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A new approach for structural optimization of series systems

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ABSTRACT: Optimization of structural systems with respect to cost is a well-known application of mathematical optimization theory. However, optimization of structures with respect cost under probabilistic reliability constraints has been subject of only very few studies. The difficulty in using reliability targets or probabilistic constraints lies in the fact that modern reliability methods themselves are formulated as a problem of nonlinear optimization. In this paper a special formulation for series systems based on the so-called first-order reliability method (FORM) is presented. It is demonstrated that the problem can be solved by a one-level optimization problem. Two examples demonstrate the algorithm indicating that the proposed formulations are comparable in numerical effort with an approach based on semi-infinite programming but are definitely superior to a two-level formulation.

1 INTRODUCTION

Effective techniques have been developed during the last 25 years to calculate the reliability of structural components and systems of structures. The evaluation of structural reliability is well known for given sets of basic uncertainty variables, state functions and deterministic parameters. The numerically expensive part typically is nonlinear optimization, at least if the well-known First or Second Order Reliability Methods (FORM/SORM) are used. The inverse reliability problem, i.e. the determination of free parameters in the state function for a given (fixed) reliability, has also found some solutions (see, for example, Der Kiureghian 1994).
Deterministic optimization with respect to cost, weight or other economical unities and certain performance criteria has been one of the prominent applications of mathematical optimization. The combination of optimization and reliability analysis can be used as a decision tool in view of the conflicting aims of safety and economy. Reliability oriented optimization of structural systems can be specified by the following task:

(RBCO) - *Reliability-Based Cost Optimization*
Minimization of total cost subject to reliability of the structural system and other constraints.

The total cost can include or do not include expected failure cost of the system and/or single components. This task has been investigated repeatedly and in part successfully. For example, Enevoldsen & Sørensen (1993, 1994) defined the top-level (optimization of cost variables) and the sub-level (reliability estimation). However, a proof for general convergence of the bi-level approach is still missing. The fact, that the failure domain depends on cost parameters, leads to mathematical difficulties of proof. The development of interactive SQP-algorithms by Peterson & Thoft-Christensen (1994, 1996) leads to an academic tool of handling numerical trouble inside the algorithms. Kirjner-Neto et al. (1995) proposed another promising approach for components within the frame work of FORM. An extension of this approach for series systems has been suggested by Polak et al. (1997).The algorithm essentially makes use of semi-infinite programming.
In the following a new approach for the minimization of total cost with (system-)reliability constraints will be developed using a mono-level approach. The mono-level approach is based on an idea by Friis Hansen & Madsen (1992). The (mathematically) correct foundation and formulation of the mono-level algorithm for structural components has been developed by Kuschel & Rackwitz (1997). The total cost can be minimized. Total cost can include the initial cost and the expected cost of failure. The reliability is bounded by a maximum failure probability or a minimum reliability index. An advantage of the mono-level ap-

proach is that reliability maximization under cost constraints is possible as well as its refinement to SORM (Kuschel&Rackwitz, 1999). Recently, the formulations were also extended to time-variant component reliability (Kuschel&Rackwitz, 1998).

This paper extends the mono-level formulation to minimizing the cost of a structural (series) system subject to constraints on the maximum FORM failure probability on each failure mode and a number of deterministic constraints on structural performances. The new mono-level formulation uses a mathematical reformulation of a problem which is similar to the one suggested by Polak et al. (1997).

The mono-level optimization of series systems will be demonstrated at two examples. The reliability-based optimization of systems cost is solved by Schittkowski's nonlinear optimization algorithm **NLPQL**.

2 BASIC FORMULATIONS OF RELIABILITY PROBLEMS

2.1 *Failure Probability of Components*

Let $\mathbf{X} = (X_1, \ldots, X_n)^T$ be a n-dimensional vector of random variables with distribution law $P_\mathbf{X}$ and distribution density $f_\mathbf{X}$. The d-dimensional vector $\mathbf{p}$ of cost parameters can involve deterministic parameters but also parameters of the distribution (law) $P_\mathbf{X}$.

