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Stochastic Modeling of Material Properties and Quality Control

M. Kersken-Bradley, R. Rackwitz

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by

M. Kersken-Bradley and R. Rackwitz

Introduction

This is one of the documents of a series of publications, prepared by individual authors but discussed within the Joint Committee on Structural Safety (JCSS), in particular within its Working Party. The series up to now consists of the following titles:

- Proposal for a Code for the Direct Use of Reliability Methods in Structural Design
  O. Ditlevsen, H.O. Madsen
- Estimation of Structural Properties by Testing for Use in Limit State Design
  M. Kersken-Bradley, W. Maier, R. Rackwitz, A. Vrouwenvelder
- Structural Performance Criteria
  L. Östlund
- Stochastic Modeling of Material Properties and Quality Control
  M. Kersken-Bradley, R. Rackwitz
- Design for Durability including Deterioration and Maintenance Procedures
  G.I. Schuëller
- Action Scenarios and Logic Trees
  R. Giannini, P.E. Pinto, R. Rackwitz
- Geometrical Variability in Structural Members and Systems
  F. Casciati, I. Negri, R. Rackwitz
- Bayesian Decision Analysis as a Tool for Structural Engineering Decisions
  O. Ditlevsen

The papers are referred to as "Working Documents" since they generally give information on the state of development of certain concepts or subjects, rather than giving approved guidelines.

This paper is concerned with the modeling of material properties in terms of their variation within a lot and between lots of a material supply. This manner of modeling allows to assess the effectiveness of compliance control and to update information accounting for specific material data; for reliability studies, the model itself may be introduced when system behaviour is of concern, else the resulting predictive distributions for a supply is sufficient.

This series of publications is intended to initiate discussions and exchange of comments. Comments may be sent to the Headquarter of IABSE, which will take care of sending these to the respective bodies of the JCSS.

Future papers of the JCSS will appear in appropriate international Engineering Journals. This series published by IABSE is closed.

The above papers are issued in honour of Professor Julio Ferry Borges, former President of the JCSS, expressing our deep appreciation and sincere thanks for successfully guiding the Joint Committee for more than 18 years.

The General Reporter of the JCSS:
M. Kersken-Bradley (until November 1990)
A. Vrouwenvelder (at present)

The President of the JCSS:
J. Schneider

LIST OF CONTENT

What is this Paper about?

This paper suggests a stochastic model for material properties. The main feature of the model is to distinguish
- the variance within lots
- the variance between lots in a supply.

This distinction gives the "structure" of a supply.

Where to use the information?

Information on the "structure" of a supply gives guidance on effective compliance control strategies as well as on the structure of stochastic models for material properties in probabilistic reliability studies.

Who may it concern?

This paper might be of special interest for specialists working on compliance rules and other control strategies for material properties.

STOCHASTIC MODELING OF MATERIAL PROPERTIES AND QUALITY CONTROL

1. INTRODUCTION

1.1 Objective

Reliability assessment of structural facilities requires realistic and operational stochastic models for the uncertainties to be accounted for in the analysis. An important class of uncertainties includes the properties of materials, in particular the strength of materials. Although a vast number of statistical investigations for the various types of materials and structural components as well as theoretical studies on specific properties of certain models is available, only a few attempts have been made to set up principles for the modeling of material properties. This report summarizes general principles of probabilistic modeling of material properties valid for the most common types of material. These models may serve for:

– the description of the variations in material properties
– the analysis of data and their updating
– the specification of procedures for quality control of materials.

It is understood that modeling is an art of reasonable simplification of reality such that the outcome, the model, is sufficiently explanatory and predictive in an engineering sense. An engineering model must be operational, i.e. it must be easy to handle in practice and code making.

1.2 Scope

Material properties vary locally in space and, possibly, in time. The time variations may be a consequence of certain aging effects, of load–induced deterioration but also of environmental factors such as temperature and humidity. In space material properties may also vary globally i.e. between objects or sets of objects. This report deals with the variability of materials in space; time variations are not included.

1.3 Scales

In general, local variations within an object and global variations between objects can be distinguished. The latter type of variability is by far and large a spatial variability of the parameters of a stochastic model for the variations within an object. It resembles very much classical parameter uncertainty which, in principle, could be removed by extensive sampling and testing. For example, the variability of the parameters mean and standard deviation of concrete cylinder strength per job or construction unit as shown in figure 1.1 is a typical form of global parameter variation. This variation primarily is the result of different production technology and production strategy of the concrete producers. Observations on global variations might be used as follows:

Given the data–based or otherwise assessed distribution of model parameters between objects, a particular parameter realization for an arbitrary object is obtained by random selection from this distribution. On site, this (prior) parameter distribution may be updated by actual data or pre–envisaged procedures of statistical quality control. Parameter variations between objects are conveniently denoted as macro scale variations. The unit of that scale is in the order of a structure or a construction unit.

