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Size effects in fibre reinforced laminated structures

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ABSTRACT: A weakest link concept is applied to failure of fibre reinforced laminates under combined stress. A new interfibre failure criterion is used. Implications for appropriate tests for the determination of the strength parameters are given. Consistency of the theory is illustrated at an example.

1 INTRODUCTION

Since the famous paper by Weibull in 1939 (Weibull 1939) the variability of strength of brittle materials and its dependence on the stresses and the size of a structure is described by the Weibull distribution. Later this distribution was strongly supported by asymptotic arguments in extreme value theory. Freudenthal (1968) associated the volume or surface elements in Weibull's theory with defects and later Batdorf and Crose (1974), Matsuo (1981) and others associated these defects with plain cracks having strongest stress singularities among all other defects such as cavities or inclusions. If the defects are randomly spaced and small compared to the volume of the structure and if there is no interaction between cracks Weibull's arguments are in fact no more needed to arrive at his distribution. It is only necessary to assume that the largest and/or most stressed defect is responsible for failure of the structure. Assume that the stress state in any point $\boldsymbol{\xi} = (\xi_1, \xi_2, \xi_3)$ of a structure can be expressed using a deterministic reference stress σ_0

$$\sigma(\xi_1, \xi_2, \xi_3) = \sigma_0 \cdot f(\xi_1, \xi_2, \xi_3) = \sigma_0 \cdot f(\boldsymbol{\xi}) \quad (1)$$

Thus the condition for fracture is $\sigma_0 \cdot f(\boldsymbol{\xi}) = r \cdot f(\boldsymbol{\xi})$. Furthermore assume that the distribution of defects in a solid can be modeled by a Poisson-field with intensity $\lambda(\boldsymbol{\xi})$. If then the probability of failure of a defect is given by a homogeneous distribution function $F_X(r)$, the probability of failure of a solid under a constant stress state is given by

$$\begin{aligned} P_f &= F_R(r) = 1 - \sum_{k=1}^{\infty} P\left(\bigcap_{j=1}^{k} \{X_k > r\}\right) \cdot p(k) \\ &= 1 - \exp\left[-\int_V \lambda(\boldsymbol{\xi})\; F_X(r)\, dV\right] \end{aligned} \quad (2)$$

where V is the volume and $p(k)$ is the probability that exactly k defects exist in the solid. For a homogeneous defect generating process (2) simplifies to

$$F_R(r) = 1 - \exp\left[-\lambda V F_X(r)\right] \quad (3)$$

This defect model is closely related to the weakest-link concept developed by Weibull, as can easily be shown by replacing $\lambda = \frac{1}{V_0}$ and $F_X(r) \approx c\,(r - r_u)^m$ in (3).

For the defect being a penny shaped crack with critical failure stress σ_{cr}, a lowest limit r_u for σ_{cr} and an arbitrary orientation, only a certain fraction of all cracks in a solid under polyaxial non-uniform stress state Σ can fail. Introducing an equivalent stress σ_{eq} to take both normal and shear stresses on the crack plane into account, the number of cracks with critical stress σ_{cr} less than or equal to σ_{eq} is assumed to be a function of $(\sigma_{cr} - r_u)$, namely

$$N(\sigma_{eq} - r_u) = \left(\frac{\sigma_{eq} - r_u}{r_c}\right)^m \quad (4)$$

Thus the probability of failure of a crack under an arbitrary stress state Σ is given by

$$F_X(\Sigma) = \frac{1}{2\pi}\int_0^{\pi}\int_0^{\pi} N(\sigma_{eq} - r_u) \times H(\sigma_{eq} - r_u)\, \sin\varphi\, d\varphi\, d\theta \quad (5)$$