The state (performance) function or failure function is denoted by $G(\mathbf{x}, \mathbf{p})$, so that $G(\mathbf{x}, \mathbf{p}) > 0$ denotes the safe state, $G(\mathbf{x}, \mathbf{p}) = 0$ the limit state and $G(\mathbf{x}, \mathbf{p}) < 0$ the failure state. $G(\mathbf{x}, \mathbf{p}) = 0$ will also be denoted by failure surface. It is assumed that $G(\mathbf{x}, \mathbf{p})$ is at least twice differentiable. The (time-invariant) failure probability then is

$$P_f(\mathbf{p}) = \int_{\{x:G(x,p)\leq 0\}} P_\mathbf{X}(d\mathbf{x}) = \int_{G(\mathbf{x},\mathbf{p})\leq 0} f_\mathbf{X}(\mathbf{x})\, d\mathbf{x}$$

provided that the probability density $f_\mathbf{X}$ exists which is assumed throughout. Moreover, it is assumed that the probability distribution functions are continuously differentiable. Especially for large n and complex state function an exact evaluation by numerical integration can require considerable computational effort or an analytical result for the integral is absent. Therefore, some special methods have been devised which can do the integration efficiently. Let a probability distribution transformation $\mathbf{T} : \mathbb{R}^n \to \mathbb{R}^n$ exist which maps the random vector $\mathbf{X} = (X_1, \ldots, X_n)^T$ into an independent standard normal vector $\mathbf{U} = (U_1, \ldots, U_n)^T$ (see, for example, Hohenbichler & Rackwitz 1981, Der Kiureghian & Liu 1986).

With the new state function $g(\mathbf{u}, \mathbf{p}) = G(\mathbf{T}^{-1}(\mathbf{u}), \mathbf{p})$ and the failure domain $\mathcal{F}_\mathbf{p} = \{\mathbf{u} : g(\mathbf{u}, \mathbf{p}) \leq 0\}$, it is:

$$P_f(\mathbf{p}) = \int_{\mathcal{F}_\mathbf{p}} P_\mathbf{U}(d\mathbf{u}) = \int_{g(\mathbf{u},\mathbf{p})\leq 0} \varphi_\mathbf{U}(\mathbf{u})\, d\mathbf{u}$$

where $P_\mathbf{U}$ is the standard normal distribution law and $\varphi_\mathbf{U}$ is the standard normal density.

2.2 *Structural Reliability Methods of Components*

Now, if the limit state function $g(\mathbf{u}, \mathbf{p}) = \boldsymbol{\alpha}^T\mathbf{u} + \beta_\mathbf{p}$ is linear, then an exact result will be $P_f(\mathbf{p}) = \Phi(-\beta_\mathbf{p})$. $\Phi(.)$ is the standard normal integral.

If $g(\mathbf{u}, \mathbf{p}) \approx \boldsymbol{\alpha}^T\mathbf{u} + \beta_\mathbf{p}$ with $\beta_\mathbf{p} = -\boldsymbol{\alpha}^T\mathbf{u}^*$ and where $\mathbf{u}^*$ is the solution of the following optimization problem

$$(\beta\mathbf{P}) \quad \textbf{minimize} \quad \|\mathbf{u}\| \qquad \text{subject to} \quad g(\mathbf{u}, \mathbf{p}) \leq 0,$$

there is (Rackwitz & Fiessler 1978)

$$P_f(\mathbf{p}) \approx \Phi(-\beta_\mathbf{P}) .$$

This approximation method is called **First-Order Reliability Method** (FORM). The solution point $\mathbf{u}^*$ of the optimization problem ($\beta\mathbf{P}$), the so called design point or β-point, defines the First-Order Reliability Index

$$\beta_\mathbf{p} = \beta(\mathcal{F}_\mathbf{p}) = \|\mathbf{u}^*\| .$$

$\boldsymbol{\alpha}$ is the vector of direction cosines of the solution point. Reference to the parameter vector $\mathbf{p}$ is omitted here and in the following whenever this is possible without loosing clarity. Various methods exist to improve this first-order estimate. In practice, such improvements are rarely necessary.

