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# Safety and Reliability

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## Time variant reliability of a column under variable loads with intermitencies

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**ABSTRACT:** The present reliability analysis of a reinforced concrete column is an extension of previous reliability studies of columns designed according to Eurocodes 1 and 2. Simpler stochastic models considered for variable actions beforehand are superseded here by more realistic square wave (jump) processes with intermitencies. Each process is characterised by a jump rate (an average number of magnitude changes) and interarrival-duration intensity (a multiple of the arrival rate and the mean duration). Considering a reference time interval 50 years the results are compared with time invariant analysis and results of previous time variant reliability studies. It appears that refined time variant reliability analysis with intermitencies may considerably improve reliability assessment of structures, particularly those exposed to variable actions of relatively short duration.

## 1 INTRODUCTION

The presented reliability analysis of a reinforced concrete column is an extension of a previous study by Holický & Vrouwenvelder (1997) based on recent theoretical development of time variant reliability analysis by Rackwitz (1993 and 1997). The study is a part of an extensive research activity on Eurocode Random Variable Models supervised by the Joint Committee for Structural Safety (JCSS). The JCSS aims at providing a standardised set of statistical models for loads and structural properties which will reflect the present state of knowledge.

Simpler stochastic models considered beforehand, when time variant variables were described by simple jump process (Holický & Vrouwenvelder (1997)), are superseded here by jump process with intermitencies developed by Rackwitz (1997). In addition some minor improvement in deterministic design of the column were accepted.

The variable actions (short and long term imposed loads and wind) are considered as stationary and ergodic random processes described by square wave (jump) sequences. Each process is characterised by a jump rate  $\lambda$  and the interarrival-duration intensity  $\rho$ , both related to one year. The total of 12 study cases of the concrete column, designed according to newly developing Eurocode 1 ENV 1991-1 (1993), ENV 1991-2-1 (1994), ENV 1991-2-4 (1995)) and Eurocode 2 (ENV 1992-1 (1993)), are analysed using the software COMREL (RCP Reliability Consulting Programs (1997)).

## 2 MODEL STRUCTURE

A model multi-storeyed structure considered in this study is schematically shown in Figure 1. It is assumed that each frame in the transversal direction of the structure may be considered as an unbraced sway frame. These frames consist of four columns at a constant transverse distance  $a_1$ ; in the longitudinal direction of the structure the frames are located within a constant distance  $a_2$  (see Figure 1).

In the following reliability analysis the edge, fully clamped column of an internal transversal frame having height  $L$  (see Figure 1) and rectangular cross-section  $b \times h$  is considered. The cross-section height

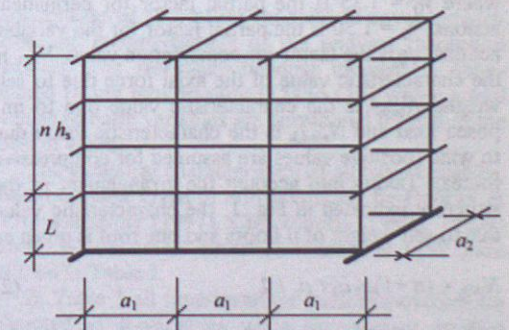


Figure 1. Transverse frame of a multi-storeyed structure.



$h$  is two times (in one study case three times) the width  $b$ . Considering different structural arrangements, a total of 12 study cases indicated in Table 1 are analysed. Further it is assumed that the storey height above the considered column is  $h_s = 3$  m, permanent load is determined as a weight of a reinforced concrete floor of a uniform equivalent thickness of 0.30 m (including weight of the slab, columns, beams, floor and cladding).

Table 1. Study cases of a reinforced column.

Study case	Number of storeys $n$	Height of the column $L$ [m]	Transversal distance $a_1$ [m]	Longitudinal distance $a_2$ [m]	Cross section dimension $b \times h$ [m $\times$ m]
1	10	6	5	5	0.35 $\times$ 0.70
2	10	3	5	5	0.30 $\times$ 0.60
3	10	9	5	5	0.35 $\times$ 0.70
4	10	12	5	5	0.45 $\times$ 0.90
5	10	6	4	5	0.30 $\times$ 0.60
6	10	6	7	5	0.35 $\times$ 0.70
7	10	6	5	4	0.30 $\times$ 0.60
8	10	6	5	7	0.40 $\times$ 0.80
9	1	6	5	5	0.25 $\times$ 0.50
10	3	6	5	5	0.25 $\times$ 0.50
11	20	6	5	5	0.45 $\times$ 0.90
12	10	6	5	5	0.25 $\times$ 0.75

