Bayesian modelling of discontinuities and piecewise trends (trend changes) improves coastal vertical land motion estimates

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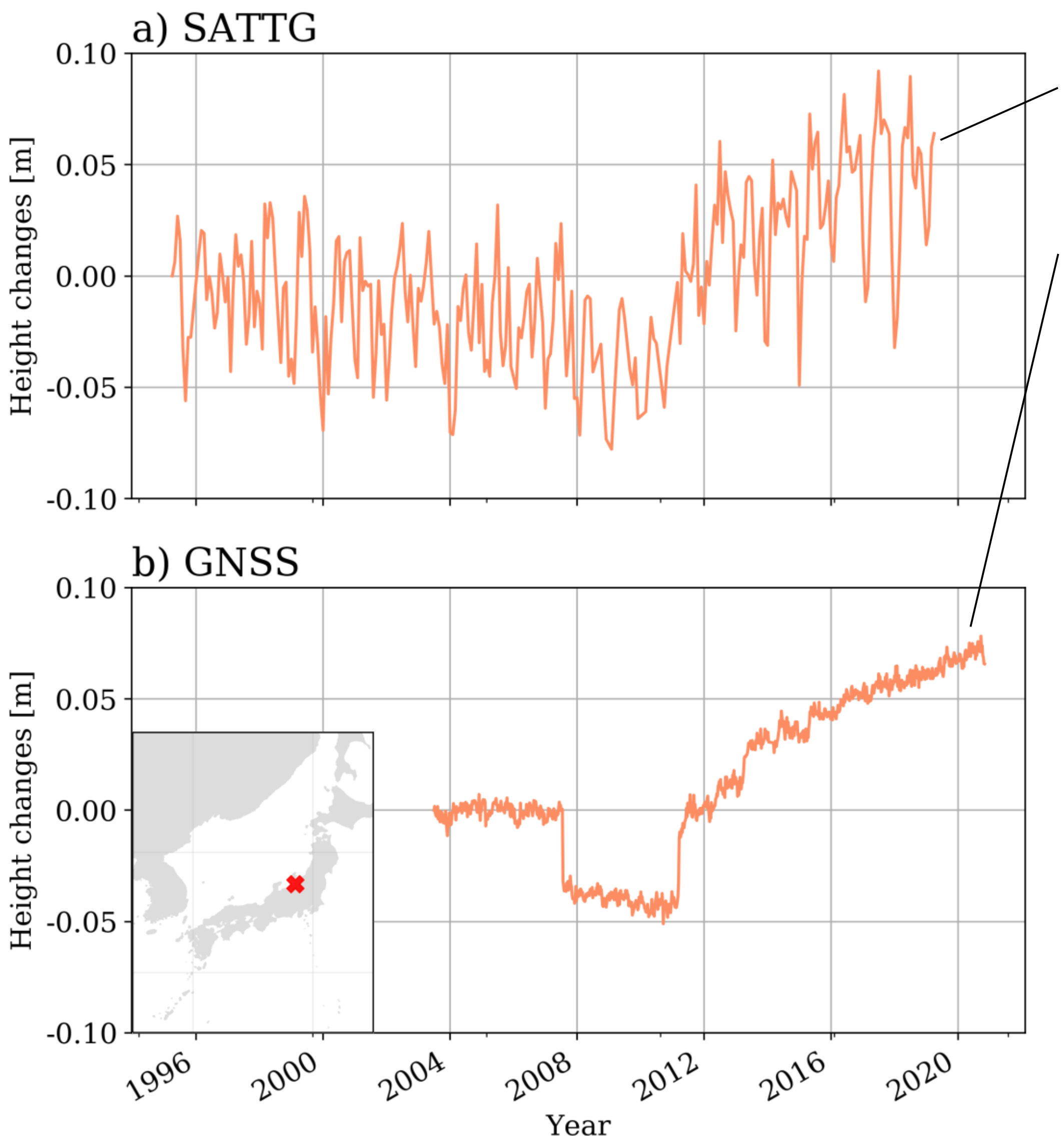
The challenge to detect time series discontinuities



- ➢ Satellite altimetry minus tide-gauge (SATTG) time series[1]
- • Proxy for height changes

- ➢ GNSS height time series (NGL, Blewitt et al., 2016)

