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INVESTIGATION OF THE ERGODICITY ASSUMPTION FOR SEA STATES IN THE RELIABILITY ASSESSMENT OF OFFSHORE STRUCTURES

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ABSTRACT

Various schemes for the computation of structural reliability in the presence of ergodic wave load processes, ergodic sea state sequences and simple (non–ergodic) random variables such as strength parameters or statistical uncertainties are investigated for both non–deteriorating and deteriorating structural properties. It is assumed that at least conditional outcrossing rates can be computed. It is found that rigorous formulations can require substantial numerical effort even in the context of FORM/SORM computation schemes as they require nested integration. However, for high reliability and not too large variabilities of the non–ergodic variables simpler computation schemes can be used. In the non–deteriorating case use of the ergodic theorem can be made. In the deteriorating case Jensen's inequality can be applied to arrive at simple and appropriate computation schemes.

NOMENCLATURE

A	= arbitrary event
b	= velocity of settlement of the platform
E[.]	= expectation
F	= failure domain
g(.)	= state function
$H_{1/3}$	= significant wave hight
J(.)	= integral of outcrossing rate with respect to time
$N^+(F,t)$	= number of the outcrossings in [0,t] through failure surface
N_0	= number of zero level upcrossings in [0,t]
P(.)	= probability of an event
P_f	= failure probability
Q	= vector of random variables / sequences
$F_{\mathbf{Q}}(\mathbf{q})$	= distribution function of **Q**
$f_{\mathbf{Q}}(\mathbf{q})$	= densitiy function of **Q**
R	= non–ergodic random variable
m_R	= mean of random variable R
σ_R	= standard deviation of random variable R
$T_\pi(.)$	= transformation of probability distributions
σ_s	= standard deviation of zero mean normal process
T	= random time to failure
T_0	= significant wave period
t	= reference time
U	= standard normal vector
X	= ergodic vector process
α	$= m_R/(\sigma_R^2 + \sigma_s^2)^{1/2}$
$\beta_G(t)$	= generalized safety index
$\Delta\,\vartheta$	= duration of a sea state
$\Phi(u)$	= standard normal integral
$\nu^+(F)$	= outcrossing rate for failure domain F
ν_0^+	= rate of zero level upcrossings

INTRODUCTION

Offshore structures are exposed to continuously randomly varying wave loads causing multi–dimensional stresses in the structural members. Other time–variant loads like current, wind, life or operation loads are also present. The structures have to withstand not only extreme loading conditions but also the long–term, material–deteriorating cyclic wave loading. The worst conditions occur when extreme loading events meet an already reduced structural capacity. Their likelihood increases with the age of the structure.

Considerable uncertainty usually exists not only with respect to the resistance properties and the structural geometry but also with respect to the actual loading environment at a particular location of the structure. Therefore, serious attempts have been made to quantify structural reliability probabilistically. This is, however, not an easy task as it requires sophisticated stochastic models for the uncertainties and non–trivial reliability computation schemes.

While uncertainties about structural properties but also about parameters of the loading can be modeled by simple random variables or vectors, the time–varying sea states must at least be modeled by a random sequence. The wave loading itself must be modeled by a random process in continuous time whose parameters depend on the random sea state sequence. The sea state sequence, in general, can be assumed to be a stationary and, given its parameters are known, even by an ergodic sequence. Also, for sufficiently long duration of the sea states, the wave elevation process can be assumed to be stationary and ergodic.

In this paper the concepts to compute reliabilities for stationary and ergodic conditions in the presence of time–invariant non–ergodic parameters will first be reviewed. Focus will be on numerical techniques like FORM/SORM. The effect of various simplifications will be studied. The considerations parallel very much those carried out recently by Bjerager [Bjerager et al., 1988]. Then, the important case of deteriorating structures will be discussed in more detail, in the framework of FORM/SORM. The reliabilities to be computed are based on the mean number of exits of the load effect process into a failure domain. Those, in turn, are estimated from the corresponding crossing rate. The reliabilities so determined are asymptotic in the sense that the results may hold no more in full rigor for relatively low reliabilities. The theoretical insights will be demonstrated at a number of numerical illustrations.