Effects of actions considered in the analysis of a built in column consist of the axial force and bending moment, denoted again by  $N$  and  $M$  with appropriate subscripts. In the design calculation, the axial force and bending moment are represented by the design values  $N_d$  and  $M_d$  respectively. The maximum design axial force  $N_{d,max}$  is given as

$$N_{d,max} = \gamma_G N_{W,k} + \gamma_Q \max \{ N_{imp,k} + \psi_0 N_{wind,k}; N_{wind,k} + \psi_0 N_{imp,k} \} \quad (1)$$

where  $\gamma_G = 1.35$  is the partial factor for permanent actions,  $\gamma_Q = 1.50$  is the partial factor for the variable actions,  $\psi_0$  is the factor for combination value,  $N_{W,k}$  is the characteristic value of the axial force due to self weight,  $N_{imp,k}$  is the characteristic value due to imposed load and  $N_{wind,k}$  is the characteristic value due to wind (positive values are assumed for compressive forces). Taking into account the arrangement of the structure indicated in Fig. 1, the characteristic value due to self weight of  $n$  floors and one roof is given as

$$N_{W,k} = (n+1) a_1 a_2 t \rho_c / 2 \quad (2)$$

where  $\rho_c$  is the weight of concrete per unit volume considered as  $0.024 \text{ MN/m}^3$ .  $N_{imp,k}$  is the characteristic value of imposed load from  $n$  storeys given as

$$N_{imp,k} = n a_1 a_2 p_{imp} \alpha_n / 2 \quad (3)$$

where according to ENV 1991-2-1: Eurocode 1 (1994) the reduction factor  $\alpha_n = [2 + (n-2) \psi_0] / n$ ; here  $n$  is the number of storeys ( $>2$ ) and  $\psi_0$  is the load combination factor according to ENV 1991-2-1: Eurocode 1 (1994). Choosing category B (Public Building) the characteristic value of floor imposed load  $p_{imp,k}$  equals  $3 \text{ kN/m}^2$ .  $N_{wind,k}$  is the wind resulting from a pressure  $C_p G p_{wind,k}$  on a vertical area equal to  $(L + nh_s) a_2$ ; multiplication by the height  $(L + nh_s)/2$  gives the overturning moment. This moment is assumed to be balanced by the normal forces in the two outer columns, so:

$$N_{wind,k} = ((1/2)(L + nh_s)^2 a_2 C_p G p_{wind,k} - 4 M_{d0}) / (3 a_1) \quad (4)$$

where  $M_{d0}$  is the first order bending moment at the bottom of the column. The characteristic value of the wind action is considered assuming the return period of 50 years as  $p_{wind,k} = 0.5 \text{ kN/m}^2$ ; the gust (exposure) factor  $G = 2.5$  and the shape factor  $C_p = 0.8 + 0.5 = 1.3$  (ENV 1991-2-4 (1995)).

In accordance with ENV 1992-1: Eurocode 2 (1993) the design value  $M_d$  of the bending moment is

$$M_d = M_{d0} + N_d (e_a + e_2) = N_d (e_0 + e_a + e_2) \quad (5)$$

where  $e_0 = M_{d0} / N_d$  is the first order eccentricity,  $e_a$  is the additional eccentricity taking into account geometric imperfections and  $e_2$  is the second order eccentricity due to deformations of the column. It is assumed that the first order moment  $M_{d0}$  is caused only by wind action and is approximately given as

$$M_{d0} = L [G C_p G p_{wind,k} (L + nh_s) a_2] / 8 \quad (6)$$

where  $L$  denotes the column height (see Fig. 1). The eccentricities  $e_a$  and  $e_2$  are determined according to Eurocode 2 (ENV 1992-1 (1993)) as

$$e_a = 1.12 L / (2 \times 200) = 0.0028 L \quad (7)$$

$$e_2 = 0.1 K_1 l_0^2 (1/r) \quad (8)$$

$$K_1 = l_0 / (20 i) - 0.75 \leq 1 \quad (9)$$

where  $l_0 / i$  ( $i$  being radius of gyration) denotes the slenderness ratio. The curvature  $1/r$  is given in Eurocode 2 (ENV 1992-1 (1993)) as

$$1/r = 2 K_2 \varepsilon_{yd} / (0.9 (h - d_1)) \quad (10)$$

$$K_2 = (N_{ud} - N_d) / (N_{ud} - N_{bal,d}) \leq 1 \quad (11)$$

where  $N_{ud}$  is the design capacity of the cross section,  $N_d$  is the design axial force and  $N_{bal,d}$  is the force which maximises the ultimate moment of the cross section; in this study (for symmetrical reinforcement)

$N_{bal,d}$  is taken as  $N_{bal,d} = 0.5 \alpha f_{cd} A_c$ , where  $\alpha$  is a coefficient taking account of long term effects on the compressive strength. The remaining variables entering equation (10), the design yield strain  $\varepsilon_{yd} = f_{yd} / E_s$  and the effective depth of cross-section  $(h - d_1)$ , are given below. The resulting values of  $N_{d,max}$  are shown in Table 2.