Herein φ and θ define the direction of the normal on the crack plane, σ_{eq} is a function of the remote stress Σ, and Heaviside's step function $H(\sigma_{eq} - r_u)$ assures that the integration is done only in that part of the volume where $\sigma_{eq} > r_u$. In a similar way Matsuo also included surface defects (Matsuo 1981). Assuming equality of the energy release rates for cracks stressed only perpendicular to their plane and for cracks under more complicated stresses the equivalent stress can, for example, be given as

$$\begin{aligned} \sigma_{eq} &= \sqrt{\left(\sigma_n{}^2 + \frac{4}{(2-\nu)^2}\tau_t{}^2\right)} \\ \sigma_n &= \sigma_1 n_1{}^2 + \sigma_2 n_2{}^2 + \sigma_3 n_3{}^2 \\ \tau_t{}^2 &= \sigma_1{}^2 n_1{}^2 + \sigma_2{}^2 n_2{}^2 + \sigma_3{}^2 n_3{}^2 - \\ & \quad (\sigma_1 n_1{}^2 + \sigma_2 n_2{}^2 + \sigma_3 n_3{}^2)^2 \\ n_1 &= \sin\varphi \cdot \cos\theta \\ n_2 &= \sin\varphi \cdot \sin\theta \\ n_3 &= \cos\varphi \end{aligned} \tag{6}$$

with σ_1, σ_2 and σ_3 the principal stresses and ν Poisson's ratio. Applying the above defect argument yields

$$F_R(\Sigma) = 1 - \exp\left[-\frac{1}{2\pi}\int_V \int_0^\pi \int_0^\pi \left(\frac{\sigma_{eq} - r_u}{r_c}\right)^m \times H(\sigma_{eq} - r_u)\sin\varphi\, d\varphi\, d\theta\, dV\right] \tag{7}$$

It is seen that expression (5) corresponds to the distribution function $F_X(r)$ in (2). More in general, for an inhomogeneous defect generating process and inhomogeneous stress and/or strength field (2) can be written as

$$F_R(\Sigma) = 1 - \exp\left[-\int_V \lambda(\boldsymbol{\xi})\, p_f(\boldsymbol{\xi})\, dV\right] \tag{8}$$

with $p_f = F_X(\Sigma(\boldsymbol{\xi}))$. For a homogeneous defect generating process the intensity $\lambda(\boldsymbol{\xi})$ can be replaced by a reference intensity $\lambda = \frac{1}{V_0}$, where V_0 is the volume of the test specimen used for the determination of the strength parameters. The strength distribution of a solid with volume V then is

$$\begin{aligned} F_R(\Sigma) &= 1 - \exp\left[-\int_V \frac{1}{V_0} \cdot p_f(\boldsymbol{\xi})\, dV\right] \\ &= 1 - \exp\left[-\frac{1}{V_0}\int_V \Phi(-\beta(\boldsymbol{\xi}))\, dV\right] \end{aligned} \tag{9}$$

where $\Phi(x)$ is the standard normal distribution function and β the (local) safety index. It is important to note that in practice the intensity parameter of the Poisson field is indirectly given by the reference volume of the tests used to determine the strength parameters. This last formulation is also suitable for any other local non-interactive failure criterion leading to brittle failure of a structural component.

