TUM School of Life Sciences

Cost-efficient fertilization strategies at the farm level

Development and application of a novel decision support system

Michael Friedrich Tröster

Vollständiger Abdruck der von der TUM School of Life Sciences der Technischen Universität München zur Erlangung des akademischen Grades eines Doktors der Agrarwissenschaften (Dr. agr.) genehmigten Dissertation.

Vorsitzender: Prof. Dr. Gerd Patrick Bienert

Prüfende der Dissertation:

1. Prof. Dr. Johannes Sauer
2. Prof. Dr. Heinz Bernhardt

Die Dissertation wurde am 08.09.2021 bei der Technischen Universität München eingereicht und durch die TUM School of Life Sciences am 30.03.2022 angenommen.

Summary

The production and sale of agricultural goods undergo intense global competition. To alleviate the price pressure resulting from heightened competition, farmers are encouraged to allocate existing resources as optimally as possible. By doing so, they act in accordance with the entrepreneurial goal of profit maximization. In daily business, this often involves decisions that need to be made at the production level. The question arises as to what level of intensity and with which combination of production factors a certain output should be targeted. In crop production, this question applies to fertilization, among other things. With the large share of the variable costs of crop production, this area certainly holds high potential for optimization. Therefore, farmers and consultants constantly seek farm-specific and cost-efficient fertilization strategies. “Fertilizer strategy” refers to the selection of a fertilizer and its temporal and quantitative placements within a crop rotation. To achieve a cost-efficient fertilizer strategy, additional aspects must be considered simultaneously: (i) the optimal intensity of all relevant nutrients, (ii) the cost-minimal selection of fertilizers, and (iii) the application costs. Due to continuous changes in the initial situation (e.g., price changes), the outlined optimization problem may not be optimally solvable without assistance.

In relation to this problem, this dissertation seeks to identify cost-efficient fertilizer strategies at the farm level. Four independent studies, which are embedded in the fundamentals of production theory and operations research, take up this question and deal with the development and evaluation of a decision support system.

In the first study, different possibilities are investigated and evaluated to appropriately implement application costs into a mathematical system, with the aim of achieving the economic optimization of fertilizer strategies. Special attention is given to individual on-farm infrastructure. In the first approach, the optimal routing is conducted by solving the underlying split delivery vehicle routing problem. A regression function, which was derived from this problem, was estimated to determine the transportation time. Then, the influence on the selection of fertilization strategies was investigated for both options and a scenario in which transportation costs were completely ignored. The results show that the derived regression function for estimating individual transport costs is preferable in terms of reliability and computational power.

The second study embeds the overarching question of the cost-efficient fertilizer strategy in the associated production theory. This study follows the approach of operations research all the way to the final developed decision support system called “IoFarm.” Considering the

application-oriented literature, a nonformal model was developed and subsequently transformed into a mathematical optimization model. Compared to the participants of a fertilization experiment, IoFarm shows an average cost advantage of €66 per hectare, with the same fertilization intensity.

Study three investigates the agronomic effects resulting from a cost-efficient fertilization strategy. For this purpose, a field trial was set up at three locations in Bavaria (Southern Germany) over several years, after which the fertilization strategy of IoFram was compared with a site-typical fertilization strategy. The statistical analysis did not reveal any significant differences in yield and quality.

Study four first focuses on identifying the differences between cost-efficient and inefficient fertilization strategies. For this purpose, the data from the aforementioned experiment were statistically analyzed. Results show that the largest influence on the design of the fertilizer strategy can be attributed to the relative price differences of the fertilizers. Thus, other differences that have been identified are not meaningful to derive general recommendations. In the second part, using IoFarm as a simulation model under changed farm conditions, farm-level influencing factors are investigated according to the *ceteris paribus* principle. Results clearly reveal that that the influence of the on-farm infrastructure on the optimal fertilization strategy is relatively small.

This dissertation contributes to the optimization of fertilization strategies at the farm level. Optimal strategies increase profits and save management time, which are particularly relevant for farmers and consultants. This work provides an important and new contribution to the understanding of cost-efficient fertilization strategies at the farm level from a scientific perspective. By modifying the objective function of IoFarm, climate-friendly fertilizer strategies can also be particularly identified. As a result, an important contribution to society can be achieved.

Zusammenfassung

Landwirtschaftliche Güter stehen in einem globalen Wettbewerb. Der daraus resultierende Preisdruck, aber auch das unternehmerische Ziel der Profitmaximierung, treiben Landwirte dazu an vorhandene Ressourcen möglichst optimal zu nutzen. Im Tagesgeschäft geht es dabei häufig um Entscheidungen die auf Ebene der Produktionsverfahren zu treffen sind. Es stellt sich regelmäßig die Frage mit welcher Intensität und mit welcher Kombination an Produktionsfaktoren ein bestimmter Output anzustreben ist. In der Pflanzenproduktion trifft diese Fragestellung unter anderem auf die Düngung zu. Mit einem großen Anteil an den variablen Kosten der Pflanzenproduktion birgt dieser Bereich durchaus hohes Optimierungspotential. Landwirte und Berater stellen sich daher mehrmals pro Saison die Frage nach der betriebsindividuellen, kosteneffizienten Düngestrategie. Unter Düngestrategie ist die Auswahl eines Düngemittels, als auch dessen zeitliche und mengenmäßige Platzierung innerhalb einer Fruchtfolge zu verstehen. Damit daraus eine kosteneffiziente Düngestrategie wird, sind zusätzliche Aspekte simultan zu beachten: (i) optimale Intensität sämtlicher relevanter Nährstoffe; (ii) kostenminimale Auswahl der Düngemittel; (iii) Berücksichtigung von Ausbringkosten. Aufgrund andauernder Veränderungen der Ausgangssituation, z.B. durch Preisveränderungen, ist das skizzierte Optimierungsproblem ohne Hilfsmittel vermutlich nicht optimal lösbar.

Diese Dissertation beschäftigt sich mit der Suche nach der kosteneffizienten Düngestrategie auf Ebene des landwirtschaftlichen Betriebes. Vier eigenständige Studien, eingebettet in die Grundlagen der Produktionstheorie, als auch des Operations Research, greifen diese Fragestellung auf und befassen sich mit der Entwicklung und Evaluierung eines Entscheidungshilfesystems.

In der ersten Studie werden unterschiedliche Möglichkeiten gesucht und evaluiert, mit denen sich Ausbringkosten angemessen in ein mathematisches System zur ökonomischen Optimierung von Düngestrategien implementieren lassen. Besondere Beachtung verdient die individuelle innerbetriebliche Infrastruktur. Die Identifikation der optimalen Routenführung erfolgt durch Lösung des zugrundeliegenden Split-Delivery Vehicle Routing Problem. Abgeleitet davon, konnte eine Regressionsfunktion zur Bestimmung der Transportzeit geschätzt werden. Für beide Möglichkeiten, als auch für ein Szenario in dem Transportkosten gänzlich ignoriert werden, wird der Einfluss auf die Auswahl von Düngestrategien untersucht. Die Ergebnisse zeigen, dass die abgeleitete Regressionsfunktion zur Abschätzung

individueller Transportkosten mit Blick auf Zuverlässigkeit und Rechenleistung vorzuziehen ist.

Die zweite Studie bettet die übergeordnete Frage nach der kosteneffizienten Düngestrategie in die dazugehörige Produktionstheorie ein und folgt der Vorgehensweise des Operations Research bis hin zum fertig entwickelten Entscheidungshilfesystem IoFarm. Auf Basis anwendungsorientierter Fachliteratur wird ein non-formales Model entwickelt und anschließend in ein mathematisches Optimierungsmodell überführt. Im Vergleich mit Teilnehmern eines Dünge-Experiments zeigen sich für IoFarm im Mittel Kostenvorteile von 66 € pro Hektar, bei gleicher Düngeintensität.

Studie drei geht der Frage agronomischer Auswirkungen nach, die sich möglicherweise durch eine kosteneffiziente Düngestrategie ergeben. Hierzu wurde an drei Standorten in Bayern (Süddeutschland) ein mehrjähriger Feldversuch angelegt in welchem die Düngestrategie von IoFarm mit einer standorttypischen Düngestrategie verglichen wurde. Die statistische Auswertung konnte keine signifikanten Ertrags- und Qualitätsunterschiede feststellen.

Studie vier konzentriert sich zuerst auf die Identifikation von Unterschieden zwischen kosteneffizienten und inneffizienten Düngestrategien. Hierzu werden die Daten aus dem bereits genannten Experiment statistisch analysiert. Der größte Einfluss auf die Ausgestaltung der Düngestrategie ist auf relative Preisunterschiede der Düngemittel zurückzuführen. Davon werden andere Unterschiede, die ebenfalls identifiziert wurden überschattet und eignen sich nur eingeschränkt um generelle Handlungsempfehlung abzuleiten. Im zweiten Teil werden betriebliche Einflussfaktoren nach dem *ceteris paribus* Prinzip untersucht, in dem IoFarm als Simulationsmodell unter veränderten betrieblichen Gegebenheiten eingesetzt wird. Dabei wird klar, dass der Einfluss der innerbetrieblichen Infrastruktur auf die optimale Düngestrategie gering ausfällt.

Diese Dissertation leistet einen Beitrag um Düngestrategien auf Betriebsebene zu optimieren. Dadurch werden Gewinne gesteigert und Managementzeit eingespart, was besonders hohe Relevanz für Landwirte und Berater hat. Aus wissenschaftlicher Sicht leistet diese Arbeit einen wichtigen und neuen Beitrag zum Verständnis kosteneffizienter Düngestrategien auf Betriebsebene. Durch Modifikation der Zielfunktion von IoFarm lassen sich auch besonders klimafreundliche Düngestrategien identifizieren. Auf diesem Weg kann zusätzlich ein wichtiger gesellschaftlicher Beitrag erzielt werden.

Content

List of Figures

List of Tables

List of Tables - Appendix

List of abbreviations

a	Year
AHL1to3	33% ammonium nitrate urea solution with 67% water
CAN	Calcium ammonium nitrate
CULTAN	Controlled uptake long term ammonium nutrition
DAP	Diammonphosphat
DM	Dry matter
DSS	Decision support system
dt	Decitonne
GIS	Geo-information system
gr. Potash	Grained potash
ha	Hectare
hl	Hectoliter
K	Potash
kg	Kilogram
Lime+Mg	Carbonic lime with magnesium
MCS	Monte Carlo simulation
Mg	Magnesium
N	Nitrogen
N_{min}	Mineral soil nitrogen
OLS	Ordinary least squares
P	Phosphate
S	Sulfur
SDVRP	Split delivery vehicle routing problem
SE	Standard error
SSA	Sulfuric acid ammonia
TSP	Triplesuperphosphate
tsr	Target seeding rate accounting for germination
ZTC	Zero transport cost

Part I

Introduction and Methods

1 Introduction

This work seeks to identify cost-efficient fertilization strategies at the farm level. Since prehistorical times and early history, this question has already been considered by mankind. At the beginning of this chapter, the history of fertilization from the early times all the way to the current research fields is discussed. Then, the research problem, its relevance, the aims of this study, and the structure of this thesis will be presented in more detail.

1.1 An ongoing story: The optimization of fertilization

Mankind has been engaged in fertilizing crops for thousands of years. Bielecke (1934, p. 7) dates the first purposeful fertilization measures back to 5000 BC or earlier. At that time, fertile land was obtained using slash-and-burn technique and used for three to four years. Albrecht Conrad Thaer (1881) reported that, in 3000 BC, organic fertilizers were used to increase yields in Egypt, which can also be proven by archaeological findings (Widmann, 2007, p. 157). Once the yield-increasing effect of fertilization was recognized, scientists began to unravel the mystery behind it and set out to find what we know today as nutrients. Around 350 BC, Aristotle proposed the "humus theory" in which he assumed that plants feed on substances that are similar to them. This theory was taken up again and expanded by Albrecht Daniel Thaer in 1809 (Bielecke, 1934).

Previous research on fertilization was based on practical experience. It was not until the introduction of the mineral theory of Carl Sprengel (1828) that fertilization began to be examined from a scientific perspective. Sprengel revolutionized humus theory and found that minerals were needed for plant nutrition. In his publication in 1828, he also wrote the following:

> *"...denn es ist nicht zu bestreiten, wenn eine Pflanze 12 Stoffe zu ihrer Ausbildung bedarf, so wird sie nimmer aufkommen, wenn nur ein einziger an dieser Zahl fehlt, und stets kümmerlich wird sie wachsen, wenn einer derselben nicht in derjenigen Menge vorhanden ist, als es die Natur der Pflanze erheischt"*[1]

Later (1855), building on the same findings, Justus von Liebig formulated the "Law of the Minimum," which laid down the foundation for the linear-limitational production function

[1] "… if a plant needs 12 nutrients for growth, it is undeniable that it will not grow if one of them is missing and grow poorly if only one of these nutrients is not present in sufficient quantity." (translation by the author)

still being used today. Justus von Liebig (1843) also provided the first comprehensive concept on mineral plant nutrition, stating that:

> *“Als Prinzip des Ackerbaus muss angesehen werden, dass der Boden in vollem Maße wiedererhalten muss, was ihm genommen wurde; in welcher Form dies Wiedergeben geschieht, ob in der Form von Exkrementen, oder von Asche oder Knochen, dieses ist wohl ziemlich gleichgültig. Es wird eine Zeit kommen, wo man den Acker, wo man jede Pflanze, die man darauf erzielen will, mit dem ihr zukommenden Dünger versieht, den man in chemischen Fabriken bereitet.”*[2]

After discovering which substances could be used for plant nutrition, Europe began to import sodium nitrate (Chile saltpeter) and Guano. Gradually, other fertilizers were developed and industrially produced, such as superphosphate (1843), various potash fertilizers from mining (1860), the first artificially produced nitrogen fertilizers (1890), and ammonia synthesis (Haber, 1908) with the Haber-Bosch process up to the first NPK compound fertilizer (1927).

Parallel to the development and production of fertilizers, research on their use was pushed forward. Hence, around the same time, the first research institutes were founded in Europe. Of particular importance was the “law of action of growth factors” formulated by Mitscherlich (1909), who found that yield growth declined with increasing fertilizer intensity. Such a discovery inevitably raised the question of the economically optimal fertilization intensity. Thus, Mitscherlich triggered a worldwide interest in researching agricultural yield functions.

However, it was only after the end of World War II that intensive use of fertilizers in agriculture began (Finck, 1991). Initially, the desired success did not always occur. The importance of secondary and trace nutrients, along with soil reaction, had not yet been established in practice or had to be investigated in greater detail. Despite this approximately 7000-year history of fertilization, numerous fields of research are still being identified and studied currently.

One of these research areas deals with basic agronomic issues related to the application of fertilizers. For example, Vilsmeier and Amberger (1980) investigated the conversion of various forms of nitrogen in the soil. Today, the slow-release nitrogen fertilizers or nitrification inhibitors play an increasingly important role in the field of agriculture, as evidenced by numerous papers (Chen et al., 2015; Mi et al., 2019; Ni et al., 2014; Noellsch et

[2] “It must be regarded as a principle of agriculture that the soil must receive back to the full extent what has been taken from it; in which form this restitution takes place, whether in the form of excrements, or of ashes or bones, is probably quite irrelevant. There will be a time when the field and the plants grown on it will be provided with fertilizer produced in chemical factories.” (translation by the author)

al., 2009; Herbst et al., 2006; Zerulla et al., 2001). In this context, it is also worth mentioning the research on the Cultan fertilization (Sedlář et al., 2011; Kozlovský et al., 2009). Other studies in this field include those on gaseous conversion losses associated with nitrogen fertilization (Vinzent et al., 2018; Kreuter et al., 2014; Sommer and Jensen, 1994), plant growth models, and models used to estimate biophysical and biochemical processes in soil, among others. The application of these models aims to optimize farm management, and although the specific objectives vary, they share similar primary objectives in terms of yields and environmental impacts of crop production.

A crop growth model that has become popular in recent years is the decision support system for agro technology transfer (DSSAT) (Jame and Cutforth, 1996). It has been adapted in numerous studies to investigate the impacts of fertilization on yield (Araya et al., 2019; Übelhör et al., 2015). Mandrini et al. (2021) used the simulation model called "Agricultural Production Systems sIMulator" (Holzworth et al., 2014) as part of their study on optimal nitrogen management in corn production. Other models, such as "Model for Nitrogen and Carbon in Agro-ecosystems" (MONICA) (Nendel, 2014), "DAISY" (Abrahamsen and Hansen, 2000), "Water and Agrochemicals in the soil, crop and Vadose Environment" (WAVE) (Vanclooster et al., 1996), or "HERMES" (Kersebaum, 1989), are mainly used to represent the complex nitrogen dynamics in the soil as accurately as possible in order to investigate various influencing factors found in the environment. In the context of nitrogen fertilization, nitrate leaching has been frequently addressed in several publications (Heumann et al., 2013, p. 399; Zhou and Butterbach-Bahl, 2014; Anger et al., 2002; Büchert et al., 2001).

Another major area of research, to which this dissertation also contributes, studies the microeconomic optimization of fertilization at the farm level. The optimal intensity of fertilization remains a relevant question in the current literature (Sihvonen et al., 2018; Xu et al., 2017; Chuan et al., 2013; Rajsic and Weersink, 2008), even though it has become a persistent concern for many decades (Kling, 1985; Baule, 1954; Mitscherlich, 1909). Other authors, meanwhile, have focused on the least-cost combination of fertilizers. For instance, Babcock (1984) developed a linear optimization model for creating least-cost blended fertilizers, while Mínguez et al. (1988) used goal programming for least-cost fertilizer combinations in sugar beet cultivation. In more recent studies, decision support systems have been developed to cover not only the subsections of a farm but rather provide whole-farm fertilizer strategies based on the principle of least-cost combination. These include FertiliCalc (Villalobos et al., 2020), Fertilizer Optimizer (Jansen et al., 2013), Smart Fertilizer (Smart

Fertilizer Management), Ecofert (Bueno-Delgado et al., 2016), and Optifer, (Pagán et al., 2015).

Another important research area is the technology of fertilizer application. Studies have also introduced the possibility of variable-rate fertilization using application maps or via real-time sensor data or remote sensing. Many authors have made relevant contributions to the development and advancement of this technology (Cummings et al., 2021; Lu et al., 2019; Fitzgerald et al., 2010). Meanwhile, other authors reported on the applicability or utility to practice (Guerrero et al., 2021; Evangelou et al., 2020; Stamatiadis et al., 2018). In some cases, they do so from specific economic perspectives (Scharf et al., 2011; Koch et al., 2004; Smit et al., 2000).

This overview shows the enormous range in which the topic of “fertilization” has been and is still being dealt with from an agricultural perspective. The following section outlines the open questions addressed by this dissertation and the overall relevance of the underlying topic.

1.2 Relevance of the topic and open questions to continue the “story”

Currently, agricultural goods are traded on a large scale worldwide, resulting in a strong competition and cost pressure due to the high degree of substitutability and the large number of producers. Related to this, farmers are forced to constantly adapt and improve their production systems. Essentially, the aim is to optimally allocate readily available production factors to ultimately achieve the entrepreneurial goal of profit maximization. In the medium term, this also affects the production program. In the short term, however, the focus is on the optimum factor input quantity and the least-cost combination of substitutable inputs. In this respect, the issue of fertilization is highly relevant for farmers. In many cases, fertilization accounts for a considerable proportion of production costs while also determining the yield and the quality of products. In Bavaria (Southern Germany), fertilization costs accounted for 29% of the variable production costs of winter wheat in 2019 (Schätzl et al., 2019). This high share clearly emphasizes the economic importance of having an optimal fertilizer strategy.

At the same time, changing environmental conditions and dynamic changes in input and output prices mean that fertilizer strategies must be constantly re-evaluated. For this reason, almost all farmers in the world would regularly ask themselves whether, how, and in what form the supply of nutrients to crops can be ensured. However, behind this question lies a very complex optimization problem, simultaneously searching for the optimal intensity of nutrient input (of all relevant nutrients) as well as the least-cost combination of the fertilizers available for this purpose. Numerous external influencing factors and supplementary requirements can drastically exacerbate the complexity. These include, among others, crop, legal and operational requirements for fertilization, field-specific transport costs, volatile prices, changing environmental conditions, various available single and compound fertilizers, and the storage function of the soil and the associated flexibility to place nutrients, in some cases completely freely, within the crop rotation. Furthermore, it can be safely assumed that the complexity of this optimization problem (Figure 1-1) exceeds the cognitive abilities of a human decision maker.

- Determination of fertilization dates (effects and losses)
- Estimation of the yield potential
- Attention to fertilizer and product prices
- Determination of the fertilizer requirement (N, P, K, …)
- Pre-purchase of fertilizers (Storage capacity)
- Permanent adjustment of the fertilization strategy during growing season

- Compliance with guidelines and legal requirements
- Attention to the crop development
- Selection of particular fertilizers (nutrients, effects, and losses)
- Preliminary fertilization with basic nutrients
- Use of compound fertilizers
- Weather and soil conditions

Figure 1-1: Optimization problem of the cost-efficient fertilization strategy (without claim to completeness).

Source: Own compilation.

To solve this optimization problem, some open questions must be addressed first:

- What influence do application costs have on the cost-efficient fertilizer strategy?
- How can fertilizer application costs be integrated into a mathematical optimization model for individual farms?
- What are the computational and data requirements of different methods to do so?

- What are the agronomic, legal, and operational requirements of a model designed for cost-efficient fertilizer strategies?
- What does the associated mathematical optimization model look like?
- What is the savings potential of such an optimization model compared to a fertilization strategy defined in the usual way?
- Does a cost-efficient fertilization strategy lead to undesirable agronomic effects?
- How does a cost-efficient fertilizer strategy differ from an inefficient one?
- What influence do different farm conditions have on the design of the cost-efficient fertilization strategy?

The answers to these questions would be highly relevant for farmers and consultants, as they have a direct impact on production costs and management time in crop production. Furthermore, the findings of this work can help reduce production efforts and increase transparency about the price worthiness of alternative fertilizers. Fertilizer producers are faced with a situation wherein farmers seek to find maximum price transparency. However, fertilizer producers can also take advantage of this knowledge and adjust their product portfolio accordingly.

This thesis aims to provide an important and new contribution to the understanding of cost-efficient fertilizer strategies at the farm level from a scientific perspective. Cost-efficient fertilization uses scarce resources efficiently, making it an important part of sustainability efforts (Tröster and Sauer, under review). This also means that the current study has social relevance that can be further extended, that is, if one applies fertilizer emissions in terms of CO2 equivalents instead of market prices, it is possible to identify particularly low-emission fertilization strategies.

1.3 Aims and structure of this thesis

The goal of this dissertation is to develop a decision support system (DSS) that satisfies the optimization problem shown in Figure 1-1. The DSS is designed to suggest a cost-efficient fertilizer strategy that can be used by farmers several times per season to improve farm profit and save management time. Another objective of this work is to assess the economic and agronomic performance of the DSS under development and to identify the potential features of a cost-efficient fertilizer strategy (e.g., in the selection of fertilizers).

In the first part of this thesis (Part I), the introduction to the research topic, an overview of the theoretical framework, and the research methods used are presented. The main part (Part II) contains four independent studies, which are presented in Chapters 3–6, respectively. A brief summary is provided at the beginning of each chapter. The first study (Chapter 3) focuses on integrating farm-specific fertilizer application costs within the framework of a mathematical optimization model. In the second study (Chapter 4), the requirements for the DSS are first described verbally followed by its implementation in mathematical form. This study also includes the economic performance evaluation. Study three (Chapter 5) presents a multi-year field trial that aims to investigate the agronomic performance of the DSS.

Meanwhile, Study four (Chapter 6) provides a detailed analysis of the characteristics of cost-efficient fertilizer strategies and examines whether general recommendations for fertilization strategies can be derived. In addition, how various farm conditions influence the design of the cost-efficient fertilization strategy is examined in this part. Part III begins with Chapter 7, which contains extended summaries of the four studies. Finally, Chapter 8 presents an overall discussion of the entire research contribution of this thesis and highlights important findings. For an initial overview, Table 1-1 provides concise information on the studies presented in Part II.

Table 1-1: Overview of the studies presented in Part II.

Topic	**Research question**	**Methods**	**Novelty**
Chapter 3: Effects of application costs on fertilizer application strategy			
Use of farm-specific application costs in optimization models	How should fertilizer application costs be considered in mathematical optimization models?	Use of route planning software to define optimal transportation routes. Derivation of a regression function to estimate individual transportation costs. Use of scenario analysis and Monte Carlo simulation to assess usability.	It is the first time that the SDVRP has been used in the context of fertilizer application. Another contribution is the deduction of a resource-friendly regression function for the accurate estimation of transport costs based on-farm characteristics.
Chapter 4: IoFarm: A novel decision support system to reduce fertilizer expenditures at the farm level			
Identifying cost-efficient fertilization strategies	How can a farm-specific cost-efficient fertilization strategy be achieved?	Classical OR approach from a nonformal model, to a formal GAMS model, to its evaluation in form of a choice experiment. The GAMS model is a two-stage deterministic dynamic optimization model that solves NLP in stage I and MINLP in stage II.	For the first time, a dynamically adaptable optimization approach is presented, which; apart from the otherwise usual aspects; also considers application costs, and its planning horizon extends over an entire crop rotation cycle.
Chapter 5: IoFarm in field test: Does a cost-optimal choice of fertilization influence yield, protein content, and market performance in crop production?			
Agronomic effects	What are the agronomic effects of a fertilizer strategy that has been optimized based on the principles of cost-efficiency?	Two-factorial, multi-site, multi-year agronomic field trial. Factor 1: Fertilizer variant (Optimization model, farm manager, and control). Factor 2: crop (winter wheat, winter barley, and silage maize).	For the first time, agronomic effects of an economically optimized fertilizer strategy (considering the least-cost combination) are reported based on a field trial.
Chapter 6: Characteristics of cost-efficient fertilization strategies at the farm level			
Knowledge of cost-effective fertilization strategies	What characterizes cost-efficient fertilizer strategies?	Differentiation of fertilization strategies from a choice experiment by employing regression and cluster analysis as well as subsequent t-test. Evaluation of external influencing factors according to the *ceteris paribus* principle using the newly built DSS as the simulation model.	For the first time, this study demonstrates the main differences between cost-efficient and inefficient fertilization strategies and provides information on the impacts of a farm's initial conditions..

2 Theoretical framework and applied methods

This chapter provides an overview of the theoretical foundations and methodological approaches that have been applied in the context of this research work. First, the problem is embedded in production theory, followed by a presentation of operations research techniques that are commonly used solve the optimization problem. Finally, methods for verifying a DSS and analyzing cost-efficient fertilization strategies are described.

2.1 Microeconomic theories and behaviors

People are permanently confronted with the task of making decisions. Due to their highly developed cognitive abilities, people are often expected to behave rationally. Therefore, it is assumed that a decision is preceded by a careful process in which alternative options are identified and evaluated according to one's own preferences. Hence, the term "rational principle" is used to refer to this idea. However, in daily life, other behaviors can be identified that rarely lead to an optimal decision. These include, for example, limited rational behavior (Simon, 1959) as well as emotional, traditional, random, and inconsistent behaviors (Brandes and Woermann, 1982, p. 16). Decisions based on these patterns of action require no or little transaction costs for obtaining information and evaluating it. Therefore, combined with decisions of little relevance, such as the selection of a seat on the bus, it would be quite economically efficient to dispense with the rational principle (Simon, 1959).

With respect to the objective of decisions, formal and substantive objectives can be distinguished (Brandes and Woermann, 1982, p. 16). A formal goal can be any conceivable objective, whereas a goal is considered substantial only if it is generally accepted, e.g., the aim to achieve prosperity. In the context of microeconomic considerations, people are usually assumed to act in a substantially rational manner; in this case, we speak of *homo oeconomicus*. Depending on personal preferences, such as desires and needs, the objectives of *homo oeconomicus*, can vary. The term "utility" is used as a measure for the satisfaction of one's wishes and needs. Therefore, a rational decision maker tries to make decisions (x) in such a way that his personal utility (U) is maximized, as shown below:

$$\max!\ U(x). \tag{2-1}$$

The concept of utility maximization is tied to the desires and needs of an individual person, thus allowing enormous flexibility. In this case, a direct transferability to companies or production processes may not be possible. At this level, goods are produced or services are offered for the purpose of generating income and profit. By generating profit in the company, the entrepreneur can satisfy numerous wishes and needs, thereby generating benefits; yet, not all wishes and needs can be satisfied by money. Nevertheless, due to the nonexisting comparability of individual utility and the considerable intersection between profit (Π) and utility (*U*), it is often assumed in production economics that *homo oeconomicus* would be a profit maximizer. This means that alternative actions (*x*) are chosen in such a way that profit (Π) is maximized, as shown in the following:

$$\max!\ \Pi\left(x\right). \tag{2-2}$$

2.2 Relevant concepts of production economics

The concept of profit maximization presented in the previous paragraph is highly advantageous, because it can define a monetary and uniform objective that can be used in models. In the context of agricultural production theory, further simplifications are applied (Mußhoff and Hirschauer, 2013, p. 144), including perfect information, homogeneity and divisibility of goods, static consideration of a production period, the absence of external effects, and the fact that the producer is the price taker.

