to voltage variation.

In fact, the proposed control approach is also able to track the sinusoidal and ramp voltage references. In the Fig. 8, it can be observed that the output voltage can track the sinusoidal references }v_{ref} = 2 \sin(10\pi t) + 51 \text{ V and } v_{ref} = -20t + 50 \text{ V with acceptable voltage tolerance.}

**B. Unknown equivalent load variation**

In this simulation, the equivalent load is stepped form 80 Ω to 40 Ω then to 60 Ω and voltage reference is 50 V. The control parameters of the cascade PI and the proposed control strategy are the same as the above simulation shown in Table II. The dynamics of output voltage for the boost converter are pictured in the Fig. 9. It can be observed that both control schemes has good voltage tracking performance before the load change.
Fig. 3. The dynamics of output voltage of PI control with constant reference 50 V

Fig. 4. The dynamics of output voltage of PI control with constant reference 50 V

Fig. 5. The dynamics of output voltage with stepped reference

(a) The proposed control strategy

(b) The PI control strategy

Fig. 6. The error between the desired value and output voltage

Fig. 7. The dynamic of disturbance observer

Fig. 8. The dynamics of output voltage with sinusoidal and ramp reference

(a) The proposed control strategy

(b) The PI control strategy
However, when the equivalent load is stepped from 80 Ω to 40 Ω, the proposed method can enhance the voltage regulation performance obviously, less dynamics overshoot and faster dynamic response in comparison with the classical PI control. Specifically, it takes more than 440 ms to achieve the transient state for the PI control while the settling time of the proposed control method is only less than 210 ms, which is reduced over half. Beyond that, the max error of output voltage is over 4 V for the PI control, but for the proposed control the error of the output voltage is limited to the small range (the max error is less than 1 V), which is reduced by more than 75%. Similarly, when the equivalent load is stepped from 40 Ω to 60 Ω, compared with PI control, the proposed method still enhance the voltage regulation performance obviously which can been seen in the Fig. 9.

Fig. 9. The dynamics of output voltage with load change

V. CONCLUSION

In this paper, a novel control strategy for dc/dc boost converters, based on the backstepping control added to disturbance observer, has been proposed. First, the average model of dc/dc boost converter is reformulated, where unknown equivalent load, input voltage, model uncertainties and unmodeled dynamics are lumped as external disturbance and the control signal only appears in the current equations. Based on the established mathematical model, a backstepping controller integrated with disturbance observer has been designed to achieve the output voltage regulation. The simulations performed by the proposed control strategy and well turned cascade PI control are given to illustrate the effectiveness of the proposed method. It is should be pointed out that the proposed control scheme exhibit more excellent performance in comparison with the classical PI control under two conditions: reference voltage variation and unknown equivalent load variation.

REFERENCES