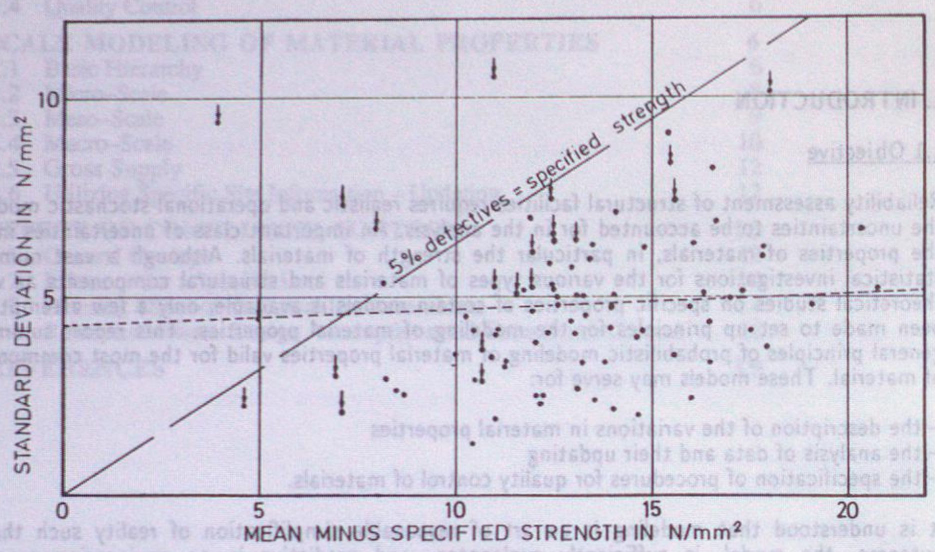

Figure 1.1: Example for observed mean minus specified strength and standard deviation of standard cube strength tests of 88 production units of concrete grade C 35 [1] (arrows indicate lots with at least one negative compliance decision according to DIN 1084)

Given a certain parameter realization for an object the next step is to model the local spatial variations within the object (= system) in terms of random processes or fields. Typically spatial correlations (dependencies) become negligible at distances much smaller than the size of the system. This is a direct consequence of the modeling procedure. It is natural to assume that the variation within the system is conditional on the variations between systems and the first type of variation is conditionally independent of the second. At this level one may speak of meso–scale variations. Examples are the spatial variation of soils within a given (not too large) foundation site or the number, size and spatial distribution of flaws along welding lines given a welding factory (or welding operator). The unit of this scale is in the order of the size of the structural elements.

At the third level, the micro–level, one has to consider rapidly fluctuating variations and inhomogeneities which basically are uncontrollable as they originate from physical facts such as the random distribution of spacing and size of aggregates, pores, cracks or particles in concrete or other materials. The scale of these variations is measured in particle sizes, i.e. in centimeters down to the size of crystals.

At this level but sometimes already at the meso–scale level the modeling process normally uses physical arguments as far as possible. The object (system) is considered as an arrangement of a large number of small elements. The statistical properties of these elements usually can only be assessed qualitatively as well as their mode of interaction. This, however, is sufficient to perform some basic operations such as extreme value, summation or intersection operations which describe the statistical interaction. The generally large number of elements greatly facilitates such operations because certain limit theorems of probability theory can be applied. The advantage of

using asymptotic concepts is that the description of the element properties can be reduced to some few essential characteristics. Size effects have to be taken into account at this level.

A useful concept is to introduce a reference volume of the material which can be chosen on rather practical grounds. It corresponds to the specified test specimen volume at which material testing is carried out. On the micro–scale level this volume may be considered as reference system for deriving parameters of virtual strength elements. On the meso–scale level, this volume may correspond to the volume of the virtual strength element, however, strength properties generally need to be transformed to in–situ strength properties. Concrete may be the most obvious example for the necessity of such additional considerations.

The reason for this concept of modeling at several levels (steps) is that it is operational, not only for probabilistic calculations but also in sampling, estimation and quality control. This way of modeling and the considerations below are only valid under certain technical standards for production and its control. At the macro–scale level it is assumed that the production process is under control. This means that the outcome of production is stationary in some sense. Should a trend develop, production control corrects for it immediately or with some sufficiently small delay so that at least for some time interval (or spatial region) whose length (size) is to be selected carefully, approximate stationarity (or even ergodicity) on the meso– and micro–scale can be assumed. Furthermore, given certain manufacturing and quality assurance standards production can be assumed to be stationary on a long term basis. Variations at the macro scale level, therefore, can be described in terms of stationary sequences. If the sequences are or can be assumed independent, it is possible to handle macro–scale variations by the concept of random variables. Stationarity may also be assumed at the lower levels. At the lower scale levels it may be necessary to use the theory of random processes (fields) especially in order to take account of significant effects of dependencies in time or space.

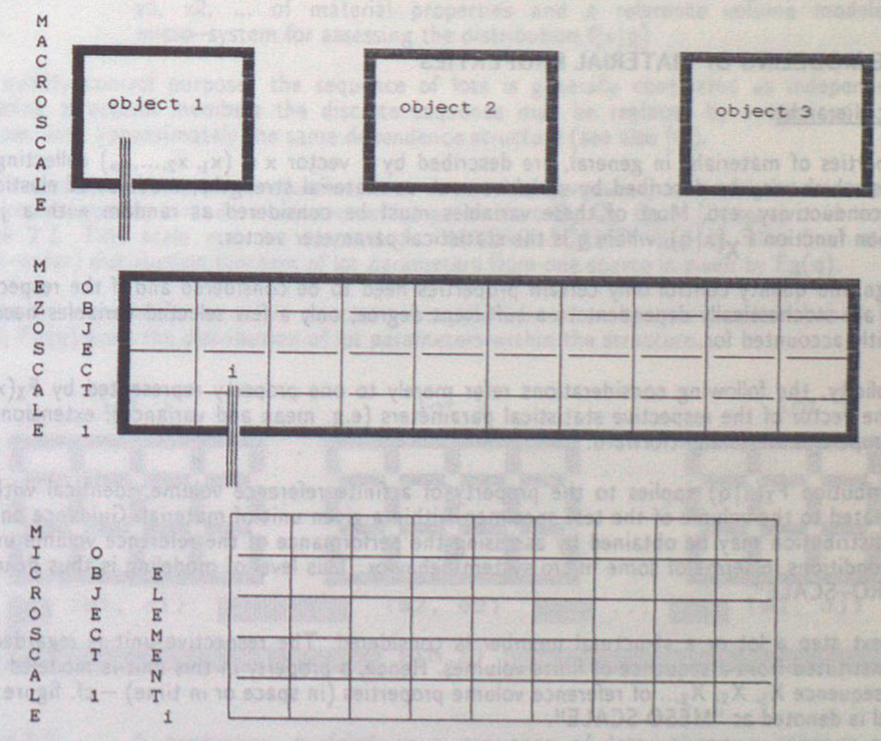

Figure 1.2: Modeling Scales

1.4 Quality Control

For material properties relevant for the structural behavior – and explicitly considered in design – quality control is required to ensure that the properties of the materials used on site comply with the assumptions made in structural design.