TIME–INVARIANT NON–ERGODIC PARAMETERS AND ERGODIC SEQUENCES (PROCESSES) FOR SEA–STATES AND WAVES

Denote by

$$E[N^+(F;t)] = \nu^+(F)\, t \qquad (1)$$

the mean number of crossings of a regular stationary and ergodic load effect process into the failure domain F during the time interval [0,t] where $\nu^+(F)$ is the corresponding outcrossing rate. Then, if the structure is initially intact, it is well known that the failure probability can be bounded by

$$P_f(t) \le P_f(0) + E[N^+(F;t)] \qquad (2)$$

or, if certain mixing conditions for the load–effect process $X(t)$ are fulfilled, can be approximated by

$$P_f(t) \sim 1 - \exp[-E[N^+(F;t)]] \qquad (3)$$

Eq. (2) can also be used as an approximation but here we are primarily concerned with the high reliability approximation in eq. (3). Next, assume that there is a sequence of stationary and ergodic sea states characterized by the vector sequence $\mathbf{Q} = (\mathbf{Q}_1, \mathbf{Q}_2, \ldots, \mathbf{Q}_j, \ldots, \mathbf{Q}_n)^T$ where n is the number of sea states in the interval [0,t]. This vector may include variables such as significant wave height, zero crossing period, wave propagation direction. The mean number of outcrossings has to be written as a conditional quantity, $E[N^+(F;t,q_j)]$, with q_j a realization of $\mathbf{Q}_j$. The failure probability is

$$P_f(t) = 1 - E_{\mathbf{Q}}[P(\bigcap_{j=1}^{n} (N^+(F;\Delta\vartheta,\mathbf{Q}_j) = 0))] \qquad (4)$$

with $n = t/\Delta\vartheta$ and $\Delta\vartheta$ the duration of each sea state. Among others, Naess has shown [Naess, 1984] that then

$$P_f(t) \sim 1 - \exp[-E_{\mathbf{Q}}[E[N^+(F;t,\mathbf{Q})]]] \qquad (5)$$

This result is best appreciated if the sequence of sea states is assumed to be independent each with duration $\Delta\vartheta$. Then, the probability that failure occurs in the interval [0,t] is given in the first line of eq. (6). It is assumed that all $\mathbf{Q}_j$ have the same statistical properties. Hence the sea state index j is omitted in the sequel for simplicity of notation.

$$\begin{aligned} P_f(t) &\approx 1 - \prod_{(n)} (E_{\mathbf{Q}}[\exp[-\nu^+(\mathbf{Q})\, \Delta\vartheta]]) \\ &\approx 1 - \prod_{(n)} (\sum_{i=1}^{m} \exp[-\nu^+(q_i)\, \Delta\vartheta]\, f_{\mathbf{Q}}(q_i)\, \Delta q_i) \\ &= 1 - \prod_{i=1}^{m} \exp[-\nu^+(q_i)\, \Delta t_i] \\ &= 1 - \exp[-\sum_{i=1}^{m} \nu^+(q_i)\, t\, f_{\mathbf{Q}}(q_i)\, \Delta q_i] \\ &\approx 1 - \exp[-E_{\mathbf{Q}}[\nu^+(\mathbf{Q})\, t]] \end{aligned} \qquad (6)$$

The integration with respect to $\mathbf{Q}$ in the first line of eq. (6) is replaced by a sum with m intervals in the second line each having a length Δq_i. $f_Q(q_i)\, \Delta q_i$ is the probability that the process is in the interval $[q_i, q_i + \Delta q_i]$. Hence, the total expected time the sequence will spend in the interval $[q_i, q_i + \Delta q_i]$ in the long run is $\Delta t_i = t\, f_Q(q_i)\, \Delta q_i$ as a portion of the time interval [0,t]. By the ergodic theorem the probability of failure is given in the third line of eq. (6). Simple mathematical manipulation leads to the fourth line. Finally, the sum with respect to $\mathbf{Q}$ is replaced by the integral in the fifth line.