For given design values of the normal forces  $N_d$  and bending moments  $M_d$ , the column cross sections are designed using a simplified formula:

$$\text{-- for } N_d < a b h f_{cd} / 2$$

$$[A_s f_{yd} (h - d_1 - d_2) + h N_d (1 - N_d / (a b h f_{cd}))] / 2 - M_d > 0 \quad (12)$$

$$\text{-- for } N_d > a b h f_{cd} / 2$$

$$K_2 [A_s f_{yd} (h - d_1 - d_2) / 2 + a b h^2 f_{cd} / 8] - M_d > 0 \quad (13)$$

$$N_{ud} = a b h f_{cd} + A_s f_{yd} \quad (14)$$

$$N_{bal,d} = a b h f_{cd} / 2 \quad (15)$$

These relationships give a good approximation of interaction diagrams derived from appropriate rules. The total reinforcement area  $A_s$  should satisfy the conditions of ENV 1992-1: Eurocode 2 (1993)

$$0.15 |N_d| / f_{yd} < A_s \quad (16)$$

$$0.003 b h < A_s < 0.08 b h \quad (17)$$

This is satisfied in all cases. Theoretical values of  $A_s$  rounded upward to the last digit are given in Table 2.

Table 2. Effects of actions for the maximum axial force  $N_{d,max}$ .

Case	$N_{d,max}$ [MN]	$M_{d0}$ [MNm]	$e_0$ [m]	$e_a 10^2$ [m]	$A_s 10^4$ [m <sup>2</sup> ]	$e_2 10^2$ [m]	$M_d$ [MNm]
1	2.075	0.329	0.159	1.7	24.9	2.2	0.410
2	2.038	0.151	0.074	0.8	19.9	0.1	0.170
3	2.111	0.535	0.253	2.5	50.2	6.0	0.715
4	2.148	0.768	0.358	3.4	31.4	0.3	1.061
5	1.857	0.329	0.177	1.7	40.2	2.8	0.412
6	2.658	0.329	0.124	1.7	37.2	1.8	0.421
7	1.660	0.263	0.159	1.7	29.2	2.9	0.339
8	2.904	0.461	0.159	1.7	31.2	1.4	0.549
9	0.306	0.082	0.268	1.7	4.9	4.9	0.102
10	0.671	0.137	0.204	1.7	10.7	4.9	0.181
11	4.735	0.603	0.127	1.7	55.6	0.7	0.716
12	2.075	0.329	0.159	1.7	33.0	1.5	0.395

### 3 DETERMINISTIC DESIGN

The following assumptions are accepted in design of column cross sections:

- symmetrical reinforcement ( $A_{s1} = A_{s2} = A_s / 2$ ) is considered only,
- the square shape of the column cross section is chosen such that  $h/b = 2$  (in the last study case  $h/b = 3$ ),
- the distance of reinforcing bars from the edge is chosen in all study cases as  $d_1, d_2 = 0.05 \text{ m}$ ,
- the assumed material characteristics for concrete class C 20/25 and reinforcing steel S 500 are

$$f_{ck} = 20 \text{ MPa}, \gamma_c = 1.5, f_{cd} = 13.33 \text{ MPa}, \alpha = 0.85 \quad (18)$$

$$f_{yk} = 500 \text{ MPa}, \gamma_s = 1.15, f_{yd} = 435 \text{ MPa} \quad (19)$$

### 4 LIMIT STATE FUNCTION

In the time variant reliability analysis the actual axial force  $N$  is considered as a simple sum of actual axial forces due to all the considered actions:

$$N = N_W + N_{imp} + N_{wind} \quad (20)$$

where  $N_W$  is the axial force due to self weight,  $N_{imp}$  is the axial force due to imposed load and  $N_{wind}$  is the axial force due to wind action (positive values are again accepted for compressive forces). The bending moment  $M$  is given by a modified equation (6) in which actual values are applied instead of the design values and a new additional eccentricity  $e_a$  is considered as  $\zeta L/2$ , where the initial sway  $\zeta$  is given in Table 3. The second order eccentricity  $e_2$  is given by modified equations (8) and (9) in which  $l_0 = L$ .