Quality control can be distinguished as production control and compliance control. It is generally acknowledged, that compliance control alone can be inefficient to ensure quality; a whole set of suitable quality assurance procedures is required which generally include compliance control. Compliance control may be pursued with the intention to

– assess individual lots and/or
– monitor the material supply.

By reference to a scale modeling of materials it can be shown, that the efficacy of strategies depends on the inherent structure of the supply. For certain supplies an assessment of the individual lot with the pretension to predict properties within the lot, proves to be inefficient.

Depending on the major objective of compliance control, this involves different forms of efficient organization of control. The significance and tasks of both internal and external control bodies are different for a compliance control mainly devoted to monitoring as compared to a compliance control for identifying inferior lots.

Finally, by means of a scale modeling a clear relation between material specifications for design purposes and compliance control may be established. The respective strategies in compliance control may also determine the suitable manner for assuming material supplies for design purposes.

2. SCALE MODELING OF MATERIAL PROPERTIES

2.1 Basic Hierarchy

The properties of materials, in general, are described by a vector $x = (x_1, x_2, \ldots, x_m)$ collecting all properties which may be described by variables such as material strengths, modulus of elasticity, thermal conductivity, etc. Most of these variables must be considered as random with a joint distribution function $F_X(x|q)$, where q is the statistical parameter vector.

For design and quality control only certain properties need to be considered and if the respective variables are stochastically dependent to a sufficient degree, only a few selected variables need to be explicitly accounted for.

For simplicity, the following considerations refer merely to one property represented by $F_X(x|q)$ with q the vector of the respective statistical parameters (e.g. mean and variance); extensions to several properties are straightforward.

The distribution $F_X(x|q)$ applies to the property of a finite reference volume, identical with or clearly related to the volume of the test specimen within a given unit of material. Guidance on the type of distribution may be obtained by assessing the performance of the reference volume under testing conditions in terms of some micro system behavior. This level of modeling is thus denoted as "MICRO–SCALE".

In the next step a lot or a structural member is considered. The respective unit is regarded as being constituted from a sequence of finite volumes. Hence, a property in this unit is modeled by a random sequence $X_1, X_2, X_3, \ldots$ of reference volume properties (in space or in time) – cf. figure 2.1. This level is denoted as "MESO SCALE".

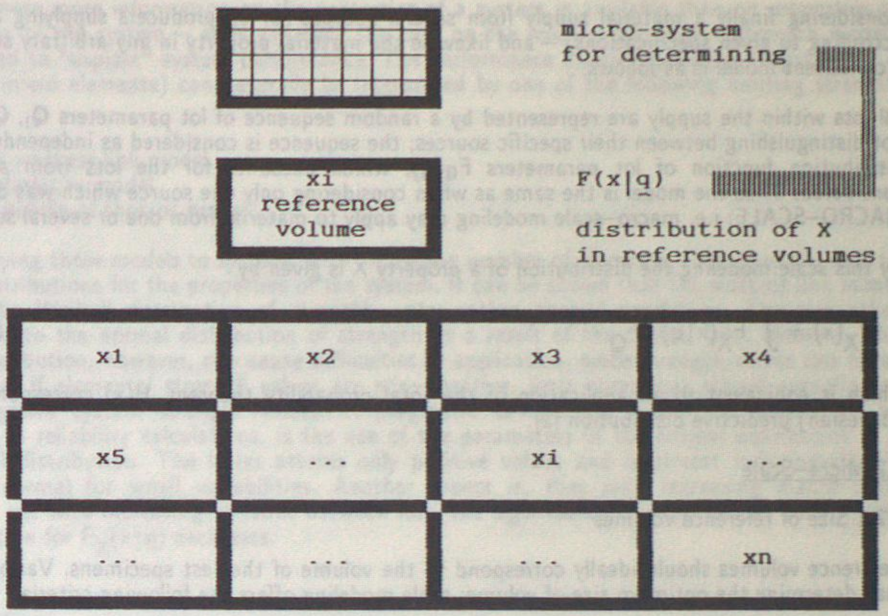

Figure 2.1: A lot modeled as a sequence of n reference volumes giving a random sequence x1, x2, ... of material properties and a reference volume modeled as a micro–system for assessing the distribution f(x|q)

For quality control purposes the sequence of lots is generally considered as independent. For modeling structural members the discrete sequence may be replaced by a continuous random process, with approximately the same dependence structure (see also [4]).

In the subsequent step the production from one source (producer) is modeled as a sequence of lots, represented by a random sequence of lot parameters $Q_1, Q_2, Q_3, \ldots$ (in space or in time) – cf. figure 2.2. This scale may be denoted as "MACRO–SCALE", but cf. also final step. The (first–order) distribution function of lot parameters from one source is given by $F_Q(q)$.

Where the material for an entire structure is supplied from one specific producer (which is rather rare), $F_Q(q)$ gives the distribution of lot parameters within the structure.

