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Edited by

N. Shiraishi
Maizuru National College of Technology, Kyoto, Japan

M. Shinozuka
University of Southern California, Los Angeles, California, USA

Y. K. Wen
University of Illinois, Urbana, Illinois, USA

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Design for optimal reliability

Norbert Kuschel & Rüdiger Rackwitz
Technische Universität München, Germany

ABSTRACT: The calculation of reliability indices for given sets of basic variables, limit state functions and deterministic parameters is a well investigated problem. The inverse problem of finding optimal designs with respect to reliability or cost has been subject of only a few studies. Reliability oriented optimization can be classified into two different tasks. The first problem is the minimization of total cost under reliability and other constraints. It has been investigated repeatedly and in part successfully. The second task is the dual optimization problem of maximization of reliability subject to maximum cost and other restrictions. It is formulated in an exact way in case of a single limit state function as a one-level optimization problem using the first order reliability method (FORM) and properties of so-called design- or β-point. Some mathematical theorems about the asymptotic behavior of reliability indices are presented and can be used to support the validity of the FORM approach. Three numerical examples demonstrate the one-level approach of reliability optimization.

1. INTRODUCTION

Effective techniques have been developed during the last 20 years to calculate approximately the reliability of structures. The evaluation of structural reliability is well known for given sets of limit state functions, basic uncertainty variables and deterministic parameters. Its numerically expensive part typically is optimization, at least if First or Second Order Reliability Methods (FORM/SORM) are used. Also, the inverse reliability problem, i.e. the determination of a free parameter in the limit state function for a given reliability, has found some solutions (see, for example, Der Kiureghian, 1994). Further, deterministic optimization with respect to cost or weight or certain performance criteria has been one of the prominent applications of mathematical optimization. The combination of optimization and reliability analysis can be used as a decision tool in view of the conflicting aims of safety and economy. Reliability oriented optimization of structures can be classified into two tasks:

a) Minimization of cost subject to reliability and other constraints,
b) Maximization of reliability subject to cost and other constraints.

In both cases (total) cost can include or do not include expected failure cost.

The first task has been investigated repeatedly and in part successfully. For example, Enevoldsen and Sørensen (1993, 1994) defined the top-level (optimization of design variables) and the sub-level (reliability estimation). However, a proof for general convergence of the two-level approach is still missing. The fact, that the failure domain depends on design parameters, leads to mathematical difficulties of proof. The development of interactive SQP-algorithms by Peterson and Thoft-Christensen (1994, 1996) leads to an academic tool of handling numerical trouble inside the algorithm. Kirjner, Polak and Der Kiureghian (1995) proposed a promising approach in the frame work of FORM. The algorithm essentially makes use of semi-infinite programming techniques.

The second problem has found very little interest in the past although it may be practically very interesting, especially for certain critical components in technical systems. It turned out that this problem is a notoriously difficult problem from a numerical point of view. In the following an approach for the reliability maximization with (total) cost constraints will be developed using a one-level approach. This optimization problem based on an idea by Friis Hansen and Madsen (1992) uses FORM only which is shown to be asymptotically correct. The total cost are bounded by maximum cost and include the initial cost and the product of failure probability and failure cost. The one-level optimization will be demonstrated at three examples. The reliability optimization subject to cost and other constraints are solved by Schittkowski's nonlinear optimization algorithm NLPQL.

2. STRUCTURAL RELIABILITY

Given is a n-dimensional vector $\mathbf{X} = (X_1, \ldots, X_n)^T$ of random variables with continuously differentiable distribution function $F_{\mathbf{X}}(\mathbf{x})$. Let further $G(\mathbf{x}, \mathbf{p})$ be the so-called limit state function and $\widetilde{\mathcal{F}}_{\mathbf{p}}$ the failure domain, with $\widetilde{\mathcal{F}}_{\mathbf{p}} = \{\mathbf{x} : G(\mathbf{x}, \mathbf{p}) \leq 0\}$. It is assumed, that $G(\mathbf{x}, \mathbf{p})$ is at least twice differentiable. The d-dimensional vector $\mathbf{p}$ is the vector of design parameters and can involve deterministic parameters but also parameters of the distribution function $F_{\mathbf{X}}(\mathbf{x})$. Reference to the parameter vector $\mathbf{p}$ is omitted in the following whenever this is possible without loosing clarity. The problem is to determine the time-invariant probability of failure:

$$P_f(\mathbf{p}) = \int_{\widetilde{\mathcal{F}}_{\mathbf{p}}} P_{\mathbf{X}}(d\mathbf{x}) = \int_{G(\mathbf{x},\mathbf{p}) \leq 0} f_{\mathbf{X}}(\mathbf{x})\, d\mathbf{x}, \qquad (1)$$

