



## Editorial

## Editorial to the special issue on Behavioral Insurance: Mathematics and Economics

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## 1. Behavioral Insurance: Mathematics and Economics

This special thematic issue of *Insurance: Mathematics and Economics* collects eight papers on “Behavioral Insurance: Mathematics and Economics”, published with open access. Each of the eight articles in this issue underwent the same strict peer-review procedure as regular submissions to *Insurance: Mathematics and Economics*. We are very grateful to the ERGO Group for generously sponsoring the open access publication fees for this special issue. We thank the Editorial Office of *Insurance: Mathematics and Economics* for the support in handling the submissions.

This Editorial briefly introduces the rapidly evolving field of Behavioral Insurance and puts the eight papers into perspective by indicating their place in and contribution to the existing literature.

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## 1.1. Aims and scope

Human behavior and decision-making under risk and ambiguity are subject to the unconscious mind, habits, and other phenomena that can induce deviations from “rationality”. In recent years, considerable empirical and experimental evidence has documented that, in particular, *insurance*-related financial decision-making often does not comply with the standard economic model of choice under uncertainty. People’s assessment of actuarial and insurance-related financial risks and the resulting behavior can often be characterized by anomalies, leading to “puzzles” and “paradoxes”.

Studies, aimed at improving our understanding of decision-making under risk and ambiguity in insurance, require state-of-the-art mathematical and economic models as well as sophisticated quantitative methods, tools, and techniques to analyze them. This special thematic issue brings together a fine collection of papers addressing this common challenge. Potential applications include a better alignment of insurers’ strategies with their clientele and shareholders when designing products, sharing information, calculating premia, and developing risk management policies. These studies may also lead to a better understanding by insureds as to how to improve their life-cycle insurance decision-making

and to an improved knowledge base for regulators concerning the behavioral mechanisms that drive the market they regulate.

### 1.2. Probability weighting and optimal underwriting and reinsurance

The first two papers in this special issue allow for a subjective element in evaluating objective probabilities. In their paper entitled “Comparative Risk Aversion in RDEU with Applications to Optimal Underwriting of Securities Issuance”, Ghossoub and He study the risk aversion implied by rank-dependent expected utility (RDEU) and its application to a contracting problem between an issuing firm and an underwriter in a public equity offering. RDEU adds a probability weighting component to the classical expected utility model to capture risk attitude towards tail events, having attracted significant interest in behavioral research (see, e.g. Quiggin, 1982, 1993, Yaari, 1987, Chew et al., 1987, Roëll, 1987; Ryan, 2006; Jin and Zhou, 2008; He and Zhou, 2011, Bernard et al., 2015, Eeckhoudt and Laeven, 2021, and the references therein).

In the first part of their paper, Ghossoub and He establish necessary and/or sufficient conditions on three comparative notions of RDEU-related risk aversion, and in particular overturn an if-and-only-if condition claimed by Quiggin (1993) in terms of comparative RDEU risk aversion. The second part of the paper applies these results to a generalization of the classical model of Mandelker and Raviv (1977) on equity issuance incorporating RDEU preferences. Under some technical assumptions, the paper proves that a firm-commitment contract is optimal if-and-only-if the issuing firm is more concerned about the tail probabilities than the underwriter is. The paper derives and streamlines the implication relationships among various notions of risk aversion, and demonstrates nicely how they can be applied to design insurance contracts for situations in which extreme events are of great concern (which classical, non-behavioral models do not address).

A major problem in actuarial science is the optimal design of (re)insurance contracts. Its general aim is to determine the optimal amount of monetary compensation to the insured as a function of the incurred loss (and other variables), also called the indemnity function. The shape of the optimal indemnity function strongly depends on the assumptions imposed, in particular those concerning preferences. Maximizing expected utility of the insured's wealth and pricing the insurance contract based on the expected value principle has been a generally accepted benchmark approach since the seminal work of Arrow (1963), Mossin (1968), and Raviv (1979). They have shown in a classical single-period setting that the optimal indemnity function usually includes a deductible. In a multi-period, dynamic setting, Briys (1986), Touzi (2000), Moore and Young (2006), Perera (2010, 2013), Zou and Cadenillas (2014), and Steffensen and Thøgersen (2019) have derived the optimal indemnity functions, mostly including a deductible, under various assumptions.