[1]ALES, PSMSL

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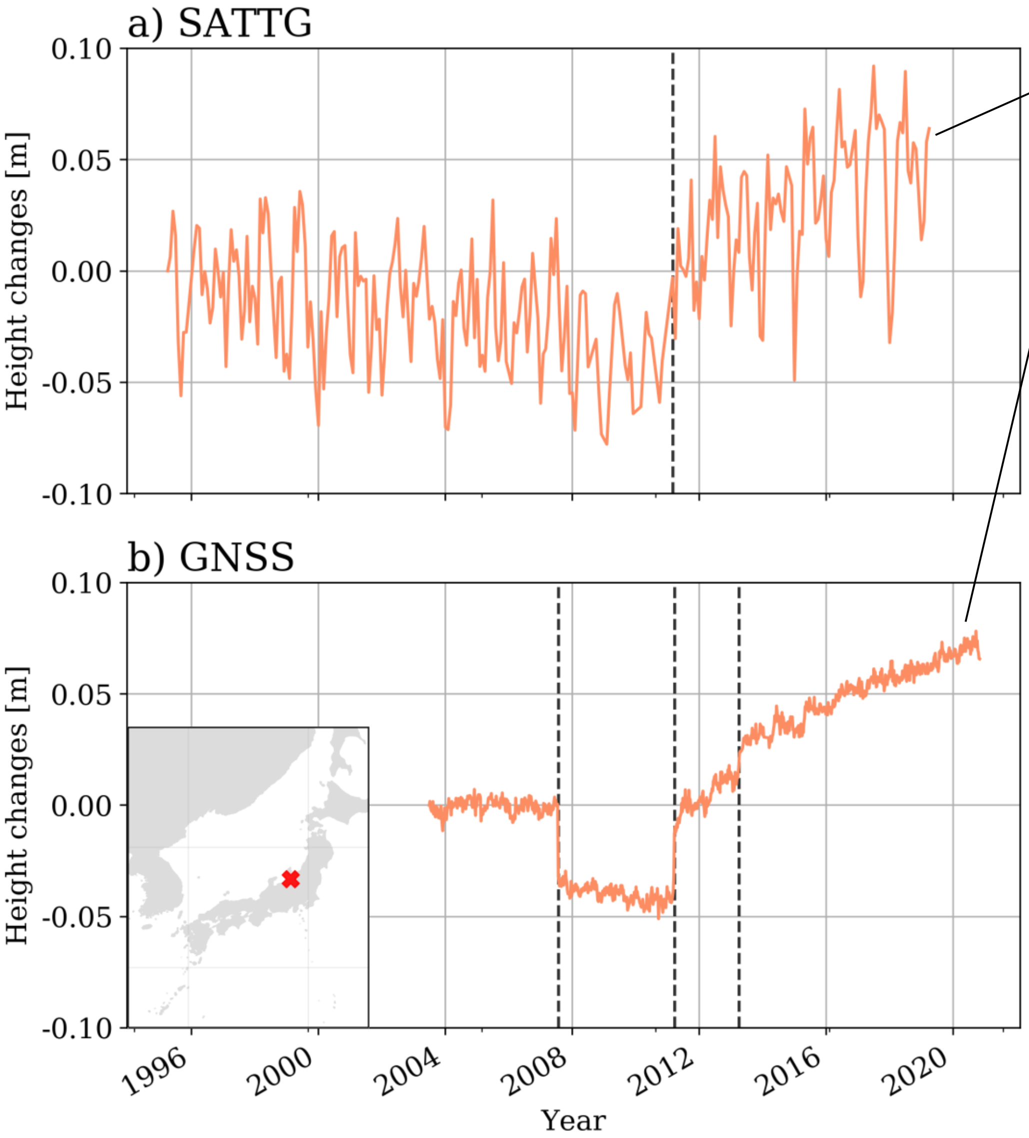

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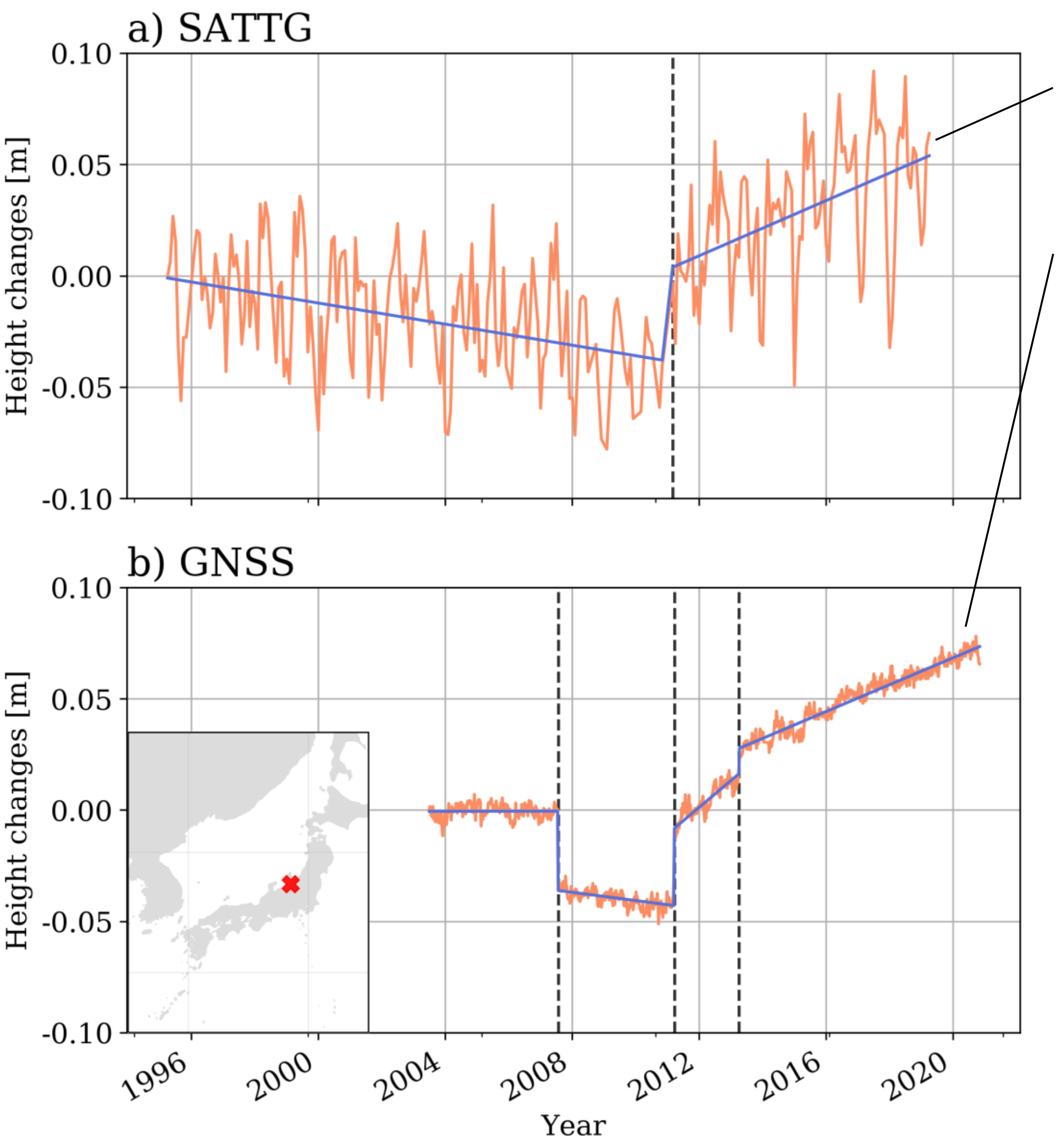

