Analog-Computer Studies on Microwave Mixing in Superconducting Weak Links

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***Abstract*—The dc characteristics of superconducting weak links, irradiated by one and two microwave frequencies and the dependence of the absorbed microwave power on the dc current, are evaluated. Within the constant voltage steps, the ac current generated in the weak link is synchronous with the incident microwaves or their mixing products. In the case of two incident microwave frequencies within the constant voltage steps, the dependence of the ratio of power-absorption variation on the impressed dc current at the two frequencies is governed by modified Manley–Rowe equations.**

I. Introduction

JOSEPHSON junctions [1]–[5] can be applied for generation, detection, mixing, and parametric amplification of microwaves [6]–[15]. It has already been shown that for ideal Josephson junctions with a sinusoidal current dependence on the quantum phase difference, general energy relations that govern the mutual conversion of the dc power and the ac powers at different frequencies are valid [14]. These equations differ from the Manley–Rowe equations [16] by an additional term accounting for the converted dc power. The energy relations show the feasability of dc pumped parametric amplifiers with Josephson junctions.

In real superconducting weak links as point contacts [17] and thin-film bridges [18], [19], a dissipative quasi-particle current flows parallel to the Josephson current. In the microwave field the dc characteristic shows a step structure [11], [18]–[23] which can be used for detecting microwave and far-infrared signals. Theoretical investigations taking into consideration the quasi-particle current agree well with experimental results [23], [24]–[27]. In this paper we extend the analog-computer studies described earlier [27] to the case of two incident microwave frequencies. The power conversion is evaluated between dc and ac for one and two incident microwave frequencies.

II. Energy Relations for Ideal Josephson Junctions

The current–voltage relation of ideal lossless Josephson junctions can be described by the equations

$$i_J(t) = I_{\max} \sin \phi(t) \tag{1}$$

$$\dot{\phi}(t) = \frac{2ev(t)}{\hbar} \tag{2}$$

where i_J is the current through the junction, $v(t)$ the applied voltage, $\phi(t)$ the quantum phase difference across the junction, and $I_{\max}$ the maximum Josephson current [1]–[5]. For low frequencies, $I_{\max}$ is independent of the voltage and depends only on the temperature and the junction geometry and material. Since the Josephson current is a nonlinear unique function of the integral of the voltage over time, the ideal Josephson junction is a nonlinear lossless inductor. With the formally introduced magnetic flux $\phi(t)$, defined by

$$\dot{\phi}(t) = v(t) \tag{3}$$

the differential inductance of the ideal Josephson junction is

$$L(\phi) = \frac{d\phi}{di_{J(\phi)}} = \frac{L_0}{\cos(2\pi\phi/\phi_0)} \tag{4}$$

with

$$L_0 = \frac{\phi_0}{2\pi I_{\max}} \tag{5}$$

$$\phi_0 = \frac{h}{2e} = 2.07 \cdot 10^{-15}\ \mathrm{V \cdot s} \tag{6}$$

where ϕ_0 is the flux quantum. The energy, stored in the Josephson junction, is given by

$$w(\phi) = \int_0^{\phi} i_J(\phi) d\phi = \frac{\phi_0 I_{\max}}{2\pi}[1 - \cos(2\pi\phi/\phi_0)]. \tag{7}$$

Contrary to other inductors, the current i_J and the energy w are periodic functions of the magnetic flux. Energy and current stay restricted when the magnetic flux goes to infinity. They are not dependent on the absolute value of the flux but only on its deviation from an integer multiple of the flux quantum ϕ_0. It is not possible therefore to interpret ϕ as stored magnetic flux, since in this case the energy would go to infinity with the absolute value of the flux. But ϕ can be considered as the rate of flux flowing across the junction. The consequence is that the ideal Josephson junction can be considered as a nonlinear lossless inductor which also supports a dc voltage without driving the current and the energy to infinity.

From (1) and (2) it follows that a dc voltage V_0 causes a sinusoidal ac current i_J with the amplitude $I_{\max}$ and the frequency

$$f_0 = \frac{1}{2\pi}\omega_0 = \frac{2eV_0}{h} = 483.6 V_0\ \mathrm{GHz/mV}. \tag{8}$$

With a Josephson junction, power conversion is possible from dc to ac, from ac to dc, and (as a consequence of its nonlinearity) between different ac frequencies.