where $P_{\mathbf{X}}(\cdot)$ is the probability law and $f_{\mathbf{X}}(\mathbf{x})$ is the probability density of $\mathbf{X}$. Analytical results for this integral are almost absent. However, let a probability distribution transformation $\mathbf{T} : \mathbb{R}^n \to \mathbb{R}^n$ exist which maps an arbitrary n-dimensional random vector $\mathbf{X} = (X_1, \ldots, X_n)^T$ into a standard normal vector $\mathbf{U} = (U_1, \ldots, U_n)^T$ with independent components (Hohenbichler/Rackwitz, 1981; Der Kiureghian/Liu, 1986). With the new limit state function $g(\mathbf{u}, \mathbf{p}) = G(\mathbf{T}(\mathbf{u}), \mathbf{p}) = G(\mathbf{x}, \mathbf{p})$ and the new failure domain $\mathcal{F}_{\mathbf{p}} = \{\mathbf{u} : g(\mathbf{u}, \mathbf{p}) \leq 0\}$, it is:

$$P_f(\mathbf{p}) = \int_{\mathcal{F}_{\mathbf{p}}} P_{\mathbf{U}}(d\mathbf{u}) = \int_{g(\mathbf{u},\mathbf{p}) \leq 0} \varphi_{\mathbf{U}}(\mathbf{u})\, d\mathbf{u}, \qquad (2)$$

where $P_{\mathbf{U}}(.)$ is the standard normal distribution law and $\varphi_{\mathbf{U}}(\mathbf{u})$ is the standard normal density. If the limit state function $g(\mathbf{u}, \mathbf{p})$ is a linear function, i.e. $g(\mathbf{u}, \mathbf{p}) = \boldsymbol{\alpha}^T\mathbf{u} + \beta$, the exact result for the failure probability is $P_f(\mathbf{p}) = \Phi(-\beta)$ where $\Phi(\cdot)$ is the standard normal distribution function.

If the limit state function is non-linear it will be approximated by a linear equation, i.e. by $g(\mathbf{u}, \mathbf{p}) \approx \boldsymbol{\alpha}^T\mathbf{u} + \beta$ with $\beta = -\boldsymbol{\alpha}^T\mathbf{u}^*$. The point $\mathbf{u}^*$ out of the standard normal distributed space is the solution point of the nonlinear optimization problem (βP):

$$(\beta P) \qquad \textbf{minimize} \quad \|\mathbf{u}\| \qquad \text{subject to} \quad g(\mathbf{u}, \mathbf{p}) \leq 0.$$

The solution $\mathbf{u}^*$ of the constrained optimization problem (βP) is called design point or β-point and defines the reliability index

$$\beta_{\mathbf{p}} = \beta(\mathcal{F}_{\mathbf{p}}) = \|\mathbf{u}^*\|. \qquad (3)$$

The n-dimensional vector $\boldsymbol{\alpha}$ is the vector of direction cosines of the β-point $\mathbf{u}^*$. Then, the failure probability can be approximated by $P_f(\mathbf{p}) \approx \Phi(-\beta_{\mathbf{p}})$.

The following important asymptotic result holds for large reliability indices. Breitung (1984) proved, that for $\beta_{\mathbf{p}} \to \infty$ is:

$$P_f(\mathbf{p}) \sim \Phi(-\beta_{\mathbf{p}}) \cdot \Pi_{i=1}^{n-1}(1 - \beta_{\mathbf{p}}\kappa_i)^{-1/2}, \qquad (4)$$

where the κ_i are the main curvatures of the limit state function $g(\mathbf{u}, \mathbf{p})$ in the solution point $\mathbf{u}^*$

3. PROPERTIES OF β-POINTS

The equation (4) indicates that $P_f(\mathbf{p}) \approx \Phi(-\beta_{\mathbf{p}})$ in fact is a first order approximation which is sufficiently accurate for most practical cases. Furthermore, the first-order approximation only requires simple differentiability of the limit state function $g(\mathbf{u}, \mathbf{p})$. Therefore, reliability analysis involves a probability distribution transformation, the search for the β-point and the evaluation of the standard normal integral. Hereby, the search for the design point $\mathbf{u}^*$ is the numerically most challenging problem. Most more recent applications use a sequential quadratic programming (SQP) algorithm specialized for the problem (βP) (see, for example Abdo/Rackwitz, 1991).

3.1 *Optimality Conditions for β-Points*

The first-order reliability index $\beta_{\mathbf{p}}$ can alternatively be used by FORM as a measure of reliability. By definition $\beta_{\mathbf{p}}$ is the minimum distance from the origin to the limit state surface in standard normal space. The following β-point theorem is easy to prove, because the β-point $\mathbf{u}^*$ is a Karush-Kuhn-Tucker-point for (βP).

Theorem 1 (β-Point-Theorem) *If $\mathbf{u}^*$, with $\mathbf{u}^* \neq \mathbf{0}$, is the solution point of optimization problem (βP), then the following two statements hold for each $\mathbf{p}$:*

a) $g(\mathbf{u}^*, \mathbf{p}) = 0,$

b) $\mathbf{u}^{*T}\nabla_{\mathbf{u}}g(\mathbf{u}^*, \mathbf{p}) + \|\mathbf{u}^*\|\|\nabla_{\mathbf{u}}g(\mathbf{u}^*, \mathbf{p})\| = 0.$

□

The equations a) and b) are necessary first-order optimality conditions for the design point. Later it will be shown that the β-point theorem is the main utility for one-level optimization algorithms in search of minimal failure probability under constraints on total costs and structural performance.

