A Multi-Vehicle Control Framework with Application to Automated Valet Parking

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Abstract—We introduce a distributed control method for coordinating multiple vehicles in the framework of an automated valet parking (AVP) system. The control functionality is distributed between an infrastructure server, called parking area management (PAM) system, and local autonomous vehicle control units. Via a vehicle-to-infrastructure (V2I) communication interface, model predictive control (MPC) decisions of the vehicles are shared with the coordination unit in the PAM. This unit in turn computes a coupling feedback which is shared with the vehicles. The control system is integrated in an automated test-system to cope with the high test requirements and short development cycles of highly automated systems. Evaluations conducted with the test-system show the functionality of the proposed distributed control method and a efficiency increase compared to an uncoordinated method in an AVP simulation environment.

I. INTRODUCTION

An automated valet parking (AVP) scenario describes the use case where a driver drops off his/her car in front of a parking area, or drop-off zone, and authorizes a parking area management (PAM) system to take control in order to autonomously guide the vehicle through the parking area and into a designated parking bay [1]. AVP is expected to be one of the first scenarios of automated driving satisfying the SAE (Society of Automotive Engineers) Level 4 requirements [2]. Level 4 autonomous driving defines cases where vehicles are able to move without any driver interaction in specific scenarios. Such systems will be available at airports, shopping malls, and at large parking areas next to places of interest.

Related works on AVP have covered the field of parking space optimization based on k-stacks [3], [4], and the security of user information [5]. Another important problem is the localization and mapping task in GPS-denied parking environments [6], and the motion planning problem. The survey in [7] presents an overview of motion planning for AVP systems considering global and local planning tasks, whereby important contributions are provided by the V-CHARGE research project [8]. However, to the best of our knowledge, none of the approaches in the literature discusses motion planning for several vehicles moving simultaneously in an AVP environment.

In order to guarantee safe and efficient movements for AVP, we developed a distributed control method for multi-vehicle coordination on predefined routes. This article focuses on the coordination phase after a target parking slot [3], [4], and a respective path [7] have been computed. The coordination task is decomposed into a local vehicle control and a global coordination task. Both units - the local vehicle control and the control coordination in the PAM infrastructure - are connected via vehicle-to-infrastructure (V2I) communication. The coordination unit determines dependencies for the exchange of control data between vehicles by constructing appropriate safety constraints for local vehicle problems, whereas the local vehicle control is responsible for efficient movements, path tracking, and the fulfillment of the safety constraints. For longitudinal and lateral control, we apply distributed model predictive control (MPC) and gain-scheduled linear-quadratic-regulators (LQR), respectively. Longitudinal predictions, resulting from each vehicle’s local MPC problem, are shared with the coordination unit. The latter collects the predictions, and a data set is forwarded to the respective neighboring vehicles which will process it in the next planning step. Fig. 1 shows an exemplary AVP scenario for multi-vehicle coordination.

Due to its environmental setup, the problem of coordinating several vehicles within parking areas is strongly related to the multi-intersection coordination scenario. The challenge of guiding vehicles autonomously through intersections has been investigated in [9] by using a reservation scheme, and an extension to the multi-intersection problem is proposed in [10]. A focus on finding optimal or quasi-optimal solutions for intersection crossing from a control perspective is, for example, proposed in [11]–[13]. Therein, vehicles negotiate a solution while approaching the intersection, and consecutively cross the intersection-zone safely and efficiently. A drawback, however, is that only a single vehicle at a time
can be in the intersection zone. The studies described in [14], [15] suggest distinguishing between several geometric regions within a single intersection, thus reducing the conservatism. It focuses on the verification of a deadlock-free intersection management algorithm. Similarly, the authors of [16] define points in the intersection area where vehicles’ paths intersect and compute an MPC-based control solution. We extended the distinction of such geometrical regions to a multi-intersection formulation. In [17] trajectories are computed for crossing two consecutive intersections using optimal control with a varying finite time of the local vehicle optimization problems. However, to guarantee a feasible solving time for embedded implementations, it is beneficial to know the optimization effort (horizon length) prior to the problem implementation. MPC is a widely used method for vehicle coordination problems, as the finite optimization problem finds close-to-optimal solutions, and constraints can be considered in the problem formulation [13], [16], [18]–[23]. However, a bottleneck of some intersection coordination problems is the requirement that the MPC horizon length has to cover the intersection area, when negotiating the solution [16], [24]. Thus, such approaches are not scalable for a multi-intersection scenario.

To overcome these challenges, we propose a distance-based formulation of the coordination problem that enables a smooth extension from a single intersection coordination problem to a complete network of intersections. Therefore, in our distributed formulation, the concept of virtual platooning [25]–[27] is adapted to the MPC problem, which combines inter-vehicle distance keeping with the intersection crossing scenario. Another distinct feature of the proposed approach is that the coordination procedure is not limited to the prediction horizon of the local MPC controllers, while it automatically reduces the number of coupling constraints between vehicles to a minimal set of safety conditions for the local MPC problem.

