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On the combination of non-stationary rectangular wave renewal processes *

R. Rackwitz

Technische Universität München, Laboratorium für den Konstruktiven Ingenieurbau, Arcisstr. 21, D-8000 München 2, Germany

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On the combination of non-stationary rectangular wave renewal processes *

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Abstract. Stationary rectangular wave renewal processes are frequently used to model structural loads. Their combination can be performed by making use of the outcrossing approach and FORM/SORM concepts. The existing theory is extended to non-stationary problems which occur in transient loading situations and whenever structural resistances experience some deterioration in time. Special computation schemes are proposed for the case when simple random variables are present including an importance sampling scheme.

Key words: structural reliability; rectangular wave processes; load combination; second order reliability method; importance sampling

1. Introduction

Rectangular wave processes are frequently used to model the time variations of occupancy loading. Such processes can also approximate other loading phenomena and, in particular, may be used to model the main characteristics in socalled missions, e.g. the journey of a ship between two places, the different sea states it experiences during the journey or the loading environment of a processing plant between the shut down periods. One of the earliest models for loads on structures is the so-called Ferry Borges/Castanheta model [1] being a random, independent and stationary sequence of rectangular waves with constant duration. The same authors proposed a concept to combine such sequences. Nonlinear combination of several such sequences in the context of FORM has been the subject of several studies (see, for example, [2], where the socalled maximum approach is used). A generalization of this model is the stationary, multivariate rectangular wave renewal process. Its combination has also been studied by several authors. Only the outcrossing approach has received more widespread attention. In Breitung/Rackwitz [3] a FORM solution is presented. In a follow up study a few considerations are given to auto- and crosswise correlated Gaussian rectangular wave processes [4]. Interesting and practically useful generalizations of the simple rectangular wave model have

* Discussion is open until June 1994 (please submit your discussion paper to the Editor, Ross B. Corotis).

been proposed by Wen [5] and others but practical computations of the outcrossing rates still had to be based on FORM except for a few special cases (see also [6,7]). Considerable improvement and simplification was achieved by applying asymptotic concepts, i.e. when failure probabilities tend to zero [8,9]. Breitung did not only show that computation of outcrossing rates essentially reduces to simple volume integral evaluations but also indicated that the optimal expansion point of the limit state surface is, in fact, the same as in time-invariant analysis. The asymptotic Poissonian nature of the crossings into the failure domain has already been shown earlier. It is noteworthy that by their probabilistic structure, combinations of rectangular wave renewal processes lead always to larger failure probabilities than the combination of simple random sequences. Important insights have been gained in Schall et al. [10] for the case when some ideal conditions on which all these computation schemes were based do no more hold. In particular, approximations were proposed if the parameters of the process are slowly varying, ergodic sequences. It was also recognized that the asymptotic Poissonian nature of crossings, which is implicit in most mentioned references except in [2], is lost in the presence of simple random variables. It is then necessary to perform a sequence of conditional reliability analyses and use the total probability theorem to obtain the unconditional failure probability. Approximations suitable for numerical analysis were proposed, i.e. the method of nested FORM/SORM integration [11]. Unfortunately, nested FORM/SORM integration can be quite involved numerically even in the stationary case and appears suitable only for small variations in the non-ergodic variables. Especially for non-stationary problems, i.e. for non-stationary processes and/or for time-variant limit state functions, nested FORM/SORM generally requires far too much numerical effort and is no more reliable. In the following an approximate computation scheme for the non-stationary crossing rates is proposed together with some indications how to deal consistently with non-ergodic variables.

2. General concepts for time variant reliability

Consider the general task of estimating the probability $P_f(t)$ that a realization $z(\tau)$ of a random state vector $Z(\tau)$ enters the failure domain $V=\{z(\tau) \mid g(z(\tau), \tau) \leqslant 0, 0 \leqslant \tau \leqslant t\}$ for the first time given that $Z(\tau)$ is in the safe domain at $\tau=0$. $g(\cdot)$ is the state function. The limit state is defined for $g(\cdot)=0$. $Z(\tau)$ may conveniently be separated into three components as

$$Z(\tau)^{\mathrm{T}} = \left(R^{\mathrm{T}}, Q(\tau)^{\mathrm{T}}, S(\tau)^{\mathrm{T}}\right) \tag{1}$$

where R is a vector of random variables independent of time, $Q(\tau)$ is a slowly varying stationary and ergodic random vector sequence and $S(\tau)$ is a vector of not necessarily stationary, but sufficiently mixing random process variables having fast fluctuations as compared to Q(t).