2 ANISOTROPIC LOCAL STRENGTH CRITERIA FOR FIBER REINFORCED PLASTICS

High strength fiber reinforced plastic (FRP) laminates show pronounced brittle fracture behavior, which suggests to apply similar considerations. Especially unidirectional layers, which are used to build up a laminate, show sudden rupture and linear behavior until failure occurs (see, e.g., Adams 1986, Flaggs and Kural 1981, Fukunaga et al. 1984, Hahn et al. 1983, Madhukar and Drzal 1991, Madhukar and Drzal 1992, Puck and Schürmann 1982, Soutis 1991). Some test series using test specimen of different size clearly show a significant influence of the size on the specimen strength. This is definitely true for fiber failures under tension. Interestingly, theoretical considerations by Harlow and Phoenix (1978) explain this as a consequence of fiber defects leading to stress concentrations but additionally to the presence of a large number of those defect generated stress concentrations in series so that the Weibull distribution fits well the observed strength of test specimens of different size. Also, transverse tensile strength shows a pronounced size effect which can be explained by the existence of fiber parallel cracks which form from defects and eventually become unstable (see especially Hahn et al. 1982, Puck and Schürmann 1982). For shear failure and for failure under compression in fiber direction and in transverse direction a size effect can also be identified but the experimental results are somewhat less conclusive (see, e.g., Puck and Schürmann). A size effect appears to be present but less strong. First of all, it is still unclear whether this is a consequence of the special experimental settings used so far or of the actual rupture process. From a micromechanical point of view dispersed damage should develop but failure is due to a macro crack formed in the most stressed and damaged zone. Once the macro crack is formed the rupture behavior is perfectly brittle due to the stress concentrations at the crack tip as indicated by high speed video records. Even if the initial damage evolution would suggest to apply one of the statistical models for parallel systems, for example the models initially developed by Daniels (1989), one must hypothesize some "weakest link" mechanism for the final rupture phase in order to explain the experimental results. At least it will produce a lower reliability bound. This encourages the use of (9) as a strength model also for FRP components under general stress. It should be pointed out that the assumption of a Poisson field is physically correct only, if the occurrence of local failure causes structural failure. The exact knowledge about the mechanisms leading to local failure is not necessary. If the distribution function of local strength is known from experiment, (9) can be used correctly. Furthermore, it can be shown that in multilayered FRP's potential redundancy is negligible for optimized structures in accordance with theory (Gollwitzer and Rackwitz 1990, Rackwitz and Cuntze 1987). A weakest link mechanism and thus a distribution function of the type (8) may, in fact, also be expected if damage, for example microcracks, develops nonlocally and essentially non-interactive. For moderate transverse compression and shear such microcracks would develop between the fibers somewhat inclined to the fiber direction. Each of them will therefore be blocked. For predominantly compression microcracks will also develop. Fiber bridging indicates another type of cracks. But also those microcracks will be blocked. Then, the randomly largest (or most compact) group of microcracks will initiate the macrocrack which typically is of different nature (parallel to the fibers). Clearly, the rupture mechanism just described does not directly correspond to Weibull's original theory nor does it directly correspond to the Poisson assumption of initial or growing defects but it is certainly similar as statistical arguments are concerned. Its different nature may be an explanation for a possibly smaller size effect in combined transverse compression and shear. Any smaller size effect implies that the reference scale is larger than for predominantly transverse tensile failure. But further carefully made experiments are needed in order to judge the validity of the above arguments and the differences in size effect depending on the type of loading. Only if there is distinct localization of damage eq. (8) may no more suitable but there is not yet sufficient experimental evidence for damage localization for the type of failures considered herein.

A fracture criterion used for a reliability analysis of FRP must take both fiber failure (FF) and interfiber failure (IFF) into account. A suitable fracture criterion for UD-FRP has been proposed by Hashin (1980) and recently improved by Puck (1992). FF and IFF are treated by two separate criteria, and IFF is assumed to take place in a crack plane parallel to the fibers, where fracture of the matrix material is most likely to occur. The FF criterion is

$$\begin{aligned} \left(\frac{\sigma_{11}}{R_{11}^t}\right)^2 &\le 1 \quad \text{for } \sigma_{11} \ge 0 \\ \left(\frac{\sigma_{11}}{R_{11}^c}\right)^2 &\le 1 \quad \text{for } \sigma_{11} \le 0 \end{aligned} \tag{10}$$

For IFF the three submodes, i.e. transverse tensile failure, shear failure parallel to the fibers and shear failure in transverse direction, are combined in

$$\left(\frac{\max\{\sigma_{nn}, 0\}}{R_{nn}^t}\right)^2 + \left(\frac{\tau_{n1}}{R_{n1} - \mu_{n1}\cdot\sigma_{nn}}\right)^2 + \left(\frac{\tau_{nt}}{R_{nt} - \mu_{nt}\cdot\sigma_{nn}}\right)^2 \le 1 \tag{11}$$

wherein the stresses σ_{nn}, τ_{n1} and τ_{nt} are acting on a plane under angle θ (see figure 1), determined by