3.2 *Sensitivity Analysis for Parameters in β-Points*

The sensitivity analysis of a structural optimization problem collects auxiliary information about the analytical structure of solution points and can help to transform the problems to a suitable formulation. It further can collect information about suitable starting values and provide insight into the cause of non-convergence of optimization algorithms.

At first the Lagrange function for the optimization problem (βP) is differentiated with respect to a parameter element p_j. Because the design-point $\mathbf{u}^*$ is a solution for (βP), it follows from the Karush-Kuhn-Tucker optimality condition:

$$\frac{\partial \beta_{\mathbf{p}}}{\partial p_j} = \frac{\partial \|\mathbf{u}^*\|}{\partial p_j} + \|\nabla_{\mathbf{u}}g(\mathbf{u}^*, \mathbf{p})\|^{-2} \cdot \left(\frac{\partial g(\mathbf{u}^*, \mathbf{p})}{\partial p_j}\|\nabla_{\mathbf{u}}g(\mathbf{u}^*, \mathbf{p})\| - g(\mathbf{u}^*, \mathbf{p})\frac{\partial \|g(\mathbf{u}^*, \mathbf{p})\|}{\partial p_j}\right)$$

It can be shown by the β-point theorem a), that the first derivative of $\beta_{\mathbf{p}}$ with respect to a parameter element p_j can be written in the following form:

$$\frac{\partial \beta_{\mathbf{p}}}{\partial p_j} = \frac{1}{\|\nabla_{\mathbf{u}}g(\mathbf{u}^*, \mathbf{p})\|} \cdot \frac{\partial g(\mathbf{u}^*, \mathbf{p})}{\partial p_j}. \qquad (5)$$

Use of equation (5) leads to the non-dimensional sensitivities, so-called elasticities $S_{p_j}^{\beta}, j = 1, \ldots, d$, of the reliability index with respect to an element of the parameter:

$$S_{p_j}^{\beta} = \frac{\partial \beta_{\mathbf{p}}}{\partial p_j} \cdot \frac{p_j}{\beta_{\mathbf{p}}} = \frac{1}{\|\nabla_{\mathbf{u}}g(\mathbf{u}^*, \mathbf{p})\|} \cdot \frac{p_j}{\|\mathbf{u}^*\|} \cdot \frac{\partial g(\mathbf{u}^*, \mathbf{p})}{\partial p_j}. \qquad (6)$$

Equation (6) can be calculated easily. The definition of $S_{p_j}^{\beta}$, of course, makes sense only if the conditions $p_j \neq 0$, $\beta_{\mathbf{p}} \neq 0$ and $\nabla_{\mathbf{u}}g(\mathbf{u}^*, \mathbf{p}) \neq \mathbf{0}$ are fulfilled. The elasticities of the reliability index with respect to an element of the cost vector can be used to investigate the importance of the parameter elements in $\mathbf{p}$.

3.3 *Asymptotic Equivalence of $\Phi(-\beta_{\mathbf{p}})$ and $P_f(\mathbf{p})$*

Breitung (1984) established the following theorem.

Theorem 2 ***(Breitung, 1984)***
If $0 < \mathcal{P}(\mathcal{F}_{\mathbf{p}})$, then holds:

$$\lim_{b \to \infty} \frac{\mathcal{P}(b\mathcal{F}_{\mathbf{p}})}{\mathcal{P}(b\mathcal{F}_{SORM,\mathbf{p}})} = 1.$$

□

It says that asymptotically a quadratic approximation of failure surface is sufficient for failure probability estimation.

Hohenbichler (1984) proved the following (weaker) theorem by using the central scaling of the failure domain $b\mathcal{F}_{\mathbf{p}} = \{b\mathbf{u} : \mathbf{u} \in \mathcal{F}_{\mathbf{p}}\}$ again. The theorem shows for the limit $b \to \infty$, that the reliability index $\beta_{\mathbf{p}}$ converges "relatively" to the exact reliability index.

Theorem 3 ***(Hohenbichler, 1984)***
If $0 < \beta(\mathcal{F}_{\mathbf{p}}) < \infty$, then holds:

$$\lim_{b \to \infty} \frac{\beta(b\mathcal{F}_{\mathbf{p}})}{\beta^E(b\mathcal{F}_{\mathbf{p}})} = 1.$$

□

In other words:

For "large" reliability indices $\beta_{\mathbf{p}}^E$ or for "small" failure probabilities $P_f(\mathbf{p})$ the geometrical reliability index $\beta_{\mathbf{p}}$ computed by optimization problem (βP) is a good approximation of the (exact) reliability index $\beta_{\mathbf{p}}^E$.