Consider first the case where only $S(\tau)$ is present. If it can be assumed that the stream of crossings of the vector $S(\tau)$ into the failure domain V is Poissonian it is well known that the failure probability $P_f(t)$ can be estimated from

$$P_f(t) \approx 1-\exp(-E[N_S^+(t)]) \leqslant E[N_S^+(t)] \tag{2}$$

with

$$E[N_S^+(t)] = \int_0^t \nu_S^+(\tau)\,\mathrm{d}\tau \tag{3}$$

for high reliability problems. $E[N_S^+(t)]$ is the expected number of crossings of $S(\tau)$ into the failure domain V in the considered time interval [0, t] and $\nu_S^+(\tau)$ the outcrossing rate. It is assumed that there is negligible probability of failure at $\tau=0$ and $\tau=t$, respectively. The upper bound in eqn. (2) is a strict upper bound but close to the exact result only for rather small $P_f(t)$. The approximation in eqn. (2) has found many applications in the past not only because of its relative simplicity but also because there has been no real practical alternative except in some special cases.

When both process variables $S(\tau)$ and time invariant random variables R are present the Poissonian nature of outcrossings is lost. Equation (2) can furnish only conditional probabilities. The total failure probability must be obtained by integration over the probabilities of all possible realizations of R. Then the equivalent to eqn. (2) is

$$P_f(t) \approx E_{\mathrm{R}}[1-\exp(-E[N_S^+(t \mid R)])] = 1-E_R[\exp(-E[N_S^+(t \mid R)])] \leqslant E_R[E[N_S^+(t \mid R)]] \tag{4}$$

In the general case where all the different types of random variables R, $Q(\tau)$ and $S(\tau)$ are present the failure probability $P_f(t)$ not only must be integrated up over the time in-variant variables R but an expectation operation must be performed over the slowly varying variables $Q(\tau)$. In Schall et al. [10] the following formula has been established in part by making use of the ergodicity theorem

$$P_f(t) \approx 1-E_R[\exp(-E_Q[E[N_S^+(t \mid R, Q)]])] \leqslant E_R[E_Q[E[N_S^+(t \mid R, Q)]]] \tag{5}$$

Equation (5) is a rather good approximation for the stationary case but must be considered as a first approximation whenever $S(\tau)$ is non-stationary or the limit state function exhibits strong dependence on τ as shown in the mentioned reference. The approximation concerns the expectation operation with respect to Q in the non-stationary case. The bounds given in eqns. (2), (4) and (5) again are strict but close to the exact result only for even smaller failure probabilities. In fact, while the approximation with respect to the expectation operation inside the exponent in eqn. (5) may be accepted also for the non-stationary case in most practical applications the expectation with respect to R must be taken outside the exponent because, depending on the relative magnitude of the variabilities of the R- and the S, Q-variables, errors up to several orders of magnitude can occur (see Schall et al. [10]). Therefore, it is of particular interest to design effective computations schemes especially in view of the fact that the R-vector can be high-dimensional (e.g. in stochastic finite elements with hundreds of variables describing random system properties).