If, however, non–ergodic time–invariant uncertain quantities $\mathbf{R}$ such as strength parameters and/or statistical uncertainties about the distribution parameters of the sea state sequence need to be considered, it is clear that one has to take the expectation with respect to the variables $\mathbf{R}$ outside the exponent ('outside' integration scheme) because the independence assumptions for the crossings in different sea states no more holds

$$P_f(t) \sim 1 - E_{\mathbf{R}}[\exp[-E_{\mathbf{Q}}[\nu^+(\mathbf{Q},\mathbf{R})\, t]]] \qquad (7)$$

The expectation operation with respect to the variable $\mathbf{Q}$ can still be performed inside the exponent ('inside' integration scheme) in good approximation in eq. (7) due to ergodicity or even independence.

The expectation operation with respect to $\mathbf{Q}$ is analytic only in a few special cases and mostly multidimensional. FORM/SORM methods are well suited to perform the expected operations with respect to $\mathbf{Q}$. In the present form they require that the integration is over standard normal variables which implies that the vector $\mathbf{Q}$ needs to be transformed appropriately [Hohenbichler/Rackwitz, 1981] and that the failure domain is given in the form

$$F = \{\mathbf{X},\mathbf{Q},\mathbf{R} \mid g(\mathbf{X};t,\mathbf{Q},\mathbf{R}) \le 0\} \qquad (8)$$

where $\mathbf{Q}$ must be transformed according to $\mathbf{Q} = T_r(\mathbf{U}_Q)$. $\mathbf{X}$ is an ergodic vector process and $\mathbf{U}_Q$ is a standard normal vector.

The expectation operation with respect to $\mathbf{R}$ is more difficult. It is, however, possible to reformulate the integral by introducing an auxiliary random variable such that FORM/SORM concepts are applicable again [Hohenbichler, Rackwitz, 1981]. Eq. (7) can be interpreted as the distribution function of the time to (first) failure. We introduce the identity

$$1 - \exp[\nu^+(r)\, t] = P(T \le t \mid r) = P(U_T \le u_T) = \Phi(u_T) \qquad (9)$$

where $\Phi(.)$ is the standard normal integral and r is a realization of $\mathbf{R}$. For the conditional failure probability this leads by rearrangement to

$$P_f(t \mid r) = P(T(r) - t \le 0) \qquad (10)$$

with $t(r)$ a realization of the random time to failure $T(r)$

$$t(r) = -\frac{1}{\nu^+(r)} \ln [\Phi(-u_T)] \qquad (11)$$

Inserting the probability distribution transformation

$$\mathbf{R} = T_r(\mathbf{U}_R) \qquad (12)$$

into eq. (11) yields

$$P_f(t) = E_{\mathbf{U}_R}[P(-\frac{1}{\nu^+(T_r(\mathbf{U}_R))} \ln[\Phi(-u_T)] - t \le 0)] \qquad (13)$$

which is precisely the form required for FORM/SORM. This is the formulation proposed in [Fujita, et. al, 1987]. It can be seen that because the quantity $\nu^+(T_r(u_R))$ already requires numerical integration by FORM/SORM two nested algorithms are required which is a serious complication. Experience also shows that this can lead to computational problems. It is, therefore, worthwhile to investigate simplifications. In particular, those simplifications are of interest which require to run only a single FORM/SORM algorithm. As the random vector $\mathbf{R}$ may introduce a strong dependence between the crossing events it is expected that the results obtained with the simplified computation scheme are conservative.

For the quantification of the numerical error a simple example is studied. Assume a stationary scalar valued normal process S and one non–ergodic normal resistance variable R. The outcrossing rate and hence the failure probability in the interval [0,t] can be calculated using classical results [see, e.g. Cramer/Leadbetter, 1967]

$$P_f(t \mid r) \sim 1 - \exp[-\nu_0^+\, t \exp[-(\frac{r}{\sigma_S})^2] \qquad (14)$$

where ν_0 is the rate of zero level upcrossings, σ_s is the standard deviation of the zero mean normal process. The statistical parameters are given in table 1.

