PARAMETRIC SIDEBAND AMPLIFICATION IN INJECTION LASERS

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If an injection laser is biased above threshold and modulated with a small signal with an angular frequency ω_s, the modulation sensitivity at ω_s can be increased if an additional large pumping current with an angular frequency $\omega_p > \omega_s$ is applied.

The existence of parametric effects in directly modulated injection lasers is suggested by the occurrence of subharmonics.[1] If an injection laser (biased above threshold current J_{th}) with internal small-signal modulation angular resonance frequency ω_0 is modulated with a sinusoidal large-signal current of an angular frequency near $2\omega_0$, subharmonic oscillations can be generated at half the modulation frequency. The generation of subharmonics results from the nonlinearity of the rate equations[2-4] and is supposed to be due to a parametric instability. We expect that there are operating conditions where instabilities are not yet possible, but parametric amplification exists. In this letter, a 2-frequency modulation will be investigated. The current applied to the laser consists of a d.c. component J_0 and two sinusoidal components with amplitudes J_p and J_s and angular frequencies ω_p and ω_s, respectively. Here, J_p corresponds to the pumping current, whereas J_s is the signal current. It is assumed that J_p is so large that the nonlinearities of the system are effective; however, J_s is small enough to be considered as a small-signal modulation.

Because of the nonlinearities, mixing components of various frequencies occur. Of these, only the term with the idler angular frequency $\omega_i = \omega_p - \omega_s$ will be taken into account. Neglecting other frequency components can be justified by the fact that only frequencies below or near the resonance frequency of the laser are of interest. If the second idler angular frequency $\omega_p + \omega_s$ is also near ω_0, we assume its suppression by an external microwave cavity. As the idler component results from mixing the large pumping current and the small signal current, it can be considered to be of small-signal type.

The laser will be described by the rate equations

$$\frac{dn}{dt} = \frac{J}{eV} - \frac{n}{\tau_{sp}} - gns \quad (1)$$

$$\frac{ds}{dt} = -\frac{s}{\tau_{ph}} + gns \quad (2)$$

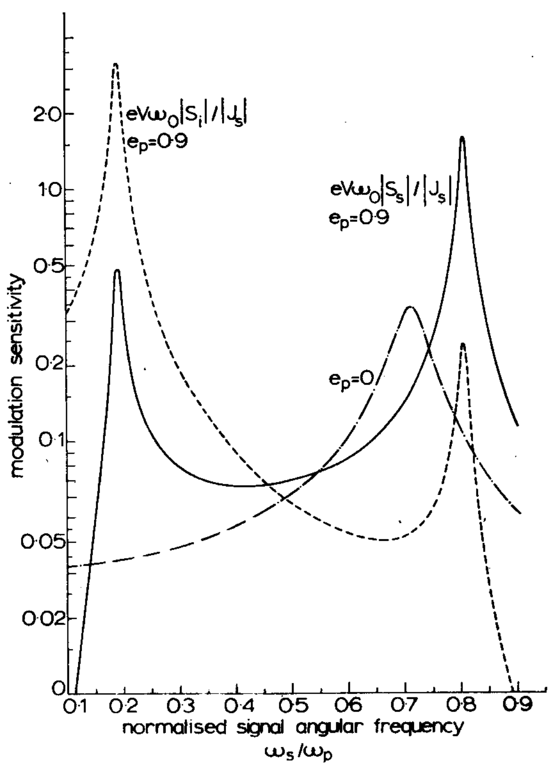

Fig. 1 *Modulation sensitivity of photon density as function of signal frequency*
$\tau_{ps} = 500$, $J_0/J_{th} = 1\cdot5$, $\omega_p/\omega_0 = 1\cdot4$

where n is the electron density, s is the photon density in the active region, J is the injection current, e is the electron charge, V is the volume of the active region, τ_{sp} is the spontaneous electron lifetime, τ_{ph} is the photon lifetime and gns is the term describing the stimulated emission. The injection current $J(t)$ has the form

$$J(t) = J_0 + \tfrac{1}{2}\{J_p \exp(j\omega_p t) + J_p^* \exp(-j\omega_p t) + J_s \exp(j\omega_s t) + J_s^* \exp(-j\omega_s t)\} \quad (3)$$

as described above. The electron and photon densities are given by

$$n(t) = N_0 + \tfrac{1}{2}\{N_p \exp(j\omega_p t) + N_p^* \exp(-j\omega_p t) + N_s \exp(j\omega_s t) + N_s^* \exp(-j\omega_s t) + N_i \exp(j\omega_i t) + N_i^* \exp(-j\omega_i t)\} \quad (4)$$

$$s(t) = S_0 + \tfrac{1}{2}\{S_p \exp(j\omega_p t) + S_p^* \exp(-j\omega_p t) + S_s \exp(j\omega_s t) + S_s^* \exp(-j\omega_s t) + S_i \exp(j\omega_i t) + S_i^* \exp(-j\omega_i t)\} \quad (5)$$