3. Conditional outcrossing rates for non-stationary rectangular wave renewal processes

If the components of a stationary rectangular wave renewal process are independent with marks S_k with distribution function $F_k(s; q, r)$, and renewal rates λ_k it has been shown that the mean number of exits into the failure domain is [3]

$$E[N^+(t_1, t_2; q, r)] = (t_2 - t_1) \sum_{i=1}^{n} \lambda_i \mathbb{P}(\{S_i^- \in \bar{V}; q, r\} \cap \{S_i^+ \in V; q, r\}) \quad (6)$$

where $\bar{V}$ and V are the safe and failure domain, respectively. S_i^+ is the total load vector when the i-th component of the renewal process had a renewal. S_i^- denotes the total load vector just before the renewal. Therefore, S_i^- and S_i^+ differ by the vector S_i which is to be introduced as an independent vector in the second set. Applying asymptotic concepts and using eqn. (6) it can further be shown that [8,9]

$$E[N^+(t_1, t_2; r, q)] \sim (t_2 - t_1) \sum_{i=1}^{n} \lambda_i \mathbb{P}(\{S \in V \mid q, r\}) \quad (7)$$

with $\mathbb{P}(\{S \in V; r, q\})$ computed as a volume integral in the usual manner by SORM. Very rarely this formula is noticeably improved for not small probabilities $\mathbb{P}(\{S \in V; r, q\})$ by replacing the term $\mathbb{P}(\{S \in V; r, q\})$ by $\mathbb{P} = \mathbb{P}(\{S_i^- \in \bar{V}; q, r\} \cap \{S_i^+ \in V; q, r\} = \mathbb{P}(\{S \in V; r, q\}) - \mathbb{P}(\{S_i^- \in V; r, q\} \cap \{S_i^+ \in V; q, r\})$ as in eqn. (6). Note that integration with respect to q is performed simultaneously with the integration with respect to s. If unconditional mean numbers of exits need to be computed as in eqn. (5) integration is also over r.

The non-stationary case is not substantially more difficult. The renewal rates $\lambda_k(\tau)$, $k = 1, 2, \ldots,$ are assumed to vary slowly in time. The distribution function of S may contain distribution parameters $r(\tau)$ varying in time and the failure domain can be a function of time, i.e. $V = \{g(s, q, r, \tau) \leq 0\}$. Then, eqn. (7) needs to be modified as

$$E[N^+(t_1, t_2 \mid r)] \sim E_Q \int_{t_1}^{t_2} \sum_{k=1}^{m} \lambda_i(\tau) \mathbb{P}(\{S \in V\} \mid q, r, \tau\})\, d\tau$$

$$= \int_{t_1}^{t_2} \int_V \sum_{i=1}^{n} \lambda_i(\tau) f_{S,Q}(s, q, \tau \mid r)\, ds\, dq\, d\tau \quad (8)$$

The time-volume integral (8) can be approximated using SORM concepts. For small $\mathbb{P}(\{S \in V \mid q, r, \tau\})$ the integrand is dominated by the probability term in the neighborhood of the most likely failure point (s^*, q^*, τ^*) in $\{S \in V \mid q, r, \tau\}$ to be determined by an appropriate algorithm. One such algorithm has been proposed by Abdo/Rackwitz [12]. Because $\lambda_k(\tau)$ is slowly varying it is drawn in front of the integral with value $\lambda_k(\tau^*)$ by virtue of the mean value theorem of integral theory. Classical FORM/SORM algorithms can be applied according to [13] after transforming the integral in eqn. (8) into a probability integral by introducing an additional uniform density $f_T(\tau) = (t_2 - t_1)^{-1}$ into eqn. (8) such that

$$E[N^+(t_1, t_2 \mid r)] = (t_2 - t_1) \sum_{k=1}^{m} \lambda_i(\tau^*) \int_{\mathbb{R}^1} \int_V f_{S,Q}(s, q, \tau \mid r) f_T(\tau)\, ds\, dq\, d\tau \quad (9)$$

With the transformation $\tau = \Phi(u_T)$ the probability integral can be determined in the usual manner.

It has been found that a slightly different scheme is advantageous. In the critical point (s^*, q^*, τ^*) the probability $\mathbb{P}(\{S \in V \mid q, r, \tau\})$ is estimated by

$$\mathbb{P}(\{S \in V \mid q, r, \tau\}) = \Phi(-\beta(\tau^*)) \times C(s^*, q^*, \tau^* \mid r) \quad (10)$$

with $C(s^*, q^*, \tau^*)$ the well known curvature correction term (in the s–q-space) in SORM.