For nonlinear lossless reactances general energy relations, given by Manley and Rowe [16], which govern the real power

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conversion between two or more incommensurable frequencies and their mixing products are valid. For Josephson junctions these equations must be extended by an additional term for the dc power [14]. For the case of two frequencies f_1 and f_2 whose ratio is not an integer, and an applied dc voltage

$$V_0 = \frac{h}{2e}(kf_1 + lf_2) \tag{9}$$

the energy relations are

$$\sum_{m=1}^{\infty}\sum_{n=-\infty}^{\infty} \frac{mP_{mn}}{mf_1 + nf_2} = -\frac{kP_0}{kf_1 + lf_2} \tag{10}$$

$$\sum_{m=-\infty}^{\infty}\sum_{n=1}^{\infty} \frac{nP_{mn}}{mf_1 + nf_2} = -\frac{lP_0}{kf_1 + lf_2}. \tag{11}$$

P_{mn} are the real powers at the frequencies mf_1+nf_2 and P_0 is the dc power. Positive sign of P_{mn} and P_0 denotes power flow towards the junction. The dc power in (10) and (11) can be treated in the same manner as the active power P_{kl} at the frequency kf_1+lf_2.

The energy relations for Josephson junctions show the possibility of lossless power conversion between different ac frequencies and dc, and also, of the possibility of parametric amplification.

Of special interest is the case where only two incommensurable frequencies f_1 and f_2 occur and the dc voltage V_0 corresponds to a frequency f_0, which is the sum or the difference of f_1 and f_2. This case can be realized by coupling the Josephson junction with a resonator which has parallel resonances at f_1 and f_2 and short-circuits all other frequencies mf_1+nf_2. As Josephson junctions have a very low impedance this assumption would be generally unrealistic. Only with tunnel junctions which have a stripline resonator structure a broad-band matching is possible. In the case

$$f_0 = f_1 + f_2 \tag{12}$$

we get from (10) and (11)

$$\frac{P_1}{f_2} = \frac{P_2}{f_2} = -\frac{P_0}{f_0}. \tag{13}$$

The dc power flowing into the Josephson junction is converted to ac power at the frequencies f_1 and f_2 and these converted powers P_1 and P_2 are in the ratio f_1 to f_2 flowing away from the junction. Equation (13) would be valid in the case of the dc-pumped negative resistance parametric amplifier with f_1 and f_2 as signal and idler frequencies. The variation of the dc pump power with the signal power P_1 can be utilized for signal detection. In the case of the negative resistance parametric amplifier signal and idler power are flowing in the same direction. If the difference frequency

$$f_0' = f_2 - f_1 \tag{14}$$

is used as pump frequency we get from (10) and (11)

$$\frac{P_1}{f_2} = -\frac{P_2}{f_2} = \frac{P_0}{f_0'}. \tag{15}$$

In this case the real signal power P_1 and the real idler power flow in the opposite direction and amplification is only possible in the up-conversion mode.

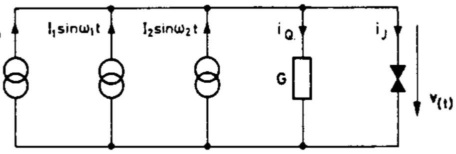

Fig. 1. Circuit model for a superconducting weak link irradiated by two monofrequent microwave sources with the angular frequencies ω_1 and ω_2.

III. Mixing of Microwaves in Real Superconducting Weak Links

Experimental investigations [18]–[23] of the dc characteristic of real superconducting weak links have shown a considerable deviation of the dependence of the constant voltage step height on the external microwave amplitude from the Bessel function behavior. The discrepancies between theory and experiment could be removed by considering the resistive feedback of the microwave field generated by the junction [21], [24]–[27]. In Dayem bridges and point contacts already at low voltages a quasi-particle current flows, which is porportional to the junction voltage and causes a conductance G parallel to the ideal junction. Tunnel junctions [28] have a parallel capacitance, which can cause hysteresis effects and instabilities in the dc I–V characteristic [29]. These effects can also be observed in point contacts with flattened points. Tunnel junctions can also act as line resonators and exhibit a step structure without microwave irradiations [6] due to the strong variation of the self-detection at the resonant frequencies. The existence of steps in the absence of external irradiated microwaves is a measure for the coupling of the Josephson junction to resonant structures. Series inductances disturb the sinusoidal current dependence on the magnetic flux. In this case a dc voltage V_0 also generates harmonics of f_0. The consequence of the locking of these harmonics of the generated frequency to the incident frequency are subharmonic steps in the dc characteristic [30].