Guaranteeing a safe operation of highly automated vehicles is one of the major challenges, and is thus subject of projects such as PEGASUS [28] and ENABLE-S3 [29]. An unfeasible amount of test kilometers is required to ensure the safety of such systems. To overcome this challenge, one trend is to move a large portion of the required testing into high-fidelity virtual simulation environments. This not only enables testing specifically challenging corner cases, but also provides the possibility of automating the testing process. Therefore, it is essential to provide a framework for distributed control applications that is suitable for being embedded in an automated test-system. We propose such an implementation, which can be seamlessly integration into an automated test-system, where agents can be added to or removed from the distributed system in a plug-and-play manner. Furthermore, changes in the environmental setup can be considered in a flexible way.

The remainder of the paper is organized as follows. Section II introduces the three layers of the complete automated test-system for validating AVP. The system under test (SUT) in the lowest layer of the test-system is the focus of this paper. The control design within the SUT is discussed in detail in Section III, where the infrastructure and autonomous vehicle functions are explained. The implementation of the control system into the automated test-system is discussed in Section IV and an evaluation is conducted in an automotive simulation environment in Section V. The paper is concluded in Section VI.

II. VALET PARKING TEST SYSTEM FRAMEWORK

This section presents a simplified overview of the test architecture used in the ENABLE-S3 project, illustrated in Fig. 2.

The top level verification and validation (V&V) databases contain scenarios to be tested, requirements which should be fulfilled during the trials, and a real-world scenes database.

The test automation layer is responsible for running the tests in an automated manner. It generates synthetic scenarios by using the scenario database, then it passes the scenarios one by one to the test platform and triggers its start. During the tests an observer records the scenario, and after completion the traces are evaluated in the scenario evaluation module. This evaluation checks the fulfillment of defined safety requirements, and the test is passed if all the requirements were met by the SUT. A final evaluation classifies the conducted test run according to its degree of reality by comparing it with real-world scenes.

The bottom layer defines the test platform, which is responsible for simulating the parking processes for the provided scenarios. It consists of a high-fidelity simulation environment and the SUT, which is a combination of the PAM system and the distributed autonomous vehicle control systems. The high-fidelity virtual environment simulates the behavior of all vehicles and possibly other moving objects based on their respective dynamic models. The simulation uses an environment model, which is provided by the scenario generation tool, and consists of map data as well as a visual database. Furthermore, the virtual simulator provides a visualization of the tested scenario, and the option of modeling sensor behaviors, e.g. LiDAR sensors, in the environment. This paper focuses on the
test platform layer and especially on the control components in the SUT, which is described in the following section.

III. DISTRIBUTED CONTROL SYSTEM DESIGN

This section describes the distributed control system of the AVP system. Fig. 3 summarizes the functionality of the subcomponents and illustrates the signal flow between them. A detailed description of the PAM functions is given in Section III-A, whilst the autonomous vehicle control functions as well as their couplings are presented in Section III-B.

A. PAM Infrastructure Functions

Similar to [30], we assume separation of the planning phase into mission planning, path planning, and trajectory planning. The outputs of the Mission Planning and Path Planning are assumed to be predefined. The former organizes the parking lot situation in the parking environment and determines the destination positions for each vehicle, while the latter computes a path, i.e. geometric waypoints, provided with the map data, and each vehicles’ start and target position. The path and map data are then shared with the control coordination unit. This unit cooperatively determines longitudinal trajectories, which will be tracked by the respective vehicle’s low level controller. A crucial property is the feasibility of a vehicle’s trajectory because safety of a scenario can only be ensured if a vehicle is able to track its computed trajectory. To achieve this requirement, we propose a decomposition of the trajectory generation process between the PAM infrastructure and the local vehicle controller. This subsection describes the control coordination procedure computed in the PAM. Based on predefined vehicle paths and trajectory predictions from local vehicle control units a vehicle adjacency will be determined with the help of a route graph. This information is used to construct distance constraints for vehicles which will result in a safe overall coordination procedure. It is achieved by the following steps:

1) Conflict Zone Detection: Intersections, i.e. points where vehicle paths cross, are a bottleneck in coordinating vehicles within the given parking environment. This is because not only a control decision has to be computed between consecutive vehicles, but also a combinatorial decision on the vehicle order. By making use of a central coordination unit, we separate the combinatorial decision, which we refer to as scheduling, from the MPC problem in the vehicles.

Fig. 4 shows possible paths in an exemplary intersection area. It is divided into five areas by grouping nearby points, where the vehicle paths either cross, merge, or diverge, thus forming potential collision points. These areas are named conflict zones $\mathcal{CZ}_i$ where $i \in \mathbb{I}_{1:5}$. Notation $\mathbb{I}_{ab}$ describes a set of integers $a,b \in \mathbb{N}$ such that the integers in the set $\{a,a+1,a+2,\ldots,b\}$ are obtained. The conflict zones of all intersections are gathered in the set $\mathcal{I}_{\text{inter}} = \{\mathcal{CZ}_j, \ldots, \mathcal{CZ}_{5N_V}\}$, where $N_V$ is the total number of intersections in the parking environment, and $j \in \mathbb{N}$ a unique ID label for each conflict zone. In addition to the intersection areas, we define a set of conflict zones $\mathcal{I}_{\text{end}}$ at the end of each vehicle’s path, where $|\mathcal{I}_{\text{end}}| = N_V$ and $N_V$ indicates the total number of moving vehicles in a coordinated parking scenario. Note that $|\mathcal{I}_{\text{end}}|$ means the cardinality of its argument set $\mathcal{I}_{\text{end}}$. Finally, we gather all these conflict zones in the set