The limit state function  $g$  may be expressed as the difference between the resistance bending moment and the actual bending moment about the cross section centre

$$g = \xi_R M_R - \xi_E M \quad (21)$$

Two coefficients of model uncertainties  $\xi_R$  and  $\xi_E$  are considered as random variables to cover imprecision and incompleteness of the relevant theoretical models. The resistance bending moment is determined taking into account (15) to (18) where actual values of basic variables are considered instead of the design values.

The statistical characteristics of all basic variables, based on data given in previous study by Holický & Vrouwenvelder (1997), CIB Reports (1989a, 1989b) and JCSS document Vrouwenvelder et al. (1997) are shown in Table 3.

In Table 3 all time-invariant random variables are denoted as R-variables while time-variant random variables are denoted as S-variables for later convenience.



Table 3. Statistical characteristics of basic variables.

Sym- bol	Name of variable	Distrib. /Type	Dimen- sion	Mean	Standard deviation
$\alpha$	reduction factor	R/R	-	0,85	0,085
$A_s$	reinforce- ment area	DET	m <sup>2</sup>	nom	0
$f_c$	concrete strength	LN/R	Mpa	30	5
$f_y$	yield strength	LN/R	Mpa	560	30
$E$	modulus of elast.	DET	Gpa	200	0
$a_1$	column distance	DET	m	nom	0
$a_2$	perpend. distance	DET	m	nom	0
$b$	width of section	N/R	m	nom	0,005
$d_1$	distance of bars	N/R	m	0,05 m	0,01
$d_2$	height of section	N/R	m	nom	0,01
$L$	height of column	DET	m	nom	0
$n$	number of floors	DET	-	nom	0
$\zeta$	initial sway <sup>(1)</sup>	N/R	rad	0	0,001 <sup>(1)</sup>
$\xi_E$	mod. unc. load	N/R	-	1,0	0,1
$\xi_R$	mod. unc. resistance	N/R	-	1,1	0,11
$\rho$	weight factor	N/R	MNm <sup>-2</sup>	0,0240	0,00192
$C_p$	shape factor	N/R	-	1,3	0,13
$G$	gust fac- tor	GUM/R	-	2,5	0,25
$p_{wind}$	wind pressure	GUM/S	MNm <sup>-2</sup>	0,00035	0,00006 <sup>(2)</sup>
$p_{impl}$	long term load	GAM/S	MNm <sup>-2</sup>	0,0006	mean $\times v$ <sup>(3)</sup>
$p_{imps}$	short term load	GAM/S	MNm <sup>-2</sup>	0,0002	mean $\times v$ <sup>(4)</sup>

Notes to Table 3:

- (1) The initial overall sway  $\zeta$  is used to calculate the additional eccentricity  $e_a$  according to equation  $e_a = \zeta L/2$ .
- (2) The mean and standard deviation correspond to the distribution of the yearly maximum.
- (3) The mean and standard deviation correspond to the random point-in time distribution;  $v^2 = (0.16+8/(a_1 a_2))(1/n + \rho_{impl}(1-1/n))$  (see CIB report (1989b)), where the coefficient of correlation of the long term loads in two floors is considered as  $\rho_{impl} = 0.5$ . The factor  $v$  also holds for one storey with  $n = 1$ .

(4) The mean and standard deviation correspond to the distribution of 24 hours (one day) maximum,  $v^2 = 50/(a_1 a_2)$ .

### 5 RELIABILITY ANALYSIS

Reliability analysis of the column is presented for four different assumptions considering variable actions as:

- time-invariant variable models for extreme wind, long and short term imposed loads - the resulting reliability indices  $\beta_0$  may be used as a first approximation of more refined analysis,
- jump processes without intermittencies for wind and long term imposed load - the resulting reliability indices  $\beta_1$  refer to Turkstra's rule (see Turkstra, (1970)), i.e. a load takes on its extreme while the other are at their point-in-time value and the resulting probabilities are summed up,
- jump processes without intermittencies for wind, long and short term imposed loads - the resulting reliability indices are denoted by  $\beta_2$ ,
- jump processes with intermittencies for wind, for simultaneously jumping long term loads and for short term loads - the resulting reliability indices are denoted by  $\beta_3$ ,
- jump processes with intermittencies with given interarrival-duration intensity for wind, independent long term loads and short term loads - the resulting reliability indices are denoted by  $\beta_4$ .

In the time invariant analysis all the variable actions are considered as time invariant processes characterised by the theoretical models indicated in Table 3. Time variant reliability analysis assumes that the variable actions (wind, short term and long term imposed loads) are stationary and ergodic random processes described by rectangular wave renewal processes (jump processes) enveloping the real load processes. In this sense all subsequent results are conservative. When no intermittencies are considered then each jump process is characterised by a jump rate  $\lambda$  (an average number of magnitude changes of square waves in a year). Two different analyses assuming jump processes without intermittencies, described in detail by Holický & Vrouwenvelder (1997) are presented here: short term imposed load is absent ( $\beta_1$ ) and short term load is present ( $\beta_2$ ).