In the following we restrict ourselves to junctions with no parallel capacitance and no resonant structure. Since the impedance of superconducing weak links is very low, the external microwave sources act as current sources. In the case of impressed current the series impedances of the junction have no effect on the dc characteristic. Fig. 1 shows the circuit model for the superconducting weak link irradiated by two microwave sources with the angular frequencies ω_1 and ω_2. The circuit is described by

$$I_0 + I_1 \sin \omega_1 t + I_2 \sin \omega_2 t = I_{\max} \sin \phi(t) + G \cdot u(t). \tag{16}$$

I_0 is the impressed dc current, I_1 and I_2 are the impressed microwave current amplitudes, and G is a conductance describing the effect of the quasi-particle current. To reduce the number of parameters we introduce the following normalized quantities:

$$\alpha_0 = \frac{I_0}{I_{\max}} \tag{17}$$

$$\alpha_1 = \frac{I_1}{I_{\max}} \tag{18}$$

$$\alpha_2 = \frac{I_2}{I_{\max}} \tag{19}$$

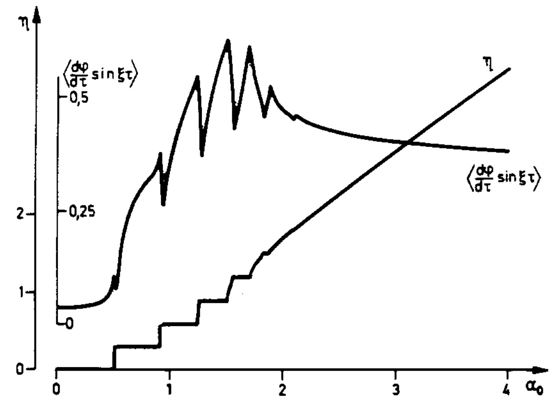

Fig. 2. Normalized dc voltage component η and normalized absorbed microwave active power $\langle (d\phi/d\tau) \sin \xi\tau \rangle$ in dependence of the normalized impressed dc current α_0 for one incident microwave frequency $\xi_1 = 0.3$ with the normalized amplitude $\alpha_1 = 0.75$.

$$\Omega = \frac{2eI_{\max}}{\hbar G} \tag{20}$$

$$\xi_1 = \frac{\omega_1}{\Omega} \tag{21}$$

$$\xi_2 = \frac{\omega_2}{\Omega} \tag{22}$$

$$\tau = \Omega t. \tag{23}$$

From (2), (20), and (23) we get the normalized voltage

$$\frac{d\phi}{d\tau} = \frac{Gv}{I_{\max}} \cdot \tag{24}$$

The normalized form of (16) is

$$\alpha_0 + \alpha_1 \sin \xi_1\tau + \alpha_2 \sin \xi_2\tau = \frac{d\phi}{d\tau} + \sin \phi. \tag{25}$$

Without external microwave radiation ($\alpha_1 = \alpha_2 = 0$) (25) can be solved exactly [29], [31]. The mean value

$$\eta = \left\langle \frac{d\phi}{d\tau} \right\rangle \tag{26}$$

of the junction voltage is given by

$$\eta = 0, \qquad \text{for } \alpha_0 \leq 1$$
$$\eta = \sqrt{\alpha_0^2 - 1}, \qquad \text{for } \alpha_0 > 1. \tag{27}$$

The ac real powers $P_i (i = 1, 2)$ flowing towards the junction at the frequencies ω_i are given by

$$P_i = \langle v(t) \cdot I_i \sin \omega_i t \rangle. \tag{28}$$

From (18), (24), and (28) we get

$$P_i = \frac{\alpha_i I_{\max}^2}{G} \left\langle \frac{d\phi}{d\tau} \cdot \sin \xi_i \tau \right\rangle. \tag{29}$$

We have calculated the normalized dc junction voltage η and the normalized real powers $\langle (d\phi/d\tau) \sin \xi_i\tau \rangle$ on the analog computer. Fig. 2 shows the results for one incident microwave frequency. The normalized frequency is $\xi = 0.3$ and the normalized amplitude is $\alpha_1 = 0.75$. The slope of the normalized voltage η has been discussed in an earlier paper [27].

We can distinguish three regions of the dc I–V characteristic, where the dependence of real power on the impressed dc current differs.

1) In the zero voltage region ($\eta = 0$) the absorbed microwave power increases with the dc current. An analytic solution can be given for a small incident microwave signal, where the zero voltage region extends from $\alpha_0 = 0$ to $\alpha_0 = 1$. For small ac amplitudes the junction inductance L depends only on the dc current. The current source I_{10} delivers an active power of

$$P_1 = \frac{1}{2} I_{10}^2 \frac{\omega^2 L^2 G}{1 + (\omega LG)^2} \tag{30}$$

into the junction. From (1)–(6), (17), and (20) it follows

$$L(\alpha_0) = \frac{1}{\sqrt{1 - \alpha_0^2}\, \Omega G} \cdot \tag{31}$$

Inserting (18), (21), and (31) into (30) yields

$$P_1 = \frac{1}{2} \frac{I_{\max}^2}{G} \frac{\alpha_1^2 \xi^2}{1 - \alpha_0^2 + \xi^2} \cdot \tag{32}$$

Since the parallel inductance L increases with the dc current, P_1 increases also.