In the following the cost function only depends on the cost parameters, i.e. $C_t = C(\mathbf{p})$. The compact subset $T_C =_{def} \{\mathbf{p} : C(\mathbf{p}) \leq C^{maximal}\}$ of the $\mathbf{p}$-space defines the admissible set of cost parameters. This gives the corollary of theorem 3.

Corollary 4 *If $0 < \beta(\mathcal{F}_{\mathbf{p}}) < \infty$, then holds:*

$$\lim_{b \to \infty} \frac{\max(\beta(b\mathcal{F}_{\mathbf{p}}) : \mathbf{p} \in T_C)}{\max(\beta^E(b\mathcal{F}_{\mathbf{p}}) : \mathbf{p} \in T_C)} = 1.$$

□

From corollary 4 follows for the reliability optimization problem with a cost constraint:

For "small" failure probabilities $P_f(\mathbf{p})$ the optimal reliability index β^*, computed by maximization of reliability index subject to the cost constraint $C(\mathbf{p}) \leq C^{maximal}$, is a good approximation of the (exact) maximum reliability index $\beta^{E^*} = \max(\beta_{\mathbf{p}}^E : \mathbf{p} \in T_C)$.

4. DESIGN-BASED OPTIMIZATION OF RELIABILITY

Many practical applications of structural optimization pursue at least three conflicting aims:

- high reliability
- low cost or weight of the structure
- good structural performance

The third option will not be dealt with explicitly, however. The cost can or cannot include the expected failure cost. Therefore, as mentioned above, two principally different types of optimization can be defined, i.e. where cost (or weight, structural performance, etc.) or reliability is optimized. The first is the optimization problem

(RCP): a constrained maximization problem where the reliability of a structure is maximized subject to given maximum cost and other structural performance requirements,

and the second is cost optimization problem

(CRP): a constrained minimization problem where the total cost, possibly including initial cost and expected cost of failure, are minimized subject to a given minimum reliability and other structural performance requirements.

Here only reliability optimization with cost constraint and other structural performance requirements (RCP) will be performed.

The reliability optimization problem (RCP) is a problem where the reliability of a structure is maximized with constraints on cost including initial cost of design and expected cost of failure, on structural performance and on design parameters, e.g. simple bounds. The reliability of the structure is obtained using first-order reliability (FORM) techniques. The solution can be viewed as a problem with two levels of optimization. The first problem (top-level) is optimization of reliability. The determination of the reliability index is the second problem (sub-level) and it is needed in the objective function (maximize reliability index) and in calculation of total cost. Instead of using a two-level approach the two optimizations can be combined into one optimization problem by use of the statements of section 3.1.

More precisely, the necessary first-order optimality condition for design points from β-point theorem are inserted into the reliability optimization problem. The necessary optimality conditions for the reliability index problem (βP) must be guaranteed by the constraints of the reliability optimization problem (RCP):

minimize	$P_f(\mathbf{p})$
subject to	$g(\mathbf{u},\mathbf{p}) = 0$
	$\mathbf{u}^T \nabla_{\mathbf{u}} g(\mathbf{u},\mathbf{p}) + \lVert\mathbf{u}\rVert \lVert\nabla_{\mathbf{u}} g(\mathbf{u},\mathbf{p})\rVert = 0$
	constraints on design / cost parameter
	constraint for total costs
	simple bounds for design parameter
	simple bounds for cost parameter,

The first order approximation of the failure probability $P_f(\mathbf{p}) \approx \Phi(-\beta_{\mathbf{p}})$ is outlined in section 3. The following optimization problems are equivalent

minimize $P_f(\mathbf{p})$ and **maximize** $\beta_{\mathbf{p}} = \lVert\mathbf{u}^*\rVert$.

The constraint related to total cost in (RCP) can be specified by $C_t(\mathbf{p},\mathbf{u}) \leq C_t^{maximal}$. The function of total expected cost $C_t(\mathbf{p},\mathbf{u})$ includes the initial cost of design and the cost of a possible failure of the structure. The cost function can be given as:

$$C_t(\mathbf{p},\mathbf{u}) = C_i(\mathbf{p})(1 - P_f(\mathbf{p})) + C_f(\mathbf{p})P_f(\mathbf{p}),$$

where $C_i(\cdot)$ is the initial cost of design and construction, $C_f(\cdot)$ is the cost of failure and $P_f(\cdot)$ is the probability of failure, which can approximated by FORM, i.e. the probability of failure is $P_f(\mathbf{p}) \approx \Phi(-\beta_{\mathbf{p}})$. Because structural failure probabilities are usually small, the simplification $1 - P_f(\mathbf{p}) \approx 1$ is admissible. The total cost function can then be written as:

$$C_t(\mathbf{p},\mathbf{u}) \approx C_i(\mathbf{p}) + C_f(\mathbf{p})\Phi(-\lVert\mathbf{u}\rVert). \quad (7)$$