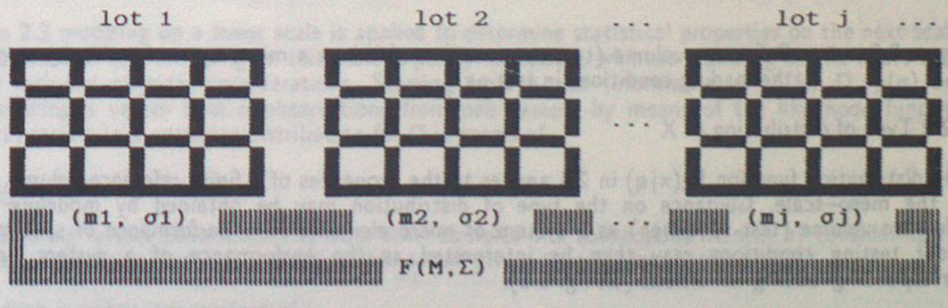

Figure 2.2: A production modeled as a sequence of lots, giving a random sequence Q1, Q2, ... of lot parameters

Considering finally a material supply from several sources (or all producers supplying a material according to given specifications) – and likewise the material property in any arbitrary structure – a consistent model is as follows:

All lots within the supply are represented by a random sequence of lot parameters Q_1, Q_2, Q_3, ..., not distinguishing between their specific sources; the sequence is considered as independent with a distribution function of lot parameters $F_Q(q)$, which accounts for the lots from all sources considered. Thus the model is the same as when considering only one source which was denoted as MACRO–SCALE; i.e. macro–scale modeling may apply to material from one or several sources.

By this scale modeling the distribution of a property X is given by

$$H_X(x) = \int F_X(x|q)\, dF_Q \qquad (2.1)$$

which is equivalent to an application of the total probability theorem. H(x) corresponds to the (Bayesian) predictive distribution [2].

2.2 Micro–Scale

2.2.1 Size of reference volumes

Reference volumes should ideally correspond to the volume of the test specimens. Various criteria may determine the optimum size of volume; scale modeling offers the following criteria:

– the sequence of volumes on the meso–scale level should be stationary, at least prior to possible "build–in" effects
– the volume should be as large as possible for decreasing the within–lot variance but, on the other hand, dimensions should not be larger than common structural dimensions in order to limit size effect corrections

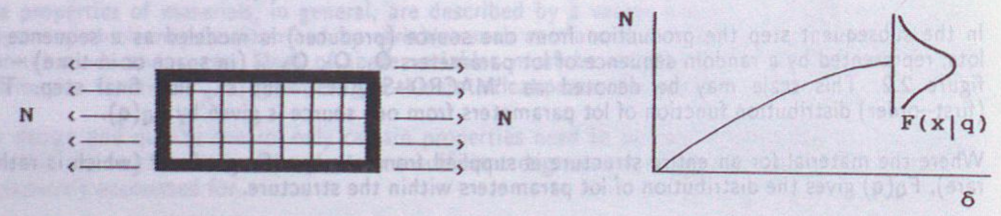

Figure 2.3: Reference volume (test specimen) mofeled as a micro system and subjected to the loading conditions in testing

2.2.2 Type of distribution of X

The distribution function $F_X(x|q)$ in 2.1 applies to the properties of a finite reference volume, i.e. to the meso–scale. Guidance on the type of distribution may be obtained by modeling the reference volume (test specimen) as a system of micro elements. The performance of specimens under testing conditions may then be interpreted as the performance of a system under corresponding loading conditions (cf. fig. 2.3).

Strength models comprise mechanical and probabilistic models for predicting the strength or other properties of a system from properties of the system elements. Their most common application is for predicting properties of structural components on the basis of well–established properties of reference volumes, i.e. more on the meso–scale level.

In cases where more information on the properties of a system is available than on properties of components of the system – as is generally the case on the micro–scale level – strength models may be used to "explain" system performance. The performance of test specimens (regarded as a system of micro elements) can generally be interpreted by one of the following limiting strength models

– Weibull's weakest link model
– the full plasticity model
– Daniel's bundle of threads model

When applying these models to systems with increasing number of elements, they generally lead to specific distributions for the properties of the system. It can be shown that the weakest link model leads to the Weibull distribution of strength under rather general conditions. The two other models lead to the normal distribution of strength as a result of the central limit theorem. The normal distribution, however, can cause difficulties in application, since strength values can never be negative. If elemental strength values are non–negative, limit operations should actually not render negative system strength values. A pragmatic compromise which has proven to be reasonable in reliability calculations, is the use of the parameters of the normal distribution in a log–normal distribution. The latter attains only positive values and is almost indistinguishable from the normal for small variabilities. Another aspect is, that with increasing macro–scale variability, i.e. with increasing variance between lots, the significance of assumptions on the type of distribution for $F_X(x|q)$ decreases.

2.3 Meso–Scale

2.3.1 Size of Lots

Lots should correspond to natural units of production (cf. 3.3.1). Various criteria may determine an optimum lot size; scale modeling offers the following criterion:

– the sequence of lots should be stationary (referring to processed material supplied from one source), presuming an effective production control

The sequence X_1, X_2, X_3, ... within a lot also should be adopted as stationary

2.3.2 Type of distribution of lot parameters

The distribution function $F_Q(q)$ applies to the lot parameters in one or several sources, i.e. to the macro–scale. Guidance on the type of distribution may be obtained by modeling the lots as random samples taken from the enlarged population with the intention to estimate statistical parameters.

As in 2.2 modeling on a lower scale is applied to determine statistical properties on the next scale. In contrast to 2.2, modeling on this scale is purely statistical, whereas modeling on the micro scale level included physical considerations. Starting from a non–informative prior for Q : p(q) and considering a vector x of n observations from one sample by means of the likelihood function $\ell(q|x)$ results in a posterior distribution for Q in terms of

$$f_Q(q|x) = \ell(x|q)\, p(q) \qquad (2.1)$$

Considering further observations x_j from other samples in a consecutive manner by considering the posterior resulting from the j–th information, will give a type of distribution for Q

– which is either independent of j
– or is established by applying asymptotic concepts

Illustration 1

For the Gaussian model a non–informative prior for M and Σ (index X omitted) is (cf. [4] and fig. 2.5)

$$p_{M,\Sigma}(m,\sigma) \sim \sigma^{-1} \tag{1}$$

and considering a first vector x of n observations from one sample (lot) by means of the likelihood function.