Then, one can write

$$E[N^+(t_1, t_2 \mid r)] \sim \int_{t_1}^{t_2} \sum_{i=1}^{n} \lambda_i(\tau \mid r) \Phi(-\beta(\tau \mid r)) \times C(s^*, q^*, \tau \mid r)\, d\tau$$

$$\approx C(s^*, q^*, \tau^* \mid r) \sum_{i=1}^{n} \lambda_i(\tau^* \mid r) \int_{t_1}^{t_2} \Phi(-\beta(\tau \mid r))\, d\tau$$

$$= C(s^*, q^*, \tau^* \mid r) \sum_{i=1}^{n} \lambda_i(\tau^* \mid r) \int_{t_1}^{t_2} \exp[\ln[\Phi(-\beta(\tau \mid r))]]\, d\tau$$

$$= C(s^*, q^*, \tau^* \mid r) \sum_{i=1}^{n} \lambda_i(\tau^* \mid r) \int_{t_1}^{t_2} \exp[f(\tau)]\, d\tau \quad (11)$$

where $f(\tau) = \ln[\Phi(-\beta(\tau))]$. The time integral in eqn. (11) is perfectly suited for application of Laplace's integral approximation. Expanding $f(\tau)$ to first and second order with derivatives

$$f'(\tau) = -\frac{\varphi(-\beta(\tau))}{\phi(-\beta(\tau))} \frac{\partial \beta(\tau)}{\partial \tau} \approx -\beta(\tau) \frac{\partial \beta(\tau)}{\partial \tau}$$

$$f''(\tau) = -\frac{\varphi(-\beta(\tau))}{\phi(-\beta(\tau))} \left[\left(\frac{\partial \beta(\tau)}{\partial \tau} \right)^2 \left(\beta(\tau) + \frac{\varphi(-\beta(\tau))}{\phi(-\beta(\tau))} \right) + \frac{\partial^2 \beta(\tau)}{\partial \tau^2} \right]$$

yields integrals which have analytical solutions. While $\partial\beta/\partial\tau$ is directly obtained as a parametric sensitivity in FORM/SORM the second derivative $\partial^2\beta(\tau)/\partial\tau^2$ must be determined numerically by a simple difference scheme. The results are

$\tau^* = t_1$ or $\dfrac{\partial \beta(\tau)}{\partial \tau} > 0$:

$$E[N^+(t_1, t_2 \mid r)] \approx C(s^*, q^*, t_1 \mid r) \sum_{i=1}^{n} \lambda_i(t_1 \mid r) \Phi(-\beta(t_1 \mid r)) \left\{ \frac{\exp[f'(t_1)t_2] - \exp[f'(t_1)t_1]}{\exp[f'(t_1)t_1] f'(t_1)} \right\} \quad (12a)$$

$\tau^* = t_2$ or $\dfrac{\partial \beta(\tau)}{\partial \tau} < 0$:

$$E[N^+(t_1, t_2 \mid r)] \approx C(s^*, q^*, t_2 \mid r) \sum_{i=1}^{n} \lambda_i(t_2 \mid r) \Phi(-\beta(t_2 \mid r)) \left\{ \frac{\exp[f'(t_2)t_2] - \exp[f'(t_2)t_1]}{\exp[f'(t_2)t_2] f'(t_2)} \right\} \quad (12b)$$

$t_1 < \tau^* < t_2$ or $\dfrac{\partial \beta(\tau)}{\partial \tau} = 0$ and $\dfrac{\partial^2 \beta(\tau)}{\partial \tau^2} > 0$:

$$E[N^+(t_1, t_2 \mid r)] \approx = C(q^* s^*, \tau^* \mid r) \sum_{i=1}^{n} \lambda_i(\tau^* \mid r) \Phi(-\beta(t_1 \mid r)) \left(\frac{2\pi}{|f''(\tau^*)|} \right)^{1/2}$$

$$\times \left\{ \Phi(|f''(\tau^*)|^{1/2}(t_2 - \tau^*)) - \Phi(|f''(\tau'')|^{1/2}(t_1 - \tau^*)) \right\} \quad (12c)$$

This modification results in more accurate exit means than eqn. (9) especially for smaller time distances $t_2 - t_1$ although the interaction between τ and the other variables is neglected. A genuine first order result does not exist but in many applications the computation of the correction factor $C(s^*, q^*, \tau^* | r)$ involving the second order derivatives in the s, q-space will yield only small improvements.