When intermittencies are considered (see Figure 2), in addition to the jump rate  $\lambda$ , each jump process is additionally characterised by the interarrival-duration intensity  $\rho$  (a product of the arrival rate and the mean duration), both expressed in terms of years. The distribution of the duration of a rectangular wave can theoretically remain unspecified. Its mean duration is  $1/\lambda$ . However, the distributions for the times between interarrivals of on-times and the

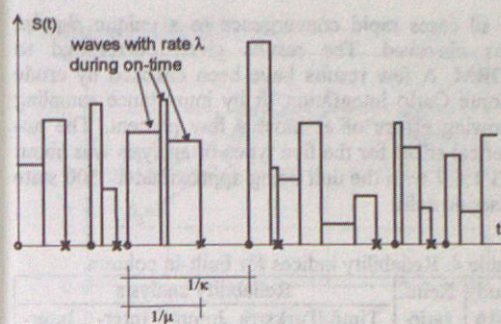


Figure 2. Rectangular wave jump process with intermittencies.

tributions of the durations of on-times must be specified.

For stationary processes and stationary intermittencies and assuming random initial conditions an upper bound failure probability is determined from

$$\begin{aligned}
 P_f(t_1, t_2) &\leq P_f(t_1) + E[N^+(t_1, t_2)] \\
 P_f(t_1) &= \sum_{i=1}^n p_i^i P_{f,i}^i(t_1) \\
 &+ \sum_{i=1}^n \sum_{j=i+1}^n p_2^{i,j} P_{f,2}^{i,j}(t_1) + \dots + p_n^{1,2,\dots,n} P_{f,n}^{1,2,\dots,n}(t_1) \\
 E[N^+(t_1, t_2)] &= \sum_{i=1}^n v_i^i \{P_i^i(t_2 - t_1)\} + \\
 &+ \sum_{i=1}^n \sum_{j=i+1}^n v_{i,j}^i \{P_2^{i,j}(t_2 - t_1)\} + \dots \\
 &+ v_{1,2,\dots,n}^i \{P_n^{1,2,\dots,n}(t_2 - t_1)\} \quad (22)
 \end{aligned}$$

where  $[t_1, t_2]$  denotes the reference time interval,  $E[N^+(t_1, t_2)]$  the mean number of outcrossings and  $v^i(t_1, t_2)$  the outcrossing rate. The  $p_k^{(k)}$  are the coincidence probabilities for the set  $\{k\}$  of loads being "on". They are computed according to a simple model proposed by Shinozuka (1981). There, it is assumed that the interarrival times of the "on"-times follow an exponential distribution with parameter  $\kappa$  and the duration of "on"-times a truncated exponential distribution with parameter  $1/(\kappa + \mu)$ . Each "on"-time is associated with only one rectangular wave so that with  $\lambda = \kappa$  the interarrival intensity is  $\rho = \lambda/\mu$  (see Figure 2). The outcrossing rate for independent amplitudes of the rectangular wave renewal processes is determined from Rackwitz (1993)

$$\begin{aligned}
 v_{\{k\}}^i(F_{\{k\}}, \tau) &= \sum_{i=1}^n \lambda_i \left[ \frac{\Phi(-\beta_{\{k\}})}{\prod_{j=1}^{n-1} (1 - \beta_{\{k\}} \kappa_{\{k\}j})^{v_{j,i}}} - \Phi(-\beta_{\{k\}}, -\beta_{\{k\}}; \rho_{\{k\}i}) \right] \\
 &= \frac{\Phi(-\beta_{\{k\}})}{\prod_{j=1}^{n-1} (1 - \beta_{\{k\}} \kappa_{\{k\}j})^{v_{j,i}}} \\
 &\quad + \sum_{i=1}^n \lambda_i \left[ 1 - \frac{\Phi(-\beta_{\{k\}}, -\beta_{\{k\}}; \rho_{\{k\}i})}{\Phi(-\beta_{\{k\}}) \prod_{j=1}^{n-1} (1 - \beta_{\{k\}} \kappa_{\{k\}j})^{v_{j,i}}} \right] \\
 &= \Phi(-\beta_{\{k\}}) \prod_{j=1}^{n-1} (1 - \beta_{\{k\}} \kappa_{\{k\}j})^{-v_{j,i}} \sum_{i=1}^n \lambda_i^{v_{j,i}} \quad (23)
 \end{aligned}$$

$\beta_{\{k\}j}$  is the reliability index and  $\kappa_{\{k\}j}$  are the main curvatures in the  $\beta$ -point according to FORM/SORM for the set  $\{k\}$  of active loads.  $\rho_{\{k\}i}$  is the correlation coefficient of the two linearized failure domains before and after a jump. As usual the reliability computations are performed in standard space results given below correspond to SORM. For the last factor can be interpreted as a first-order correction to the jump rates then denoted by  $\lambda^i$  improvement as compared to the asymptotic one derived by Breitung (1993). The improved model is demonstrated in Figure 3.