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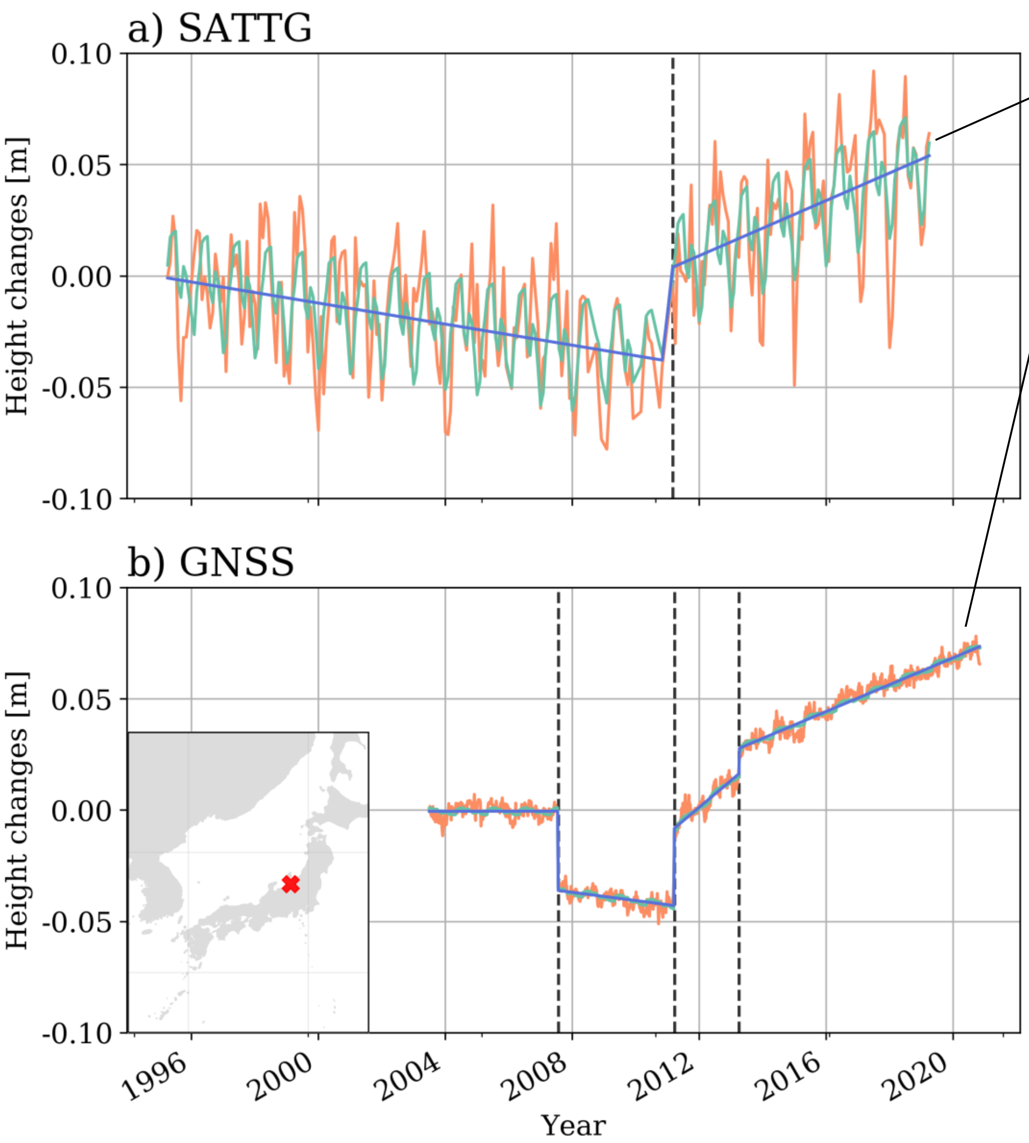

