Coordinated trimodal skip-stop plans optimisation for urban rail transit operations under time-variant demand

Master's Thesis

A thesis presented in part fulfilment of the requirements of the Degree of Master of Science in Transportation Systems at the Department of Civil, Geo and Environmental Engineering, Technical University of Munich.

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Submitted on Munich, 21.09.2021

Declaration

I hereby confirm that the presented thesis work has been done independently and using only the sources and resources as are listed. This thesis has not previously been submitted elsewhere for purposes of assessment.

Mueang, Chiang Mai, THAILAND, 21.09.2021, Signature

Abstract

Nowadays, there are numerous transit systems where skip-stop operation is the only possible acceleration strategy to move people faster. The conventional skip-stop scheme, also called the A/B scheme, has been rivalled by a family of flexible skip-stop schemes which have the ability to adapt to time-variant demand. Nevertheless, flexible skip-stop schemes are relatively complex because stopping patterns vary from one train service to another. This research attempts to extend the A/B scheme whose stopping pattern is fixed by station, and it is potentially easier for the users to comprehend and remember than a flexible skip-stop plan. The extension of the A/B scheme aims to enable some capability to adapt to a time-variant demand pattern by a coordination of skip-stop schemes across time periods. A novel optimisation framework is devised for this purpose, and a genetic algorithm is applied to solve the combinatorial optimisation problem. Various station-based skip-stop schemes are then experimented in a case study on the circle Mass-Rapid-Transit Blue Line in Bangkok, Thailand. Perturbations on provided demand distribution were necessary to produce an artificial time-varying demand pattern. However, this perturbation technique leads to a limitation making coordinated skip-stop schemes inefficient. Nevertheless, the optimisation results can be considered valid for at least a simple skip-stop plan optimisation under static demand. The most optimal skip-stop scheme found was the simplest, conventional the A/B scheme which could save total passenger travel time by 2.6%.

Acknowledgements

First of all, I would like to thank Prof. Dr. Constantinos Antoniou and Mohammad Sadrani, M.Sc., for their kind and unparalleled support from early on. Their valuable comments and guidance in the early stage of this thesis have greatly helped me align my understandings in this particular research field.

Secondly, I would like to thank the Federal Government of Germany, Real Invest International GmbH, and ADAC Stiftung who have financially supported my studies in the M.Sc. Program in Transportation Systems at the Technical University of Munich through the Deutschland-stipendium program.

Thirdly, I would like to thank the Mass Rapid Transit Authority of Thailand (MRTA) for the data provided for this research. It plays a vital role in displaying the real-world applicability of the stop-plan optimisation model developed in this study. I hope that the results of this research are able to provide preliminary insights into the venture of making transit systems even better for patrons.

Fourthly, I would like to thank all my friends and families who have gave me moral support and much more during this difficult time of COVID-19 pandemic. I would like to thank Dhapjan-Langbein family for their unmatched hospitality and kindness during my short-but-memorable stay in Munich, Germany where this thesis topic was developed. I would like to leave special thanks to Natapit Roongsrisoothiwong who has always been supportive of me and has kindly lent out his high-calibre personal computer for computational purposes.

Last but not least, I would like to express my appreciation and gratitude towards all my family members. They genuinely care for my well-beings, particularly my elder brother who has dedicated his time and effort to designing and building a brand-new office which has greatly improved my work environment at home.

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1. Introduction

1.1. Background Information

1.1.1. Mass Rapid Transit Systems

Public transit systems play vital roles side-by-side with the urbanisation and economic development of cities and metropolises around the world. They are one of the most economically efficient means of passenger transportation made for urban areas. Technological advancements in construction nowadays allow greater possibilities for high-performance urban transit systems. However, when it comes to high-performance transit systems, large civil structures are required for them to operate with physically segregated, dedicated right of way. Mass Rapid Transit (MRT) systems are an example thereof. It is one of the marketing names for high-performance urban rail-based transit systems; its synonymous system names are such as subway, metro, underground, U-Bahn (German). Hereinafter, this type of transit system will be referred to as MRT. There are distinct characteristics which MRT systems apart from other public transit mode; hence, more description is given below.

Vuchic (2007) classified urban transit modes into 3 groups based on the level of protection from external traffic entities/users. These levels of protection are referred to as Right of Way (ROW) categories. ROW A offers full isolation/protection such that there is zero interaction with other modes of traffic, while the ROW B and C categories offer partial protection and no protection, respectively. The ROW A category is associated with the term 'high-performance' mode mentioned above owing to the use of exclusive infrastructure. On the other hand, road-bound transit such as bus or streetcar systems are of lower ROW C category since they suffer from regularly occurring issues alongside general road traffic. Transit systems having dedicated right of way (ROW A) have numerous operational advantages compared to ROW C modes: high line-capacity-to-space utilisation ratio, competitive normal operation speed, high safety and reliability etc.

According to the definition of ROW categories, not only urban transit (MRT) but also long-haul modes such as commuter rail transit, suburban rail transit, regional transit belong to the ROW A category. Infrastructure for long-haul modes generally includes siding tracks at some intermediate stations for express transit services to overtake more-frequently stopping ones. If the demand allows, third or fourth track may be planned to increase line capacity. In contrast, the functional characteristics of urban transit such as short station spacing and short line length, double-track infrastructure (one for each direction) suffices for most MRT systems. Moreover, MRT systems face with tighter space constraints in densely-populated areas; therefore, most of the existing infrastructural systems predominantly include elevated or subterrestrial tracks and stations. This makes such systems extremely difficult to undergo an infrastructural upgrade. Erecting large transport civil structures with minimal detrimental effects on existing urban developments remains as constructional and economical challenges (Flyvbjerg et al., 2014). Therefore, research on innovative operational strategies aiming to achieve more efficient transit operations is important.

The transit mode of interest for this research is the more infrastructurally constrained MRT in which train overtaking is not possible at any point of the system. All in all, categorisation of ROW and other mode-specific characteristics have significant implications on transit operations modelling and optimisation in the following section.

1.1.2. Passenger Rail Transit Operations Optimisation

Passenger rail transit operations optimisation (PRTO) is a highly attractive field of research, which spans across various aspects of transit operations. Transit operations by themselves are complex products resulted from a hierarchical planning process comprising multiple decision levels as shown in Figure 1 (more details in the review paper by Lusby et al. (2011)). Research works in this field fall into either planning problems (strategic and tactical) or control problems (operational) (Sadrani et al., 2021). Optimisation methodologies in each of these problem families have different optimisation scopes and hence different objectives.

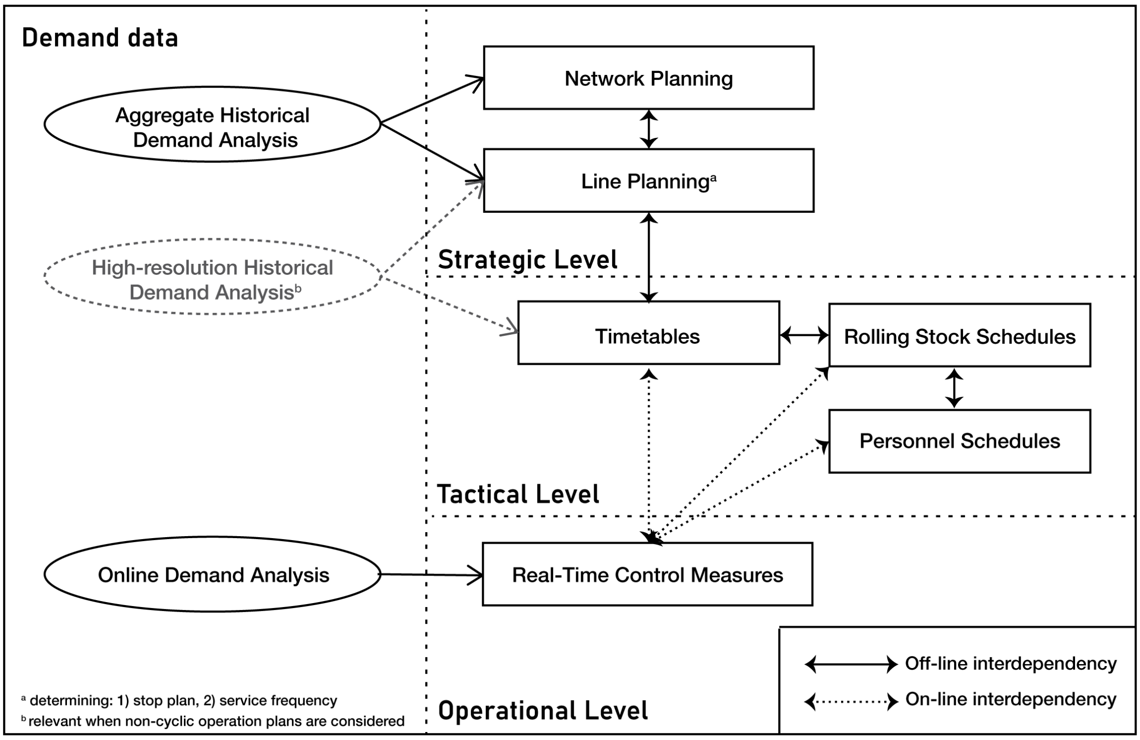


Figure 1 Decomposition of Railway Operation Planning Process

Planning problems concern with high-level decision makings about transit operation plans under certain idealised conditions assumed. The highest level of planning is the network planning. *Network planning* entails every consideration necessary to the realisation of transport project infrastructure, and the output of this process is the boundary conditions of the subsequent line planning step. *Line planning* deals with the determination of the two attributes describing a transit line: stop pattern and frequency. Since it is possible that a solution obtained from the *line planning* step is suboptimal due to some constraints imposed by the preceding network planning step, feedback from the line planning could be coordinated to the higher network planning. Similar recursive process occurs at every step in Figure 1 and therefore double-headed arrow lines are used to pictorially represent this interdependency. The solutions to planning problems are operation plans which are used for a long time horizon before they get updated with new boundary conditions. The length of the planning horizon corresponds to the hierarchical order of each planning step (infrastructural changes from network planning may take decades, and organisational changes from line planning may take years). Therefore, the aggregate summary of historical demand collected over a sufficiently long time period and further analyses of the demand are crucial for these steps.

The step further from line planning is *timetabling* which is the central component in this schema. A timetable defines the departure and arrival times of every train at every station in a line; therefore, it defines the final transport product and serves as the interface between the demand and supply. Conventionally, timetabling does not seek to improve the efficiency of an operation plan but rather attempts to allocate all train movements to the infrastructure in a conflict free manner (Caprara et al., 2002). Therefore, the level of service is defined in the line planning step, and timetabling does not belong to the strategic planning level. Consequently, the conventional timetabling approach produces easy-to-remember *cyclic timetables* which are ubiquitously used. In the last decade, the scope of the timetabling problem has been broadened to considering *non-cyclic timetables* (Figure 2) which can respond better to time variations of demand (Barrena et al., 2014a,b). When non-cyclic timetables are considered, it is imperative that the service frequency will not be constant over the planning horizon. The relaxation of periodicity in the timetabling problem potentially provides benefits when demand is not constant with respect to time (Shang et al., 2016). Under a constraint on the number of train services, non-cyclic timetables show substantial merits to minimising the number of stranded passengers due to system oversaturation in during peak times (Niu and Zhou, 2013; Zhang et al., 2018). Non-periodic timetabling has also been applied to a train corridor with multiple stopping patterns (Niu et al., 2015). Loosely speaking, non-periodic timetabling methodologies adjust the service frequency such that more services are provided in the time of high demand, reducing the total *waiting time* of passengers.

In PRTO, passenger *waiting time* and *in-vehicle travel time* are the two main travel time components. More comprehensively, researchers have additionally considered access and egress time to and from the system (Lee et al., 2014; Gu et al., 2016) as changing stopping patterns could mean that transit patrons will use another neighbouring station instead. While timetabling primarily concerns with the minimisation of passenger *waiting time*, the minimisation of *in-vehicle travel time* is possible only in the *line planning* step in which an efficient stop plan is sought after. At a constant level of transport supply (i.e. constant service frequency), these two time components are contradictory in the *line planning* step, particularly in the making of stop plan. As the number of stops in a train line increases, the travel time increases. Conversely, passengers at the skipped stations experience longer waiting time.

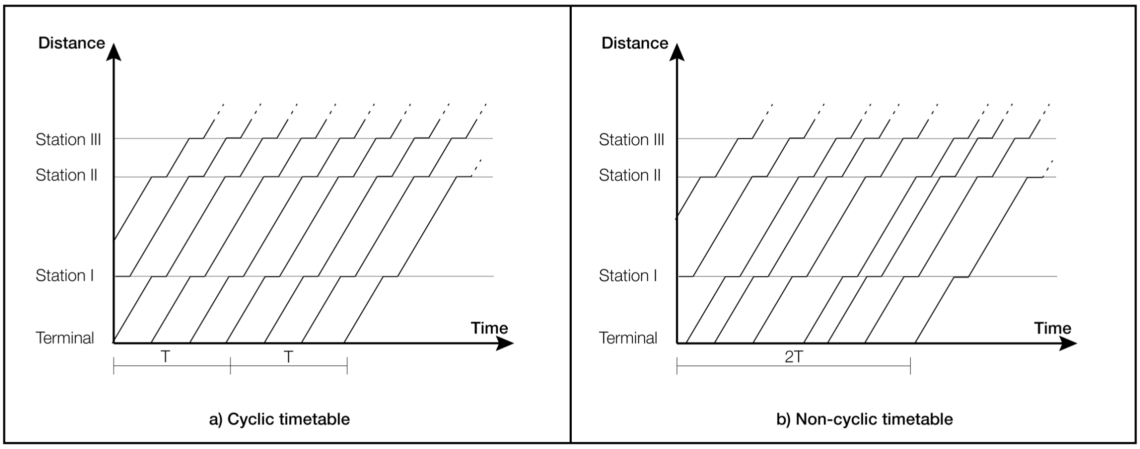


Figure 2 Time-space diagrams: a) cyclic timetable; b) non-cyclic timetable

This is the first trade-off of passenger time illustrated by the left part of Figure 3. The second trade-off in the decision about service frequency is more challenging to take into account in an optimisation formulation since the underlying time units of passenger costs and operator costs are not the same ($[time \cdot pax]$ and $[time \cdot transitunit]$). Whether the operator cost considered is train run time (Cao et al., 2014) or energy consumption of train runs (Wang et al., 2014), incorporating these operator costs to the passenger costs needs an extra step of determining normalising factors for dimensional compatibility of the objective function. Generally, researchers consider passenger time costs in the modelling and assume a constant service frequency.

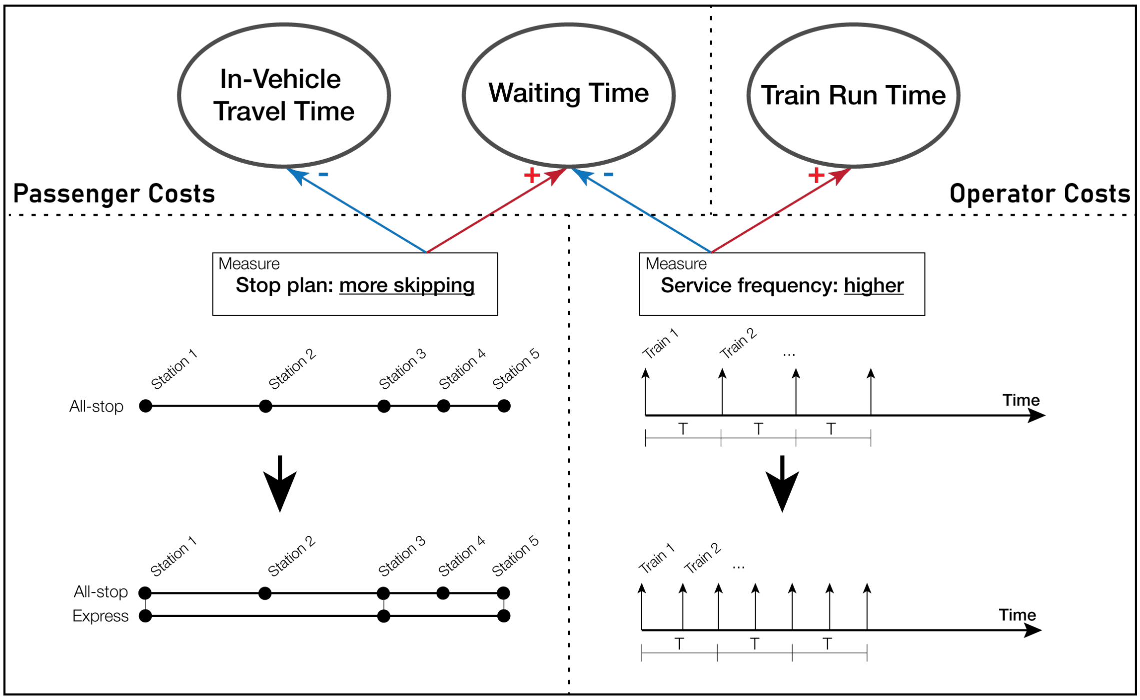


Figure 3 Cause-effect relationships of adjustments in stop plan and service frequency

To tackle the railway operation planning problem in a more comprehensive way, a number of researchers have proposed optimising operation plans on joint problems which combine the *line planning and timetabling* problems. This type of problem formulation attempts to find the best combination of stop plan and timetable (note that the goal for the timetabling part of the joint problem is the same as the goal of determining *non-cyclic timetables*). Dong et al. (2020) have proposed an integrated optimisation framework that deals with the joint problem, and it additionally includes train run time in the objective function. It was shown that the optimised plan results in lower waiting time, lower delay time (the increase from the no-stop running time) and lower train run time. These merits are even more significant in the case of demand oversaturation as the optimised stop plan utilises the train capacity more efficiently, reducing the number of stranded patrons due to trains being full. Qi et al. (2021) have proposed an integer linear programming model to solve the joint problem. If the time-dependent demand distribution with the highest possible temporal resolution is available, the desired departure times of patrons are then known. The study from Qi et al. (2021) highlights the importance of time-dependency consideration, for it reveals these time-specific demand characteristics that cannot be observed in a static demand distribution.

The main problem in *line planning* is stop-plan optimisation which strives to find an optimal stop plan that efficiently serves the given demand patterns. There are chiefly 3 types of stop plan that may be considered to reduce *in-vehicle travel time* relative to standard all-stop operations (Vuchic, 2007). However, the only stopping strategy applicable to systems where overtaking is not possible (MRT systems) is skip-stop (SS) strategy (Figure 4), besides express-local and zonal operations in which train overtakings need to be planned. The so-called conventional `A/B` skip-stop services are provided by train services alternating between `A` and `B` trains which stop at stations of the corresponding station types. Skip-stop plans can be adopted to increase service quality in terms of *in-vehicle travel time* without any substantial investment for infrastructure. The impacts arisen from an implementation of SS that are discussed in the literature and reported from real-world cases will be given in the next chapter on literature review.

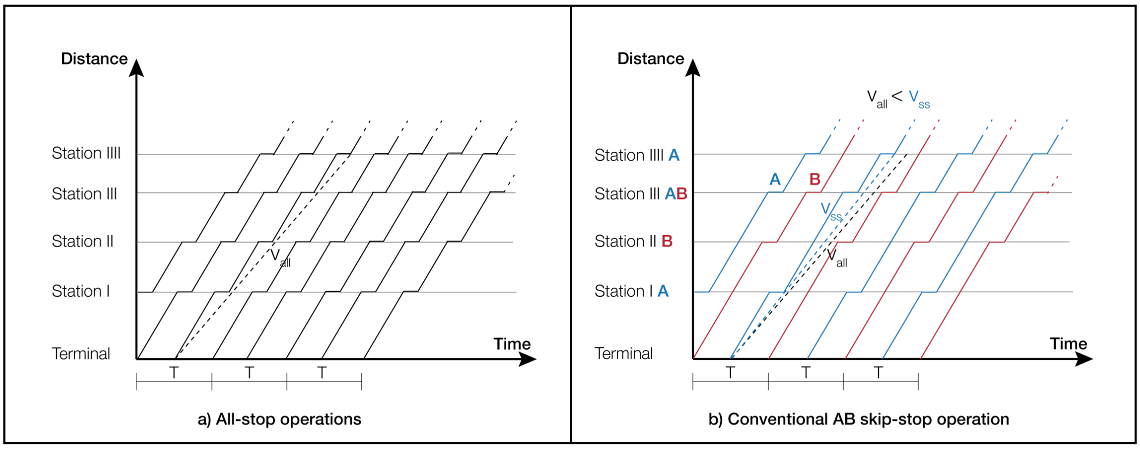


Figure 4 Time-space diagrams: a) all-stop operation; b) `A/B` skip-stop operation

1.2. Structure of Thesis

1.2.1. Research Gap

There have been research efforts to improve the travel time saving further by considering *non-cyclic SS operations* that are identified as *flexible skip-stop operations* given in Figure 5. In a flexible skip-stop operation, each train is permitted to have a unique stopping pattern which is adapted to time-variant demand (requires high-resolution demand data, Figure 1). A flexible SS scheme has degrees of freedom in the adjustment of spatio-temporal availability of services unlike the conventional SS scheme which assumes no time variation of demand patterns. The solution quality in terms of passenger time costs in flexible SS is hence higher than an conventional `A/B` SS operation when there is some degree of time variation of demand.

Some researchers proposed that conventional `A/B` skip-stop optimisation can be independently executed for multiple SS planning periods to adjust the plan to the time variation of demand (Lee et al., 2014). Although the optimality of multi-period approach should theoretically be greater compared to optimising on single period of the time-aggregate demand, any two SS plans are allowed to be completely different. It is obvious that using two completely different stop plans applied on a line over different periods is impractical since it can be confusing to transit patrons. Hence, system complexity will be of concern when applying an SS approach over a day.

Flexible SS operation poses major inconvenience to transit patrons. Flexible SS plans are hard to recognise due to the irregular schedules. Research works in the direction of flexible SS mostly rely on the assumption of effective passenger information system given by a multitude of information dissemination media such as in-station announcement, personal telecommunication devices, electronic signs etc. (Cao et al., 2014; Wang et al., 2014). Bounded rationality of transit patrons is potentially the main reason for its impracticality regarding service design.

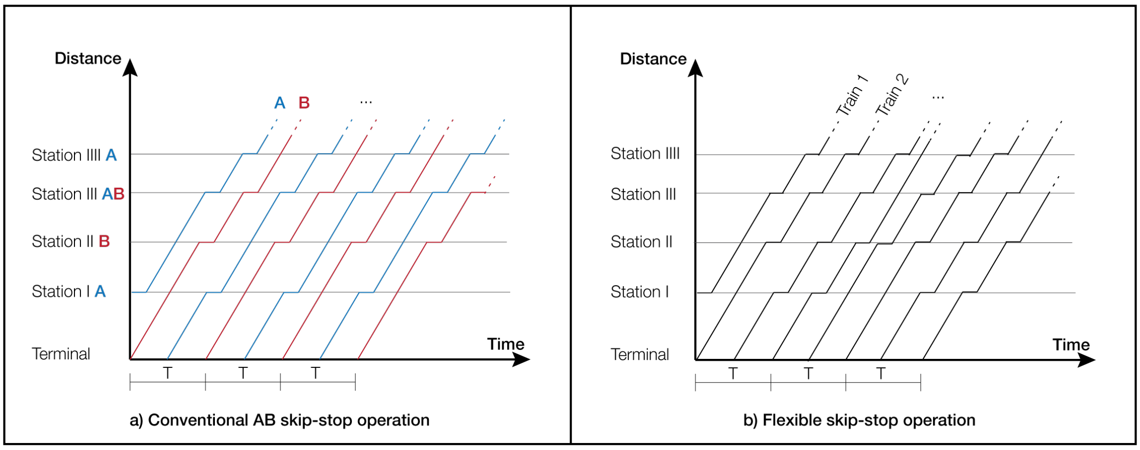


Figure 5 Time-space diagrams: a) `A/B` skip-stop operation; b) flexible skip-stop operation

1.2.2. Research Objectives

This study endeavours to find a compromise between the conventional and the flexible SS operation schemes, an operation framework that has some ability to adjust the schedules to time-variant demand while some extent of service regularity is persisted for passengers' convenience. The optimisation framework proposed in this study considers a coordination between SS schemes where each scheme is subject to a different OD demand distribution. In addition to the conventional `A/B` scheme, trimodal SS schemes are to be included for experimentation.

The foundation of this study is formed around the notion of bounded rationality that transit patrons, as human beings, have limited cognitive capacity, imperfect information, and need to make decisions under time constraints. To avoid passenger confusion, the optimisation framework should not produce a solution with high operational complexity close to a flexible SS plan. To this end, transit operation models for this SS planning problem are created in order that various effects on patrons travel time are quantified and compared between the standard all-stop, conventional `A/B` and other unexplored SS schemes.

Note that this work only considers the joint problem of *line planning* and *timetabling*. Non-recurring, unpredictable disturbances are not covered in this planing step. They need to be managed by the other family of strategy for operation control. Nevertheless, disturbances is typically not an issue in MRT systems (ROW A) but rather in unprotected bus, tram systems (ROW B, C) (Gkiotsalitis and Cats, 2021). The disturbance-related problems of ROW B,C modes are such as bunching, schedule sliding, overcrowding, and one of the control strategies which can cope with them is stop-skipping. Stop-skipping as a control strategy is not to be confused with skip-stop operation as they belong to different levels of operation planning (see Figure 1).