$$\mathcal{I} = \mathcal{I}_{\text{inter}} \cup \mathcal{I}_{\text{end}}.$$  

The previous steps are carried out before the start of the coordination procedure and the locations of the conflict zones in the set $\mathcal{I}_{\text{inter}}$ depend on the geometry of the parking environment. Furthermore, we assume that paths are tracked accurately such that a vehicle’s bounding box does not laterally
pass a conflict zone other than its allocated path. The elements in \( \mathcal{I} \) will serve as reference zones for coordinating the vehicles passing through a zone. This enables coordinating the vehicles approaching from different sides (intersection scenario) and vehicles coming from the same lane (vehicle following scenario) towards a critical zone \( \mathcal{CZ}_i \in \mathcal{I} \).

**Remark 1 (Conflict zones):** The set in (1) can be extended by further zones, \( \mathcal{CZ}_8 \), at arbitrary positions where collisions between vehicles can occur. This is for example the case if a vehicle crosses an accommodating lane to navigate into a parking bay. The coordination procedure is not affected by this action. Therefore, we refrain from explaining this part, for the sake of simplicity of presentation.

2) **Graph Representation:** The set

\[
V = \{v_1, v_2, \ldots, v_{N_V}\} \tag{2}
\]

contains all vehicles that are coordinated by the PAM, where each vehicle \( v_i \in V \) is labeled with a unique ID. Now, we can define a multigraph \( \mathcal{G}_{\text{route}} = (V_\text{route}, E_\text{route}) \) with vertices \( V_\text{route} \in \mathcal{I} \) and edges \( E_\text{route} = V_\text{route} \times V_\text{route} \), labeled with \( v_i \). An edge-vertices pair in \( \mathcal{G}_{\text{route}} \) indicates that a vehicle’s path connects two conflict zones, while the edge direction represents the driving direction of vehicle \( v_i \) between those zones. Every time a vehicle has passed a conflict zone \( \mathcal{CZ}_j \in \mathcal{I} \) the corresponding edge is removed from \( \mathcal{G}_{\text{route}} \). During execution time, the sets \( \mathcal{I}, V \), and \( \mathcal{G}_{\text{route}} \) are continuously updated to cope with changes, such as vehicle poses and picked-up or newly dropped-off vehicles.

3) **Adjacency Definition:** The resulting \( \mathcal{G}_{\text{route}} \) is used to determine a set of crossing orders

\[
\mathcal{O} = \{o_i = (v_j, v_k, \ldots) \mid \forall i \in \mathbb{I}:|i|; j, k \in \mathbb{I}:N_V\} \tag{3}
\]

It represents the logical sequence in which vehicles must cross the respective conflict zone, while the actual timing, i.e. the resulting trajectory, will be determined by the distributed longitudinal control laws (Subsection III-B). The scheduling orders in (3) are determined on a first-come-first-served basis, where the sequences through the conflict zones in an intersection area are sorted according the vehicles’ distances to the respective intersection. Alternative heuristics to generate such scheduling orders on road networks are discussed in [31].

Finally, we determine the adjacency matrix \( A_{\text{adj}} = (a_{ij})_{N_V \times N_V} \) with \( a_{ij} \in \mathcal{T} \), and

\[
\mathcal{T} = \{(m_1, \ldots, m_p, \ldots, m_q) \cup \emptyset \mid q \in \mathbb{N}, \forall p \in \{1, \ldots, k\} : m_p \in \mathbb{I}\} \tag{4}
\]

This implies, \( a_{ij} \) defines a dependency between two vehicles \( v_i \) and \( v_j \) if \( v_i \) is a successor of \( v_j \) in any scheduling order \( o_k \), where \( i, j \in \mathbb{I}:N_V \), and \( k \in \mathbb{I}:|i| \). Thus, an element \( a_{ij} = \mathcal{CZ}_k \) of \( A_{\text{adj}} \) is labeled with the conflict zone both vehicles will pass through. In those cases where vehicle \( v_i \) is a successor of \( v_j \) for several conflict zones, for example if \( v_i \)'s route partially overlaps with that of \( v_j \), \( a_{ij} \) becomes an \( n \)-tuple, i.e. \( a_{ij} = (\mathcal{CZ}_k, \mathcal{CZ}_l, \ldots) \) with \( k, l \in \mathbb{I}:|i| \). In the following we refer to the first element of a non-empty \( n \)-tuple by using the notation \( a_{ij} \), which is the closest conflict zone \( v_i \) and \( v_j \) have in common.