In their paper entitled “Optimal Reinsurance with Multiple Reinsurers: Distortion Risk Measures, Distortion Premium Principles, and Heterogeneous Beliefs”, Boonen and Ghossoub explore a one-period optimal reinsurance design problem for an insurer seeking reinsurance from multiple reinsurers. The insurer is assumed to apply probability weighting when assessing the total, retained risk exposure after reinsurance. Furthermore, the reinsurers are supposed to use probability weighting when computing their reinsurance premia. The insurer and reinsurers are allowed to differ in their beliefs concerning the underlying, “true” probability distribution. The insurer minimizes the risk exposure, while satisfying the so-called no-sabotage and no-ex post moral hazard conditions. The authors also analyze the effect on the optimal design of a premium budget—a maximum amount spent on reinsurance. Interestingly, the authors show that the optimal indemnity functions in their general setting are of the familiar “layer-type” form,

which appears to be a robust finding for optimal indemnity functions under probability weighting (see, e.g. Cui et al., 2013, and the references therein).

### 1.3. Habit formation and optimal non-life insurance demand

In the literature on dynamic optimal insurance design, the insured's preferences are typically described by an additive time-separable utility function, assuming that the instantaneous satisfaction only depends on the current level of consumption. In reality, however, it may be reasonable that the insured's past consumption also influences current decision-making. Habit formation is a popular approach to allow for the insured's instantaneous utility to depend upon past consumption. This concept, where a consumer's instantaneous utility depends upon the consumption history, has been successfully used to analyze asset prices and optimal consumption and investment; see, e.g. Sundaresan (1989), Constantinides (1990), Abel (1990), Detemple and Zapatero (1991), Munk (2008), van Bilsen et al. (2020a,b), and the references therein. Furthermore, Ben-Arab et al. (1996) study optimal insurance demand under habit formation.

In their paper “Demand for non-life insurance under habit formation”, Li, Tan and Wei analyze the optimal design of non-life insurance contracts. More specifically, they derive optimal indemnity functions for non-life insurance contracts in the life-cycle consumption problem of an individual who is supposed to decide on consumption, saving and the demand for insurance, and who exhibits habit formation. In various aspects, they differ from and extend the model of Ben-Arab et al. (1996), such as the definition of the loss, the assumptions on the utility function and the admissibility of a general class of indemnity functions. Considering the expected value premium principle, the authors show that the optimal indemnity function includes a deductible. Furthermore, under the assumption of an exponential utility function, they obtain a fully explicit solution. The authors find that the insurance coverage is typically reduced by habit formation. This phenomenon is of particular interest and may offer a partial explanation for the empirical evidence of global underinsurance. They also study the case where only proportional insurance is available, derive the corresponding solution, and investigate the welfare loss from such a suboptimal situation compared to the optimal unconstrained consumption strategy and indemnity function.

### 1.4. Behavioral theories and optimal annuitization

Within the behavioral insurance literature, there is a large and growing number of researchers interested in what is generally known as the “annuity puzzle” and its assorted solutions. Whereas most consumers actively seek and acquire protection against quite a wide variety of visible economic risks, it seems they are less inclined to protect against so-called longevity risk, at least at the high levels predicted by rational additive utility theory, that is, within the Yaari (1965) model. A multitude of research papers have been written in the last 50 years, either trying to “extend and generalize” or to “break and negate” the theoretical predictions contained in Yaari (1965). Now, this is not a mere academic matter revolving around economic or insurance theory. Indeed, billions of consumers around the world must decide how to allocate trillions of investable wealth at retirement, as the government provision of Defined Benefit (DB) pensions continues to decline.

These low levels of voluntary annuitization are also intimately linked with the reduced levels of discretionary spending in retirement, appropriately called the “consumption puzzle”. In other words, individuals reach retirement and not only avoid annuities—which would ensure ample resources for the rest of their life—but they also avoid consumption, perhaps out of fear of exhausting

those same resources. In loose terms, it is the equivalent of owning an expensive car, but not insuring it properly or at all, and then not driving it for fear it might befall an accident. These twin puzzles are ripe for behavioral theories which require rationalizations beyond classical models (i.e., positive aspects). But the twin puzzles also beg for interventions and educational efforts (i.e., normative aspects) that break through behavioral obstacles. This can be achieved via the design of better decumulation products or clearer explanations for how consumers should behave and allocate wealth in retirement.

Three papers in this special issue address these matters directly and head-on. The article by Chen, Hieber and Rach entitled “Optimal Retirement Products under Subjective Mortality Beliefs” focuses on personal longevity risk perceptions and how they might differ from objective population risk measures. In particular, they show that heterogeneity of mortality beliefs will lead some retirees to favor annuities in which payments are guaranteed, while others might prefer “natural tontines” in which longevity risk is pooled and shared. And, while the discounted value and payoff of a tontine converges to that of a natural annuity in the absence of market frictions, small pools and real-world considerations result in a dispersion of outcomes. Chen, Hieber and Rach advance the tontine versus annuity dialogue and provide conditions under which one is preferred to the other.