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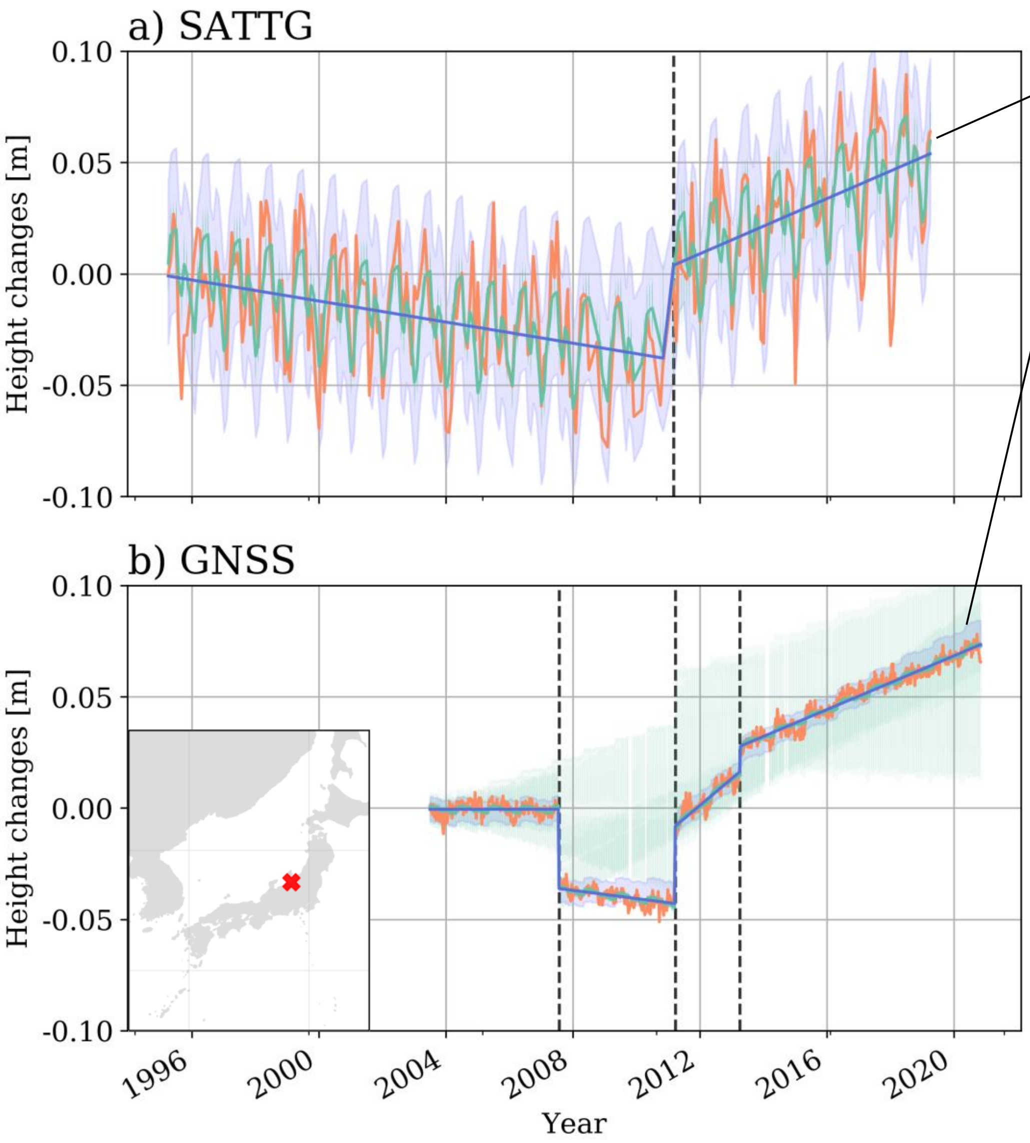

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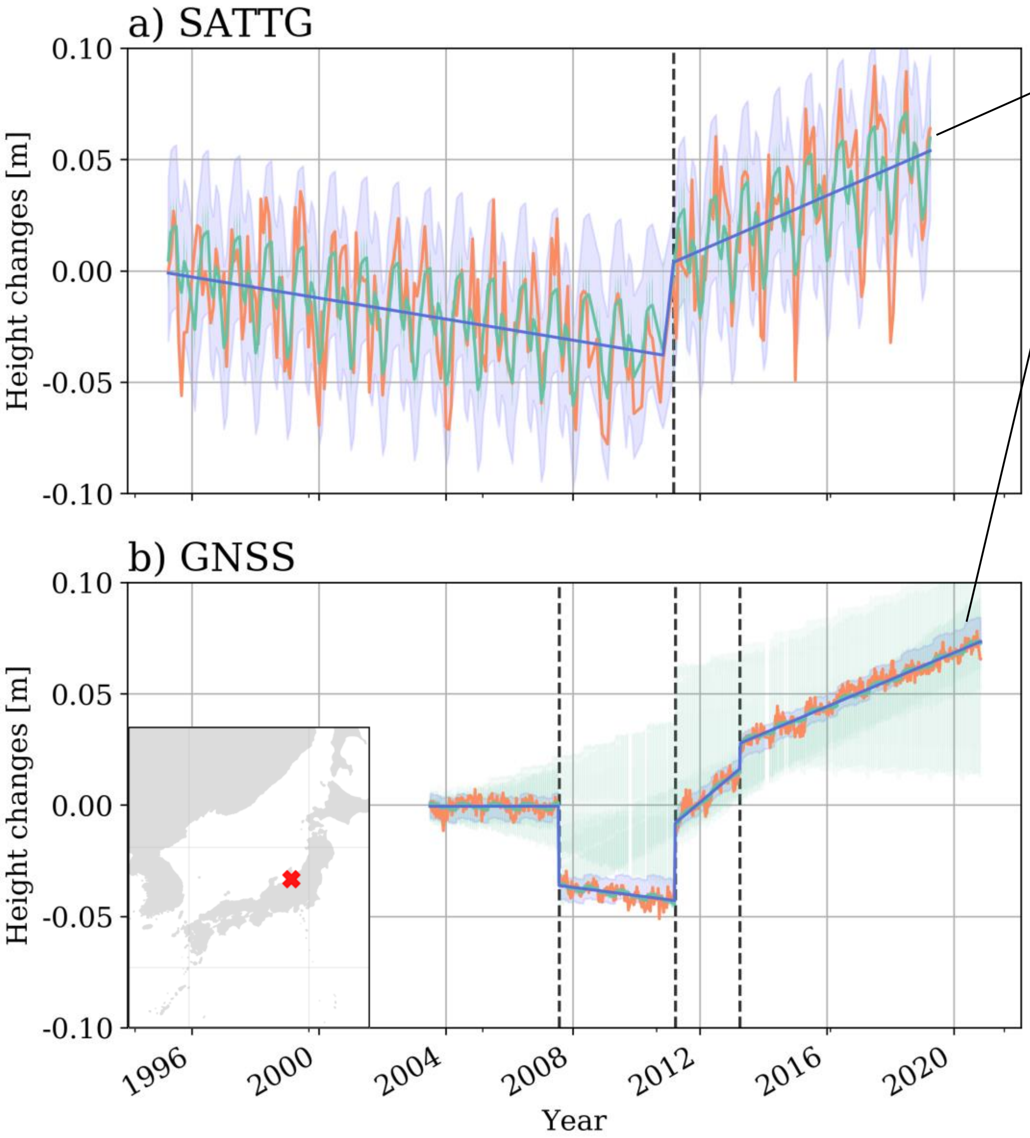