Amplitude dependencies as in Table 3 can be introduced by making the mean of the imposed long term load uncertain, for example, as a normally distributed variable with mean zero. Then,  $\rho_{impl} = 0.5$  corresponds to a standard deviation of this uncertain mean of about 70% of the original standard deviation and the standard deviation of  $p_{impl}$  given the mean is also only about 70% of the original value of  $(0.16+8/(a_1 a_2))$ .

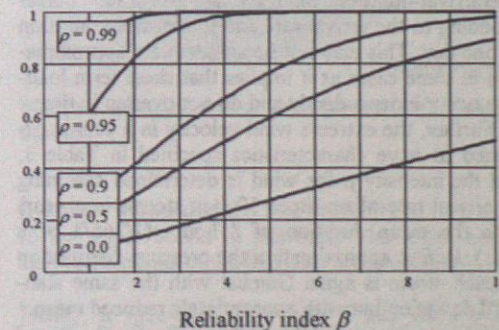


Figure 3. Ratio of improved and asymptotic domain probabilities over reliability index  $\beta$  for various  $\rho$ .



A lower bound for the failure probability can also be given as the point-in-time failure probability

$$P_f(t_1, t_2) \geq \max_{\tau} \left\{ \sum_{i=1}^n P_i^i P_{f,i}^i(\tau) + \sum_{i=1}^n \sum_{j=i+1}^n P_2^{i,j} P_{f,2}^{i,j}(\tau) + \dots + P_n P_{f,n}^{1,2,\dots,n}(\tau) \right\} \geq \sum_{i=1}^n P_i^i P_{f,i}^i(t_i^*) + \sum_{i=1}^n \sum_{j=i+1}^n P_2^{i,j} P_{f,2}^{i,j}(t_{i,j}^*) + \dots + P_n P_{f,n}^{1,2,\dots,n}(t_{1,2,\dots,n}^*) \quad (24)$$

where, due to stationarity,  $t_{i(k)}^*$  can be taken as  $t_i$ . Formulae (23) to (24) are given here for the first time whereas formula (22) can already be in Shinozuka (1981) in different notation.

The following parameters are considered here:

- ♦ long term imposed load:  $\lambda = \kappa = 1/7$  [1/year],  $\rho = 3$ ,
  - ♦ short term imposed loads:  $\lambda = \kappa = n$  [1/year],  $\rho = n/365$ ,
  - ♦ wind load:  $\lambda = \kappa = 10$  [year],  $\rho = 10/365/3 = 0.009$ ,
- where  $n$  denotes number of storeys (equal to 1, 3, 10 and 20 - see Table 1).

Thus, the long term imposed load is assumed to change simultaneously in all storeys of a building in average every 7 years and its mean duration is 5.25 years. The influence of the parameter  $\rho$  of long term imposed load is not very important. The assumption of  $\rho = 3$  may be a little unrealistic as it still produces off-times of 1.75 years.

The short term load is assumed to be present in each storey of the building during one day in a year. Approximately (neglecting possible dependence and time overlapping of the load in different storeys), the short term load is considered as one process with the number of magnitude changes in a year  $\kappa = n$  and the interarrival-duration intensity  $\rho = n/365$  corresponding to the arrival rate and to the mean duration of one day. This may not be an adequate approximation in some cases as it implies that short term loadings occur independently and do not overlap in time.

Further, the extreme wind velocity in a year is assumed to have characteristics specified in Table 3, and the intensity  $\rho$  for wind is determined assuming an arrival rate of on-times 10 (ten storms in a year) with the mean duration of 8 hours ( $1/365/3$  of a year). In first approximation the pressure distribution in each storm is again Gumbel with the same standard deviation but with appropriately reduced mean.

The resulting reliability indexes  $\beta_0$ ,  $\beta_1$ ,  $\beta_2$ ,  $\beta_3$  and  $\beta_4$  for the 12 study cases are given in Table 4, assuming a life time of 50 years.