Variable	Distribution	Mean	S.D.
R	Normal	(varies)	1–10
S	Normal Process	0	1.0

Table 1: Statistical parameters

In this case the 'inside' integration is analytic. The unconditional failure probability according to eq. (8) is

$$P_f(t) \sim 1 - \exp[-\nu_0^+\, t\, \frac{\sigma_S}{(\sigma_R^2+\sigma_S^2)^{1/2}} \exp[-\frac{1}{2}\alpha^2]] \qquad (15)$$

where $\alpha = m_R/(\sigma_R^2+\sigma_S^2)^{1/2}$. For the 'outside' integration according to eq. (7) it is possible to apply FORM/SORM for the expectation operation with respect to the non–ergodic variable R.

The generalized safety indices $\beta_G(t) = -\Phi^{-1}[P_f(t)]$ for the different integration methods are plotted in figure 1 versus the ratio σ_R/σ_S for a constant value of $\alpha = 2$. The upper curves represent 'outside' integration. If σ_R tends to infinity the coefficient of variation of R approaches the value $1/\alpha = 0.5$. As expected, the difference in the safety indices for the 'inside' and 'outside' integration increases with σ_R. It approaches zero if the resistance is deterministic. Because the outcrossings all depend on the resistance variable R the above difference must grow for a larger number of zero–crossings $N_0 = \nu_0 t$ as shown in the figure. From figure 2 it is seen that the error made by the 'inside' integration decreases with higher standardized thresholds α. Thus, the 'inside' integration for non–ergodic variables yields slightly conservative results except in extreme cases (N_0 in figure 1 and 2 large). Furthermore, if interest is in the sensitivity of the probability with respect to parameters or in an optimal set of design variables rather than the absolute value of the safety index, the 'inside' integration can be shown to be sufficiently accurate. If, however, the parameter vector is an ergodic vector sequence as, for example, the vector $\mathbf{Q}$ describing the variations between sea–states, the 'outside' integration yields unconservative results.

As a general conclusion we have that the order of integration should be correct unless one is satisfied with the upper bound for the failure probability which is obtained by the 'inside' integration scheme for all uncertain parameters.

As a side remark it should be mentioned that in system reliability analysis where, for example, k components must fail in order to cause system failure the time to failure for each component can be evaluated as in eq. (11) but conditioned not only on the vector $\mathbf{R}$ but also on the already failed components $T_k(T_1, T_2, \ldots T_{k-1} \mid r)$. Then, the componental failure events are conditionally independent. System failure occurs if a certain criterion, e.g. structural instability, is reached and the probability of failure along a path including ℓ elements is

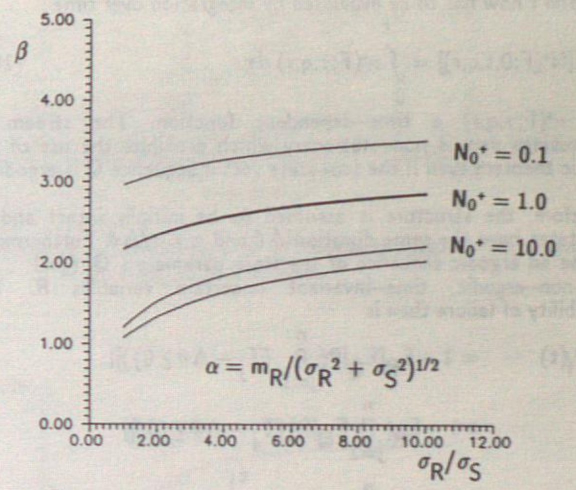

Figure 1: $\beta_{outside}$ and β_{inside} for $\alpha = 2$

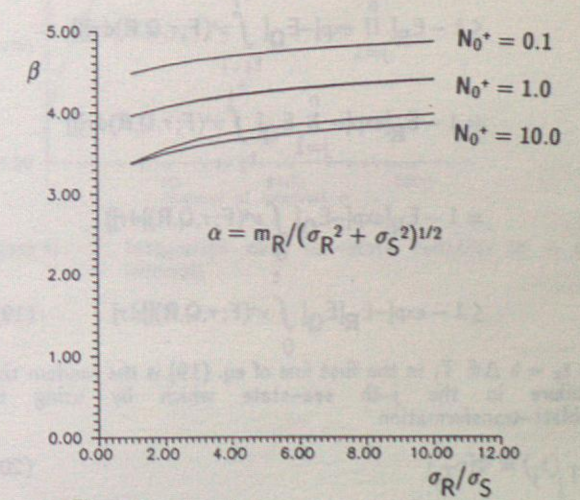

Figure 2: $\beta_{outside}$ and β_{inside} for $\alpha = 4$

$$P_f(t) = P(\sum_{k=1}^{\ell} T_k(T_1, T_2, \dots T_{k-1}|r) - t \le 0) \qquad (16)$$

The quantities T_k all have to be represented as in eq. (11).