$$\begin{aligned} \sigma_{nn}{}^2 &= \left[\frac{1}{2}(\sigma_{22} + \sigma_{33}) + \frac{1}{2}(\sigma_{22} - \sigma_{33})\cos 2\theta + \tau_{23}\cdot\sin 2\theta\right]^2 \\ \tau_{nt}{}^2 &= \left[-\frac{1}{2}(\sigma_{22} - \sigma_{33})\sin 2\theta + \tau_{23}\cdot\cos 2\theta\right]^2 \\ \tau_{n1}{}^2 &= \frac{1}{2}(\tau_{12}{}^2 + \tau_{13}{}^2) + \frac{1}{2}(\tau_{12}{}^2 - \tau_{13}{}^2)\cos 2\theta + \tau_{12}\cdot\tau_{13}\cdot\sin 2\theta \end{aligned} \tag{12}$$

The angle θ must be found by maximizing the left-hand side of (11). The material strength parameters in (10) and (11) are R_{11}^t and R_{11}^c for tensile and compressive strength in fiber direction, R_{nn}^t for tensile strength in transverse direction, R_{n1} and R_{nt} for the shear strength parallel to the fibers and in transverse direction, respectively. Note that R_{nt} cannot be determined by a τ_{23}-test. The parameters μ_{n1} and μ_{nt} have only in part a mechanical interpretation. Their inclusion in (11) leads to a better fit to experimental data.

Using this set of criteria, it is possible to identify the failure mode. As the stress state in laminated structures usually is calculated using the Finite Element Method (FEM), the structures' probability of failure $P_{f,S}$ can be calculated by regarding every

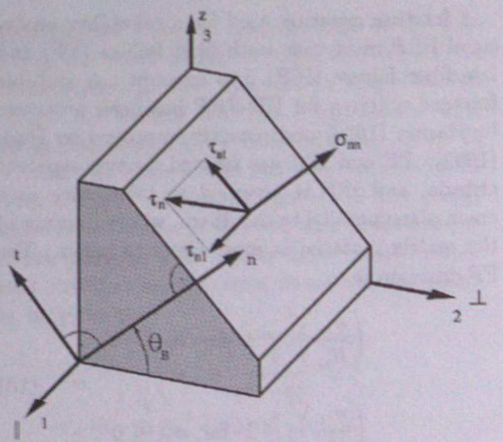

Figure 1: Definition of the coordinate systems for a lamina. The system 1-2-3 points with axis 1 (or ||) in fibre direction, with axis 2 (or ⊥) in transverse direction and with axis 3 (or z) through the thickness of the laminate.

element of the FEM mesh as a failure element of a series system, composed of the two failure modes in each layer of the laminate. The event F_i is defined as failure of element i, i.e. $P(F_i) = P_{f,i}$. If $P(F_i)$ is small for all elements and if the events are only weakly dependent, Ditlevsen bounds can be used to calculate $P_{f,S}$ (Ditlevsen 1979).

$$P_{f,S} = P\left(\bigcup_{i=1}^{n} F_i\right) = \begin{cases} \leq P(F_1) + \sum_{i=2}^{n} \{P(F_i) - \max_{j<i}\{P(F_i \cap F_j)\}\} \\ \geq P(F_1) + \sum_{i=2}^{n} \max\left\{0, P(F_i) - \sum_{j<i}^{n} P(F_i \cap F_j)\right\} \end{cases} \tag{13}$$

Herein the number n of failure events F_i depends on the number of elements in the FEM-mesh. Therefore $P_{f,i}$ must be calculated taking the volume V_i of element i into account as $P_{f,S}$ must be independent from the FEM-meshing. The probabilities $P_{f,i}$ can conveniently be determined by FORM/SORM methods (Hohenbichler et al. 1987). From (10) and (11) it follows that the corresponding limit states are given for FF by

$$g(\mathbf{X}) = \left(\frac{\sigma_{11}}{R_{11}^{t}}\right)^2 - 1 = 0$$

or

$$g(\mathbf{X}) = \left(\frac{\sigma_{11}}{R_{11}^{c}}\right)^2 - 1 = 0 \tag{14}$$