2.3 *Optimality Conditions for β-Points*

The first-order reliability index $\beta_\mathbf{p}$, i.e. the minimum distance from the origin to the limit state surface in standard normal space, can alternatively be used as a measure of reliability. If $\mathbf{u}^*$ is an optimal point for (βP), the β-point is a Kuhn-Tucker-point. The following theorem is proved in Kuschel & Rackwitz (1997)

Theorem 1 ***(β-Point-Theorem)***
If $\mathbf{u}^*$, with $\mathbf{u}^* \neq 0$, is the solution point of optimization problem ($\beta\mathbf{P}$), then the following two statements hold for each $\mathbf{p}$:

a) $g(\mathbf{u}^*, \mathbf{p}) = 0$,

b) $\mathbf{u}^{*T}\nabla_\mathbf{u} g(\mathbf{u}^*, \mathbf{p}) + \|\mathbf{u}^*\| \|\nabla_\mathbf{u} g(\mathbf{u}^*, \mathbf{p})\| = 0$.

□

The equations a) and b) are necessary (first-order) optimality conditions for the β-point. Kuschel & Rackwitz (1997) have shown that the β-Point-Theorem is the main utility for the mono-level-formulation of component problems. In the next section it is demonstrated that the β-Point-Theorem is also the main utility for the mono-level-optimization of series systems.

3 SOME APPROACHES FOR STRUCTURAL OPTIMIZATION OF SYSTEMS

An extension of the mono-level approach for the RBCO-problems of structural series systems will be derived and discussed. Reliability is to be analyzed by the method of first order (FORM). Consider the optimal design of a *(separable) series system*, in which a separate constraint is imposed on the probability of each failure mode. In essence, separable series systems show no or no significant correlation among failure modes. Therefore, they must be considered as idealized series systems.

First the bi-level formulation of the RBCO problem for separable series systems by Polak et al. (1997) is presented. The

- continuously differentiable cost function:
  $C_{sys} : \mathbb{R}^d \to \mathbb{R}$

is deterministic. The cost function C_{sys} does not contain expected damage or failure cost in the formulation suggested by Polak et al. (1997). The set of functions $g_k : \mathbb{R}^n \times \mathbb{R}^d \to \mathbb{R}$, $k = 1, \ldots, s$, is the set of the continuously differentiable status functions. The continuously differentiable

- reliability functions of first order:
  $\gamma_l : \mathbb{R}^n \times \ldots \times \mathbb{R}^n \to \mathbb{R}, l = 1, \ldots, t,$

describe the FORM-approximations to prescribed boundaries of the (system) failure probabilities.

Thus the (time-invariant) RBCO problem for separable series systems in the formulation by Polak et al. (1997) is defined by

SYS-SE

$$\begin{aligned}
&\textbf{minimize} && C_{sys}(\mathbf{p}) \\
&\textbf{subject to} \; : && \gamma_l(\mathbf{u}_1, \ldots, \mathbf{u}_s) \leq 0 \quad , l = 1, \ldots, t \\
& && \mathbf{u}_k = \mathit{arg\,min} \left\{ \|\mathbf{u}\|^2, g_k(\mathbf{u}, \mathbf{p}) \leq 0 \right\} \\
& && \qquad\qquad , k = 1, \ldots, s \\
& && H(\mathbf{p}) \overset{=}{\underset{\leq}{}} \mathbf{0}_{m'+m} \\
& && \mathbf{p}^l \leq \mathbf{p} \leq \mathbf{p}^u .
\end{aligned}$$

Here $H(\cdot)$ denotes a constraint vector of m' equality constraints and m inequality constraints of structural performance requirements on the cost vector $\mathbf{p}$ and $\mathbf{u}_1, \ldots, \mathbf{u}_s, \mathbf{u} \in \mathbb{R}^n$ are vectors of independent, standard normal distributed variables.

The determination of the failure probabilities of structural systems in standard space is treated in detail in Hohenbichler & Rackwitz (1983) and Hohenbichler et al. (1987). Here, in agreement with the requirement of separability we assume that the system failure probability is well estimated by the sum of componential failure probabilities.

The bi-level cost optimization problem for separable systems **(SYS-SE)**, defined above, can be simplified in the special case of only one state function (in detail examined by Kuschel & Rackwitz 1997). The optimization problem **(CRP)** has to consider the reliability-relevant constraint $\Phi(-\|\mathbf{u}\|) \leq P_f^{maximum}$.