NON–STATIONARY CASE WITH NON–ERGODIC PARAMETERS AND ERGODIC SEQUENCES FOR SEA–STATES

The analysis of deteriorating structures or when non–stationary load–effect processes are present is more complicated. Similar to eq. (2) the upper bound for the failure probability is

$$P_f(t) \le P_f(0) + \int_0^t E_R[E_Q[\nu^+(F;\tau,Q,R)]]d\tau \qquad (17)$$

The mean number of outcrossings according to eq. (1) conditioned on q and r now has to be evaluated by integration over time

$$E[N^+(F;0,t,q,r)] = \int_0^t \nu^+(F;\tau,q,r)\, d\tau \qquad (18)$$

with $\nu^+(F;\tau,q,r)$ a time–dependent function. The stream of outcrossings now is non–stationary which prohibits the use of the ergodic theorem even if the sea–state vector sequence **Q** is ergodic.

As before, the structure is assumed to be initially intact and all sea–states have the same duration $\Delta\vartheta$ and $n = t/\Delta\vartheta$. Furthermore, assume an ergodic sequence of sea state parameters Q_j (j=1, ...,n) and non–ergodic, time–invariant uncertain variables **R**. The probability of failure then is

$$\begin{aligned} P_f(t) &= 1 - E_R[E_Q[P(\bigcap_{j=1}^{n} \{T_j - \Delta\vartheta \ge 0\})]] \\ &\approx 1 - E_R[\prod_{j=1}^{n} E_Q[P(\{T_j - \Delta\vartheta \ge 0\})]] \\ &\sim 1 - E_R[\prod_{j=1}^{n} E_Q[\exp[-\int_{t_{j-1}}^{t_j} \nu^+(F;\tau,Q,R)d\tau]]] \\ &\le 1 - E_R[\prod_{j=1}^{n} \exp[-E_Q[\int_{t_{j-1}}^{t_j} \nu^+(F;\tau,Q,R)d\tau]]] \\ &= 1 - E_R[\exp[-\sum_{j=1}^{n} E_Q[\int_{t_{j-1}}^{t_j} \nu^+(F;\tau,Q,R)d\tau]]] \\ &= 1 - E_R[\exp[-E_Q[\int_0^t \nu^+(F;\tau,Q,R)]d\tau]] \\ &\le 1 - \exp[-E_R[E_Q[\int_0^t \nu^+(F;\tau,Q,R)]]d\tau] \end{aligned} \qquad (19)$$

where $t_k = k\,\Delta\vartheta$. T_j in the first line of eq. (19) is the random time to failure in the j–th sea–state which by using the Rosenblatt–transformation

$$F_{T_j}(t_j) = \Phi[u_{T_j}] \qquad (20)$$

can be represented by

$$t_j = J_j^{-1}(-\ln[\Phi[u_{T_j}]],q,r) \qquad (21)$$

and t_j a realization of T_j and where

$$J_j = \int_{t_{j-1}}^{t_j} \nu^+(F;\tau,q,r)\, d\tau \qquad (22)$$

and U_{T_j} is a standard normal variable.

A numerical evaluation of the rigorous formulation in the first line of eq. (19) appears not feasible even if FORM/SORM concepts are applied because the mean number of crossings must be determined for each sea–state (e.g., for a duration of $\Delta\vartheta = 3.5$ [h] there are about 2500 sea state in one year). Simplifications are necessary.