N_0, S_0, N_p and S_p are known from large-signal calculations:[4]

$$N_0 = \frac{J_{th}}{eV}\tau_{sp} \quad (6)$$

$$S_0 = \frac{\tau_{ph}}{eV}(J_0 - J_{th}) \quad (7)$$

$$S_p = a\phi(a) S_0 e^{j\varphi} \quad (8)$$

$$N_p = ja\omega_p \tau_{ph} N_0 e^{j\varphi} \quad (9)$$

with

$$\phi(a) = 2I_1(a)/aI_0(a) \quad (10)$$

where $I_0(a)$ and $I_1(a)$ are the modified Bessel functions of order 0 and 1 and a and φ are given by

$$e_p = \left\{\left(\frac{\omega^2}{\omega_0^2} - \phi(a)\right)^2 + \omega^2 \tau_{ph}^2 \left(\phi(a) + \frac{1}{\omega_0^2 \tau_{ph} \tau_{sp}}\right)^2\right\}^{\frac{1}{2}} \quad (11)$$

$$\tan\varphi = \omega\tau_{ph}\left(\phi(a) + \frac{1}{\omega_0^2 \tau_{ph}\tau_{sp}}\right)\left(\frac{\omega^2}{\omega_0^2} - \phi(a)\right)^{-1} \quad (12)$$

with

$$e_p = J_p/(J_0 - J_{th}) \quad (13)$$

$$J_{th} = eV/g\,\tau_{sp}\tau_{ph} \quad (14)$$

$$\omega_0^2 = (J_0/J_{th} - 1)/\tau_{sp}\tau_{ph} \quad (15)$$

The amplitudes N_s, S_s, N_i and S_i are unknown. Substituting eqns. 3–5 into eqns. 1 and 2, the equations can be linearised with respect to the amplitudes of signal and idler frequency. All large-signal amplitudes are known from eqns. 6–9, and terms containing products of two small-signal amplitudes are neglected. Thus, a linear equation system is obtained that relates the electron and photon densities of signal and idler frequency to the signal current:

$$A\begin{pmatrix} S_s \\ S_i^* \\ N_s \\ N_i^* \end{pmatrix} = \begin{pmatrix} J_s/eV \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (16)$$

with

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \quad (17)$$

$$A_{11} = \begin{pmatrix} gN_0 & gN_p \\ gN_p^* & gN_0 \end{pmatrix} \quad (17a)$$

$$A_{12} = \begin{pmatrix} gS_0 + \dfrac{1}{\tau_{sp}} + j\omega_s & gS_p \\ gS_p^* & gS_0 + \dfrac{1}{\tau_{sp}} - j\omega_i \end{pmatrix} \quad (17b)$$

$$A_{21} = \begin{pmatrix} gN_0 - \dfrac{1}{\tau_{ph}} - j\omega_s & gN_p \\ gN_p^* & gN_0 - \dfrac{1}{\tau_{ph}} + j\omega_i \end{pmatrix} \quad (17c)$$

$$A_{22} = \begin{pmatrix} gS_0 & gS_p \\ gS_p^* & gS_0 \end{pmatrix} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (17d)$$

Solving eqn. 16 for S_s and S_i yields

$$\begin{pmatrix} S_s \\ S_i^* \end{pmatrix} = (A_{11} - A_{12} A_{22}^{-1} A_{21})^{-1} \begin{pmatrix} J_s \\ 0 \end{pmatrix} \quad . \; . \; . \quad (18)$$

Calculations of the normalised modulation sensitivity $eV\omega_0 S_s/J_s$ and $eV\omega_0 S_i/J_i$ at signal and idler frequency have been carried out for different values of the parameters ω_p/ω_0, τ_{sp}/τ_{ph} and J_0/J_{th}. In Fig. 1, typical results are demonstrated for the modulation sensitivity of the photon density for both the signal and the idler frequency as function of the normalised signal angular frequency ω_s/ω_p. Practical values are chosen for the parameters: $\tau_{sp}/\tau_{ph} = 500$, $J_0/J_{th} = 1{\cdot}5$ and $\omega_p/\omega_0 = 1{\cdot}4$.

For $e_p = 0$, i.e. without pumping current, the signal modulation sensitivity has the well known shape, with a maximum at the small-signal resonance frequency. Obviously, the idler component does not exist. Quite a different situation arises if an additional pumping current ($e_p = 0{\cdot}9$) with an angular frequency $\omega_p = 1{\cdot}4\,\omega_0$ is applied. The signal modulation sensitivity (full curve) shows two resonance peaks at $\omega_s/\omega_p = 0{\cdot}81$ and $0{\cdot}19$, respectively. These two resonance frequencies also occur in the idler modulation sensitivity (broken curve). As an essential result of this calculation, it is shown that the maximum signal modulation sensitivity is significantly increased by applying a pumping current. Thus, an amplification of the modulation sensitivity has been obtained. Since only the modulation sidebands, and not the optical carrier, are amplified, this effect is called optical sideband amplification in contrast to the well known parametric quantum amplification of the optical carrier.

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