The reliability-evaluating function γ then is:

$$\gamma(\mathbf{u}) = \Phi(-\|\mathbf{u}\|) - P_f^{maximum}$$

This simplifying application of the RBCO-problem **(SYS-SE)** leads directly to a bi-level formulation for the reliability-based cost optimization, as indicated by Kirjner-Neto et al. (1995).

Polak et al. (1997) developed an algorithm on the basis of semi-infinite optimization in order to solve the optimization problem **(SYS-SE)**. However, this structural optimization method is limited to cost functions without expected failure costs due to their problem structure.

The same problem can also be studied in the mono-level formulation. The optimization aim is the minimization of the

- function of expected total cost
  $C_{G,sys} : \mathbb{R}^{s \cdot n+d} \to [0, \infty)$

$$\begin{aligned}
C_{G,sys}(\mathbf{u}_1, \ldots, \mathbf{u}_s, \mathbf{p}) \;&=\; C_0 + C(\mathbf{p}) \\
&\quad + \sum_{k=1}^{s} L_k(\mathbf{p}) \cdot \Phi(-\|\mathbf{u}_k\|) ,
\end{aligned}$$

where C_0 are fixed construction cost and $C(\mathbf{p})$ are variable construction cost. The possible failure cost $L_k(\cdot)$ can be considered separately for each individual failure mode. The total expected failure cost are calculated analogously to Kuschel & Rackwitz (1997) by the sum of all expected failure costs of each individual failure mode. If the failure event in each mode causes the same damage costs $L(\mathbf{p})$, then:

$$\sum_{k=1}^{s} L(\mathbf{p}) \cdot \Phi\left(-\|\mathbf{u}_k\|\right) = L(\mathbf{p}) \cdot \sum_{k=1}^{s} \Phi\left(-\|\mathbf{u}_k\|\right)$$

Now the RBCO-problem for separable series systems is to be formulated by the use of mono-level approach. First the reliability constraints of the bi-level problem **(SYS-SE)** are reformulated individually for the various boundaries. Thus it follows:

$$\Phi\left(-\|\mathbf{u}_k\|\right) \;\leq\; P_{f,k}^{maximum}$$

$$\gamma_k(\mathbf{u}_1,\ldots,\mathbf{u}_s) \;=\; \Phi\left(-\|\mathbf{u}_k\|\right) - P_{f,k}^{maximum} \quad , k=1,\ldots,s$$

In the next step the mono-level formulation for the reliability-based cost optimization problem for series systems with separable reliability conditions for each failure mode results in analogy to the optimization problem **(CRP)** (see Kuschel & Rackwitz 1997) by integrating the equality constraints a) and b) of the β-point-theorem in:

CRP-SYS

$$\textbf{minimize} \quad C_0 + C(\mathbf{p}) + L(\mathbf{p}) \cdot \sum_{k=1}^{s} \Phi\left(-\|\mathbf{u}_k\|\right)$$

$$\textbf{subject to} \; : \; \Phi\left(-\|\mathbf{u}_k\|\right) \leq P_{f,k}^{maximum} \quad , k=1,\ldots,s$$

$$g_k(\mathbf{u}_k,\mathbf{p}) = 0 \quad , k=1,\ldots,s$$

$$\mathbf{u}_k^T \nabla_{\mathbf{u}} g(\mathbf{u}_k,\mathbf{p}) + \|\mathbf{u}_k\| \|\nabla_{\mathbf{u}} g(\mathbf{u}_k,\mathbf{p})\| = 0 \quad , k=1,\ldots,s$$

$$H(\mathbf{p}) \overset{=}{\underset{\leq}{}} \mathbf{0}_{m'+m}$$

$$\mathbf{u}_k^l \leq \mathbf{u}_k \leq \mathbf{u}_k^u \quad , k=1,\ldots,s$$

$$\mathbf{p}^l \leq \mathbf{p} \leq \mathbf{p}^u$$

If different failure cost are associated with each failure mode the objective function is

$$C_0 + C(\mathbf{p}) + \sum_{k=1}^{s} L_k(\mathbf{p}) \cdot \Phi\left(-\|\mathbf{u}_k\|\right) ,$$

where $L_k(\mathbf{p})$ $(k = 1,\ldots,s)$ are specific failure costs of each failure mode k.