$$\ell(m,\sigma|x) \sim \sigma^{-n} \exp\left(-\frac{1}{2\sigma^2}\left(\nu s^2 + n(m-\bar{x})^2\right)\right) \tag{2}$$

which accounts for the normal distribution of X, given m and σ (cf. sec. 3.2), results in a posterior distribution for M and Σ which is normal–gamma [2,3]:

$$f(m,\sigma|\underline{x}) = k\,\sigma^{-(n+1)} \exp\left(-\frac{1}{2\sigma^2}\left(\nu s^2 + n(m-\bar{x})^2\right)\right) \tag{3}$$

where $\bar{x}$ and s are the sample mean and standard deviation from the observations $\underline{x}$ and n the sample size, with ν the degree of freedom. The normalizing constant k is defined by

$$k = (n/2\pi)^{1/2}\,(1/2\,(\nu/2))^{-1}\,(\nu s^2/2)^{\nu/2} \tag{4}$$

Considering further observations x_j from other samples in a consecutive manner by considering the posterior resulting from the (j–1)–th observation as the prior for dealing with the j–th observation, the normal–gamma–type distribution is maintained, with updated parameters $\bar{x}'$, s', n' and ν'.

If the mean $\bar{x}_i$ and the standard deviation s_i is available from $i = 1 - ...\kappa$ samples each with n_i observations, then

$$\bar{x}' = \Sigma\, n_i \bar{x}_i \,/\, n$$

$$n' = \Sigma\, n_i$$

$$s'^2 = \frac{1}{\nu'}\left(\Sigma\left(n_i \bar{x}_i^2 + \nu_i s_i^2\right) - n\bar{x}'^2\right)$$

$$\nu' = \Sigma\left(\nu_i + \delta(n_i)\right) - \delta(n)$$

where

$$\delta(n) = \begin{cases} 0 \text{ for } n_i \text{ or } n = 0 \\ 1 \text{ for } n_i \text{ or } n > 0 \end{cases}$$

2.4 Macro Scale

2.4.1 Production volume

The production volume consists of the lots from one or several sources. Ultimately this may be the gross supply according to sec. 2.5. Various criteria may determine maximum/optimum extent of sources to be covered by a unique statistical model; scale modeling offers the following criteria

- the assumption of ergodicity should be applicable
- exclusion of some sources should not alter the parameters of the model significantly
- when different models for different sets of sources are introduced their admissibility has to be verified

2.4.2 General Model

Type and parameters of the distribution of **Q** must be estimated from previous observations of the lot parameters. In general, estimation yields a certain purely data–based distribution whose shape and parameters depend on the particular production regimes under study. Under the assumption of effective production control it appears natural to assume that most of the marginal parameter distributions are unimodal.

As a general rule, types of distribution which are suitable for a simple updating procedure to make use of actual local information should be preferred. This includes a suitable measure of ponderation for the distribution of **Q**, e.g. in terms of equivalent sample sizes. It can best be made by virtue of Bayes theorem. The distribution of **Q** then serves as a prior distribution. An appropriate choice of the distribution functions frequently is the choice of the natural conjugate prior distribution in Bayesian estimation theory if those exist. The point estimates of the lot parameters should be unbiased and statistically stable, i.e. have negligible statistical uncertainty. Maximum likelihood estimators should be used. Application of Bayesian concepts is particularly useful because the distribution of **Q** should be updated once observations on actual lots become available (cf. sec. 2.6). If the parameters of the lots form an independent sequence, the sample size is sufficiently large and the distributions of the k components of **Q** are unimodal the asymptotic theory of maximum–likelihood–estimation may also be applied

$$\mathbf{Q} \sim N_k(\hat{\mathbf{q}}, \hat{\Sigma}) \tag{2.2}$$

where $N_k(.;.)$ is the k–dimensional normal distribution function, $\hat{\mathbf{q}}$ the maximum–likelihood–estimator as the solution of max $\{L(\mathbf{x}|\mathbf{q})\}$ and [2]

$$\hat{\Sigma}^{-1} \approx \left\{-\frac{\partial^2 \ln L(\mathbf{x}|\mathbf{q})}{\partial q_i\, \partial q_j}\bigg|_{\mathbf{q}=\hat{\mathbf{q}}}\, ;\ i,j = 1,\ldots,k\right\} \tag{2.3}$$

$L(\mathbf{x}|\mathbf{q}) = \Pi\, f_{\mathbf{X}}(\mathbf{x}|\mathbf{q})$ is the joint likelihood function and **x** the observation matrix. The density function of **X** may, as mentioned, be assessed from micro scale considerations.

Illustration 2

By the statistical modeling of sec. 2.2.3 the distribution of M and Σ was determined as normal–gamma, which is the conjugate prior to the normal distribution. In 2.2.3 the parameters of the normal–gamma distribution ($\bar{x}'$, s', n', ν') were determined from sample parameters $\bar{x}_j$ and s_j (from samples size n_j) To account for data $\bar{x}_i$ and s_i from different populations, maximum likelihood estimators prove to be efficient. The corresponding estimators for the parameters – now denoted as ($\bar{x}'$, s', n_e, ν_e) – are given below.