1.2.3. Organisation of Contents

Subsequently, a review of literature related to skip-stop operation optimisation is elaborated in Chapter 2. Then, the development of the mathematical models for SS operation are presented along with the solution algorithm in Chapter 3. In chapter 4, the optimisation model is then applied in a case study on a circle MRT line in Bangkok, Thailand. Optimisation results are described and discussed also in Chapter 4. Lastly, the study is concluded in Chapter 5

2. Literature Review

2.1. Effects of Skip-Stop Operations

2.1.1. Comparisons between Skip-Stop and All-Stop Operations (Vuchic, 2007)

This section summarises the effects of skip-stop operation compared to all-stop operation, which have been discussed in the book from Vuchic (2007). Firstly, transit patrons experience fewer stops along their journeys, and this can be inferred as increased patron comfort. Secondly, higher operating speed is achieved through station skipping, and the operation cycle time is consequently reduced. Hence, service frequency is increased with the same fleet and personnel. The increase of service frequency implies that waiting time at A/B stations is reduced. Therefore, the line capacity of any origin-destination (OD) pairs of two A/B stations is increased. In a nutshell, in-vehicle travel time is reduced for most patrons, and waiting time is reduced only for patrons travelling from a A/B station to another.

Adverse effects from SS operation also arise in exchange for the aforementioned benefits, and they occur to patrons using exclusive A or B stations. Firstly, patrons travelling from or to an exclusive station are offered half of the provided services: their waiting time is double of A/B stations on average. Secondly, for a trip where the origin and destination stations are of different types, patrons face substantial inconvenience of transfer at an intermediate A/B station. Moreover, if there is no A/B station between the origination and destination, the situation is exacerbated by the need to reverse travel also known as backtracking. This case is observed in short OD pairs and all of adjacent A-B stations; hence, short-distance travel is generally discouraged with an introduction of SS operation. Lastly, SS operation increases system complexity and could lead to patrons using a wrong type of train. Passenger information systems may need an enhancement to ensure maximal comprehension about the offered SS operation scheme. All of these effects are summarised in Table 1, and the most affected OD pair types are illustrated by Figure 6.

2.1.2. Effects from Real-World Cases

As explained earlier, skip-stop operation can be considered to speed up a transit system by reducing passenger in-vehicle travel time, and this has been the main motivation for an SS implementation. According to Zhang et al. (2017), the SS operation scheme was invented in 1947 for the Chicago Metro system; later on, it was introduced to the metro systems in Philadelphia and New York. The aim was to increase the passenger in-vehicle travel time during rush hours. However, the Chicago Transit Authority had been gradually eliminating SS services in the 1990s until all services became all-stop in 1995. Although increased travel time had to be shouldered, the Chicago Transit System then observed a significant increase of ridership of 34 to 50 percent at former exclusive A and B stations as the frequency doubled

OD pair type	OD Notation	Travel time	Waiting time	Transfer	Confusion[a]
Type 1: All-stop sta. to all-stop sta.	A/B →A/B	Decreased ↓	Decreased ↓	No	No
Type 2: Exclusive sta. to all-stop sta. or between exclusive stations of the same type	A →A/B B →A/B A →A B →B	Decreased ↓	Increased ↑	No	No
Type 3: All-stop sta. to exclusive sta.	A/B →A A/B →B	Decreased ↓	Increased ↑	No	Yes
Type 4: Between exclusive stations of different types – forward-tracking transfer	A →B B →A	Decreased ↓	Substantially increased ↑↑ (transfer time)	Yes	Yes
Type 5: Between exclusive stations of different types – back-tracking transfer	A →B B →A	Increased ↑ or Decreased ↓	Substantially increased ↑↑ (transfer time)	Yes	Yes

[a] Whether there is a risk of boarding a train of a wrong type

Table 1 Effects of SS operation on each demand segment

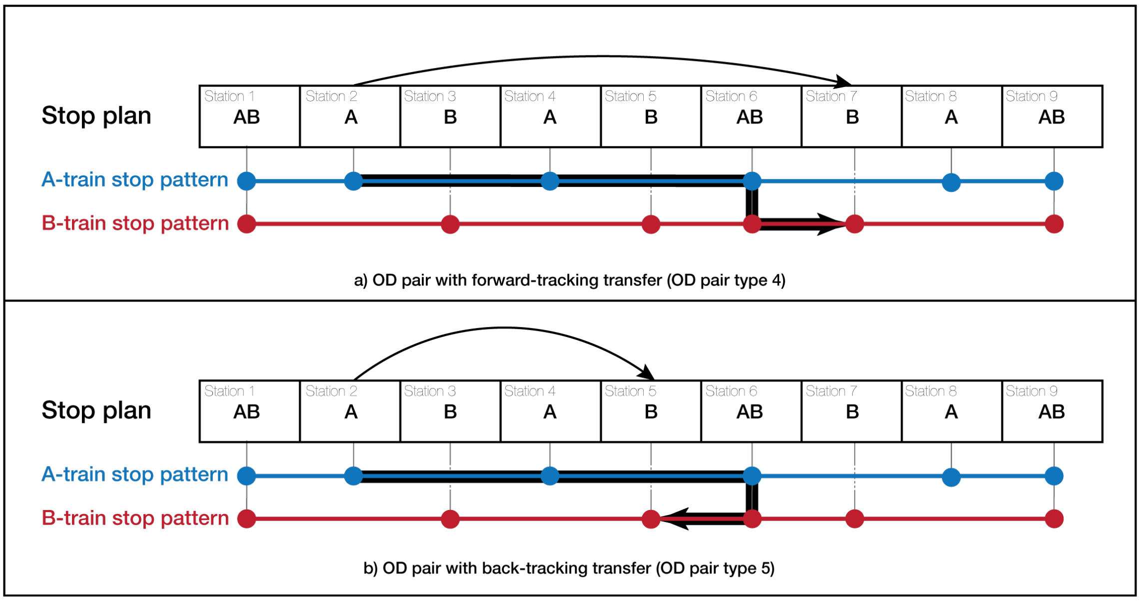


Figure 6 OD pair types requiring transfer in the conventional A/B skip-stop scheme

L. Skip-stop services in Philadelphia's and New York's systems have also been discontinued primarily due to long waiting time at exclusive stations, resulting in all-stop operation nowadays.

On the other hand, the authority of Santiago Metro implemented SS operation in 2007 on its system to tackle overcapacity problems (Freyss et al., 2013). Initially, an SS approach was applied to the less loaded direction during morning peak hours. It was reported that transit patrons adopted it well. The stop plan changes in the initial phase benefited both passengers and the transit operator: skipping of each station reduced 47 seconds of travel time which accumulated to 4.3 minutes of time saving for the most distant station pair; service frequency increased from 34.5 trains to 38 trains/hr with the same fleet; and the operational cost has dropped. These manifestations of operational benefits led to the expansion of the SS scheme to both service directions during both peak times on multiple transit lines. It has been reported that demand elasticity holds for the case of Santiago as the observed number of trips that require a transfer was found to be lower than predicted. Additionally, the importance of demand elasticity in regards of changing stop patterns has also been reflected by the research work by Parbo et al. (2018).

2.2. Optimisation Problem Formulations

2.2.1. Continuous Approximation Approach for a Justification of Skip-Stop Operations

At the network planning step, it is accustomed that engineers and planners base the network design on the standard all-stop approach. In consequence, the infrastructure plans of urban transit projects are finalised under the all-stop scheme, and the infrastructural boundary conditions for the line planning step are fixed. However, if there exists a planning framework which integrates the network planning step down to the line planning step, higher cost-effectiveness of investment could be obtained since the infrastructure will be designed jointly with the determination of the optimal operational scheme. Therefore, it is crucial to realise also the interdependency between network planning and line planning.

There have been research efforts trying to tackle this large-scope problem with the help of some strong assumptions for problem simplification. Some of the major assumptions found in the pioneering works are 1) spatially homogeneous demand distribution, and 2) uniform distribution of stations. As a consequence, `A/B` stations density can be used as a continuous independent variable to represent a SS plan solution. In doing so, the mathematical problem becomes continuous which can be more computationally tractable than a combinatorial optimisation problem. Freyss et al. (2013) constructed such a mathematical formulation and provided sensitivity analysis results which suggest network characteristics that favour skip-stop operation over all-stop operation. The formulation is based on the objective function of travel time, waiting time (transfer time included) and energy consumption of train operation. It provides `A/B` stations density as the sole indication regarding optimal SS plan, given the following input parameters: line length, total stop density, fleet size, total number of passengers. It has been empirically demonstrated that skip-stop operation requires 1) high service frequency to avoid high passenger waiting time, 2) long average trip distance (represented by long line length), 3) numerous stops for it to have substantial advantages over its all-stop counterpart, and these findings agree with the classical literature about SS operation in Vuchic (2007). Later on, Gu et al. (2016) adopted the modelling approach and expanded its extent to analyse other urban transit modes under a multitude of alternative stop schemes.

This advanced research attempted to determine the most suitable operational scheme for a transit mode based on the combined generalised costs of passengers' time, agency's operation and infrastructure. In this work, novel schedule-coordinated SS scheme (Figure 6) was studied along with conventional `A/B` uncoordinated SS and express-local schemes. The rationale for a schedule coordination between transit unit types is to minimise passenger transfer time. For railway systems, this novel coordinated SS scheme requires an upgrade from single track to double tack due to the contradiction between service concept and safety headway in railways. Greater number of skip-stop modes (transit unit types `A/B/C/...`) is also allowed as a free variable. Overall, the coordinated SS scheme was found to be superior to the conventional one, and coordinated SS operation with a larger number of skip-stop modes is incrementally suitable with respect to the combined effect of demand density and trip length. Both of these works provide mathematical models which can provide powerful insights for strategic network planning supported by low-level operational scheme consideration, despite the strongest assumption of uniform passenger distribution.

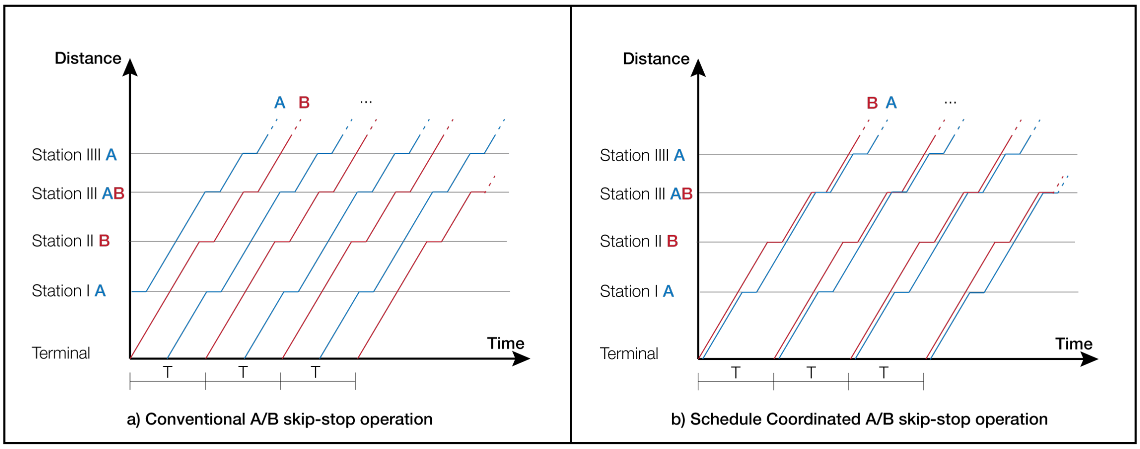


Figure 7 Time-space diagrams: a) AB skip-stop operations; b) flexible skip-stop operations

Recently, enhanced continuous approximation models that account for spatially heterogeneous demands have been proposed. Continuous demand density function, which varies with 1-dimensional coordinates of origin and destination, is used instead of constant demand density scalar. The publication from Mei et al. (2021) provides the first continuous approximation model consider heterogeneous demands. The main findings are bifold: optimal station spacing and distance between two transfer stations are inversely proportional to the local-trip demand density function; optimal station spacing and distance between two transfer stations are proportional to the local cross-sectional patron flows. The sensitivity analysis results also comply with its precedent studies from Vuchic (2007) in that SS operation is particularly effective in systems under high-demand loads. The latest research considering heterogeneous demands from Fan and Ran (2021) compares all-stop, uncoordinated SS and 12 variants of schedule-coordinated `A/B` SS schemes. The simplest variant of the coordinated scheme offers equal headways of `A` and `B` trains, while the other 11 schemes offer `A/B` services of different headway ratios. This research was formulated around the concept that asymmetric deployment of `A/B` trains could be beneficial for schedule coordination. Nonetheless, it was found that the basic coordinated SS scheme with equal headways performs best among all. Since an SS schedule coordination for railways requires extra tracks, its performance is lower than uncoordinated SS scheme when demand density is low to medium.

Continuous approximation studies considerably contribute to the field of operation optimisation especially in the step of network planning. These studies do not attempt to find an optimal SS plan given a set of transit system attributes and demand patterns (which are not specifiable in the pioneering models), but to justify the time-cost optimality of SS operation over all-stop or other operational schemes in an idealised setting. It is true that the incorporation of heterogeneous demands allows the model to return optimal station spacing and `A/B` stop spacing, which can be used to determine stop locations. These models cannot break free from the underlying assumptions about the fixed arrangement of exclusive stations (skip-stop bay). Therefore, they do not serve as stop-plan design algorithms but as high-level preliminary evaluation of operation scheme.

2.2.2. Combinatorial Optimisation for Cyclic Skip-Stop Plans

The conventional `A/B` SS scheme is the simplest cyclic SS scheme where train stop patterns are repeated. The service frequency at any station is constant over time; therefore, they may be called station-based SS plans. Unlike the problem definitions of the continuous approximation studies elaborated above, stop plan optimisation problems are formulated in pursuit of the most time-cost efficient stop plans that suit passenger demand distribution. This group of research works is the most relevant to this thesis topic; thus, generous amount of details thereof are included. There are few studies that focus on conventional `A/B` stop-plan optimisation. All of the reviewed research works in this subsection are summarised in Table 2.

Lee et al. (2014) formulated a combinatorial optimisation model with four sets of constraints. Their studies consider access-egress time in the cases of OD pairs that require a transfer (refer to Table 1). Depending upon the origin-destination locations, affected patrons may opt to use any next closest station that eliminates the need for transfer if the access or egress time to origin or destination station, respectively, is lower than transfer time. Every set of constraints for this problem is created to prevent train collision or any train movements that violate system safety headway; however, each of them allows a different level of solution flexibility. The restrictiveness of constraint sets is asserted by another important system characteristic that is virtually impossible to quantify: the ease of stop-plan comprehension. For instance, the most restrictive constraint set coerces a different type of exclusive station to reside anywhere between 2 exclusive stations of the same type, which results in a stop plan that may appear uniform and hence easy to memorise. The second most restrictive set relaxes the first one by only prohibiting adjacent exclusive stations of the same type, which still allows convenient access to another type of station in cases of compulsory-transfer OD pairs. The 2 least restrictive sets allow any configuration of stop patterns. The second least restrictive set constrains on uniform departure headway at terminal station, whereas the least restrictive set does not. Under the least restrictive constraint set, the departure headway at the terminal could become asymmetric ($\mathtt{A}\rightarrow\mathtt{B}$ headway $\neq$ $\mathtt{B}\rightarrow\mathtt{A}$ headway), thereby permitting a larger number of consecutive skipping stations of the same type. The optimisation results verify that the most flexible (or least restrictive) constraint set allows for the most optimal skip-stop plan with respect to the objective function. A different formulation of the same problem was presented in research work by Yang et al. (2019). It uses a fleet size constraint to define the service headway instead of a predefined value in Lee et al. (2014). To prevent backtracking OD, the constraint on the density of A/B stations is used to avoid long sequences of exclusive stations. Segmented time saving results from a numerical experiment on Line 6 of Beijing Urban Transit System correspond to Table 1.

Huang et al. (2017) proposed a variant of A/B SS plan for bus-rapid transit operations which suffer reliability problems due to unbalanced passenger loading in one direction. Note that bus-rapid transit generally is categorised into the ROW B category as it has to ride through intersections where road space is shared with general traffic. Following the problem definition, the optimisation model is direction-specific, and 3 strategies in terms of SS directions are studied: unidirectional SS in the peak direction with an all-stop plan for the non-peak direction, unidirectional SS in the non-peak direction with an all-stop plan for the peak direction, and bidirectional SS plan. It was shown that the optimised SS plan in the peak direction can realise overwhelmingly superior transit operations. Bus bunching occurrence is significantly reduced and so is the passenger travel time. Moreso, the optimised bidirectional SS plan performs even better than the mentioned unidirectional plan. The standard deviations of bus cycle time and passenger loading decrease in relation to all-stop operations.

Jamili and Pourseyed Aghaee (2015) were concerned with the uncertainty of passenger demand and devised a robust optimisation approach for SS plans. A reliability level would need to be given to such a model. A special characteristic of this model is the adjustable number of skip-stop modes, and this can be from 1 (all-stop plan) to the total number of trains (flexible skip-stop plan). The results show that, due to the higher flexibility of solutions, a plan with a higher number of train stop patterns performs more optimally. For this reason, this study can also be classified as a flexible SS study. However, they did not discuss the complexity of those multi-pattern plans, which inevitably has an implication in real-world applications.

All of these studies consider rather short planning horizons of one to few hours, and optimise for a single SS plan for the respective time period. None of them have explored the possibilities of a trimodal (A/B/C) scheme or any schemes of higher orders in conjunction with SS plan transition across time-variant continuum of demand.

Publication	Objective function	Type of demand data	Scope of modelling	Decision variables	Constraints
Lee et al. (2014)	**Passenger costs** travel time, waiting time, transfer time, access-egress time	Static demand	Not specified	**Combinatorial opt.** ✓Stop plan	• Minimum safety headway • Maximum headway for passenger demand • 4 separate sets of constraints allowing different levels of flexibility of stop patterns
Jamili and Pourseyed Aghaee (2015)	**Operator costs** Train run time	Static demand	Unidirection	**Combinatorial opt.** ✓Stop plan	• Minimum safety headway • Vehicle capacity
Huang et al. (2017) (for bus-rapid-transit systems)	**Passenger costs** travel time, waiting time, transfer time	Static demand	Bidirection, Direction-specific stop pattern	**Combinatorial opt.** ✓Stop plan	• Minimum headway forbidding overtaking • Compulsory stops at both line termini
Yang et al. (2019)	**Passenger costs** travel time, waiting time, transfer time	Static demand	Unidirection	**Combinatorial opt.** ✓Stop plan	• Minimum safety headway • Fleet size • Minimum density of AB stations (to minimise the number of backtracking OD pairs) • Compulsory stops at both line termini

Table 2 Summary of problem formulations for conventional skip-stop plan optimisation in the literature

2.2.3. Combinatorial Optimisation for Non-Cyclic Skip-Stop Plans

As mentioned in the introduction chapter, non-cyclic skip-stop plans allow each individual train service to have its own stopping pattern. As a result, the optimisation problem for this kind of strategies is much larger than the problem for station-based SS plans, meaning that there is greater solution flexibility for it to adapt to dynamic demand. Hence, an optimised flexible SS plan virtually always performs better than its optimised `A/B` SS plan counterpart. For this reason, numerous researchers paid attention to the flexible SS operations optimisation. Despite the argument stated in the research gap section, reviews on several flexible SS plan optimisation studies are given in this subsection. All of the reviewed research works in this subsection are summarised in Table 3.

The first flexible SS model recognised here is from Sogin et al. (2012). The formulated model was used to find stop patterns of 14 passenger train services on the Metra Union Pacific North Line (Chicago, USA) running from 6 AM to 9 AM. Since it is a regional train line where service frequency is not as high as in an MRT system, the stop plan solutions were deemed feasible in spite of the lack of safety headway constraint. This study considered only direct trips for demand fulfilment, while unserved demands are considered as a penalty term in the objective function. Another simplification is the consideration of static demand, which does not fully leverage the potential of flexible SS scheme. Cao et al. (2014) formulated a flexible SS scheme that prevents waiting time longer than twice of the service headway. In this formulation, safety headway was not considered as a constraint, but any two consecutive stations may not be skipped by the same train, which can be feasible when there is enough line capacity remaining and assuming that the subsequent and precedent trains do skip approximately the same number of stations. An important feature of this formulation is the constraint that any two consecutive trains may not skip the same station; it eliminates the need for transfer and long waiting time. To illustrate, when a patron is first served with a train that skips the desired destination station, the patron may use the next train which is guaranteed to stop at the intended destination station. Although static demand was considered for just one train in the case study, it is by its virtue a flexible SS, for it is a train-based stop plan. Moreover, the objective function also incorporates train running time to represent the operator cost besides passenger waiting time and in-vehicle travel time. The mixed-integer optimisation problem formulation from Wang et al. (2014) not only solves for stop plans but also travel speed, departure and arrival time of each train. The solution method proposed for this complex problem is a bi-level approach, as a result, which solves for a stop plan, fixes the respective stop plan and then deploys a sequential quadratic programming solver for shifting of train run trajectories. This was proposed to be executed in a real-time rolling horizon setting for reactive adjustment of stop-scheduling strategy. Unlike other studies that assume a constant stop-dwelling time, this study adopted a dwelling time function which is non-linear with respect to boarding and alighting passengers. An energy consumption model was created for the inclusion of operator cost. Transfers are not allowed in this model, and there is no guarantee on the longest waiting time that a skipped OD pair will have to experience. Only maximum departure headway is specified for a waiting time guarantee, which is effective only at stopped stations. This is another major drawback of flexible SS schemes when the Cao et al.'s (2014) constraint is absent (see non-cyclic timetable optimisation study by Dong et al. (2020)).

The following models exploit actual time-dependent demand data. Zhang et al.'s (2017) proposed a model where train departure time is also a free variable, upsizing the optimisation problem. The settled solution approach has the departure time variable substituted by simulated values according to the most restrictive constraint. Constraints on service headway were set up to ensure reasonable waiting time. They also studied different scenarios where a portion of patrons mistakenly board a wrong train that skips their destination stations. Confused patrons are assumed to leave the train at the subsequent station and wait for the next train that fulfils their needs. Since the possibility of confusion only occurs to low-demand OD pair of which the destination station is skipped, the benefits of the SS scheme overrule even at the scenario of 100% confused passengers. Lastly, Zhao et al. (2021) constructed a flexible SS plan optimisation model that features constraints similar to Cao et al.'s work 2014 but under time-variant demand.

After conducting the literature review, a multitude of details in the modelling efforts to produce the most well-round models with precise representation of transit operations have been given. The flexible skip-stop plans may face with user acceptance issues regarding the ease of use, especially when applying it to a system that has long been under an all-stop scheme. Most importantly, none of the studies have explored any opportunities of using the more comprehensible `A/B` scheme in an attempt to fit the stop plan to time-variant demand.

Publication	Objective function	Type of demand data	Scope of modelling	Decision variables	Constraints
Sogin et al. (2012)	**Passenger costs** travel time, cost of unfilled demand	Static demand	Unidirection	**Combinatorial opt.** ✓Stop plan	• Train movement constraints based on a graph-based solution representation
Cao et al. (2014)	**Passenger costs** travel time, waiting time **Operator costs** train run time	1 static demand pattern for 1 train	Unidirection	**Combinatorial opt.** ✓Stop plan	• Two consecutive trains may not skip the same station • Two consecutive stations may not be skipped by the same train • Compulsory stops at both line termini
Wang et al. (2014)	**Passenger costs** travel time, waiting time **Operator costs** Energy consumption of train runs	Static demand (rolling horizon approach for real-time solutions)	Unidirection (cyclic line)	**Combinatorial opt.** ✓Stop plan **Sequential quadratic programming** ✓Segment speed ✓Departure time	• Minimum safety headway • Maximum headway for passenger demand • Turnaround capacity at termini • Vehicle capacity (stranded passengers are taken into account)
Zhang et al. (2017)	**Passenger costs** travel time, waiting time	Time-variant demand	Bidirection	**Combinatorial opt.** ✓Stop plan **Simulated instead of continuous opt.** ✓Departure time	• Minimum safety headway • Maximum headways for passenger demand and allowable waiting time • Turnaround capacity at termini
Zhao et al. (2021)	**Passenger costs** travel time, waiting time **Operator costs** Train dwell time	Time-variant demand	Unidirection	**Combinatorial opt.** ✓Stop plan	• Two consecutive trains may not skip the same station • Two consecutive stations may not be skipped by the same train • Compulsory stops at both line termini • Minimum safety headway

Table 3 Summary of problem formulations for flexible skip-stop plan optimisation in the literature

3. Methodology

3.1. Purposes of Studies on Trimodal Skip-Stop Schemes

The proposal of trimodal skip-stop scheme revolves around two motives. First of all, most of the station-based schemes are the conventional `A/B` scheme consisting of 2 modes of train services according to the literature review. Considering an additional service mode `C` to the conventional `A/B` scheme provides a large variety of operational possibilities. The bimodal (`A/B`) scheme has $\binom{2}{1} + \binom{2}{2} = 3$ types of stations, while a trimodal scheme may have up to $\binom{3}{1} + \binom{3}{2} + \binom{3}{3} = 7$ types. The additional mode makes any trimodal schemes more flexible than the conventional A/B scheme, making them worth investigating. It is intended here to empirically study different strategies for designing this trimodal scheme and compare it with the conventional `A/B` scheme.

Secondly, it is admitted that an `A/B` SS plan is only optimal for a certain static demand pattern. When the demand pattern varies, the stop plan may become sub-optimal. Independently determining another stop plan for the varied demand pattern will result in a potentially different plan. Although it is the way to achieve the most optimal plans, this means that a station would have to shift in station type as time passes by, which may become impractical. This study takes a different approach to this problem. Instead of choosing a station type in each stationary demand period, the proposed method here entails choosing a predefined bundle of station types across varying-demand periods. Both the conventional approach and the proposed station-type coordination approach are depicted by Figure 8. An appropriate number of bundles of station types are determined by engineers and planners for the final solution to be comprehensible. Indeed, bundling station types between 2 bimodal schemes is logical in an attempt to eliminate uncoordinated cross-period patterns (Figure 8b). Moreover, bundling the bimodal scheme with a trimodal scheme is a possible solution that is worth investigating (Figure 8c). In the following section, an optimisation framework that jointly designs station types for multiple demand periods is elaborated.