![Fig. 5: Example scenario with six vehicles and pre-computed paths. Labels at circles are the conflict zone IDs. Scheduling orders for affected conflict zones are shown on the right.](image)

**Illustrative Example 1:** In Fig. 5 we present an example coordination scenario with the vehicle set

\[
V = \{v_1, v_2, \ldots, v_6\} \tag{5}
\]

and four intersections leading to the conflict zones

\[
\mathcal{I} = \{\mathcal{CZ}_1, \mathcal{CZ}_2, \ldots, \mathcal{CZ}_{20}\}. \tag{6}
\]

The example illustrates the states of the coordination process at a certain time instant. The paths, provided by the path planning unit, are marked with black lines and the resulting route graph \( \mathcal{G}_{\text{route}} \) is shown in Fig. 6. Each conflict zone \( \mathcal{CZ}_i \in \mathcal{I} \) that a vehicle will pass is represented by a vertex and the vehicles’ paths are represented by labeled edges. The scheduling unit determines an order \( o_i \) for each vertex. The resulting ordering is shown in Fig. 5 and finally, the scenario’s adjacency matrix is given as follows:

\[
A_{\text{adj}} = \begin{pmatrix}
\mathcal{CZ}_{17} & \mathcal{CZ}_{20} & \mathcal{CZ}_{19,15} & \mathcal{CZ}_8 & \mathcal{CZ}_9 & \mathcal{CZ}_6 \\
\mathcal{CZ}_8 & \mathcal{CZ}_{17} & \mathcal{CZ}_{19,15} & \mathcal{CZ}_9 & \mathcal{CZ}_8 & \mathcal{CZ}_9 \\
\mathcal{CZ}_{19,15} & \mathcal{CZ}_9 & \mathcal{CZ}_8 & \mathcal{CZ}_{17} & \mathcal{CZ}_{19,15} & \mathcal{CZ}_9 \\
\mathcal{CZ}_9 & \mathcal{CZ}_8 & \mathcal{CZ}_{19,15} & \mathcal{CZ}_9 & \mathcal{CZ}_8 & \mathcal{CZ}_{17} \\
\mathcal{CZ}_8 & \mathcal{CZ}_{17} & \mathcal{CZ}_{19,15} & \mathcal{CZ}_9 & \mathcal{CZ}_8 & \mathcal{CZ}_9 \\
\mathcal{CZ}_9 & \mathcal{CZ}_8 & \mathcal{CZ}_{19,15} & \mathcal{CZ}_9 & \mathcal{CZ}_8 & \mathcal{CZ}_9
\end{pmatrix}
\]
4) Coupling Constraint: Given the adjacency definition $A_{adj}$, coupling constraints for each vehicle’s MPC problem are determined by the PAM. During the coordination, each vehicle, $v_i \in V$, shares the prediction vector of its distance state, 

$$d_i(t \mid |t) = (d_i(t + 1|t), d_i(t + 2|t), ..., d_i(t + M|t)) \in \mathbb{R}^m,$$  

with the PAM. The state $d_i$ is the vehicle’s distance to the end of its path, and $M$ is the prediction length. The notation $d_i(t + k|t)$ means the predicted value of state $d_i$ for time step $t + k$, with $0 \leq k \leq M$, at the discrete time step $t$. Transformation $T(CZ, i)$ defines the distance from the position where $v_i$’s path enters a conflict zone $CZ_j$ to the end of its path. Now, the coupling constraint $c_i$ for each vehicle $v_i \in V$ is calculated by determining the most critical predecessor vehicle with 

$$j_k = \arg\min_{j \in I_1 : N_v} \{d_i(t + k|t - 1) - [T(a_{ij}, i) + d_j(t + k|t - 1) - T(a_{ij}, j)]\},$$  

where $k \in I_1 : M - 1$, and $a_{ij}$ refers to the conflict zone at row $i$ and column $j$ of $A_{adj}$. Given Equation (8), the coupling constraint is given by 

$$c_i(d_i(t + k|t)) = d_i(t + k|t) - [T(a_{ij}, i) + d_j(t + k|t - 1) - T(a_{ij}, j^k)],$$  

with $k \in I_1 : M - 1$.

For each predicted time step $t + k$, $c_i(d_i(t + k|t))$ is the closest distance of all predecessor vehicles of $v_i$ with respect to their common conflict zone and information from the previous time step, $t - 1$. Fig. 7 illustrates the coupling constraint formulation in Equation (8) and (9) for a determined predecessor $v_j$ of vehicle $v_i$ with respect to the conflict zone $a_{ij}$ at one prediction time step $t + k$. Note that the procedure is the same if vehicles approach the conflict zone from the same lane.

B. Autonomous Vehicle Functions

The local vehicle controller is decomposed into a longitudinal control and lateral control. In this subsection, we describe the implementation of the high-level control, whose output will be passed to the low-level control units to ensure the hardware actuation, i.e. acceleration (throttle and brake), and steering.

1) Longitudinal Motion Model and Control Design: A vehicle $v_i$ is modeled using the states $d_i$, $v_i$, and $a_i$. States $v_i$ and $a_i$ are the velocity and acceleration of vehicle $v_i$, respectively. The continuous linear-time-invariant (LTI) dynamics of the longitudinal vehicle motion is summarized as:

$$\begin{pmatrix} d_i \\ \dot{v}_i \\ \dot{a}_i \end{pmatrix} = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{1}{\tau_t} \end{pmatrix} \begin{pmatrix} d_i \\ v_i \\ a_i \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \frac{1}{\tau_t} \end{pmatrix} u_i,$$  

(10)

where $\tau_t$ is a time constant for throttle and brake actuation, and $u_i$ is an exogenous control input, which represents the desired acceleration [32]–[34].