If means $\bar{x}_i$ and standard deviations s_i from $i = 1...\kappa$ samples (sources) are available, then with

$$m_i = \bar{x}_i\,; \qquad h_i = 1/s_i^2$$

$$\bar{h} = \frac{1}{\kappa}\sum_1^{\kappa} h_i\,; \qquad \bar{h} = \frac{1}{\kappa}\sum_1^{\kappa} h\, h_i;$$

$$\hat{h} = \frac{1}{\kappa}\sum_1^{\kappa} h_i m_i\,; \qquad h = \frac{1}{\kappa}\sum_1^{\kappa} h_i m_i^2$$

maximum–likelihood estimation renders the following parameters:

$$\bar{x}' = \hat{h}/\bar{h}$$

$$n_e = (h - \hat{h}^2/\bar{h})^{-1}$$

$$s' = \bar{h}^{-1/2}$$

$$\nu_e \approx (h\,\bar{h} - \tilde{h})^{-1}$$

The parameters n_e and ν_e are subject to a particular interpretation. They describe the heterogeneity of the enlarged population in terms of equivalent sample sizes which would result in random sample means and standard deviations with the same statistics as the lot parameters in the supply. The corresponding density or distribution of X within the population considered is obtained from eq. 2.1

$$h_X(x) = \int_0^{\infty}\int_{-\infty}^{\infty} f(x|m_X,\sigma_X)\, f(m_X,\sigma_X)\, dm\, d\sigma \tag{2.4}$$

For the posterior distribution for M and Σ given in illustration 1 this results in a central t–distribution with ν_e degrees of freedom describing X within the supply,

$$H_X(x) = T_{\nu_e}\left(\frac{x-\bar{x}}{s}\sqrt{\frac{n_e}{n_e+1}}\right) \tag{2.5}$$

2.5 Gross Supply

The gross supply comprises all material produced (and controlled) according to given specifications, within a country or groups of countries. The macro–scale model may be used if

– the number of producers is large
– differences between producers can be considered as approximately random.

Otherwise the distribution of lot–parameters within the gross supply may be obtained as a multimodal distribution from the distributions of lot–parameters from the individual sources. Allowance may be made for different production volumes by pondering the distributions from the different sources according to their fraction of production in the total supply.

2.6 Utilizing Specific (Site) Information – Updating

The scale modeling allows for specific information to be included as appropriate. Examples are as follows.

For a specific lot observations x on a material property are available from a limited number of n_0 random samples. The supply from which the lot originates is described by the distribution $F_Q(q)$ of lot parameters derived according to sec. 2.4. This distribution is considered as prior distribution for Q within the lot. The posterior distribution is then obtained from

$$f_Q(q|x) = k\, f_Q(q)\, L(q|x)$$

For a specific structure it is known that materials are only supplied from one specific producer. Then it is sufficient to consider only the distribution of $F_Q(q)$ of lot parameters as relevant for this producer, which may be estimated on the basis of compliance control records.

For the Gaussian model, where $q_1 = m$ and $q_2 = \sigma$ the distribution of lot parameters is given by eq. 2.6, with parameters $\bar{x}'$, s', n_e and ν_e. The observations x render estimates $\bar{x}_0$ and s_0 for the mean of the standard deviation within the lot concerned.

The posterior distribution is likewise normal–gamma, as given by eq. 3 in illustration 1 with updated parameters:

$$\bar{\bar{x}}'' = \frac{1}{n'}(n_e\,\bar{x}' + n_0\,\bar{x}_0)$$

$$n'' = n_e + n$$

$$s'' = \frac{1}{\nu'}(n_0\bar{x}_0^2 + n_e\bar{x}'^2 + \nu_0 s_0^2 + \nu_e s'^2 - n'\bar{x}'^2)$$

$$\nu'' = (\nu_e + 1) + (n_0 + 1) - 1 = \nu_e + \nu_0 \tag{2.6}$$

The predictive distribution of X within the lot concerned is given by eq. (3.9) with aforementioned updated parameters.

3. EFFECT OF COMPLIANCE CONTROL

3.1 General Concept

Assuming a clear correspondence between the properties considered in compliance tes[...] those used in design, compliance control may be performed

a) in order to assess individual lots and/or
b) for monitoring properties of a material supply

In compliance control a "statistic" z is derived from the observations x. By means of a rule d(z) depending on z, a lot is classified as compliant or non–compliant. In the case of non–compliance, certain procedures are requested, e.g. further testing, removal or further safety checking. The uncertainty of the decisions depends on the specific decision rule d(z).

Where non–compliant lots are removed (immediately or in the wake of further procedures) the supply is reduced to those lots which are finally accepted during compliance control. This modification of supply is referred to as "filtering effect" and the supply is denoted by "filtered supply" [5]. In terms of structural reliability this means that instead of the failure probability P(F) accounting for the original material supply, a conditional failure probability may be considered:

$$P(F|A) = \frac{P(F \cap A)}{P(A)} \tag{3.1}$$

wherein F denotes the failure event and A the event of compliance during control. The extent to which the conditional event of compliance actually reduces the failure probability depends on

– the capability of the compliance specifications to identify "poor" lots, which is reflected by the operation characteristic (cf. sec. 3.3.2)
– but also by characteristics of the material supply which affect the acceptance rate and thus determine the degree to which compliance control actually improves the supply (cf. sec. 3.3.3)

For monitoring a material supply (b), the pretension to assess individual lots could more or less be dropped; instead, data on the material supply are collected with the intention to ensure that the material supply is the same or is superior to the supply assumed for design purposes. Only in cases of a non–compliant supply, correcting procedures would actually be required.

A further discussion of these two options is pursued in sec. 3.4, where it is shown, that monitoring of the supply is always required.

3.2 Inference for a Specific Lot

If for a specific lot observations x on a material property from compliance testing are available, the lot can be assessed with regard to the predictive distribution for X.

Obviously, this is not the usual approach in compliance control but the situation is relevant for e.g. further safety checking in cases of built–in lots which are identified as non–compliant, or for redesign where information from compliance testing is utilized. Conceptually, this approach could be used to derive decision criteria d(z) as discussed in sec. 3.4.

Also, provided the lot can be regarded as representative for the supply from which it originates, the information on the supply can only be regarded as prior information for the lot. The predicted distribution of X within a lot is then conditioned on the observation vector x (or equivalently on estimators for the lot parameters $\hat{q}$ provided that this estimator is sufficient):

$$H(x|\hat{q}) = \int F(x|q)\, dF_Q(q|\hat{q}) \tag{3.2}$$

Hence, the procedure is an updating procedure as described in sec. 2.6. For non–compliant lots sufficient information should be available such that any prior information is dominated by the actual data.