4 NUMERICAL EXAMPLES

The above theory is illustrated at two examples. The minimization of cost under reliability constraints of series systems is carried out by a non-commercial program package **COSTREL** based on

- the non-linear optimization algorithm **NLPQL** by Schittkowski (1985) and on
- tools of **STRUREL**® by RCP GmbH (1998).)

The programs written in Fortran enable to handle simple cost optimization as well as cost optimization with expected failure cost included. The first task, in general, is relatively easy, inexpensive and reliable although the required number of iterations is a little larger than for simple component analysis. The case with expected failure cost included is more involved as it requires additional non-linear scaling of the objective function to reach convergence. Also, it is substantially more expensive than simple cost optimization.

4.1 *Portal Frame*

A first simple example is the frequently studied *rigid-plastic portal frame* with three different failure modes. The design objective is to minimize the deterministic parameter p which denotes the mean of the third plastic moment.

Hence, the one-dimensional cost parameter is the mean of M_3 and given by $\mathbf{p} = p$. The elements of the 7-dimensional random vector $\mathbf{X} = (M_1,\ldots,M_5,X_6,X_7)^T$ are independent and log-normally distributed. The mean values and standard deviations are chosen as shown in the table below.

Mean values and standard deviations of random variables					
Variable	$M_{1/2}$	M_3	$M_{4/5}$	X_6	X_7
Unit	[kNm]	[kNm]	[kNm]	[kN]	[kN]
Mean value	134.9	p	134.9	50	40
Stand. dev.	13.49	13.49	13.49	10	10

The portal frame can fail in the tree different types of failure modes, shown in the figure 3. Then, the three state functions G_k, $k = 1,\ldots,3$, are given by the principle of work. Given is the random vector $\mathbf{X}$, the deterministic cost parameter and the height $h = 5$ [m] of the frame. The state functions can be formulated as:

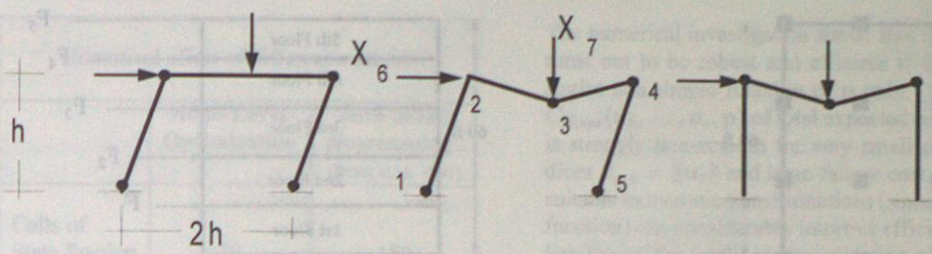

Figure 1. Failure modes of the portal frame

- sway mode

$$G_1(\mathbf{X},\mathbf{p}) = M_1 + M_2 + M_4 + M_5 - hX_6$$

- vertical mode

$$G_2(\mathbf{X},\mathbf{p}) = M_2 + 2M_3 + M_4 - hX_7$$

- combined mode

$$G_2(\mathbf{X},\mathbf{p}) = M_1 + 2M_3 + 2M_4 + M_5 - h\left(X_6 + X_7\right)$$

The variable construction cost are

$$C(\mathbf{p}) = p \cdot 2.5 \text{ [CU/kNm]} .$$

At first, the total cost only depend on the construction cost $C(\cdot)$, i.e. the (expected) failure cost are not included. The problem does not contain further constraints on the cost parameters. The maximum admissible failure probability $P_f^{maximum}$ in each mode is 10^{-4}.

The optimal cost parameter p^* of the rigid-plastic frame is

$$p^* = 137.13$$

and the optimal cost are after a few iterations

$$C^* = 342.83 \quad \text{[CU]}.$$

The local reliability indices are:

$$(\beta_1^*, \beta_2^*, \beta_3^*) = (3.839, 4.048, 3.719)$$

and corresponding failure probabilities are

$$\left(P_{f,1}(p^*), P_{f,2}(p^*), P_{f,3}(p^*)\right) = \left(0.62 \cdot 10^{-4}, 0.25 \cdot 10^{-4}, 1.00 \cdot 10^{-4}\right)$$

fulfilling the reliability requirements. If, however, expected failure cost are included with

$$L = 1000 \text{ [CU]}$$

and the same reliability constraints are maintained the optimal total expected costs are

$$C_{G,sys}^* = 343.02 \quad \text{[CU]} .$$

4.2 *Five Story Building*

The second example is *a five story building* with 16 square columns per story (see Figure 1), which was already used by Polak et al. (1997). The design objective is to minimize the sum of the cross-sectional areas of the columns. It is assumed that the elastic modulus E of the columns to be designed is random, and that the five story building is subjected to five random quasi-static loads F_k, $k = 1,\ldots 5$ (see Figure 2).