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Aim:

- ➢ Simultaneous and automatic estimation of such time series components for different types of geodetic data

- ➢ Improved estimation of vertical land motion in SATTG and GNSS data

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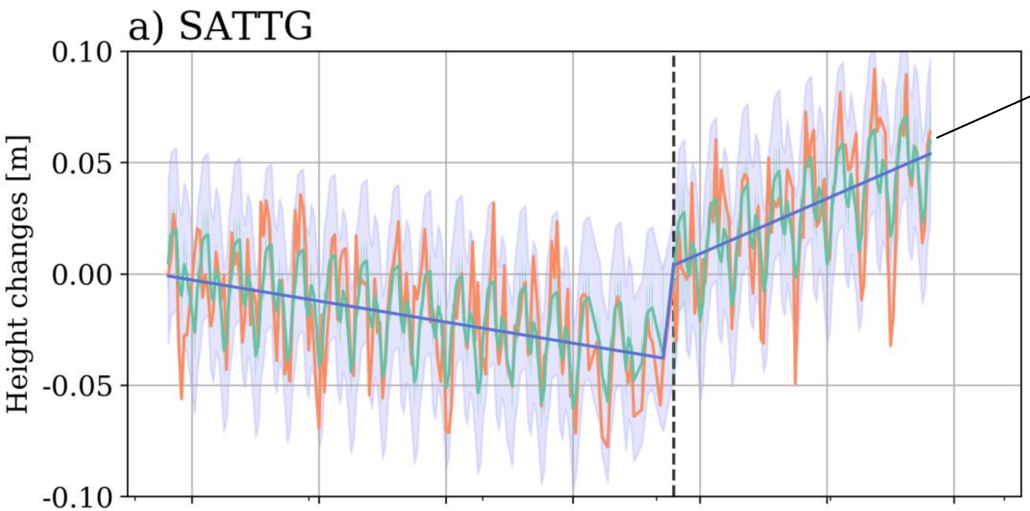
The challenge to detect discontinuities time series



Examples of widely applied existing algorithms:

- Hector (Bos et al., 2013)
    - No simultaneous estimation of components
- MIDAS (Bewitt et al., 2016)
    - No estimation of trend changes
    - Limited information of change points (e.g. uncertainty of positions and sizes of jumps)

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Bayesian modelling of discontinuities and piecewise trends: An application to vertical land motion

Model formulation:

$$y(t) = \underbrace{o(t)}_{\text{offsets}} + \underbrace{g(t)}_{\text{trends}} + \underbrace{seas}_{\text{seasonal}} + \underbrace{\eta}_{\text{noise term}}$$

offsets + trends + seasonal + noise term

Bayes-theorem:

$$\underbrace{P(\theta|y)}_{\text{Posterior distribution}} = \frac{P(y|\theta)P(\theta)}{P(y)}$$

Posterior distribution

Component parametrization and prior distributions:

- Trends g(t) : $k + \sum_{j:t>s_j} h_j$ $\quad k, \vec{h} \sim \mathcal{N}(\mu_{k,\vec{h}}, \sigma^2_{k,\vec{h}})$
- Offsets o(t): $o + \sum_{j:t>s_j} p_j$ $\quad o, \vec{p} \sim \mathcal{N}(\mu_{o,\vec{p}}, \sigma^2_{o,\vec{p}})$
- Seasonal term: $\sim\vec{m}$ $\quad \vec{m} \sim \mathcal{N}(\mu_{\vec{m}}, \sigma^2_{m})$
- AR1 – noise: $\sim\varphi, \epsilon$ $\quad \varphi, \epsilon \sim HalfNormal({\sigma_{\varphi,\epsilon}}^2)$
- Change point (CP) epoch: $\vec{s}$ $\quad \vec{s} \sim \mathcal{N}(\mu_{\vec{s}}, s^2), \quad with \quad \mu_{\vec{s}} \sim U(0, t_{max})$
- CP 1,0 switch function $\vec{q}$ $\quad \vec{q} \sim Ber(\mu_{\vec{q}}),\ \mu_{\vec{q}} \in [0,1]$