4. Integration with respect to time invariant variables R

If there are time-invariant random vectors $\boldsymbol{R}$ several possibilities exist the most straightforward being numerical integration. However, even for small dimensions of $\boldsymbol{R}$ the computational effort can be considerable. But, the computational problem can be reformulated as

$$P_f(t) = P(T(\boldsymbol{R}) - t \leqslant 0) = P(g(\boldsymbol{R}, U_T) \leqslant 0) \tag{13}$$

where $T(\boldsymbol{R})$ is a random lifetime with realizations $t(\boldsymbol{r})$ given by the solution to the equation

$$E[N_S^+(t(\boldsymbol{r}) | \boldsymbol{r})] + \ln(-\Phi(u_T)) = 0 \tag{14}$$

and u_T a realization of an auxiliary standard normal variable. Then eqn. (13) is appropriate for classical volume integration by FORM/SORM provided that the quantity $E[N_S^+(t(\boldsymbol{r})|\boldsymbol{r})]$ can be calculated. This formulation is always exact to the order of computation level (FORM or SORM). However, in the case of time varying limit state functions and/or non-stationary processes the numerical solution of eqn. (14), which must be performed at least twice in each iteration, becomes rather time consuming because of repeated calculations of $E[N_S^+(t(\boldsymbol{r})|\boldsymbol{r})]$ and the convergence of the algorithm may become slow if not unreliable. Therefore, it can be recommended only in special cases.

Another method is based on Laplace's integral approximations. For convenience, it is assumed that R is an uncorrelated, standardized Gaussian vector which can always be achieved by a probability distribution transformation. Equation (5) can be rewritten as

$$\begin{aligned} P_f(t) &= E_{\boldsymbol{R}}\left[1 - \exp(-E_{\boldsymbol{Q}}[E[N_S^+(t | \boldsymbol{Q}, \boldsymbol{R})]])\right] \\ &= \int_{\mathbb{R}^{n_r}} (1 - \exp(-E_{\boldsymbol{Q}}[E[N_S^+(t | \boldsymbol{Q}, \boldsymbol{r})]]))\varphi_{\boldsymbol{R}}(\boldsymbol{r})\,\mathrm{d}\boldsymbol{r} \\ &= \int_{\mathbb{R}^{n_r}} (\varphi_{\boldsymbol{R}}(\boldsymbol{r}) - \exp(-E_{\boldsymbol{Q}}[E[N_S^+(t | \boldsymbol{Q}, \boldsymbol{r})]] - \ln(\varphi_{\boldsymbol{R}}(\boldsymbol{r}))))\,\mathrm{d}\boldsymbol{r} \\ &= \int_{\mathbb{R}^{n_r}} \left(\varphi_{\boldsymbol{R}}(\boldsymbol{r}) - \frac{1}{(2\pi)^{n_r/2}} \exp\left(-E_{\boldsymbol{Q}}[E[N_S^+(t | \boldsymbol{Q}, \boldsymbol{r})]] - \tfrac{1}{2}\boldsymbol{r}^{\mathrm{T}}\boldsymbol{r}\right)\right)\mathrm{d}\boldsymbol{r} \end{aligned} \tag{15}$$

$\varphi_{\boldsymbol{R}}(\boldsymbol{r})$ denotes the probability density function of $\boldsymbol{R}$. The integral of the term in parenthesis is now approximated using asymptotic concepts although they do not strictly apply here. The asymptotic solution is

$$P_f(t) \approx \exp\left[-\tfrac{1}{2}(\boldsymbol{r}^{*\mathrm{T}}\boldsymbol{r}^*)\right]\left[1 - \exp(-C\Phi(-\beta(\boldsymbol{r}^*)))(|\det \boldsymbol{G}(\boldsymbol{r}^*)|)^{-1/2}\right] \tag{16}$$

by making use of Lemma 8.3.2 in [14] and where $\boldsymbol{G}(\boldsymbol{r}^*)$ is the Hessian matrix of C $\Phi(-\beta(\boldsymbol{r}))$ $+\frac{1}{2}\boldsymbol{r}^{\mathrm{T}}\boldsymbol{r}$ with respect to $\boldsymbol{r}$ taken in $\boldsymbol{r}^*$. Provided that $\boldsymbol{R}$ is made uncorrelated only the diagonal of $\boldsymbol{G}$ needs to be determined, i.e.