As mentioned above, a cost-efficient fertilizer strategy is an important approach in maximizing farm profit. In this respect, profit from crop production can be represented as follows:

$$\Pi\left(x\right)=R\left(x\right)-C\left(x\right), \tag{2-3}$$

where profit depends on a revenue function *R(x)* and a cost function *C(x)*. The following applies to the revenue function *R(x)*:

$$R\left(x\right)=p\times y\left(x\right), \tag{2-4}$$

where *p* is the product price, and *y(x)* is the corresponding yield function. In crop production, numerous factors *x* influence yield *y*; thus, the yield function should be extended as follows:

$$y=f\left(x_1...x_n\,|\,x_{n+1}...x_m\right). \tag{2-5}$$

Given that the focus is on fertilizer selection, the variable factors x_1 to x_n denote individual fertilizers (incl. time of application and quantity), while all other growth factors are considered fixed. As plant growth depends on nutrients rather than fertilizers, only an indirect relationship exists between fertilizer use and yield. Therefore, the individual nutrient contributions u_{ns} (N, P, K, Mg, S, etc.) are relevant to the crop production function. These nutrient contributions can be defined for each fertilizer x_n via the known nutrient content of the fertilizers ($SUP_{n,ns}$), as shown below:

$$\sum_n (u_{ns}) = x_n \times SUP_{n,ns} . \tag{2-6}$$

Therefore, Eq. (2-5) also be represented as follows:

$$y = f\left(u_{ns} \middle| x_{n+1} ... x_m\right). \tag{2-7}$$

To determine the optimal relationship between nutrient input and yield, it is necessary to define the functional form for Eq. (2-7). Crop production functions are usually found in the form of a linear-limitational, quadratic, or asymptotic functional form Frank et al. (1990). In this thesis, the concept of linear-limitational plant production function, which can be traced back to Justus von Liebig, is used. Although asymptotic functional forms dominate the literature for the most part, this concept is also considered in parallel in many studies. It has been often stated that the coefficient of determination of a linear-limitational function is only insignificantly worse. In fact, some studies have used the linear-limitational function to better describe the relationships compared to other common crop production functions (Stone et al., 2010; Bäckman et al., 1997; Grimm et al., 1987). Omitting the nitrogen dynamics in the soil can help derive a linear relationship between plant yield and the nutrient content in the plant, which describes the linear part of the production function. Once the nitrogen dynamics of the soil are incorporated via measured or estimated values and an adjusted upper yield limit is defined in parallel, a site-specific linear-limitational production function is obtained. The production function established in this way does not require expensive and site-specific data collection in field experiments. Therefore, it can be used in an uncomplicated way across sites. Furthermore, it is suitable for complex mathematical optimization models, as linear functions require considerably less computing power than quadratic or asymptotic functions. For the reasons mentioned above, the current work is based on the linear-limitational production function that can be traced back to Justus von Liebig. Figure 2-1 shows a representation of this function.

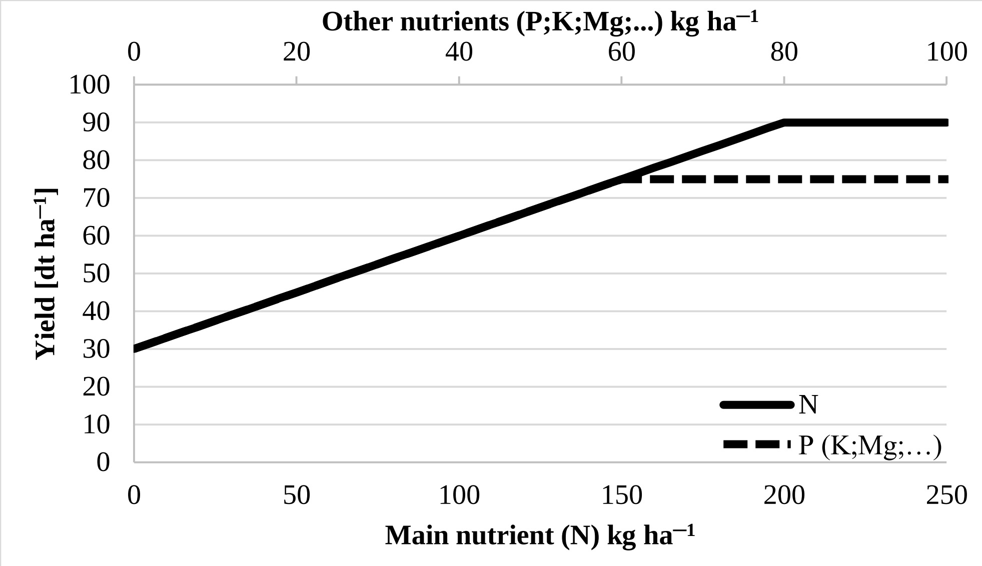


Figure 2-1: Linear-limitational plant production function for multiple nutrients.

The plant production function shown in Figure 2-1 is limited by a maximum. As a rule, it is assumed that this limit is set by a limiting growth factor that corresponds to the genetic yield potential of the plant under ideal growth conditions. Then, the maximum yield (*YEX*) is defined by the site-specific yield expectation of the farmer to allow the user to respond to changing growth conditions in the later optimization model. This is a dynamic process that can be adjusted several times per season. This results in the following crop production function:

$$\left.\begin{array}{l} y\left(u_{ns}\,|x_{n+1}\ldots x_m\right)\le \dfrac{u_{ns}}{UEXT_{ns}} \\ y\left(u_{ns}\,|x_{n+1}\ldots x_m\right)\le YEX \end{array}\right\} \min = y\,, \tag{2-8}$$

where u_{ns} denotes the nutrient quantity, and $UEXT_{ns}$ the nutrient requirement of the crop (per yield unit). Now consider the cost function *C(x)*, which is also a part of Eq. (2-3). Assume that adjustments to fertilizer strategy can be made without making changes in the fixed assets, which means that only variable costs need to be considered. In this case, the cost function *C(x)* consists the price of fertilizer q_n and the variable application costs for fertilizer m_n, each in relation to the amount of fertilizer used x_n. All other influencing factors are considered to be fixed:

$$C\left(x_1\ldots x_n\,|x_{n+1}\ldots x_m\right)=\sum\nolimits_n\left(q_n\times x_n+m_n\times x_n\right). \tag{2-9}$$

At this point, it is already known that fertilizers x_1 to x_n contribute differently to nutrient supply (compare (2-6)) and that the technically efficient path of nutrient supply is given by the crop production function (2-8). Hence, the following question arises: What is the least-cost combination of fertilizers to achieve a given yield *y*? Ultimately, this issue has already been illustrated in Eq. (2-5). Furthermore, it is presented graphically in simplified form, using the combination of two fertilizers x_1 and x_2 (Figure 2-2 to Figure 2-4).

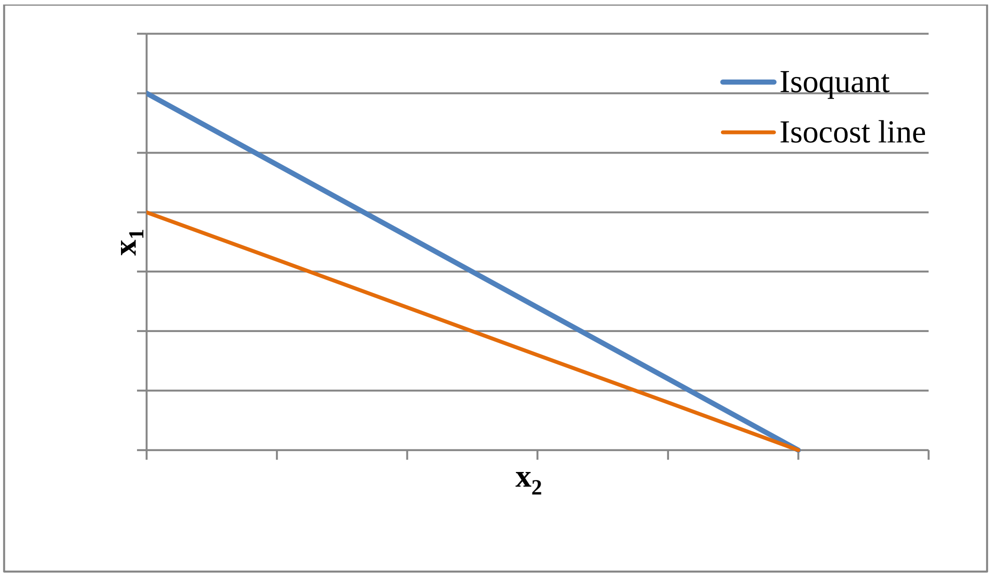


Figure 2-2: Least-cost combination at the linear marginal rate of substitution.

Figure 2-2 shows the isoquant for two fully substitutable fertilizers x_1 and x_2. All combinations of the two fertilizers on this line produce the same yield. This, however, is a highly simplified case, because more than two fertilizers are usually needed to meet the nutrient requirements of a crop. The complete substitutability of fertilizers is also only theoretical because of different nutrient compositions and different nutrient forms. As shown by the intersection with the isocost line, such substitution relationships lead to corner solutions. In the above case, fertilization with fertilizer x_2 alone is preferred for cost reasons. The representation of decreasing marginal rates of substitution is often found in the literature (Figure 2-3).

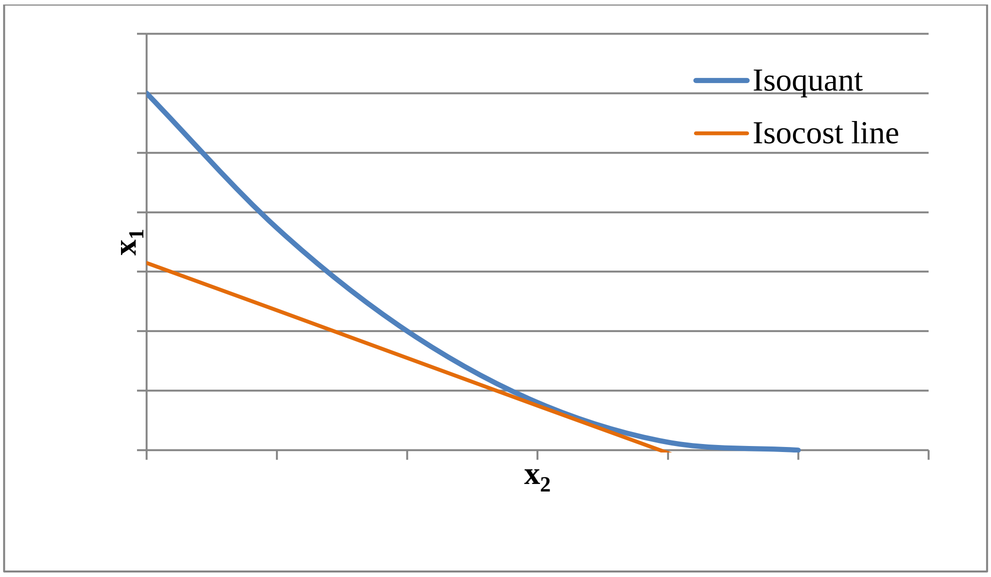


Figure 2-3: Least-cost combination with decreasing marginal rate of substitution.

Once again, any combination of the fertilizers x_1 and x_2 produces the same yield, which is represented by the isoquant. However, even a small amount of x_1 leads to a considerable reduction of x_2. Hence, the least-cost combination of both fertilizers is determined by the intersection with the isocost line. Such substitution effects are plausible, such as when nitrogen fertilizers with different forms of nitrogen are compared. However, technically efficient combinations of fertilizers are often relevant for the least-cost combination of fertilizers, especially considering the fact that several nutrients are considered in parallel (Figure 2-4):

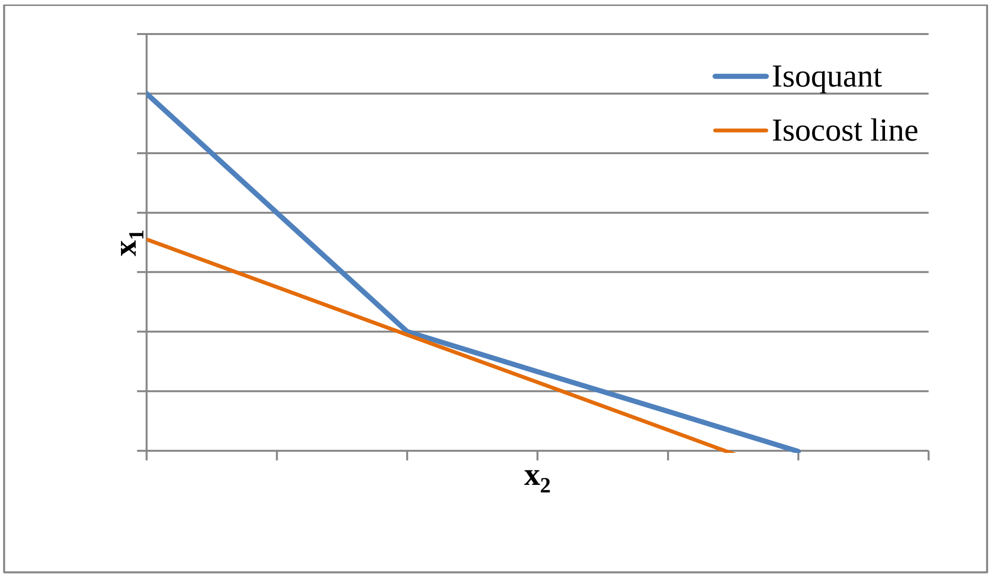


Figure 2-4: Least-cost combination with piecewise linear marginal rate of substitution.

Figure 2-4 illustrates the isoquant for the combination of two different compound fertilizers. Fertilizer x_1 contains 20% nitrogen and 7% each of potash (K_2O) and phosphorus (P_2O_5).

Meanwhile, fertilizer x_2 contains 15% each of the above nutrients. The linear substitution of both fertilizers is possible up to a certain point, at which one of the nutrients becomes the limiting factor for plant growth. In this case, the substitution of another unit of x_1 for x_2 leads to the fact that more x_2 must be used (e.g., to satisfy the N demand of the crop). This kink in the isoquant is also called a technically efficient combination (Mußhoff and Hirschauer, 2013, p. 167; Nicholson and Snyder, 2008, p. 113). In the case presented, this point also corresponds to the least-cost combination, because the isocost line intersects here. In actual scenarios, numerous fertilizers must be considered to identify the least-cost combination. Therefore, isoquants are not to be understood as simple lines, but as multidimensional structures (compare Debertin, 1986, p. 113), in which the substitution relationships between the fertilizers (as shown) take different forms.

The course of isocost lines will be considered in more detail after discussing the potential forms of isoquants. Isocost lines are typically displayed as straight lines, as shown in Figure 2-2 to Figure 2-4. Each combination of fertilizers on the isocost line leads to identical fertilizer expenditures. Now, let us consider the variable application costs as another component of the cost function (2-9). Tröster et al. (2019) (see paragraph 3.4) stated that application costs for fertilizer are nonlinear. For example, differences in application costs of fertilizers can be attributed to variations in the specific weights of fertilizers. Each independent fertilizer application commonly requires a fixed amount of setup time. Differences in the nutrient concentration of fertilizers cause changes in the absolute amount of fertilizer applied among alternative fertilizer strategies. In our case, nonlinear application costs mean that the isocost line cannot be a straight line. Furthermore, entry costs in the form of setup time cause a step change of the isocost line. The same applies to additional technical requirements for fertilization, e.g., due to minimum application rates. The consequences for the least-cost combination are shown in Figure 2-5.

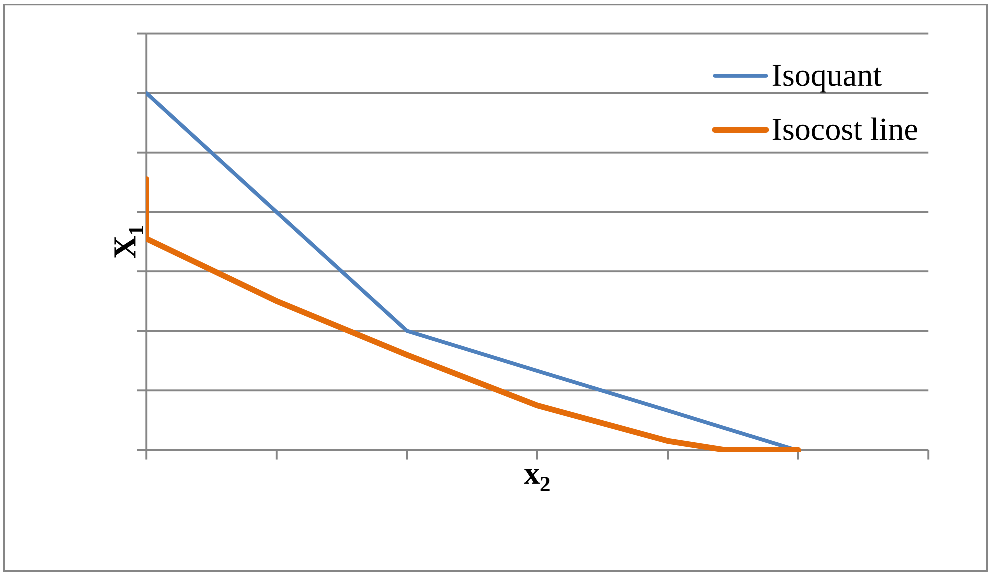


Figure 2-5: Least-cost combination in the case of nonlinear application costs.

Figure 2-5 now shows an example of the use of two fertilizers x_1 and x_2 with an irregular course of the isocost line. As can be seen, the total costs of application increase abruptly as soon as a combination of both fertilizers is used. This means that less fertilizer can be purchased with the same financial budget. In the above example, this also implies that the least-cost combination is no longer at the kink of the isoquant, but now, only fertilizer x_2 is used. Considering application costs makes it much more difficult to determine the optimum from a mathematical point of view. In addition, the application costs m_n are themselves a function dependent on the choice of fertilizer x_n:

$$m_n = f\left(x_1 \ldots x_n\right). \tag{2-10}$$

As displayed in Eq. (2-10), the application m_n costs of a fertilizer x_n depend on the choice of fertilizers or the fertilizer strategy as a whole. However, the application costs themselves influence the choice of fertilizer, thereby implying that there exists a feedback loop within the optimization problem, which can only be solved simultaneously.

2.3 Expansion path concept for optimal fertilizer strategies

In the previous section, the least-cost combination of fertilizers was discussed in detail. As the least-cost combination represents an optimization of input costs, it has a potential impact on the optimal factor input quantity. The relationship between optimal intensity and least-cost combination is described by the expansion path. It is essential to follow the expansion path to a maximum of profit in order to achieve a cost-efficient fertilization strategy. Figure 2-6 presents a typical illustration of the expansion path in the literature.

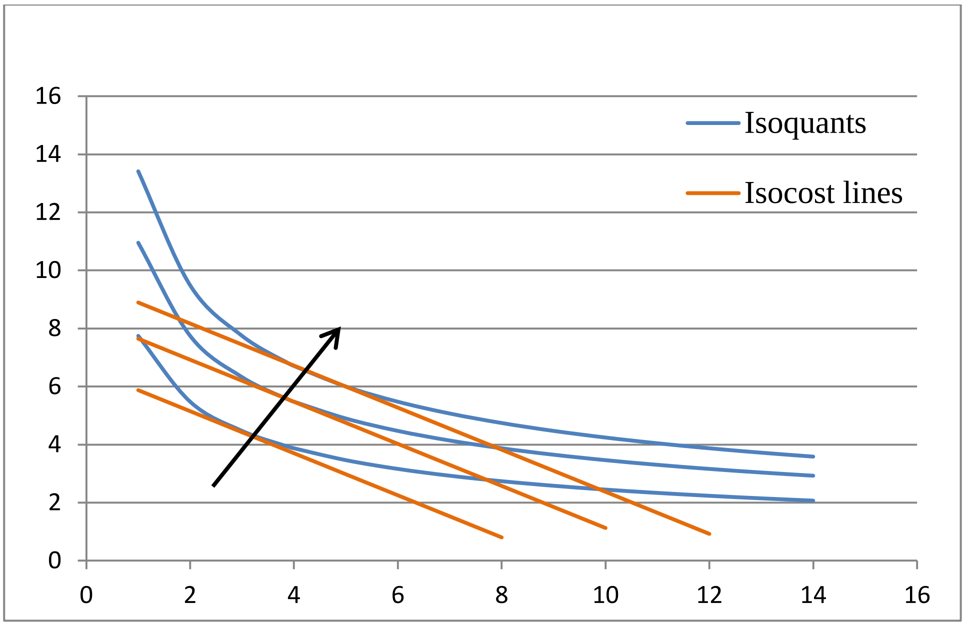


Figure 2-6: Classic illustration of the expansion path.

Author's own illustration, derived from (Mußhoff and Hirschauer, 2013, p. 168).

As shown in Figure 2-6, the cost-efficient ratio of input factors remains constant regardless of the level of production. Therefore, the expansion path connects these intersections in the form of a straight line. Hence, the profit maximization can also be done in two steps: (i) finding the least-cost combination, and (ii) finding the optimal input intensity given the input ratio (least-cost combination). However, this stepwise approach is not suitable for identifying a cost-efficient fertilizer strategy. This is because, as shown earlier (Figure 2-4), the isoquants do not behave uniformly; instead, there are abrupt or gradual changes in the isoquants. For this reason, the course of the isoquants can change at different production levels. In microeconomic theory, this is shown, for example, by inferior inputs that become less important as production intensity increases (compare Nicholson and Snyder, 2008, p. 329). Furthermore, the expansion path to the cost-efficient fertilizer strategy is characterized by nonlinear isocost lines. This has already been demonstrated using nonlinear application costs (Figure 2-5). Given that the application costs themselves are dependent on the fertilizer combination (compare Eq. (2-10)), the same principle applies to the isocost lines as to the isoquants: their course can change at different production levels. Consequently, the expansion path is nonlinear and can have various least-cost combinations at different production levels (see Figure 2-7). Therefore, the simultaneous optimization of the optimal input intensity and least-cost combination is mandatory in the process of identifying a cost-efficient fertilization strategy.

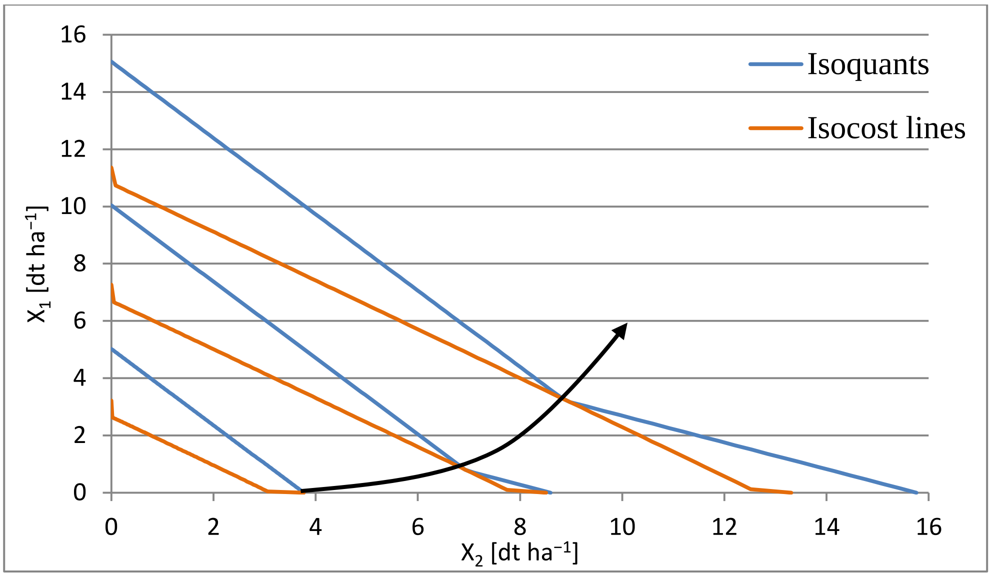


Figure 2-7: Cost-efficient fertilizer strategy based on the expansion path.

Author's own illustration, derived from (Nicholson and Snyder, 2008, p. 328)

The concept of the expansion path is graphically transferred to the cost-efficient fertilizer strategy in Figure 2-7. However, the illustration is still very far from reality, as only two variable inputs (fertilizers) are regarded. Specifically, two compound fertilizers are considered as inputs: fertilizer x_1 containing 20% nitrogen, 7% each of phosphorus (P_2O_5), and potash (K_2O). Fertilizer x_2 contains 15% each of the above nutrients. Both fertilizers are used for the production of winter wheat. For simplicity, only the nutrients N, P, and K are considered, and a linear relationship between yield and nutrient input is assumed, where 1 dt yield = 2.51 kg N, 1.04 kg P_2O_5, and 1.67 kg K_2O. Under actual conditions, the dimensionality of this optimization problem increases considerably, and the decision variable x is defined over the following sets:

- *t* Year
- *tm* Month
- *cr* Crop
- *fz* Fertilizer
- *f* Field

In addition, these sets are also considered relevant (completely or partially) for yield y, product price p, fertilizer price q, and application costs m. Accordingly, the profit function from Eq. (2-3) is established in detail as follows:

$$\Pi = \sum_{t,f,cr} \left(y_{t,f,cr} \times p_{t,cr} \right) - \sum_{t,tm,cr,fz,f} \left(x_{t,tm,cr,fz,f} \times q_{t,tm,fz} \right) - \sum_{t,tm,cr,fz,f} \left(x_{t,tm,cr,fz,f} \times m_{t,tm,fz} \right) \quad (2\text{-}11)$$

The first part of this profit function present the total revenue, which consists of the sum product of revenue $y_{t,f,cr}$ with the product price $p_{t,cr}$. Note that the yield $y_{t,f,cr}$ is itself a function of the variable $x_{t,tm,cr,fz,f}$. The middle part calculates the total cost of buying the fertilizers. Meanwhile, the sum product of fertilizer input $x_{t,tm,cr,fz,f}$ and application cost $m_{t,tm,fz}$ (in the right part) summarizes the cost of fertilizer application. It should be emphasized here that $m_{t,tm,fz}$ is a function dependent on $x_{t,tm,cr,fz,f}$. Due to the concatenation of functional relationships and the dimensionality of the decision variable $x_{t,tm,cr,fz,f}$, it is unlikely that such a complex problem can be solved by a human decision maker in an optimal way (Amann, 2019, p. 19). Therefore, the question arises as to which method can be used to optimally solve the optimization problem.

2.4 Finding the optimal fertilizer strategy

Solving optimization problems is a central task of operations research, and this field of study offers various methods for this purpose. Given that the optimization problem at hand can be represented as a mathematical model, the following methods are particularly considered: exact optimization methods and heuristics and metaheuristics.

One of the exact optimization methods is linear programming (LP), first described by Kantorovich (1960) in 1939. From this, further exact optimization methods have been developed, such as integer programing (IP), mixed integer programming (MIP), nonlinear programming (NLP), and a combination of mixed-integer and nonlinear programming (MINLP). The exact optimization methods specifically search for the mathematically optimal solution and have the distinct advantage of being able to provide information regarding the optimality of the solution (e.g., whether the solution found is indeed a global optimum). In comparison, heuristics and metaheuristics only search for the best possible solution with reasonable effort and do not specifically search for a mathematically optimal solution; they also do not allow any statement on the optimality of the solution found (Suhl and Mellouli, 2013, p. 13).

In current optimization software, both methods mentioned above are often combined: heuristics are used in the so-called pre-solve methods to reduce the size of the optimization problem so that the exact optimization methods can be subsequently applied in a more efficient manner. Numerous solvers included in the GAMS software package (GAMS Development Corporation, 2016) offer this possibility. Moreover, the variety of solvers included in GAMS facilitates the easy switching among different model categories (LP, MIP, NLP, MINLP, and others). For these reasons, the exact optimization methods were used for the implementation in this study, along with the software package GAMS. Below is the typical structure of an LP (modified after Andrei, 2013, p. 109).

$$\textit{minimize} \qquad f(x) \tag{2-12}$$

$$\textit{subject to:} \qquad g(x) \leq 0 \tag{2-13}$$

$$h(x, y) = 0 \tag{2-14}$$

$$x \in \left[x^L, x^U\right] \tag{2-15}$$

As shown above, $f:\mathbb{R}^n \to \mathbb{R}$, $g:\mathbb{R}^n \to \mathbb{R}^m$, and $h:\mathbb{R}^n \to \mathbb{R}^p$ represent linear functions, and the solution space of the decision variable *x* is limited by the lower and upper bounds referred to as *L* and *U*, respectively. Usually, *x* is defined as non-negative. If the objective function (2-12) or one of the constraint functions (2-13) or (2-14) is nonlinear, it is an NLP. If additional integer criteria are considered, it is an MINLP, whose structure can be represented below (modified after Floudas, 2011, p. 618).

$$\textit{minimize} \qquad f(x, y) \tag{2-16}$$

$$\textit{subject to:} \qquad g_i(x, y) \leq 0 \tag{2-17}$$

$$h(x, y) = 0 \tag{2-18}$$

$$x \in \left[x^L, x^U\right] \tag{2-19}$$

$$y \in \left[y^L, y^U\right] \cap N^q \tag{2-20}$$

As shown above, at least one of the functions $f:\mathbb{R}^n \to \mathbb{R}$, $g:\mathbb{R}^n \to \mathbb{R}^m$, and $h:\mathbb{R}^n \to \mathbb{R}^p$ is nonlinear. The decision variables *x* and *y* are constrained by the lower and upper bounds referred to as *L* and *U*, respectively. In addition, *N* is the set, and *q* is the number of integers or binary variables.

The already described interrelationships of the present optimization problem necessarily lead to a nonlinear model structure. There is a technical lower limit for the application rate of fertilizers in practice (e.g., 0.8 dt ha^{-1}); hence, the optimization problem also includes semiconditional variables[3]. Thus, a practical model for the economic optimization of fertilizer strategies falls under the MINLP category given that such variables lead to binary constraints within the model. MINLPs are widely applicable due to the combination of discrete and nonlinear content; however, solving such problems is also tremendously challenging (Lee and Leyffer, 2012, p. vii; Bussiek and Pruessner, 2003) and rarely succeeds optimally.

The choice of MINLP solver has a large impact on whether or in what time a problem can be solved optimally (or with a relative objective gab < 0.1%) (Kronqvist et al., 2019). This fact also applies to the model developed in the current work. To solve the emergent model as efficiently as possible, a performance test with different MINLP solvers was carried out at an early stage of the model development. MINLP solvers use specialized NLP and MIP subsolvers to solve these kinds of subproblems. Furthermore, the optimization model itself was also built in two stages. Stage I was used to eliminate inefficient solutions, also known as sequential decision making (Amann, 2019, pp. 19–20). Table 2-1 provides an overview of the solvers used in both model stages. It presents a selection of the MINLP and NLP solvers included in the GAMS software package in January 2016, which were combined in this step. A temporary GAMS license with all full versions of the solvers was available for this test.

The results of the performance test from Table 2-1 clearly highlight the differences among various solvers. As can be seen, not every solver is equally suited for solving the MINLP model at hand. A large part of the tested solver combinations could not find a simple solution or a valid solution for the optimization problem. Only the first five solver combinations, shown in Table 2-1, have been found to be useful. The best valid optimization result, at a relatively low time cost, was obtained with the ANTIGONE solver (Misener and Floudas, 2013). This solver additionally required a license for the MIP solver CPLEX (IBM Corporation, 2017) and the NLP solver CONOPT (ARKI Consulting and Development A/S, 2016a). Therefore, this solver package was purchased and used for further model development in the current study.

[3] The extent of this variable is either 0 or any value between a lower and upper limit.

Table 2-1: Solver performance test.