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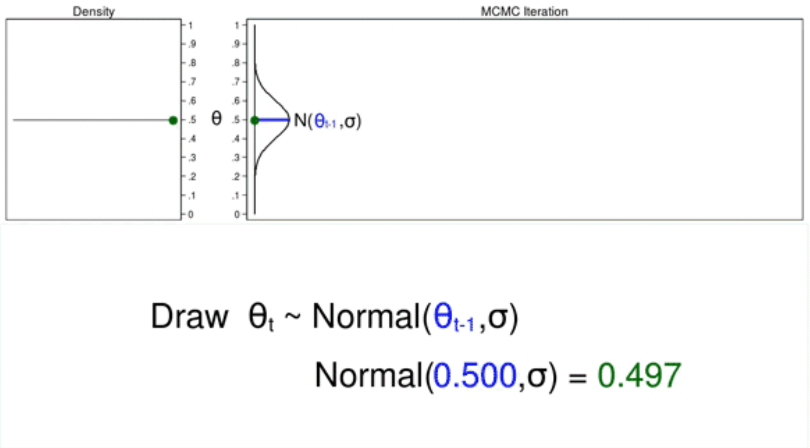

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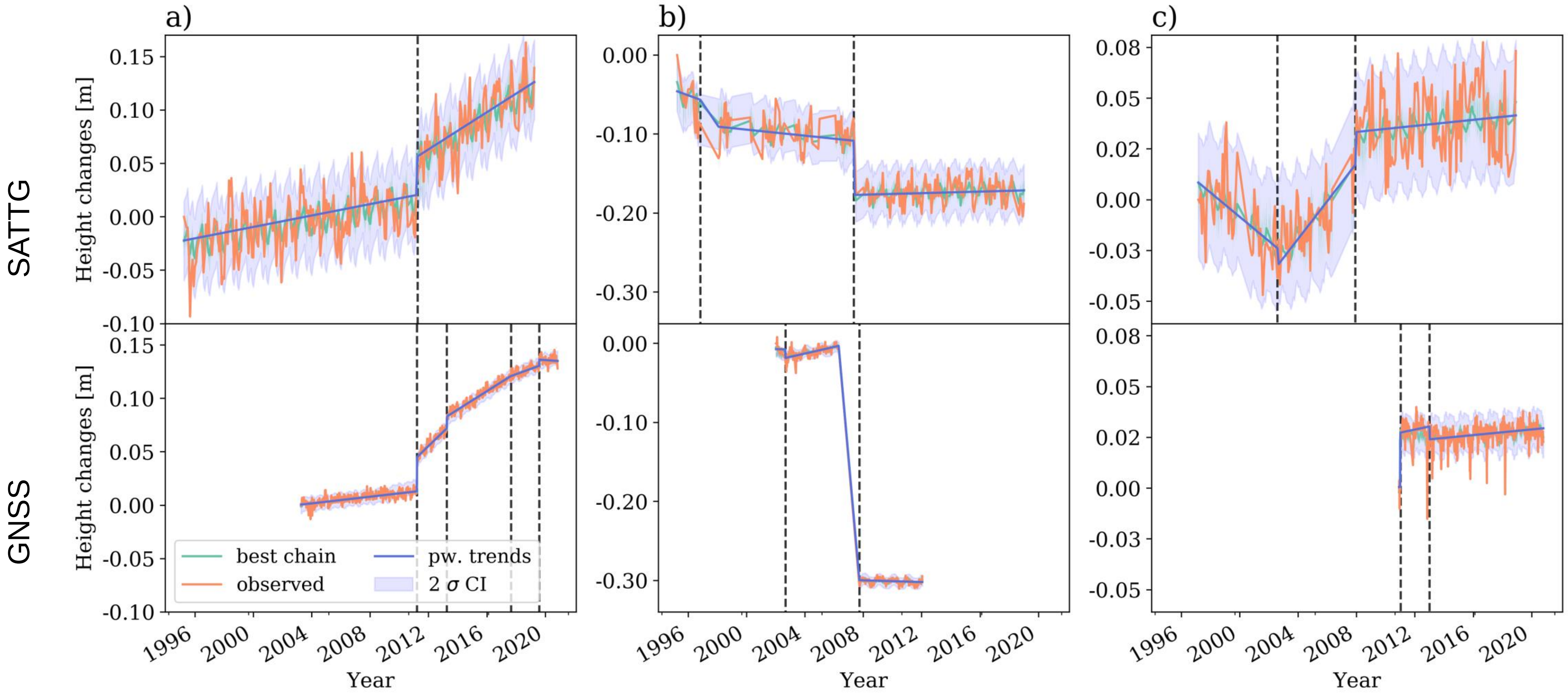
- Markov chain Monte Carlo Sampling to estimate the posterior distributions

- No-U-Turn (NUTS) sampler
- Metropolis-within-Gibbs

- For more details please see paper or Neal et al., 1993, Hoffman and Gelmann, 2014, etc.

Bayesian modelling of discontinuities and piecewise trends: An application to vertical land motion

Does piecewise trend estimation outperform linear trend estimation (without accounting for change points)?