Within the context of life annuities, the article by Bernard, De Gennaro and Levante entitled “Optimal Annuity Demand for General Expected Utility Agents” examines how to measure and calibrate the welfare benefits of annuitization within a variety of additive utility-based objective functions. Thus, while it is clear that real-world annuity take-up and demand should be larger than what is observed in practice, back to Yaari (1965), a matter that has not received much attention in the literature is the relative magnitude of the annuity welfare gains. Stated differently, the authors address the question: How much do consumers lose by not allocating to annuities within their retirement or drawdown portfolio? The annuity equivalent wealth (AEW) is the go-to metric used in the insurance and economics literature to calibrate welfare gains, and Bernard, De Gennaro and Levante test the robustness of an easy-to-use analytic approximation to the AEW. Perhaps by reporting and widely disseminating the AEW, consumers will be able to evaluate, rank and choose the right annuity (or even tontine) for them.

Finally, the article by Zhang, Purcal and Wei entitled “Optimal Life Insurance and Annuity Demand under Hyperbolic Discounting when Bequests are Luxury Goods” augments the twin controls of consumption and annuitization with a third lever, namely the demand for life insurance to satisfy bequest motives. After all, if the reason retirees shun annuities is due to bequest motives, then life insurance must surely be introduced into such a discussion and model. This decision trinity—consumption, annuities and insurance—is analyzed within a dynamic life-cycle framework, and then calibrated using data from Switzerland. Zhang, Purcal and Wei are able to graft cutting-edge economic concepts on time-inconsistency and hyperbolic discounting into behavioral insights with testable implications.

### 1.5. Loss aversion and return smoothing

In many countries, participating life insurance contracts with an embedded guarantee on the yearly portfolio return have had the biggest market share for decades. However, many years of eroding interest rates have ultimately revealed the risks involved in this long-term cliquet-style option. At first sight, the consequences are most severe for insurance companies because they have to stand up to their guarantees. On the other hand, as insurance companies have adapted their portfolio strategies, often to more conserva-

tive investments due to solvency considerations, the savings of the existing insured individuals are now also affected. Another consequence has been a change in product design for new consumers: yearly guarantees have been lowered, removed, or replaced by other guarantees. Removing the cliquet-style option can, in fact, be beneficial for both insurance companies and insured persons: the former being freed from this obligation, the latter hoping for more attractive returns once insurance companies have more freedom in their investment decisions. However, the reality is that in many countries consumers still appreciate a year-by-year guarantee, and it is difficult to convince them of the attractiveness of alternatives.

This real environment provides the background for the paper entitled “Return Smoothing in Life Insurance from a Client Perspective” by Russ and Schelling. In their paper, the authors demonstrate that return smoothing alone (without guarantees) can significantly increase the product attractiveness for investors. Particularly, the paper demonstrates that under realistic assumptions, many loss-averse long-term investors prefer products without guarantees, but with smoothed returns. The results are valid for long-term investors who, besides the terminal value, also evaluate potential annual changes in the account value. The proposed model provides evidence that long-term investors consider potential annual value changes already when making the investment decision. In this way, the paper offers an explanation for the popularity of traditional participating life insurance products with return smoothing in Germany.

### 1.6. Self-excitation in insurance

Deviations from traditional views concerning risk can be obtained by using preference representations that go beyond the standard economic model of choice under uncertainty. The first seven papers in this special issue assume either general or non-standard preferences. As an alternative, one may aim at directly modeling a resulting phenomenon, without imposing explicit, potentially too narrow, assumptions about the underlying preferences or economic circumstances that drive the phenomenon of interest. The paper by Swishchuk, Zagst and Zeller entitled “Hawkes Processes in Insurance: Risk Model, Application to Empirical Data and Optimal Investment” follows the latter modeling approach to describe the statistical behavior of insurance claims and analyzes the corresponding implications.

For this purpose, the authors use the class of self-exciting, or Hawkes, processes, named after Hawkes (1971). These point processes with stochastic jump intensities have been applied in finance, for example, to capture financial contagion potentially caused in part by herding behavior; see, e.g. Ait-Sahalia et al. (2015), Hawkes (2017), and the references therein. Interestingly, the contribution by Swishchuk, Zagst and Zeller represents a first application using real *insurance* data. The data are obtained from a large German Insurance group, comprising claim occurrences induced by “legal expenses insurance”, with triggering features. Furthermore, exploiting functional central limit theorem results, the paper also analyzes ruin probabilities via diffusion approximations in this setting. Finally, the paper analyzes the optimal investment implications of Hawkes processes, in particular when compared to the benchmark Poisson process. The paper demonstrates that the clustering features of Hawkes processes induce a higher risk level and therefore a lower risk profile, i.e., more conservativeness in the investment strategy of the insurer.

### 1.7. Concluding remarks

We hope that the eight papers collected in this special issue spur further developments in this challenging and fascinating area of research.