In all cases rapid convergence to a unique  $\beta$ -point was observed. The results given correspond to SORM. A few results have been checked by crude Monte Carlo integration or by importance sampling showing errors of at most a few percent. The numerical effort for the five types of analysis was about 1:3:1:6:7 with the unit being approximately 500 state function calls.

Table 4. Reliability indices for built-in column.

Study case	Reinf. ratio [%] $A_s/bh$	Reliability analysis				
		Time invar.	Turkstra rule	Jump proc.	Intermitt.	Intermitt.
		$\beta_0$	$\beta_1$	$\beta_2$	$\beta_3$	$\beta_4$
1	1.02	5.34	5.31	5.30	5.98	-
2	1.11	5.95	6.03	6.03	6.69	-
3	2.05	5.07	5.05	5.04	5.76	-
4	0.78	4.06	3.99	3.97	4.83	-
5	2.23	5.27	5.34	5.33	6.08	-
6	1.52	6.08	6.07	6.07	6.69	-
7	1.62	5.38	5.35	5.35	6.04	-
8	0.98	5.57	5.55	5.54	6.21	-
9	0.39	2.85	2.70	2.60	3.75	3.75
10	0.86	3.69	3.59	3.55	4.48	4.29
11	1.37	5.59*	5.69*	5.64*	6.32*	-
12	1.76	5.62*	5.68*	5.66	6.35	-

The reliability indices are within a wide range from 2.6 to 6.7 (similarly as in previous studies by Vrouwenvelder & Siemens (1987) and Holický & Vrouwenvelder (1997)). When judging the results one has to know whether long and short term occupancy loading are "loading" or "resistance" variables. These loads can have a stabilizing effect. Whenever one of them is a "loading" variables this is indicated by the superscript "a".

The time invariant calculations for  $\beta_0$  and  $\beta_1$  are approximate in the sense that the extreme value distributions described by gamma-distributions are replaced by Gumbel distributions with the same mean and variance.  $\beta_0$  is in general far too inaccurate and should be considered as a first approximation only. In all study cases  $\beta_0$ ,  $\beta_1$ , and  $\beta_2$  (no intermittencies) differ insignificantly and all are always smaller than  $\beta_3$  (intermittencies). This is due to dominating wind load. However,  $\beta_2$  and  $\beta_3$  can generally be markedly different from  $\beta_0$  and  $\beta_1$ . Note, that in study cases 9 and 10 the reliability indices  $\beta_0$ ,  $\beta_1$  and  $\beta_2$  (in study case 9 also  $\beta_3$  and  $\beta_4$ ) are less than 3.8, which is the value recommended in ENV1991-1: Eurocode 1 (1994).

Figure 4 shows the reliability indices  $\beta_2$  and  $\beta_3$  versus reinforcement ratio. It appears that for reinforcement ratio less than 1%, both reliability indices (and as follows from Table 4 that also  $\beta_0$ ,  $\beta_1$  and  $\beta_2$ ) considerably decreases and may be unsatisfactory.

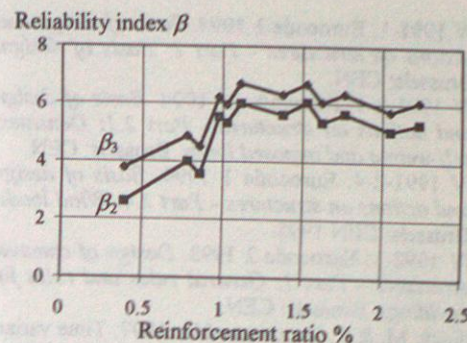


Figure 4. Reliability indexes  $\beta_2 < \beta_3$  versus reinforcement ratio.

## 6 STUDY CASE 10

The assumption that long term imposed load changes in all storeys of a building (in average every 7 years) simultaneously may not be realistic. In the following this hypothesis is changed to another extreme assumption. The long term imposed loads in particular storeys are mutually independent. So, in study case 10, having three storeys, three independent long term imposed loads are considered together with the short term imposed load and wind load. Thus, in total  $2^3 - 1 = 31$  different "load combination cases" have to be studied, i.e. any one load "on" and all others "off", any two loads "on" and the others "off", ..., all 5 different loads "on". The probability of failure for all loads being "off" is neglected. For the fourth type of analysis there are only 7 "load combinations". For convenience of presentation only these combinations are shown in the table 5. It is seen that larger contributions to the total failure probability come only when wind is active. In this case occupancy loading has a stabilizing effect. The computations with simultaneous jumps ( $\beta_3$ ) result in larger  $\beta$ -values than with independent jumps. Detailed analysis shows that this is due to a larger total jump rate but also due to the correlation between loads for simultaneous jumps.