The cross-section p_k denotes the area of the column the kth floor. Hence, the vector of the cost parameters is given by $\mathbf{p} = (p_1,\ldots,p_5)^T$. The elements of the random vector $\mathbf{X} = (E, F_1,\ldots,F_5)^T$ are independent and logarithmically normal distributed with a C.o.V of 10%. The mean value of the modulus of elasticity E is 4000 [kpsi] and the mean values of the random loads are chosen as shown in the table below. Given is the random vector $\mathbf{X}$ and the deterministic cost vector $\mathbf{p}$. The displacements v_k, $k = 1,\ldots,5$ can be computed by solving the following linear system of equations, which is obtained by assuming that the structure is linear-elastic.

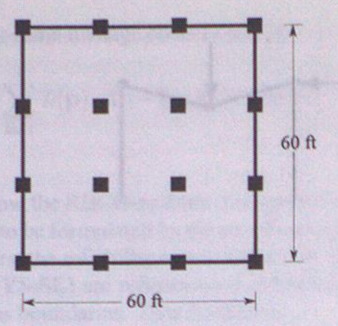

Figure 2. Cross section of 5-story-building to be designed

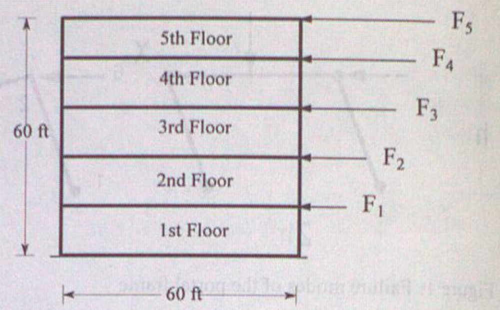

Figure 3. Front view of 5-story-building to be designed

Mean values of random loads					
Load	F_1	F_2	F_3	F_4	F_5
Mean [kpsi]	53.7	64.5	77.4	92.9	111.5

$$\frac{16E}{H^3}\cdot P\cdot\begin{bmatrix} v_1\\ v_2\\ v_3\\ v_4\\ v_5\end{bmatrix}=\begin{bmatrix} F_1\\ F_2\\ F_3\\ F_4\\ F_5\end{bmatrix}$$

where the matrix P is defined by

$$P=\begin{bmatrix} p_1^2+p_2^2 & -p_2^2 & 0 & 0 & 0\\ -p_2^2 & p_2^2+p_3^2 & -p_3^2 & 0 & 0\\ 0 & -p_3^2 & p_3^2+p_4^2 & -p_4^2 & 0\\ 0 & 0 & -p_4^2 & p_4^2+p_5^2 & -p_5^2\\ 0 & 0 & 0 & -p_5^2 & p_5^2\end{bmatrix}.$$

The state functions G_k, $k=1,\ldots,5$, depend directly on the displacements v_k and can be defined by the following formulas:

$$\begin{aligned} G_1(\mathbf{X},\mathbf{p}) &= 0.002\cdot h - v_1\\ G_k(\mathbf{X},\mathbf{p}) &= 0.002\cdot h-(v_k-v_{k-1}),\\ &\qquad\qquad ,k=2,\ldots,5\end{aligned}$$

where h denotes the story height, thus it is

$$h=144in=12ft=60ft/5\,.$$

The variable construction cost depend on the sum of the areas of the cross-sections. Hence the (expected) total cost are

$$C_{G,sys}(\mathbf{p})=C(\mathbf{p})=(p_1+p_2+p_3+p_4+p_5)\cdot 1[\mathrm{CU/in}^2],$$

where [CU] is a currency unit. The cost optimization problem does not contain further constraints on the deterministic cost parameter $\mathbf{p}$. The total costs exclusively depends on the construction cost $C(\cdot)$, i.e. the (expected) failure cost are not included. The maximum admissible failure probability $P_f^{maximum}$ is given by 0.00621. The corresponding minimum admissible reliability index is 2.500.