The longitudinal MPC law is computed solving the following optimization problem

$$\begin{equation} \label{eq:longitudinal_mpc} L_i^{x^g} \left( x^g_i(t), u_i(t) \right) = \min_{U_i} \sum_{k=1}^{M} \| \Delta_x x^g_i(t + k|t) \|^2_{Q_i} + \sum_{k=0}^{M-1} \| u_i(t + k|t) \|^2_{R_i}, \end{equation}$$  

s.t. 

$$x^g_i(t + k + 1|t) = A^{g} x^g_i(t + k|t) + B^{g} u_i(t + k) \quad k \in \mathbb{Z}_{0 : M - 1}$$  

(11b)

$$x_i(t|t) = x_i(t)$$  

(11c)

$$v_{min} \leq v_i(t + k|t) \leq v_{max} \quad k \in \mathbb{Z}_{0 : M}$$  

(11d)

$$a_{i, min} \leq a_i(t + k|t) \leq a_{i, max} \quad k \in \mathbb{Z}_{0 : M}$$  

(11e)

$$c_i \left( d_i(t + k|t) \right) \geq d_s \quad k \in \mathbb{Z}_{0 : M}.$$  

(11f)

Here, $x_i(t + k|t)$ is the state prediction for time $t + k$ at the current discrete time step $t$. The model (11b) is attained by discretizing (10) using a sampling time $T_s^{g}$. The optimized variables vector is of the form $U_i = (u_i(t), u_i(t + 1), ..., u_i(t + M - 1)) \in \mathbb{R}^m$ with the horizon length $M$. After optimizing (11) in each time step $t$, the control law $U_i(1) = u_i(t)$ determines the desired acceleration input to the vehicle, and the procedure is repeated in consecutive time steps in a receding horizon fashion. The objective function in (11a) consists of a state error cost and a control input cost. The longitudinal reference vector in the state error vector, $\Delta_x x^g_i(t + k|t) = x^g_i(t + k|t) - x^g_{i, ref}$, is of form

$$x^g_{i, ref} = (d_{i, ref}, v_{ref}, 0)^T,$$  

(12)

with the reference velocity $v_{ref}$ and the reference distance

$$d_{i, ref} = \begin{cases} 0 & \text{if } v_i \text{ has no predecessor} \\ d_s + d_{slack} & \text{if } v_i \text{ has a predecessor}. \end{cases}$$  

(13)

If the reference distance in (13) becomes active, we add an additional slack value $d_{slack}$ to the inter-vehicle safety distance $d_s$ in order to avoid a reference at the constraint bound. In (11a), we use the notation $\| \alpha \|^2_\beta \triangleq \alpha^T P \alpha$ with a vector $\alpha \in \mathbb{R}^n$. 

Fig. 7: Illustration of the coupling constraint formulation using an example with one conflict zone $CZ_i$, and two vehicles approaching form different directions (dotted lines).
Fig. 8: Single track vehicle model, showing the vehicle model variables [36] and the lateral distance and heading angle errors with respect to the path reference [35].

$R^*$ and a matrix $P \in \mathbb{R}^{n \times n}$. The weighting matrices in (11a) are

$$Q_i = \text{diag}(q_{i1}, q_{i2}, q_{i3}) \in \mathbb{R}^{3 \times 3} \text{ and } R_i \in \mathbb{R}.$$  \hspace{1cm} (14)

Here $Q_i \succeq 0$ and $R_i > 0$ are positive semi-definite and positive definite matrices, respectively.

Among the model-based equality constraint (11b) problem (11) considers inequality box constraints to limit the vehicle’s velocity (11d) and acceleration (11e). Finally, (11f) represents the coupling constraint, which ensures that vehicle $i$ keeps a minimum safety distance $d_s$ to its predecessor vehicles. It thus forms the interface between the local vehicle control and the global PAM coordination decisions.

2) Lateral Motion Model and Control Design: We use the following continuous time matrix equation to model the lateral dynamics [35], [36]:

$$\dot{x}^l = A^l x^l + B^l \delta,$$

where

$$x^l = \left[ \begin{array}{c} v_x \\ 0 \\ 0 \end{array} \right], \quad A^l = \left[ \begin{array}{ccc} 0 & v_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -\frac{j_C f + j_C r}{j_x v_x} \end{array} \right], \quad B^l = \left[ \begin{array}{ccc} 0 & 0 & v_x \\ 0 & -v_x & 0 \\ \frac{l_C f}{j_x} & 0 & \frac{l_C r - l_C f}{j_x} \end{array} \right].$$

Here, $B^l_{1,1}$, $B^l_{2,1}$, and $B^l_{3,3}$ are the first, second, and third column of $B^l$, respectively, $d_{le}$ is the lateral distance error, $\theta_{he}$ is the heading angle error, $\psi$ is the yaw rate, $v_x$ is the longitudinal velocity, $C_f$ and $C_r$ are the cornering stiffness of the front and rear axle, respectively, $J_x$ is the moment of inertia around the yaw axis, $l_f$ and $l_r$ are the distances between the center of gravity (CoG) and the front and rear axle, respectively. Fig. 8 illustrates the model parameters of the bicycle model. $\kappa$ is the path reference curvature, $\beta$ the vehicle sideslip angle, and $\delta$ the steering angle control input. Therefore, the pair $(A^l, B^l_{1,1})$ is used to design the controller. The vehicle sideslip angle is not used as a system state as its estimation results in considerable error.