It is easy to verify that the following optimization problem (RCP) will maximize the reliability of a structure subject to a given maximum cost: Thus, the complete optimization problem for (RCP) is:

maximize	$\lVert\mathbf{u}\rVert$
subject to	$g(\mathbf{u},\mathbf{p}) = 0$
	$\mathbf{u}^T \nabla_{\mathbf{u}} g(\mathbf{u},\mathbf{p}) + \lVert\mathbf{u}\rVert \lVert\nabla_{\mathbf{u}} g(\mathbf{u},\mathbf{p})\rVert = 0$
	$h_i(\mathbf{T}(\mathbf{u}),\mathbf{p}) = 0 \quad, i = 1, ..., m'$
	$\widetilde{h}_j(\mathbf{T}(\mathbf{u}),\mathbf{p}) \leq 0 \quad, j = m' + 1, ..., m$
	$C_i(\mathbf{p}) + C_f(\mathbf{p})\Phi(-\lVert\mathbf{u}\rVert) \leq C_t^{maximal}$
	$(\mathbf{x}^l, \mathbf{p}^l) \leq (\mathbf{T}(\mathbf{u}), \mathbf{p}) \leq (\mathbf{x}^u, \mathbf{p}^u),$

where $h_i(\cdot,\cdot)$, $\widetilde{h}_j(\cdot,\cdot)$ are m' equality and $m - m'$ are inequality constraints for the design vector and the parameters, e.g. structural performance requirements on parameters $\mathbf{p}$. The $(n+d)$-dimensional vectors $(\mathbf{x}^l, \mathbf{p}^l)$, $(\mathbf{x}^u, \mathbf{p}^u) \in \mathbb{R}^n \times \mathbb{R}^d$ are simple lower and upper bounds for the random vector $\mathbf{x} = \mathbf{T}(\mathbf{u})$ and the cost parameter $\mathbf{p}$. The relation " $\leq$ " for the simple bounds in the optimization problem is a vector relation and is defined by the ordinary order relation for the individual components of a vector.

Therefore, the inverse reliability optimization problem can completely be solved using FORM concepts and a special generalization of the inverse reliability problem $\max(\beta^E(b\mathcal{F}_{\mathbf{p}}) : \mathbf{p} \in T_C)$ is the optimization problem (RCP). The first-order approximation of the failure probability, i.e. $P_f(\mathbf{p}) \approx \Phi(-\beta_{\mathbf{p}})$, in the cost optimization problem (CRP), however, can only lead to rough approximations which has to be observed taken into account when judging the approaches by, for example, Der Kiureghian et al. (1995) (see also theorem 2).

5. NUMERICAL EXAMPLES

In the following section three examples for reliability optimization (RCP) are presented. The maximization of reliability index is carried out by a non-commercial PC/DOS program package based on the non-linear optimization algorithm NLPQL by Schittkowski (1985) and on tools of STRUREL, especially the probability distribution transformation and distribution routines (see SYSREL 9.0, RCP GmbH, 1997). It consists of a main program and various tools, e.g.. for sensitivity analysis, pre-evaluation of active/ inactive constraints and suitable choice of starting points. The main program and the routines mentioned above are written in FORTRAN.

5.1 Reinforced Concrete Beam

A rectangular reinforced concrete beam with parameters, $\mathbf{p} = (w, d, a_s)$ is considered in the first example (see Friis Hansen/Madsen, 1992, but with modified parameters) with some other cost parameters and two constraints on parameters w, d and a_s:

Table 5.1: Cost parameter of reinforced concrete beam

Variable	Symbol	Unit
Width of Concrete Beam	w	m
Effective Depth of Concrete Beam	d	m
Reinforcement Area of Beam	a_s	m^2

The distributions and stochastic characteristics of uncorrelated variables of the uncertain design vector $\mathbf{x} = (T_s, T_c, M_b, K)$ are given by:

Table 5.2: Design parameter of reinforced concrete beam

Variable	Symbol [Unit]	Distribution	Mean / Standard derivation (Parameters)
Steel Yield Stress	T_s [MPa]	N	360.0 / 36.0
Conc. Com. Strength	T_c [MPa]	LogN	40.0 / 6.0
Appl. Bend. Moment	M_b [MNm]	Gumbel	0.05 / 0.003
Model Uncertainty	K [-]	Rec	(0.5, 0.667)

The limit state function in terms of the random vector (T_s, T_c, M_b, K) and of the parameter (w, d, a_s) is given by:

$$G(\mathbf{x},\mathbf{p}) = \left(1 - K\,\frac{a_s T_s}{w d T_c}\right) a_s\, d\, T_s - M_b\,.$$

The failure cost C_f is estimated as:

$$C_f = 50\,000 CU \qquad \textit{(CU = currency unit)}\,.$$

The reinforced concrete beam has a fixed span of 5 m and the initial cost is given by

$$C_i(\mathbf{p}) = 5_{[m]}\,(800_{[CU/m^3]} \cdot wd + 2000_{[CU/m^3]} \cdot a_s)\,.$$

Two constraints are given, a maximum admissible area of reinforcement in relation to the total area of the concrete section and a lower bound for the area of the beam:

$$a_s \leq 5\% \cdot wd$$

and

$$0.01 \leq wd.$$

The maximal permitted total cost of the reinforced beam is 145.00 CU. The transformed standard normal vector elements are bounded by -30.00 and 30.00. The width w and the effective depth d of the beam have the lower bound 0.15 m and the upper bound 0.50 m, respectively. The area of the steel reinforcement a_s must be within the interval $(10^{-4}, 10^{-2})$.