Upper and lower (approximate) bounds of the failure probability  $P_f$ , defined by (22) and (24), are indicated in Figure 5 in a parameter study concerning interarrival-duration intensity  $\rho$  of wind load within a range from 0.005 to 0.015 (the value assumed in previous calculation was 0.009) implying a change in the duration of the storms from roughly 50% (4 hours) to 150% (12 hours).

Figure 5 shows that the upper and lower bounds are very close to each other. For example, for  $\rho = 0.009$  (used in previous analysis) it holds

$$4.48 < \beta < 4.63 \quad (25)$$

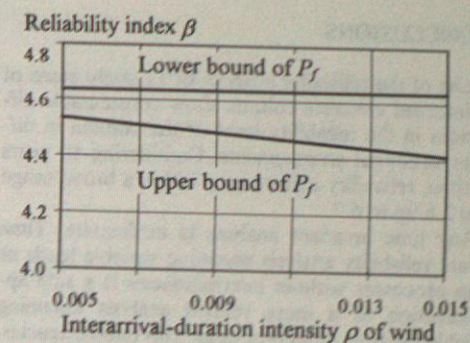


Figure 5. Lower and upper bounds of failure probability  $P_f$  for study case 10 ( $\beta_3$ ).

Table 5. Load combinations for study case 10 for  $\beta_3$  (sequence is wind, ps, pl).

Case	Active load	$P_f\{k\}$	$p\{k\}$
1	xxx	3.93E-07	5.45E-05
2	xoo	1.09E-05	2.21E-03
3	oxo	2.44E-23	2.02E-03
4	oox	3.27E-23	7.37E-01
5	xxo	5.04E-07	1.82E-05
6	xox	1.70E-06	6.64E-03
7	oxx	2.72E-23	6.06E-03

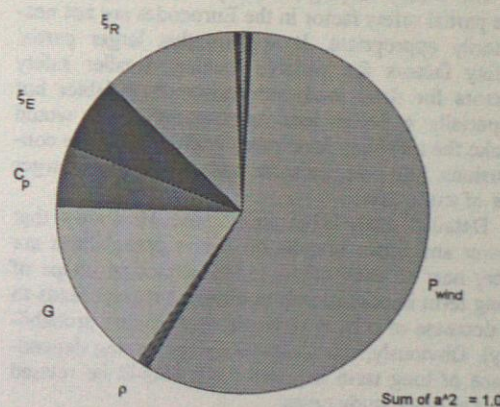


Figure 6.  $\alpha$ -values of variables ( $\alpha_{pL1} = \alpha_{pL2} = \alpha_{pL3} = 0.0109$ ).

It can be shown that the true reliability is very close to the upper bound in this case. The lower bound also corresponds to the initial failure probability.

Finally, it follows from Figure 6 that wind is the dominating variable and live loads are "resistance" variables. The corresponding  $\alpha$ -values are positive but small.



## 7 CONCLUSIONS

Results of the reliability analysis of 12 study cases of a reinforced concrete column show considerable differences in the reliability level of the column in different structural arrangements. Considering 50 years life time, reliability indices vary within a broad range from 2.6 up to 6.7.

Any time invariant analysis is inadequate. Time variant reliability analysis assuming variable loads as jump processes without intermittencies is a safe approximation to a more refined analysis assuming variable loads as jump processes with intermittencies. The latter type of analysis may considerably improve reliability assessment of structures, particularly those exposed to variable actions with relatively low inter-arrival-duration intensity. It is further emphasised that the effect of stabilising occupancy loading can only be taken into account consistently by assuming jump processes with intermittencies.

It appears that the reliability level of reinforced concrete columns designed according to Eurocodes may in some cases be insufficient. In other cases, depending on actual structural arrangements, it may become uneconomical to follow the codes. The obtained reliability indices  $\beta$  seem to be dependent on reinforcement ratio. In particular, lower  $\beta$ -values were obtained for reinforcement ratios below 1%.

The widely varying reliability indices suggest that the partial safety factor in the Eurocodes are not necessarily appropriate. It appears that larger partial safety factors for variable loading, smaller safety factors for dead load and resistance variables but especially a better load combination rule would make the reliability level more uniform. Definite conclusions, however, must be based on a much larger number of study cases.

Detailed analysis of study case 10 shows that lower and upper bounds on failure probabilities are very near to each other. Independence of jumps of long term imposed loads in different storeys leads to a decrease of  $\beta$  by 0.15 (increase of failure probability). Obviously, the assumptions concerning dependence of long term imposed loads should be revised also in other study cases.

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**KEYWORDS:** Component reliability, concrete column, variable load, jump process, intermittencies.



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