2) In the regions $n\xi < \eta < (n+1)\xi$ between the constant voltage steps P_1 decreases with increasing dc current. For this behavior we suggest the following qualitative explanation. If the junction voltage has a dc component, the Josephson current i_J oscillates between $-I_{\max}$ and $+I_{\max}$ and causes a variation of the junction voltage between $(I_0 + I_{\max})/G$ and $(I_0 - I_{\max})/G$. In the interval from $-I_{\max}$ to 0 the current i_J flows across G in the same direction as I_0. Therefore, the junction voltage and, according to (2), the phase velocity $\dot{\phi}(t)$ are raised and this interval is of a shorter duration than the interval of $i_J(t)$ from 0 to $I_{\max}$, where the junction voltage and $\dot{\phi}(t)$ are lowered. Consequently, $i_J(t)$ has a positive mean value. For small η the phase velocity $\dot{\phi}(t)$ is lowered in the vicinity of $i_J = I_{\max}$ strongly enough so that i_J approximates $I_{\max}$ for the most time of the period. From (1) and (4) we see, that the junction inductance gets to infinity as i_J goes to $I_{\max}$. It follows from (30) that for small η the mean value of P_1 is a maximum. We note that this explanation is justified only for small α_1 and small η, because only in this case the time dependence of the junction inductance is not altered by the incident microwave signal, and averaging (30) over a slowly varying $L(t)$ becomes possible.

3) For incident microwaves the dc I–V characteristic exhibits constant voltage steps at $\eta = n\xi$. Within the nth step the frequency of the incident microwaves coincides with the nth harmonic of the frequency generated by the dc voltage. The Josephson junction is synchronized by the frequency of the incident microwaves.

Within the first step P_1 increases with increasing dc power. If I_{10} is the complex amplitude of the current impressed at the angular frequency ω_1 into the parallel circuit of the loss conductance G and the ideal Josephson junction, and if I_{1J} is the complex amplitude at ω_1 of the current flowing into the ideal junction, the real powers P_{1G}, flowing from the radiation source I_{10} into G, and P_{1J}, flowing from the ideal

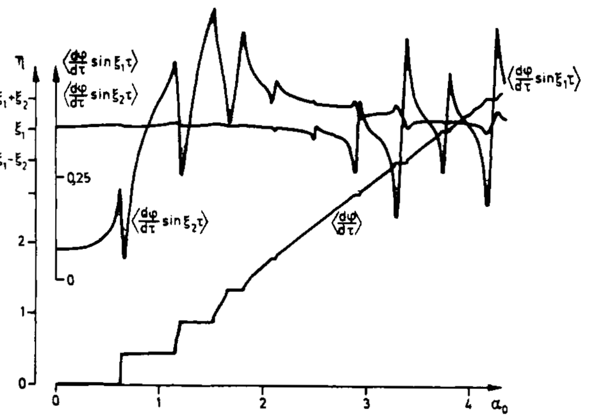

Fig. 3. Normalized dc voltage component η and normalized absorbed microwave active powers $\langle (d\phi/d\tau) \sin \xi_1\tau \rangle$ at the normalized frequency ξ_1 and $\langle (d\phi/d\tau) \sin \xi_2\tau \rangle$ at the normalized frequency ξ_2 in dependence of the normalized impressed dc current α_0 for two incident microwave frequencies $\xi_1 = 3.6$ and $\xi_2 = 0.45$ with $\alpha_1 = \alpha_2 = 0.75$.

Josephson junction into G, are

$$P_{1G} = \frac{1}{2G} \left(|I_{10}|^2 - \mathrm{Re}\,[I_{10} I_{1J}^*] \right) \tag{33}$$

$$P_{1J} = \frac{1}{2G} \left(|I_{1J}|^2 - \mathrm{Re}\,[I_{10} I_{1J}^*] \right). \tag{34}$$

For weak resistive feedback the absolute value of I_{1J} is constant and equal to the maximum Josephson current I_{max}. If also the impressed current amplitude is held constant P_{1G} and P_{1J} depend only on the phase difference between I_{10} and I_{1J}. Since the terms depending on this phase difference are equal in (33) and (34) the changes of P_{1G} and P_{1J} with the phase difference are equal. Within the step, the microwave current generated in the junction is synchronous with the incident microwave radiation. For constant dc current the phase difference between I_{10} and I_{1J} is also constant. Increasing the dc current leads to a higher microwave power flowing from the Josephson junction towards G. This raises the ac voltage at G and therefore also the power flowing from the current source to G.