Stage I[1] NLP Solver	Stage II[2] MINLP Solver	Objective value [€]	MINLP Solver Status	MINLP solving time [sec]
Antigone[4]	Antigone[4]	459346	Integer Solution	78
Conopt[6]	SBB[14]	453357	Integer Solution	383
Conopt[6]	Dicopt [Conopt[6], SCIP[15]]	441029	Integer Solution	50
SCIP[15]	Couenne[7] [Baron[5]]	424213	Integer Solution	287
SCIP[15]	Baron[5]	424213	Integer Solution	298
SCIP[15]	Couenne[7] [SBB[14]]	453468	Feasible Solution[17]	1220
Couenne[7]	SBB[14]	453412	Feasible Solution[17]	601
SNOPT[16]	SBB[14]	253100	Intermediate Non Integer	121
SCIP[15]	Dicopt[8] [Conopt[6], SCIP[15]]	462382	Intermediate Non Integer[17]	726
Couenne[7]	Dicopt[8] [Conopt[6], SCIP[15]]	453412	Intermediate Non Integer[17]	600
Knitro[10]	Knitro[10]	-2444841	Locally Infeasible	481
Local Solver[11]	SBB[14]	58072775	Locally Infeasible	607
Path[13]	SBB[14]	-574908	Intermediate Infeasible	100609
Minos[12]	SCIP[15]	-	No Solution	0
Conopt[6]	SCIP[15]	-	No Solution	2
IPOPTH[9]	SCIP[15]	-	No Solution	100
Baron[5]	SBB[14]	-	No Solution	104
Couenne[7]	SCIP[15]/SBB[14]		No Solution	500
Couenne[7]	Alphaecp[3]	-	No Solution	608
SCIP[15]	SBB[14]	-	No Solution	26210

Remarks: Used hardware: Intel i7-4790K CPU 4.00 GHz; 16 GM RAM. Used software: GAMS 24.8.1 for MS Windows (10 Pro). The solvers in square brackets are the used NLP and MIP sub-solvers. Foodnotes: 1) Stage I = relaxed NLP Model; 2) Stage II = full MINLP Model; 3) (Westerlund and Lundqvist, 2001); 4) (Misener and Floudas, 2013); 5) (Tawarmalani and Sahinidis, 2005); 6) (ARKI Consulting and Development A/S, 2016a); 7) (Belotti et al., 2006); 8) (Vecchietti and Grossmann, 2016); 9) (Wächter and Biegler, 2006); 10) (Byrd et al., 2006); 11) (Innovation 24, 2016); 12) (Murtagh and Saunders) 13) (Dirkse and Ferris, 1995); 14) (ARKI Consulting and Development A/S, 2016b); 15) (Gamrath et al., 2016); 16) (Gill et al., 2013); 17) Integer conditions violated.

Thus far, Section 2.4 describes only one necessary step to solve an optimization problem in accordance with the guidelines of operations research. This is the third of a total of five steps (Taha, 2017, p. 40), as listed below.

- Definition of the problem
- Construction of the model
- Solving the model
- Validation of the model
- Implementation and presentation of the solution

The steps listed above were followed in sequence during the optimization problem processing. The first step involved using a nonformal model to identify the optimization problem and its exact description. Here the nonformal model could be converted into a mathematical model (construction of the model), because the components and relationships of the optimization problem are sufficiently known. Meanwhile, step number three (solving the model) has already been explained in detail. Numerous model runs were performed under changing input parameters to validate the model, focusing particularly on the plausibility of the model output. Next, an experiment in which the test persons competed directly with the optimization model was performed to validate economic performance. The participants' task was to achieve a cost-efficient fertilization strategy. The validation of the agronomic performance and a first implementation of the optimization model in practice was performed via field tests conducted over several years. More information on the validation activities can be found in Section 2.7.

2.5 Decision support systems in crop production

In the literature, the term “decision support system” has been described by numerous authors (Valencia-García et al., 2018; Turban et al., 2011; Power, 2002; Sprague and Carlson, 1982). In fact, there is a wide agreement among authors on the definition, which is accurately reflected by Zámečníková and Kreslíková (2016, p. 73) as follows:

> *“Decision Support System (DSS) is a computer-based information system or subsystem that supports business or organizational decision-making activities. DSSs serve the management, operations, and planning levels of an organization and provides help in decision making process. […] Decision support systems can be either fully automated, human [driven] (Author’s note) or a combination of both.” (Zámečníková and Kreslíková, 2016, p. 73).*

Thus, a DSS is not a completely autonomous optimization system, but a system over which the user still has influence. For this reason, any external intervention in the decision-making process is possible, along with the flexible consideration of new information. Certainly, the objective of a DSS is to help the user arrive at the best possible decisions. Figure 2-8 shows the schematic structure of a DSS.

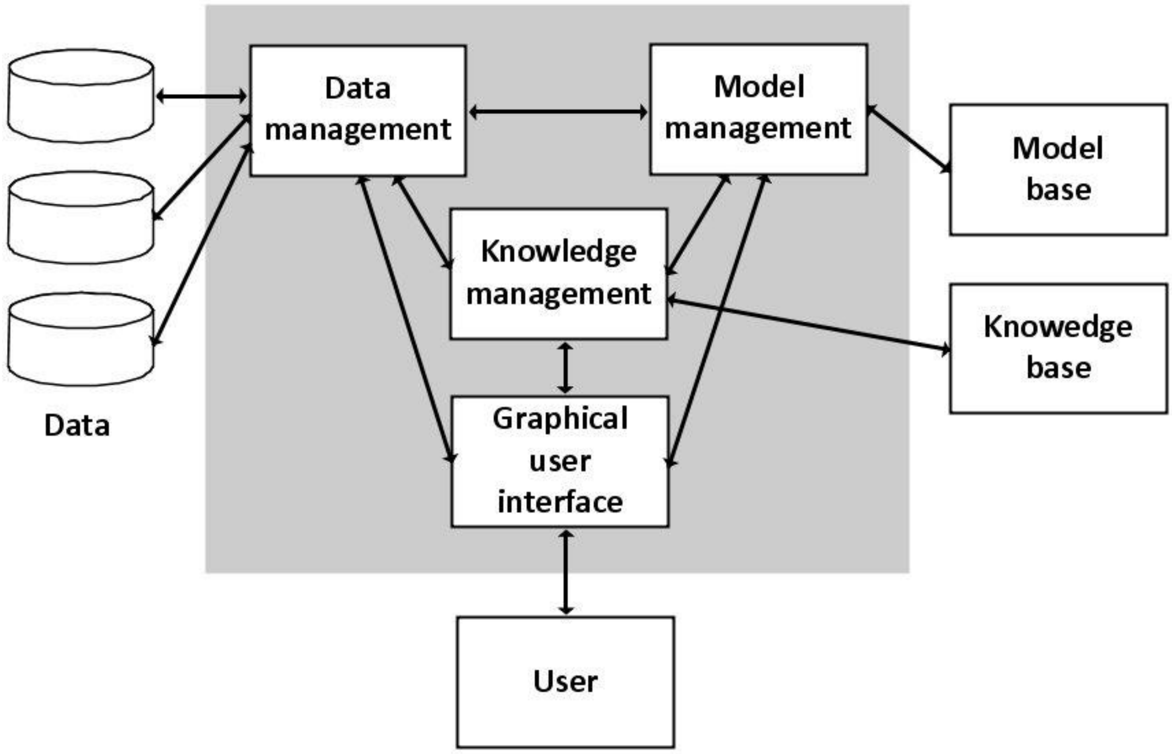


Figure 2-8: Schematic representation of a DSS (unchanged: Hujer, 2011).

A DSS consists of several components, namely, data management subsystem, model management subsystem, knowledge management subsystem (optional), and user interface (Hujer, 2011; Turban et al., 2011; Ragsdale, 2001). The data management subsystem is commonly used to provide and, if necessary, process all internal and external information. The optional knowledge management subsystem contains theoretical principles, correlations, and empirical values that contribute to the process of problem solving and are, for example, displayed to the user. Sometimes, artificial intelligence can also be used to integrate this knowledge into the solution process (Hujer, 2011). The subsystems mentioned so far communicate directly with the model management subsystem. There are numerous possibilities involved in creating the model management subsystem, and optimization or simulation software is often used. Ultimately, the DSS user can access the mentioned subsystems via a user interface to make changes or to simply view information for decision making.

The DSS is only considered for serious problems, because its construction usually requires extensive resources. DSS are particularly useful in solving poorly organized or unstructured problems (Turban et al., 2011) that are not sufficiently solvable by human decision-makers due to their complexity or the abundance of information (Valencia-García et al., 2018; Power, 2013, p. 36). Valencia-García et al. (2018, preface) wrote about the benefits of DSS as follows:

> *“Proper application of DSS increases productivity, efficiency and effectiveness and gives many businesses a competitive advantage …”*

In general, productivity, efficiency, and competitiveness are relevant objectives in all sectors of the economy, including agriculture. For example, in crop production, simulation models are often used to evaluate the impacts of different management practices and derive decisions from them. These include, for example, the prominent crop growth models “Agricultural Production Systems sIMulator” (APSIM, Holzworth et al., 2014), “Cropping system simulation model” (CropSyst, Stöckle et al., 2003), and “DSSAT” (Hoogenboom et al., 2019). These examples of DSSs are extensively used worldwide to support decisions in crop production. The list of successful DSSs in this sector is long. Some of them, such as SIMSEPT (Kluge et al., 2006) for predicting *Septoria tritici* and *Septoria nodorum* in wheat and SYMBLIGHT (Kleinhenz et al., 2007) for predicting the first occurrence of *Phytophthora infestans* in potatoes, are used directly by farmers. Other examples of DSS can also be found in the field of fertilization, such as FertiliCalc (Villalobos et al., 2020), Ecofert (Bueno-Delgado et al., 2016), Optifer (Pagán et al., 2015), or “Nutrient Expert for Wheat” (Chuan et al., 2013). The optimization model developed in this thesis also represents a DSS and addresses a problem that affects almost every farm. The tool, which is based on-farm-level conditions, suggests a cost-efficient fertilizer strategy to the farm manager. According to Power (2013, pp. 35–37), DSS can be divided into the following categories:

- Communications-driven DSS
- Data-driven DSS
- Document-driven DSS
- Knowledge-driven DSS
- Model-driven DSS

A model-driven approach is appropriate for the identification of a cost-efficient fertilizer strategy. It focuses on the integrated optimization model, which helps in making rational and efficient decisions.

Despite all the advantages, many DSSs are unable to establish themselves in practice. Rose et al. (2016) considered this observation and analyzed the following requirements that should be met by a proposed DSS for it to be considered useful in practice: low computational costs, high performance, minimum data requirement on the users' part, trust in the developer, and high degree of user-friendliness. During the development and evaluation of the optimization model, these requirements were considered to create the broadest possible acceptance for the new DSS.

2.6 Consideration of farm-specific fertilizer application costs

It is assumed that the application costs of fertilizers influence the cost-efficient fertilizer strategy. However, application costs are considerably farm-specific. They are influenced by several factors, including road network, field structure, mechanization, application rates, and utilization costs of the production factors, among others. In principle, the fertilizer application process is straightforward and can be captured analytically. This process can be further divided into the following subfunctions Baey-Ernsten (2011): setup time, loading time, field work time with turning and loss time, and transport time. These add up to the application costs of fertilization, along with the costs of the production factors.

In the following, the characteristics of the respective subfunctions are only briefly discussed; detailed descriptions are given in Chapter 3. The costs caused by setup times represent a fixed amount per fertilization measure; thus, their share of the costs is not proportional to the application rate. The costs for loading fertilizer can approximately be regarded as proportional to the application rate. The costs of field work consist of the tractor and labor costs associated with field work and the variable costs of the fertilizer spreader (proportional to the application rate). The duration of field work is largely independent of the application rate per hectare due to the automated dosing system of modern fertilizer spreaders. Therefore, it is assumed that tractor and labor costs are proportional to the area being processed. In contrast to the subfunctions of the application costs described thus far, the transport costs cannot be represented without considering the field structure and the road network of a farm. This relationship is explained further using Figure 2-9.

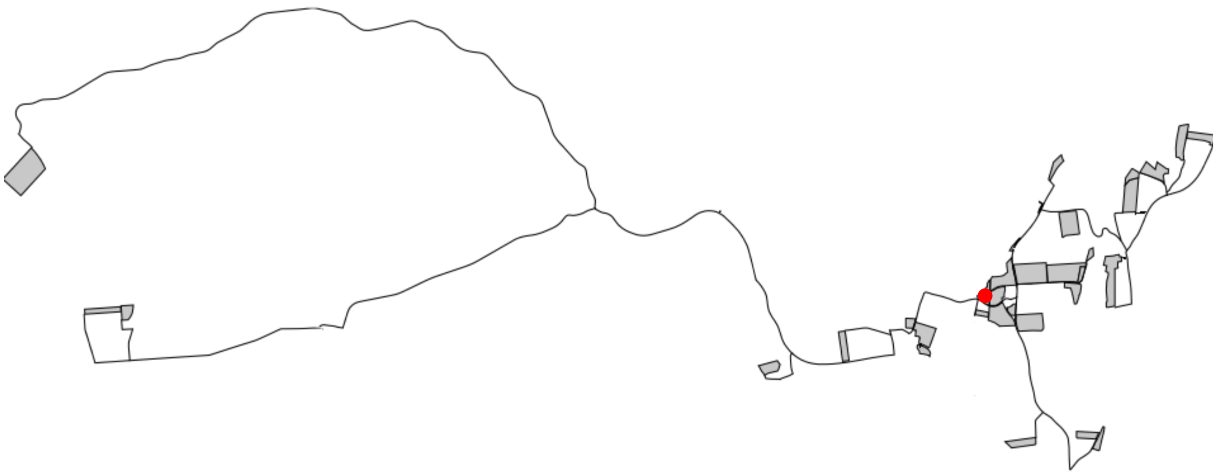


Figure 2-9: Illustration of the field structure and road network of an exemplary farm.

Remarks: The red dot marks the farm location. The figure contains all farm-to-field connections on the shortest route. Created: (Machl, 2018); Data basis: Tröster.

At this point, it is assumed that fertilizing measures are carried out in a single-stage work process, which means that the loading of the fertilizer spreader takes place at the farm location (red dot). Next, let us assume that fertilization is to be performed on eight of the field plots shown in Figure 2-9, for which a total of 2.5 spreader fillings are required. In this case, the farmer has to plan three tours. Furthermore, the farmer will try to keep the transport costs as low as possible; thus, it is also important to minimize the sum of farm–field and field–field trips. At the same time, however, the farmer must ensure two things: (i) that the capacity of the fertilizer spreader is not exceeded in any tour and (ii) the required amount of fertilizer is spread on all field plots at the end. The problem described here is called split delivery vehicle routing problem (SDVRP) (Dror and Trudeau, 1990), in which the the optimal route will change depending on the selection of road network, field structure, field pieces, mechanization, fertilizer and fertilizer rate per hectare. This also affects the ratio of farm-to-field to field-to-field trips.

One way to capture the transportation cost portion of application costs on a farm-by-farm basis is to use the SDVRP. However, it also has disadvantages in the form of high data requirement (all farm–field and field–field routes are needed) and the enormous computational effort needed to solve a SDVRP (Archetti et al., 2011b). Thus, an alternative solution must be found to implement the transportation costs within the framework of a mathematical optimization model. Starting from an existing farm, 125 randomized farms with different road networks and field structures were formed. The SDVRP was applied to this set of randomized farms under different constellations of application rate per hectare and field selection, thereby resulting in a total of 3,500 model runs with optimized routing. Then, in the next step, farm- and measure-specific parameters were identified that have considerable influence on field–field or farm–field trips. Using these parameters, a linear regression function could then be established for both trips, with which the transport cost can ultimately be estimated for each individual farm.

2.7 Verification of a DSS and output analysis

It would be helpful to compare the proposed DSS with the otherwise usual decision-making process to check the performance and usability the former (Taha, 2017, p. 41). This reveals potential differences in solutions, such as those in terms of input, output, or feasibility. This information is crucial for evaluating performance and may also reveal where adjustments are needed to improve feasibility in practice.

This work verifies a proposed DSS, which represents an agricultural production process. This project is particularly challenging due to numerous dynamic influencing factors (e.g., prices and weather) and long production cycles (crop rotation cycle). Therefore, a two-part approach is chosen for the verification: the first part examines the impact on the total cost of fertilization, while the second part addresses the potential agronomic impact of this DSS. This approach facilitates an unbiased comparison between DSS and the typical decision-making process. This is because dynamic influencing factors can be more easily controlled or fixed using the split approach.

The volatile and unknown price development of fertilizers represents a major influencing factor, on the total cost of a multi-year fertilizer strategy. Price information only becomes available successively in actual application scenarios. This situation leads to the fact that fertilizers are used without knowledge of the future price development. This can have both positive and negative effects on the overall costs of the fertilizer strategy. Therefore, to determine the economic performance of the optimization model, a choice experiment was conducted wherein the price development of fertilizers (dynamic influence factor) was fixed for all actors.

Furthermore, the fixed specifications for nutrient input allow the assumption that the fertilizer strategies to be compared do not differ significantly in terms of output. This experiment, hereafter referred to as the “fertilizer quiz,” ensures identical conditions, thus allowing an unbiased view of the total costs of different fertilizer strategies. The participants’ task was to plan fertilization as cost-efficiently as possible for a simplified farm with three field plots and three crops over a three-year period. Table 2-2 provides a reduced representation of the fertilizer quiz, showing one of a total of nine planning segments. The fertilizer quiz itself is available for download[4], and further information can also be found in Chapter 4.

Table 2-2: Excerpt from the fertilizer quiz: Planning segment silage maize 2016.

	2015					2016											
	AugI	SepI	OctI	NovI	DecI	JanII	FebII	MarII	AprII	MayII	JunII	JulII	AugII	SepII	OctII	NovII	DecII
Fertilizer																	
CAN[1] (27N)																	
Urea (46N)									3.1								
…																	
DAP[2] (18N 46P)																	
…																	
Potash (40K 6MgO 5S)																	
Kieserite (25MgO 20S)									2.0								
Burned lime (90CaO)																	

	AugI	SepI	OctI	NovI	DecI	JanII	FebII	MarII	AprII	MayII	JunII	JulII	AugII	SepII	OctII	NovII	DecII
N (minus losses)									126								
P_2O_5																	
K_2O																	
MgO									50								
S									40								
Lime effect (CaO)									−143								

	N	P_2O_5	K_2O	MgO	S	CaO
Actual (in total)		0	0	50		−143
Target (in total)		99	0	85		
Actual (in vegetation)	126	0	0	50	40	
Target (in vegetation)	123	0	0	43	25	
Status	ok	ok	ok	ok	ok	

Remarks: In the first section of the planning matrix, the selection and timing of fertilization measures was performed by specifying the amount of fertilizer in dt per hectare. The associated nutrient quantities are added up on a monthly basis in the middle section. Nutrient quantities in fields with a gray background are not counted. In the lower part of the table, the quiz participant receives a status overview and can determine whether the fertilization planning meets the requirements. Abbreviations: 1: Calcium ammonium nitrate, 2: Diammonium phosphate.

The growing conditions must be controlled in the process of comparing the agronomic performance of a crop production DSS with an otherwise standard decision-making process.

[4] Link to the fertilizer quiz (last access 14.07.2021): https://drive.google.com/file/d/14rBHNKKDuBq8oyeeVUXuek2id1B9z_Dw/view?usp=sharing

Crop rotation, variety choice, crop protection intensity, weather, and location are examples of dynamic influencing factors that should ideally not differ in such a comparison. Therefore, to identify potential agronomic effects, a three-year field trial was conducted on a total of three sites in Bavaria, Southern Germany: Geiselsberg, Triesdorf, and Roggenstein. The experiment was designed as a two-factor, split-plot design. The first factor reflected the fertilizer variant and was kept stationary at the plot level over the entire period. The following fertilizer variants were tested:

- Control (without any fertilization)
- Farm manager- mineral
- Farm manager- mineral and organic (only at the Triesdorf site)
- Optimization model- mineral
- Optimization model- mineral and organic (only at the Triesdorf site)

Factor two represents the cultivated crop. During the trial period, the crop rotation of winter barley, silage maize, and winter wheat was grown once on each plot. During the entire trial period, 297 plots were harvested to determine potential variations in quality or yield. Statistical analysis was performed using slit-plot ANOVA and subsequent post-hoc tests. Figure 2-10 presents a visual impression of the field trial.

Figure 2-10: Field trial at the Triesdorf site in the spring of 2016.

Remarks: Winter wheat in the foreground, winter barley in the middle, and silage maize in the background shortly after emergence.

Then, the fertilizer quiz was used once more to determine the characteristics of cost-efficient fertilizer strategies. Following the detailed processing of the data obtained from this quiz, it was possible to describe each individual fertilizer strategy with 675 variables. In a trial and error procedure, linear regression analysis was used to identify variables showing significant influences on the proposed fertilizer strategy's cost-efficiency. In addition, the fertilizer strategies of the quiz participants were divided into three clusters based on their total costs, after which post-hoc tests were used to identify group differences in fertilizer use. Next, the developed DSS was used as a simulation model to determine the influence of farm conditions on fertilizer strategy. Based on the conditions of an actual farm, certain parameters were changed, such as farm size, infrastructure, soil fertility, and the availability of organic fertilizers according to the *ceteris paribus* principle to calculate the adapted fertilizer strategies. By comparing these solutions, the influence of the tested parameters on the fertilization strategy could thus be determined.

Part II

Contributing manuscripts

3 Effects of application costs on fertilizer application strategy

This is a pre-copyedited, author-produced version of an article published in Computers and Electronics in Agriculture following peer review. The version of record [Michael F. Tröster, Hubert Pahl and Johannes Sauer (2019): Effects of application costs on fertilizer application strategy. *Computers and Electronics in Agriculture* 167] is available online at: https://doi.org/10.1016/j.compag.2019.105033

Authors' contributions: Michael Tröster is the main author of this contribution. Michael Tröster developed the research question, designed and conducted the analysis and wrote the manuscript. Hubert Pahl and Johannes Sauer contributed to reviewing and editing of the manuscript. Johannes Sauer provided supervision and software resources and was helpful in discussing the results. The authors would like to thank the editor and two anonymous referees for their useful comments.

Abstract

To optimize production activities, it is important to understand the associated costs. If the optimization is carried out using mathematical instruments, the production costs are implemented in the form of a restriction. The functional form is critical not only to ensure accuracy but also to facilitate computing power and input data requirements. The present study documents the development of a cost function for fertilizer application. Three potential ways to address transportation costs within the whole cost function are observed: (i) calculating minimal transportation time using a "split delivery vehicle routing problem" (SDVRP), (ii) estimating transportation time using a regression model, and (iii) neglecting transport costs altogether. In Section 3.3, the costs of fertilizer application and their influence on the fertilizer application strategy are compared. Despite minimal differences in the application cost values, all methods lead to comparable results. A further investigation reveals additional factors that influence the reliability of decision-making, for example, price relations. The computational power and data input demands are explored as well. In this respect, the SDVRP method was identified as the most resource-demanding option. We conclude that the performance of the regression method is the most reliable for optimizing the fertilizer application strategy using mathematical instruments. The present study may support researchers focusing on farm logistics or related cost functions, such as spraying, sowing or manure application.

Keywords

SDVRP; application costs; transport costs; route-planning; fertilizer strategy

3.1 Introduction

The complexity of decision-making in management and production contexts is a key challenge for farmers all over the world. Decisions often depend on many variables and decisions influence the level of success or failure. Decision support systems (DSS) are valuable instruments that facilitate decision-making based on broad and objective criteria. Their application is influenced by performance, data requirements, user-friendliness as well as other limitations such as computing power (vgl. Rose et al., 2016). Sensor-based determination of crop nitrogen demand (Fitzgerald et al., 2010), genomic selection in animal breeding (VanRaden, 2008) and computing of feed rations with the least costs (Waugh, 1951) are examples of established applications of DSS in farm management. In particular, we are interested in minimizing feed costs and adapting the idea to plant nutrition requirements. Our research contributes to technical and allocative productivity in land management and facilitates the formulation of farm-specific optimal fertilizer application strategies.

The process of optimizing a farm's fertilizer strategy begins with the optimal nutrient input requirements within a season or an entire rotation period and the selection of suitable fertilizers. The costs of fertilizer application greatly affect the optimal strategy. For example, fertilizer application strategies that meet the nutrient requirements of crops with the least possible number of doses are preferred when the costs of fertilizer application are high. This highlights the significance of application costs. Transport costs are part of the application costs. They are highly influenced by farm infrastructure. A common approach for accounting for transport costs is the use of mean on-farm transportation time (KTBL, 2019). Transport costs could be addressed using route-planning software, which have the ability to calculate detailed transport costs. Some examples can be found in transport-intensive agricultural sectors, including the dairy industry (Basnet et al., 1996). However, route planning is very resource-intensive. Thus, a more straightforward approach for addressing on-farm transport costs during fertilizer application is required. It is also necessary to evaluate the usability based on mathematical optimization models. The hypothesis is that using farm-specific data on infrastructure to estimate transport costs rather than integrating them in route-planning software would not significantly affect the fertilizer application strategy. Under certain conditions, we expect that disregarding transport costs does not significantly affect the optimal solution.

The present study evaluates the importance of transport costs and develops a cost function for fertilizer application (or other production inputs). This function must be appropriate for a DSS at farm level within the framework of mathematical optimization methods. This means it is (i) sufficiently precise to obtain a reliable decision on fertilizer application strategy, (ii) resource-friendly in terms of computational power and (iii) meets minimal requirements of farm-specific data. To address the problem, we compared three approaches for determining transport costs. First, we generated a cost function that takes into account a split delivery vehicle routing problem ("SDVRP method"). Second, we replaced route planning with a regression model ("regression method"). Finally, we included a scenario in which the transport costs are disregarded entirely ("zero transport cost method").

To date, the SDVRP method has rarely been used to optimize agricultural production. Vougioukas et al. (2012) used the SDVRP method to minimize in-field transportation time of robotic crop-transport aids. Frisk et al. (2018) used it as an instrument for optimizing animal welfare and economic costs in pre-slaughter logistics. However, within the transportation science and logistics, SDVRP is recognized and widely applied (see Latiffianti et al., 2018; Eldrandaly and Abdallah, 2012). In our application, we expected that the SDVRP method would be useful due to its potential to precisely compute transport time. However, its application is limited due to its computing power demands. The other two approaches were deemed more appropriate within the framework of mathematical optimization methods. The aim of our study is to identify an appropriate approach to address transport costs within the comprehensive optimization of fertilizer application strategies at farm level.

3.2 Materials and methods

The subject of our study is a working procedure for mineral fertilizer application, in which both fertilizer application and transportation occur in one combined step.

Cost functions of complex procedures are often analyzed using regression models. An overview of the methods for determining cost functions can be found in Adnan and Jian (2006). Known costs and less complex working procedures, as in the case of fertilizer application, are therefore preferably determined analytically. Moreover, there is a possibility to break down the overall function into individual quantifiable subfunctions. A description of the method has been provided by (Baey-Ernsten, 2011). Accordingly, the required working time can be divided into setup time; fieldwork time, including turning and lost time; loading time; and transport time. In addition to the variable costs of mechanization and based on one's own analysis of the procedures in practice, the following subfunctions emerge:

- CWP: Work preparation, respective follow-up work (setup time)
- CLS: Loading the fertilizer spreader (loading)
- CFW: Cost of completion of fieldwork (fieldwork)
- CVS: Variable costs of the fertilizer spreader (fieldwork)
- CYF: Transport costs from farmyard to field (transport)
- CFF: Transport costs from field to field (transport)

If the factors and conditions of the farm remain unchanged, only variable costs are important for the decision-making. Therefore, the fixed costs are not considered. To evaluate the user-dependent costs in the subfunctions, sufficient knowledge of individual workflows in a farm and associated costs are required. For this purpose, primary data of a sample farm are used, which were supported by secondary data (KTBL, 2016, 2005). The sample farm is a northern Bavarian mixed farm with 50 ha of arable land and mechanization as indicated (see Appendix, Table A 3-1). The in-house infrastructure presented by the farm-specific distance table (*Ci*) has a major influence on transport costs. The distances are based on the shortest distances in terms of time (min). The advantage of a time-based specification is related to the simultaneous consideration of track quality, distance, and maximum speed of the transportation unit.

3.2.1 Breakdown of the cost function into quantifiable subfunctions

Fertilizer application costs are related to the number of applied spreader fillings (*asf*):

$$asf\left(hf, ar\right) = \frac{hf \times ar}{SW \times Q} \tag{3-1}$$

The number of applied spreader fillings is determined by hectares fertilized (*hf*), application rate (*ar*), spreader capacity (*Q*), and the specific fertilizer weight (*SW*). The specific weight ranges from 75 to 170 kg hl^{-1}. Therefore, the fertilizer itself influences the required number of spreader fillings. Application cost needs to be expressed in Euros (€) per hl. Although fertilizer costs are based on a mass unit, here they are expressed in Euros per 100 kg. To avoid possible confusion, we assume a specific weight of 100 kg per hl. This is justifiable because we just look for the general structure of application costs at this stage and neglect the optimal selection of fertilizers.

Eq. (3-2) presents the personnel costs for the preparation and follow-up work of a fertilization measure (were Setup time = *ST* and Wage entitlement = *WE*).

$$CWP = ST \times WE \tag{3-2}$$

Eq. (3-3) determines the costs for loading the fertilizer spreader (were Loading time = *LT* and Variable costs of the vehicle used for loading = *VL*).

$$CLS\left(hf, ar\right) = asf \times LT \times \left(VL + WE\right) \tag{3-3}$$

Eq. (3-4) shows the cost of the work completion in the field. It is assumed that fertilizer application rate (*ar*) and working speed (*WS*) are independent, which leads to a constant work time per hectare. This reflects the current status of fertilizer spreader technology. For nonproductive turning and lost times, the theoretical rate of work is reduced by *TT*. Fechner (2014) found that *TT* depends on various factors like shape and size of the field, agility of the machine and machining direction. According to him *TT* ranges from 8% to 27%. Identifying field individual values for *TT* takes a lot of effort with little effect on the total application costs. Thus, we decided to use a uniform value of 20% for *TT*. (Working width = *WW*; Variable cost of tractor = *VT*).

$$CFW\left(hf\right) = \frac{hf}{\left[WS \times 1000 \times WW \times \left(1 - TT\right) \times 10000^{-1}\right]} \times \left(VT + WE\right) \tag{3-4}$$

The total variable costs of the fertilizer spreader are described using Eq. (3-5). (Variable cost of fertilizer spreader = *VS*).

$$CVS\left(hf, ar\right) = asf \times Q \times VS \tag{3-5}$$

In Eqs. (3-6) and (3-7), the transport costs are presented as dependent functions of the unknowns *tx* and *ty*. They correspond to the transport times of farm–field (*tx*) or field–field trips (*ty*).

$$CYF\left(tx\right) = tx \times \left(VT + WE\right) \tag{3-6}$$

$$CFF\left(ty\right) = ty \times \left(VT + WE\right) \tag{3-7}$$

For additional information, see Appendix, Table A 3-1.