The results of the optimization algorithm NLPQL are for the (RCP)-problem:

Table 5.3: Optimal results of reliability maximization for reinforced concrete beam

	Reliability maximization
Starting values	
$(u^0_{T_s}, u^0_{T_c}, u^0_{M_b}, u^0_K)$	(-0.25, -0.25, 1.00, 0.25)
(w^0, d^0, a^0_s)	(0.150, 0.182, 0.00109)
Optimization results	
Final Failure Probability	$1.59 \cdot 10^{-6}$
Final Reliability Index	4.659
$(u^*_{T_s}, u^*_{T_c}, u^*_{M_b}, u^*_K)$	(0.36, -3.86, 2.27, 1.24)
(w^*, d^*, a^*_s)	(0.150, 0.215, 0.0016)
Number of calls	
Function-calls	89
Gradient-calls	26

5.2 Steel Column

The second example is a steel column with cost parameter $\mathbf{p} = (b, d, h)$:

Table 5.4: Cost parameter of steel column

Variable	Symbol	Unit
Mean of Flange Breadth	b	mm
Mean of Flange Thickness	d	mm
Mean of Height of Steel Profile	h	mm

The steel column has a constant span of 7500 mm. The function of total cost $C_t(\mathbf{p},\mathbf{u})$ includes no failure cost, i.e. $C_f = 0$, and has the following form:

$$C_t(\mathbf{p},\mathbf{u}) = C_i(\mathbf{p}) = (bd + 5_{[mm]} \cdot h) \cdot {}_{[CU/mm^2]}$$

The two by two uncorrelated design variables of the uncertain vector $\mathbf{x} = (F_s, P_1, P_2, P_3, B, D, H, F_0, E)$ and the stochastic characteristics are given by:

Table 5.5: Design parameter of steel column

Variable	Symbol [Unit]	Distri-bution	Mean / Standard derivation
Yield Stress	F_s [MPa]	LogN	400 / 35
Dead Weight Load	P_1 [N]	N	500000 / 50000
Variable Load	P_2 [N]	Gumbel	600000 / 90000
Variable Load	P_3 [N]	Gumbel	600000 / 90000
Flange Breadth	B [mm]	LogN	b / 3
Flange Thickness	D [mm]	LogN	d / 2
Height of Profile	H [mm]	LogN	h / 5
Initial Deflection	F_0 [mm]	N	30 / 10
Youngs Modulus	E [MPa]	Weibull	21000 / 4200

The limit state function in terms of the random vector $\mathbf{x}$, the parameter (b, d, h) and auxiliary functions $\mathcal{A}_s, \mathcal{M}_s, \mathcal{M}_i, \mathcal{E}_b, \mathcal{P} = P_1 + P_2 + P_3$ is defined by:

$$G(\mathbf{x}, \mathbf{p}) = F_s - \mathcal{P}\left(\frac{1}{\mathcal{A}_s} + \frac{F_0}{\mathcal{M}_s} \cdot \frac{\mathcal{E}_b}{\mathcal{E}_b - \mathcal{P}}\right).$$

where

$$\begin{aligned} \mathcal{A}_s &= 2BD, && \text{(area of section)} \\ \mathcal{M}_s &= BDH, && \text{(modulus of section)} \\ \mathcal{M}_i &= \frac{1}{2}BDH^2, && \text{(moment of iteria)} \\ \mathcal{E}_b &= \frac{\pi^2 E \mathcal{M}_i}{s^2}, && \text{(Euler buckling load)} \end{aligned}$$

Table 5.6: Results of reliability maximization of steel column with maximum cost from 4 000,00 CU to 13 000,00 CU

Maximal cost	Optimal cost vector $\mathbf{p}^*$			Reliability index
$C^{maximal}$	b^*	d^*	h^*	$\beta^*_{\mathbf{p}}$
4000.00	200.00	17.50	100.00	3.132
5000.00	200.00	22.50	100.00	4.961
6000.00	200.00	27.50	100.00	6.369
7000.00	216.67	30.00	100.00	7.427
8000.00	250.00	30.00	100.00	8.249
9000.00	283.33	30.00	100.00	8.967
10 000.00	316.67	30.00	100.00	9.605
11 000.00	350.00	30.00	100.00	10.180
12 000.00	383.33	30.00	100.00	10.709
13 000.00	400.00	30.00	200.00	11.065

No other constraints on cost and design parameters are given in the example. The nine elements of the transformed standard normal vector $\mathbf{u}$ must be within the interval $(-30.00, 30.00)$. The means of flange breadth b and flange thickness d have the lower bounds