3.2. Problem Statement

All notations and their descriptions used in this mathematical model are summarised in Appendix A. The unit of each variable/parameter is given in square brackets $[unit]$ after its appearance.

Consider a double-track MRT system without siding tracks. There are N number of stations whose indices are given by station set $\mathbb{N} = \{1, 2, ..., N\}$. Stop-spacing vector $\mathcal{S}$ of size $N - 1$ is used to describe the spatial relation of the stations. In $\mathcal{S}$, the distance between a pair of adjacent stations n and $n + 1$ is given and denoted as d_n $[m]$ $\forall n \in \mathbb{N} - \{N\}$.

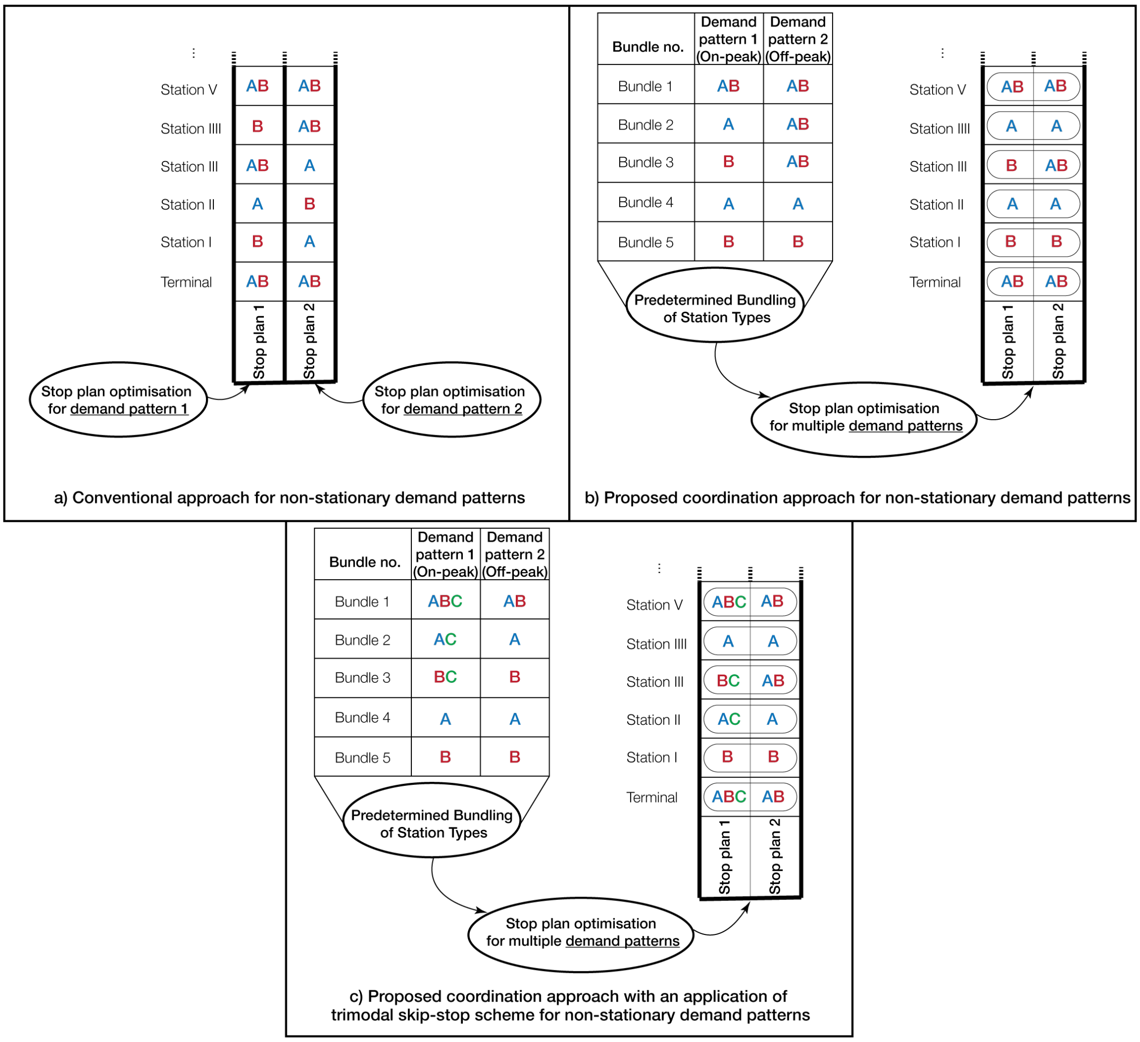


Figure 8 Conventional and proposed strategies for station-based SS plan optimisation under time-variant demand

In this model description, subscripts $i, j \in \mathbb{N}$ respectively represent the origin (O) and destination (D) stations of passenger trips. This MRT system is subject to time-variant passenger demand which is stipulated to be a collection of K' static OD demand distributions. Therefore, the time-variant demand distribution is represented by 3-dimensional OD demand matrix $\boldsymbol{\mathcal{D}}'$ where a scalar passenger arrival rate $\lambda'_{i,j,k'}$ $[pax/s]$ for every OD pair i, j and every demand period k' in $\mathbb{K}'$, where $\mathbb{K}' = \{1, 2, ..., K'\}$ is demand period set. Un-boldfaced-and-subscripted OD demand matrix notation $\mathcal{D}'_{k'}$ is used to represent the static demand distribution at demand period k'. Each demand period k' starts at $o^{sta}_{k'}$ and ends at $o^{end}_{k'}$. The continuum of demand data should be complete, meaning that $o^{end}_{k'} = o^{sta}_{k'+1}$ should hold $\forall k' \in \mathbb{K}' - \{K'\}$. The interval lengths of demand periods need not be uniform.

Train movements in this model follow deterministic riding time functions of distance. The following parameters are involved: acceleration rate α $[m/s^2]$, deceleration rate β $[m/s^2]$, maximum operating speed v^{max} $[m/s]$, turnaround headway at termini h^{turn} $[s]$, and safety headway h^{min} $[s]$. To sum up for travel time between stations, a value for minimum dwell time $t^{dwell,min}$ $[s]$ also needs to be specified. The roles of these parameters are elaborated primarily in Subsection 3.2.5.

3.2.1. Assumptions

This transit operation model relies on the underpinning assumptions delineated below.

Assumption 1 *Demand is inelastic with respect to varied travel time. Although it has been reported that OD demands that are affected by SS transfers plummet after the implementation of SS operation (Freyss et al., 2013), this model does not shift passenger demand from one OD pair to a more convenient one (Lee et al., 2014; Huang et al., 2017) or tamper with $\mathcal{D}'$ in any ways.*

Assumption 2 *Trains have infinite passenger-carrying capacity. This assumption simplifies the model because waiting patrons are never stranded when a stopping train is full. However, if that is the case, it rather harms the representation accuracy. Therefore, this model is to its disadvantage when it deals with crowded transit systems with high passenger volumes.*

Assumption 3 *Train movements are fully deterministic, meaning that giving a certain distance between two stops to the travel time model (Section 3.2.5) always produces the same travel time.*

Assumption 4 *Stop-dwelling time is equal to constant $t^{dwell,min}$ for all stopping trains. This is once again for the sake of model simplicity since stop-dwelling time is usually affected by number of alighting-boarding passengers, number of doors, etc (Wang et al., 2014).*

Assumption 5 *Train movements through skipped stations do not involve any deceleration or coasting. Trains proceed with the holding speed of v^{max} if the speed has been reached before the station.*

Assumption 6 *Every station spacing d_n in S is equal to or larger than the distance needed to reach the maximum operating speed from a complete stop $\frac{(v^{max})^2}{2\alpha}$ and the distance needed to come to a complete stop from the maximum operating speed $\frac{(v^{max})^2}{2\beta}$. This assumption is created for some simplification in Subsection 3.2.5.*

Assumption 7 *As a result of model simplification, service headways between different train modes are equal among all stations of the same type (same stopping modes). In the conventional `A/B` scheme, this assumption holds when the numbers of `A` stations and `B` stations between two transfer stations are equal. Otherwise, if either of the train modes stops more frequently between two transfer stations, the mode with more stopping will arrive later at* *the downstream transfer station, causing the headway to be wider than it is at the upstream station. At this stage, this difference is neglected; consequently, the passenger distribution between stopping train modes is uniform with respect to the location of the station.*

Assumption 8 *In the cases that a transfer is required, transfer stations are prohibited to be located upstream of the origin station, meaning that the train departing at the origin station has to follow the resultant trip direction.*

3.2.2. Predetermined Skip-Stop Settings

There are several parameters regarding SS schemes that need to be defined prior to initialisation of optimisation process.

Firstly, after an investigation on the demand dynamics, a number of SS periods K and their time intervals $[p_k^{sta}, p_k^{end}]$ have to be decided $\forall k \in \mathbb{K} = \{1, 2, ..., K\}$. The demand parameters need to be aggregated according to the SS periods. Similarly, the continuum of demand data should be complete, meaning that $p_k^{end} = p_{k+1}^{sta}$ should hold $\forall k \in \mathbb{K} - \{K\}$. Time-weighted average of static passenger demand distributions within SS period k can be expressed as the following formula:

$$\mathcal{D}_k = \frac{1}{p_k^{end} - p_k^{sta}} \left[\Big(\sum_{k'=f}^{g} \mathcal{D}'_{k'} \cdot (o_{k'}^{end} - o_{k'}^{sta}) \Big) + \mathcal{D}'_{f-1} \cdot (o_{f-1}^{end} - p_k^{sta}) + \mathcal{D}'_{g+1} \cdot (p_k^{end} - o_{g+1}^{sta}) \right], \tag{3.1}$$

where $o_{f-1}^{sta} < p_k^{sta} \leq o_{f-1}^{end}$ and $o_{g+1}^{sta} \leq p_k^{end} < o_{g+1}^{end}$ inequalities hold, implying that f and g are respectively indices of the first and last demand periods that are fully covered by SS period k. The result of the summation within the square brackets in Equation 3.1 is geometrically represented by the hatched area in Figure 9. Each $\mathcal{D}_k$, $\forall k \in \mathbb{K}$, makes up multi-SS-period demand matrix $\boldsymbol{\mathcal{D}}$. Ideally, the time-variation of demand distributions within each SS period should be as minimal as possible in order to keep aggregation errors small.

Secondly, the number of SS modes M_k in each SS period t has to be specified. Each SS mode set $\mathbb{M}_k$ contains SS mode labels following the alphabetical order starting at `A`; e.g., if $M_k = 3$, $\mathbb{M}_k = \{$`A, B, C`$\}$. The operational sequence is also alphabetically ordered by train modes and repeats itself starting with the first mode `A` (`[A/B/C/A/B/C/A/...]` for $M_k = 3$). For the convenience of explanation, a special type of operator is introduced for any non-numeric element m_k in $\mathbb{M}_k$. Simply adding integer I to m_k results in the I^{th} mode after m_k; e.g., if $m_k =$ `A`, then $m_k + 1 =$ `B`. Subtraction also works vice versa. Indeed, adding the total number of SS modes to a mode label always results in the same mode label $m_k + M_k = m_k$.

Thirdly, the average service frequency h_k^{ave} $[s]$ of each SS period k has to be specified. The departure headway between these train modes at terminal stations is uniformly fixed at the constant of h_k^{ave}, and this makes the problem setting resemble the third constraint

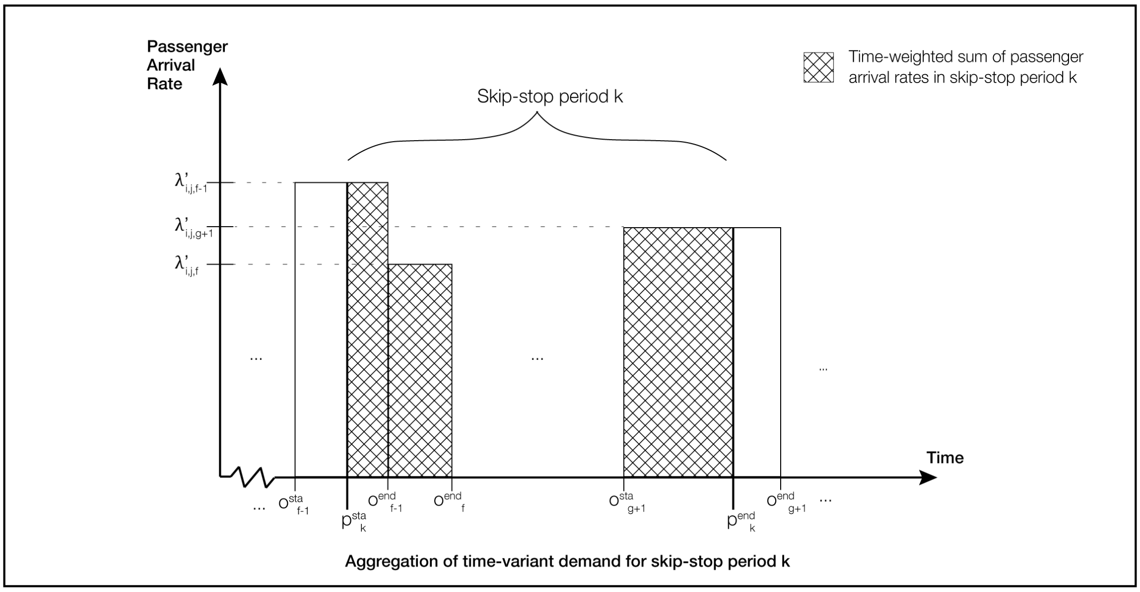


Figure 9 Aggregation of time-variant demand distributions for an arbitrary OD pair i, j for SS period k
zz

scenario in Lee et al.'s 2014 model. Despite the fact that releasing this condition would allow more optimal solutions (the fourth constraint scenario from Lee et al. (2014)), this condition is deemed necessary as for consideration of turn-around capacity. The shortest time for trains to turn-around at termini in MRT systems is generally larger than minimum safety headway, but it is most likely smaller than service headway.[1] For this reason, a constraint on turn-around time need not be applied when departure headways are kept at a constant interval.

Fourthly, a so-called SS station type bundling strategy has to be specified in station-type policy matrix $\mathcal{P}$. Matrix $\mathcal{P}$ is composed of a number of binary entries. If binary entry b_{x,k,m_k} takes the value of 1, trains of mode m_k in SS period k then stop at every station of bundle number x; otherwise, the trains skip the stations. An example $\mathcal{P}_{ex}$ is provided as follows,

[1] According to information provided by Bangkok Mass Rapid Transit Authority of Thailand, turn-around headway is 2–3 minutes, while minimum safety headway is 2 minutes for the MRT Chalerm Ratchamongkol Line (Bangkok's MRT Blue Line)

given $T = 2, M_1 = 3, M_2 = 2$.

$$\begin{array}{cc} & \begin{array}{ccccc} \text{1.A} & \text{1.B} & \text{1.C} & \text{2.A} & \text{2.B} \end{array} \\ \mathcal{P}_{ex} = & \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{array}{l} \text{Bundle 1} \\ \text{Bundle 2} \\ \text{Bundle 3} \\ \text{Bundle 4} \\ \text{Bundle 5} \end{array} \end{array} .$$

This $\mathcal{P}_{ex}$ is equivalent to the table in Figure 8c in the first step of the optimisation process. There are $\sum_{k=1}^{K} M_k$ columns in station-type policy matrix where each column represents a train mode in a certain SS period. Each row represents a station bundle and defines which train modes in all SS periods stop at a station of its bundle. The stop plan in this problem is the same for both directions of train runs (opposite to so-called bidirectional skip-stop plan where stop plans are allowed vary per direction). For consistency, Bundle 1 is always to be set as the all-stop bundle. The total number of bundles is denoted by X, and station-type bundle set is denoted by $\mathbb{X}$.

Lastly, skippable station set $\mathbb{S} \subset \mathbb{N}$ has to be specified. The existence of a station index in $\mathbb{S}$ indicates that the station may take any type of bundle declared in station-type policy matrix $\mathcal{P}$; otherwise, it is constrained to be only of all-stop bundle (Bundle 1).

3.2.3. Objective Function

This optimisation problem seeks after the optimal station types across SS periods, which is wholly considered as finding the optimal station-type bundles of all skippable stations in $\mathbb{S}$. Vector $\mathbf{X}$ is used to represent a solution to this problem. It comprises bundle number $x_s \in \mathbb{X}$ for each skippable station s in $\mathbb{S}$. Its counterpart $\mathbf{X}'$ is $\mathbf{X}$ itself with non-skippable stations included; hence, $\mathbf{X}'$ a superset of $\mathbf{X}$.

The quality of $\mathbf{X}$ is defined by two passenger time components: in-vehicle travel time y^{tt} and at-platform waiting time y^{wt} of all passengers in the system according to an a-priori $\boldsymbol{\mathcal{D}}'$. The following optimisation problem set up is to minimise the total travel time defined as the sum of in-vehicle and waiting times:

$$\min_{\mathbf{X}} z = \min_{\mathbf{X}} \left[y^{tt} + y^{wt} \right] \tag{3.2}$$

A diagrammatic overview of the optimisation framework is given in Figure 10. Up to this point, most of the components in data input and data preprocessing have been described.

Data preprocessing is executed only once at the initialisation of the optimisation process, and the resulting parameters from this step are used to set the objective function which is decomposed into the waiting time model (Subsection 3.2.4) and travel time model (Subsection 3.2.5). The search space is constrained by only one constraint described in Subsection 3.2.6.

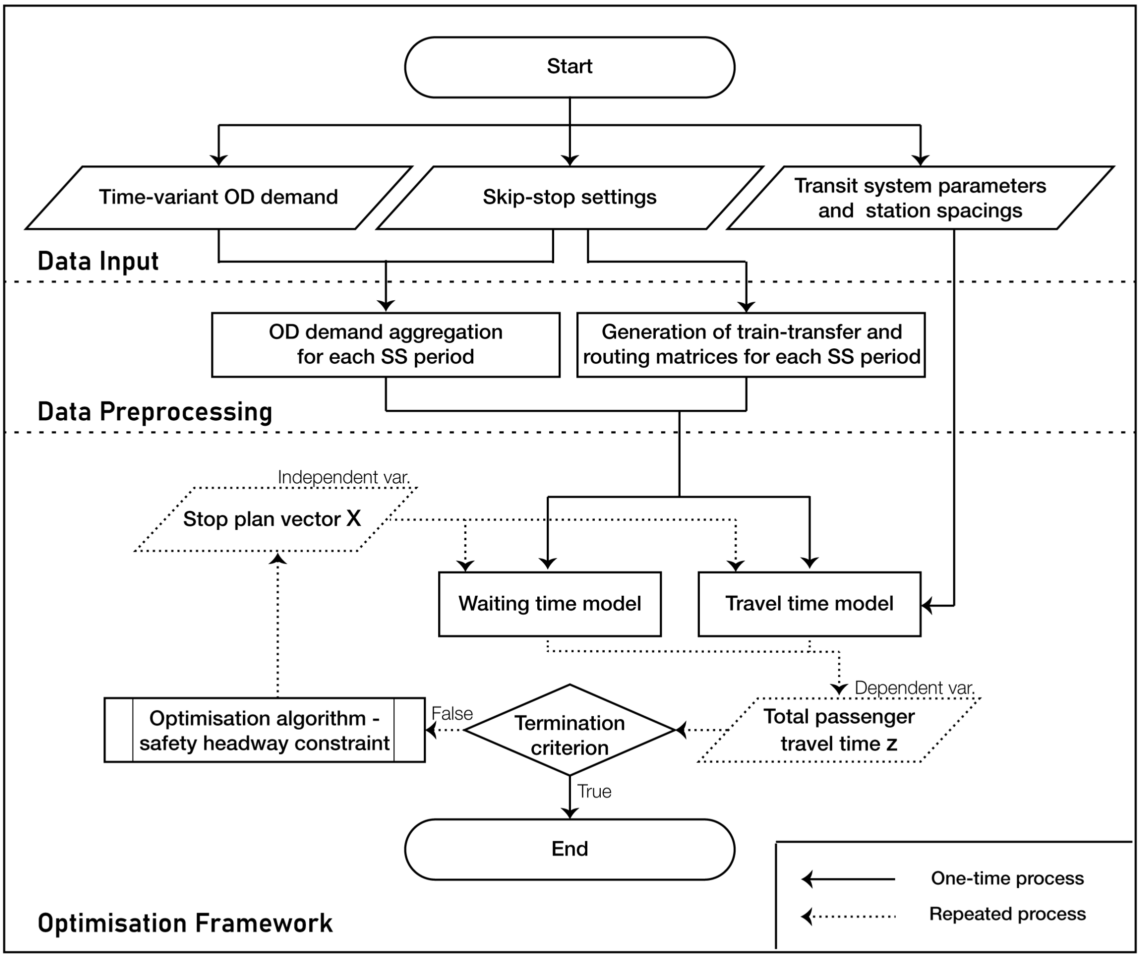


Figure 10 Flow chart for the optimisation framework

3.2.4. Waiting Time Model

In SS period k, consider the time interval of $h_k^{ave} M_k$ which is the amount of time alternating train-mode operations take to repeat their patterns. Based on a set of predetermined SS settings and a solution vector $\mathbf{X}'$, headway h_{n,k,m_k}^{wait} of train mode m_k in SS period k for station n is determined by Equation 3.3 according to Assumption 7. Essentially, headway h_{n,k,m_k}^{wait} is the length of time from the departure of a precedent stopping mode to the departure of stopping mode m_k. In cases that train mode m_k skips the station, the headway for that mode is set to 0.

$$h_{n,k,m_k}^{wait} = \begin{cases} h_k^{ave}\big(1 + (1 - b_{x'_n,k,m_k-1}) + (1 - b_{x'_n,k,m_k-1})(1 - b_{x'_n,k,m_k-2}) + ... \\ +(1 - b_{x'_n,k,m_z k-1})...(1 - b_{x'_n,k,m_k-(M_k-1)})\big), & \text{if } b_{x'_n,k,m_k} = 1 \\ 0, & \text{if } b_{x'_n,k,m_k} = 0 \end{cases} \tag{3.3}$$

The summation of headways of all modes at any station n is always equal to the length of the time interval: $\sum_{m_k \in \mathbb{M}_k} h^{wait}_{n,k,m_k} = h^{ave}_k M_k$.

With the information of headways at a station, waiting time can be calculated by cumulative arrival counts method from the queuing theory. When the passenger arrival rate is constant, the average waiting time of passenger is equal to half of the headway as in many optimisation works (Lee et al., 2014; Cao et al., 2014; Zhang et al., 2017; Yang et al., 2019; Sadrani et al., 2021). The total passenger waiting time in demand period k at station n is given by Equation 3.4 and illustrated in Figure 11.

$$t^{wt}_{n,k} = \frac{p^{end}_k - p^{sta}_k}{h^{ave}_k M_k} \sum_{m_k \in \mathbb{M}_k} \left(\frac{h^{wait}_{n,k,m_k}}{2} \cdot h^{wait}_{n,k,m_k} \sum_{j \in \mathbb{N}} \lambda_{n,j,k}\right) \tag{3.4}$$

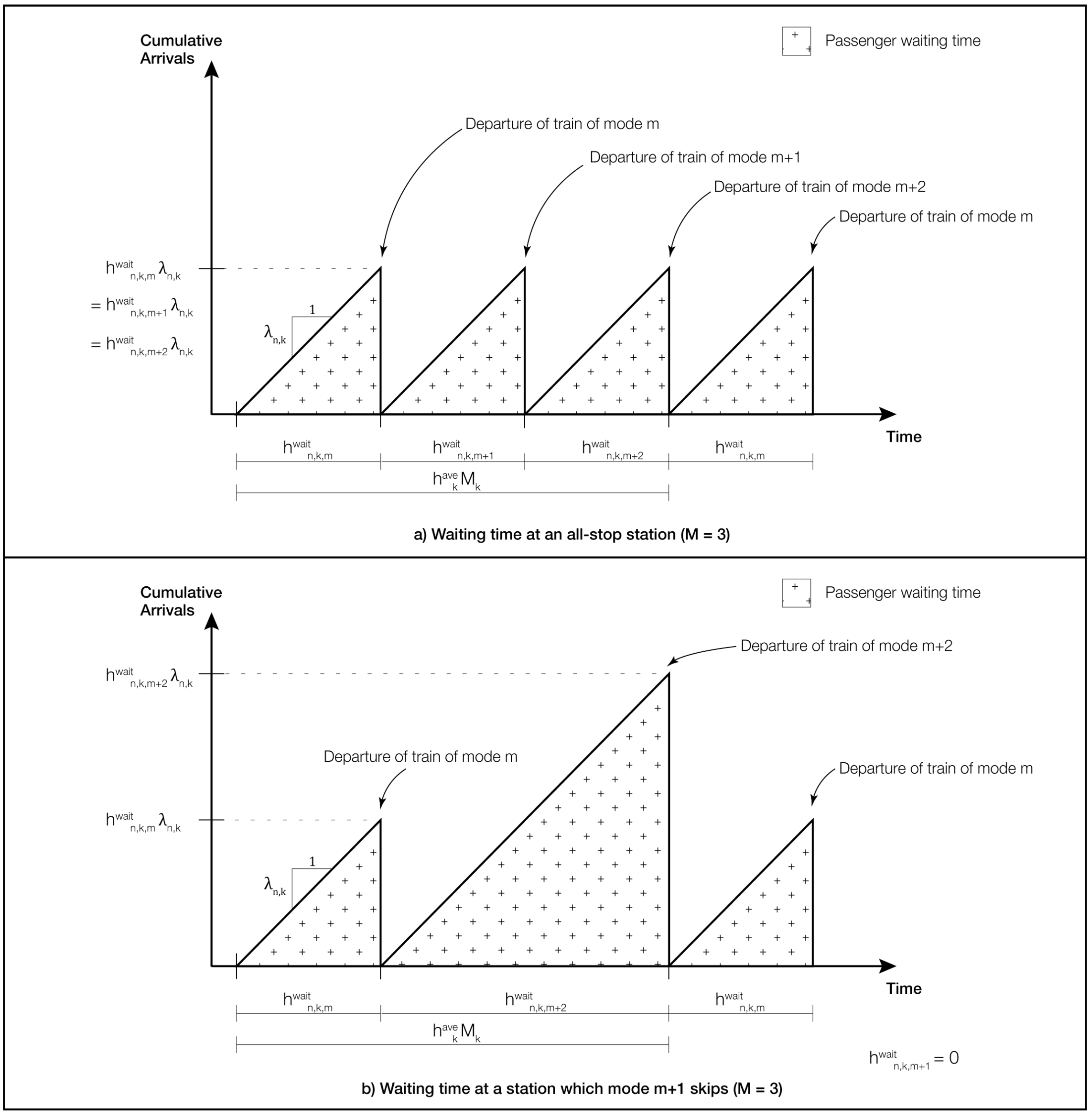


Figure 11 Service headways and the passenger waiting time at station n

Nevertheless, Equation 3.4 holds only when all stations are of the same station type. The service headway at a station provided by Equation 3.3 cannot be applied to all patrons arriving at the station since some of them may be unable to use certain modes to reach their destinations that are exclusive stations. This invalidates the term $h^{wait}_{n,k,m_k} \sum_{j \in \mathbb{N}} \lambda_{n,j,k}$ in Equation 3.4. To solve this problem, the service headway information has to take the OD demand data structure instead ($\cancel{h^{wait}_{n,k,m_k}} \to h^{wait}_{i,j,k,m_k}$). In order to calculate the waiting time of each individual OD pair, service headway has to be recast into a OD data structure, and this process comprises two steps which have to be done only once and they are irrelevant to $\mathbf{X}$. Before that, the definition of mode pair, direct pair and transfer pair is laid since passengers of an OD pair may have more than 1 possible route to get from O to D. Note that a route in this problem statement differentiates itself from others by the use of different train modes.