Besides, these eight papers—six of which interestingly begin with the word “optimal” in their title—signal to IME readers and the profession at large that dynamic optimization techniques continue to play a robust and important role, even in a behavioral future. Optimization is not quite ready for retirement!

## 2. Table of contents

1. Laeven, Roger J. A., Moshe A. Milevsky, Matthias Scherer, Rudi Zagst and Xun Yu Zhou. “Editorial to the Special Issue on Behavioral Insurance: Mathematics and Economics.”
2. Ghossoub, Mario and Xuedong He. “Comparative Risk Aversion in RDEU with Applications to Optimal Underwriting of Securities Issuance.”
3. Boonen, Tim and Mario Ghossoub. “Optimal Reinsurance with Multiple Reinsurers: Distortion Risk Measures, Distortion Premium Principles, and Heterogeneous Beliefs.”
4. Li, Wenyuan, Ken Seng Tan and Pengyu Wei. “Demand for Non-Life Insurance under Habit Formation.”
5. Chen, An, Peter Hieber and Manuel Rach. “Optimal Retirement Products under Subjective Mortality Beliefs.”
6. Bernard, Carole, Luca De Gennaro Aquino and Lucia Levante. “Optimal Annuity Demand for General Expected Utility Agents.”
7. Zhang, Jinhui, Sachi Purcal and Jiaqin Wei. “Optimal Life Insurance and Annuity Demand under Hyperbolic Discounting when Bequests are Luxury Goods.”
8. Russ, Jochen and Stefan Schelling. “Return Smoothing in Life Insurance from a Client Perspective.”
9. Swishchuk, Anatoliy, Rudi Zagst and Gabriela Zeller. “Hawkes Processes in Insurance: Risk Model, Application to Empirical Data and Optimal Investment.”

## 3. Review of the 23rd IME conference in Munich

After one year of global pandemic, with restrictions putting a full stop to academic gatherings, professional and social contacts, writing a report about an international conference with around 400 participants almost feels like describing a crime scene.<sup>1</sup> Being positive, let us describe what we have missed over the last year and hope to soon experience again. The 23rd conference of this renowned series was hosted in Germany for the first time and took place from July 10 to 12, 2019. The gathering was organized by the Chair of Mathematical Finance at the Technical University of Munich (TUM) with the organizing committee consisting of Lexuri Fernández, Bettina Haas, Yevhen Havrylenko, Peter Hieber, Matthias Scherer, Lorenz Schneider, Büsra Temoçin and Rudi Zagst.

The program opened with addresses from Wolfgang A. Herrmann (TUM president) and Mark Klein (Chief Digital Officer of the ERGO Group). Both emphasized the success of the cooperation between TUM and the ERGO Group within the framework of the ERGO Center of Excellence in Insurance. The first keynote speaker was Paul Embrechts (ETH Zürich), who discussed “The Fundamental Theorem of Quantitative Risk Management”. He particularly pointed out the danger of accepting inaccurate distributional assumptions to achieve mathematical simplicity. In a very entertaining way, Moshe A. Milevsky (York University Toronto) provided a lecture entitled “500 Years of Annuity Mispricing”. A survey on stochastic programming in portfolio optimization was given by William T. Ziemba (University of British Columbia and London School of Economics). The more recent history of dependence modeling using Vine Copulas was jointly discussed by Kjersti Aas

(Norwegian Computing Center) and Claudia Czado (Technical University of Munich) in their talk “Vine Copulas in Finance and Insurance”. Christian Gollier (Toulouse School of Economics) made interesting philosophical considerations on the relevance of discounting with regard to the transfer of wealth between generations. The final keynote speech was delivered by Hansjörg Albrecher (Université de Lausanne). His presentation was entitled “Models in Non-Life Insurance: From the Casting to the Catwalk” and dealt with current developments in the field of loss-event modeling. Almost 300 participants were active as speakers. The talks were classified by the topics Data Science in Insurance, Life and Non-Life Insurance, Quantitative Risk Management, Asset Liability Management, and Emerging Risks. This resulted in a very interesting and diverse program of presentations, organized in nine parallel tracks.

Besides the extensive scientific program, there was also time for social exchange and a first impression of Munich. For this purpose, a nocturnal guided tour through Munich’s city center and a classic Bavarian dinner in the English Garden were offered. There was great praise for the catering by the Sheraton Munich Arabellapark. A surprise to the participants was the premiere of the documentary film about the mathematician and pacifist Emil J. Gumbel, to which Lexuri Fernández and Matthias Scherer contributed as scientific advisors. A particular highlight of the congress was the ceremonial farewell of Rob Kaas as Managing Editor of the IME journal. Roger Laeven and Hans-Ulrich Gerber gave an entertaining and very personal review of his many years of service to IME.

## Declaration of competing interest

There is no competing interest.

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