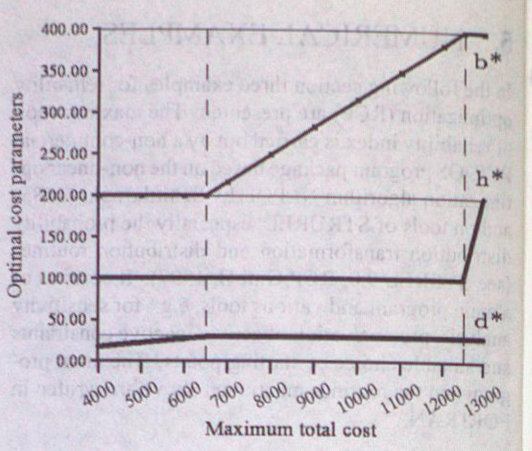

Figure 5.1: Dependence of optimal cost vector elements b, h and h on maximal admissible total cost of the steel column

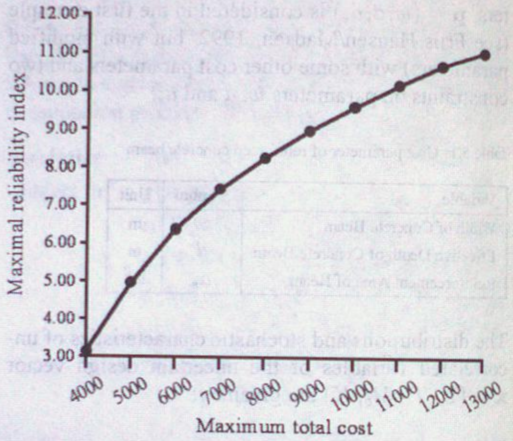

Figure5.2: Dependence of reliability index on maximal admissible total cost of the steel column

200 mm resp. 10 mm and the upper bounds 400 mm resp. 30 mm. The interval (100[mm], 500[mm]) defines the admissible mean height h of the T-shaped steel profile.
The results of the reliability optimization of steel column with various maximal permitted (total) cost are given by table 5.6.
The increase of the parameters b, h and h depends on the maximum cost and the importance of the individual parameters for the reliability of the structure, i.e. the value of gradients of limit state function. It is easy to see, that at first the mean of flange thickness and only subsequently the mean of flange breadth increases to the upper bonds 30.00 and 400.00. The mean of height of steel column remains at the lower bound up to maximum cost from 12 500 CU.Figure 5.1 illustrates this.

It is further seen that higher maximal cost $C^{maximal}$ lead to an increase of the maximum reliability index. But figure 5.2 shows also that the reliability indices $\beta^*_{\mathbf{p}}$ from the (RCP)-problem decrease exponentially for lower maximum cost.

5.3 *Pile Sheet Wall Stability*

The stability of a pile sheet wall with the deterministic cost parameter $\mathbf{p} = (a_1, eps, t, l_1, l_2, h_{01}, h_{02})$ is considered in the last example. The parameters are given by table 5.7.

Table 5.7: Cost parameter of pile sheet wall stability

Variable	Symbol	Unit	Bounds
1st Anchor Force	a_1	kN/m	(275, 325)
Anchor angle	eps	rad	(0.1, 1.0)
Depth of sheet wall l	t	m	(2.5, 3.0)
Horizo. Length of 1st Anchor	l_1	m	(10, 20)
Horizo. Length of 2nd Anchor	l_2	m	(10, 20)
Depth of 1st Anchor Head	h_{01}	m	3.5
Depth of 2nd Anchor Head	h_{02}	m	(7, 10)
Excavation Depth	h	m	14

For simplicity of illustration the function of total cost $C_t(\mathbf{p}, \mathbf{u})$ includes no failure cost, i.e. $C_t(\mathbf{p}, \mathbf{u})$=$C_i(\mathbf{p})$. It can be written in the following form:

$$C_t(\mathbf{p}, \mathbf{u}) = (2t + l_2 + 4h_{02}) \cdot B \leq 47.00\,CU.$$

B is a coefficient close to one. The uncertain vector $\mathbf{x} = (Phi, G, C, Q)$ and the stochastic characteristics of design variables are given in table 5.8. The friction angle Phi and the cohesion C are correlated by $\rho(Phi, C) = -0.5$. All other uncertain parameters are uncorrelated. The depth of the foot of the second anchor is defined by $h_2 = h_{02} + l_2 \sin(eps)$.

Table 5.8: Design parameter of pile sheet wall stability

Variable	Symbol [Unit]	Distri-bution	Mean / Standard derivation
Friction Angle	Phi [rad]	LogN	0.52 / 0.039
Specific Weight of Soil	G [kN/m^3]	N	20 / 2
Cohesion	C [kN/m^2]	LogN	10 / 2.5
Service Load on ground	Q [kN/m^2]	Gumbel	10 / 4