The second line is exact when the sea state sequence is, in fact, an independent sequence. If the dependence structure of the sea state sequence is such that there is

$$P(\bigcap_{i=1}^{n} \{A_i\}) \ge \prod_{i=1}^{n} P(\{A_i\}) \qquad (23)$$

with A_i an arbitrary event the second line even represents an upper bound to the first line. The independence assumption for the sea state sequence also allows to interchange the expectation with the product operator. In the third line the assumed mixing property of the wave process given the parameters **Q** = **q** is used. Substantial further simplification can be achieved by Jensen's inequality (Rao, 1973)

$$E[g(X)] \ge g(E[X]) \qquad (24)$$

for g(.) a convex function from below. Its first application in the fourth line of eq. (19) results in the most significant simplification as it allows integration with respect to time over the whole interval. Its application is based on purely mathematical arguments. Simple algebraic manipulations lead to the fifth and sixth line. In the last line Jensen's inequality is applied a second time. The approximation of the last line can be improved by retaining the integration with respect to **R** outside the exponent in analogy to the ergodic case. The integration can now be performed by FORM/SORM in analogy with the procedure in eqs. (9) to (13). It is

$$P_f(t) \sim E_R[P(J^{-1}(-\ln[\Phi[-u_T]], R) - t \le 0)] \qquad (25)$$

where

$$J = \int_0^t E_Q[\nu^+(F;\tau,Q,r)\, d\tau$$

The inversion of the integral with respect to the upper integration limit has to be performed numerically. Finally, a lower bound for the failure probability is obtained if in the third line of eq. (19) n = 1 and the time integral is extended from 0 to t.

In order to illustrate the numerical differences of the various approximations in eq. (19) a simple example is studied. It is assumed that the threshold function decreases linearly with time. One may view it as the air gap between the mean water level and the lowest level of installations in a platform which settles with time according to $r(t) = r^*(1 - bt)$. b is the velocity of settlement. r^* is the initial value of the air gap. The time dependent failure domain is

$$F = \{S,B\,|\,r^*(1-bt) - s(t) \le 0\} \qquad (26)$$

and the expected conditional number of crossings is [Cramer, Leadbetter, 1967]

$$\begin{aligned} E[N^+(F;t_{j-1},t_j,H_{1/3},T_0,b)] &= \int_{t_{j-1}}^{t_j} 1/T_0 \exp[-8(\frac{r^*(1-b\tau)}{H_{1/3}})^2]\, d\tau \\ &= \frac{H_{1/3}}{4}\frac{(2\pi)^{1/2}}{b\, r^*\, T_0}(\Phi[(1-bt_{j-1})\frac{r^*\,4}{H_{1/3}}] - \Phi[(1-bt_j)\frac{r^*\,4}{H_{1/3}}]) \end{aligned} \qquad (27)$$

where T_0 is the zero crossing period and $H_{1/3}$ the significant wave hight. The sea state parameter $H_{1/3}$ is modeled by a Weibull distribution. For the purpose of this illustration T_0 is assumed to be deterministic. The parameters used in the example are collected in table 2.

Variable	Distribution	Mean	S.D.	units
$H_{1/3}$	Weibull	0.86	0.62	[m]
B	Normal	0.05/0.15	0.05/0.15	[1/s]
t_0	determ.	4.4		[s]
r^*	determ.	1.0/1.5		[m]

Table 2: Statistical and deterministic parameters

Figure 3 shows the approximations of the failure probability which is given in the first line of eq. (19). If both the ergodic **Q** ($H_{1/3}$) and non–ergodic **R** (B) variables are integrated 'outside' and the time integration is extended over the whole interval an unconservative, lower bound for the probability of failure is obtained. If, however, the expectation operation with respect to the ergodic and non–ergodic variable is performed 'inside' (last line of eq. (19)) the result will be a upper bound. If the expectation is performed correctly 'inside' the exponent with respect to the ergodic variables a sharper upper bound is obtained. Note that upper and lower bound differ approximately by one order of magnitude. The quality of Jensen's inequality as an approximation is also investigated, i.e. the results according to the third and the sixth line eq. (19) are compared for n = 6 sea–states. Remember that in each sea state with duration $\Delta\vartheta$ and fixed value of the sea–state parameter **Q** = **q** all crossings for the wave process out of the failure domain F depend on this parameter. Hence, the integration with respect to this variable within the interval $\Delta\vartheta$ has to be carried out outside the exponent. In figure 3 the exact results according to the third line are shown for sea state durations $\Delta\vartheta = 1$ [year]. If, however, the number of sea states and/or the durations of the sea states become small, rapid convergence of the results in the sixth line is observed (figure 4). This behavior is best appreciated if the exponent is expanded into an exponential series with only the linear term retained because the argument of the exponent is close to zero which is true for $\Delta\vartheta \to 0$ as $n \to \infty$. Then with