Mode pairs are possible passenger routes. The set of mode pairs of an OD demand pair depends on both station types of origin and destination stations. The total number of mode pairs is equal to the number of stopping train modes at the origin station multiplied by the number of stopping train modes at the destination station. Mode pairs can be classified into two groups:

- **A direct pair** is a major passenger route which relies on only 1 train mode for direct connections. A direct pair is denoted by the single alphabet representing the train mode `[directMode]`. There are totally M_k possible direct pairs in an SS scheme.
- **A transfer pair** is a possible but less preferable passenger route using two train modes. Passengers take the first train mode at the origin station to reach a transfer station, then take the second train mode to reach the destination station. A transfer pair is denoted by ordered mode alphabets with a right arrow in between `[1stTrainMode→2ndTrainMode]`. Transfer pairs have to be considered as ordered pairs of train modes because when $M_k \geq 3$, a transfer pair and its opposite transfer pair (e.g. `[A→B]` and `[B→A]`) are no longer equivalent. There are totally ${}^{M_k}P_2 = \frac{M_k!}{(M_k-2)!}$ possible transfer pairs in an SS scheme.

The illustration of time components with mathematical notations of each case is provided in Figure 12.

The first step involves creating train-transfer table $\mathcal{T}_k$ for SS period k. It provides three properties for each transfer pair between train modes: 1) conflict modes, 2) transfer time, 3) transfer station types. Conflict modes (1) are direct pairs that render the transfer pair irrelevant. The existence of at least 1 of conflict modes overrides the need of the transfer. Conflict modes of a transfer pair are the origin and destination modes and all train modes in between; e.g., the conflict modes of `[B→A]` are all of the modes `[B, C, A]` when $M = 3$. Transfer time (2) indicates the amount of time that patrons need to wait at the intermediate station when taking the transfer pair, and it is the number of conflict modes subtracted by 1. Transfer station types (3) are the enumeration of all possible station types at which the transfer pair can accommodate its intermediation. Transfer stations basically have to have at least both of the two train modes of the transfer pair stopped. For example, the transfer station types for transfer pair

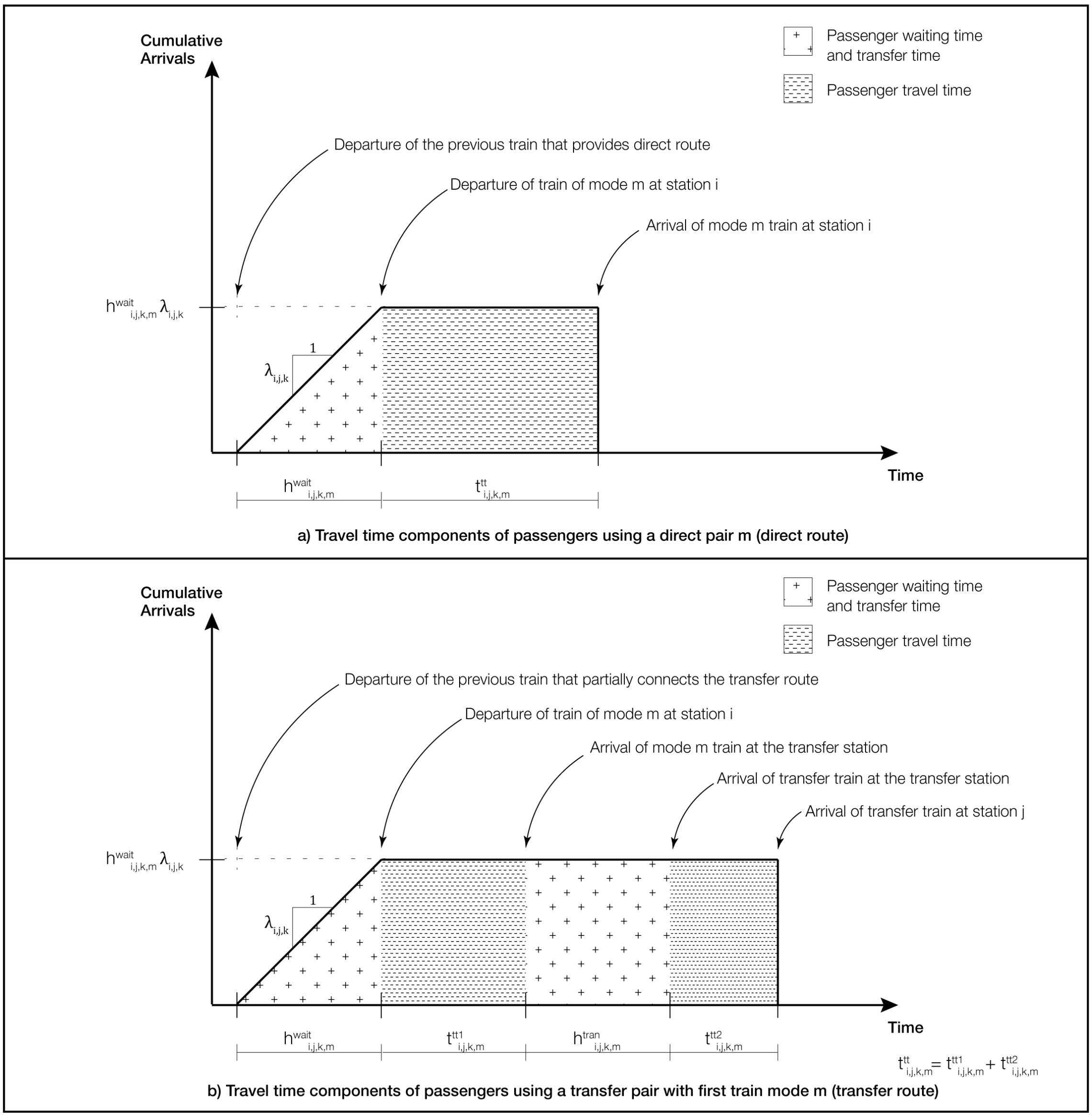


Figure 12 Travel time components of passengers arriving over a time interval of $h^{wait}_{i,j,k,m}$: a) case of direct pair; b) case of transfer pair

`[A→C]` are `[A/B/C]` (Bundle 1) and `[A/C]` (Bundle 2). For exemplification, the station-type policy matrix $\mathcal{P}_{ex}$ defined in Subsection 3.2.2 is further used specifically the part of SS period 1. In the SS period, there are 5 distinct station types: Bundle 1: `[A/B/C]`; Bundle 2: `[A/C]`; Bundle 3: `[B/C]`; Bundle 4: `[A]`; and Bundle 5: `[B]`. The train-transfer table $\mathcal{T}_{ex,1}$ following $\mathcal{P}_{ex}$ is given by Table 4, where there are ${}^3P_2 = \frac{3!}{(3-2)!}$ = 6 transfer pairs in total.

The second step involves the creation of routing table $\mathcal{R}_k$ of X rows representing the origin station types and X columns representing the destination station types for SS period k. A cell in $\mathcal{R}_k$ defines mode pairs which shall be used for a certain OD pair, and the following steps are taken to construct one cell:

1. **Enumerate all mode pairs**

Transfer pair	Conflict modes	Transfer time	Transfer station types
A→B	`A, B`	1	`[A/B/C]` (Bundle 1)
A→C	`A, B, C`	2	`[A/B/C]` (Bundle 1), `[A/C]` (Bundle 2)
B→A	`B, C, A`	2	`[A/B/C]` (Bundle 1)
B→C	`B, C`	1	`[A/B/C]` (Bundle 1), `[B/C]` (Bundle 3)
C→A	`C, A`	1	`[A/B/C]` (Bundle 1), `[A/C]` (Bundle 2)
C→B	`C, A, B`	2	`[A/B/C]` (Bundle 1), `[B/C]` (Bundle 3)

Table 4 Train-transfer table $\mathcal{T}_{ex,1}$ given $\mathcal{P}_{ex}$

2. **Eliminate irrelevant transfer pairs**: For each transfer pair, if there is at least 1 direct pair that belongs to the conflict modes according to $\mathcal{T}_k$ (Table 4), eliminate the transfer pair.

3. **Eliminate suboptimal transfer pairs**: If there are any 2 transfer pairs sharing the same mode of first train, eliminate the transfer pair that has a higher transfer time according to $\mathcal{T}_k$ (Table 4).

4. **Conclude the transfer routes**: If there are any 2 transfer pairs sharing the same mode of second train, two possible cases arise:

   a. If waiting time (waiting time for first train) weighs adversely higher than transfer time (waiting time for second train), keep both transfer pairs.

   b. Otherwise, if transfer time weighs equally or higher than waiting time, eliminate the transfer pair that has a higher transfer time according to $\mathcal{T}_k$ (Table 4).

   A mathematical proof regarding step 4. is given in Appendix B.

The routing table $\mathcal{R}_{ex,1}$ following $\mathcal{P}_{ex}$ and $\mathcal{T}_{ex,1}$ is given as Table 5, assuming that waiting time and transfer time weigh equally.

The identification of routing strategy then allows transformations of service headways to take the form of OD data structure h^{wait}_{i,j,k,m_k} given a solution $\mathbf{X}$. Notation h^{wait}_{i,j,k,m_k} can be enunciated as headway of stopping mode m_k at station i bounding for station j (direct pair, Figure 12a) or a transfer station (transfer pair, Figure 12b). As explained earlier, this service headway governs the amount of waiting time for the first train.

Transfer headway h^{tran}_{i,j,k,m_k} is the time interval from the time that transferring patrons alight from m_k (`1stTrainMode`) train at a transfer station to the departure time of the transfer train

O \ D	A/B/C sta.			A/C sta.			B/C sta.			A sta.			B sta.		
A/B/C sta.	A	~~A→B~~	~~A→C~~	A		~~A→C~~		~~A→B~~	~~A→C~~	A				~~A→B~~	
	~~B→A~~	B	~~B→C~~	~~B→A~~		~~B→C~~		B	~~B→C~~	~~B→A~~				B	
	~~C→A~~	~~C→B~~	C	~~C→A~~		C		~~C→B~~	C	~~C→A~~				~~C→B~~	
A/C sta.	A	~~A→B~~	~~A→C~~	A		~~A→C~~		A→B	~~A→C~~	A				A→B	
	~~C→A~~	~~C→B~~	C	~~C→A~~		C		~~C→B~~	C	~~C→A~~				~~C→B~~	
B/C sta.															
	~~B→A~~	B	~~B→C~~	~~B→A~~		~~B→C~~		B	~~B→C~~	~~B→A~~				B	
	~~C→A~~	~~C→B~~	C	~~C→A~~		C		~~C→B~~	C	C→A				~~C→B~~	
A sta.	A	~~A→B~~	~~A→C~~	A		~~A→C~~		A→B	~~A→C~~	A				A→B	
B sta.															
	~~B→A~~	B	~~B→C~~	~~B→A~~		B→C		B	~~B→C~~	B→A				B	

Table 5 Routing table $\mathcal{R}_{ex,1}$ given $\mathcal{P}_{ex}$ and $\mathcal{T}_{ex,1}$

($\texttt{2ndTrainMode}$), where station i is the origin station of those patrons. The amount of transfer time is predetermined by the type of transfer pair. Transfer headway is obtained by multiplying h_k^{ave} by the transfer time of the transfer pair given in $\mathcal{T}_k$, or it is 0 for direct pairs. Service headways and transfer headways of each station-type OD pair is derived from train-transfer table $\mathcal{T}_k$ and the respective routing table $\mathcal{R}_k$, and they are organised in headway table denoted by $\mathcal{H}_k$. The headway table $\mathcal{H}_{ex,1}$ following $\mathcal{T}_{ex,1}$ and $\mathcal{R}_{ex,1}$ is given as Table 6. As shown in Tables 5 and 6, the service headways for an OD pair does not only depend on the station type of the origin station but also the station type of the destination station. Equation 3.4 is accordingly corrected, resulting in Equation 3.5.

The transfer headway described above is only for forward-tracking transfers. The transfer headway for back-tracking transfers, on the other hand, requires additional terms because the transfer trains run in the direction opposite to the first train. These additional terms are the following:

$$
\begin{aligned}
t^{tt}_{l,N,k,m_k} + h^{turn} + t^{tt}_{N,l,k,m_k} - v(h^{ave}_k M_k), &\text{if } i > j \\
t^{tt}_{l,1,k,m_k} + h^{turn} + t^{tt}_{1,l,k,m_k} - v(h^{ave}_k M_k), &\text{if } j > i
\end{aligned}
$$

where t^{tt}_{l,N,k,m_k} is the travel time of train mode m_k from transfer station l to the terminal station N , h^{turn} is the turnaround headway, and v is the largest whole number that makes the resulted transfer headway positive.

O \ D	$\leftarrow m$	**A**/**B**/**C** sta.		**A**/**C** sta.		**B**/**C** sta.		**A** sta.		**B** sta.	
headway type $\rightarrow$		h^{wait}_m	h^{trans}_m	h^{wait}_m	h^{trans}_m	h^{wait}_m	h^{trans}_m	h^{wait}_m	h^{trans}_m	h^{wait}_m	h^{trans}_m
A/**B**/**C** sta.	A	h^{ave}	-	h^{ave}	-	0	-	$3h^{ave}$	-	0	-
	B	h^{ave}	-	0	-	$2h^{ave}$	-	0	-	$3h^{ave}$	-
	C	h^{ave}	-	$2h^{ave}$	-	h^{ave}	-	0	-	0	-
A/**C** sta.	A	h^{ave}	-	h^{ave}	-	h^{ave}	h^{ave}	$3h^{ave}$	-	$3h^{ave}$	h^{ave}
	C	$2h^{ave}$	-	h^{ave}	-	$2h^{ave}$	-	0	-	0	-
B/**C** sta.	B	$2h^{ave}$	-	0	-	$2h^{ave}$	-	0	-	$3h^{ave}$	-
	C	h^{ave}	-	$3h^{ave}$	-	h^{ave}	-	$3h^{ave}$	h^{ave}	0	-
A sta.	A	$3h^{ave}$	-	$3h^{ave}$	-	$3h^{ave}$	h^{ave}	$3h^{ave}$	-	$3h^{ave}$	h^{ave}
B sta.	B	$3h^{ave}$	-	$3h^{ave}$	h^{ave}	$3h^{ave}$	-	$3h^{ave}$	$2h^{ave}$	$3h^{ave}$	-

Table 6 Headway table $\mathcal{H}_{ex,1}$ given $\mathcal{T}_{ex,1}$ and $\mathcal{R}_{ex,1}$

$$
\begin{aligned}
t^{wt}_k &= \frac{p^{end}_k - p^{sta}_k}{h^{ave}_k M_k} \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{m_k \in \mathbb{M}_k} \big(\frac{h^{wait}_{i,j,k,m_k}}{2} \cdot h^{wait}_{i,j,k,m_k} \lambda_{i,j,k} + h^{tran}_{i,j,k,m_k} \cdot h^{wait}_{i,j,k,m_k} \lambda_{i,j,k} \big) \\
&= \frac{p^{end}_k - p^{sta}_k}{h^{ave}_k M_k} \sum_{i=1}^{N} \sum_{j=1}^{N} \lambda_{i,j,k} \sum_{m_k \in \mathbb{M}_k} h^{wait}_{i,j,k,m_k} \big(\frac{h^{wait}_{i,j,k,m_k}}{2} + h^{tran}_{i,j,k,m_k} \big)
\end{aligned} \tag{3.5}
$$

Ultimately, the total waiting time across SS periods can be simply summed up as in Equation 3.6.

$$
y^{wt} = \sum_{k=1}^{K} t^{wt}_k \tag{3.6}
$$

3.2.5. Travel Time Model

There are two cases of speed profiles of train movements between two stops. Case I is where the maximum operating speed v^{max} could not be reached within the acceleration range such that deceleration has to start at a $v^{peak} < v^{max}$. On the other hand, Case II is where v^{max} could be reached and held for a certain amount of time before deceleration. Figure 13 illustrates these two speed profile cases, and the acceleration-deceleration time formulae for Case I and Case II are given by Equation 3.7 and Equation 3.8 (Vuchic, 2005), respectively. These speed profiles are simplifications of reality in that they are discontinuous at the junctures between zero acceleration and non-zero constant acceleration.

$$t_n^{\alpha,I} + t_n^{\beta,I} = \sqrt{\frac{2d_n(\alpha+\beta)}{\alpha \cdot \beta}} \tag{3.7}$$

$$t_n^{\alpha,II} + t_n^{\beta,II} = v^{max}(\frac{1}{\alpha} + \frac{1}{\beta}) \tag{3.8}$$

A possible way to tackle this problem is to divide all station spacings in $\mathcal{S}$ into two cases with the help of a distance-based threshold d^{thres} such that all stop-stop distances smaller than d^{thres} fall into Case I, while the complement falls otherwise into Case II. This distance-based threshold in Equation 3.9 is obtained by equating the right-hand-side terms of Equations 3.7 and 3.8.

$$d^{thres} = \frac{(v^{max})^2}{2(\alpha+\beta)}\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\right) \tag{3.9}$$

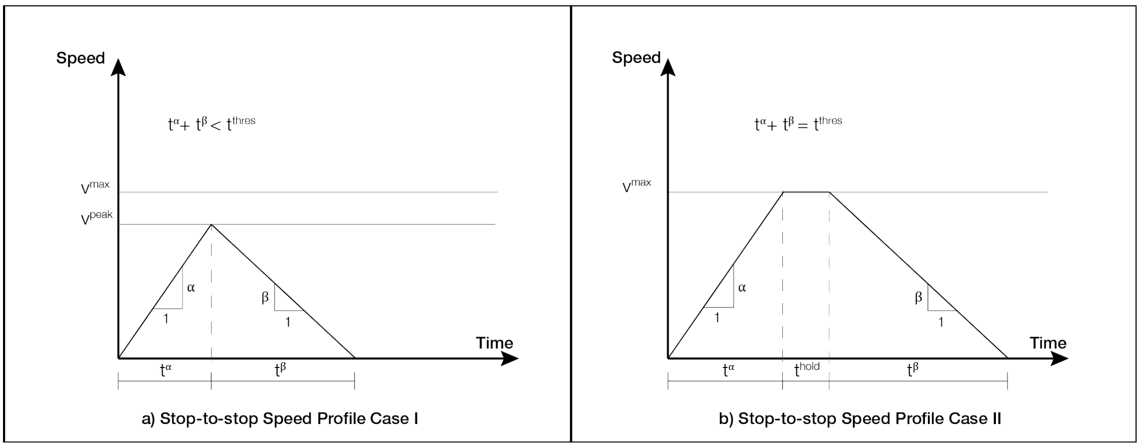


Figure 13 Generic speed profiles for this study: a) Speed Profile Case I; b) Speed Profile Case II

In accordance with Assumptions 3–5, the calculation of travel time between every adjacent station pair served by train mode m can be completed by the formula summary provided in Table 7. These formulae, however, are based on the case that trains stop at both n and $n+1$. When at least one station in a pair of consecutive stations is skipped, the boundary conditions of motions change. The departure-to-departure travel time from station n to station $n+1$ by train mode m_k in SS period k can be broken down into 4 cases depending on the skip-stop variables of both stations. Table 8 enumerates all these possibilities, and this is where effects on travel-time originate due to a variation in stop plan.

According to Assumption 6, trains always reach the maximum operating speed v^{max} before passing through a skipped station. This assumption is virtually always valid because any d_n is most likely larger than $\frac{(v^{max})^2}{2\alpha}$ or $\frac{(v^{max})^2}{2\beta}$. In other words, trains movements through skipped stations are always at v^{max}. Hence, in Cases 2-4 in Table 8, distance and time functions are assumed to follow the formulae of Speed Profile Case II even if $d_n < d^{thres}$.

Using Table 8, the departure-arrival travel time t^{tt}_{i,j,k,m_k} for the OD pair i, j by using a direct pair of mode m_k in SS period k is expressed as the following equation:

$$t^{tt}_{i,j,k,m_k} = \left(\sum_{n=i}^{j-1} t^{tt}_{n,n+1,k,m_k} \right) - t^{dwell}_j \tag{3.10}$$

In cases of transfer pairs, the transfer station has to be firstly located according to the type of transfer pair. The eligible transfer station types of the transfer pair are provided in $\mathcal{T}_k$. If the transfer station is located between the origin and destination stations, this case is called forward-tracking transfer. On the other hand, if there is no transfer station in between, a transfer station is to be located beyond the destination station; this case is called back-tracking transfer. Assumption 8 prohibits any locations of transfer stations upstream of the origin station. The location of a transfer station does not affect travel time in case of forward-tracking transfer; however, it does in case of back-tracking transfer. The transfer station that is closest to the destination station shall be selected. An analytical form of travel time for OD pairs with a transfer is not presented; however, it follows the same principle of departure-arrival travel time used in Equation 3.10.

Once the departure-arrival travel time t^{tt}_{i,j,k,m_k} for an arbitrary OD pair i, j, an arbitrary SS period k, an arbitrary train mode m_k is formulated (regardless of need for a transfer), the total passenger travel time in SS period k is as follows:

$$\begin{aligned} t^{tt}_k &= \frac{p^{end}_k - p^{sta}_k}{h^{ave}_k M_k} \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{m_k \in \mathbb{M}_k} t^{tt}_{i,j,k,m_k} \cdot h^{wait}_{i,j,k,m_k} \lambda_{i,j,k} \\ &= \frac{p^{end}_k - p^{sta}_k}{h^{ave}_k M_k} \sum_{i=1}^{N} \sum_{j=1}^{N} \lambda_{i,j,k} \sum_{m_k \in \mathbb{M}_k} h^{wait}_{i,j,k,m_k} t^{tt}_{i,j,k,m_k} \end{aligned} \tag{3.11}$$

Finally, the total passenger travel time across SS periods is obtained by the following summation.

$$y^{tt} = \sum_{k=1}^{K} t^{tt}_k \tag{3.12}$$

Distance functions		
Travel distance components	**Speed Profile Case I:** $d_n < d^{thres}$	**Speed Profile Case II:** $d_n \geq d^{thres}$
Acceleration distance	$d_n^{\alpha}(d_n) = \frac{\beta d_n}{\alpha+\beta}$	$d_n^{\alpha} = \frac{\beta d^{thres}}{\alpha+\beta} = \frac{(v^{max})^2}{2\alpha}$
Holding-speed distance	$d_n^{hold} = 0$	$d_n^{hold}(d_n) = d_n - d^{thres}$
Deceleration distance	$d_n^{\beta}(d_n) = \frac{\alpha d_n}{\alpha+\beta}$	$d_n^{\beta} = \frac{\alpha d^{thres}}{\alpha+\beta} = \frac{(v^{max})^2}{2\beta}$

Time functions		
Travel time components	**Speed profile Case I:** $d_n < d^{thres}$	**Speed profile Case II:** $d_n \geq d^{thres}$
Acceleration time	$t_n^{\alpha}(d_n) = \sqrt{\frac{2\beta d_n}{\alpha(\alpha+\beta)}}$	$t_n^{\alpha} = \frac{v^{max}}{\alpha}$
Holding-speed time	$t_n^{hold} = 0$	$t_n^{hold}(d_n) = \frac{d_n - d^{thres}}{v^{max}}$
Deceleration time	$t_n^{\beta}(d_n) = \sqrt{\frac{2\alpha d_n}{\beta(\alpha+\beta)}}$	$t_n^{\beta} = \frac{v^{max}}{\beta}$
At-stop dwell time	$t_n^{dwell} = t^{dwell,min}$ (Assumption 5)	

Table 7 Summary of fundamental formulae for travel time calculation

Case no.	Skip-stop variables		Departure-departure travel time $t^{tt}_{n,n+1,k,m_k}$
	b_{x_n,k,m_k}	b_{x_{n+1},k,m_k}	
Case 1	1	1	$t_n^{\alpha} + t_n^{hold} + t_n^{\beta} + t_{n+1}^{dwell}$
Case 2	0	1	$\left(\frac{d_n - d_n^{\beta}}{v^{max}}\right) + t_n^{\beta} + t_{n+1}^{dwell}$ (Assumption 6)
Case 3	1	0	$t_n^{\alpha} + \left(\frac{d_n - d_n^{\alpha}}{v^{max}}\right)$ (Assumption 6)
Case 4	0	0	d_n / v^{max} (Assumption 6)

Table 8 Departure-departure travel time differentiated by skip-stop variables of two adjacent stations

3.2.6. Safety Headway Constraint

The sole constraint in this model is the constraint on the time gap between consecutive trains. Since trains of different modes depart at termini at equal headways of h_k^{ave}, the difference between cumulative travel times of any 2 train modes from a terminus to any station may not be greater than the remaining headway capacity $h_k^{ave} - h^{min}$ of any SS period k. The same statement is mathematically expressed in Equation 3.13. It is noted that only one direction of operations needs to be considered since a stop plan in this problem is used for both directions of train runs. A violation of the constraint considering cumulative travel time in one direction implies the violation of the constraint similarly formulated for the opposite direction.

$$\sum_{n=1}^{c} t^{tt}_{1,n+1,k,m_k} - \sum_{n=1}^{c} t^{tt}_{1,n+1,k,m_k-1} \geq h_k^{ave} - h^{min}, \ \forall c \in \mathbb{N} - \{N\}, \quad \forall m_k \in \mathbb{M}_k, \forall k \in \mathbb{K} \tag{3.13}$$

3.2.7. Alternative Objective Function

Since the total number of passengers may considerably vary among different SS periods, the average waiting time (Equation 3.14) and the average travel time (Equation 3.15) of SS period k can be alternatively considered as the objective function instead. The use of average values prevents numerical dominance of an SS period that is long or under high demand over other smaller SS periods.

$$\bar{y}^{wt} = \sum_{k=1}^{K} \frac{t_k^{wt}}{(p_k^{end} - p_k^{sta}) \sum_{i=1}^{N} \sum_{j=1}^{N} \lambda_{i,j,k}} \tag{3.14}$$

$$\bar{y}^{tt} = \sum_{k=1}^{K} \frac{t_k^{tt}}{(p_k^{end} - p_k^{sta}) \sum_{i=1}^{N} \sum_{j=1}^{N} \lambda_{i,j,k}} \tag{3.15}$$

3.3. Solution Method: Genetic Algorithm

With regard to the discrete nature of independent variable, this problem is a combinatorial optimisation problem: a NP-hard decision problem where full-enumeration rapidly becomes computationally prohibitive as problem size grows. The size of the search space in this decision problem is X^S where X is the number of possible station-type bundles and S is the number of skippable stations.