3.2.2 SDVRP model for transport time determination

It is a logistic challenge to find the most time-saving route that accounts for in-house infrastructure, application rate and spreader capacity. To address this challenge, route planning has to consider the option of multiple runs to fields. The logistic challenge corresponds to the SDVRP, which was first formulated by Dror and Trudeau (1990). It is a variant of the vehicle routing problem, which is derived from Dantzig and Ramser (1959). Such kinds of problems are often NP-hard, which means they are not solvable in polynomial time. Investigations for this can be found in a study by Archetti et al. (2011b, S 748). NP-hard problems place particularly high demands on computing power and cannot be solved satisfactorily if problem sizes increase. Using the General Algebraic Modeling System (GAMS) (GAMS Development Corporation, 2016), an appropriate SDVRP optimization model was developed to address such logistic problems. It is a mixed integer programming model, which is solved using the commercial solver CPLEX (IBM Corporation, 2017) (see Appendix, Table A 3-2 for further information on the SDVRP model).

We used the route-planning model to determine the transport times for *tx* and *ty*. After determining the values, Eqs. (3-6) and (3-7) are incorporated unaltered into the cost function:

$$cf\left(hf, ar\right) = CWP + CLS + CFW + CVS + CYF + CFF \quad (3\text{-}8)$$

3.2.3 Importance of transport costs with regard to the optimal fertilizer strategy

To reveal the significance of transport costs, we need to apply Eq. (3-8) under several different conditions. In addition to a varying spread rate and a varying number of fertilized sites, we need to account for farm-specific infrastructure, such as plot sizes and distances. However, detailed empirical data on farm infrastructure for running multiple SDVRPs for a representative set of farms is unavailable. To use an SDVRP, we need to know all distances between all fields of a farm, which is a vast amount of information. Machl et al. (2016) developed a GIS-based instrument for calculating the shortest farm–field connections in all Bavarian farms. Modifying the tool to calculate field–field distances can help in addressing information gap in the future; however, since the necessary data contained sensitive geocoded information, access was denied. This is why we used the so-called informed guess: We examined available data for a single farm to generate a random set of 125 farms. Within a bootstrapping procedure, the route optimization model was used in 125 independent runs. At the beginning of each run, both the sizes of the field and the farm–field or field–field distances (parameter c_{jk}) were defined based on triangle-distributed random numbers[5]. In this way, we randomly generated 125 farms, for which the influence of a varying spread rate (10, 80, 200, and 320 kg ha^{-1}) was investigated. Optimal routes were calculated for each of the four fertilizer levels, taking into account different field combinations:

- All fields in the farm
- All cereal fields with wheat and barley (6 from 9 fields)
- All fields with wheat and corn (6 from 9 fields)
- All fields with barley and corn (6 from 9 fields)
- All fields with wheat or barley, or corn (respectively 3 from 9 fields)

There was a total of 3,500 model runs (possible number of problem combinations). Since the SDVRP model was not always capable of finding an optimal route, we set a time limit of 120 s per run. This time limit prevents the solver from endless iterations. Farms with no defined optimal route within this time limit were excluded from further analyses. As a criterion, a relative deviation between the objective value (*obj*) and the best possible result[6] of more than 7% was selected. Overall, 55 companies exhibited a relative deviation of more than 7%. From the remaining 70 random farms, there were 1960 model runs, which revealed optimized routes. In addition to the time required for the entire overall route (*obj*), the results also include the time required for the sum of all farm–field (*tx*) and field–field trips (*ty*). Based on such knowledge on transport time, we are able to evaluate the significance of transport costs. Table 3-1 compares the proportions of the application costs (*cf*) for all components of Eq. (3-8) in consideration of the fertilizer costs.

[5] For the distance Parameter (c_{jk}), we used 0 min as the minimum, 4.5 min as the median, and 32.3 min as the maximum. The plot size (Parameter ha_j) has the following specifications: minimum 0.1, median 1.4, and maximum 7.4 ha.

[6] The Solver CPLEX was set to a 120-s time limit. After the time has expired, the solver returns the best found result, together with the relative deviation, to the best possible result “relative gap”.

Table 3-1: Fertilizer application costs.

Fertilized fields [pcs] or [%]	Application rate (ar) [kg ha^{-1}]	(1) Fertilizer [%]	(2) CWP [%]	(3) CLS [%]	(4) CFW [%]	(5) CVS [%]	(6) CYF [%]	(7) CFF [%]
1 Field	80	64.3**	21.2**	1.0	6.1	0.0	7.4**	0.0
	200	80.4**	11.2**	1.2	3.1	0.1	4.0*	0.0
	320	85.8**	7.6**	1.3	2.0	0.1	3.2*	0.0
33% of all Fields	80	78.8*	7.4*	1.2	7.5	0.1	2.5*	2.6*
	200	88.7*	3.3*	1.4	3.4	0.1	2.4	0.7
	320	91.5*	2.2	1.4	2.2	0.1	2.2	0.5
67% of all Fields	80	82.8*	3.7	1.3	7.9	0.1	2.2	2.1
	200	90.7	1.6	1.4	3.4	0.1	2.0	0.8
	320	92.8	1.0	1.4	2.2	0.1	2.0	0.5
100% of all Fields	80	84.4	2.5	1.3	8.0	0.1	1.7	2.1
	200	91.3	1.1	1.4	3.5	0.1	1.8	0.8
	320	93.2	0.7	1.4	2.2	0.1	1.9	0.5

Note: The percentages are mean values of 70 randomly created farms, which differ in plot size and distances. The fertilizer price is €21.50 per 100 kg. ** Range > 20%; * Range > 5%. Source: Own compilation.

Table 3-1 is useful for determining the major drivers of application costs. In addition to the fertilizer cost itself, *CFW* (costs for completion of fieldwork) dominate the cost-composition. This is followed by *CWP* (costs for work preparation) and transport costs if we sum up the corresponding columns 6 and 7. Consequently, we are able to highlight scenarios where transport costs could have a real impact on the optimal fertilizer application strategy. A threshold of more than 3.1% on total fertilization costs is exceeded in all scenarios where only a single field is fertilized and in the scenarios with a low application rate of 80 kg ha^{-1}. Some mean values of Table 3-1 are labeled with an asterisk. A double asterisk denotes a range of more than 20%, and a single asterisk indicates a range of more than 5% in the original data. The relevance of farm–field transport costs (*CYF*) increased considerably in individual cases. A detailed look at the data reveals that *CYF* is of great importance when only a single small and far-flung field is involved. Since we are dealing with percentage values, the fertilizer prices have a major impact on the contents of Table 3-1. We used a fertilizer price of €21.5 per 100 kg. A higher fertilizer price would minimize the impact of the application cost in general, and vice versa. This will be covered separately.

3.2.4 Estimation of field–field and farm–field transport time

We need to assume that transport time has an effect on the optimal fertilizer application strategy. Instead of using an SDVRP, we are now looking for a straightforward approach for estimating transport time based on farm-specific information.

Estimating "*ty*"

Based on the 70 model runs with a low application rate of 10 kg ha^{-1}, we obtain the shortest connection among all the fields in each farm[7]. This farm-specific value for *ty* was set for each random farm to be equivalent to 100%. Therefore, it is possible to express the proportions of field–field trips (*rty*) for all the investigated variations of a random farm. In case none of the fields or one field is fertilized, there are no field–field transport trips. The proportion of field–field trips is therefore equal to 0%. In Figure 3-1(a-d), the proportional field–field trips are depicted in relation to the application rate and the proportions of the fertilized farmlands. With regard to the proportion of field–field trips, three influential factors can be identified: (i) field–field trips can only occur when more than one field is fertilized, (ii) a higher share of fertilized farmland leads to an increase in field–field trips, and (iii) field–field trips display a wider dispersion and slightly decrease with increase in fertilizer application rate. To examine correlations based on a regression analysis, a dataset was formed using the following information: proportions of field–field trips, proportions of fertilized farmland, spreader fillings per hectare and number of fertilized fields. Since the dataset was not equally weighted with regard to the independent variable "number of visited fields", this was corrected by the duplication of underrepresented cases. In sum, the dataset contained 4,200 observations[8].

[7] For a total farm size of 50 ha and a spreader capacity of 2000 L, only 0.25 spreader fillings are required at this application rate. Therefore, the optimal route does not include additional farm–field trips.

[8] Explanation: 70 farms under four different levels of fertilization, each with three results for the fertilization of a combination of nine fields, six fields, three fields, one field, and zero fields.

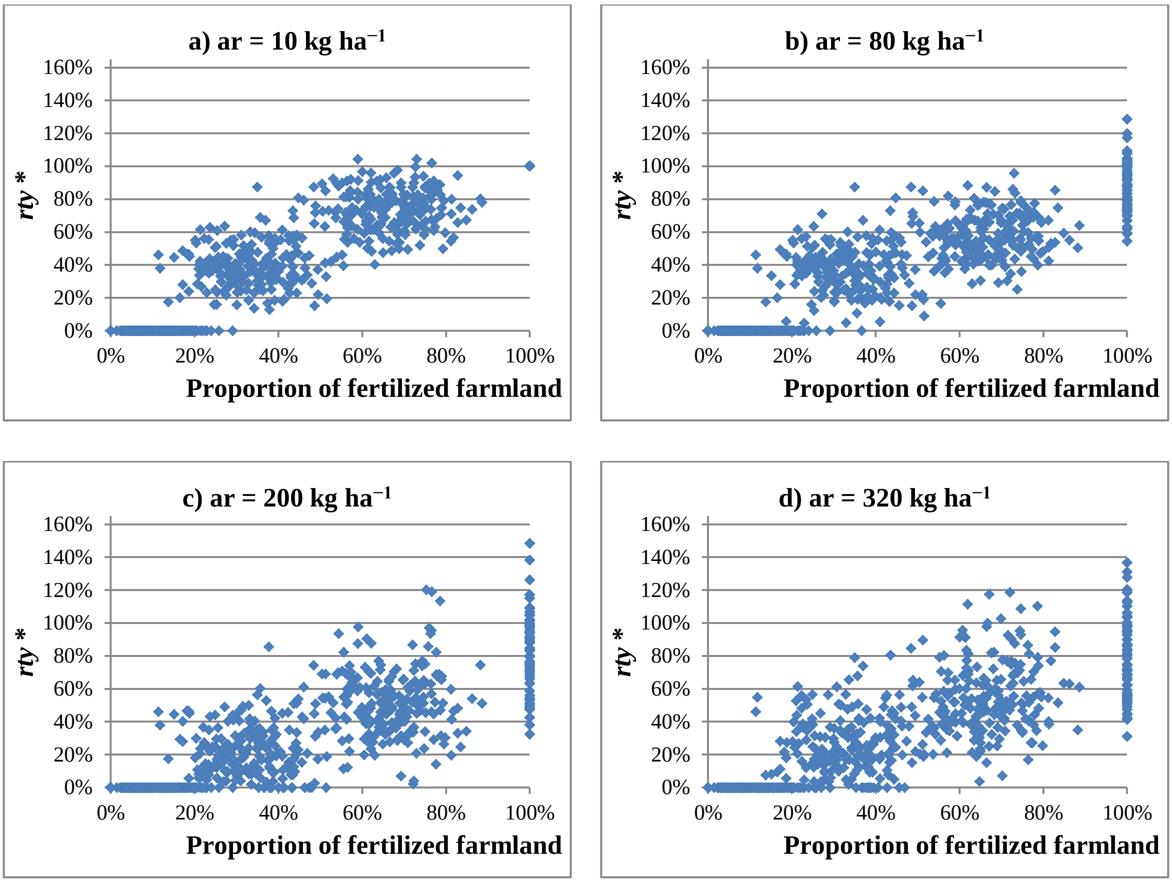


Figure 3-1(a-d): Proportions of field–field trips (*rty) at the respective farm-specific optimal routes depending on the proportion of fertilized farmland.

Figure 3-1(a–d) leads to the assumption of a linear relationship between the proportions of field–field trips and fertilized farmland. In addition, we recognize a lower effect of the application rate. The application rate is relevant for the number of spreader fillings per hectare. This shows how often a field has to be visited. To account for such observations, the following regression model was developed to estimate the transport times required for field–field trips (*rty*):

$$rty_s = \beta_1\left(\frac{hf_s}{\sum_i HA_i}\right) + \beta_2\left(\frac{asf_s}{hf_s}\right) + \varepsilon_s \qquad s \in i = 1,\ldots n; hf_s > 0 \qquad (3\text{-}9)$$

Index "*s*" indicates a dynamic set of scenarios consisting of fertilized fields in combination with an application rate. The dependent variable is determined by the proportion of the fertilized farmland (first term) and the number of spreader fillings per hectare. Since field–field trips only occur when more than one field is involved, a binary variable would be necessary to account for that. Binary variables may considerably complicate mathematical optimization problems. As this was our main application, we decided to disregard the binary variable. The consequence was only a minor decrease in the coefficient of determination. The results of the regression model are presented in Table 3-2.

Table 3-2: Estimation of field–field trips for varying fertilization measures.

Model	**OLS**			
Proportion of field–field trips	rty_s	(Dependent variable)		
Proportion of fertilized farmland	β1	0.9418***	SE	(0.0056)
Spreader fillings per hectare	β2	–0.7298***	SE	(0.0366)
R^2		0.91	n	4,200

Remarks: *** $p < 0.001$; SE: Standard error; n: Investigations; no constant term. Source: Own compilation.

The regression model according to Eq. (3-9) describes the dependent variable rty_s. As expected, the proportion of fertilized farmland has a positive impact on the number of field–field trips required. An increase in the number of spreader fillings per hectare results in a slight decrease in the number of field–field trips. The reason is that direct farm–field trips are more frequent. Using the regression method, we only obtain the proportion of field–field trips based on a farm-specific parameter (*TF*). *TF* is the sum of all field–field trips on an optimal route, and rty_s is equal to 100% in such a case. To estimate the transport durations during field–field trips in any farm, *TF* must be known. To be precise, the value would have to be calculated using a tour-planning model. In practice, the value is estimated based on the experiences of a farm manager. The following applies for the duration of field–field trips:

$$ty_s = rty_s \times TF \qquad s = 1, \ldots n \qquad (3\text{-}10)$$

Estimating *tx*

The estimation of the time spent on farm–field trips, tx_s, is based on the average farm–field distance (FY_s). FY_s is based on a farm–field distance table FD_i and additionally weighted by the plot sizes (HA_i) of the associated fields:

$$FY_s = \frac{\sum_s \left(FD_i \times HA_i\right)}{\sum_s HA_i} \qquad s \in i = 1, \ldots n \qquad (3\text{-}11)$$

Supplying large fields with production inputs requires more transport trips between farms and fields. *FY* is doubled and multiplied by the number of necessary spreader fillings to obtain the transport time *tx*. The regression model for transport time is presented in Eq. (3-12):

$$tx_s = \beta_1 \times asf_s \times 2 \times FY_s + \varepsilon_s \qquad s = 1,\ldots n \qquad (3\text{-}12)$$

This simple model is quite useful for estimating the time requirement for farm–field trips. Table 3-3 shows the statistic.

Table 3-3: Estimation of farm–field trips for varying fertilization measures.

Model	**OLS**			
Time for farm–field trips	tx_s	(Dependent variable)		
$asf_s \times 2 \times FY_s$	β1	0.9961***	SE	(0.0032)
R²	0.96	n	4,200	

Remarks: *** $p < 0.001$; SE: Standard error; n: Investigations; no constant term. Source: Own compilation.

In reality, the applied spreader fillings (*asf*) have to be an integer number calculated for each field separately. Even if a full spreader filling for one field is not required, it is necessary to visit the field at least once. This can happen either through a direct farm–field trip or indirectly by a field–field trip. Using *asf* as a continuous variable (as in the present case) would lead to the underestimation of transport time, particularly if far-flung fields are involved. The coefficient of determination is rarely influenced. Therefore, we are able to disregard integer variables, which would pay off when we apply the regression method in the context of a mathematical optimization model.

3.2.5 Zero transport costs

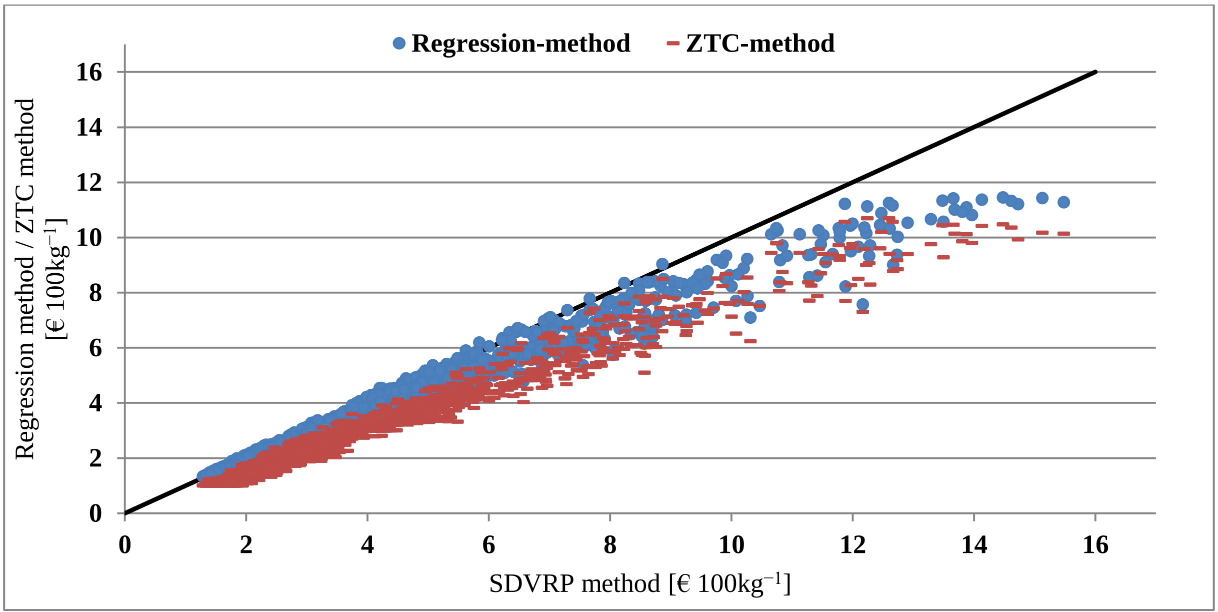
Another potential way of dealing with the transport costs is to neglect them. The analysis in Table 3-1 shows that transport costs are of little importance, at least in most of the observed scenarios. Therefore, a meaningful impact on the fertilizer strategy is debatable. Excluding the transport costs facilitates further work in two ways: there is no need to consider detailed information on farm infrastructure, and the functional aspects of the application costs become easier to determine with regard to computational power. However, under unfavorable conditions, such an approach could lead to a suboptimal fertilizer application strategy.

In the following section, we compare the three designated options and check the robustness of their results.

3.3 Results

The results of the present study show the influence of the different approaches on the total cost of fertilizer application and the optimal fertilizer application strategy. Furthermore, we address their potential application within a DSS.

The three approaches are as follows: the first variant, using the logistic optimization model, is "SDVRP method" (see Section 3.2.2), the second variant is "regression method" (see Section 3.2.4), and the third variant, which disregards the transport costs, is "zero transport cost method" or abbreviated as "ZTC method" (see Section 3.2.5). The total costs of fertilizer application are calculated using Eq. (3-8) and are presented in Figure 3-2.



Remarks: Results calculated at an application rate lower than or equal to 10 kg ha−1, and scenarios with a total fertilizer amount of less than 15% of the spreader capacity are omitted due to their insignificance for praxis.

Figure 3-2: Total application costs: Regression method and ZTC method versus SDVRP method.

The x-axis of Figure 3-2 shows the application costs for 100 kg fertilizer based on the SDVRP method. The values calculated based on the regression method (dots) and the ZTC method (lines) are on the y-axis. The regression method underestimates the fertilizer application costs in the upper range, which has already been explained at the end of Section 3.2.4. Of course, the ZTC method underestimates the fertilizer application costs in general, and the degree of underestimation increases as the application costs increase. When fertilizer application costs are greater than €8 per 100 kg, the validity of the regression method and the ZTC method deteriorates. This is often consistent in situations where only one far-flung field is fertilized.

In such cases, transport costs are considerably affected which is not adequately considered by the regression method and failed by the ZTC method. Based on the data presented in Figure 3-2, we obtain a standard deviation of €0.49 100 kg^{-1} for the residues of the SDVRP method compared with those of the regression method. Table 3-4 compares the total fertilization costs of the three methods applied on a test nitrogen fertilizer application scenario.

Table 3-4: Influence of application costs on a test nitrogen fertilizer strategy.

		(1)	(2)	(3)	(4)	(5)	(6)
Method		SDVRP		Regression		ZTC	
1. Dose	kg ha^{-1}	200	320	200	320	200	320
2. Dose	kg ha^{-1}	200		200		200	
3. Dose	kg ha^{-1}	200	320	200	320	200	320
Fertilizer Price	€ 100kg^{-1}			21.50			
Application costs	€ 100kg^{-1}	2.19	1.66	2.19+σ	1.66-σ	1.78	1.24
Total costs	€ ha^{-1}	**142.15**	148.22			**139.68**	145.55
+ σ				145.10			
- σ					**145.06**		

Remarks: σ = A standard deviation of €0.49 100 kg^{-1}. Source: Own compilation

In the first two columns, two potential fertilizer application strategies are tested using the SDVRP method. In option 1, the nitrogen fertilizer application is divided into three equal doses, each containing 200 kg ha^{-1} calcareous ammonium nitrate (CAN). The application costs according to the SDVRP method are €2.19 per 100 kg. In option 2, a total of 640 kg ha^{-1} CAN is applied in two equal doses. Here, 6.7% more nitrogen is applied as fertilizer to compensate for losses caused by the stronger aggregation of fertilization. This is because an aggregation of nitrogen fertilization increases the risk of losses. Nevertheless, the additional amount of 6.7% is based on an expert opinion. Therefore, we tested different scenarios to adjust for local conditions. Even if the application costs of option 2 are considerably lower, at €1.66 per 100 kg, option 1 is the preferred fertilizer strategy due to the lower total costs of €142.15 per ha. Both fertilizer application strategies are also compared using the regression method (columns 3 and 4) as well as the ZTC method (columns 5 and 6). To simulate potential deviations in the estimation of application costs using the regression method, we increased the application costs of the initial cheaper strategy (option 1) by a standard deviation of €0.49 per 100 kg^{-1}. In addition, we reduced the application cost of Option 2 by one standard deviation. The ZTC method does not include transport costs. The manipulation of the application costs in the case of the regression method leads to a change in fertilizer application strategy, even if the cost difference per hectare is just only cents. In contrast, neglecting the transport costs completely does not influence the fertilizer application strategy (please compare the total costs in Table 3-4). In Table 3-5 the influence of application costs on the strategy of basic fertilization is analyzed. The structure is similar with the aforementioned example (Table 3-4).

Table 3-5: Influence of application costs on a basic fertilizing strategy.

		(1)	(2)	(3)	(4)	(5)	(6)
Method		SDVRP		Regression		ZTC	
PK (16 + 16) (25 €100kg^{-1})	kg ha^{-1}	320		320		320	
Potash 40 (€27.50 100kg^{-1})	kg ha^{-1}		128		128		128
Triple superphos 46 (€40.50 100kg^{-1})	kg ha^{-1}		111		111		111
Potash	∑ kg ha^{-1}	51.2	51.2	51.2	51.2	51.2	51.2
Phosphorus	∑ kg ha^{-1}	51.2	51.2	51.2	51.2	51.2	51.2
Application costs*	€ 100kg^{-1}	1.66		1.66+ σ		1,24	
Application costs**	€ 100kg^{-1}		3.32		3.32–σ		2.59
Appl. costs***	€ 100kg^{-1}		3.75		3.75–σ		2.92
Total costs	€ ha^{-1}	**83.31**	88.70			**83.97**	86.85
+ σ	€ ha^{-1}			**86.89**			
- σ	€ ha^{-1}				87.52		

Remarks: The application costs are listed separately for the different fertilizer quantities: * 320 kg ha^{-1}, ** 128 kg ha^{-1}, *** 111 kg ha^{-1}. The prices of fertilizers are placed in the brackets (January 2017). σ = A standard deviation of €0.49 100 kg^{-1}. Source: Own compilation

It is possible to apply phosphorus and potash with a compound fertilizer (columns 1, 3, and 5). Alternatively, both nutrients can be applied separately (columns 2, 4, and 6). The SDVRP solution shows that the application of the compound fertilizer is preferable. This solution remains unchanged for the other two methods, and the optimal fertilization strategy remains the same.

To check whether the results of the regression method and ZTC method are robust, a Monte Carlo simulation (MCS) was carried out. For that purpose, we changed the fertilizer application costs by adding normally distributed random numbers with a mean value of zero and a standard deviation of €0.49 100 kg^{-1}. Compared to the original fertilizer strategy from the SDVRP method, the results remained unchanged in more than 92% of cases for nitrogen application. For basic fertilizer application, the results remained unchanged in more than 97% of cases. The results of this simulation depended heavily on (i) the additional amounts of nitrogen used in the case of the stronger aggregation of nitrogen fertilizer strategy and (ii) the relationships among the prices of the different basic fertilizers. We analyzed the relationships using the aforementioned MCS in combination with a stepwise increase in the nitrogen level or a modified price for the fertilizer “PK (16 + 16)” (see Table 3-6 and Table 3-7).

Table 3-6: Reliability of decision-making for a nitrogen fertilizer strategy.

Additional nitrogen	Regression method	ZTC method	Financial error
[%]	Consistent decisions [%]		[€ ha^{-1}]
0.2%	76.2	100.0	3.0
1.2%	63.7	100.0	1.5
2.3%	50.6	0.0	0.0
3.4%	63.9	100.0	1.6
4.5%	75.9	100.0	3.1
5.6%	85.6	100.0	4.6
6.7%	**92.0**	**100.0**	**6.1**
7.8%	95.7	100.0	7.6

Remarks: Comparison of double- and triple-stage nitrogen nutrition. The first column indicates the add-on of nitrogen to balance nitrogen losses caused by stronger aggregated nitrogen nutrition. Source: Own compilation

The starting point for the analysis in Table 3-6 (in bold) is equal to the additional nitrogen supply for a double-stage nitrogen fertilizer strategy used in Table 3-4. At this point, we have a high reliability in the decision-making for both methods. The decisions of the regression method in comparison with the original fertilizer strategy from the SDVRP method would be equal in 92.0% of all cases. We simply call this “consistent decisions”. If the necessary nitrogen add-on would be 2.3%, the regression method would yield 50.6% consistent decisions, whereas the ZTC method would yield 100% inconsistent decisions. The last column shows the financial error triggered by a wrong decision. With an additional nitrogen supply of 2.3%, the costs for the extra fertilizer are equal to the cost savings due to a cheaper fertilizer application. At this stage, both alternative fertilizer strategies (see Table 3-4) would result in the same total costs for fertilizer and fertilizer application: Option 1 has lower cost for the fertilizer but higher costs for application. In option 2 this is reversed. The financial error is zero, because both options lead to the same costs. In Table 3-6, the probability of a wrong decision is highly correlated with a small financial error and, therefore, is acceptable from a management perspective.

Table 3-7: Reliability in decision-making for a basic fertilizer strategy.

Fertilizer price	Regression method	ZTC method	Financial error
[€ 100kg^{-1}]	Consistent decisions [%]		[€ ha^{-1}]
25.0	**97.2**	**100.0**	**3.4**
25.3	91.1	100.0	2.4
25.6	79.6	100.0	1.5
25.9	61.6	0.0	0.5
26.2	59.7	100.0	0.5
26.5	78.0	100.0	1.4
26.8	90.7	100.0	2.4
27.1	97.2	100.0	3.3

Remarks: Comparison of single and compound basic fertilizer strategy. The first column indicates the price for the compound fertilizer. Other prices remain steady. Source: Own compilation

The optimal fertilizer application strategy is often a case of selecting a fertilizer among various substitutable fertilizers. In such a case, the driving factor would be the relative price distance with regard to the nutrient content. In Table 3-7, we increased the price of “PK (16 + 16)” while the prices of the substitutable fertilizers remained steady. The result and interpretation are similar to those of the aforementioned example.

3.4 Discussion

The results of the present study indicate that the described methods lead to comparable results. The method has a minimal impact on the selection of the optimal fertilizer application strategy. The financial error within the nitrogen fertilizer example ranges from €0.0 up to €7.6 per hectare (see Table 3-6). The error within the basic fertilizer example is between €0.0 and €3.4 per hectare (see Table 3-7). In case of a wrong decision caused by the regression method or ZTC method, the financial error is low. Under the considered circumstances, the ZTC method has an extremely narrow error margin. Within the margin, the decisions are 100% incorrect (with low financial damages) and otherwise 100% correct. Under conditions of a higher significance of the transport costs, the error margin of the ZTC method would increase. In that respect, the regression method is more robust.

The ZTC method is a very simple approach for decision-making. It proves useful as long as the variable transport costs are small and the farm–field distances are homogenous. However, when using the method within a DSS, there is no information about the distance between the farm and the corresponding field or set of fields. Therefore, the solver cannot distinguish between near and far-flung fields. This has a significant disadvantage as the field-specific transport costs are unobserved and we disregard a major driver for a field-specific fertilizer strategy. We cannot opt for a differentiated solution such as spreading a more expensive compound fertilizer to the far-flung fields (saving transport costs) and using cheaper single nutrient fertilizers on the nearer fields (saving fertilizer expenses). The regression method does not hold back this field-specific information completely. As soon as a field is selected for a fertilization approach, its farm–field distance and its plot size are used to calculate the weighted farm–field distances (see Eq. (3-11)). Differentiated solutions, therefore, are possible, but they would not be applied often, since a fixed setup time influences entry costs for each separate fertilization measure. The flexibility of the regression method in considering a wider range of farm-specific situations compelled us to apply it within the context of a DSS.