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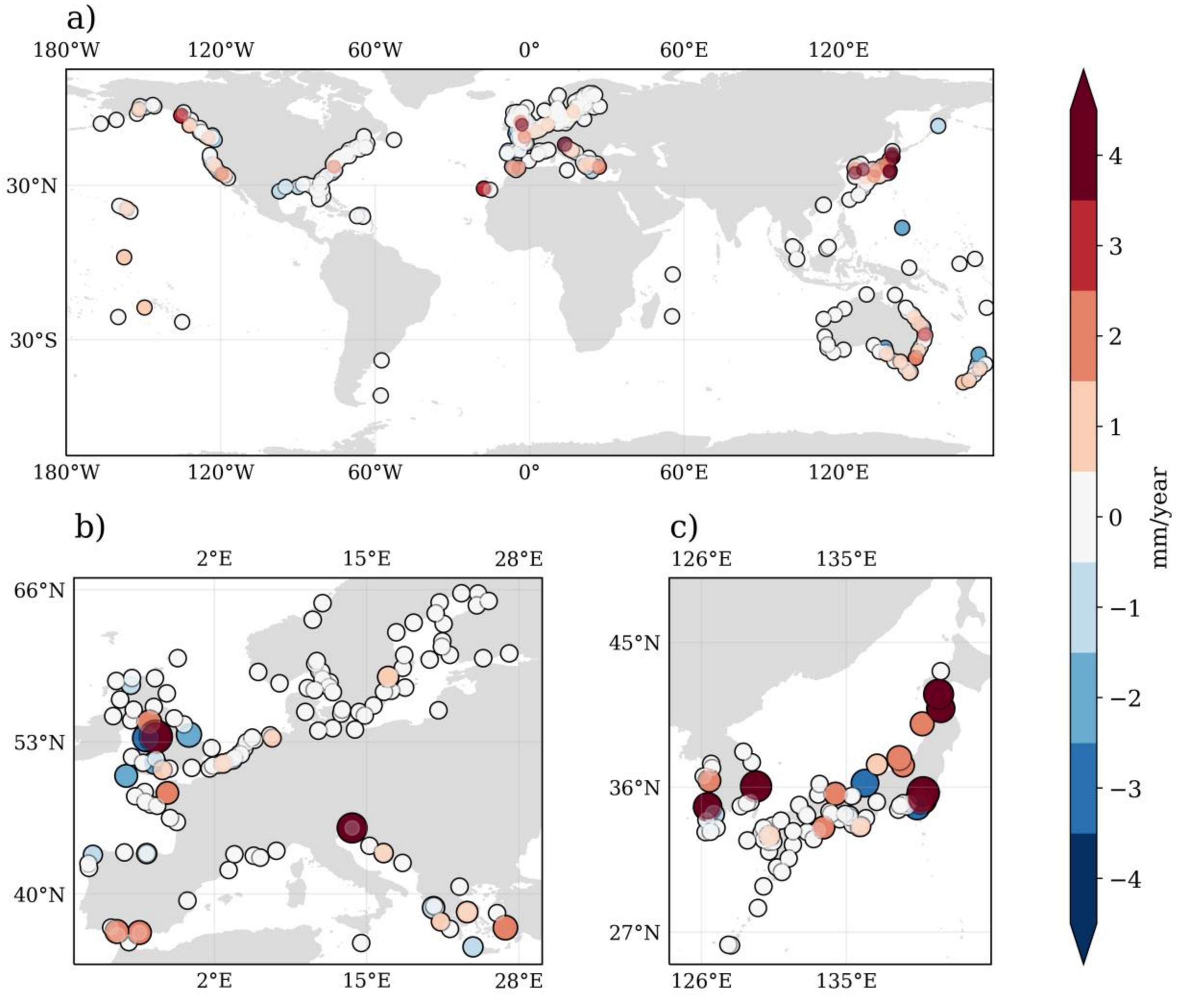
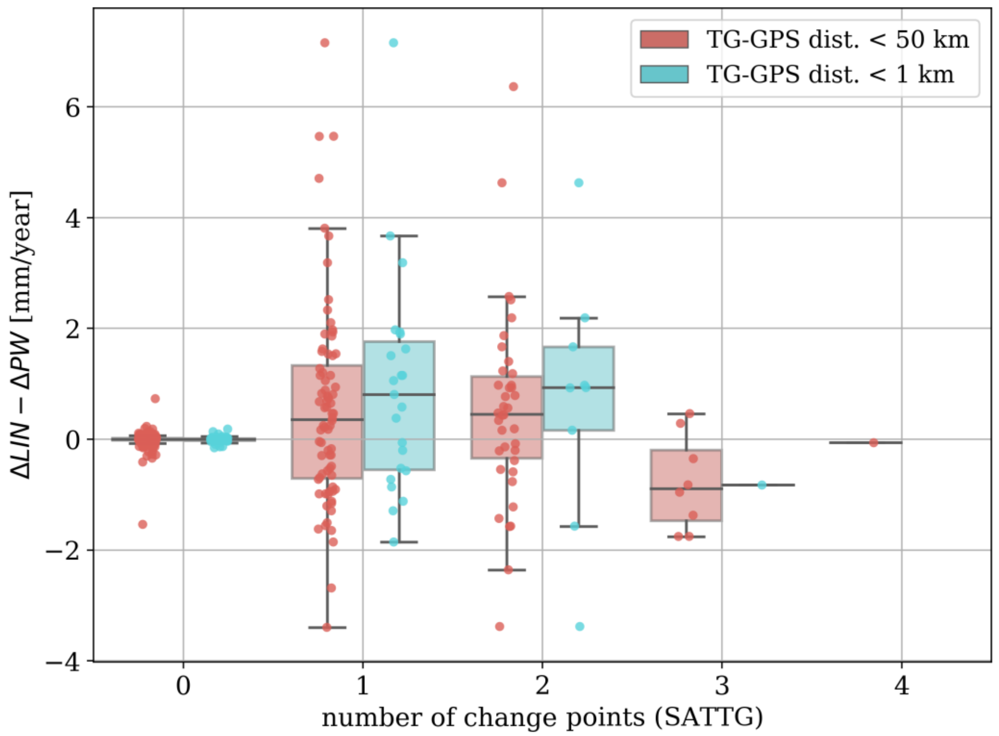