The reviewed stop-plan optimisation studies in Section 2.2 also similarly dealt with NP-hard problems. Genetic algorithms have been the predominant solution methods for numerous the cyclic(Lee et al., 2014; Huang et al., 2017; Yang et al., 2019) and non-cyclic (Sogin et al., 2012; Wang et al., 2014; Zhang et al., 2017; Zhao et al., 2021) skip-stop plan optimisation problems. A standard genetic algorithm is then implemented for this problem. Genetic algorithm is a one of the nature-inspired meta-heuristic algorithms. More details about engineering applications of genetic algorithms can be found in the book chapter by Bozorg-Haddad et al. (2017).

The concept of genetic algorithms originates from the principle of evolution by natural selection that was first theorised by Charles Darwin in 1859. The core concept explains that organisms having various genetic traits are not equally fit when they inhabit in the same environment. The fitter individuals continue to thrive better and reproduce individuals that have genetic traits that are potentially favourable in the environment, while the weaker individuals tend to be outgrown and go extinct. In the process of reproduction, mutations that gradually alter genetic traits occur randomly over generations. Naturally, random mutations can result in more favourable or less favourable genetic traits. After some generations, the latest, most evolved generations are most likely composed of individuals which are fitter than those in the first, primitive generations.

Basic terminology for genetic algorithms is firstly described, then a standard procedure of genetic algorithms is given. **Terminology:** Chromosome is the the term in genetic algorithms for a solution vector, a 1-dimensional array of independent variables. An independent variable is called a gene whose domain can take a form of discrete or continuous set. Correspondingly, a chromosome is composed of multiple genes, and one chromosome represents a complete solution for the problem. A generation is a collection of chromosomes that coexist together in an iteration; they compete for survival. Fitness function provides an indicator of solution quality which conventionally known as objective function value, but fitness function is originally formulated to be maximised. **Algorithm:** For the first iteration, an initial generation is usually randomly generated from the uniform probability distribution. Let G denote the number of chromosomes in a generation, and S the number of genes in a chromosome or simply chromosome size. The algorithm schema is provided in Figure 14 as parts of the optimisation framework. The core process of the algorithm is divided into 3 sequential parts as follows:

1. **Natural selection**: Fitness value for each chromosome is evaluated and used for random selection of parental chromosomes. Chromosomes with higher fitness values are more probable to be selected as parents. More precisely, the selection probability is a function of fitness value, and it can be evaluated in various ways such as proportionate selection, ranking selection and tournament selection. The proportionate selection approach is also known as roulette wheel selection. A number of parental chromosomes $R < G$ are drawn from the probability distribution characterised by the selection probabilities. All of these R parental chromosomes survive through the next generation, and they are used in the next step. The other $G - R$ chromosomes are simply discarded at this point.

2. **Crossover**: Since every generation must comprise the same number of chromosomes, missing $G - R$ chromosomes are then to be reproduced by a genetic operator called crossover operator. In the standard genetic algorithm, parental chromosomes are once again randomly eliminated from the parent population based on the predefined crossover probability. This elimination process is omitted in the implementation for this study, which is the same as setting the crossover probability equal to 1.0. There are various types of crossover operators for selection such as 1-point crossover, 2-point crossover, uniform crossover. The 1-point crossover method involves a random variable for the position of crossover point along chromosome strand of length S drawn from the uniform probability distribution. Two parental chromosomes are drawn from the pool. Both of them are cut into two parts at the position of crossover point, and the divided parts are swapped between chromosomes. An instance of this operation generates 2 offspring chromosomes. The crossover operator repeats until $G - R$ slots in the next generation are filled. The results from the crossover operator are then further manipulated in the next step.

3. **Mutation**: Every newly reproduced chromosome from a crossover operator has to undergo the mutation operator. Given a hyperparameter for mutation probability P_m,

every gene has the equal probability of mutation which is a random change of independent variable value (gene). When a gene should be mutated, its original value is replaced by a draw from the uniform distribution over the feasible domains. Given a small mutation probability (e.g., 0.05), most of the reproduced chromosomes are expected to slightly alter.

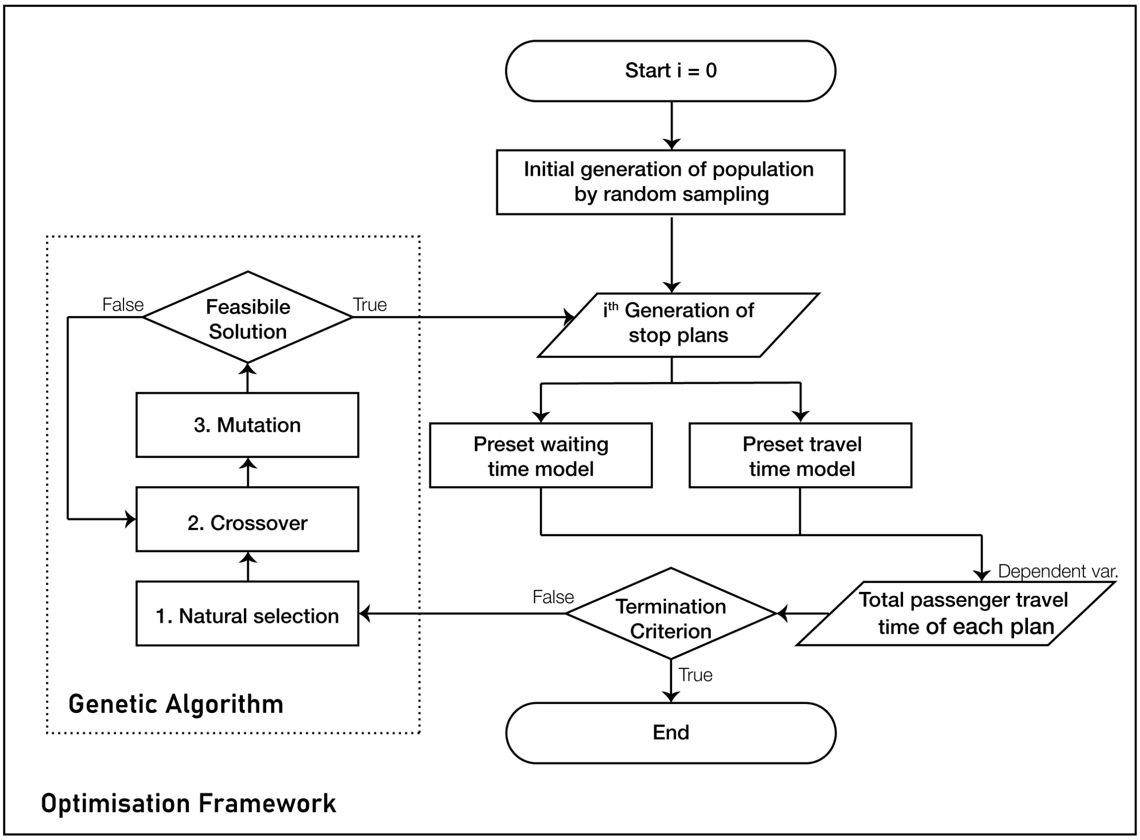


Figure 14 Optimisation framework diagram omitting the presetting of waiting time and travel time models

There are a number of hyperparameters to be specified before execution of the algorithm. Hyperparameters are those parameters specific to the optimisation algorithm, which may affect convergence performance of the algorithm. When a meta-heuristic optimisation algorithm is applied, tuning of hyperparameters is fostered since it may leap the algorithm towards a better fit to the specific optimisation problem in terms of convergence or time efficiency. As introduced above, there are notations for size of population or generation M and number of parental chromosomes $R < G$, which are the first two natural-number hyperparameters to be aware of. The method for the calculation of selection probability and the type of crossover operator are also hyperparameters themselves, not to mention the associated hyperparameters that come along with the method. Lastly, the quotient hyperparemeter of mutation probability P_m takes any value in the closed interval between 0 and 1.

Lastly, an exploitative modification to the genetic algorithm's natural selection operator is devised. In standard genetic algorithms, the selection of parental population is purely probabilistic in a way that makes the algorithm rather non-greedy or explorative. The main reason for this explorative behaviour can be attributed to the fact that the most optimal chromosomes in a generation are easily lost, and this loss may subsequently lead to a divergence. The exploitative modification used in the algorithm is to simply immortalise a number of best chromosomes $r < R$ in each generation. These chromosomes are forced to be parental chromosomes, automatically circumventing all the genetic operators. Suppose that a descent is not achieved after a several generations, the two most optimal chromosomes earlier found are kept unchanged and continue to be immortal until more optimal ones are found. This modification allows the algorithm to concentrate the search to the space surrounding the r most optimal solutions, improving the exploitation behaviour of the algorithm. There are other $R - r$ probabilistically selected parental chromosomes which still constitute the exploration behaviour.

3.3.1. Tailoring the Algorithm to the Formulated Problem

A gene in this SS plan optimisation problem represents a station that can take any bundle in the finite-discrete station-type bundle set $\mathbb{X}$. Due to the nature of discrete non-cardinal variable, the feasible domain of each independent variable is not explicitly known. The effects of a change in one gene propagate from the station to others downstream. Therefore, the feasibility of solution is considered only after the all crossover and mutation operations are complete for a chromosome. If a chromosome represents a feasible solution according to Equation 3.13, the chromosome is then assigned to the next generation (Figure 14).

The objective function for the formulated problem in Equation 3.2 is to be minimised, but genetic algorithms are formulated for maximisation of the fitness function as the objective function. One can simply think of applying a negative sign to the objective function 3.2 in order to convert it to fitness function. Nevertheless, fitness function is not allowed to a take negative value; therefore, a positive additive term that is sufficiently large has to be also applied. A straightforward norm is to use the objective function value of the all-stop solution in which every station takes Bundle 1. This follows the rationale that the total travel time resulted from the all-stop plan should be larger than a skip-stop plan that adapts to the demand distribution well, making the majority of passengers travel faster while the few wait longer. However, a randomly generated stop plan could be a skip-stop plan that does the opposite, making the objective function even larger than that of the all-stop solution. A coefficient $c > 1.0$ is applied to the total travel time of the all-stop plan to ensure that the fitness function $F(\mathbf{X})$ stays in the positive range. The larger coefficient c makes the survival probabilities of the weak ones and the strong ones more even out; therefore, a good value should not be just large enough to ensure non-negativity of the fitness function.[2] The sign of the fitness function is not a problem when ranking selection is used. The optimisation problem is then represented by an alternative form of fitness function maximisation in Equation 3.16.

$$\max_{\mathbf{X}} F(\mathbf{X}) = \max_{\mathbf{X}} cz(\mathbf{X}_{all-stop}) - z(\mathbf{X}) \tag{3.16}$$

[2] $c = 1.10$ was empirically found to suffice the purpose of preventing negativity of fitness function

4. Case Study: MRT Blue Line in Bangkok

In this chapter, a case study is presented to demonstrate the benefits of the proposed operational scheme.

4.1. Input Parameters

The Mass Rapid Transit Authority of Thailand MRTA has courteously provided passenger demand data and system characteristics data for Bangkok MRT Blue Line, officially known as the Chaloem Ratchamonkhon Line, for this research. The demand data are the daily passenger counts organised in the format of OD matrix from the 1st to the 31st of May 2021. The system characteristics parameters are maximum speed, safety headway, turnaround headway. Apart from the mentioned data types, inter-station distance was manually measured in Google Maps. A summary of all input parameters is provided in Table 9.

Parameter	**Value**	**Source**
Static OD demand distribution $\mathcal{D}$	N/A	MRTA
Station spacing $\mathcal{S}$	N/A	Google Maps
Safety headway h^{min}	$2\ min$	MRTA
Turnaround headway	$2-3\ min$	MRTA
Maximum speed	$80\ km/h$	MRTA
Perturbed OD demand distribution	N/A	Assumption
Minimum dwell time $t^{dwell,min}$	$30\ sec$	
Acceleration rate α	$0.9\ m/s^2$	
Deceleration rate β	$1.0\ m/s^2$	

Table 9 Input parameters for the case study on Bangkok MRT Blue Line

Nonetheless, a few additional assumptions have to be made to attain a complete set of input parameters. The provided static OD demand distribution is randomly perturbed to some extent in order to create a OD demand distribution varied from the original one. This perturbed distribution is then coupled with the original one to form a synthetic time-variant demand distribution. The minimum dwell time, constant acceleration and deceleration rates are taken from Cao et al. (2014). Moreover, the weights of waiting time and transfer time are assumed to be equal to 1 (used in the determination of routing table in Section 3.2.4).

4.1.1. Selection and Perturbation of the OD Passenger Demand Distribution

Despite the prevalence of COVID-19 in Bangkok, the major variation in travel behaviour can be well differentiated by the official workdays in May 2021 announced by the Government of Thailand. There are 13 public holidays and 17 official holidays in May 2021 in Thailand. The total number of patrons in the system averaged on the workdays is 99541 ($s = 7150$), whereas the total volume drops to 49847 ($s = 7811$) when averaged on the holidays. The standard deviations of the passenger OD counts among different days, visualised in Figure 15a, show that the variation among workdays and the variation among holidays are considerable smaller than among the whole month. Furthermore, daily passenger OD demand distribution is normalised by the max-min method to focus solely on the spatial relativity in the demand distribution, eliminating the demand intensity. The standard deviations of the normalised passenger OD counts reveal that the spatial distribution of demand varies less among official workdays compared to public holidays. Hence, the main (static) passenger demand distribution $\mathcal{D}^{ori*}$ used in this study is the average of OD demand distributions on the official workdays in May 2021 where the system is more heavily loaded and the spatial distribution varies slightly between these days. The asterisk sign in the notation $\mathcal{D}^{ori*}$ indicates that this OD demand distribution needs an additional conversion which is explained in the next subsection.

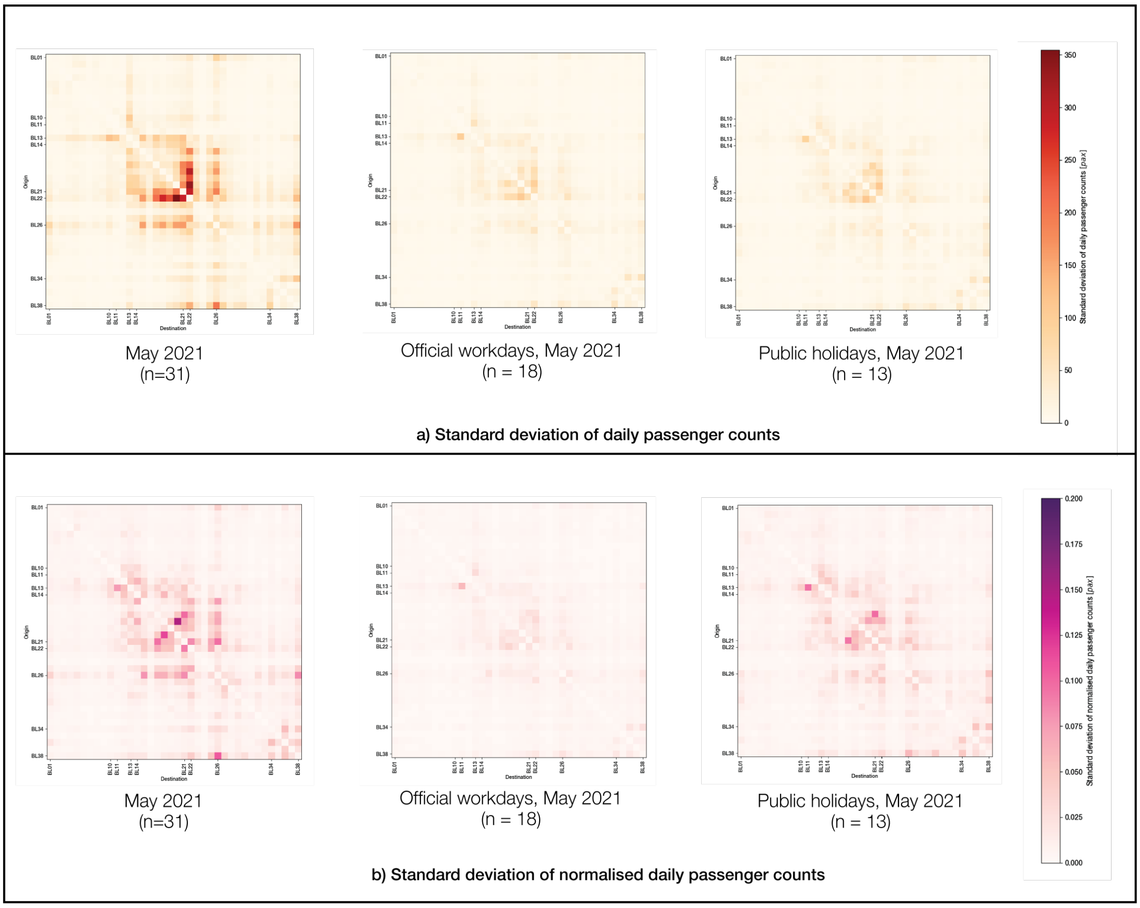
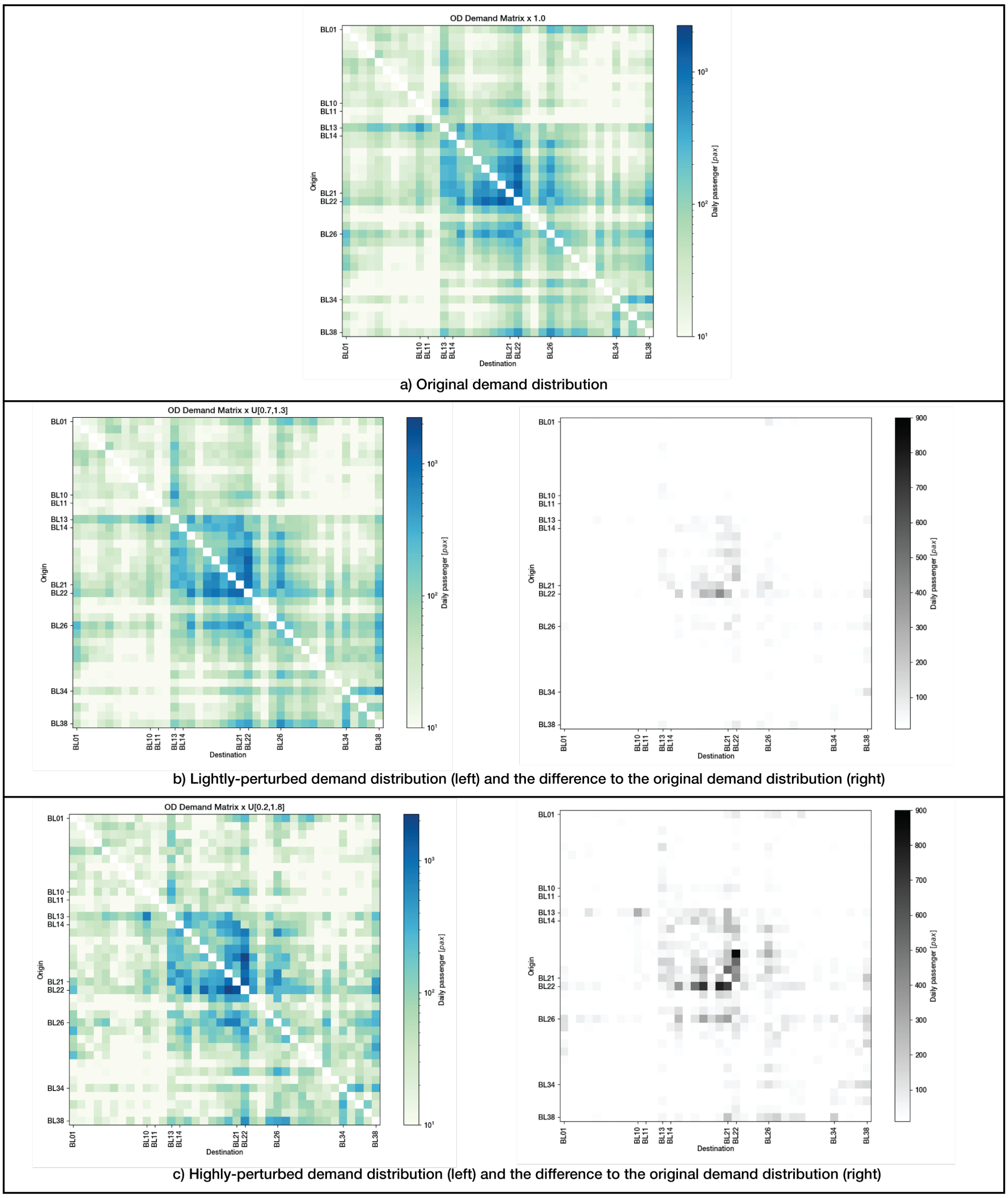


Figure 15 Standard deviations of the daily passenger demands in May 2021: a) daily passenger counts; b) normalised daily passenger counts



a) Original demand distribution

b) Lightly-perturbed demand distribution (left) and the difference to the original demand distribution (right)

c) Highly-perturbed demand distribution (left) and the difference to the original demand distribution (right)

Figure 16 Demand distributions used in this case study

A major drawback in this case study is zero knowledge about hourly variation of passenger demand within a day. As mentioned earlier, random perturbations are introduced to the original passenger OD demand distribution to imitate a time-dependent variation. Perturbation random values contained in matrix Q are drawn from uniform probability distribution with a predefined range centred at the multiplication identity (1.0). Two synthetic (static) demand distributions are created with the random-variable ranges of $[0.7, 1.3]$ and $[0.2, 1.8]$, and they are named lightly-perturbed and highly-perturbed demand distributions, respectively.

In this study, each experimental setting is composed of 2 static demand distributions corresponding to $K = 2$ skip-stop periods. The interval of each SS period is assumed to be 1 hour long $p_k^{end} - p_k^{sta} = 3600, \forall k \in \{1, 2\}$. Thus, there are totally two demand scenarios used in this case study as given in Table 10 shown in Figure 16.

Demand scenario	SS period 1	SS period 2
Demand Scenario 1: Original and lightly perturbed demand distributions (Figure 16b)	$\mathcal{D}^1 = \mathcal{D}^{ori*}$	$\mathcal{D}^2 = Q \circ \mathcal{D}^{ori*}$ where $q_{i,j} \sim U(0.7, 1.3) \forall i, j \in \mathbb{N}$
Demand Scenario 2: Original and highly perturbed demand distributions (Figure 16c)	$\mathcal{D}^1 = \mathcal{D}^{ori*}$	$\mathcal{D}^2 = Q \circ \mathcal{D}^{ori*}$ where $q_{i,j} \sim U(0.2, 1.8) \forall i, j \in \mathbb{N}$

Table 10 Demand data used in this case study

4.1.2. Dealing with the Demand Data of Circle Lines with Branches

The MRT Blue Line is the only ring line encompassing the centre of Bangkok with a branch towards the western suburban area called Bang Khae. A transit map focusing on this line is provided in Figure 17a. At the first glance, an obvious problem arisen is the incompatibility of the line geometry and the optimisation problem formulation; however, the formulation is still applicable to this line after a modification of input demand distribution.