The SDVRP method is certainly optimal for determining individual transport time. When logistics are of most importance, the SDVRP method is preferable because it offers guidance on the optimal route. If the SDVRP based cost function is used in a whole-farm context, the corresponding model offers additional optimization options. For example, through a simultaneous optimization of cultivation planning, transport times can be reduced overall. This could be very useful from the perspective of landscape planning and land consolidation (Harasimowicz et al., 2017). However, using an SDVRP model for route planning places very high demands on databases and computing power. As previously mentioned, SDVRP models are often NP-hard. Therefore, they cannot always be optimally solved even based on small model sizes. This has already been shown in the model runs of the present study. Accordingly, for more extensive applications, specialized SDVRP algorithms are required. Here, the two-phase algorithm according to Jin et al. (2007) and the integration of tabu search by Archetti et al. (2006) should be mentioned. State-of-the-art SDVRP algorithms are capable of solving problems with up to 288 subjects (fields) within a timeframe of 1,422 s (Archetti et al., 2011a). Other routing problems with only 41 subjects are still not solvable within 7,200 s (see study by Ozbaygin et al., 2018). Numerous farms surpass such limits. Implementing such a specialized algorithm in a large DSS would tremendously increase the problem size. Due to the complexity of such models, however, it is debatable whether the additional optimization potential can be achieved due to the exponential increase in model size.

Another factor for discussion is the database of the present study. We generated data on farm infrastructure based on informed guesses. This approach was necessary because access to empirical data was not possible. Although our dataset is plausible, it would be sensible to repeat the analyses with empirical data in the future. As already mentioned, the prerequisite for an SDVRP model is a table containing complete farm–field and field–field distances. A GIS-based tool developed by Machl et al. (2016) could facilitate the generation of a suitable dataset. Another challenge is processing the empirical data within an SDVRP model because the farm size has a significant impact on whether the solver is able to find a solution or not. This is a barrier not only for an empirically grounded analysis but also for application within a DSS at a farm level. The feasibility of the SDVRP method is currently low. In agriculture, it is not always prudent to implement the routing guidance of an SDVRP model. An example is the traffic carrying capacity of the soil. It could influence whether a field can be driven on with a full spreader or not. In addition, farmers would not be able to realize the optimal route in numerous cases due to various external factors.

3.5 Conclusions

We present three potential strategies for determining farm-specific application costs for mineral fertilizers. In principle, all of them can be integrated into a DSS for optimized fertilizer application planning. The selection depends on the available data and the goal of the optimization tool. The regression method is particularly suitable for the optimization of fertilization at farm level. Estimating the transport times provides a good indication of the costs of fertilizer application, and the number of input parameters required is manageable. Under circumstances where the farm-specific input data is not available, the ZTC method provides acceptable results. If transport time is an important factor at the farm level (e.g. widely distributed fields), the ZTC method is not appropriate. The SDVRP method is resource-intensive in terms of computational power and input data requirements. It has the potential to generate optimal fertilizer strategies and simultaneously reveal the optimal route. The method could facilitate the saving of transport resources. It also has great potential in the optimization of future applications. However, under current conditions, the potential cannot be exploited adequately. In contrast, the DSS would reach a new level of complexity and thus often cannot be solved to our satisfaction.

The use of SDVRP for fertilizer application decisions contributes to literature on transport issues in agricultural contexts. The relevance of the original question could be extended to further working procedures. Therefore, a direct transfer to the application of plant protection products or grain sowing would be appropriate.

3.6 Appendix

Table A 3-1: Farm-specific parameters and variables that influence fertilization costs.

	Explanation	Data	Unit
ST	Setup time (per fertilizer and month of use)	1.25	h
WE	Wage entitlement	€20.00	h^{-1}
WS	Ø Working speed in the field	12.00	km h^{-1}
WW	Working width	21.00	m
Q	Volume of the fertilizer spreader	20	hl
TT	Turning and lost time on the field working time (*SD)	20%	
VS	Variable costs of the fertilizer spreader (*SD)	€0.015	hl^{-1}
VT	Variable costs of the tractor (*SD)	€12.97	h^{-1}
VL	Variable costs of the load vehicle (*SD)	€12.97	h^{-1}
LT	Loading time per spreader filling	0.2	h
TF	Time required to run all fields one after another	1	h
SW	Specific weight of the fertilizer	75 – 170	kg hl^{-1}
FD_i	Farm–field distance table for each field (Index i)	var.	min
HA_i	Plot size for each field (Index i)	var.	ha
ar	Application rate per hectare within each fertilization measure	var.	kg ha^{-1}
nf	Number of fertilized fields within each fertilization measure	var.	pcs.
hf	Fertilized acreage within each fertilization measure	var.	ha
tx	Time for all farm–field trips within each fertilization measure	var.	h
ty	Time for all field–field trips within each fertilization measure	var.	h

Note: The parameters are abbreviated in capital letters and the variables in lower case. *SD = Secondary data.

Table A 3-2: Sets, parameters, and variables for developing an SDVRP model at farm level.

Sets		
j	Designation of the field plots (F1–Fn) and the farm (B)	{B, F1… Fn}
i_j	Subset of j; all field plots	{F1… Fn}
h_j	Subset of j; field plots for one planned fertilization measure	$h_j \subseteq i_j$
v	Number of possible tours with a maximum of M tours, e.g., $M = \left\lceil \sum_{j \in i} A \times ha_j \times Q^{-1} \right\rceil + 2$	{T1…TM}
vv_v	Dynamic subset of v, with the minimum number of tours L $L = \left\lceil \sum_{j \in h} A \times ha_j \times Q^{-1} \right\rceil$	{T1… TL}; $vv_v \subseteq v$
Alias		
k	Alias for Set j	
Parameters		
A	$A = ar \times SW^{-1}$ fertilizer quantity in hl per hectare	ϵ Z+
Q	Volume of the fertilizer spreader in hl	ϵ Z+
ha_j	Field size of the field plots	ϵ Z+
c_{jk}	Distance between two points	ϵ Z+
Variables		
x_{vjk}	Tour planning; x = 1 means run from j to k, where j $\neq$ k	x_{vjk} ϵ {0,1}
y_{vj}	Indicator for visiting fields	y_{vj} ϵ {0,1}; $y_{vv0} = 1$
u_{vj}	Dummy variable for limiting sub-tours	u_{vj} ϵ Z+
w_{vj}	Fertilizer application per tour and per field	w_{vj} ϵ Z+; $w_{v0} = 0$
obj	$obj = tx + ty$; Objective value = Distance in minutes	Minimize!

Input of Table A 3-2 is used in line with Jin et al. (2007) to build the following SDVRP:

Min $$obj = \sum_{j=0}^{n}\sum_{k=0}^{n}\sum_{v=1}^{M} x_{vjk}c_{jk} \qquad j = 0,1,...n; v = 1,2,...M \tag{3-13}$$

s.t. $$\sum_{k=0}^{n} x_{vjk} = \sum_{k=0}^{n} x_{vkj} = y_{vj} \qquad j,k = 1,2,...n; v = 1,2,...M \tag{3-14}$$

$$u_{vj} - u_{vk} + nx_{vjk} \leq n-1 \qquad j \in h, v = 1,2,...M \tag{3-15}$$

$$w_{vj} \leq \left(ha_j A\right) y_{vj} \qquad j \in h \tag{3-16}$$

$$\sum_{v=1}^{M} w_{vj} = \left(ha_j A\right) \qquad v = 1,2,...M \tag{3-17}$$

$$\sum_{j=0}^{n} w_{vj} \leq Q \tag{3-18}$$

The total transport time of the fertilizer measure is minimized by the target function (3-13). The restrictions (3-14) facilitate proper tour planning. Each element of j, which is visited once within a tour, must also be departed from again. The binary variable y_{vj} is assigned with the value 1 if a field is actually visited within the tour. The restrictions (3-15) ensure that the number of sub-tours within a tour does not exceed the accepted maximum. Due to the inequalities (3-16), a fertilizer delivery w_{vj} is only possible if the corresponding field y_{vj} has actually been visited in the tour. Restrictions (3-17) and (3-18) ensure that the total demand for fertilizer in all fields is satisfied and the maximum volume of the fertilizer spreader is not exceeded within the individual tours.

4 IoFarm: A novel decision support system to reduce fertilizer expenditures at the farm level

This is a pre-copyedited, author-produced version of an article published in Computers and Electronics in Agriculture following peer review. The version of record [Michael F. Tröster and Johannes Sauer (2021): IoFarm: A novel decision support system to reduce fertilizer expenditures at the farm level. *Computers and Electronics in Agriculture* 188] is available online at: https://doi.org/10.1016/j.compag.2021.106322.

Authors' contributions: Michael Tröster is the main author of this contribution. Michael Tröster developed the research question and worked out the conceptual framework. Michael Tröster designed the decision support system, conducted the programming and the performance evaluation and wrote the manuscript. Johannes Sauer contributed in the development of the conceptual framework and was helpful in reviewing and editing of the manuscript. Johannes Sauer provided supervision and software resources. The authors would like to thank the editor and referees for their useful comments.

Abstract

Farmers and consultants make the best use of existing resources to increase profitability, and this begins at the operating resources level. Existing literature primarily deals with the optimal intensity of fertilization. A multitude of different fertilizers, with fluctuating price ratios, however, promises an additional economic optimization potential that can be achieved by cleverly combining these fertilizers to form a least-cost combination. This potential is extended if individual costs of fertilizer application are considered in parallel. The objective of the IoFarm decision support system is to take up this overall complex and to determine individual fertilizer strategies for farms, which also determines the economic optimization potential. In terms of methodology, the solution to this problem is based on mixed integer nonlinear programming (MINLP). The restrictions and parameters used for this were taken from the literature and where necessary were derived in a simplified form in order to keep usability a practical tool. The subsequent evaluation of the economic performance was carried out by conducting an experiment. The participants were asked to define a fertilizer strategy for a simplified farm with three fields and three crops over three years. Despite the considerable amount of time it took the testers to conduct this investigation, IoFarm performed 19% better in terms of costs. Our results show surprisingly clearly the complexity

of this decision-making process and the previously unused monetary and time optimization potential behind it. The agronomic performance of IoFarm has already been confirmed in another study.

Keywords

Decision support tool, sustainable intensification, fertilizer recommendation, fertilizer application; profit maximization, least-cost combination, economic modeling

4.1 Introduction

Recent literature focuses on efficiency and sustainability of fertilization, and fertilization has also become a topic of social interest. The farm manager's primary goal is profit maximization. Figures from the Bavarian agricultural state institute (LFL) show that the average proportional costs of fertilizer measured against the variable costs of winter wheat production—ignoring factor costs—were 30% (Schätzl et al., 2019), which highlights the significance of an optimal fertilizer strategy in order to maximize farm profit.

Earlier economic studies (Mitscherlich, 1909; Baule, 1954; Kling, 1985; Smit et al., 2000) also took up this topic, but the authors' approaches and priorities differ, which is not surprising, since farm profit is influenced on two levels: (i) the optimal intensity of inputs (nutrients) and (ii) the least-cost combination of substitutable inputs (fertilizers). The second aspect is only covered by a few studies, which are reviewed in Section 4.3. In our opinion, this aspect deserves closer attention from an economic point of view:

- A variety of single and compound fertilizers is available on the market
- Continuous change in price relations
- Interdependence between fertilizer strategy and total application costs

To achieve an optimal fertilizer strategy, it is necessary to simultaneously care about the optimal intensity and the least-cost combination of inputs. The goal of this study is to develop a decision support system (DSS) at the farm-level capable of meeting these expectations. Furthermore, product and input prices as well as environmental influences are dynamic, which is why the DSS needs to consider these inputs dynamically as well. Additionally, two more features will differentiate the DSS from existing tools: (i) the system considers the nutrient demand of an entire rotation and (ii) application costs are included in the optimization process.

Since the factors and conditions for each farm are different, for example, the availability of organic fertilizers or rotation and yields, an optimal fertilizer strategy is always farm specific. IoFarm addresses this problem at the farm level and farmers benefit through increased profit. In addition, outsourcing this problem causes a reduction in valuable management time. IoFarm helps to reduce the input of resources, such as energy or fuel, which is also a valuable benefit for society.

4.2 Conceptual framework

The starting point is the economic principle of utility maximization. We apply a common framework and replace utility with profit and the according target function is as follows:

$$\pi(x) = R(x) - C(x) \tag{4-1}$$

Profit π is dependent on a revenue- and cost-function of fertilizer input *x*.

$$R\left(x\right) = p \times y(x) \tag{4-2}$$

Revenue *R* is calculated by product price *p*, multiplied by product quantity *y*.

$$y = f(x_1 \ldots x_n) \tag{4-3}$$

The product quantity is a function of several inputs *xn*. Relevant to us is the input of different fertilizers needed to fulfill the nutritional requirements of *y*.

$$C(x) = (q \times x) + (m \times x) \tag{4-4}$$

C is the corresponding cost-function, with fertilizer price *q* and fertilizer application costs *m*, each multiplied by the fertilizer quantity *x* and summarized.

$$x = f(x_1 \ldots x_n) \tag{4-5}$$

$$m = f(x_1 \ldots x_n) \tag{4-6}$$

Fertilizer quantities *x* as well as application costs m are dependent on the factor combination of *x* represented in functional form. Application costs will influence profit maximization. High application costs will favor fertilizers with a high nutrient content. Application costs are a neuralgic part of this optimization problem, and they depend heavily on individual farm infrastructure. Further information according to the functional form and relevance of applications costs are in Tröster et al. (2019).

To answer our research question, we need to identify the optimal nutrient input and the cheapest combination of fertilizers. This concept is known as the expansion path and is displayed in Figure 4-1 in different variants.

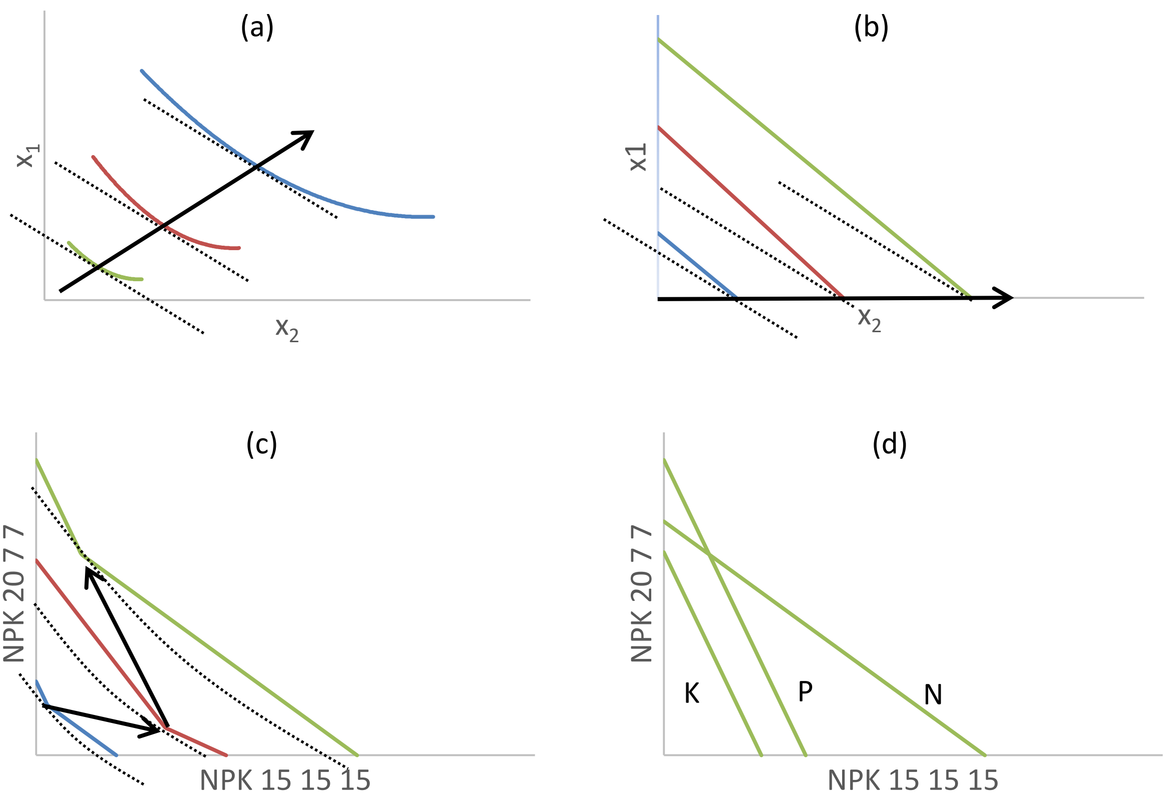


Figure 4-1: Expansion path: a) Standard theory b) linear isoquants c) modified to optimize the fertilizer strategy at the farm level d) isolated view on an isoquant separated by nutrients.

Figure 4-1a shows the standard version of this concept, with curved isoquants and linear isocost lines. This concept is true for the case of two factors with a decreasing substitution rate. Literature defines two more isoquants applicable for our research question: (i) isoquants with a linear substitution rate and (ii) isoquants with a limiting relationship of factors (Debertin, 2012b, pp. 151–178). The first option is true if we focus on a single nutrient and compare two completely substitutable fertilizers (Figure 4-1b). The course of the isoquants and isocost lines is linear and this will result in a single fertilizer solution. But as we know, plants need multiple nutrients in parallel. Figure 4-1c shows this in the form of two compound fertilizers containing different concentrations of nitrogen (N), phosphor (P) and potash (K) in a fixed composition. Fertilizer x_1 is an NPK 20 7 7, fertilizer x_2 an NPK 15 15 15. A linear substitution between these two fertilizers is possible up to a certain point where one of the nutrients becomes limiting. At this point, there is a kink in the course of the isoquant (Figure 4-1d). This kink indicates a “technical efficient combination” (Mußhoff and Hirschauer, 2013, p. 167) of both fertilizers. To identify the least-cost combination, we need to add isocost lines, which are usually assumed to be linear, which will change as soon as we also consider application costs. Fertilizer applications costs are nonlinear (Tröster et al., 2019). Thus, the course of these isocost lines cannot be generalized. Knowing that a combination of fertilizers increases application costs, it is clear that the isocost line is closer to the origin in this type of case. We decided to display this relationship by integrating a bent isocost line in Figure 4-1c. Due to the irregularly shaped isocost lines and isoquants, the optimal input combination jumps between the different levels of input intensity. Linking the least-cost combinations, we obtain the expansion path indicated in Figure 4-1c by a sequence of arrows.

As part of the expansion path, the functional form of the input and output relation is important for determining optimum input intensity. Crop response functions to nutrients are usually expressed as linear with an upper limit, quadratic or as asymptotic functions (Frank et al., 1990). This study uses a linear function with an upper limit to describe the relation between nutrient inputs and product output.

After transformation into a revenue function, Figure 4-2a shows this concept of the “broken stick” in combination with linear costs for fertilization. The upper limit is either defined by a nutrient in minimum or by a local yield potential. The economic optimum is reached when the slope of costs is equal to the slope of revenues. If all nutrients are sufficiently available, the economic optimum in this case also corresponds to the revenue maximum (yield potential times price). But, as previously mentioned, costs for fertilization—including application costs—are nonlinear. Figure 4-2b provides an example of this. Here again, the economic optimum is defined by comparing the slopes of the cost and revenue function. The yield potential is not reached due to an increase in fertilization costs. This increase could be explained by a necessary change in the fertilizer strategy. The concept in Figure 4-2b provides the possibility of specifying the optimal nutrient input intensity, whereas a concept with linear costs is only able to use the yield potential either completely or not at all.

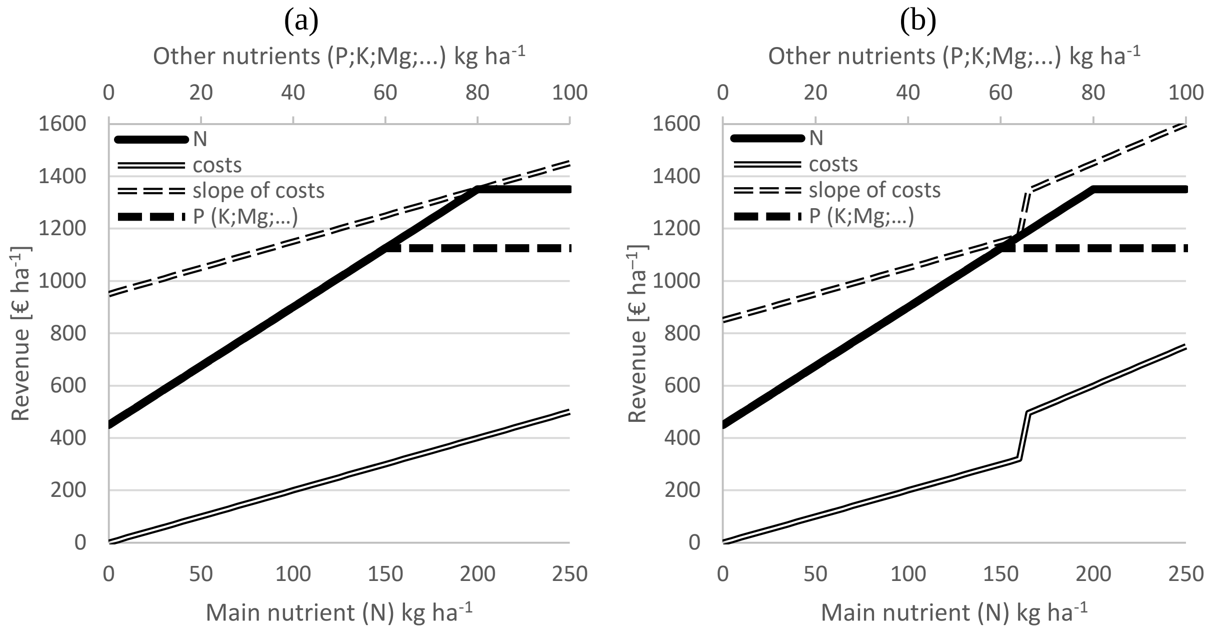


Figure 4-2: Linear crop revenue function with an upper limit.

"Under a given input price relation and production technology, the expansion path shows the cheapest possibility to expand production" (Mußhoff and Hirschauer, 2013, p. 169). Assuming the output price, we are able to transform the expansion path into a profit function to identify the optimal factor input. This optimum is valid for the intensity and the least-cost combination of inputs. Of course, price changes or an adjustment in production technology will lead to a new optimum. Finding this optimum is important for profit maximization, but a huge variety of fertilizers and volatile prices complicate the solution process in actual practice. Operations research (OR) is a scientific field that specializes in this type of complex issue. The classical problem-solving process according to (Mariappan, 2013, chapter 1.5) is as follows: Formulate the problem (verbal model), construct the mathematical model, verify the model, implement and evaluate recommendations. DSS often fall back on OR and this problem-solving process. For the practical usability and acceptance of a DSS, Rose et al. (2016) determine several demands. Of special relevance are: performance expectancy; ease of use; trust and relevance to user.

4.3 Existing tools

A large number of tools for fertilization planning are available all over the world. These tools differ in objective, quality, origin (scientific, commercial or private), etc. This section covers a selection of DSS relevant to farmers and is classified into: (i) tools identifying nutrient demands and (ii) tools blending a fertilizer or proposing a fertilizer strategy. Subsection (iii) differentiates IoFarm from the explored tools.

4.3.1 DSS identifying nutrient demands

These kinds of tools are built to identify the nutrient demand of crops or entire crop rotations. The "ImageIT" (Yara International ASA) mobile app uses pictures of crops to estimate the current N demand in kg per hectare. Optical sensors added to machines, such as the "GreenSeeker" (N Tech Industries), "N sensor" (YARA) or "Crop sensor ISARIA" (Fritzmeier Umwelttechnik & Claas), are able to do the same thing in real time, which is important for sub-area fertilization. "Fertilizer Removal by Crop" (IFA Productions Inc., 2012) allows the user to select a crop, insert an expected yield and the program returns the corresponding nutrient demand. This kind of simple application is popular, but the utility is limited. Since fertilization is regulated in many countries, there are also official tools specialized institutes provide. For example, a Bavarian farmer has to determine the nutrient requirements for his crops according to a scheme provided by LFL (Offenberger and Wendland). This scheme helps to identify the nutrient demand of crops after taking environmental effects and organic fertilization into account. Selecting fertilizers and allocating them is up to the farmer. This kind of fertilization planning is currently the usual approach for the average Bavarian farmer.

4.3.2 DSS blending a fertilizer or proposing a fertilizer strategy

These tools go beyond building on identifying nutrient demands. Babcock (1984) studied the optimal fertilizer composition to satisfy the nutrient demand for a single crop. His tool, based on linear programming, provided the user with a blending scheme to obtain a least-cost compound fertilizer. Recent studies have come up with comparable tools. "Optifer" (Pagán et al., 2015) and "Ecofert" (Bueno-Delgado et al., 2016) are designed for use in fertigation systems. They calculate the cheapest possible mix of fertilizers to get a nutrient solution that meets the needs of a single crop. "Fertilizer Optimizer" (Jansen et al., 2013) is a mobile application specialized for African countries. It helps to find an optimal fertilizer strategy within a limited budget. The outcome does not include a time schedule for the fertilizer application. Commercial farm management systems (FMS) built an own category of tools. They are widely used and therefore have a high impact on modern farm management. They usually include modules for whole-farm fertilization planning. The FMS "365FarmNet" (365FarmNet GmbH) contains the "YARA Plan" (YARA GmbH & Co. KG). This instrument proposes a fertilizer strategy based on own-company products. The user receives information regarding which fertilizer to use, including its quantity and timing. “NextFarming,” another FMS, offers a module called “NextDüngeplanung” (FarmFacts GmbH). According to the description, the tool uses linear programming to identify a preferable selection of fertilizers, but more than this is not revealed. Input parameters and output show that the tool aims to balance site-specific nutrient demand annually, adjusted against the previous year’s nutrient balance, which hampers basic fertilization in advance or at the end of a rotation period. The tool adds up the costs for fertilizer, but neither these costs nor the costs for application are included in the optimization process. “Smart Fertilizer” (Smart Fertilizer Management) is a commercial web application that specializes in optimizing fertilization in multiple farming systems. Similar to the previous tool, it provides a site-specific fertilizer schedule. The optimization mechanism and the included restrictions are not accessible to the user; thus, our analysis is based on input and output. To our knowledge, site-specific application costs as well as an aggregated basic fertilization within a rotation period are not adequately addressed.

4.3.3 Differentiation from existing tools

The following bullet points describe the functionality of IoFarm:

- Selection of fertilizers as per least-cost combination
- Allocation of fertilizer doses, considering growing stage and crop demands
- Dynamic input of yield expectations, market prices and site-specific parameters
- Observation period: three years in advance
- Observed nutrients: N, P, K, S, Mg and pH balancing
- Enables the aggregated use of P, K, Mg and liming within a rotation period
- Site-specific nonlinear fertilizer application costs are considered

The last two bullet points are what distinguishes IoFarm from other tools. The possibility of an aggregated basic fertilization and the consideration of application costs at the field level are unique. On the one hand, IoFarm has more open space to allocate basic nutrients during the rotation, which generates additional optimization potential. On the other hand, application costs are added to the target function, which influences fertilizer selection as well.

4.4 Design requirement and meta-planning

Following the typical OR problem solving process, this section collects and describes the requirements of the IoFarm optimization model in verbal form. The verbal model serves as a guideline for the subsequent mathematical model. To maintain usability, it is sometimes necessary to simplify the approach. The trade-off between scientific demands and practicality will be the subject of discussion at the corresponding position.

4.4.1 Price relationships and fertilizer information

In order to guarantee an extensive optimization potential, it is necessary to include all relevant fertilizers, which also includes the following information: price, nutrient content, N form, specific weight and acidification potential (appendix: Table A 4-2, Table A 4-3 and Table A 4-4). In order to guide the DSS on the expansion path, price updates for fertilizers and products are important. We decided to ask regularly for fertilizer quotes. The price estimation of harvest products is based on commodity futures less the transport costs to spot market.

4.4.2 Weather

Weather is a key factor in fertilization and a lot of decisions in farming depend on weather conditions. IoFarm uses weather information to derive N mineralization and nitrate leaching (see Section 4.4.4). Determining factors are soil temperature and the climatic water balance (CWB). The CWB expresses the connection between rainfall and evaporation. A negative value indicates that the evaporation is greater than the rainfall over a certain period of time. Both parameters are constantly recorded by common weather stations. The data is available online (Agrarmeteorologie Bayern, 2019). Site-specific weather conditions for an upcoming month are considered by averaging values based on long term records from the closest weather station. These average weather parameters are updated with the real values for the previous month.

4.4.3 Legal restrictions

In the European Union, the use of fertilizers is regulated by state legislation. The German sectoral legislation (BMEL, 2006) is supplemented by federal regulations (Wendland et al., 2018), amended in 2016 with entry into force in 2017. There is still ongoing work in some affected regulations. For this reason, we did not consider legal restrictions for the moment. At any rate, IoFarm is conceptualized to be of fundamental interest to farmers worldwide. In this regard, implementing country-specific legal restrictions before using IoFarm in actual practice is unavoidable.

4.4.4 Plant and soil

Plant needs and the soil properties are key factors for a successful fertilizer strategy. Sustainable plant nutrition has to consider multiple nutrients, particularly N, P, K, Mg, S and a compensation of acidification through liming. N and S are vulnerable to leaching and should be placed close to the crop demand. On many soils, P, K, Mg and lime are suitable for preliminary fertilization (Finck, 2007), which leads also to a difference in managing these nutrients in actual practice.

Basic fertilizer management and pH balancing

Preliminary fertilization offers additional optimization potential, and a holistic concept has to consider the demands of whole-crop rotation for these nutrients. The option of preliminary fertilization requires a multi-period model. P, K and Mg demands equal the amount of nutrients removed by the entire crop rotation, though differences in soil fertility must be respected. The soil fertility is determined through soil testing and categorized in five levels ranging from “Very poor” (level A) to “Very high” (level E). When determining nutrient requirements, nutrient levels on poor soils are enriched by adding a supplement and vice versa. This approach is intended to balance soil fertility. For details, refer to (Wendland et al., 2018, pp. 27–31). Similar approaches can also be found in (Zorn et al., 2007, pp. 58–66). In addition, it is good agricultural practice to forgo the optional preliminary fertilization in case of poor soil conditions. Instead, the lacking nutrient should be applied on an annual pattern to maintain the yield potential.

Some fertilizers cause the soil to become acidic, and this acidification has to be balanced by adding lime, unless the soil pH is unfavorably high, anyway. Acidic fertilizers lead to increased lime demand and thus acidification is important information in order to identify the least-cost combinations of fertilizers.

General aspects of N fertilization

The N supply during vegetation should preferably adapt to the N uptake of the crop. To control for that, we need to estimate the N uptake of the crop and the N dynamics of the soil. Literature provides several models dealing with this. Typical models are “HERMES” (Kersebaum, 1989), “DAISY” (Abrahamsen and Hansen, 2000; Hansen et al., 1991), “WAVE” (Vanclooster et al., 1996) and “MONICA” (Nendel, 2014). Integrating these kinds of specialized models is preferable from a scientific viewpoint. In practice, this is hampered by the necessary input parameters. Temporal and financial expenditures for data collection remain in conflict with conditions for a DSS of high usability (Rose et al., 2016). These circumstances require a pragmatic approach to estimate the N uptake and N dynamic based on data available from the usual operating procedure in plant production.