The simplified lateral model $\dot{x}^l = A^l x^l + B^l \delta$ is discretized using the sampling time $T^l_s$ and the MATLAB command DLQR is used to design the Discrete-time Linear Quadratic Regulator (DLQR) gains. The weighting matrices $Q^l = \text{diag}(q_{l1}, q_{l2}, q_{l3}) \in \mathbb{R}^{3 \times 3}$ and $R^l \in \mathbb{R}$  \hspace{1cm} (19)

are used to calculate the DLQR gains, which are positive semi-definite and positive definite, i.e. $Q^l \succeq 0$ and $R^l > 0$.

The model linearization, discretization, and DLQR gain calculations are done at three vehicle speeds: $v_1$, $v_2$, and $v_{max}$. Using these DLQR gains a speed-dependent gain scheduling is used so that the controller is robust to variations in the vehicle speed. The gain scheduling uses convex summation of the DLQR gains, as shown below:

$$G_{dlqr} = \begin{cases} G_1, & \text{if } v_{min} \leq v_x \leq v_1 \\ (v_x - 1)G_1 + (2 - v_x)G_2, & \text{if } v_1 < v_x \leq v_2 \\ (v_x - 2)G_2 + (3 - v_x)G_3, & \text{if } v_2 < v_x \leq v_{max} \\ G_3, & \text{if } v_x > v_{max} \end{cases}$$

The lateral control input $\delta$ is generated using the following equation:

$$\delta^l = G_{dlqr} \bar{x}^l$$

$$\delta = \begin{cases} \delta(t), & \text{if } -\delta_{max} \leq \delta \leq \delta_{max} \\ \pm \delta_{max} & \text{if } \pm \delta > \delta_{max}. \end{cases}$$

In (22), $-\delta_{max}$ and $\delta_{max}$ are the lower and upper limits of the vehicle steering angle. The vehicle yaw rate and speed, which are required to calculate the control input, are measured by vehicle sensors.

IV. INTEGRATION OF THE DISTRIBUTED CONTROL SYSTEM INTO THE TEST PLATFORM

In order to test the growing complexity of distributed systems, it is crucial to use a systematic way of integrating the system functions. It should be possible to seamlessly test any changes in the distributed system. Changes may for example occur in the test environment or in the system components. To account for this, we propose the implementation method described below, utilizing emerging standards for road network topology descriptions, and a variable amount and type of subsystems. The test-system components in Fig. 2 are embedded in a Robotic Operating System (ROS) [37] network. The parking area management (PAM) unit and each vehicle controller are realized as separate ROS-nodes. We use the commercial virtual environment VTD\textsuperscript{1} to simulate the parking scenario, and implemented a VTD-to-ROS unit to read data from and a ROS-to-VTD unit to send data to the virtual environment. The scenario generation unit in Fig. 2 provides a map in the de facto standard format openDRIVE [38], populated with a certain number of vehicles, as well as a scenario file, which contains initial scenario conditions. The generated openDRIVE map is interpreted by the VTD environment, and is at the same time used by the PAM ROS-node. The scenario file specifies the IDs of the vehicles and distinguishes between each vehicle’s status, i.e. dropped-off, parked, or pick-up-requested. If a vehicle has the status parked, it is not handled by the PAM, whereas dropped-off and pick-up-requested vehicles will be controlled by the SUT during the test scenario. The PAM ROS-node is programmed in C++ such

\textsuperscript{1}https://vires.com/vtd-vires-virtual-test-drive/
that it can handle an arbitrary number of vehicles according to the scenario file. We used MATLAB and Simulink to design and develop the vehicle control unit, i.e. the lateral control and the longitudinal MPC. The MPC law is converted into a quadratic programmin (QP) formulation and passed to the integrated qpOASES solver [39]. The controller setup is then exported as a standalone C++ ROS-package using MATLAB’s Robotics System Toolbox and Embedded Coder. In order to achieve the simulation of several vehicles, we use the ROS-launch concept. Thereby, several ROS-nodes with varying names and parameters can be launched using the same ROS-package. The necessary extensible markup language (XML) based launch-file is automatically generated based on the information included in the scenario-file. This method enables the simulation of a varying number of vehicles in different simulation scenarios. Fig. 9 illustrates the described setup and presents a simplified structure of the ROS network. The first four messages in the table are defined using standard ROS-message types, whereas the coordination message is a user-defined ROS message tailored to our MPC problem. For communication with VTD (VTD bus), internal runtime data bus (RDB) messages are used. Note that all ROS messages in the table are published separately in the namespace of each vehicle.