Before other complicated issues, a skippable station set $\mathbb{S}$ can be established with a simple strategy that eliminates all termini and intermediate stations connected to other transit lines from the station set $\mathbb{N}$. Those stations are made non-skippable because they are major points of connection where service frequency should be maximised. Based on the original demand distribution, the passenger counts between these non-skippable stations account for 17%, and the passenger counts of all OD pairs associated with the non-skippable stations account for 76% of the total.

Train operations on this circle line with a branch are not different from any regular diametrical lines. The closure of circular shape at Tha Phra (BL01) interchange station does not have turning infrastructure. This means that trains travelling westward from Itsaraphap (BL32) to Tha Phra (BL01) can only progress straightforward to Bang Phai (BL33), and southbound trains from Charan 13 (BL02) to Tha Phra (BL01) likewise cannot proceed further to BL32 or BL33. For this reason, Tha Phra (BL01) interchange station can be considered as a mere spatial coincidence of line terminus and intermediate station. This consideration brings forth the transit-line topological model where the interchange station (BL01) decomposed into two independent stations, shown in Figure 17b. Let all stations apart from the interchange station on the circular segment be the ring part (BL02 – BL32) and the rest be the branch part (BL33 – BL38). Also, let the station index starts at 1 at the terminal station BL01_NS and runs increasingly in the clockwise direction; as a result, the station index for the intermediate station BL01_WE then becomes 33.

The decomposition of the interchange station creates another subsequent problem. The

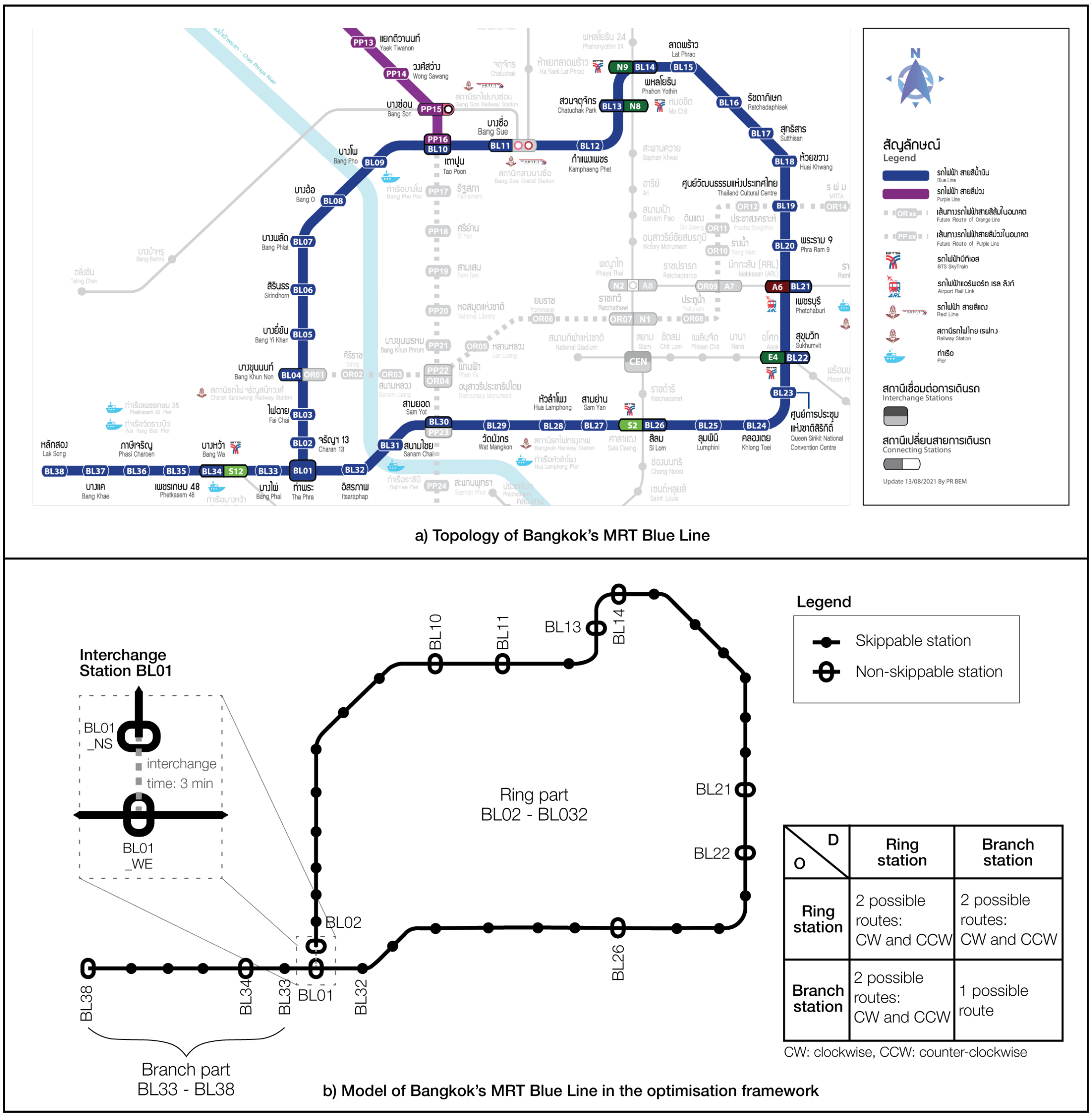


Figure 17 Modelling of a circle MRT line: the Blue Line in Bangkok: a) Route map of the Blue Line, source: MRTA; b) Model of the Blue Line in the optimisation framework

demand or ridership data are usually obtained from the automated fare collection system which counts and the number of passengers at the ticket-control gates and coordinates the check-in point and check-out point of every trip. Naturally, the check-in and check-out points at the interchange station cannot count number of transfers between west-east trains and north-south trains. This means that the original OD demand matrix provided by MRTA does not contain information about transfers. Therefore, OD demand pairs having a relation with the interchange station have to be decomposed and reorganised in order to make the OD demand matrix in line with the model in Figure 17b. This subsection is dedicated to describing the calculation procedure for this decomposition of demand about the interchange station.

Essentially, the decomposition process attempts to determine which of the decomposed stations is used by a related OD pair. A trip from or to a station in the ring part can be made in either the clockwise direction or counter-clockwise direction, whereas a trip within the branch part only has one straightforward direction. Although many of OD pairs with a relation with the ring part have an obviously optimal travel direction where the opposite direction is substantially suboptimal (e.g., BL01 – BL02 trip is best done in the clockwise direction while the connection in the counter-clockwise direction is overwhelmingly long), there are cases with ambiguity in travel directions. Ambiguous cases occur in OD pairs where the travel time in both directions are relatively the same (such as trips from BL01 to BL21 or from BL30 to BL10).

The logistic function is used to systematically evaluate the probability (or proportion) of passengers using the clockwise direction $P_{i,j}(CW)$. This probability is based on average travel time that is calculated by dividing station distance by an average line speed (Equation 4.1) which is assumed to be $7.5\ m/s$ for this case.[1] For any OD pair where $i > j$ and both i and j stations are not in the branch part, the clockwise trip always involves a transfer from BL01_-WE to BL01_NS, which is assumed to take $180\ sec$ of average transfer time. The formula of the logistic function is given in Equation 4.2. The steepness constant c of $1/200$ was deem appropriate after a number of experimentations, and the numerical result is given in Figure 18.

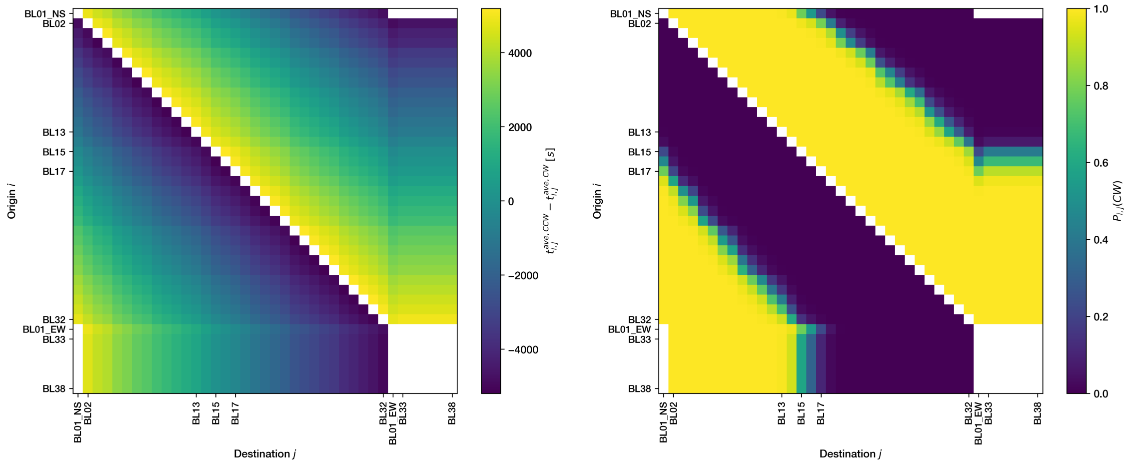


Figure 18 Dealing with the ambiguity of trip directions: left) travel time difference between counter-clockwise and clockwise trips; right) probability of choosing the clockwise direction for the OD pair

$$t_{i,j}^{ave,CW} = \begin{cases} \frac{1}{7.5}\left(\left(\sum_{n=i}^{j-1} d_n\right),\right. & \text{if } i < j \text{ and } (i < 33 \text{ or } j < 33) \\ \frac{1}{7.5}\left(\sum_{n=i}^{33-1} d_n + \sum_{n=1}^{j-1} d_n\right) + 180, & \text{if } i > j \text{ and } (i < 33 \text{ or } j < 33) \end{cases} \tag{4.1}$$

$$P_{i,j}(CW) = \frac{1}{1 + e^{-c(t_{i,j}^{ave,CCW} - t_{i,j}^{ave,CW})}}, \text{if } i < 33 \text{ or } j < 33 \tag{4.2}$$

[1] The resulted average travel time can be easily checked against the timetable in order to adjust for a representative average speed

The probability of passengers using the counter-clockwise direction is simply the complement of the above probability .

$$P_{i,j}(CCW) = 1 - P_{i,j}(CW) \tag{4.3}$$

These probabilities are applied to the OD demand distribution to subdivide it into two directionalised portions. The decomposition procedure consists of two main steps as visually aided in Figure 19 and described in what follows:

0. **OD demands within the branch part (Figure 19b):** Branch-to-branch OD pairs remain unchanged.

1. **OD demands about the interchange station (Figure 19b):**

   a. Carryover OD demands between the original interchange station (BL01) and the branch part to the decomposed intermediate station (BL01_WE)

   b. Associate the probabilities of travel directions with OD demands between the interchange station (BL01) and the ring part. Demands originating at the interchange station remain at terminus BL01_NS if they are best done in the clockwise direction; otherwise, they are assigned to the counterpart station BL01_-WE. Inversely, OD demands ending at the interchange station remain at terminus BL01_NS if they are best done in the counter-clockwise direction; otherwise, they are assigned to the counterpart station BL01_WE.

2. **OD demands requiring a transfer (Figure 19c):** This step resembles 1.b, but, unlike the earlier step, decomposing OD demands that are best done with a transfer at the interchange station produces double number of passenger counts.

   a. Consider specifically the lower triangular elements of the demand matrix excluding the elements related to the previous step. If any OD pairs among these elements are to be done in the clockwise direction, a transfer from the intermediate station BL01_WE to its counterpart station BL01_NS is needed. Corresponding to the probabilities of choosing the clockwise direction shown in Figure 18, the lower-left yellow trapezoidal area is these OD pairs that most likely need the transfer. The passenger counts of OD pair $i \rightarrow j$ is carried over to other two OD demand pairs: $i \rightarrow 33$ and $1 \rightarrow j$.

   b. The same operation is applied to the upper triangular elements of the demand matrix, but the directions are all the opposite.

Note that the OD demand pairs surrounding the main diagonal line that remain in the same positions in Figure 19c are those that do not require a transfer.

Finally, the superposition of the results of the two steps is the decomposed demand distribution shown in Figure 20 which can be readily used in the optimisation framework. The zero

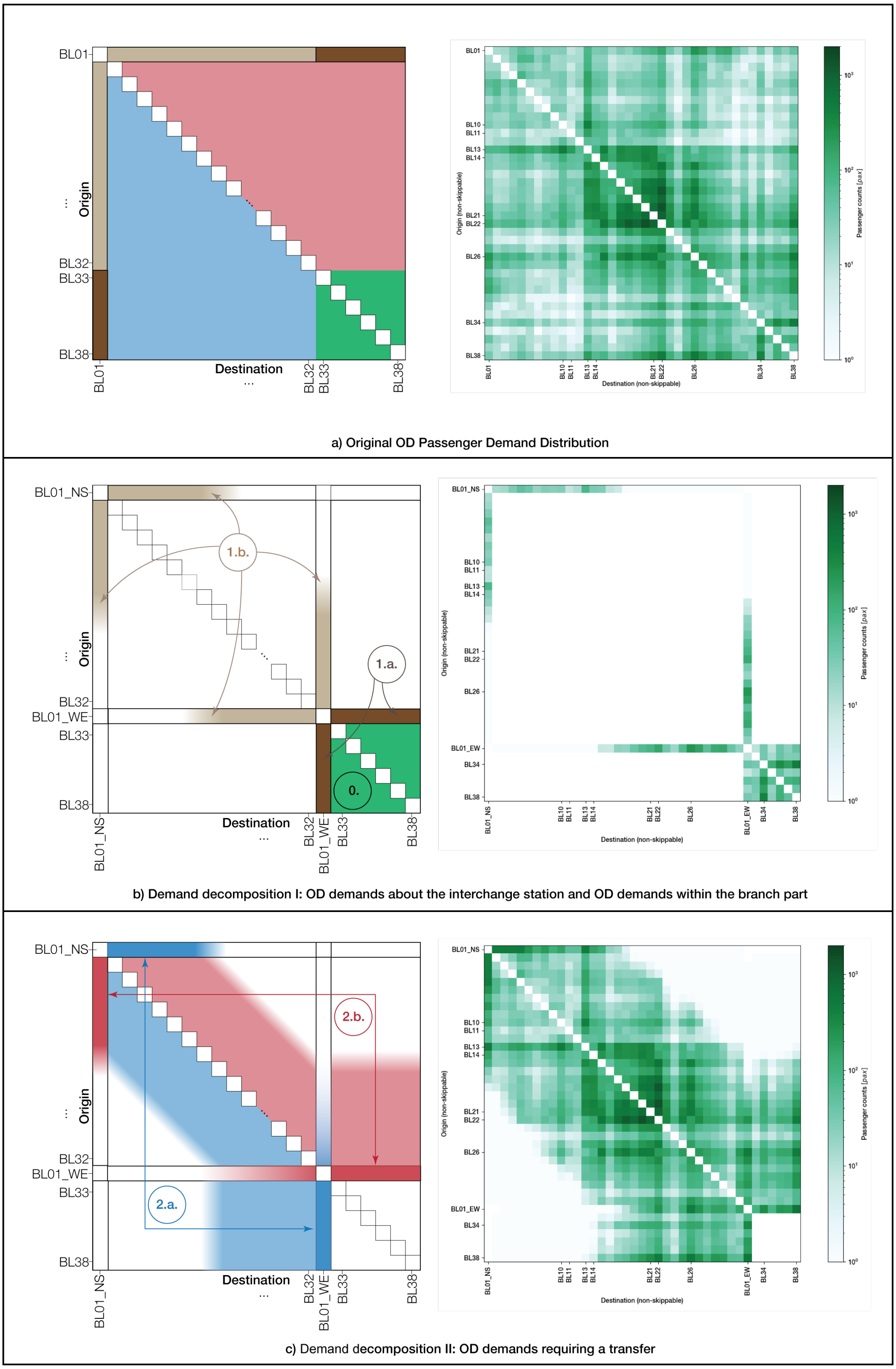


Figure 19 Demand decomposition procedure: a) Original OD Demand Distribution; b) Step 0 and Step 1; c) Step 2

elements in the far upper and lower triangular parts are those trips that need a transfer at the interchange station; hence, they are summed to the OD demand pairs that are directly related to both the terminus BL01_NS and the intermediate station BL01_WE.

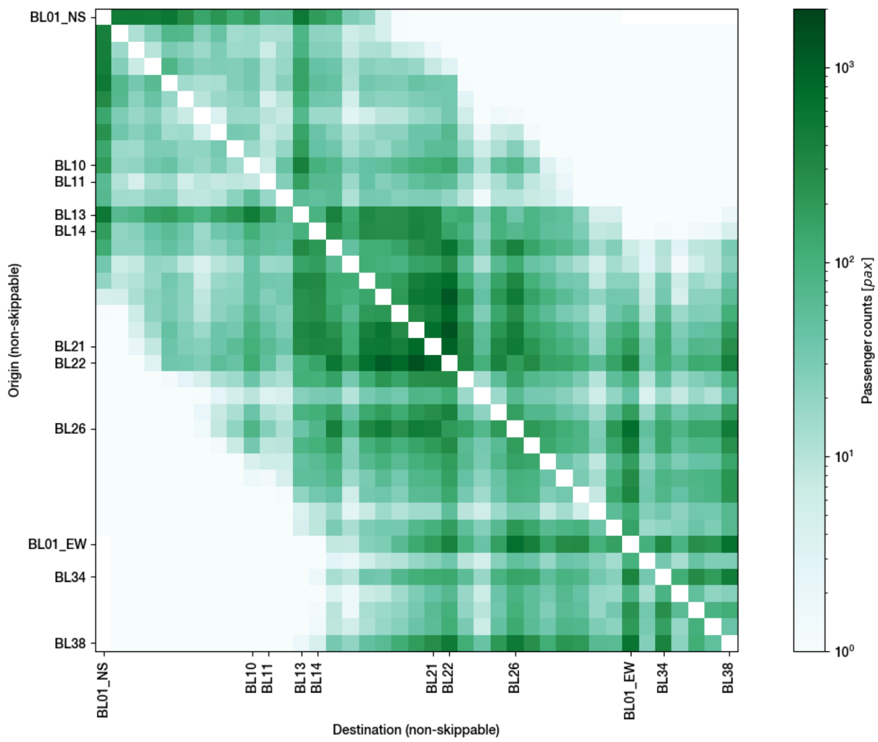


Figure 20 Decomposed passenger OD demand distribution (original)

Daily passenger counts are divided by operating hours of the Bangkok MRT system to obtain the passenger arrival rate. According to the official website of the MRT Blue Line operator, the line operates from 5:30 - 12:00 am on every workday. The daily passenger counts are then divided by $\frac{(24.0-5.5)\cdot 3600}{F}$ where F is an assumed peak-load factor. It is assumed that the passenger load during peak hours is $F = 2$ times higher than the daily average, making the converting divisor equal to $33000 \ \frac{sec}{workday}$.

4.1.3. Skip-Stop Scenarios and Hyperparameters for the Genetic Algorithm

There are totally 9 SS settings considered for stop-plan optimisation in this case study. They are various structures of SS operations under the same demand parameters and in the same transit system described previously, and each of them corresponds to an experiment number. The optimisation results of each setting are then compared against the other SS settings to shed light on the advantages of one over the others. The service frequency is assumed to be equal to $250 \ [s]$ for both SS periods and all SS settings $h_k^{ave} = 250 \ [s]$, $\forall k \in \{1, 2\}$. The optimisation algorithm for each of them is set with the same hyperparameters listed in Table 11. The only termination criterion is defined by maximum number of generations (iterations). A description on the tuning of these hyperparameters is omitted.

The SS setting of each experiment is described in the following and depicted in Figure 21.

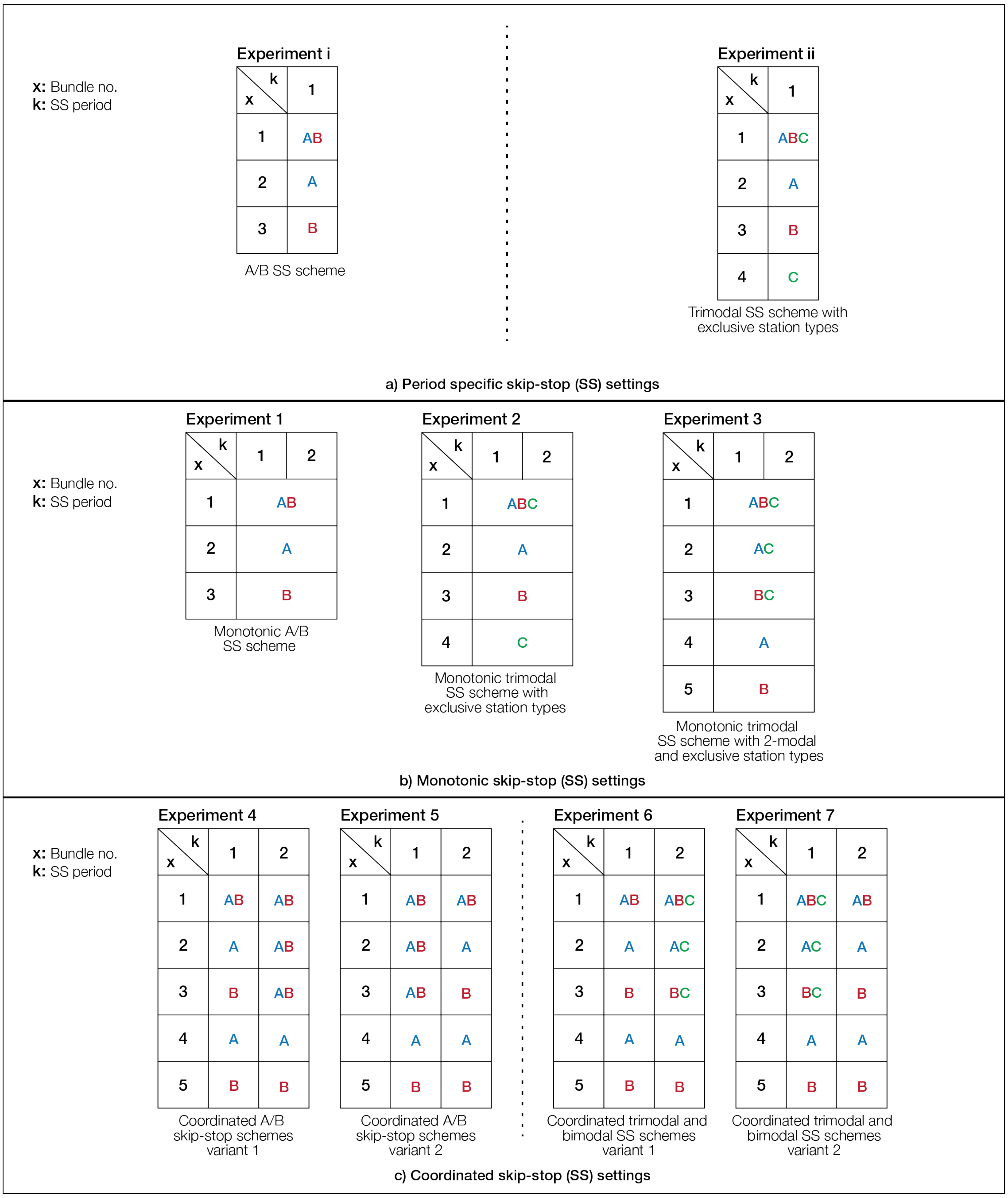


Figure 21 SS settings used for the case study

The first two settings, in experiments i and ii, consider only one SS period at a time; therefore, 3 instances of optimisation run have to be executed for each experiment, correspondingly to the 3 static demand distributions: the original, lightly-perturbed, and highly-perturbed static demand distributions.

The next three SS settings, in experiments 1-3 (Figure 21b), are monotonic SS schemes in which stopping patterns do not change between the first and the second SS periods: SS setting 1 is the conventional `A/B` scheme; SS setting 2 is a trimodal scheme with the all-stop station type and 3 exclusive station types `[A]`, `[B]`, `[C]`; SS setting 3 is a trimodal scheme

Genetic operators	
Natural selection operator	proportionate selection with the exploitative modification
Crossover operator	1-point crossover
Mutation	uniform mutation
Numerical hyperparameters	
Maximum number of generations	$100\ generations$
Population size G	$20\ chromosomes$
Parental population size R	$6\ chromosomes$
Number of immortal parents r	$2\ chromosomes$
Mutation probabilityP_m	0.3
All-stop solution coefficient c	1.10

Table 11 Genetic algorithm hyperparameters for all SS experiments

with two 2-mode station types `[A/C]`, `[B/C]` and 2 exclusive station types `[A]`, `[B]` beside the all-stop station type. The monotonic schemes do not have an ability to adapt to time-variant demand; therefore, the resulted stop plans are expected to be relatively inferior in time-variant demand scenarios compared to the coordinated schemes.

The last four settings, in experiments 4-7 (Figure 21c), involve a coordination of two different SS schemes. These experiments come in pairs where SS schemes of one are swapped to make the other complementary one. SS Setting 4, in short, is `A/B` schemes with station-type bundles that allow transformation from an exclusive station type to the all-stop type, and the counterpart SS Setting 5 allows the opposite transformation. Next, experiments 6 and 7 involve a coordination between the `A/B` scheme and the trimodal scheme in SS setting 3. The coordination pairs the exclusive station types of the conventional `A/B` scheme with the corresponding two-mode and exclusive station types of the trimodal scheme. Additionally, the same information in the forms of station-type policy matrices $\mathcal{P}$ is given in Appendix C.