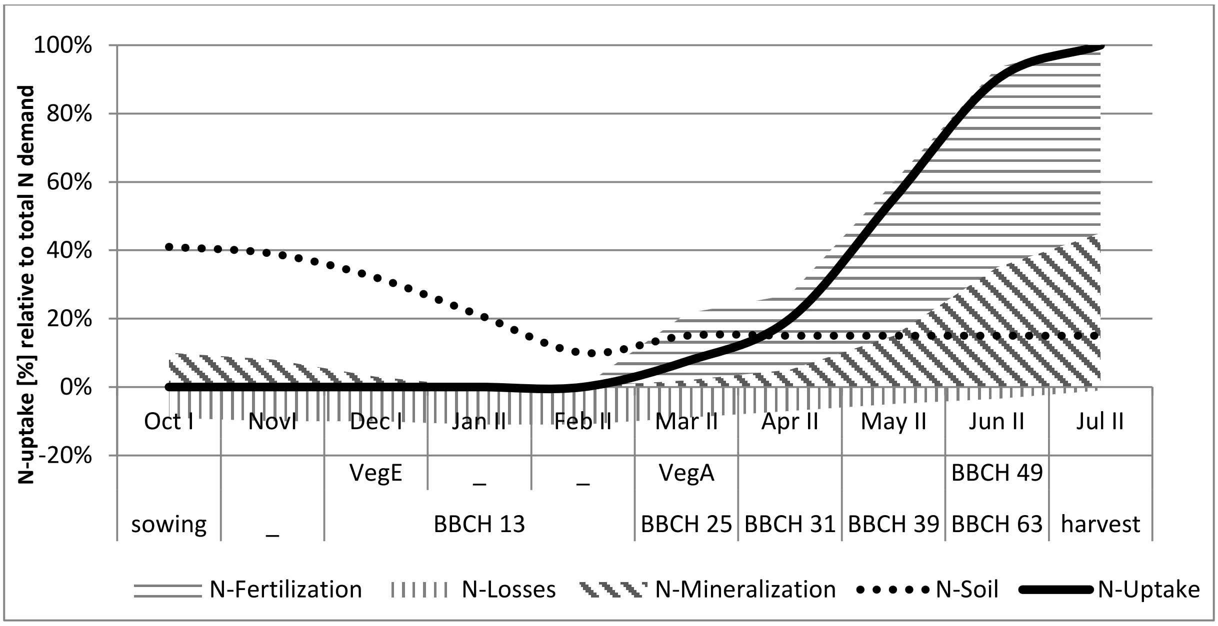


Figure 4-3: Cumulative N uptake of winter wheat and soil N dynamics.

(Oct I = October of seed year; Jul II = July of harvest year; VegE = End of vegetation period; VegA = Start of vegetation period; BBCH = Code for growing stage (Meier, 2018))

Figure 4-3 displays this approach by means of winter wheat production. Until BBCH 13 (three leaves unfolded), winter wheat is able to cover his N uptake (solid line) out of the N depot available from the grain (Lütke Entrup, 2000, p. 299). At the same time, there is usually a big pool of residual N available in the soil (N soil = dotted line) and there is no need for additional N fertilization. N soil is the source for N uptake and negative N soil levels are not feasible. Furthermore, during the start of vegetation (VegA), we recognize a need for N fertilization to fill up the minimum level of N soil. From VegA until flowering (BBCH 63), it is necessary to hold this minimum N soil level. Doing otherwise would lead to a lack of N and thus reduce the yields. In addition, the N soil level is also influenced by N losses (leaching = vertical lines), N mineralization (mineralization = diagonal lines) and N fertilization (horizontal lines). Less relevant and omitted due to their opposite effects are N deposition and denitrification.

Estimating the N uptake of plants

To cover the N uptakes of wheat, barley and maize, we had to summarize the findings of Lütke Entrup (2000, p. 229), Reiner and Dörre (1992, p. 91), Waldren and Flowerday (1979, p. 396). This result in a percentage N uptake is assigned to important growing stages. To use this information in different growing years, a dynamic assignment of growing stage and growing month needs to be made within a season. This approach helps to fit the N uptake to a calendrical basis and enables the model to combine fertilization measures carried out in different crops at the same time, which may be beneficial due to reduced application costs.

To transform this percentage value into a monthly N uptake, we need to know the total demand per season, which is ideally expressed by a crop yield response function to N. There are many studies (for example: Grimm et al., 1987; Bäckman et al., 1997) dealing with this topic, but N dynamics within the soil and local conditions such as soil quality are not consistently observed. As a result, the emerging response functions are valid solely for the analyzed locations and the corresponding year. To overcome this, we put crop yield in context with the N content of the entire plant, including roots. The N content of the roots is derived from the N content of straw (cereals) or the entire plant (silage maize) as published by Wendland et al. (2018, pp. 77–78). With reference to Lütke Entrup (2000, p. 298), we expect that the dry matter of cereal roots equals 20% of the dry matter of grain and straw. In case of silage maize, this value is reduced to 10%. Thus, we obtain a linear relationship between yield and N uptake:

- Winter wheat = 2.46 kg dt^{-1} at 86% DM
- Winter barley = 2.30 kg dt^{-1} at 86% DM
- Silage maize = 1.40 kg dt^{-1} at 100% DM

This linear concept of yield and nutrient demand was introduced earlier in Figure 4-2. To exclude implausible yields, an upper limit in the form of yield expectation is necessary. This upper limit is defined by the farmer and can be adjusted monthly. Of course, misjudgments are a serious external factor that can negatively impact the entire system. However, farmers can adjust their estimates at very short notice.

Now it is important to distinguish between N uptake and N fertilization. Higher yields and thus higher N uptake require a disproportionate supply of N (compare Mitscherlich, 1909). An increasing N fertilization favors N losses and may influence the N dynamic in the soil in general. To obtain a customized recommendation for N fertilization, we need to estimate the following side effects:

- Gaseous N emissions at fertilizer application
- Transformation speed of different N Forms
- N losses by nitrate leaching
- N mineralization

Estimation of gaseous N losses after fertilizer application

Typical chemical forms of N in fertilizers are nitrate, ammonium and urea. After fertilizer application, urea and ammonium will transform into nitrate. This process is accompanied by gaseous N emissions in the form of ammonia. The amount of N emissions heavily depends on environmental conditions and thus literature reports N losses on different levels (Hutchings et al., 2019; Kreuter et al., 2014; Ni et al., 2014; Sommer and Jensen, 1994). There is a consensus that urea is more sensitive to gaseous N emissions than ammonium. After evaluating different references, gaseous N emissions of urea are defined at 11.5% and for ammonium these losses are 8.5%. These parameters are easy to adapt to individual conditions or new insights.

Estimating the transformation speed of various N forms

The duration of the transformation from urea or ammonium to nitrate is termed “N speed.” As we know today, plants are able to use urea and ammonium directly. But the crop response is delayed in comparison with nitrate. Fertilizers with nitrification inhibitors slow down the N speed even further. To make sure that the crop’s N uptake is always covered by available N, only nitrate is considered as fully available. This approach is particularly important at the start of the growing season where low soil temperatures reduce the speed of transformation. N speed is dependent on the original N form, soil temperature (Vilsmeier and Amberger, 1980), soil moisture, soil pH value and the use of nitrification inhibitors (NI) (Chen et al., 2015; Herbst et al., 2006; Irigoyen et al., 2003; Zerulla et al., 2001). Under Central European conditions, N form, soil temperature and the use of NIs are the main drivers for N speed. Therefore, we refer to the research of Vilsmeier and Amberger, who show the relationship between soil temperature and N speed. An increase in soil temperature does facilitate the transformation speed considerably. Since their findings are based on only three different soil temperatures, we had to estimate the intermediate values. To do this, we used an OLS regression. For the transformation speed of amides to ammonium, the best coefficient of determination was obtained using a logarithmic functional form (Eq. (4-7); R2=0.9987). The transformation speed of half of the ammonium to nitrate, however, was better represented by an exponential function (Eq. (4-8); R2 = 0.9832):

$$TS_{Amide \longrightarrow Ammonium} = -1.292 \times \ln(ST) + 4.9142 \quad (4\text{-}7)$$

$$TS_{50\% Ammonium \longrightarrow Nitrate} = -372.11 \times ST^{-1.338} \quad (4\text{-}8)$$

TS represents the transformation speed in days, either for amide to ammonium or for 50% of ammonium to nitrate depending on the soil temperature ST in degrees Celsius. Specialized models such as DAISY (Abrahamsen and Hansen, 2000) could provide a better estimation, but need much more input. Zerulla et al. (2001) demonstrate that NIs considerably diminish the transformation speed from ammonium to nitrate. Low soil temperatures support the effect. Their study compares the effect of NIs at three levels of soil temperature. Using several linear OLS regressions, we determined for each temperature level by what factor the use of NI reduces the transformation rate. These factors were applied to the results of Vilsmeier and Amberger, resulting in Eq. (4-9):

$$TS_{50\% Ammonium \xrightarrow{+NI} Nitate} = 856.09 \times ST^{-1.175} \quad (4\text{-}9)$$

Based on this information, IoFarm selects appropriate N fertilizers and separates N fertilization into reasonable doses. Special annual conditions such as a low soil temperatures or a rapid crop growth will be answered with an adjusted N fertilization.

Estimation of nitrate leaching

The relationship between CWB (see Section 4.4.2) and nitrate leaching is often described in studies on nitrate leaching (compare Büchert et al. (2001) and Anger et al. (2002)). In a monthly comparison of CWB and nitrate leaching, e.g., in Anger et al. (2002, p. 644), it is obvious that in phases with low CWB, nitrate leaching is also reduced or stops.

According to a worldwide meta-study by Zhou and Butterbach-Bahl (2014), the average leaching losses of maize are 23% in relation to the N fertilization applied and 17% for wheat. Büchert et al. (2001) showed the nitrate leaching for maize in a study from Schleswig Holstein. Depending on the intensity of the fertilization, leaching ranged from 16% to 28% in terms of N applied. In our view, a corridor of 0% to 30% of the leaching losses can be observed. We use this corridor in conjunction with monthly values for CWB to estimate the nitrate losses over time based on the following assumptions: A CWB less than or equal to −80 means there is no leaching. Leaching increases linearly as CWB increases. The upper limit of the corridor is reached at a CWB greater or equal to 130, which means that 30% of the soil N is lost due to leaching within a one-month period. Despite clear correlations, we are currently unable to prove our assumptions. However, several arguments have led us to apply this scheme for estimating nitrate leaching: (i) The reduction of input to CWB increases usability; (ii) Under Central European conditions, only a small part of the leaching takes place during vegetation; (iii) Inaccuracies are compensated by soil testing twice a year.

Estimation of N mineralization

Mineralization is affected by the organic N pool in the soil, soil pH, soil temperature and moisture (Heumann et al., 2013, p. 399). The cultivated crop itself has also an impact (Seith, 2015). We expect that the monthly rate of N mineralization can be estimated thru: (i) a sitespecific mineralization potential, (ii) a crop-specific mineralization factor and (iii) soil temperature. We assume that N mineralization is stopped at a soil temperature equal to or lower than zero degrees Celsius. Following vańt Hofft́s rule, mineralization is doubled by an increase in soil temperature of 10 Kelvin:

$$Q = \left(\frac{R_2}{R_1} \right)^{\left(\frac{10K}{ST_2 - ST_1} \right)} \tag{4-10}$$

Q = Reaction speed factor corresponding to a temperature increase of 10 Kelvin; R_n = Reaction speed at soil temperature ST_n

To avoid division by zero, R_1 has to be 1 at a soil temperature of 0° C. To double mineralization by a temperature increase of 10 Kelvin, Q has to be 2. Knowing these parameters, we are able to restructure the formula toward R_2:

$$R_2 = 2^{\left(\frac{ST_2}{10} \right)} \tag{4-11}$$

Inserting a soil temperature (ST_2) of 0° C would result in an R_2 value of 1. But, as already mentioned, we assume that N mineralization stops at this point. Under Central European conditions, an increase in soil temperature is often linked to a decrease in soil moisture. Both a lack of water and a transgression of the biological temperature optimum hamper further facilitation of N mineralization. To cover this, R_2 needs to be manipulated as follows:

$$
\begin{aligned}
&if: (R_2 \geq 1) \wedge (R_2 \leq 3) \rightarrow R_2^{'} = R_2 - 1 \\
&elseif: R_2 > 3 \rightarrow R_2^{'} = 2 \\
&else: R_2^{'} = 0
\end{aligned}
\tag{4-12}
$$

R_2 is 0 for a soil temperature equal to or lower than 0° C. With increasing soil temperature, the mineralization rate speeds up, reaching its upper limit for R_2, which is 2 at a soil temperature of about 16° C. Based on average soil temperatures, we calculate a site-specific value for R_2 every month. The sum of all R_2 values over a year is equal to 100%, which enables us to interpret the monthly R_2 values as a percentage value. Multiplying (i) the annual mineralization potential in kg per hectare with (ii) the crop-specific N mineralization factor and with (iii) the percentage value of R_2, we obtain an estimate for the monthly N mineralization for a specific place underneath a specific crop.

Calibrating the N dynamics

The above system reproduces the N dynamics on a monthly scale. Estimates for N uptake, N losses, N mineralization and N speed are used to calculate the N soil level for each month. The N fertilization is the only external variable in this system. It is also the only variable with which we can directly influence the relationships shown in Figure 4-3. N fertilization controls the N soil level and thus the availability of N for plant nutrition. Of course, the reliability of this approach suffers from a lot of estimates and assumptions. Therefore, the calculated N soil level is updated based on soil testing results. Soil testing takes place well in advance of the first N fertilization and post-harvest. This approach minimizes possible shortcomings to an acceptable level, which is a good trade-off between usability and accuracy.

Calibration should be carried out using an established N fertilizer system before using this system on a farm for the first time. Our empirical example relates to Bavaria, so we use the official system (Wendland et al., 2018, pp. 26–30) established there to calibrate IoFarm. Calibration is necessary to adjust the site-specific N mineralization potential as well as the mineralization factors for different crops relative to winter barley. Due to this calibration, the level of total N fertilization is well adapted to the reference system. We use the OLS method to calibrate the system. The residual values are developed by comparing the suggested fertilizer level of IoFarm and the reference system. The sum of these error squares is minimized by changing the calibration factors. After an initial calibration, IoFarm is ready to use for the corresponding farm.

Estimating the S demand

The S demand for crops is usually covered by sulfate, which is very susceptible to leaching. S should therefore be applied during the vegetation period and parallel to the main N uptake. To ensure this, a percentage link between total N uptake and S demand, in combination with a suitable time window for a useful S fertilization, is needed. The practical recommendation for S fertilization is around 20 kg per hectare for the crops mentioned. With a ratio of 1 to 10 between S fertilization and N uptake, our crops are allocated around 20 kg of S per hectare per year. To satisfy the needs of other crops, this ratio needs to be crop-specific and adjustable.

4.4.5 Further restrictions

The feasibility of model decisions may be hampered due to unforeseen external effects. Affected restrictions may significantly limit the optimization potential. Example: A fertilizer application scheduled for March could fail due to the soil's insufficient traffic-carrying ability or a labor shortage. The omitted fertilization in March mathematically points to the fact that the appropriate restriction is not fulfilled as expected. The consequence would possibly be a drastic decline in the modeled yield expectation. But a temporary lack of nutrients does not immediately reduce the yield potential. Plants are able to compensate for a temporary shortage and missing nutrients are absorbed at a later point. To achieve this flexibility within IoFarm, it is necessary to exclude certain former restrictions, for example, the monthly N restriction. The N demand for March is no longer important in April. Instead, higher-ranking restrictions ensure that the total nutrient demands within a growing season (N, S) or an entire rotation (P, K, Mg) are satisfied. Temporary shortages in the past are then covered by an additional supply of nutrients. This procedure is also important to enable a subsequent increase in yield expectations, for example, due to good weather conditions. Because the farmer regularly updates his yield expectations, the system remains subject to external control.

Today's fertilizer spreaders control the application rate by volume flow rate and driving speed. The maximum driving speed and the minimum diameter of the dosing opening creates a minimum application rate per hectare. This limit was set to 80 kg ha^{-1}.

Optional restrictions (not yet implemented): (i) Storage capacity and ongoing availability of organic fertilizers; (ii) Pre-purchase and fertilizer storage (this automatically includes a hedge on the price development of fertilizers and would therefore lead to a bias in the economic assessment of IoFarm).

4.5 Model development

The previously described verbal model needs to be specified in the form of a formal optimization model. In this section, we identify appropriate optimization methods and software, followed by a detailed presentation of the IoFarm model.

4.5.1 Identifying the appropriate methods and software for optimization

The scope and complexity of this problem exceed the limits for an informal optimization strategy. The structure of the problem is explicit and formal. In this case, it is useful to choose between methods based on mathematical or heuristic programming. Heuristics are often powerful instruments for finding acceptable solutions to huge problems, but they are not able to guarantee an optimum outcome. Mathematical programming is preferred if an optimum solution is desired. Babcock (1984), for example, used an LP model to solve a fertilizer blending problem. Mínguez et al. (1988) used LP as part of a Goal Programming model to optimize fertilizer use on sugar beets in Spain. The described fertilizing problem also contains integer variables and nonlinear terms, which makes it a mixed integer nonlinear problem (MINLP). MINLP models are extremely difficult to solve. It is known that the applied solver exerts a tremendous influence on performance and the optimality of the solution. The GAMS modeling language provides the opportunity for switching among a large number of solvers. Therefore, GAMS is an excellent instrument in MINLP modeling. An early stage performance evaluation among all MINLP solvers within GAMS (Version 24.8.1) revealed that ANTIGONE is the preferred solver for IoFarm. The solver choice changed during the modeling process. The latest version of IoFarm is solved using SCIP.

4.5.2 Model flow

The flowchart in Figure 4-4 is intended to provide an initial overview of the IoFarm model structure. In parallel, it becomes visible which exogenous inputs feed the model. Since the model runs in parallel to actual farming processes, these exogenous inputs are constantly updated. Real-world observations successively replace the assumptions made at the beginning of a multi-year planning period (crop rotation cycle). With each run of IoFarm, the workflow shown in Figure 4-4 is passed through. This procedure usually happens once a month during the growth period. The exogenous parameters are also updated at this time interval. For example, as soon as crop yields are known, they are fixed. At the same time, a system of slack variables ensures that violations of affected restrictions (including the production function) due to differences between modeled yields and observed yields are bypassed. Fertilization measures that were suggested by IoFarm and have already been carried out in practice are also fixed and represent exogenous inputs for the model from this point on.

To obtain a solution close to optimality, it was necessary to divide the solving procedure into two stages. An overview of this is provided in Figure 4-4. Stage I is mainly a relaxation of the original MINLP problem. All integer conditions are ignored, and the application costs were defined in a simplified way. The results of stage I serve as initial variable values at the beginning of stage II. Fertilizers not used during the actual month and the last two months in stage I are no longer available in stage II. This reduction in model size enables us to introduce semiconditional variables in stage II for the current decision month. The model is still able to respond to the additional integer conditions by selecting from a number of fertilizers that have been beneficial in the past. This workaround is necessary to find solutions close to optimality, although we do lose some optimization potential.

In addition to the two-stage design, the model is additionally divided into a static and dynamic deterministic model. A lot of parameters are uncertain at the start of a planning horizon. Weather, prices, yield expectations and all dependent parameters are affected. Instead of real values we use expected values, which changes over time, if more and more of these values become known. At the same time, decisions regarding variables in the past are fixed, which steadily reduces model size. To achieve this, we save the model results in an EXCEL document using the GAMS data exchange tools.

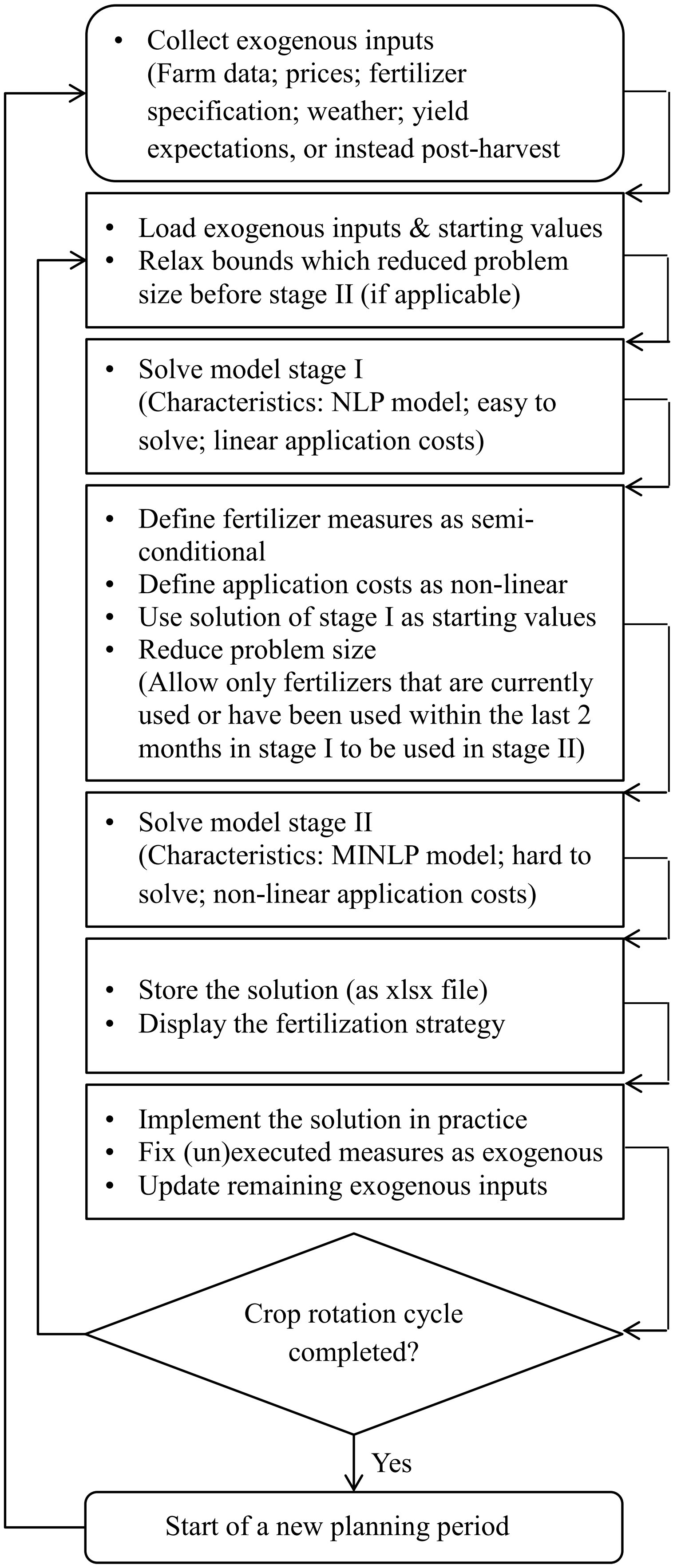


Figure 4-4: IoFarm flowchart.

4.5.3 Stage I: Relaxed NLP model

The following section describes the mathematical structure of the IoFarm model. It should be noted that the model is initially presented in simplified form in the main part. Only the most important restrictions are picked out for the sake of easier comprehensibility. Indices are not listed in this section, either. Variables and parameters that do not have indices even in the full model are highlighted in bold in the simplified model. Variables are shown in lower-case letters and external parameters in upper-case letters (see appendix Table A 4-1 for an overview of acronyms). The role of restrictions not explicitly shown is described under the keyword "*secondary restrictions*." In addition to the simplified presentation of the model, there are consistent cross-references to the detailed restrictions in the appendix, where the complete mathematical structure can be reviewed.

Target function

Eq. (4-13) contains the objective function. IoFarm is designed as a maximization problem and therefore the total objective function value *obj* is maximized. It depends on the total revenue of the plant production *typ* minus the total expenditure for fertilizer *tfc* and the necessary application costs *tsc*:

$$\boldsymbol{obj} = \boldsymbol{typ} - \boldsymbol{tfc} - \boldsymbol{tsc} \tag{4-13}$$

The three main variables are influenced not only by factors external to the model but also to a large extent by the quantity, type and timing of the fertilizers applied. The interaction of quantity, type and time of fertilization is called fertilizer strategy. It is expressed in the *fu* variable. The value of this variable also indicates the solution path for cost-optimal fertilization and is thus decisive for the user. There is a central relationship between *fu* and the three main variables. This relationship will be made clear in the following sections.

Relationship between total revenue and fertilizer strategy

Of course, the chosen fertilizer strategy *fu* exerts an influence on crop yields. Therefore, Eq. (4-14) (alternatively appendix, Eq. (4-29)), first shows the composition of *typ*, the total revenue of plant production within the period under consideration. The total revenue *typ* is composed of the sum product of *ty* (crop yield) and *PPH* (price or expected price of the crop):

$$\boldsymbol{typ} = \sum [(ty \times PPH) \tag{4-14}$$

The annual yield of the individual crop *ty* is also determined within the model (Eq.(4-15); alternatively appendix Eq. (4-30)). It is composed of the modeled yield per hectare *y*, the slack variable *yslk* and the crop-specific area under cultivation *CRP*:

$$ty = \sum[(y + yslk) \times CRP] \tag{4-15}$$

Secondary restrictions: The slack variable *yslk* is needed to compensate for differences between the modeled yield per hectare *y* and the real yield *YPH*. In reality, there will always be deviations at this point. It is therefore important to be able to replace the modeled yield *y* (endogenous) with the real yield *YPH* (exogenous). If the real yield is higher than the modeled yield, restrictions are violated afterward, and the model becomes infeasible. The slack system is only used if the real yield is already known. In this case, it eliminates possible violations, and the model remains feasible. In parallel, nutrient requirements are adjusted to the real nutrient removal. Further information on the slack system can be found in the appendix (Eqs. (4-33), (4-34) and (4-36)). Eq. (4-16) describes the input–output relationship of nutrients and yield, forming the production function (for more details see appendix Eqs. (4-31) and (4-32)). This production function defines the modeled yield per hectare y as a linear function between nutrient input *u* [kg $ha^{-1}a^{-1}$] and nutrient removal *UEXT* [kg dt^{-1}]. The restriction is also constructed as an inequality. Thus, *y* can remain below the farmer's yield expectation if the marginal costs are higher than the marginal benefit. Since this restriction is valid for several nutrients at the same time, y is also restricted if only one nutrient is deficient:

$$y \le \frac{u}{UEXT} \tag{4-16}$$

Secondary restrictions: Other methods are used to determine the S nutrient input. The S requirement is proportionally oriented to the N requirement that is determined in parallel (appendix Eq. (4-35)). As already mentioned, the modeled yield y is limited by the farmer’s yield expectation *YEX* (appendix Eq. (4-32)). The already introduced system of slack variables enables the real yield *YPH* to excide this upper limit. *YEX* can be continuously adjusted as an external parameter, which allows the model to react dynamically to changes in yield expectations. As soon as the real yield *YPH* is known, this yield is taken over in the parameter *YEX*. In order to take into account the soil content of P, K and Mg, the nutrient input *u* is again corrected in terms of quantity and application period, if necessary. (appendix Eq. (4-39)). Parallel to the modeled yield per hectare *y*, the necessary nutrient input u in [kg ha^{-1}] was determined in Eq. (4-16). In order to include the different sizes of field pieces in the further process, *u* is to be converted into the new variable *w* (Eq.(4-17)). For this purpose, *u* is multiplied by the respective cultivation area (*CRP*), so that variable *w* now indicates the nutrient requirement in kg per field piece:

$$w \geq u \times CRP \tag{4-17}$$

Eq. (4-17) (alternatively appendix, Eq. (4-43)) refers only to the nutrients N and S. Modification is necessary for nutrients such as P, K and Mg, which can be balanced, since in this case, fertilization is possible within the framework of crop rotation (Eq. (4-18); alternatively appendix Eq. (4-37)). The variable *w* must be summarized for the period of crop rotation. With the parameters *AYN* and *MYN* the need of the balanceable nutrients is adapted to the soil content (compare Section 4.4.4). Altogether the equation ensures that within a crop rotation, the entire need of balanceable nutrients is covered. Thus, an aggregated fertilization of the balanceable nutrients is also possible:

$$\sum(w) \geq \sum\{[(u + uslk + AYN) \times MYN] \times CRP\} \tag{4-18}$$

The variable *uslk* can be traced back to the same slack system as in yield calculation, i.e., it serves to balance the nutrient requirements according to the modeled yield and the nutrient requirements according to the actual yield (appendix Eqs. (4-33), (4-34), (4-36)). This system gives us the flexibility to update the model post-harvest with real yields without violating prior constraints.

Eqs. (4-17) and (4-18) define the requirements of the nutrients N and S, or P, K and Mg per field and year in the variable *w*. In Eq. (4-19) (alternatively appendix Eq. (4-38)), this requirement is covered by selecting a fertilizer strategy *fu*. Mathematically, this is done by multiplying and summing up *fu* with the nutrient content of the fertilizer *SUP*:

$$w = \sum(fu \times SUP) \tag{4-19}$$

Secondary restrictions are linked to the use of fertilizers: maximum available quantities of organic fertilizers (appendix Eq. (4-42)), reasonable timeframes for using different fertilizers (appendix Eqs. (4-39), (4-44), (4-50)) and requirements for compensating the lime-consuming effect (appendix Eqs. (4-40), (4-41)).

This restriction system is not yet sufficient for a plant-appropriate supply of N. In addition to the total N requirement, its distribution during the year must also be taken into account. The variable *w* is distributed over a period of months by multiplying it by the factor *NDM*, which results in variable wm (Eqs. (4-20)). *NDM* indicates on a monthly basis what proportion of the total N demand a crop typically requires:

$$wm \geq w \times NDM \tag{4-20}$$

Of course, it would not be appropriate to apply N once a month, which is why the restriction is also implemented as an inequality. The possibility of combining the N fertilization in several applications as well as integrating the natural N dynamics of the soil complicates the situation considerably. Eq. (4-20) is therefore replaced by Eq. (4-45). *Secondary restrictions*: Parallel to Eq. (4-20) or (4-45), it is ensured that a site-specific soil supply of N in the form of nitrate may not fall below a certain level (appendix Eq. (4-46)). For this purpose, the soil nitrate supply must be estimated within the model (appendix Eq. (4-48)). This estimated value is adjusted to the actual measured value at the time of the soil testing by using another slack system (appendix Eqs. (4-49), (4-47)).