The optimal cost are

$$C^*_{G,sys}=1183.71\quad[\mathrm{CU}],$$

which are determined by the optimal cost parameter

$$(p_1^*,p_2^*,p_3^*,p_4^*,p_5^*)\ =\ (292.04, 272.45, 246.86, 212.04, 160.32)\ .$$

The local reliability indices are in each failure mode:

$$(\beta_1^*,\beta_2^*,\beta_3^*,\beta_4^*,\beta_5^*)\ =\ (2.50, 2.50, 2.50, 2.50, 2.50)\ .$$

The results are the same as in Polak et al. (1997). The numerical effort for semi-infinite optimization (see Polak et al. 1997) and the mono-level optimization can be compared in the following table. The table shows that the mono-level optimization will require somewhat larger numerical effort than the method by Polak et al. (1997) and this conclusion appears to hold also in other examples. However, it is to be noticed that with the semi-infinite algorithm suggested by Polak et al. (1997) only RBCO problems without failure cost can be solved.

Numerical effort of different methods		
	Mono-Level-Optimization	"semi-infinite programming" (Polak et.al. 1997)
Calls of State Fuction	**30**	150
Gradients of		
random variables	**390**	78
cost parameters	**30**	65

5 SUMMARY AND CONCLUSION

A mono-level reliability-based cost optimization of series systems (**CRP-SYS**) based on FORM-concepts given maximum failure probabilities for each failure mode and, possibly, some other constraints is derived. The great advantages of the new mono-level optimization problem (**CRP-SYS**) are:

- a well-known standard non-linear optimization algorithm, e.g. a SQP-algorithm, can be used to solve the problem,
- the formulation of the optimization task is especially simple,
- the method appears locally stable and robust and
- scaling problems for complicated problems are handled by standard optimization routines.

The mono-level formulation of series systems has, no doubt, some disadvantages (see also Madsen & Friis Hansen 1992 or Kuschel & Rackwitz 1997). As in componential optimization:

- the numerical calculation of second derivatives of the state function is required and
- a probability distribution transformation from the standard normal **u**-space to the original space **x** must be included explicitly, because a standard space formulation is required for the mono-level approach to work.

Some additional numerical effort sometimes is necessary in order to achieve convergence, for example, by:

- selection of good starting values and
- monotonic transformations of the objective function and/or reliability constraints.

The numerical investigation show: that the algorithm turns out to be robust and efficient if the cost constraint is a simple function of $\mathbf{p}$ only. The function $C_{G,sys}(\mathbf{u}_1,\ldots,\mathbf{u}_s,\mathbf{p})$ of total expected cost, however, is strongly non-smooth for very small reliability indices $\beta_{\mathbf{p},k}=\|\mathbf{u}_k\|$ and large failure cost. In this case suitable monotonic transformations (smoothing of cost function) can considerably improve efficiency and reliability of the nonlinear optimization algorithm. A sensitivity analysis of the cost function $C_{G,sys}$ and an investigation of importance measures or (see Kuschel & Rackwitz 1997) can also help to retain numerically non-zero gradients: For very large reliability indices, say $\|\mathbf{u}_k\|\geq 7.0$, $k=1,\ldots,s$, the total expected cost are simply the initial cost $C(\mathbf{p})$.

An extension to SORM exists for componential RBCO-problems (see Kuschel & Rackwitz, 1999) and, conceptually, appears straightforward for series systems. However, such an extension is of little practical interest and would be rather time consuming. The minimum cost problem is formulated for an arbitrary set of separable state functions. Hence, the present mono-level formulation (**CRP-SYS**) is restricted to separable failure modes only. A generalization to multiple dependent failure modes (unions of failure modes) appears possible and is presently under study. The alternative problem of optimizing reliability of separable or dependent series systems under total or individual cost constraints with expected failure cost included or not also appears possible from a conceptual point of view. One must expect relatively large numerical effort, at least for approaches along the lines discussed in the above.

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