3.3 Filtering Effect of Statistical Lot Assessment

3.3.1 Compliance Specifications

A basic element of statistical compliance testing is the definition of a lot. In practice lots correspond to e.g.

- a daily production of ready–mix concrete
- structural steel from one melt processed according to the same conditions
- ... m^3 of masonry grade
- etc.

Generally, these lots may be chosen such that they can be considered as quasi–stationary, although they may display dependencies with or without systematic trends on a long–term basis.

For statistical compliance testing the following specifications are required – referred to as test plan:

a) properties to be assessed
b) definition of lot (lot size)
c) the sampling procedure (sample size, sampling technique)
d) testing (fabrication of specimens where necessary, testing procedure, rules for calculating a statistics z
e) decisions to be taken (criteria for acceptance, rejection or further procedures

Properties may be qualitative or quantitative (the corresponding tests then refer to attributive or variable testing). Where possible, an assessment of quantitative properties is recommended – in view of more efficient statistics; for quantitative properties statistics (z) referring to statistical parameters are recommended for the same reason.

As a general rule, a sampling technique by which each element of the lot has the same probability of being sampled ("random sampling") is always correct.

Where knowledge on the inherent structure of the lot is available, this should be utilized, rendering more efficient sampling techniques, e.g.:

- sampling at weak points, when trends are known
- sampling at specified intervals
- stratified sampling
- extreme value sampling

The larger efficiency results in e.g. smaller sample sizes for obtaining the same filtering capability of a test.

3.3.2 Operation Characteristics

A common way to describe the efficiency of a given test plan is by the so–called operation characteristic as a conditional probability.

$$L(q) = P(A|\ d(z),\ q) \tag{3.3}$$

which describes the probability of accepting a lot with Q = q, assessed by a statistics z and a decision rule d(z). The statistic z can be multidimensional (see, also [6]).

Operation characteristics measure the potential filtering capacity of compliance specifications. For an assessment of the effect of compliance testing, however, the corresponding material supply needs to be considered.

3.3.3 Concept of a Filtered and Unfiltered Supply

For the assessment of a material supply by means of sample parameters recorded in the context of previous compliance tests, there are two possibilities concerning the available data:

1. the data refer to ALL lots subjected to testing
2. the data refer only to the accepted lots

In the first case, estimation of $F_Q(q)$ renders a description of the original supply prior to any rejections – it is denoted as the "unfiltered" supply (US).

In the second case, application of the aforementioned estimates renders a description of the supply as it passes compliance testing, i.e. the "filtered" supply (FS). Hereby it is presumed, that rejected lot are not incorporated into structures (cf. however sec. 3.4).

If all data are available then an empirical assessment of the filtering effect of the existing compliance specifications may be pursued by comparing the US and FS in terms of the respective distributions, the distribution of lot parameters or of the resulting distributions for X – cf. sec. 2.4.2 and 2.4.3.

Mathematically, the FS ($f_Q(q|A)$) can be derived from the US ($f_Q(q)$) by:

$$f_Q(q|A) = \frac{f_Q(q)\ L(q)}{\int f_Q(q)\ L(q)\ dq} \tag{3.4}$$

where the normalizing constant in the denominator represents the proportion of accepted lots, given a compliance rule d(z). For example, with the operation characteristic $L(m,\sigma)$ the probability of acceptance P(A), with the frequentistic meaning of an acceptance rate h(A) is obtained from

$$P(A|d(z)) = \iint f(m,\sigma)\, L(m,\sigma)\, dm\, d\sigma \qquad (3.5)$$

where $f(m,\sigma)$ describes the distribution of M and Σ in the unfiltered supply.

Where $f(m,\sigma)$ is modeled by a normal–gamma distribution (or normal distribution) analytical solutions are possible for some particular decision rules [7,8]. For more complex rules, generally simulation methods are required.

$P(A|/d(z))$ as related to the sequence of units of production can also be estimated from

$$h(A) = \frac{\text{Number of lots accepted}}{\text{Number of lots tested}}$$

However, an empirical assessment is limited to the assessment of existing compliance specifications (or slight modifications to existing specification). Furthermore it may be noted, that in many cases only data from accepted lots are recorded.

Hence, for an assessment of the filtering effect of new specifications or for estimating the effect of anticipated alterations in the supply (US, e.g. due to new technologies) – analytical (or numerical) methods need to be applied.

3.4 Tentative Relation between Design Assumptions and Compliance Control

3.4.1 Basis

In the model for specifying material properties for design the material supply is described by the distribution of lot parameters $F_Q(q)$ which may apply to

1. the unfiltered supply (US), i.e. the original supply prior to any compliance control
2. the filtered supply (FS), i.e. disregarding all non–compliant lots according to a given decision rule d(z).

Combining the distribution of lot–parameters in the selected supply with the distribution $F_Q(x|q)$ of X within lot, renders the distribution of X within the supply: $H(x)$ or $H(x|A)$ – cf. eq. (2.1) – either of which may be assumed in design.

Prior to any choice of supply (i.e. US or FS) for design, we may recall the two (possible) objectives of compliance control mentioned in sec. 3.1. Relating the two objectives to the assumed distribution $H(x)$ assumed for design results in the following criteria:

(a) the predictive distribution of X within every lot and/or
(b) the distribution of X within the actual supply

shall not render less favorable event probabilities (or design values) than the distribution $H_X(x)$ assumed for design. In the following these two options are discussed.

3.4.2 Criteria related to the individual lot

Criterion (a) of 3.4.1 which related the predictive distribution of X within a lot to the distribution assumed for design renders a decision rule for compliance testing.