and for interfiber failure by

$$g(\mathbf{X}) = \left(\frac{\max\{\sigma_{nn}, 0\}}{R_{nn}^{t}}\right)^2 + \left(\frac{\tau_{n1}}{R_{n1} - \mu_{n1} \cdot \sigma_{nn}}\right)^2 + \left(\frac{\tau_{nt}}{R_{nt} - \mu_{nt} \cdot \sigma_{nn}}\right)^2 - 1 = 0 \tag{15}$$

and were the strength parameters must be related to the reference volume V_0 in (9). If the stress state is not constant over the finite element, the variations of stresses must be taken into account appropriately as $\Phi(-\beta)$ increases exponentially with increasing stress (see figure 2).

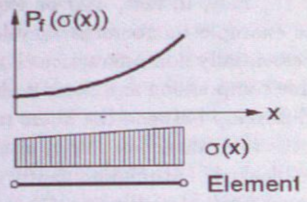

Figure 2: Exponential growth of $P_f = \Phi(-\beta)$ for a linearly increasing stress.

If a constant stress state — corresponding, for example, to the center of gravity of the element — is used the element mesh must be refined until the error due to the assumption of constant stress becomes sufficiently small.

3 DISCUSSION AND DETERMINATION OF PARAMETERS

The basic assumption of the theory outlined above is the existence of defects which are the nuclei of cracks leading to fracture, be it due to transverse tension, shear or compression. There are several implications of the theory. It is not important to identify the exact nature of the crack initiating defect. If it is the same defect leading to failure in the three submodes of IFF, it follows that the parameters in (14) and (15) should be highly correlated if not functionally dependent. Unfortunately direct experimental verification of this hypothesis appears impossible as specimens cannot be tested twice. A second implication arises from the fact that the strength of an embedded layer is dominated by internal defects, whereas the strength of the usual test specimen is most likely dominated by surface defects. It follows that the test specimen must be such that the effect of surface defects is excluded, or in other words test specimen must be suitable laminates. Therefore, most of the available test results are inadequate. A third implication is that the size effect should be the same for all three IFF submodes.

The assumption that crack generating defects determine the strength of UD-FRP layers is not new. For transverse tensile stresses and shear stresses Hahn, Erikson and Tsai (1982) not only hypothesized fiber parallel cracks as the strength determining defects. They could also show that by using the energy release rate approach a failure criterion similar to (11) would result. They further concluded that the strength distribution should be a Weibull distribution whose shape parameter is the same for each of the IFF submodes, using the following distribution function for each of the submodes:

$$F_R(r) = 1 - \exp\left\{-\left(\frac{\sigma_{eq}}{\sigma_{cr}}\right)^{\alpha}\right\} \tag{16}$$

However, their experiments with a single artificial defect in an UD-layer for determining the critical energy release rates k_{Ic}, k_{IIc} and k_{IIIc} were contradictory. The parameter α was different for every critical energy release rate. This contradiction can be resolved by introducing a lower strength limit τ, which can be different for each of the IFF submodes. If, for example, a three-parameter Weibull distribution function

$$F_X(x) = 1 - \exp\left[-\left(\frac{x-\tau}{w-\tau}\right)^{k}\right] \tag{17}$$

were used, the scaling of the measured strength S_1 from a test specimen with volume V_1 to the strength S_0 of the reference volume V_0 must be done by

$$\frac{S_0 - \tau}{S_1 - \tau} = \left(\frac{V_1}{V_0}\right)^{\frac{1}{k}}$$

or

$$S_0 = \left(\frac{V_1}{V_0}\right)^{\frac{1}{k}} \cdot (S_1 - \tau) + \tau \tag{18}$$

As an illustration table 1 shows the influence of τ on test results for the fiber parallel tensile strength of a FRP. For a mean value $\mu = 1500$ and C.o.V. = 0.06 quite different sets of parameters are estimated for arbitrarily selected values of τ.