V. NUMERICAL EVALUATION

To run the tests, we distribute the test architecture of Fig. 9 on two PCs, as shown in Fig. 10. The Simulation PC runs the test management system (see Fig. 2), the VTD simulation environment, and the PAM ROS-node. It is equipped with an Intel Core i7 – 6700K 4 core processor and 32 GiB of RAM memory. The vehicle control ROS-nodes are launched on a separate Control PC with an Intel Core i7 – 4790K 4 core processor and 32 GiB of RAM memory. We allocate each control process (ROS-node) to a designated core of the CPU, such that two processes share one core. This guarantees the real-time computation of the MPC optimization problem, with an average solving time of 5ms. The two PCs are connected via an Ethernet connection.

In Fig. 11, we give insights into the test platform visualization during a trial. Figure 11a shows the visualization of the VTD environment. The plot in Fig. 11b visualizes all objects (moving and parked vehicles) in the scenario using the ROS-rviz tool, and displays the paths provided by the PAM pathplanning unit. The bottom plot in Fig. 11c illustrates a randomly generated openDRIVE map of the trial from the scenario generation tool.

Now, we illustrate the functionality of the coordination and control method, and highlight the benefits compared to an uncoordinated scenario by evaluating the simulation results. Table II introduces the control parameters used for the evaluations.

Note that the actual maneuvering into/out of the parking bay may also require backward driving and thus the velocity constraint should also allow $v_{min} < 0$. However, as this paper focuses on the coordination procedure within the parking area, we avoid backward driving.

First, the longitudinal coordination control is investigated. We thus illustrate the step-response of a single vehicle in Fig. 12. The vehicle starts from standstill and receives a reference

| TABLE I: Explanation of the most important ROS messages in the multi-vehicle control framework. |
|---|---|---|
| Signal name in Fig. 9 | Message type | Explanation |
| pose | geometry_msgs/Pose | position and heading |
| measurements | std_msgs/Float64 | velocity |
| path | nav_msgs/Path | heading rate |
| driving command | ackermann_msgs/AckermannDrive | waypoints computed by pathplanner in PAM |
| coordination | ctrl/Ctrl_msg | steering and acceleration input |
| VTD bus | Runtime Data Bus (RDB) | distance predictions of predecessor, reference values and constraints for MPC |
| VTD internal user interaction at run-time | | |

Fig. 10: Distributed implementation of the test-system in our lab.
velocity of $1.1\, \text{m/s}$, which increases to $2.9\, \text{m/s}$ and then steps back to $1.1\, \text{m/s}$. The weights $Q_i$ and $R_i$ of the MPC problem in (11) are chosen such that we avoid an overshoot in the vehicle’s velocity, enabling a safe inter-vehicular distance from possible successor vehicles.

In the next step, we evaluate the lateral tracking control. Fig. 13 shows a reference path in the global coordinate system and the measured positions of a vehicle, while tracking this path. We find good tracking performance for kinematically feasible curves (segments A and E in Fig. 13) and accurate path following for straight segments (segment C). As the path is computed globally in the PAM, there is no kinematic feasibility guarantee for local vehicle tracking. The evaluation, however, shows a stable and adequate tracking behavior for infeasible references (segments B and D).

In order to illustrate the coordinated behavior of several vehicles in the distributed control framework, we simulate a platoon of homogeneous vehicles in the scenario introduced in Fig. 11. Four vehicles ($v_1, v_2, v_3, v_4$) start from standstill in the area marked with a single $*$ in Fig. 11c. The resulting acceleration and velocity profiles of vehicles $v_2$ and $v_4$ are plotted in Fig. 14. At the intersection, marked with $**$, vehicle $v_3$ leaves the platoon by turning right ($t = 27s$) to possibly park at a different location. In the further course, the vehicle adjacencies change in the PAM algorithm and thus vehicle $v_4$ has the incentive to reduce the distance to vehicle $v_2$. The bottom plot of Fig. 14 shows an approximately constant inter-vehicle distance of $\approx 4\, \text{m}$ between vehicles $v_1$ and $v_2$. The upper line in the same plot illustrates the inter-vehicle distance between vehicles $v_2$ and $v_4$, which is $\approx 9\, \text{m}$ (chassis dimensions excluded), when vehicle $v_3$ is in between them, and reduces after $v_3$ leaves the platoon.

Now, we evaluate the performance of the proposed distributed coordination control by simulating the coordinated parking processes. Vehicles $V = \{v_1, v_2, \ldots, v_7\}$ are randomly placed in the map as illustrated in Fig. 11c, and the PAM algorithm allocates a free parking spot to each vehicle, as well as a respective path. We simulate a set of five trials with different initial positions. Each trial runs with three different control methods. First, a solo-driving method is tested, where only one vehicle moves at a time. Second, an uncoordinated scenario is simulated. Each vehicle moves in an uncoordinated manner, and if a collision at an intersection would occur, the right of way is granted to the vehicle that arrives first. All other vehicles involved in the conflict have to stop. This
method is supposed to simulate the behavior of human-driven cars in parking environments. The third method represents the coordinated control method proposed in this paper. As performance measures, we evaluate the required acceleration for each vehicle in a trial. To do so, we integrate the positive acceleration values of each vehicle over time ($\sum t a_i(t) > 0$). Furthermore, we analyze the duration of the parking process, referred to as time to park ($ttp$). Fig. 15 shows five trials and compares the acceleration effort of the three methods described above. Vehicles start from standstill at the beginning of each trial. The left bar of a set of three shows the solo-driving results, the middle bar is the uncoordinated scenario, and the right bar is the coordinated method. As the solo-driving method does not require any interaction, it has the lowest acceleration effort for each trial. We find that the coordinated method outperforms the uncoordinated method in most cases. In some cases, the coordinated algorithm is outperformed by the uncoordinated one if vehicles are granted the right of way (e.g. $v_1$ in trial 2 and $v_7$ in trial 5). However, the overall performance of all vehicles in the respective trial shows better results using the coordinated method. In trial 5, the performance of the coordinated and uncoordinated methods is similar. This is due to the vehicles’ initial positions, which only require a small amount of vehicle interaction. To evaluate the amount of interaction, Table III lists the number of vehicle stops during a simulation scenario. A higher number of stops using the uncoordinated control method requires a higher interaction amount using the coordinated control method.