$$\boldsymbol{G}(\boldsymbol{r}^*) \approx \mathrm{diag}\left\{C\frac{\varphi(-\beta(\boldsymbol{r}^*))}{\|\nabla g_r(\boldsymbol{r}^*)\|}\left(\frac{\beta(\boldsymbol{r}^*)}{\|\nabla g_r(\boldsymbol{r}^*)\|}\left(\frac{\partial g(\boldsymbol{r}^*)}{\partial r_i}\right)^2 - \frac{\partial^2 g(\boldsymbol{r}^*)}{\partial r_{i^2}}\right) + 1;\quad i = 1, 2, \ldots, n_1\right\} \tag{17}$$

where the constant C collects all factors determined above and where $\partial\beta(\boldsymbol{r})/\partial r_i = \partial g(\boldsymbol{r})/\partial r_i \|\nabla g_r(\boldsymbol{r}^*)\|^{-1}$. Numerical analysis shows that the conditional mean number of exits is rather sensitive to the conditioning variable $\boldsymbol{r}$. The critical point $\boldsymbol{r}^*$ must, in principle, be found by an appropriate search algorithm. If, however, the upper bound solution in eqn. (5) has been determined beforehand it suffices to use $\boldsymbol{r}^*$ from this computation in first approximation. The advantage of this method is that given $\boldsymbol{r}^*$ known only n_r additional function calls are necessary to determine the second derivative in eqn. (17) numerically. By numerical studies and not unexpected for theoretical reasons the second order correction by $|\det \boldsymbol{G}(\boldsymbol{r}^*)|^{-1/2}$ usually is not significant. With and without the correction term eqn. (16) furnishes only relatively crude approximations with better quality whenever the reliability problem is dominated by the s-variables. Therefore this scheme can only be recommended for small to very small variabilities of the r-variables.

Alternatively and arbitrarily exact, the expectation operation in eqn. (5) can be performed either by crude Monte Carlo integration or, more efficiently, by importance sampling. Provided the important region for integration is known by $\boldsymbol{r}^*$ it is

$$E_{\boldsymbol{R}}\left[1 - \exp\{-E_{\boldsymbol{Q}}[E[N_S^+(t | \boldsymbol{Q}, \boldsymbol{R})]]\}\right] = \int_{\mathbb{R}^{n_r}} \left[1 - \exp\{-E_{\boldsymbol{Q}}[E[N_S^+(t | \boldsymbol{Q}, \boldsymbol{r})]]\}\right] \frac{\varphi_{\boldsymbol{R}}(\boldsymbol{r})}{h_{\boldsymbol{R}}(\boldsymbol{r})} h_{\boldsymbol{R}}(\boldsymbol{r})\,\mathrm{d}\boldsymbol{r} \tag{18}$$

where $h_{\boldsymbol{R}}(\boldsymbol{r})$ is the sampling density. Then,

$$E_{\boldsymbol{R}}\left[1 - \exp(-E_{\boldsymbol{Q}}[E[N_S^+(t | \boldsymbol{Q}, \boldsymbol{R})]])\right] = \approx \frac{1}{N}\sum_{i=1}^{N}\left[1 - \exp\{-E_{\boldsymbol{Q}}[E[N_S^+(t | \boldsymbol{Q}, \boldsymbol{r}_i)]]\}\right] \frac{\varphi_{\boldsymbol{R}}(\boldsymbol{r}_i)}{h_{\boldsymbol{R}}(\boldsymbol{r}_i)} \tag{19}$$

