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SHORT COMMUNICATION

DISCUSSION TO: X. CHEN AND N.C. LIND, "FAST PROBABILITY INTEGRATION BY THREE-PARAMETER NORMAL TAIL APPROXIMATION", STRUCTURAL SAFETY, 1 (4) (1983) 269–276

R. Rackwitz and S. Gollwitzer

Technical University Munich, Munich (West Germany)

Presentations and conclusions of the proposal of a "new fast probability integration" method need, in the writers' opinion, clarification and limitation of its domain of validity. The principal idea in the paper is to perform a transformation of a non-normal vector (in contrast to a statement in the paper, this must be restricted to independent vectors) into a normal one with an improper distribution function, such that the curvatures of the failure surface in the approximation point, introduced by the probability distribution transformation, become negligible or even vanish. This is, in principle, an interesting idea. Its application appears suitable if the basic uncertainty vectors are independent and the original failure surface is a hyperplane. It would be interesting to see whether it is possible to relax these ideal conditions, which might be met only occasionally in applications.

The authors, however, appear to have overlooked some work which offers an alternative, general solution to the same problem. It can be split into two steps. First, a probability preserving transformation $\boldsymbol{X} = \boldsymbol{T}(\boldsymbol{Y})$ such that

$$P_f = P(g(\boldsymbol{X}) \leqslant 0) = P(g(\boldsymbol{T}(\boldsymbol{Y})) \leqslant 0) \qquad (1)$$

is performed. Ref. [1] discusses such a transformation. Second, the design point y^* is found and a quadratic approximation of the failure surface is made in this point. The probability content is estimated by

$$P_f \approx \psi(\beta). \qquad (2)$$

ψ is the distribution function of a quadratic form in normal variates. Details for computing eqn. (2) may be found in Ref. [2]. Moreover, in Ref. [3] it is proved that quadratic approximations are asymptotically ($\beta \to \infty$) correct, in which case we have

$$P_f \sim \phi(-\beta) \prod_{i=1}^{n-1} (1 - \beta\kappa_i)^{-1/2} \qquad (3)$$

where the κ's are the main curvatures of the failure surface in the design point.

It is emphasized that formulae (2) and (3) take into account both types of curvatures, the one possibly originating from the probability distribution transformation and the other possibly inherent in the physical context. As pointed out in Ref. [2], the use of quadratic forms also produces simply calculated bounds to the true failure probability by taking circumscribing and inscribing rotational forms, provided that a unique design point exists and the relevant domains meet some convexity requirements.

The authors' remarks that the R-F algorithm can involve serious numerical errors unless double precision is used and that it is

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less fast then the new one indicates a certain unfamiliarity with numerical calculations. In fact, the search algorithm used in the paper is virtually the R-F algorithm. The second remark is believed to be incorrect unless it is meant in the sense that the search for the β-point can be truncated earlier (at points y^* at which $g(\boldsymbol{T}(y^*)) \neq 0$), since curvature information can be included already in a non-stationary point. As an answer to the first remark it is proposed to use separate formulae or expansions for the lower and upper tail of the distribution functions or, almost always sufficient, to work as far as possible with $\ln F(\boldsymbol{x})$ or the corresponding logarithmic expansion. Furthermore, from a numerical aspect the formulation and the execution of any suitable algorithm might usually best be done entirely in the y-space, i.e. by replacing each variable X_i in $g(\boldsymbol{x})$ by $\boldsymbol{T}(\boldsymbol{Y}) = F_i^{-1}[\phi(Y_i)|x_1,\ldots,x_{i-1}]$. It is only the load combination part in the original R-F algorithm which requires that non-normal variables be replaced locally by equivalent normal variables in the original space.

Finally, any proposal or requirement for improvement of first-order reliability estimates should, from a practical point of view, be judged with due consideration of the additional numerical effort and, for high reliability problems at least, in terms of absolute rather than relative errors and with sensible respect to the problem of choosing realistic models for the uncertainty variables.

A number of numerical errors and misprints in the paper have been indicated to the authors.

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