The limit state function can be assessed by the limit equilibrium method and is given below in terms of the uncertain vector (Phi, G, C, Q), the deterministic parameters (h_2, h) and the cost parameters $(a_1, eps, t, l_1, l_2, h_{01}, h_{02})$ in the following form:

$$\begin{aligned} G(\mathbf{x}, \mathbf{p}) = {} & \mathcal{F}_E \cdot \cos(Phi/2\text{-}\mathcal{C}_{12}) \\ & + \mathcal{F}_a(\cos(Phi\text{-}\mathcal{C}_{12}\text{-}eps) - \nu\cos(Phi\text{-}\vartheta\text{-}eps)) \\ & + (\mathcal{C}_1 + \nu\mathcal{C}_2)\cos(Phi) + \mathcal{C}_{12}\sin(Phi/2\text{-}\vartheta\text{-}\pi/4) \\ & + (\mathcal{G}_1 + \mathcal{P}_1)\sin(Phi\text{-}\vartheta) + (\mathcal{G}_2 + \mathcal{P}_2)\sin(Phi\text{-}\pi/4) \end{aligned}$$

where $\mathcal{F}_E$, $\mathcal{F}_a, \nu, \mathcal{P}_1, \mathcal{P}_2$, $\mathcal{G}_1, \mathcal{G}_2, \mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_{12}$ are auxiliary functions of the geometrical and stochastic variables not given herein. The anchor lengths are subject to optimization and there is $l_1 \geq l_2 + 1$[m]. As in the last example the interval $(-30.00, 30.00)$ is the admissible domain of the n standard normal elements.
The numerical results for the best stability of the pile sheet wall given the cost constraint and the other constraints are :

Table 5.9: Optimization results for pile sheet wall

	Reliability maximization
Optimization results	
Final Failure Probability	$1.75 \cdot 10^{-5}$
Final Reliability Index	4.138
$(u^*_{Phi}, u^*_G, u^*_C, u^*_Q)$	(-4.09, 0.35, -0.50, 0.06)
$(a^*_1, eps^*, t^*, l^*_1, l^*_2, h^*_{01}, h^*_{02})$	(400, 0.1, 2.5, 20, 14, 3.5, 7)
(h^*_2, h^*)	(8.40, 14)
Number of calls	
Function-calls	192
Gradient-calls	114

It is seen that all design variables are either at the lower or upper bound except l_2. Because uncertainty in the friction angle dominates the problem, the parameter l_2 is the parameter which is least sensitive to variations in the friction angle.

6. SUMMARY AND CONCLUSION

A one-level reliability optimization (RCP) based on FORM given maximum expected total cost and, possibly, some other constraints is derived.
The great advantages of a one-level optimization problem (RCP) are:

- a well-known standard non-linear optimization algorithm, e.g. a SQP-algorithm, can be used to solve the problem,
- scaling problems for complicated problems are handled by standard optimization routines,
- the methods appear locally stable and robust.
- the formulation of the optimization task is especially simple

The formulation has, no doubt, some disadvantages (see also Madsen and Friis Hansen, 1992):

- a standard space formulation is required in order to perform the FORM analysis. A probability distribution transformation from the standard normal **u**-space to the original space **x** must be included explicitly. Those probability distribution transformations may require additional numerical effort and can cause numerical problems in extreme cases (numerical inversion of distribution functions).
- the numerical calculation of second derivatives of the limit state function is required,
- monotonic transformations of the objective function and the cost constraints sometimes are necessary in order to achieve convergence.
- good starting values sometimes are required to achieve convergence.

If the cost constraint is a simple function of $\mathbf{p}$ only the algorithm turns out to be very robust and efficient. The cost function $C_t(\mathbf{p}, \mathbf{u})$, however, is strongly non-smooth for very small reliability indices $\beta_{\mathbf{p}}$ and large failure cost. In this case suitable monotonic transformations can improve efficiency and reliability of the algorithm considerably. For small failure probabilities cost constraints including the expected failure cost should also be transformed so that the cost constraint $C_t^{maximal} - C_t(\mathbf{p}, \mathbf{u})$ possesses numerically non-zero gradients in the entire parameter domain. A sensitivity analysis of the cost function and an investigation of importance measures or of elasticities can help to retain numerically non-zero gradients: For very large reliability indices, say $\|\mathbf{u}\| \geq 10$, the total expected cost are approximated simply by the initial cost $C_i(\mathbf{p})$.

In summary, the one-level formulations for reliability optimization proposed in the paper is limited to FORM formulations. However, the asymptotic correctness of reliability optimization with cost constraints is proved by a corollary showing the asymptotic equivalence of first-order and exact reliability indices for small failure probabilities. The structure of the resulting optimization problem is rather simple. An extension to SORM formulations appears to be not straightforward. Only one optimization algorithm, preferably a SQP-algorithm, is necessary for the solution of the design-oriented problem (RCP). Nevertheless, the problem can have constraints with widely varying gradients and even zero gradients in extreme cases. Therefore, several numerical "tricks", e.g. transformations of constraint functions or determination of "good" starting values, must be applied in order to achieve convergence of the numerical optimization algorithm. With these auxiliary tools it can be stated that the used NLPQL-algorithm works quite efficiently.
The maximum reliability problem is formulated for one state function only. Hence the (RCP)-formulation is restricted to one failure mode only. The generalization to multiple failure modes (unions of failure modes) appears possible and will be investigated.

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