The sampling density (standard space) can conveniently be chosen as the standard normal density with mean $\boldsymbol{r}^*$ from the upper bound solution eqn. (5) and covariance matrix $\boldsymbol{I}$. A crude conservative estimate is already obtained for $\boldsymbol{r}_i = \boldsymbol{r}^*$ (compare also with eqn. (16)). If the Hessian in eqn. (17) has already been determined further adjustments with respect to the covariance matrix of the sampling density are possible but have been found insignificant. From extensive testing of examples it is concluded that the scheme according to eqn. (19) is rather robust provided that an efficient and reliable algorithm is available to locate the critical point (s^*, q^*, τ^*) for every simulated $\boldsymbol{r}$.

5. Summary and conclusions

Conditional non-stationary crossing rates for rectangular renewal wave processes are computed in part making use of asymptotic concepts. Hereby, the parameters of the wave processes may depend on slowly fluctuating random and ergodic sequences. Numerical analysis is performed by applying classical FORM/SORM concepts. For time integration a separate computational step is proposed. Some effort is spent to remove the conditioning by simple random variables by appropriate numerical schemes. It is found that a importance sampling scheme is the numerically by far most satisfying numerical method in the stationary as well as the non-stationary case. It, nevertheless, must be admitted that time-variant reliabilities are considerably more difficult and laborious to determine by FORM/SORM than time-invariant reliabilities.

References

[1] J. Ferry Borges and M. Castanheta, Structural safety, Laboratorio Nacional de Engenharia Civil, Lisbon, 1971.

[2] R. Rackwitz and B. Fiessler, Structural reliability under combined random load sequences, *Comp. & Struct.*, 9 (1978) 484–494.

[3] K. Breitung and R. Rackwitz, Nonlinear combination of load processes, *J. of Struct. Mech.*, 10(2) (1982) 145–166.

[4] R. Rackwitz, Reliability of systems under renewal pulse loading, *J. of Eng. Mech.*, ASCE, 111(9) (1985) 1175–1184.

[5] Y.-K. Wen, *Structural Load Modeling and Combination for Performance and Safety Evaluation*, Elsevier, Amsterdam, 1990.

[6] M. Shinozuka, Stochastic characterization of loads and load combinations, *Proc. 3rd ICOSSAR*, Trondheim, 23–25 June, 1981, *Structural Safety and Reliability*, T. Moan and M. Shinozuka (Eds.) Elsevier, Amsterdam, 1981.

[7] K. Schrupp and R. Rackwitz, Outcrossing rates of marked poisson cluster processes in structural reliability, *Appl. Math. Modelling*, 12 (1988) 482–490.

[8] K. Breitung, Asymptotic approximations for multinormal integrals, *J. Eng. Mech.*, ASCE, 110(3) (1984) 357–366.

[9] K. Breitung, Asymptotic approximations for the maximum of the Poisson square wave processes, Berichte Zuverlässigkeitstheorie Bauwerke, Technical University Munich, 69 (1984) 59–82.

[10] G. Schall, M. Faber and R. Rackwitz, The ergodicity assumption for sea states in the reliability assessment of offshore structures, *J. Offshore Mechanics and Arctic Engineering*, Transact. ASME, 113(3) (1991) 241–246.

[11] F. Guers and R. Rackwitz, Time-variant reliability of structural systems subject to fatigue, *Proc. ICASP 5*, Vancouver, May, 1987, Vol. I, pp. 497–505.

[12] R. Abdo and R. Rackwitz, A new beta-point algorithm for large time-invariant and time-variant reliability problems, *Proc. 3rd IFIP WG 7.5 Conference*, Berkley, USA, 1990, A. Der Kiureghian, P. Thoft-Christensen (Eds.) Springer-Verlag, pp. 1–12.

[13] O. Hagen and L. Tvedt, Parallel system approach to vector outcrossings, *Proc. 10th OMAE Conf.*, Stavanger, June 23–28, 1991, Vol. II, pp. 165–172.

[14] N. Bleistein and R.A. Handelsman, *Asymptotic Expansions of Integrals*, Dover, 1986.

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