4.2. Optimisation Results

Due to the the stochastic nature of the employed meta-heuristic optimisation algorithm, the genetic algorithm, each optimisation run is unique and may produce completely different final stop plan results. Therefore, each experiment was repeated 5 times. The objective function of the best chromosome in each generation is then plotted in Figures 22 and 23, where solid lines represent the mean and the coloured opaque areas represent the 95% confidence intervals (based on student's t-statistics). It is evident that the standard deviation of the best objective functions converges and become relatively stable from generation 50 onwards. Also, the rate of descent has almost completely diminished. Based on these two facts, the most optimal stop plans from these experimental runs are accepted. Nonetheless, these plans are most likely not the global optimal solutions since they can be truly proven only by full enumeration. The most optimal stop plans from all experiments except coordinated schemes are given in Appendix D.

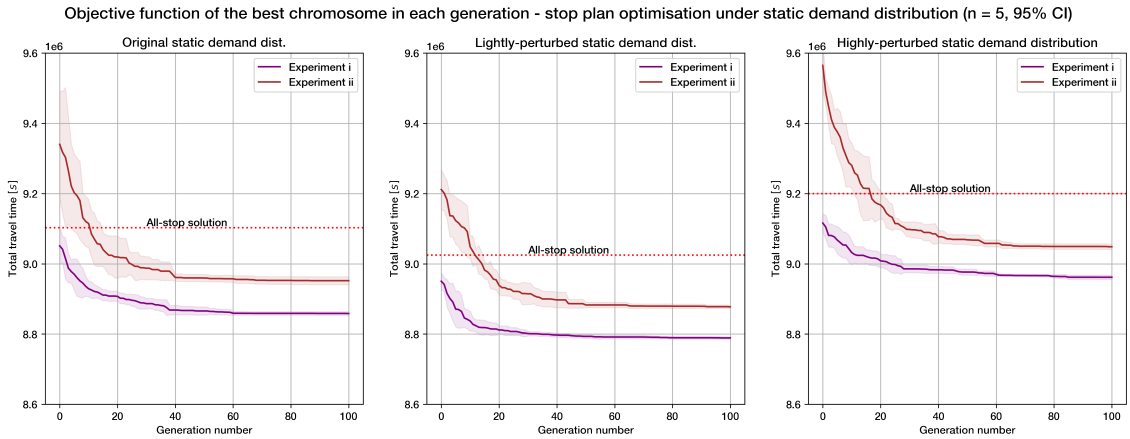


Figure 22 Convergence plots for experiments i and ii

An effect of perturbations on the original demand distribution is displayed in Figure 22. The objective function of the all-stop scenario, represented by red dotted line, varies as the demand distribution is perturbed. However, the objective functions of the most optimal plans are also affected but in roughly the same proportion as the all-stop solution. This signifies that the perturbation approach produces somewhat the same effect as numerical scaling (multiplication by a scalar).

The results of experiments i and ii are combined according to Demand Scenarios 1 and 2 for comparison with all other experiments. The travel-time-saving performances of the monotonic schemes (exp. 1–3) are closely related to that of period-specific schemes (exp. i and ii). In Demand Scenario 2, the combined result of experiment i performs about 1% better than the monotonic scheme in experiment 1. Although the difference is marginal, this corresponds to the hypothesis described in Section 3.1 that independently optimising for another stop plan under a varied demand pattern would result in more optimal total travel time. Bundling the station types across the time periods, as in the monotonic schemes, constrains the solution space and would lead to less optimal solution. The optimisation results of the two demand scenarios are majorly almost identical.

Among the monotonic schemes, the worst performing scheme here is the trimodal scheme in SS setting 2 with only exclusive stations, and the best performing scheme is the conventional `A/B` scheme in SS setting 1. The performance of the trimodal scheme with exclusive and 2-mode station types in SS setting 3 is in-between.

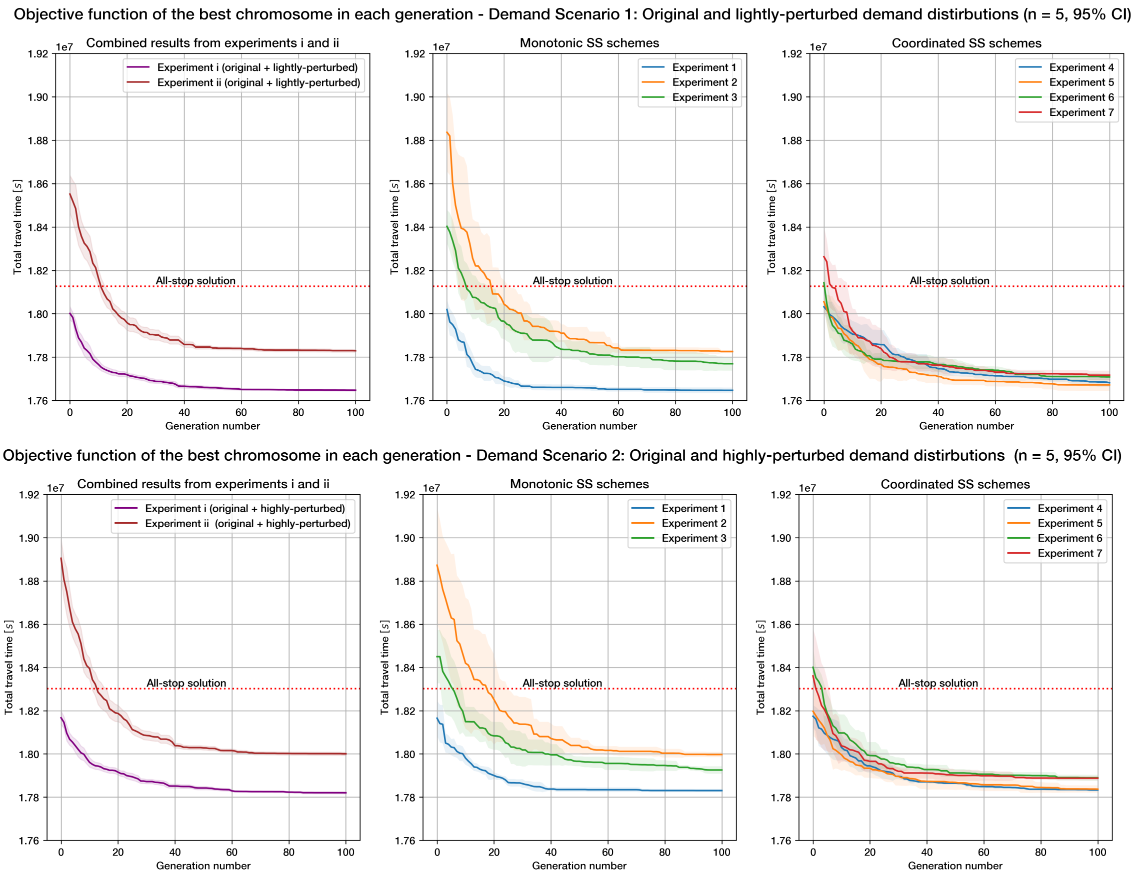


Figure 23 Convergence plots for all experiments: top) Demand Scenario 1; bottom) Demand Scenario 2

The coordinated schemes in experiments 4 and 5 can be seen as variants of the `A/B` scheme as they perform almost the same as stop plans from experiment 1. The hybrids coordinated schemes in experiments 6 and 7 are not as optimal as the simple `A/B` scheme. To summarise, all of the experimented coordinated schemes are all inferior to the monotonic `A/B` scheme in experiment 1 regarding both the total travel time and the complexity of stop plan. However, these findings are based on the use of uniform random variables for demand perturbations. As there are countless ways of demand perturbation to experiment, coordination of SS plans is not yet concluded to be an ineffective strategy based on these optimisation results. If a transit system is assumed to be subject to a different pattern of demand variation, or real time-dependent demand data is available, then these coordinated approaches may be reinvestigated. Nonetheless, the results from monotonic SS schemes may be validly discussed based on a presumption that the imitated time-variant demand patterns are rather static.

For both demand scenarios, the monotonic `A/B` scheme achieves a reduction of total travel time of approximately 2.6% from the all-stop plan. In comparison, the case study of Lee et al. (2014) presented the most optimal `A/B` SS plan that achieves a reduction of about 20 %. It is true that different transit system characteristics, service level parameters, model-specific assumptions can make a difference in the total time saving, but this is by far an extreme difference. An explanation to this is given in the next section where stop plans are discussed against the demand distribution of this circle line.

4.3. Discussion on the Optimised Stop Plans

In this section, the most optimal stop plans from experiments 1-3 are analysed, focusing on average travel time improvement from the all-stop scenarios. Four types of OD pairs are differentiated as follows:

- **Type I**: pairs of two all-stop stations.
- **Type II**: pairs of an all-stop station and an exclusive station or a two-mode station.
- **Type III**: pairs of two exclusive stations, pairs of two two-mode stations, or pairs of an exclusive station and a two-mode station, that do not require a transfer.
- **Type IV**: pairs of two exclusive stations, pairs of two two-mode stations, or pairs of an exclusive station and a two-mode station, that require a transfer.

To begin with, the analysis results of the plans from experiment 1 are given in Figure 24. The patterns are similar among the two demand scenarios; hence, they are collectively discussed. The passenger counts of each OD pair type over the SS periods of 2 hours are given in the parentheses.

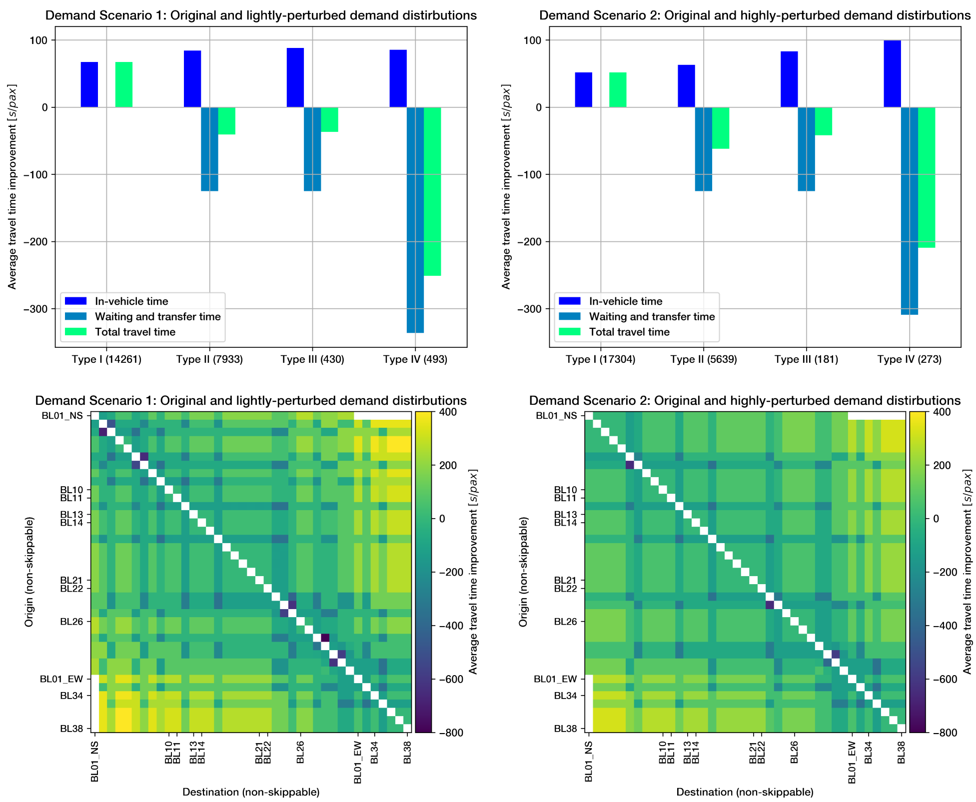


Figure 24 Average travel time improvements from the all-stop scenarios of the most optimal solutions from Experiment 1

The bar charts indicate that a number of patrons have to wait longer at platforms, while only patrons associated with Type I OD pairs do not experience any change in waiting time. The average waiting time improvement of Types II and III is exactly equal to $-\frac{h^{ave}}{2}$ in accordance with Assumption 7, while the average change in waiting time of Type IV is significantly larger due to transfer requirements. Most of the patrons experience shorter in-vehicle time, but not all of them do so. When in-vehicle time and waiting time are summed up, only patrons using Type I OD pairs benefit from the SS scheme as all other types offer longer travel time on average. Since, the number of patrons associated with Type I OD pairs is the largest, this SS scheme benefits the system as a whole. This corresponds to the principle of SS operations 'slowing the few to make the many travel faster'.

However, there are blind spots that come with the SS scheme where some patrons experience longer in-vehicle time in contrast to the majority. The average travel time improvement is also given in the structure of the colour-coded OD matrices, where brighter yellow colour represents larger total travel time improvement. Dark spots, indicating large increases of average travel time, along the main diagonal line can be noticed. These spots represent enormous time impedance due to back-tracking transfers which occur due to consecutive exclusive stations of different types. As a result, the transit system would lose some short-haul demands if the scheme is implemented. The number of these dark spots are not too large or the demands are low enough that the average in-vehicle time improvement of Type IV is still positive as in the bar chart.

These colour-coded matrices show that the OD pairs that benefit most from the scheme are those of long-haul trips as in the brighter upper right and lower left areas of the matrices. However, the demand distributions of any circle lines (Figure 20) would always evades these brighter areas. This is the explanation to why any SS scheme for a circle line may perform worse than it would in a diametrical line system with some long-haul demand loads.

Next, the optimisation results of the trimodal scheme from experiment 2 is given in Figure 25. In this scheme, making a station skipped by some trains instantaneously increases the total waiting time of the patrons associated with the station by three times. This significant increase is overwhelming compared to the benefits in in-vehicle time. Here, the dark spots scattered in the colour-coded matrices are not due to back-tracking transfers, but they are purely due to excessively long waiting time combined with long forward-tracking transfers. Given that the colour coding among Figures 24 through 26 use the same numerical range, the upper-right and lower-left areas are significantly dimmer as a result of the smaller number of skipped stations. All of the mentioned effects are accordingly reflected by the bar charts.

Lastly, the more balanced trimodal scheme from experiment 3 is briefly presented. Every aspect of the time-saving performance given in Figure 26 is in halfway between the two schemes discussed above. The two-mode station types are the options that do not exceedingly affect waiting time and transfer time, compared to the exclusive station types.

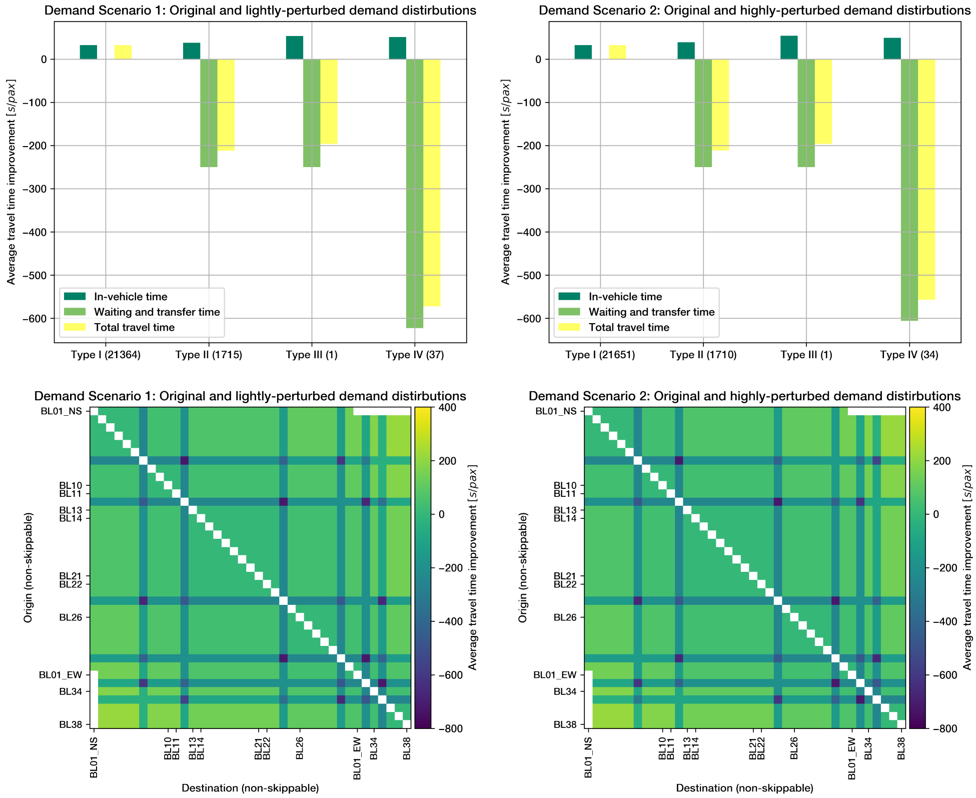


Figure 25 Average travel time improvements from the all-stop scenarios of the most optimal solutions from Experiment 2

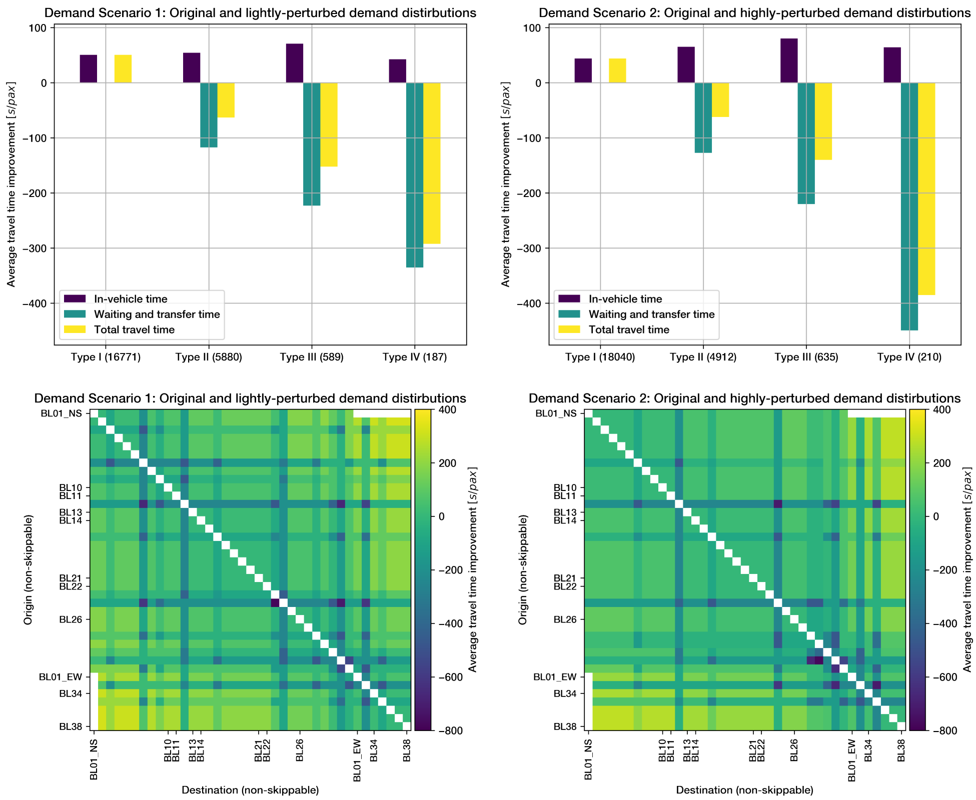


Figure 26 Average travel time improvements from the all-stop scenarios of the most optimal solutions from Experiment 3

5. Conclusion

A skip-stop scheme can accelerate transit operation from the standard all-stop approach when it is properly optimised according to the demand distribution in the transit system. By simply making some trains skip lightly-loaded stations, the total travel time of the whole system can be decreased while a small number of passengers using those skipped stations have to wait longer. This simple acceleration strategy can be done even in transit systems where train overtaking is not possible.

In the existent literature, there are two families skip-stop schemes as introduced before: the conventional `A/B` SS scheme and flexible SS schemes. These two families differ in that the stopping pattern of each train service is unique to other train services in a flexible SS scheme, whereas the stopping pattern of the conventional SS scheme is fixed by stations. The conventional `A/B` scheme is, indeed, not the only possible station-based SS scheme. This study proposes an optimisation formulation that can generalise the characteristics of station-based SS schemes; so, the formulation is not limited to the `A/B` scheme. It is open for any station-based SS schemes with an arbitrary number of train modes, arbitrary number and patterns of station types. A number of trimodal schemes, which have never been explored, are then experimented in a case study; however, this is not the main research objective.

A drawback of the conventional scheme is the inability to adapt to time-variant demand, which is what flexible SS schemes were created for. This optimisation formulation primarily aims to enable the ability to adapt to time-variant demand by using coordinated station-based SS schemes. The term coordination pertains to the judgement about the following:'from a certain station type, which station type is allowed to transform to when demand distribution varies?' This judgement is to be done by engineers employing this optimisation program.

The experimental results from the case study on the circle MRT Blue Line in Bangkok, Thailand are based on a demand data which has a major limitation: the lack of actual time-variant demand patterns. Static daily OD demand distributions were only available. A simple approach of demand perturbation was consequently used on the original static demand distribution to create an artificial variation of demand through time. This approach may be too simplistic to represent real dynamics of time-variant demand in a real-world system. Therefore, it is difficult conclude about the coordinated schemes for real-world implications at this point.

With the current artificial time-variant demand patterns, the monotonic `A/B` SS scheme was found to be the best performing scheme in terms of reduction in total travel time and solution complexity, as one would easily see. The results of the hybrid trimodal scheme in experiment 3 are relatively positive compared to the trimodal scheme in experiment 2. The hybrid scheme may be worth further research, for it has the flexibility to differentiate three levels of service frequency at a station.

Due to the time constraints of this research and limited computational power, this study is concluded without sensitivity analysis on other transit system characteristics or service frequency parameters. Nonetheless, a novel demand-data decomposition method for circle transit lines had to be devised prior to optimisation process, and this is another humble contribution in addition to the main research objective. Later on, the mismatch of the decomposed demand distribution to the average travel time improvement matrices was shown to suggest that the merit of an SS scheme may not be fully leveraged as in a straight diametrical transit line with a large proportion of long-haul demands.

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A. Methodology Appendix: Notations

Parameter arrays		Parameter array entries	
$\boldsymbol{\mathcal{D}}'$	time-variant demand matrix	$\lambda'_{i,j,k'}$	passenger OD arrival rate from origin i to destination j at demand period k' $[pax/s]$
$\boldsymbol{\mathcal{D}}$	SS-period demand matrix	$\lambda_{i,j,k}$	weighted average passenger OD arrival rate from origin i to destination j in SS period k $[pax/s]$
$\mathcal{S}$	stop-spacing vector	d_n	distance between stations n and $n+1$ $[m]$
$\mathcal{P}$	station-type policy matrix	b_{x,k,m_k}	binary value indicating stopping of train mode m in SS period k at stations of Bundle x
Finite sets		**Size of finite sets**	
$\mathbb{N}$	station set	N	total number of stations
$\mathbb{S}$	skippable station set	S	total number of skippable stations
$\mathbb{K}'$	demand period set	K'	total number of demand periods
$\mathbb{K}$	skip-stop period set	K	total number of SS periods
$\mathbb{M}_k$	skip-stop mode set for SS period k	M_k	total number of SS modes for SS period k
$\mathbb{X}$	station-type bundle set	X	total number of bundles
Operational parameter scalars		**Indices**	
α	constant acceleration rate $[m/s^2]$	i	origin station index belonging to $\mathbb{N}$
β	constant deceleration rate $[m/s^2]$	j	destination station index belonging to $\mathbb{N}$
$t^{dwell,min}$	minimum dwell time $[s]$	n	arbitrary station index belonging to $\mathbb{N}$
h^{min}	safety headway $[s]$	k'	demand period index belonging to $\mathbb{K}'$
v^{max}	maximum operating speed $[m/s]$	k	SS period index belonging to $\mathbb{K}$
h_k^{ave}	average service headway in SS period k $[s]$	x	station-type bundle index belonging to $\mathbb{X}$
h^{turn}	turnaround headway at termini $[s]$	m_k	SS alphabetical mode index (or label) belonging to $\mathbb{M}_k(\texttt{A},\texttt{B},\texttt{C},\ldots)$
Temporal parameters			
$o_{k'}^{sta}$	start time of demand period k' $[s]$	p_k^{sta}	start time of SS period k $[s]$
$o_{k'}^{end}$	end time of demand period k' $[s]$	p_k^{end}	start time of SS period k $[s]$

Table 12 Notations for the mathematical model for coordinated SS operations: boundary conditions, including operational conditions and predetermined skip-stop conditions, and indices

Objective function			
y^{wt}	Sum of total at-platform waiting time and transfer time of all passengers across all SS periods in $\mathbb{K}$ $[s \cdot pax]$	$\bar{y}^{wt}$	Unweighted sum of average at-platform waiting time and transfer time of all SS periods in $\mathbb{K}$ $[s]$
y^{tt}	Sum of total in-vehicle travel time of all passengers across all SS periods in $\mathbb{K}$ $[s \cdot pax]$	$\bar{y}^{tt}$	Unweighted sum of average in-vehicle travel time of all SS periods in $\mathbb{K}$ $[s]$
Independent variable arrays		**Independent variable array entries**	
$\mathbf{X}$	independent variable vector	x_s	bundle number for station $s \in \mathbb{S}$
$\mathbf{X}'$	bundle-number vector	x'_n	bundle number for station $n \in \mathbb{N}$