Considering Eq. (4-21) (alternatively appendix Eq. (4-50)), the N requirement *wm* during the year is now ensured by selecting a suitable fertilizer strategy *fu*, wherein the fertilizers are multiplied by their transformation speed *NSM* and totaled. The binary parameter *TWN* excludes timeframes in which N fertilization is not useful. The satisfaction of *wm* takes place simultaneously while satisfying the previously mentioned requirements for other nutrients:

$$wm = \sum[(fu \times NSM) \times TWN] \tag{4-21}$$

Relationship between total expenditures for fertilizers and fertilizer strategy

The total expenditure of the fertilizers used, referred to as *tfc*, plays a central role in the optimal fertilizer strategy. They correspond to the sum product of the fertilizer input *fu* and the monthly updated fertilizer price *PF* (Eq. (4-22); alternatively appendix Eq. (4-54)):

$$\boldsymbol{tfc} = \sum(fu \times PF) \tag{4-22}$$

Relationship between total costs of fertilizer application and fertilizer strategy

In order to consider different costs when applying solid and liquid fertilizers, a distinction is made between the application costs of solid (*spdc*) and liquid fertilizers (*spyc*). The total sum of both variables then represents the total cost of fertilizer application *tsc* (Eq. (4-23); alternatively, appendix Eq. (4-55)):

$$\boldsymbol{tsc} = \sum(spdc) + \sum(spyc) \tag{4-23}$$

A detailed consideration of the application costs leads to a disproportionately high computational effort for IoFarm. Therefore, in stage I of IoFarm, a simplified approach to application costs is chosen (Eqs. (4-24), (4-25); alternatively appendix Eqs. (4-56), (4-57)). The variables *spdc* and *spyc* represent the application costs per fertilization measure. *SLS* describes the cost per fill (*spd*, *spy*) caused by setup time as well as loading and spreading fertilizer. Transport costs are added in the second part of the equation. The field-yard distance *C* [min] is converted into hours. A return trip is calculated and charged with the variable costs for the tractor *VT* and the personnel costs *WE* for each filling. Integrating the field-yard distance *C* ensures that a fertilizer strategy adapted to the transport distance is proposed:

$$spdc = \boldsymbol{SLS} \times spd + \sum [spd \times C \times 2 \times (\boldsymbol{VT} + \boldsymbol{WE})] \tag{4-24}$$

$$spyc = \boldsymbol{SLS} \times syd + \sum [syd \times C \times 2 \times (\boldsymbol{VT} + \boldsymbol{WE})] \tag{4-25}$$

Secondary restrictions: The number of fillings *spd* or *spy* depends on the density of the fertilizer and the tank volume (appendix Eqs. (4-58), (4-59)).

4.5.4 Stage II: Final MINLP model

The previous NLP model becomes an MINLP model during the transition to stage II. For this purpose, Eqs. (4-24) and (4-25) (respectively appendix (4-56) and (4-57)) are replaced. Instead of the simplified presentation of the application costs, the individual cost components are integrated in detail, including setup time, loading, field work and transport. The last item depends largely on the farm infrastructure and is difficult to determine. Tröster et al. (2019) developed an approach for integrating application costs comprehensively and individually for each farm. This approach is adapted for use in mathematical models. A description is not given here, but reference is instead made to the original publication. The corresponding restrictions can be found in the appendix (Eqs. (4-73) to (4-79)). *Secondary restrictions:* This method requires the total area-per-fertilization measure in order to determine the application costs, which uses two auxiliary restrictions (appendix Eqs. (4-52), (4-53)) to determine them. Another aspect that affects the transport costs is the number of fillings per field piece (appendix Eq. (4-72)). In addition, mainly for mathematical reasons, binary information is needed to indicate whether fertilization is taking place (appendix Eqs. (4-70), (4-71)).

Furthermore, stage II of IoFarm ensures that the fertilizer strategy is practically feasible in relation to the application rates. For this purpose, the variable *fu* is divided by the corresponding area (*CRP*) (appendix (4-51)), which gives the variable *sfu*, which indicates the fertilizer use in dt per hectare. In case a fertilizer is used, *sfu* cannot be zero meaning that we can limit *sfu* using the binary variable bin and an externally defined lower, (*LO*) or upper limit (*UP*). This is done in Eqs. (4-26) and (4-27) (alternatively appendix Eqs. (4-68), (4-69)). *sfu* hereby becomes a semiconditional variable that can take either the value zero or values between *LO* and *UP* (mineral fertilizers 0.8 to 15 dt ha^{-1}; lime 3 to 50 dt ha^{-1}; organic fertilizers 12.5 to 50 t ha^{-1}):

$$sfu \geq bin \times LO \tag{4-26}$$

$$sfu \leq bin \times UP \tag{4-27}$$

4.6 Performance evaluation

We created a hypothetical example to evaluate the economic performance of IoFarm. Interested readers will find this “fertilizer quiz” as supplementary material to this article. An introductory video explains task and rules in detail. The following bullet points outline the “fertilizer quiz” in brief:

- The aim is to find a cost-effective fertilizer strategy
- Applications costs are also exemplified and considered
- The rotation consists of barley, wheat and maize (50 ha each)
- The planning period is three years in advance
- Seasonal and overall nutrient demands are given
- Different single and compound fertilizers are available
- Full price and weather information within the planning period are given

The quiz was distributed via email and social media. A large part of the reach was achieved via mailing lists of alumni associations of higher agricultural education institutions and universities. In addition to the fertilizer strategy, the participants were asked for the following information: (i) time needed to complete the quiz; (ii) financial self-assessment of their own solution; (iii) level of experience. Level of experience was defined as follows: Expert = Person possessing either scientific experience in plant nutrition or economic optimization models; Farmer = Person with at least five years of professional experience in agriculture and plant nutrition; Student = Student with advanced knowledge in economic optimization models and plant nutrition; “Others” (not included). The results of this experiment are displayed in

Table 4-1. This experiment only allows a differentiation of fertilization costs. The benchmark for the financial evaluation is the cost of IoFarm's fertilizer strategy. Due to time and cost constraints, the different fertilization strategies of the participants were not tested in the field. Potential differences in yield or quality are unobserved. The fertilization requirements were identical for IoFarm and the participants, which means: identical specifications for nutrient requirements, uniform time windows for fertilization measures, same minimum application rates per hectare, etc. Due to these clear specifications, the potential for significant yield differences is very low. Nevertheless, in order to be able to make valid statements on potential yield or quality differences, IoFarm was compared with farm-usual fertilization strategies in a separate field trial (Tröster and Sauer, 2021a). For those who want to try it on their own, the quiz is available online[9]. The fertilizer strategy of IoFarm is in the appendix Table A 4-5.

On average, the valid participants spent €340 $ha^{-1}a^{-1}$ for fertilizer and application. The solution for IoFarm performed significantly better at €274 $ha^{-1}a^{-1}$. For the financial evaluation, we compared the participants' results with the benchmark set by IoFarm; see Table 4-1, "Cost difference" column. The self-assessment is presented in the last column. The participants had to estimate the amount per hectare and year in which their solution falls short compared to IoFarm's solution. To structure this, the following options were given: 0–5; 5–15; 15–30; 30–60; 60–120 or>120 € $ha^{-1}a^{-1}$. Compared to the average of the valid participants, IoFarm achieved a cost advantage of €66 $ha^{-1}a^{-1}$. The self-assessment of the participants also shows that they see potential in optimizing their fertilizer strategy. In direct comparison, however, it can be seen that the financial optimization potential is even underestimated. Although, our "fertilizer quiz" is simplified and not comparable to real conditions in fertilizer management with lots of fields and crops, it was difficult and time consuming for the participants. On average, they needed 81 min to take the quiz. Due to fluctuating prices and yield expectations, a farmer needs to repeat this process several times within a single planning period. Projecting this information onto real-world farm conditions, fertilizer management is a very time-consuming job. Outsourcing this job to a computer program is therefore a real timesaver.

[9] https://drive.google.com/file/d/14rBHNKKDuBq8oyeeVUXuek2id1B9z_Dw/view?usp=sharing

Table 4-1: Results of the fertilizer quiz.

		Time needed [min]	Cost difference [€ ha^{-1}]		Self-assessment [€ ha^{-1}]
	Min	30.0	20.8	Min	5-15
Expert	Avg	83.8	57.4	Mod	5-15
n = 4 + 1*	Max	150.0	94.3	Max	15-30
	Sd	52.4	26.6		
	Min	30.0	10.0	Min	5-15
Farmer	Avg	76.9	64.0	Mod	30-60
n = 18 + 1*	Max	180.0	127.6	Max	60-120
	Sd	37.7	27.5		
	Min	45.0	45.4	Min	5-15
Student	Avg	90.8	76.6	Mod	30-60
n = 6 + 1*	Max	230.0	113.5	Max	60-120
	Sd	63.9	23.8		
	Min	30.0	10.0	Min	5-15
Total	Avg	80.9	65.8	Mod	30-60
n = 28 + 3*	Max	230.0	127.6	Max	60-120
	Sd	47.0	27.3		

Remarks: "Cost difference in € ha^{-1}" compares the participants solution with the one of IoFarm. "Self-assessment": The participants had to evaluate their solution in comparison to IoFarm's using a prescribed scale. Min = Minimum, Avg = Average, Max = Maximum, Sd = Standard deviation, Mod = Modus. *) Invalid solutions.

4.7 Discussion

An experiment was used to evaluate IoFarm's performance. The main people involved were experienced farmers and agricultural students with experience in the field of mathematical optimization as well as some experts. This selection may entail a selection bias. However, it is to be expected that the participants are by and large tough opponents due to their education and experience. Unfortunately, the desired participation was not achieved, which is probably due to the enormous amount of time required and the lack of incentive for the participants. Even in the case of a lucrative incentive, there is a risk that participants will solve the quiz without actually paying attention to minimizing fertilization costs. In order to include just motivated participants, we decided not to offer any material incentives. The fertilizer quiz is available online as additional material to give the reader the option of testing IoFarm's performance in a self-experiment.

The relevance and feasibility of the suggested fertilization strategies depend on many factors. IoFarm is designed to be very flexible and can accept changes that a farmer makes in fertilization strategy. In addition, the fertilizer strategy can be easily adapted to individual farm requirements via additional restrictions, such as the consideration of labor capacities. Due to identical requirements in our experiment, we may assume an equivalent relevance and feasibility of all fertilization strategies. The results of the fertilizer quiz are impressive and show the financial optimization potential that can be achieved with IoFarm in actual practice. For a 150 ha farm without organic fertilizers, the costs for fertilizer and application could be reduced by 19% in comparison. This proportion remains largely stable even with increasing farm area, so that larger farms benefit from it much more in absolute terms than smaller farms. As a rule, organic fertilizers are applied on the farm's own land. Organic fertilizers are natural compound fertilizers. High percentages per hectare reduce the need for mineral fertilizers or, in extreme cases, even make them obsolete. Therefore, the potential for optimization diminishes as the proportion of organic fertilization in the total fertilizer application increases. Thus, farms with a large area of land without in-house organic fertilizers benefit most. These estimations can also be made for saving management time in fertilization planning. No information is available for directly comparing the financial and temporal effects with other DSS mentioned in literature. Thus, none of the cited articles (Jansen et al., 2013; Bueno-Delgado et al., 2016; Pagán et al., 2015; Mínguez et al., 1988; Babcock, 1984) examine the extent to which costs can be saved by applying the respective optimization tools compared to a standard operating solution. Among the commercial providers of software-based solutions, only the "Smart Fertilizer" website contains statements on this topic, where they mention cost savings of 60% and an increase in income of 40%. However, no proof of this is provided. Sensor-based measures for optimizing fertilization were also mentioned at the beginning of this article. The manufacturers of these systems expect profit increases of €20 to €30 ha^{-1} by improving the N efficiency. Evangelou et al. (2020) even show in their field trial for N fertilization in maize a savings potential of €33 to €92 ha^{-1}. However, sensor- based systems are not in competition with IoFarm but could be used in parallel.

Another point of discussion is the way in which biological, chemical and agronomic processes have been taken into account. Here, numerous simplifications were accepted during the model development (see Section 4.4). We decided that a field trial was indispensable in order to determine potential influences on yield and quality of harvested products. This field trial was conducted between 2015 and 2018 (see Tröster and Sauer, 2021a). In addition, integrating and testing other ways of determining fertilization requirements in IoFarm should be considered in the future: combination with site-specific fertilization approaches based on sensors, maps or satellites would be promising.

IoFarm is a mathematical optimization model in which the nutrient requirements are explicitly specified. In practice, it can be observed that the nutrient requirements of a crop are rarely satisfied exactly to 100% without direct effects on yield or quality. This observation is taken into account in less explicit approaches such as goal programming. Mínguez et al. (1988) used goal programming for a similar optimization problem in fertilizing sugar beets. Their scope only referred to a single crop year. This approach should be viewed rather critically when considering a period of several years. There is the risk that nutrients could be dosed below the actual requirement level several years in a row. IoFarm leverages the advantages of goal programming solely for allocating basic nutrients. Fertilization within the framework of crop rotation is permitted provided that the soil is adequately supplied with nutrients.

The structure as a mathematical optimization model with integer variables and nonlinear conditions leads to the fact that IoFarm places high demands on computing capacity. Compared to alternative heuristic approaches, however, the advantage is that the optimality of a solution can be judged by the gap in the potential optimum. For the application of IoFarm under practical conditions with more than three crops and numerous field plots, it has to be checked whether heuristic approaches can possibly lead to better results in terms of cost savings or computing power.

4.8 Conclusions

At approximately €66 $ha^{-1}a^{-1}$, the saving potential IoFarm obtains turns out to be surprisingly high when comparing the average participant in the fertilizer quiz. At €10 $ha^{-1}a^{-1}$, even the best individual participant lags behind the IoFarm result. In addition to the financial advantage, valuable management time can be saved, since fertilizer selection can be outsourced. These results highlight the benefits of this type of a DSS for farmers and consultants. The first version of this type of optimization tool for individual farms has been successfully developed and fills the gap in the area of these management tools. IoFarm is primarily focused on profit maximization, which is often associated with negative environmental impacts. However, IoFarm draws part of its optimization potential from a fertilizer strategy that is as efficient and resource-saving as possible and thus a considerable contribution to sustainability is achieved.

We published a field trial of several years to show the influence on yield and quality in comparison to common fertilizer strategies (Tröster and Sauer, 2021a). In the future, we want to use IoFarm to answer further questions, such as, under which conditions is using compound fertilizer economically reasonable? How does the fertilizer strategy change in the case of a very homogeneous or very heterogeneous supply of the soil with basic nutrients? How does the fertilizer strategy change if the overall target function is used to optimize the greenhouse gas balance? These questions are relevant for the fertilizer industry, policy makers and for society as a whole.

4.9 Appendix

Table A 4-1: Overview of model acronyms.

Sets			
cr	Crop	swg	Specific weight
f	Field	t	Time in years
fz	Fertilizer	tm	Time in month
nu	{N,P_2O_5,K_2O,MgO,S,CaO}	ttm	Alias of set tm
Subset of (Set)			
afz (fz)	Min. fertilizer without lime	mfz (fz)	Mineral fertilizer
bns (nu)	{N, P_2O_5, K_2O MgO}	mfz2 (mfz)	mfz2 = mfz≠{AHL1zu3}
bnu (nu)	{ P_2O_5, K_2O MgO}	ofz (fz)	Organic fertilizer
lfz (fz)	Lime fertilizer	sfz (fz)	N fertilizers; soil active
lqfz (fz)	Liquid leaf fertilizer		
Variables			
$bin^{b}_{t,tm,cr,fz,f}$	Decision on fu {0/1}	$spdha^{+}_{t,tm,mfz}$	Spreader fillings per hectare
$fu^{+}_{t,tm,cr,fz,f}$	Fertilizer usage	$spy^{+}_{t,tm,lqfz}$	Sprayer loads
$hf^{+}_{t,tm,mfz}$	Acreage per measure	$spy0^{+}_{t,tm,lqfz}$	Binary indicator of spy
$hff^{+}_{t,tm,mfz,cr,f}$	Acreage per measure &field	$spyc^{+}_{t,tm,lqfz}$	Spray costs
$nfn^{+}_{f,t,ttm,cr}$	N fertilization as nitrate	$spycFF^{+}_{t,tm,lqfz}$	Cost of field to field trips II
$nso^{+}_{f,t,tm,cr,nu}$	Nitrate spillover	$spycFW^{+}_{t,tm,lqfz}$	Costs of fieldwork stage II
obj	Objective value	$spycLT^{+}_{t,tm,lqfz}$	Costs of load time stage II
$sslk_{f,t,tm,cr}$	Slack var. for soil-nitrate	$spycST^{+}_{t,tm,lqfz}$	Costs of setup time stage II
$sfu^{+}_{t,tm,cr,fz,f}$	Specific fertilizer usage	$spycVC^{+}_{t,tmlq}$	Costs of sprayer stage II
$sfu0^{+}_{t,tm,cr,fz,f}$	Binary indicator of sfu	$spycYF^{+}_{t,tm,lqfz}$	Costs of farm field trips II
$snm^{+}_{f,t,tm,cr}$	Soil nitrate minimum	$spyha^{+}_{t,tm,lqfz}$	Sprayer fillings per hectare
$snn_{f,t,tm,cr}$	Soil-nitrogen as nitrate	tfc^{+}	Total fertilizer costs
$spd^{+}_{t,tm,mfz}$	Spreader loads	tsc	Total spread and spray costs
$spd0^{+}_{t,tm,mfz}$	Binary indicator of spd	$ty^{+}_{cr,t}$	Total yield
$spdc^{+}_{t,tm,mfz}$	Spread costs	typ	Total crop revenue
$spdcFF^{+}_{t,tm,mfz}$	Cost of field to field trips II	$u^{+}_{t,f,cr,nu}$	Specific nutrient demand
$spdcFW^{+}_{t,tm,mfz}$	Costs of fieldwork stage II	$uslk_{t,f,cr,nu}$	Nutrient demand slack var.
$spdcLT^{+}_{t,tm,mfz}$	Costs of load time stage II	$w^{+}_{f,t,cr,nu}$	Crop nutrient demand
$spdcST^{+}_{t,tm,mfz}$	Costs of setup time stage II	$wm^{+}_{f,t,tm,ttm,cr,nu}$	Nitrate demand per month
$spdcVC^{+}_{t,tm,mfz}$	Costs of spreader stage II	$y^{+}_{t,f,cr}$	Modeled yield
$spdcYF^{+}_{t,tm,mfz}$	Costs of farm field trips II	$yslk_{t,f,cr}$	Slack var. for yield

Remarks: A superscript + indicates a positive variable; a superscript b indicates a binary parameter or variable.

Table A 4-1 is continued on next page

Table A 4-1: Overview of model acronyms (continued).

Parameter			
$AYN_{f,t,nu}$	Addition to basic nutrients	$SYN^{b}_{tm,cr}$	Timeframe S use
$BFF_{f,t,bnu}$	Basic fertilization factor	TF	Time to reach all fields
C_{f}	Farm-field distance	TFA	Total farm area
$CRP_{f,t,cr}$	Cropping plan	$TLIST_{t,tm}$	Serial number for t and tm
HA_{f}	Plot size for each field	TSTAMP	Current time stamp for t, tm
LO_{fz}	Lower boundary for sfu	TT	Share of turning & lost time
LT	Loading time per load	$TWBF^{b}_{tm,cr}$	Timeframe: basic fert. use
$LYN^{b}_{f,t,nu}$	Lime usage yes/no	$TWBFF^{b}_{tm,cr}$	TWBF + low soil fertility
MSNL	Minimum soil nitrate level	$TWLA^{b}_{tm,cr}$	Timeframe: lime use
$MYN_{f,t,nu}$	Factor on basic nutrients	$TWLN^{b}_{lqfz,tm,cr}$	Timeframe: liquid N use
$NDM_{t,tm,cr}$	Monthly plant nitrate need	$TWN^{b}_{tm,cr,t}$	Timeframe: Nitrogen use
$NL_{t,tm}$	Nitrate leach in percentage	$TWOF^{b}_{tm,cr}$	Timeframe: organic fert.
$NMOB_{f,t,tm}$	Natural nitrate mobilization	$TWSN^{b}_{tm,t,cr}$	Timeframe: solid N fert.
$NSM_{fz,ttm,t,tm}$	Nitrate supply monthly	$UEXT_{nu,cr}$	Nutrient extraction
$OFA_{ofz,t}$	Organic fertilizer amount	$ULFZ_{t,tm,lfz}$	Use of lime in past 3 mos.
$PEF_{t,tm,cr,fz,f}$	Fixed values for sfu	$ULQFZ_{t,tm,lqfz}$	Use of lqzf in past 3 months
$PF_{fz,t,tm}$	Price of fertilizers	$UMFZ_{t,tm,mfz}$	Use of mfz in past 3 months
$PPH_{cr,t}$	Crop price	UP_{fz}	Upper bound for sfu
Q	Spreader/sprayer volume	$VEG^{b}_{t,cr}$	Start of vegetation
$SABS_{f,t,tm}$	Soil NH_4^+ in spring	VL	Variable cost: load vehicle
$SAPH_{f,t,tm}$	Soil NH_4^+ post-harvest	VS	Variable cost: spreader
$SNBS_{f,t,tm,*}$	Soil nitrate in spring	VT	Variable cost: tractor
$SNPH_{f,t,tm,*}$	Soil nitrate post-harvest	WE	Wage entitlement
SLS	Cost for: setup+load+spread	WS	Working speed in field
ST	Setup time	WW	Working width
$SUP_{fz,nu}$	Nutrient supply of fertilizer	$YEX_{cr,t}$	Yield expectation
$SW_{mfz,swg}$	Specific weight of fertilizer	$YPH_{f,t,cr}$	Yield post-harvest

Remarks: A superscript + indicates a positive variable; a superscript b indicates a binary parameter or variable

Equations Stage I

$$obj = typ - tfc - tsc \tag{4-28}$$

$$typ = \sum\nolimits_{cr,t} \left(ty_{cr,t} \times PPH_{cr,t} \right) \tag{4-29}$$

$$ty_{cr,t} = \sum\nolimits_{f} \left(\left(y_{t,f,cr} + yslk_{t,f,cr} \right) \times CRP_{f,t,cr} \right) \ \forall cr,t \tag{4-30}$$

For Eq. (4-30) applies: As long as $YPH_{f,t,cr}$ is unknown, $yslk_{t,f,cr} = 0$.

$$y_{t,f,cr} \leq \frac{u_{t,f,cr,bns}}{UEXT_{bns,cr}} \ \forall t, f, cr, bns \tag{4-31}$$

$$y_{t,f,cr} \leq YEX_{ct,t} \ \forall t, f, cr \tag{4-32}$$

$$yslk_{t,f,cr} = \frac{uslk_{t,f,cr,bns}}{UEXT_{bns,cr}} \ \forall t, f, cr, bns \tag{4-33}$$

Eq. (4-33) only applies if $YPH_{f,t,cr} > 0$.

$$yslk_{t,f,cr} = YPH_{f,t,cr} - y_{t,f,cr} \ \forall t, f, cr \tag{4-34}$$

Eq. (4-34) only applies if $YPH_{f,t,cr} > 0$.

$$u_{t,f,cr,nu\{S\}} \geq u_{t,f,cr,nu\{N\}} \times UEXT_{nu\{S\},cr} \ \forall t, f, cr \tag{4-35}$$

$$uslk_{t,f,cr,nu\{S\}} = uslk_{t,f,cr,nu\{N\}} \times UEXT_{nu\{S\},cr} \ \forall t, f, cr \tag{4-36}$$

Eq. (4-36) only applies if $YPH_{f,t,cr} > 0$.

$$\begin{aligned} &\sum\nolimits_{cr,t} \left(w_{f,t,cr,bnu} \right) \geq \\ &\sum\nolimits_{cr,t} \left(\left(\left(u_{t,f,cr,bnu} + uskl_{t,f,cr,bnu} + AYN_{f,t,bnu} \right) \times MYN_{f,t,bnu} \right) \times CRP_{f,t,cr} \right) \forall f, bnu \end{aligned} \tag{4-37}$$

For Eq. (4-37) applies: As long as $YPH_{f,t,cr}$ is unknown, $uslk_{t,f,cr,bnu} = 0$.

$$w_{f,t,cr,bnu} = \sum\nolimits_{tm,fz} \left(fu_{t,tm,cr,fz,f} \times SUP_{fz,bnu} \right) \ \forall f, t, cr, bnu \tag{4-38}$$

$$\begin{aligned} &\left(u_{t,f,cr,bnu} + AYN_{f,t,bnu} \right) \times MYN_{f,t,bnu} \times CRP_{f,t,cr} \times BFF_{f,t,bnu} \leq \\ &\sum\nolimits_{tm,fz} \left(fu_{t,tm,cr,fz,f} \times TWBFF_{tm,cr} \times SUP_{fz,bnu} \right) \ \forall t, f, cr, bnu \end{aligned} \tag{4-39}$$

Eq. (4-39) only applies if the soil supply with P, K or Mg is low (see Section 4.4.4). In this case, the factor $BFF_{f,t,bnu} > 0$ and a placed fertilization must be carried out annually within a given timeframe *TWBFF*. Thus, this restriction only intensifies the original restriction from Eq. (4-37). In order to prevent subsequent non-compensable violations, Eq. (4-39) is canceled as soon as an actual harvest quantity is determined, i.e., when $YPH_{f,t,cr} > 0$.

$$0 \leq \sum\nolimits_{t,tm,cr,fz} \left(fu_{t,tm,cr,fz,f} \times SUP_{fz,nu\{CaO\}} \times LYN_{f,t,nu\{CaO\}} \right) \ \forall f \tag{4-40}$$

Eq. (4-40) is only relevant for fields where liming is allowed $\left(LYN_{f,t,nu\{CaO\}}=1\right)$. In this case, the lime supply must be at least neutral in the course of a crop rotation period.

$$0=\sum\nolimits_{tm,cr,lfz}\left(fu_{t,tm,cr,lfz,f}\right)\ \forall t,f \qquad (4\text{-}41)$$

Eq. (4-41) only applies if $LYN_{f,t,nu\{CaO\}}=0$. In this case, Eq. (4-40) becomes ineffective. Eq. (4-41) then ensures that no special lime fertilizers are applied to this field at any time.

$$0\geq\sum\nolimits_{tm,cr,f}\left(fu_{t,tm,cr,ofz,f}\right)-OFA_{ofz,t}\ \ \forall t,ofz \qquad (4\text{-}42)$$

$$w_{f,t,cr,nu\{N,S\}}\geq u_{t,f,cr,nu\{N,S\}}\times CRP_{f,t,cr}\ \ \forall t,f,cr,nu\{N,S\} \qquad (4\text{-}43)$$

Eq. (4-43) only applies if $CRP_{f,t,cr}\neq 0$. With regard to the nutrient sulfur, the equation is only taken into account as long as $YPH_{f,t,cr}$ is unknown.

$$w_{f,t,cr,nu\{S\}}=\sum\nolimits_{tm,fz}\left(fu_{t,tm,cr,fz,f}\times SYN_{tm,cr}\times SUP_{fz,nu\{S\}}\right)\ \forall f,t,cr,nu\{S\} \qquad (4\text{-}44)$$

$$\begin{aligned}
&w_{f,t,cr,nu\{N\}}\times NDM_{t,tm,cr}=\\
&\begin{pmatrix} wm_{f,t,tm=1,ttm=1,cr,nu\{N\}}-nso_{f,t,tm=1,cr,nu\{N\}}\\ +\left(CRP_{f,t,cr}\times\left(SABS_{f,t,tm=1}+SAPH_{f,t,tm=1}+NMOB_{f,t,tm=1}\right)\right)\\ -\left(sslk_{f,t,tm=1,cr}\times CRP_{f,t,cr}\right)\end{pmatrix}+\\
&\begin{pmatrix} wm_{f,t,tm=1,ttm=2,cr,nu\{N\}}+wm_{f,t,tm=2,ttm=2,cr,nu\{N\}}-nso_{f,t,tm=2,cr,nu\{N\}}+nso_{f,t,tm=1,cr,nu\{N\}}\times\left(1-NL_{t,tm=1}\right)\\ +\left(CRP_{f,t,cr}\times\left(SABS_{f,t,tm=2}+SAPH_{f,t,tm=2}+NMOB_{f,t,tm=2}\right)\right)\\ -\left(sslk_{f,t,tm=2,cr}\times CRP_{f,t,cr}\right)\end{pmatrix}+\\
&\ldots\\
&\begin{pmatrix} wm_{f,t,tm=1,ttm=17,cr,nu\{N\}}+\ldots+wm_{f,t,tm=17,ttm=17,cr,nu\{N\}}-nso_{f,t,tm=17,cr,nu\{N\}}+nso_{f,t,tm=16,cr,nu\{N\}}\times\left(1-NL_{t,tm=16}\right)\\ +\left(CRP_{f,t,cr}\times\left(SABS_{f,t,tm=17}+NMOB_{f,t,tm=17}\right)\right)\\ -\left(sslk_{f,t,tm=17,cr}\times CRP_{f,t,cr}\right)\end{pmatrix}\\
&\forall f,t,tm,ttm,cr,nu\{N\}
\end{aligned} \qquad (4\text{-}45)$$

For Eq. (4-45) applies: The variable *sslk* is only considered if $\left(SNPH_{f,t,tm,*}>0\wedge CRP_{f,t,cr}>0\right)$ or if $\left(SNBS_{f,t,tm,*}>0\wedge CRP_{f,t,cr}>0\right)$, which is always the case when new soil test results are available. The equation is divided into a total of 17 segments, one segment per fertilization month *tm* (August to December in the year of cultivation and January to December in the year of harvest). Only the segment corresponding to the current fertilization month is taken into account. *SAPH* is only defined for segments 1-5, i.e., for tm ={1, ... 5}. The natural N mineralization is integrated via the parameter *NMOB*. The variable *nso* allows the transfer of N surpluses from one fertilization month to another within a cropping period. Losses in the form of leaching are deducted using the parameter *NL*. The variable *wm* thus indicates the planned nitrate supply per fertilization month and crop. Since N fertilizers do not necessarily contain nitrate, or contain other N forms besides nitrate, the complete conversion into nitrate is often spread over several months. For this reason, it was necessary to define the variable *wm* two-dimensionally, by the indices *tm* and *ttm*. Thus, it is possible to show the nitrate effect of an N fertilization based on the fertilizer's speed of action (*NSM*) over several fertilization months. Thus, IoFarm is able to control the timing and at the same time does not overlook possible utility costs, for example, losses due to nitrate leaching. An example will help to better understand the purpose of Eq. (4-45): Assume that winter wheat has a nitrogen requirement of 40 kg in May, which is defined by $w \times NDM$. However, this requirement does not necessarily have to be covered by N fertilization in May alone. Part of it can be covered from a previous fertilization measure. For example, N may have already been applied in March, but was not fully absorbed. After deducting losses, this N is still available in May. In addition, soil N is mineralized during this period. In May, therefore, only a reduced fertilization, possibly even no fertilization at all, would be necessary to meet the N requirement of 40 kg.