Fig. 3 shows the dc current dependence of η and the normalized real powers $\langle (d\phi/d\tau) \sin \xi_1\tau \rangle$ and $\langle (d\phi/d\tau) \sin \xi_2\tau \rangle$ flowing towards the junction at the normalized frequencies ξ_1 and ξ_2 for two incident microwave signals of different frequencies. From (10) and (11) we expect a power conversion between the dc power and the powers at frequencies ω_1 and ω_2, if the dc voltage component across the junction corresponds to a combination frequency of ω_1 and ω_2. It has to be noted, that the energy relations are only valid for ideal Josephson junctions with no quasi-particle currents. Additionally, in the case of impressed current, the junction voltage is rich in harmonics and mixing products, so that the ratios of the powers flowing into the ideal junction cannot be determined from the power relations alone. Only in the case of strong damping by the loss conductance, when the frequencies of the incident signals and the generated frequency ω_0 dominate, (10) and (11) give a possibility to estimate the power ratios. At the normalized voltages $\eta = \xi_1 - \xi_2$ and $\eta = \xi_1 + \xi_2$ the powers P_1 and P_2 exhibit a strong dependence on the dc current.

If the dc current is varied within a constant voltage step the dc power flow into the loss conductance remains constant, since the voltage across G is constant. The change of the dc power equals the change of the power converted by the Josephson junction into ac power. If only the frequencies ω_1 and ω_2 occur across an ideal Josephson junction, (13) and (15) are valid for $\eta = \xi_1 + \xi_2$ and $\eta = \xi_1 - \xi_2$. Contrary to these equations, according to Fig. 3, all powers flow towards the junction as a consequence of the power absorption in real Josephson junctions. Also the ratios of the real powers P_1 and P_2 do not agree with (13) and (15) because a considerable amount of the incident microwave power is directly absorbed by the loss conductance G. Only the ratio of the power variations at the normalized frequencies ξ_1 and ξ_2, when α_0 is changed with a step, shows a relation to the power ratios in the ideal case. For equal normalized amplitudes α_1 and α_2 the ratio of the power variations is given by (13) and (15). Within the step at the sum frequency $\xi_1 + \xi_2$, the variations of the powers P_1 and P_2 with α_0 are, according to (13), in the same direction. Within the step at the difference frequency $\xi_1 - \xi_2$ the variations of P_1 and P_2 with α_0 are in opposite direction according to (15).

The zero-voltage region and the region between the steps can be interpreted in the same way as in Fig. 2. We note, that for steps, corresponding to one frequency or its harmonics $\eta = m\xi_1 (\eta = n\xi_2)$ only the real power $P_1(P_2)$ of the signal incident at this frequency is strongly dependent on α_0. The real power $P_2(P_1)$ at the other frequency shows only weak bumps within these steps. This result also shows a relevance to the energy relations. In the small signal case, when the time dependence of the junction inductance results only from the dc voltage component across the junction, these bumps disappear.

IV. Conclusion

The general energy relations for ideal Josephson junctions suggest the application of Josephson junctions for frequency conversion and parametric amplification. One possible method for parametric amplification is to measure the absorbed power P_2 at a higher frequency ω_2 as it depends on the amplitude of a signal with a lower frequency ω_1.

For a high loss conductance G we see from Figs. 2 and 3 that a considerable part of the microwave powers is absorbed by the loss conductance. Therefore, parametric amplification is not possible for high loss conductances. In the case of a low loss conductance, due to the higher resistive feedback, more harmonics and mixing products are generated in the Josephson junction and a considerable amount of the power is converted into parasitic frequencies.

For the use in parametric amplifiers, the Josephson junction must be matched in a broad frequency band to a low load impedance. Such a load can be for example the resonator formed by a tunnel junction with strip line geometry. A broad-band matching is necessary for the following reason: If a Josephson junction is only matched to the load at the frequency ω_0, corresponding to the applied dc voltage V_0, the load conductance will damp the Josephson junction at ω_0 and the junction will possibly oscillate at two frequencies ω_1 and ω_2, the sum of which is ω_0. To avoid such parametric instabilities a broad-band matching of Josephson junctions is necessary.

A higher junction impedance can be achieved by lowering the maximum current I_{max} or by going to higher frequencies.

As the ideal Josephson junction is a nonlinear inductor, its impedance is inversely proportional to the frequency.

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