Intermediate variables			
Waiting time model		**Travel time model**	
$\mathcal{T}_k$	train-transfer table for SS period k	t_n^{α} (d_n^{α})	acceleration time (distance) for the distance between station n and station $n+1$ $[s]$
$\mathcal{R}_k$	routing table for SS period k	t_n^{β} (d_n^{β})	deceleration time (distance) for the distance between station n and station $n+1$ $[s]$
$\mathcal{H}_k$	headway table for SS period k	t_n^{hold} (t_n^{hold})	riding time (distance) at constant speed of v^{max} for the distance between station n and station $n+1$ $[s]$
h_{n,k,m_k}^{wait}	time interval from the departure of train mode $m_k - 1$ to the departure of train mode m_k (headway) at station n in SS period k $[s]$	t_n^{dwell}	dwell time at station n $[s]$
h_{i,j,k,m_k}^{wait}	time interval from the departure of train mode $m_k - 1$ to the departure of train mode m_k (headway) at station i in SS period k, where m_k bounds for either station j or an transfer station in case of transfer route $[s]$	d^{thres}	distance threshold for the demarcation of station spacing d_n between Speed Profile Cases I and II $[s]$
h_{i,j,k,m_k}^{tran}	time interval from the departure of train mode m_k to the departure of the transfer train mode predefined by $\mathcal{T}_k$ (headway) at a transfer station of arbitrary location in SS period k (only becomes non-zero when the route is a transfer route) $[s]$	t_{i,j,k,m_k}^{tt}	departure-arrival travel time of m_k train in SS mode k from station i to station j (which can be either directly or through a transfer where m_k is the first train mode of the transfer pair) $[s]$
t_k^{wt}	total waiting time and transfer time in SS period k $[s \cdot pax]$	t_k^{tt}	total passenger travel time in SS period k $[s \cdot pax]$

Table 13 Notations for the mathematical model for coordinated SS operations: objective function and intermediate variables

B. Methodology Appendix: Concluding Transfer Routes

Suppose that there are 2 possible transfer routes, for an OD pair, that share the same transfer-train (second train) mode. The main difference between these two routes is in first-train mode. Consequently, one route involves the first train-mode that departs just qh^{ave} ahead of the transfer mode, while the first train mode of the other route leaves a larger gap of transfer time rh^{ave} where $r > q$ and $r, q \in \{1, 2, 3, ...\}$. The question is, which first-train modes should patrons use to travel from from O to D? Recall that M is the total number of SS train modes, $q, r < M$, and the average departure headway notation h^{ave} is reduced to h for conciseness only in this chapter. Consider the following two possible transfer strategies:

1. **Both transfer routes are recommended** (corresponding to 4.a):

$$\text{Total waiting time: } \frac{\lambda}{2}\big[((M-(r-q))h)^2 + ((r-q)h)^2\big]$$
$$\text{Total transfer time: } \lambda\big[(M-(r-q))h \cdot rh + (r-q)h \cdot qh\big]$$

2. **Only the transfer route with smaller transfer time gap is recommended** (corresponding to 4.b):

$$\text{Total waiting time: } \frac{\lambda}{2}\big[(Mh)^2\big]$$
$$\text{Total transfer time: } \lambda\big[Mh \cdot qh\big]$$

The mathematical derivation of the following equation demonstrates that the sums of total waiting time and total transfer time of the two transfer strategies are equal:

$$\frac{\lambda}{2}\big[((M-r+q)h)^2 + ((r-q)h)^2\big] + \lambda\big[(M-r+q)h \cdot rh + (r-q)h \cdot qh\big] = \frac{\lambda}{2}\big[(Mh)^2\big] + \lambda\big[Mh \cdot qh\big]$$

$$\frac{1}{2}\big[(M^2+r^2+q^2-2Mr+2Mq-2qr)h^2+(r^2-2qr+q^2)h^2\big]+\big[(Mr-r^2+qr)h^2+(qr-q^2)h^2\big] = \frac{M^2h^2}{2} + Mqh^2$$

$$\frac{M^2}{2} + \frac{r^2}{2} + \frac{q^2}{2} - Mr + Mq - qr + \frac{r^2}{2} - qr + \frac{q^2}{2} + Mr - r^2 + qr + qr - q^2 = \frac{M^2}{2} + Mq$$

$$\frac{M^2}{2} + Mq = \frac{M^2}{2} + Mq$$

The next derivation presented in the following is used to demonstrate that the total transfer time of the first strategy is always longer than that of the second strategy:

$$\begin{aligned}
\lambda\big[(M - (r - q))h \cdot rh + (r - q)h \cdot qh\big] &> \lambda\big[Mh \cdot qh\big] \\
(Mr - r^2 + qr)h^2 + q(r - q)h^2) &> Mqh^2 \\
Mr - r^2 + qr + q(r - q) - Mq &> 0 \\
M(r - q) + r(r - q) + q(r - q) &> 0 \\
(M + q + r)(r - q) &> 0
\end{aligned}$$

Since it has been defined that $r > q$, the inequality $M + q + r > 0$ always holds. This automatically deduces, on the contrary, that the total waiting time of the first strategy is always shorter than that of the second strategy.

In conclusion, the total waiting time and total transfer time of both strategies is equal, but each strategy results in different proportions of total waiting time and total transfer time. Therefore, if the waiting time for first train weighs higher, the first strategy is to be chosen over the second strategy, and vice versa. In the case that waiting time and transfer time weigh equally, the second strategy may be preferred because it is simpler to implement than the first strategy.

C. Case Study Appendix: Station-Type Policy Matrices

SS setting i

$$\mathcal{P}_i = \begin{array}{c} \begin{array}{cc} \text{1.A} & \text{1.B} \end{array} \\ \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \end{array}$$

SS setting ii

$$\mathcal{P}_{ii} = \begin{array}{c} \begin{array}{ccc} \text{1.A} & \text{1.B} & \text{1.C} \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

SS setting 1

$$\mathcal{P}_1 = \begin{array}{c} \begin{array}{cccc} \text{1.A} & \text{1.B} & \text{2.A} & \text{2.B} \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

SS setting 2

$$\begin{array}{c} \begin{array}{cccccc} \text{1.A} & \text{1.B} & \text{1.C} & \text{2.A} & \text{2.B} & \text{2.C} \end{array} \\ \mathcal{P}_2 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

SS setting 3

$$\begin{array}{c} \begin{array}{cccccc} \text{1.A} & \text{1.B} & \text{1.C} & \text{2.A} & \text{2.B} & \text{2.C} \end{array} \\ \mathcal{P}_3 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

SS setting 4

$$\begin{array}{c} \begin{array}{cccc} \text{1.A} & \text{1.B} & \text{2.A} & \text{2.B} \end{array} \\ \mathcal{P}_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

SS setting 5

$$
\begin{array}{cc}
 & \begin{array}{cccc} \text{1.A} & \text{1.B} & \text{2.A} & \text{2.B} \end{array} \\
\mathcal{P}_5 = & \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}
\end{array}
$$

SS setting 6

$$
\begin{array}{cc}
 & \begin{array}{ccccc} \text{1.A} & \text{1.B} & \text{2.A} & \text{2.B} & \text{2.C} \end{array} \\
\mathcal{P}_6 = & \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}
\end{array}
$$

SS setting 7

$$
\begin{array}{cc}
 & \begin{array}{ccccc} \text{1.A} & \text{1.B} & \text{1.C} & \text{2.A} & \text{2.B} \end{array} \\
\mathcal{P}_7 = & \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}
\end{array}
$$

D. Case Study Appendix: Experimental Results

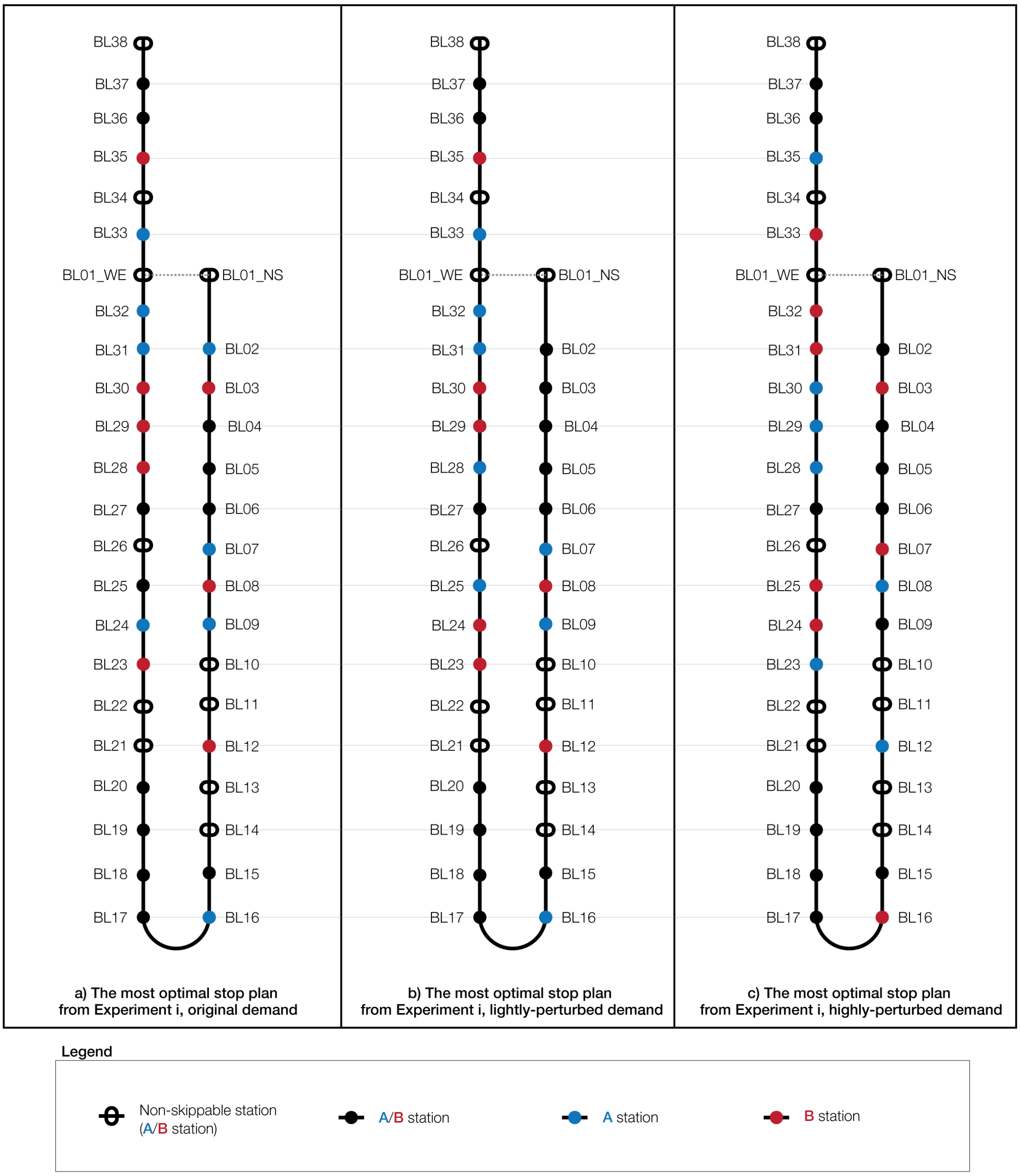

Figure 27 The most optimal stop plans from experiment i

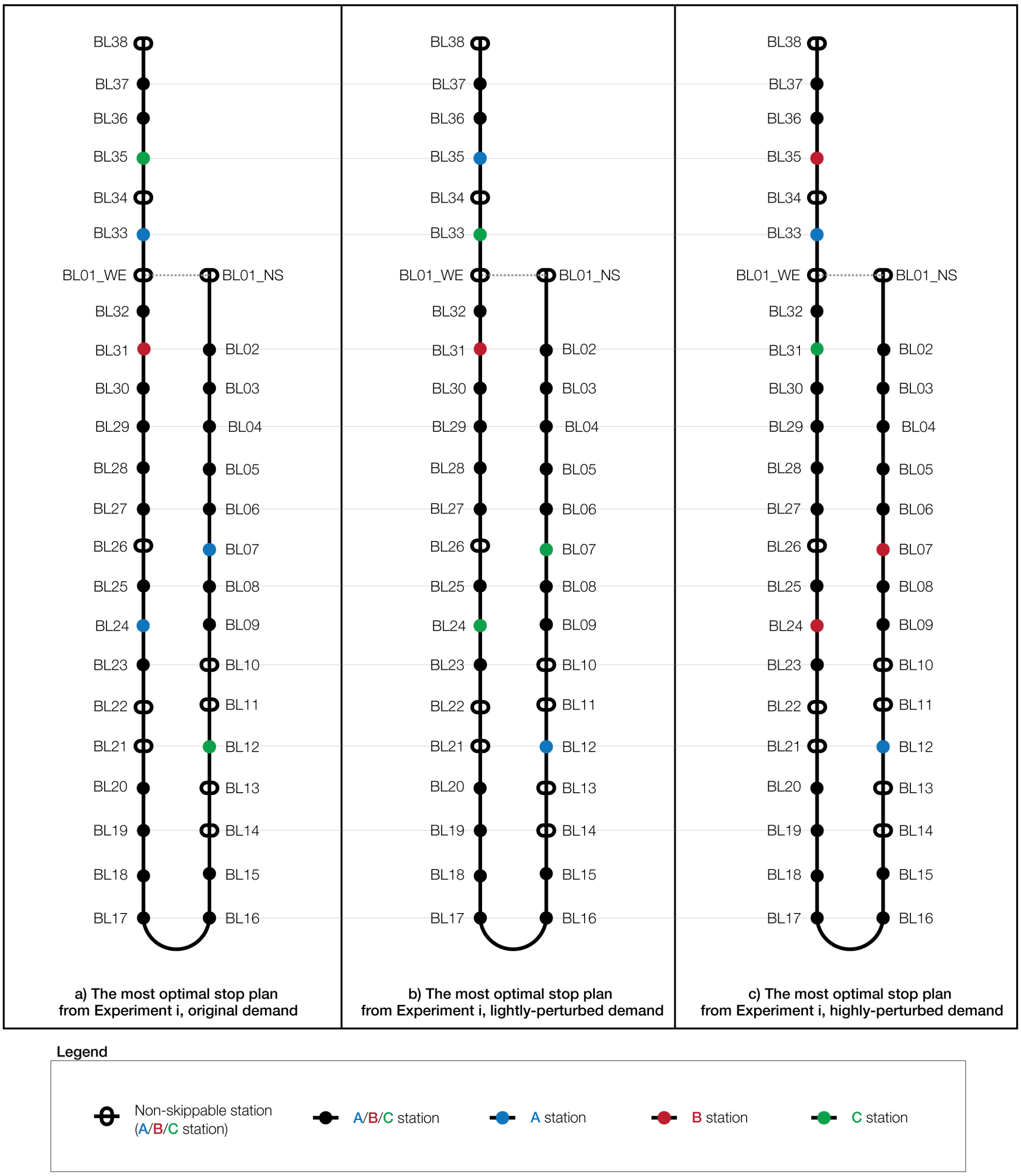

Figure 28 The most optimal stop plans from experiment ii

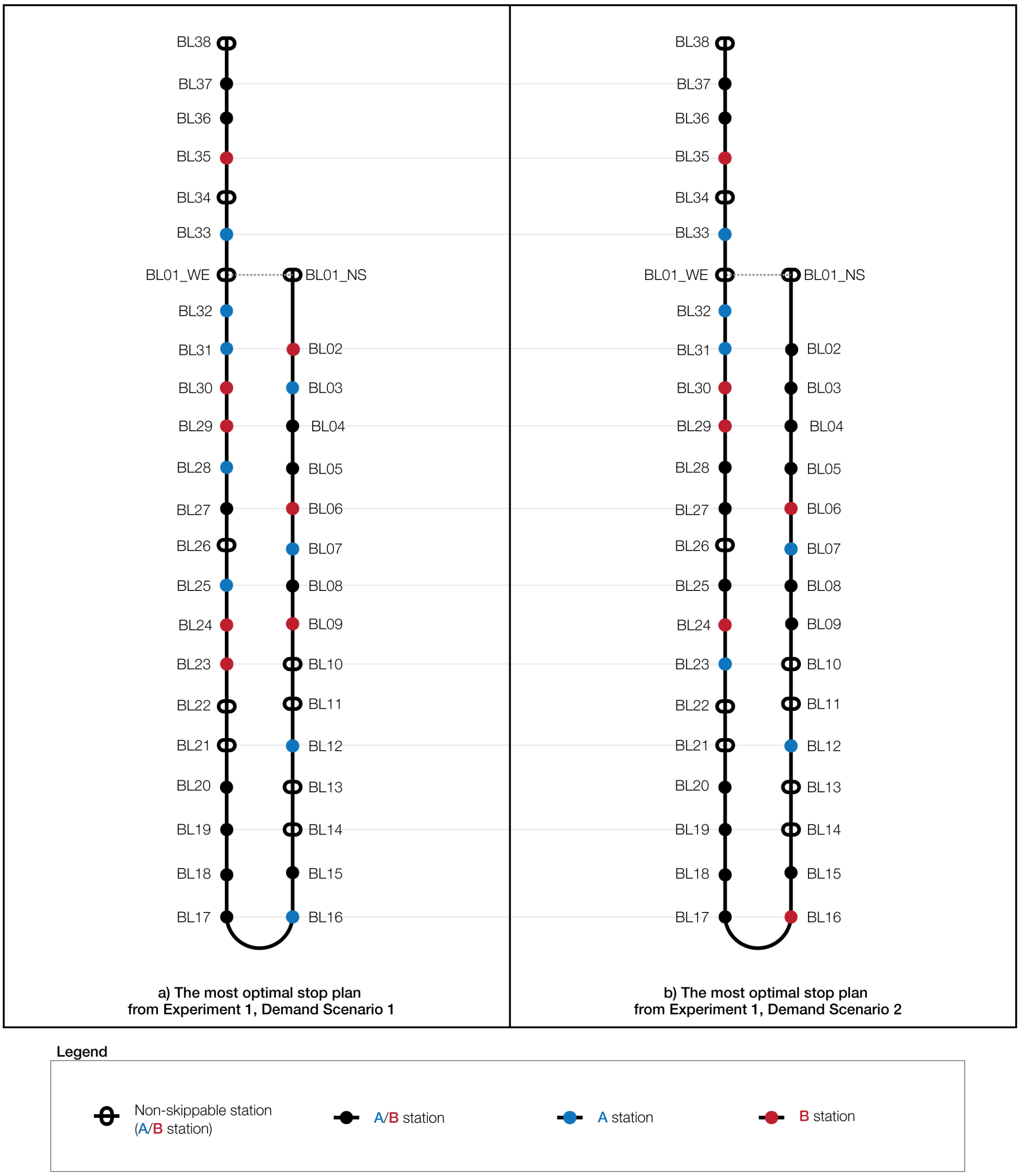

Figure 29 The most optimal stop plans from experiment 1

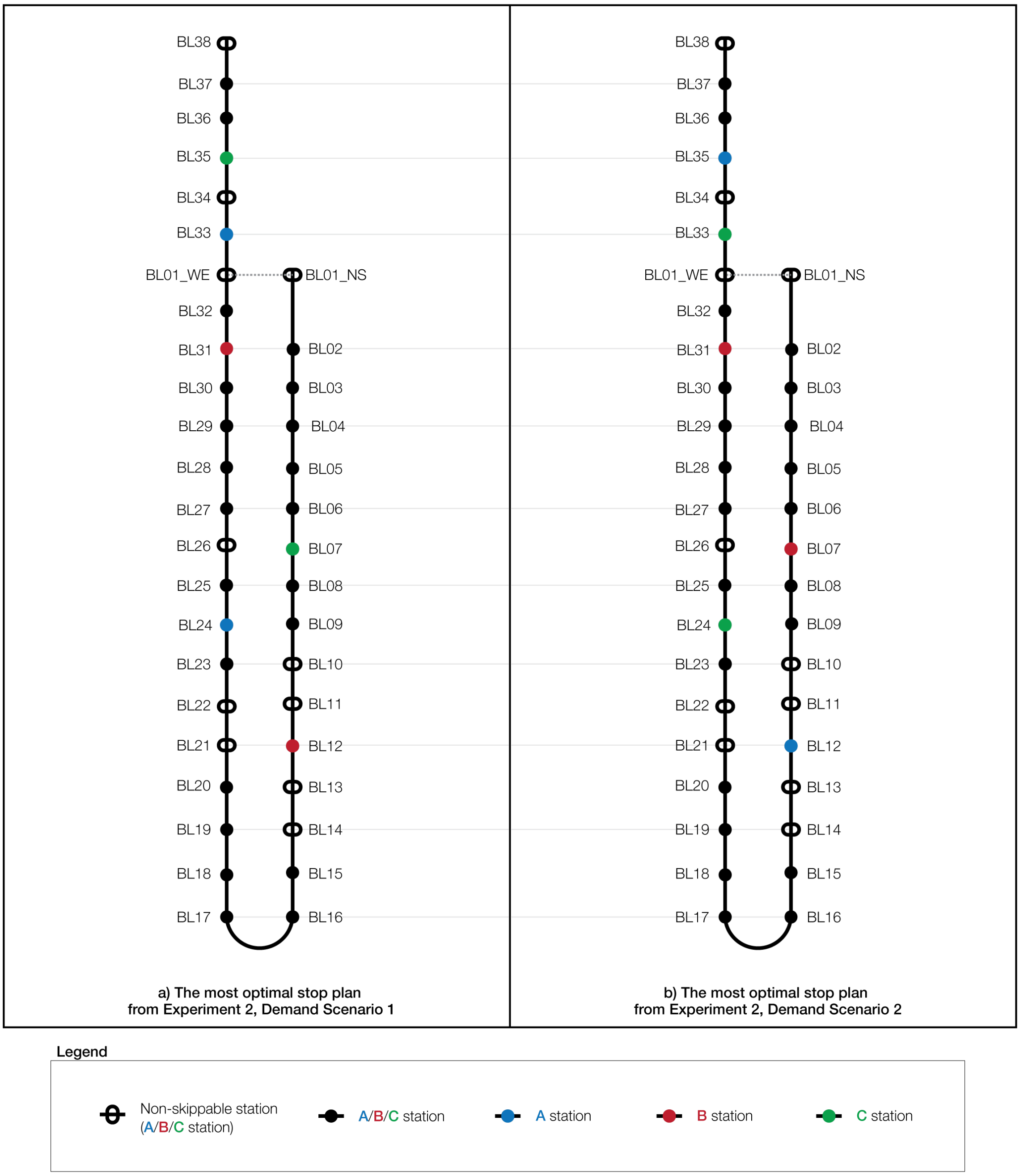

Figure 30 The most optimal stop plans from experiment 2

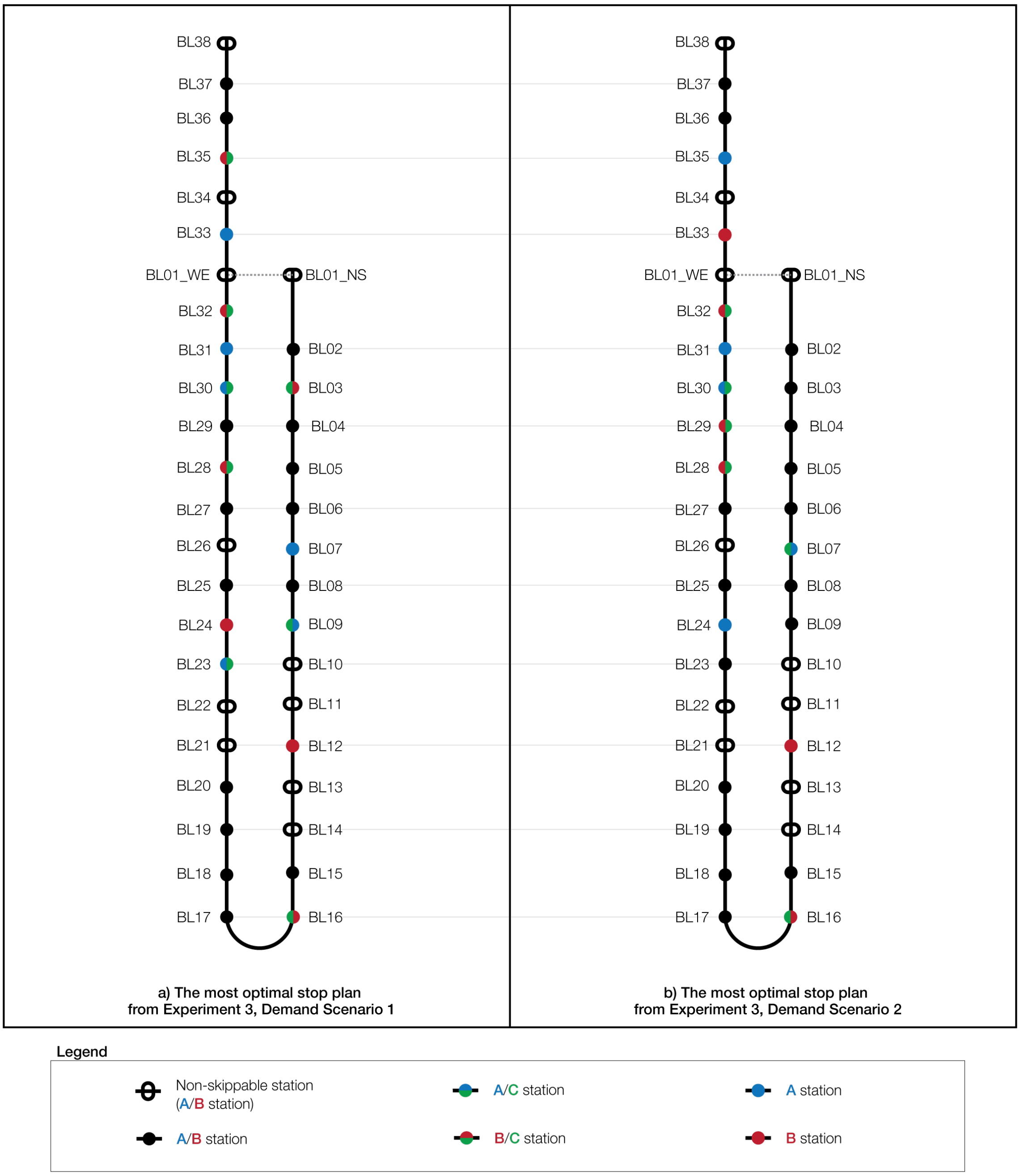

Figure 31 The most optimal stop plans from experiment 3