$$E_Q[\exp[-g(Q)]] > E_Q[1 - g(Q)] = 1 - E_Q[g(Q)] \qquad (28)$$

the inequality sign in the fourth line of eq. (19) can be replaced by an equality sign at least asymptotically. The rate of convergence must depend on the relative variability of the wave process and the sea state sequence. It should be large for small sea state variabilities as compared to the variability of the wave process. Unfortunately, this is not the case for many sea state environments. But the number of sea states is generally large enough to justify the approximation of the sixth line in eq. (19). For independent sea states the last result allows to calculate the unconditional failure probability with a rather effective integration scheme. In [Bjerager, et al., 1988] it has been shown that even for highly positively correlated sea states this conclusion holds. Finally, for high reliability levels the error by using the last line in eq. (19) can be shown to be small and conservative which parallels the findings for the ergodic case discussed before.

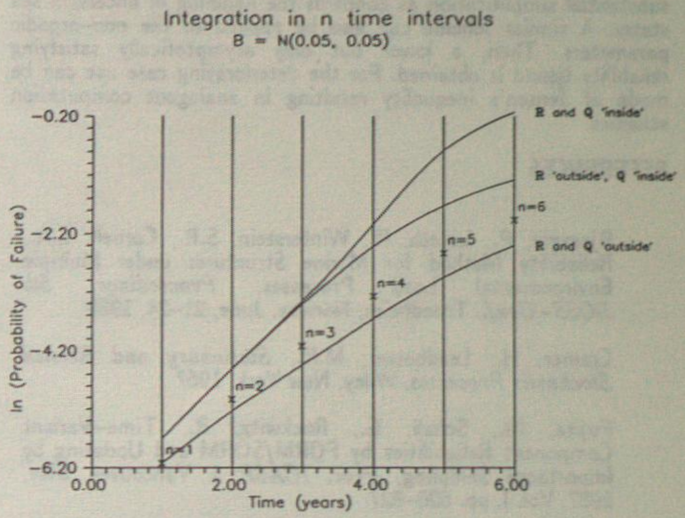

Figure 3: Approximation of failure probability with various integration schemes

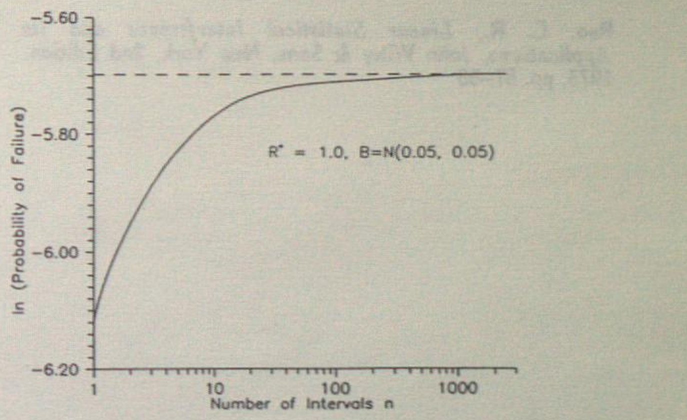

Figure 4: Integration over sea–state variables in n time intervals

CONCLUSIONS

Rigorous computation schemes for the reliability of structural components of offshore installations can be quite involved and numerically expensive even in the context of FORM/SORM. Simple approximations and bounds can, however, be derived. In the non–deteriorating case the ergodic theorem furnishes the basis for substantial simplification as concerns the handling of uncertain sea states. A similar scheme can also be applied to the non–ergodic parameters. Then, a lower but only asymptotically satisfying reliability bound is obtained. For the deteriorating case use can be made of Jensen's inequality resulting in analogous computation schemes.

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