- Improvement of piecewise trend estimation vs. strictly linear trend estimation for 387 co-located GNSS/TG stations **by 0.44 mm/year** (on average)



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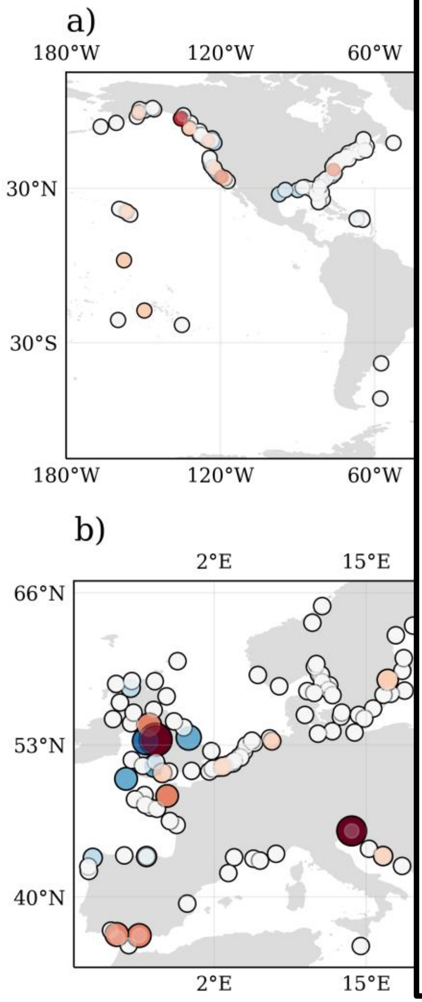

Thank You!

For more information see (soon):

- Study Submitted to Journal of Geodesy
- **Code availability:** https://github.com/oelsmann/discotimes GPLv3

Julius.oelsmann@tum.de

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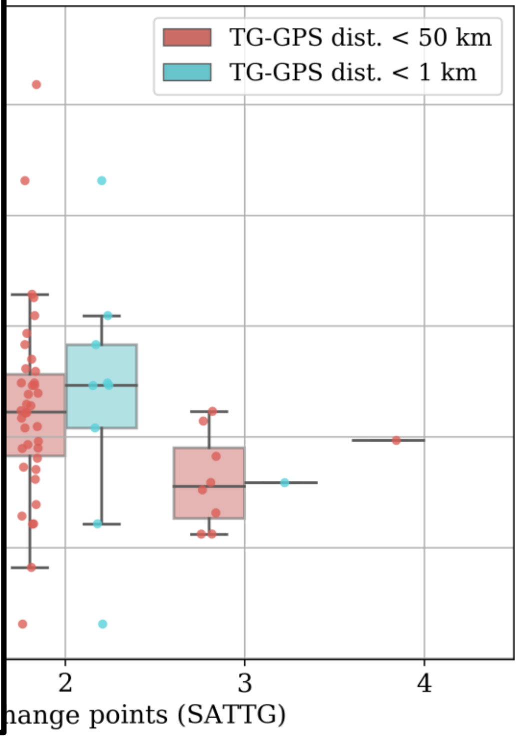

References

Blewitt G, Kreemer C, Hammond WC, Gazeaux J (2016) Midas robust trend estimator for accurate gps station velocities without step detection. Journal of Geophysical Research: Solid Earth 121(3):2054–2068, DOI 10.1002/2015JB012552

Bos M, Fernandes R, Williams S, Bastos L (2013) Fast error analysis of continuous gnss observations with missing data. Journal of Geodesy 87(4):351–360, URL http://nora.nerc.ac.uk/id/eprint/501636/

Gazeaux J, Williams S, King M, Bos M, Dach R, Deo M, Moore AW, Ostini L, Petrie E, Roggero M, Teferle FN, Olivares G, Webb FH (2013) Detecting offsets in gps time series: First results from the detection of offsets in gps experiment. Journal of Geophysical Research: Solid Earth 118(5):2397–2407, DOI https://doi.org/10.1002/jgrb.50152,

Hoffman MD, Gelman A (2014) The no-u-turn sampler: Adaptively setting path lengths in hamiltonian monte carlo. Journal of Machine Learning Research 15(47):1593–1623, URL http://jmlr.org/papers/v15/hoffman14a.html

Neal RM (1993) Probabilistic inference using markov chain monte carlo methods

Oelsmann, J., Passaro, M., Dettmering, D., Schwatke, C., Sánchez, L., and Seitz, F.: The zone of influence: matching sea level variability from coastal altimetry and tide gauges for vertical land motion estimation, Ocean Sci., 17, 35–57, https://doi.org/10.5194/os-17-35-2021, 2021.

Oelsmann, J., Passaro, M., Dettmering, D., Schwatke, C., Sánchez, L., and Seitz, F.: (In preparation) Bayesian modelling of discontinuities and piecewise trends (trend changes) improves coastal vertical land motion estimates, Journal of Geodesy