This may be obtained by e.g. comparing the probability associated with the design value x^* in the assumed distribution $H_1(x^*)$ with the corresponding probability in the predicted distribution $H_2(x^*|q)$

$$H_1(x^*) \geq H_2(x^*|q)$$

This comparison renders limits for $\hat{q}$, which may be used as decision rule d(z).

Illustration 3

As an example the simple Gaussian model is used, with known within lot variance σ. The distribution of X assumed in design is given by

$$H_1(x) = \Phi\left[\frac{\bar{x}'-x}{\sigma}\sqrt{\frac{n_e}{n_e+1}}\right] = \Phi\left[\frac{\bar{x}'-x}{\sigma_1}\right]$$

implicitly assuming an unfiltered supply. The predictive distribution of X, accounting for the prior information from the supply and depending on the estimator $\bar{x}''$ for the mean is

$$H_2(x|\hat{q}) = \Phi\left[\frac{\bar{\bar{x}}'-x}{\sigma}\sqrt{\frac{n''}{n''+1}}\right], \quad \bar{\bar{x}}'' = (\bar{x}_0\, n + \bar{x}'\, n_e)/n'', \quad n'' = n + n_e$$

Equating probabilities associated with a design value $x^* = \bar{x}' - \alpha\,\beta\,\sigma_1$ renders a limit for the sample mean $\bar{x}_0$

$$\bar{x}_0 \geq \bar{x}' - \alpha\beta\sigma\sqrt{\frac{n''+1}{n''}}$$

corresponding to the decision rule

$$d(z):\begin{cases}\bar{x}_0 \geq \bar{x}' - \lambda(n)\,\sigma \text{ then accept}\\ \text{otherwise reject}\end{cases}$$

Here α is the so–called sensitivity factor ($\alpha \approx 0.8$) and β the target safety index. If the distribution of X assumed in design would be based on the filtered supply ($H(x|A)$ instead of $H(x)$) for a given decision rule, this would render more severe limits for the sample mean $\bar{x}_0$.

3.4.3 Criteria related to the supply

Criterion (b) can only be pursued by monitoring the supply, referring in particular to the supply from individual sources (producers). Monitoring the supply involves

- observations from the previous supply
- prediction of the future supply based on the previous observations (which only will differ in case of trends)
- correction measures or adaptation of measures, such that criterion (b) may be assumed to be observed.

Assuming now that material specifications for design are based on the unfiltered supply (cf. no.1 in 3.4.1), the following considerations apply:

- if the actual (unfiltered) supply of a producer is "as good as" or "better than" the assumed supply – e.g. in terms of design values – criterion (b) is observed without need for decision rules involving defacto–rejections of lots
- if the unfiltered supply of a producer is "worse" than the assumed supply, then decision rules involving defacto–rejections are required, such that his filtered supply corresponds to the assumed supply
- if the filtered supply of a producer (applying a given decision rule), is "worse" than the assumed supply, more stringent criteria are required

In the other case, i.e. material specifications for design are based on a filtered supply (cf. no.2 in 3.4.1), implicitly accounting for a given decision rule, analogous consideration nevertheless apply:

- if the actual (unfiltered) supply of a producer is "as good as" or "better than" the assumed supply – criterion (b) is observed without need for defacto–rejection of lots
- if the filtered supply of a producer (applying a given decision rule), is "worse" than the assumed supply, more stringent criteria are required
- if the filtered supply of a producer (applying given decision rule), is "better than" the assumed supply, less severe criteria could be adopted.

A convenient measure for a comparative assessment of supplies is the (fictitious) acceptance rate h(A). The original supply assumed for design purposes will be associated with an acceptance rate h'(A) for a given decision rule. Where a filtered supply is assumed for design purposes, a rate h''(A) may be determined by subjecting the filtered supply again to compliance control.

Assuming an initial phase of production where given decision rules (involving defacto–rejections) are applied, the following strategy may be adopted after this initial phase:

	Material specifications for design are based on an unfiltered supply	a filtered supply
no defacto–rejections for	$h(A) \geq h'(A)$	$h(A) \geq h''(A)$
rejection according to more stringent rules required if	$h(A) \ll h'(A)$	$h(A) < h''(A)$
otherwise reject according to given rule		

Assuming an unfiltered supply requires fewer adaptation of decision rules than the filtered supply, for which a further domain for rejection according to less stringent rules could be added.

3.4.4 Conclusions

Where compliance criteria only refer to the supply (b), defacto–rejections are dispensable where the actual supply is superior to the supply assumed for design purposes. Monitoring of the supply is always required, i.e. also for criteria relating to individual lots (a).

Decision rules derived on the basis of predictive distributions for lots only render reasonable acceptance rates if the variability between lots is of the same magnitude (or larger) than the variability within a lot. This situation is consistently reflected by a noticeable filtering effect of compliance control in the supply.

Hence, objective and corresponding strategies in compliance control should depend on the structure of the supply:

A. For supplies with small within–lot variability as compared to the variability between lots, compliance control may effectively identify inferior lots. Procedures for non–compliant lots are always required. Monitoring of the supply is required to support the assessment of lots. For economic reasons the design specifications may be based on the filtered supply.

B. For supplies with large within–lot variability as compared to the variability between lots, compliance control is effectively pursued only in terms of monitoring the supply. Defacto–rejections are not required for superior supplies. Design specifications are preferably based on the unfiltered (original) supply.

Compliance control with the major emphasis on monitoring of the supply (B) is mainly concerned with collecting information on the supply. On the basis of data records, an external control body may exempt a producer from defacto–rejections and is mainly in charge of monitoring the supply. For compliance control with major emphasis on the assessment of individual lots (A) adherence to procedures required in case of non–compliance is essential. This has to be ensured by an adequate organizational form of the internal control. An external control may check adherence to this procedure (possibly involving an independent assessment of lots at random) and should be concerned with monitoring the producers supply.

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