Table 1: Influence of the shift parameter τ on the scale parameter k of the three parameter Weibull distribution function

τ	0	500	750	1000
w	1540.0	1539.4	1538.7	1537.0
k	20.7	13.58	10.04	6.51

Further, if the hypothesis of a single crack is true and therefore there must be a common shape parameter in the Weibull distributions for the strength parameters of the different IFF submodes, the different C.o.V. observed for these modes can only be explained by the existence of lower limits τ for the strength parameters, as $C.o.V. = \sigma/(\mu - \tau)$.

Unfortunately, the shift parameter τ can be accurately estimated only from rather large samples. Even then considerable statistical uncertainties may remain making further statistical reasoning difficult. Alternatively, the following method is proposed. The scale parameter k is estimated from two types of tests with different specimen sizes V_1 and V_2 by

$$k = \frac{\ln V_2 - \ln V_1}{\ln(S_1 - \tau) - \ln(S_2 - \tau)} \tag{19}$$

As the parameter τ influences k, a third type of test with different specimen volume V_3 must be performed. Then the parameter τ must be selected such that k has the same value for any pair of V_1, V_2 and V_3 applying (19). If then for a fourth type of test with specimen volume V_4 the strength can be predicted correctly using (18) and the estimated parameters k and τ one can conclude that the material strength can be modeled by the Weibull distribution function correctly.

This method certainly is expensive but it seems to be the only way to verify the applicability of the Weibull theory to a material by experiment. The proposed estimation procedure must be performed for every resistance quantity required by the applied fracture criterion. The results then can be used to verify the interpretation of the defect as a single crack. On the other hand this or a similar series of experiments appears to be the only way to falsify the above theory. It is also the only way

Table 2: Mechanical properties of CFRP used for the example (left table) and set of basic variables for the reliability analysis of the strut (right table).

Variable	Mean value	C.o.V.
E_{11}	170000	-
E_{22}	9000	-
ν_{12}	0.28	-
$\alpha_{T,1}$	$-3.0 \cdot 10^{-6}$	-
$\alpha_{T,2}$	$4.0 \cdot 10^{-5}$	-
R_{11}^{t}	2900	0.04
R_{11}^{c}	1620	0.08
R_{22}^{t}	60	0.10
R_{22}^{c}	165	0.09
R_{12}	86	0.04

Variable	Mean value	C.o.V.	model
Tension	209000	0.1	Lognormal
Bending	3600	0.1	Lognormal
Temperature	-100	0.05	Lognormal
R_{11}^{t}	2900	0.04	Weibull
R_{11}^{c}	1620	0.08	Weibull
R_{nn}^{t}	60	0.10	Weibull
R_{n1}	86	0.04	Weibull
R_{nt}	71	0.1	Weibull
μ_{n1}	0.15	0.1	Lognormal
μ_{nt}	0.15	0.1	Lognormal

Table 3: Probability of failure for one layer boundary, including both FF and IFF.

Meshsize (FEM)	Scaling	Failure components	range of β	P_f
208	yes	3328	$2.81 \leq \beta \leq 2.81$	≤ 0.0025
416	yes	6656	$2.81 \leq \beta \leq 2.81$	≤ 0.0025
208	no	3328	$-0.22 \leq \beta \leq 0.52$	≤ 0.5854
416	no	6656	$-36.62 \leq \beta \leq 0.40$	≤ 1.0000

Table 4: Results for the entire strut, including both FF and IFF.

Meshsize (FEA)	208	416
Failure components	19998	39936
Componential β	$4.34 \leq \beta \leq 10.96$	$4.40 \leq \beta \leq 11.13$
System bounds (all comp.)	$2.33 \leq \beta \leq 2.34$	$2.34 \leq \beta \leq 2.34$
System prob. of failure	$0.0096 \leq P_f \leq 0.0098$	$0.0096 \leq P_f \leq 0.0096$

to show that there are different shape parameters in an appropriately modified theory.

As test results produced by the proposed method are not yet available, an individual scaling of the test data can tentatively be employed as an approximation of (9). For the special case of a small stress gradient over the structure the size effect can then be included in a different way. Instead of scaling the individual strength values to a reference volume V_0 and then integrating (9) over the volume V of an element, the scaling for each strength parameter is performed from test specimen size to the volume V using (18). By this method the interaction of σ_{nn} and the τ_{n1} or τ_{nt} possibly is not scaled correctly.