Table IV presents an overall evaluation for each trial, considering both the acceleration effort and the parking duration. Rows labeled $\Sigma$ sum the acceleration integrals of all

Table III: Number of required vehicle stops during simulation with the uncoordinated control method.

<table>
<thead>
<tr>
<th>trial 1</th>
<th>trial 2</th>
<th>trial 3</th>
<th>trial 4</th>
<th>trial 5</th>
</tr>
</thead>
<tbody>
<tr>
<td># stops</td>
<td>11</td>
<td>9</td>
<td>10</td>
<td>6</td>
</tr>
</tbody>
</table>

Fig. 14: Coordinated driving in a platoon, where vehicle $v_3$ leaves the platoon around $t = 27s$.

Fig. 15: Performance comparison of different control coordination methods for 7 vehicles and 5 trials with different initial conditions.
TABLE IV: Overall performance evaluation of the parking process with different control methods.

<table>
<thead>
<tr>
<th></th>
<th>solo-driving</th>
<th>control method</th>
<th>control method</th>
<th>coordinated</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ttp(s)</td>
<td>Σacc</td>
<td>ttp(s)</td>
<td>Σacc</td>
</tr>
<tr>
<td>trial 1</td>
<td>334.33</td>
<td>1166.1</td>
<td>68.47</td>
<td>4257.5</td>
</tr>
<tr>
<td>avg.</td>
<td>41.791</td>
<td>145.76</td>
<td>51.386</td>
<td>532.19</td>
</tr>
<tr>
<td>trial 2</td>
<td>366.08</td>
<td>1118</td>
<td>61.97</td>
<td>3782.7</td>
</tr>
<tr>
<td>avg.</td>
<td>45.76</td>
<td>139.75</td>
<td>47.404</td>
<td>472.84</td>
</tr>
<tr>
<td>trial 3</td>
<td>385.67</td>
<td>1041.9</td>
<td>66.15</td>
<td>4318.1</td>
</tr>
<tr>
<td>avg.</td>
<td>48.209</td>
<td>130.24</td>
<td>52.455</td>
<td>539.76</td>
</tr>
<tr>
<td>trial 4</td>
<td>391.27</td>
<td>1179.3</td>
<td>70.69</td>
<td>3134.7</td>
</tr>
<tr>
<td>avg.</td>
<td>48.909</td>
<td>147.41</td>
<td>48.587</td>
<td>391.84</td>
</tr>
<tr>
<td>trial 5</td>
<td>280.86</td>
<td>1127.5</td>
<td>57.07</td>
<td>2482.9</td>
</tr>
<tr>
<td>avg.</td>
<td>35.108</td>
<td>140.94</td>
<td>36.48</td>
<td>310.36</td>
</tr>
</tbody>
</table>

Vehicles in columns with Σacc, and list the total time of the parking maneuvers in columns with ttp. The total time is the summation of the parking duration of all vehicles in the solo-driving method, as the vehicles are driven one by one. On the contrary, it is the maximum value of the parking process duration among all simulated vehicles for the uncoordinated and coordinated methods, as the vehicles are driven simultaneously. Rows with avg. list the average values of all vehicles in the respective scenario. Due to its extensive total time to park, the solo-driving method is unrealistic for real-world application. Furthermore, we find that the uncoordinated method is outperformed by the coordinated method in terms of parking duration and acceleration effort. This underlines the strength of the coordinated method, resulting in an optimized acceleration effort as well as a low time consumption.

VI. CONCLUSIONS

This paper presents a control framework for coordinating autonomous vehicles in parking areas by distributing the trajectory generation between vehicles and an infrastructure. Potential collision areas, called conflict zones, are used for the coordination procedure. We integrate this system into an automated test-system. The integration allows seamless variations of the system under test, such as changing agents or environments - factors which play an important role for testing highly automated systems.

Simulations illustrate the functionality and behavior of the integrated distributed control system. Furthermore, the increase in efficiency of the distributed coordination control is shown and compared with an uncoordinated method. Future work includes the integration of uncertain behavior into the distributed setup. Uncertainty can result for example from an environmental perspective such as pedestrians in the parking area. Furthermore, among the control coordination, higher layer planning levels, i.e. mission planning and path planning, have a significant impact on the performance of the coordination procedure. Therefore, investigating appropriate algorithms for those layers shall be considered in future work.


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