For fiber failure $P_{f,i}$ is then given by

$$F_R(\Sigma) = P\left[\left(\frac{\sigma_{11}}{R_{11}^{t}{}^{*}}\right)^2 \leq 1\right]$$

or (20)

$$F_R(\Sigma) = P\left[\left(\frac{\sigma_{11}}{R_{11}^{c}{}^{*}}\right)^2 \leq 1\right]$$

and for interfiber failure by

$$F_R(\Sigma) = P\left[\left(\frac{\max\{\sigma_{nn},0\}}{R_{nn}^{t}{}^{*}}\right)^2 + \left(\frac{\tau_{n1}}{R_{n1}{}^{*} - \mu_{n1}\cdot\sigma_{nn}}\right)^2 + \left(\frac{\tau_{nt}}{R_{nt}{}^{*} - \mu_{nt}\cdot\sigma_{nn}}\right)^2 \leq 1\right] \quad (21)$$

where Σ is the remote stress and the superscript $*$ indicates individually scaled resistances.

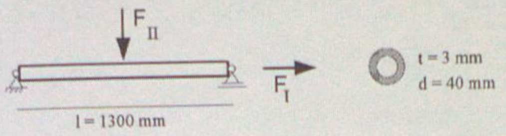

Figure 3: Example: a strut under longitudinal tension F_I, a bending force F_{II} in the middle and constant temperature $\Delta T = -100°C$.

4 ANALYSIS OF A STRUT UNDER RANDOM LOADING

The method as proposed above is illustrated at a strut under non-constant stress state. The geometry and the loads are shown in figure 3, the stochastic model and the material properties of the carbon fiber reinforced plastic (CFRP) are given in table 2. The stacking sequence of the laminate is $(90°/0°/0°/90°)$ with $0°$ pointing in the longitudinal direction of the strut, the thickness of the layers is $t_i = (0.15/1.35/1.35/0.15)$ (all in [mm]).

One half of the strut is divided into 16 elements in radial direction and into 13 elements in longitudinal direction (208 in total). As a second case, the strut is divided into 26 elements in longitudinal direction (416 in total). To demonstrate the effect of the Weibull-size-effect the structure is analyzed with the two different meshes. The reliability analysis has to be performed at each boundary of a layer for every finite element and both FF and IFF must be considered. As an example the influence of the Weibull size effect on the probability of failure is demonstrated for one layer boundary. The results for the outer boundary of the 0°-layer are presented in table 3.

The first column shows the number of elements of the FEM-model for the strut, the second indicates whether the Weibull-scaling of the strength variables has been taken into account. The third column contains the number of failure components for the series system, the fourth shows the bounds of β, and the last column gives the upper bound of system-P_f. It should be mentioned that the intersection probabilities in (13) of at most the first 800 larger probability events had to be computed in order to obtain the rather narrow reliability bounds. It can be seen, that ignoring the scaling of resistances produces severe errors.

To verify the assumption of constant stress in the elements the probability of failure has been evaluated for the entire strut. For 208 (416) elements having four β-points each (due to the geometrical model of the shell-plate-element), eight layer boundaries and two limit state functions (20) and (21) the number of failure components in the series system is 19968 (39936). The results are given in table 4.

5 SUMMARY AND CONCLUSION

It has been shown that the size effect of brittle materials can easily be included in structural reliability analysis by the use of a weakest link concept. The theory of Poissonian non-interacting defects is reviewed. Links to the theory of Weibull are established. Applications to high strength fiber reinforced plastics are then discussed. The implications of the assumption that a single defect is responsible for all types of fiber and interfiber failure are discussed in detail. In particular, a suitable fracture criterion for fiber reinforced plastics requires new types of tests for the determination of material strength parameters. Some alternative concepts are outlined briefly. Application of the theory in FEM-calculations is discussed and the feasibility to account for the size effect over the full structure is demonstrated using modern FORM methods.

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