$$snm_{f,t,tm,cr} = \left(\frac{\left(nso_{f,t,tm,cr,nu\{N\}} + wm_{f,t,tm=1,ttm=1,cr,nu\{N\}} \right)}{CRP_{f,t,cr}} \right) - MSNL \quad \forall f,t,tm,cr \tag{4-46}$$

Eq. (4-46) is only valid as of the start of vegetation ($\forall tm \geq VEG_{t,cr}$), starting in March at the earliest ($\forall tm \geq 8$), or from the month in which the need for N arises ($NDM_{t,tm,cr} > 0$). Points of time in the past are likewise faded out by using the equation only if the following applies: $TSTAMP \leq TLIST_{t,tm}$.

$$nfn_{f,t,ttm,cr} = \sum_{tm} \left(\frac{wm_{f,t,tm,ttm,cr,nu\{N\}}}{CRP_{f,t,cr}} \right) \quad \forall f,t,ttm,cr \tag{4-47}$$

$$snn_{f,t,tm,cr} = NMOB_{f,t,tm} + nfn_{f,t,tm,cr} - \left(\left(u_{t,f,cr,nu\{N\}} + uslk_{t,f,cr,nu\{N\}} \right) \times NDM_{t,tm,cr} \right) + \left(\frac{nso_{f,t,tm-1,cr,nu\{N\}} \times \left(1 - NL_{t,tm-1}\right)}{CRP_{f,t,cr}} \right) \quad \forall f,t,tm,cr \tag{4-48}$$

$$sslk_{f,t,tm,cr} = snn_{f,t,tm,cr} - nfn_{f,t,tm,cr} - SNPH_{f,t,tm,"NSommer"} - SNBS_{f,t,tm,"NFruehjahr"} \quad \forall f,t,tm,cr \tag{4-49}$$

Eq. (4-49) only applies if $SNPH_{f,t,tm,*} > 0 \vee SNBS_{f,t,tm,*} > 0$.

$$wm_{f,t,tm,ttm,cr,nu\{N\}} = \sum_{fz} \left(fu_{t,tm,cr,fz,f} \times NSM_{fz,ttm,t,tm} \right) \times \mathrm{TWN}_{tm,cr,t} \quad \forall f,t,tm,ttm,cr,nu\{N\} \tag{4-50}$$

$$sfu_{t,tm,cr,fz,f} = \frac{fu_{t,tm,cr,fz,f}}{CRP_{f,t,cr}} \quad \forall t,tm,cr,fz,f \tag{4-51}$$

$$hff_{t,tm,mfz,cr,f} \times sfu_{t,tm,cr,mfz,f} = fu_{t,tm,cr,mfz,f} \quad \forall t,tm,mfz,cr,f \tag{4-52}$$

$$hf_{t,tm,mfz} = \sum_{cr,f} (hff_{t,tm,mfz,cr,f}) \quad \forall t,tm,mfz \tag{4-53}$$

$$tfc = \sum_{t,tm,cr,fz,f} \left(fu_{t,tm,cr,fz,f} \times PF_{ft,t,tm} \right) \tag{4-54}$$

$$tsc = \sum_{t,tm,mfz} \left(spdc_{t,tm,mfz} \right) + \sum_{t,tm,lqfz} \left(spyc_{t,tm,lqfz} \right) \tag{4-55}$$

$$spdc_{t,tm,mfz} = SLS \times spd_{t,tm,mfz} + \sum_{f} [spd_{t,tm,mfz} \times C_f \times 2 \times (VT + WE)] \quad \forall t,tm,mfz \tag{4-56}$$

Eq. (4-56) applies only for $\forall mfz \neq \{AHL1zu3\}$

$$spyc_{t,tm,mfz} = SLS \times spy_{t,tm,mfz} + \sum_{f} [spy_{t,tm,mfz} \times C_f \times 2 \times (VT + WE)] \quad \forall t,tm,mfz \tag{4-57}$$

Eq. (4-57) applies only for $\forall mfz = \{AHL1zu3\} \wedge TWLN_{lqfz,tm,cr} = 0$

$$spd_{t,tm,mfz} = \sum_{cr,f,swg} \left(\frac{fu_{t,tm,cr,mfz,f}}{SW_{mfz,swg}} \right) \times Q^{-1} \quad \forall t,tm,mfz \tag{4-58}$$

$$spy_{t,tm,lqfz} = \sum_{cr,f,swg} \left(\frac{fu_{t,tm,cr,lqfz,f}}{SW_{lqfz,swg}} \right) \times Q^{-1} \quad \forall t,tm,lqfz \tag{4-59}$$

For Eq. (4-59) applies: If $TWLN_{lqfz,tm,cr} = 1$, then $spy_{t,tm,lqfz} = 0$.

Bounds in Stage I

These bounds ((4-60) – (4-67)) are defined before the solve statement of stage I:

$$if: TSTAMP > TLIST_{t,tm} \rightarrow sfu_{t,tm,cr,fz,f} = PEF_{t,tm,cr,fz,f} \tag{4-60}$$

Eq. (4-60) transfers fertilization measures that have already been carried out to the PEF parameter. PEF is then available to fix past fertilization measures in the model.

$$if: TWBF_{tm,cr} = 1 \rightarrow sfu_{t,tm,cr,afz,f} \leq 15; else: sfu_{t,tm,cr,afz,f} = 0 \tag{4-61}$$

$$if: TWSN_{tm,cr} = 1 \rightarrow sfu_{t,tm,cr,sfz,f} \leq 15; else: sfu_{t,tm,cr,sfz,f} = 0 \tag{4-62}$$

$$if: TWSN_{tm,cr} = 1 \rightarrow sfu_{t,tm,cr,lqfz,f} \leq 2.5; else: sfu_{t,tm,cr,lqfz,f} = 0 \tag{4-63}$$

$$if: TWLA_{tm,cr} = 1 \rightarrow sfu_{t,tm,cr,lfz,f} \leq 50; else: sfu_{t,tm,cr,lfz,f} = 0 \tag{4-64}$$

$$if: TWOF_{tm,cr\{WW,WG\}} = 1 \rightarrow sfu_{t,tm,cr\{WG,WW\},ofz,f} \leq 30; else: sfu_{t,tm,cr\{WW,WG\},ofz,f} = 0 \tag{4-65}$$

$$if: TWOF_{tm,cr\{SM\}} = 1 \rightarrow sfu_{t,tm,cr\{SM\},ofz,f} \leq 50; else: sfu_{t,tm,cr\{SM\},ofz,f} = 0 \tag{4-66}$$

Eqs. (4-61) to (4-66) check on a crop-specific basis whether reasonable time windows for different fertilization measures ($TWBF$ to $TWOF$) apply. If yes, a technically reasonable upper limit is defined, otherwise this upper limit is set to 0.

$$if: YPH_{f,t,cr} = 0 \rightarrow uslk_{t,f,cr,nu} = 0 \tag{4-67}$$

Eq. (4-67) controls the slack variable $uslk$ as long as no actual yield YPH has been determined. Otherwise, an uncontrolled negative expansion of $uslk$ would be possible.

Equations in Stage II

$$sfu_{t,tm,cr,fz,f} \geq bin_{t,tm,cr,fz,f} \times LO_{fz} \tag{4-68}$$

$$sfu_{t,tm,cr,fz,f} \leq bin_{t,tm,cr,fz,f} \times UP_{fz} \tag{4-69}$$

Eqs. (4-68) and (4-69) are only considered if $TSTAMP = TLIST_{t,tm}$. Thus, the number of binary variables is reduced to a minimum for computational reasons.

$$sfu0_{t,tm,cr,fz,f} \times sfu_{t,tm,cr,fz,f} = sfu_{t,tm,cr,fz,f} \quad \forall t,tm,cr,fz,f \tag{4-70}$$

$$spd0_{t,tm,mfz2} \times spd_{t,tm,mfz2} = spd_{t,tm,mfz2} \quad \forall t,tm,mfz2 \tag{4-71}$$

$$spdha_{t,tm,mfz} \times hf_{t,tm,mfz} = spd_{t,tm,mfz} \quad \forall t,tm,mfz \tag{4-72}$$

$$\begin{aligned} spdc_{t,tm,mfz2} &= spdcST_{t,tm,mfz2} + spdcLT_{t,tm,mfz2} + spdcFW_{t,tm,mfz2} + spdcVC_{t,tm,mfz2} \\ &\quad + spdcYF_{t,tm,mfz2} + spdcFF_{t,tm,mfz2} \quad \forall t,tm,mfz2 \end{aligned} \tag{4-73}$$

$$spdcST_{t,tm,mfz} = ST \times WE \times spd0_{t,tm,mfz} \quad \forall t,tm,mfz \tag{4-74}$$

$$spdcLT_{t,tm,mfz} = spd_{t,tm,mfz} \times LT \times \left(WE + VL\right) \quad \forall t,tm,mfz \tag{4-75}$$

$$spdcFW_{t,tm,mfz} = \left(\frac{hf_{t,tm,mfz}}{\left(WS \times WW \times \left(1 - TT\right) \times 0.1\right)} \times \left(WE + VT\right) \right) \times spd0_{t,tm,mfz} \quad \forall t,tm,mfz \tag{4-76}$$

$$spdcVC_{t,tm,mfz} = spd_{t,tm,mfz} \times Q \times VS \quad \forall t,tm,mfz \tag{4-77}$$

$$\begin{aligned} spdcYF_{t,tm,mfz} &= 0.9960552 \times spd_{t,tm,mfz} \times 2 \\ &\quad \times \left(\frac{\sum_f \left(HA_f \times C_f \times \sum_{cr} \left(sfu0_{t,tm,cr,mfz,f} \right) \right)}{\left(hf_{t,tm,mfz} + 0.001 \right)} \right) \times \left(WE + VT\right) \quad \forall t,tm,mfz \end{aligned} \tag{4-78}$$

$$spdcFF_{t,tm,mfz} = \left(0.9418345 \times \frac{hf_{t,tm,mfz}}{TFA} - 0.7297727 \times spdha_{t,tm,mfz} \right) \times TF \times \left(WE + VT\right) \quad \forall t,tm,mfz \tag{4-79}$$

Eqs. (4-71) to (4-79) are repeated in order to be able to consider other costs involved in applying liquid fertilizers, if necessary. For this purpose, the variables or indices in the corresponding restrictions are replaced as follows: In all variables, the letter combination *spy* replaces *spd*, which results in new variables. *lqfz* replaces indices *mfz* and *mfz2*.

Bounds in Stage II

These bounds ((4-80)–(4-83)) are defined after the solve statement of Stage I and before the solve statement of stage II:

$$if\ : TSTAMP > TLIST_{t,tm} \rightarrow sfu_{t,tm,cr,fz,f} = PEF_{t,tm,cr,fz,f} \tag{4-80}$$

Eq. (4-80) is a repetition of Eq. (4-60) and simply ensures that no important information is accidentally lost between the two stages.

$$if\ : \sum\nolimits_{cr,f} \left(sfu_{t,tm,cr,fz,f} + sfu_{t,tm-1,cr,fz,f} + sfu_{t,tm-2,cr,fz,f} \right) = 0 \rightarrow bin_{t,tm,cr,fz,f} \wedge sfu_{t,tm,cr,ft,f} = 0 \tag{4-81}$$

Eq. (4-81) checks on a farm-wide basis which fertilizers have been selected in stage I currently and in the last two months. Any fertilizers that were not used in this period are not considered in stage II in the current fertilizer month.

$$if\ : TSTAMP = TLIST_{t,tm} \wedge sfu^{"StageI"}{}_{t,tm,cr,fz,f} = 0 \rightarrow sfu_{t,tm,cr,fz,f} = 0 \tag{4-82}$$

Eq. (4-82) checks on a field-specific basis which fertilizers were not currently used in stage I and also excludes them for stage II in the current fertilizer month.

$$if\ : TSTAMP \leq TLIST_{t,tm} \wedge sfu^{"StageI"}{}_{t,tm,cr,fz,f} \geq LO \rightarrow sfu_{t,tm,cr,fz,f} = sfu^{"StageI"}{}_{t,tm,cr,fz,f} \tag{4-83}$$

Eq. (4-83) refers to the present and the future. Fertilization measures that have exceeded the technically necessary lower limit (*LO*) in stage I are fixed in stage II. However, with each new model run, stage I is able to readjust future fertilization measures, so future fertilization measures in stage II remain flexible for adaptation.

Table A 4-2: Fertilizer Price [€ dt^{-1}] development (August 2015 - April 2017).

Fertilizer / Date	Aug15	Sep15	Oct15	Nov15	Dec15	Jan16	Feb16	Mar16	Apr16	May16	Jun16	Jul16	Aug16	Sep16	Oct16	Nov16	Dec16	Jan17	Feb17	Mar17	Apr17
N Fertilizer																					
Calcium ammonium nitrate (27N)	26.6	26.6	26.6	26.6	27.2	27.2	26.6	26.6	26.6	24.9	21.5	19.5	19.5	18.9	18.9	19.2	19.2	21.5	21.5	23.2	23.2
Calcium ammonium nitrate (27N 4Mg)	26.6	26.6	26.6	26.6	27.2	27.2	26.6	26.6	26.6	24.9	21.5	19.5	19.5	18.9	18.9	19.2	19.2	21.5	21.5	23.2	23.2
Ammonium nitrate urea solution (28N)	30.2	30.2	30.2	30.2	26.5	26.5	26.8	24.2	24.2	26.5	23.5	18.0	18.0	19.5	19.5	19.8	19.8	24.5	24.5	21.5	21.5
Urea (46N)	34.2	34.2	34.2	34.2	33.5	33.5	33.8	33.0	33.0	31.5	28.5	29.0	29.0	25.6	25.6	27.0	27.0	32.0	32.0	33.9	33.5
Calcium ammonium nitrate (24N 6S)	27.6	27.6	27.6	27.6	28.2	28.2	27.6	27.6	27.6	26.6	23.0	20.5	20.5	19.9	19.9	20.2	20.2	23.0	23.0	24.6	24.6
Ammonium sulphate nitrate (26N 13S)	29.9	29.9	29.9	29.9	30.2	30.2	29.9	29.9	29.9	28.9	25.0	21.0	21.0	22.2	22.2	22.5	22.5	25.7	25.7	27.2	24.2
Sulfuric acid ammonia (21N 24S)	23.5	23.5	23.5	23.5	23.8	23.8	23.8	24.0	24.0	23.5	24.0	18.5	18.5	19.1	19.1	19.3	19.3	19.5	19.5	21.0	21.0
Urea (33N 12S)	33.5	33.5	33.5	33.5	36.0	36.0	36.5	36.8	36.8	36.5	37.8	25.0	25.0	26.0	26.0	26.2	26.2	28.5	28.5	29.5	29.5
ENTEC (26N 13S)	34.9	34.9	34.9	34.9	35.5	35.5	34.9	34.9	34.9	34.9	34.9	34.9	34.9	26.7	26.7	27.0	27.0	29.0	29.0	30.6	30.6
Stabilized urea (46N)	42.5	42.5	42.5	42.5	41.6	41.6	42.0	42.3	42.3	42.0	42.3	29.0	29.0	29.0	29.0	29.4	29.4	34.0	34.0	36.0	36.0
Water + Ammonium nitrate urea solution (9N)	9.7	9.7	9.7	9.7	8.5	8.5	8.6	7.7	7.7	8.5	7.5	5.8	5.8	6.2	6.2	6.3	6.3	7.8	7.8	6.9	6.9
NP Fertilizer																					
Diammon phosphate (18N 46P)	51.5	51.5	51.5	51.5	51.0	51.0	49.9	49.0	49.0	48.0	46.0	39.5	39.5	39.5	39.5	38.0	38.0	40.5	40.5	42.7	42.5
NP (20N 20P 2S)	38.9	38.9	38.9	38.9	38.9	38.9	38.9	38.9	38.9	38.0	36.0	29.5	29.5	29.9	29.9	29.9	29.9	31.0	31.0	31.7	31.7
NPK Fertilizer																					
ENTEC (15N 5P 20K 2MgO 8S)	50.5	50.5	50.5	50.5	50.9	50.9	51.5	51.5	51.5	51.5	51.5	51.5	51.5	45.9	45.9	46.2	46.2	48.6	48.6	49.0	49.0
NPK (15N 15P 15K 2S)	38.9	38.9	38.9	38.9	38.9	38.9	38.9	38.9	38.9	38.9	38.9	29.5	29.5	30.9	30.9	31.2	31.2	31.9	31.9	32.7	32.7
NPK (20N 8P 8K 3MgO 4S)	37.0	37.0	37.0	37.0	37.5	37.5	37.5	37.5	37.5	37.5	37.5	37.5	37.5	29.0	29.0	29.2	29.2	30.4	30.4	30.5	30.5
NPK (23 5 5 +6S)	36.5	36.5	36.5	36.5	37.0	37.0	37.0	37.0	37.0	37.0	37.0	37.0	37.0	27.0	27.0	27.2	27.2	28.5	28.5	28.7	28.7
P and PK Fertilizer																					
Triple superphosphate (46P)	41.2	41.2	41.2	41.2	41.2	41.2	41.5	39.0	39.0	41.5	39.0	28.0	28.0	39.5	39.5	39.5	39.5	40.5	40.5	40.5	40.3
PK (16P 16K 2MgO 7S)	27.9	27.9	27.9	27.9	27.9	27.9	27.9	27.9	27.9	27.9	27.9	24.5	24.5	25.5	25.5	25.5	25.5	25.0	25.0	25.3	25.3
K Fertilizer																					
Potash (40K 6MgO 5S)	28.9	28.9	28.9	28.9	29.2	29.2	29.2	29.2	29.2	29.2	29.2	26.2	26.2	27.6	27.6	27.6	27.6	27.5	27.5	27.2	27.2
Kainite (11K 5MgO 4S)	13.6	13.6	13.6	13.6	13.9	13.9	13.9	13.9	13.9	13.2	13.9	13.2	13.2	12.9	12.9	12.9	12.9	13.8	13.8	13.5	13.5
Mg & S Fertilizer																					
Kieserite (25MgO 20S)	29.0	29.0	29.0	29.0	29.0	29.0	29.0	29.0	29.0	29.0	29.0	26.5	26.5	26.5	26.5	26.5	26.5	26.9	26.9	27.5	27.5
Lime Fertilizer																					
Carbonic lime (2S 50CaO)	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4
Carbonic lime (14MgO 53.4CaO)	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0
Burnt lime (90CaO)	11.7	11.7	11.7	11.7	11.7	11.7	11.9	11.9	11.9	11.9	11.9	11.9	11.9	11.9	11.9	11.9	11.9	11.9	11.9	12.2	12.2

Remarks: Sales prices of fertilizers were regularly requested from the local agricultural retailer (Schiebel, 2015 - 2018).

Table A 4-3: Fertilizer Price [€ dt^{-1}] development (May 2017 - December 2018).

Fertilizer / Date	May17	Jun17	Jul17	Aug17	Sep17	Oct17	Nov17	Dec17	Jan18	Feb18	Mar18	Apr18	May18	Jun18	Jul18	Aug18	Sep18	Oct18	Nov18	Dec18
N Fertilizer																				
Calcium ammonium nitrate (27N)	23.0	20.5	19.5	19.5	19.5	20.9	20.9	21.9	21.9	21.9	21.9	21.9	21.0	21.0	21.0	21.0	21.0	21.0	21.0	21.0
Calcium ammonium nitrate (27N 4Mg)	23.0	20.5	19.5	19.5	19.5	20.9	20.9	21.9	21.9	21.9	21.9	21.9	21.0	21.0	21.0	21.0	21.0	21.0	21.0	21.0
Ammonium nitrate urea solution (28N)	21.5	21.0	20.0	20.0	20.0	23.0	23.0	23.7	23.7	23.7	23.7	23.7	23.7	23.7	23.7	23.7	23.7	23.7	23.7	23.7
Urea (46N)	33.5	29.5	26.5	26.5	26.5	29.5	29.5	29.2	29.2	29.2	29.2	29.2	29.2	29.2	29.2	29.2	29.2	29.2	29.2	29.2
Calcium ammonium nitrate (24N 6S)	24.6	21.5	20.5	20.5	20.5	22.4	22.4	23.4	23.4	23.4	23.4	23.4	22.5	22.5	22.5	22.5	22.5	22.5	22.5	22.5
Ammonium sulphate nitrate (26N 13S)	27.2	26.0	22.5	22.5	22.5	24.4	24.4	24.9	24.9	24.9	24.9	24.9	24.0	24.0	24.0	24.0	24.0	24.0	24.0	24.0
Sulfuric acid ammonia (21N 24S)	21.0	21.0	20.0	20.0	20.0	20.5	20.5	22.9	22.9	22.9	22.9	22.9	22.9	22.9	22.9	22.9	22.9	22.9	22.9	22.9
Urea (33N 12S)	29.5	29.5	27.0	27.0	27.0	28.0	28.0	29.5	29.5	29.5	29.5	29.5	29.5	29.5	29.5	29.5	29.5	29.5	29.5	29.5
ENTEC (26N 13S)	30.6	30.6	27.9	27.9	27.9	29.2	29.2	29.2	29.2	29.2	29.2	29.2	29.2	29.2	29.2	29.2	29.2	29.2	29.2	29.2
Stabilized urea (46N)	36.0	36.0	31.0	31.0	31.0	31.0	31.0	35.5	35.5	35.5	35.5	35.5	35.5	35.5	35.5	35.5	35.5	35.5	35.5	35.5
Water + Ammonium nitrate urea solution (9N)	6.9	6.7	6.4	6.4	6.4	7.4	7.4	7.6	7.6	7.6	7.6	7.6	7.6	7.6	7.6	7.6	7.6	7.6	7.6	7.6
NP Fertilizer																				
Diammon phosphate (18N 46P)	43.0	41.5	39.6	39.6	39.6	38.5	38.5	39.5	39.5	39.5	39.5	39.5	41.0	41.0	41.0	41.0	41.0	41.0	41.0	41.0
NP (20N 20P 2S)	32.5	31.5	29.9	29.9	29.9	30.7	30.7	30.7	30.7	30.7	30.7	30.7	30.7	30.7	30.7	30.7	30.7	30.7	30.7	30.7
NPK Fertilizer																				
ENTEC (15N 5P 20K 2MgO 8S)	49.0	49.0	45.5	45.5	45.5	46.0	46.0	47.6	47.6	47.6	47.6	47.6	47.6	47.6	47.6	47.6	47.6	47.6	47.6	47.6
NPK (15N 15P 15K 2S)	32.7	32.7	29.9	29.9	29.9	30.7	30.7	32.2	32.2	32.2	32.2	32.2	32.2	32.2	32.2	32.2	32.2	32.2	32.2	32.2
NPK (20N 8P 8K 3MgO 4S)	30.5	30.5	29.5	29.5	29.5	30.5	30.5	30.2	30.2	30.2	30.2	30.2	30.2	30.2	30.2	30.2	30.2	30.2	30.2	30.2
NPK (23 5 5 +6S)	28.7	28.7	27.5	27.5	27.5	28.7	28.7	30.2	30.2	30.2	30.2	30.2	30.2	30.2	30.2	30.2	30.2	30.2	30.2	30.2
P and PK Fertilizer																				
Triple superphosphate (46P)	40.3	40.3	39.5	39.5	39.5	40.2	40.2	40.5	40.5	40.5	40.5	40.5	40.5	40.5	40.5	40.5	40.5	40.5	40.5	40.5
PK (16P 16K 2MgO 7S)	25.3	25.3	25.0	25.0	25.0	25.7	25.7	25.9	25.9	25.9	25.9	25.9	25.9	25.9	25.9	25.9	25.9	25.9	25.9	25.9
K Fertilizer																				
Potash (40K 6MgO 5S)	27.2	27.2	26.8	26.8	26.8	27.2	27.2	27.2	27.2	27.2	27.2	27.2	27.2	27.2	27.2	27.2	27.2	27.2	27.2	27.2
Kainite (11K 5MgO 4S)	13.5	13.5	13.5	13.5	13.5	13.5	13.5	13.5	13.5	13.5	13.5	13.5	13.5	13.5	13.5	13.5	13.5	13.5	13.5	13.5
Mg & S Fertilizer																				
Kieserite (25MgO 20S)	27.5	27.5	26.2	26.2	26.2	27.5	27.5	27.9	27.9	27.9	27.9	27.9	27.9	27.9	27.9	27.9	27.9	27.9	27.9	27.9
Lime Fertilizer																				
Carbonic lime (2S 50CaO)	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.5	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4	4.4
Carbonic lime (14MgO 53.4CaO)	4.0	4.0	4.0	4.0	4.0	4.0	4.0	3.9	3.9	3.9	4.0	3.9	3.9	3.9	3.9	3.9	3.9	3.9	3.9	3.9
Burnt lime (90CaO)	12.2	12.2	12.2	12.2	12.2	12.2	12.2	12.5	12.5	12.5	12.5	12.5	12.5	12.5	12.5	12.5	12.5	12.5	12.5	12.5

Remarks: Sales prices of fertilizers were regularly requested from the local agricultural retailer (Schiebel, 2015 - 2018).

Table A 4-4: Fertilizer composition, lime effect and specific weight.

Fertilizer / Content	NO3- [%]	NH4+ [%]	Urea [%]	Stabilized NH4+ [%]	Stabilized Urea [%]	P2O5 [%]	K2O [%]	MgO [%]	S [%]	CaO [$kg\ dt^{-1}$]	Specific weight [kg/L]
N Fertilizer											
Calcium ammonium nitrate (27N)[1,6]	13.5	13.5								-15.0	1.0
Calcium ammonium nitrate (27N 4Mg[3,6])	13.5	13.5						4.0		-9.0	1.0
Ammonium nitrate urea solution (28N)[2,6]	7.0	7.0	14.0							-28.0	1.3
Urea (46N)[1,6]			46.0							-46.0	0.8
Calcium ammonium nitrate (24N 6S)[3,6]	12.0	12.0							6.0	-34.0	1.1
Ammonium sulphate nitrate (26N 13S)[1,6]	7.0	19.0							13.0	-49.0	1.0
Sulfuric acid ammonia (21N 24S)[4,6]		21.0							24.0	-63.0	1.0
Urea (33N 12S)[2,6]		10.4	22.6						12.0	-54.0	0.8
ENTEC (26N 13S)[1,6]	7.5			18.5					13.0	-49.0	1.0
Stabilized urea (46N)[2,6]					46.0					-46.0	0.8
Water + Ammonium nitrate urea solution (9N)	2.2	2.2	4.5							-9.0	1.0
NP Fertilizer											
Diammon phosphate (18N 46P)[1,6]		18.0				46.0				-36.0	1.0
NP (20N 20P 2S)[1,6]	8.2	11.8				20.0			2.0	-31.0	1.1
NPK Fertilizer											
ENTEC (15N 5P 20K 2MgO 8S)[1,6]	6.9			8.1		5.0	20.0	2.0	8.0	-14.0	1.2
NPK (15N 15P 15K 2S)[1,6]	6.0	9.0				15.0	15.0		2.0	-15.0	1.1
NPK (20N 8P 8K 3MgO 4S)[5,6]	9.0	11.0				8.0	8.0	3.0	4.0	-21.0	1.1
NPK (23 5 5 +6S)[6]	10.7	13.3				5.0	5.0		6.0	-23.0	1.0
P and PK Fertilizer											
Triple superphosphate (46P)[6]						46.0				-1.0	1.1
PK (16P 16K 2MgO 7S)[7,6]						16.0	16.0	2.0	7.0	6.0	1.1
K Fertilizer											
Potash (40K 6MgO 5S)[8,6]							40.0	6.0	5.0	0.0	1.1
Kainite (11K 5MgO 4S)[6]							11.0	5.0	4.0	0.0	1.2
Mg & S Fertilizer											
Kieserite (25MgO 20S)[8,6]								25.0	20.0	0.0	1.3
Lime Fertilizer											
Carbonic lime (2S 50CaO)[9]									2.0	50.0	1.7
Carbonic lime (14MgO 53.4CaO)[9]								14.0		53.4	1.7
Burnt lime (90CaO)[9]										90.0	1.0

Remarks: The specification of fertilizers can be found in the following sources: [1] (EuroChem Agro GmbH); [2] (SKW Stickstoffwerke Piesteritz GmbH); [3] (YARA GmbH & Co. KG); [4] (DOMO Caproleuna GmbH); [5] (Borealis L.A.T GmbH); [6] (Wendland et al., 2018); [7] (METRAC Handelsgesellschaft mbH); [8] (K+S Minerals and Agriculture GmbH); [9] (DüKa Düngekalkgesellschaft mbH). Note: Nutrient contents are subject to minor changes over time, especially if natural products are included (last update: March 2017).

Table A 4-5: IoFarm fertilizer strategy (application rate [dt ha^{-1}]).

			Field 1	Field 2	Field 3
1st rotation period			**Maize**	**Barley**	**Wheat**
	Aug I	Carbonic lime (14MgO 53.4CaO)	3.04	3.00	
	Feb II	Kieserite (25MgO 20S)			1.05
	Feb II	Carbonic lime (2S 50CaO)		3.00	
	Mar II	Calcium ammonium nitrate (27N 4Mg)			0.80
	Mar II	Urea (46N)		2.58	0.85
	Mar II	Diammon phosphate (18N 46P)		1.08	
	Apr II	Urea (46N)	2.53		
	Apr II	Sulfuric acid ammonia (21N 24S)	1.03		
	May II	Urea (46N)			1.07
	May II	Diammon phosphate (18N 46P)		0.80	1.39
	May II	Sulfuric acid ammonia (21N 24S)		0.80	
	Jun II	Calcium ammonium nitrate (27N 4Mg)		0.80	1.93
	Aug II	Potash (40K 6MgO 5S)			4.71
2nd rotation period			**Wheat**	**Maize**	**Barley**
	Aug I	Triple superphosphate (46P)		3.23	1.3
	Aug I	Carbonic lime (14MgO 53.4CaO)		3.00	
	Oct I	Carbonic lime (14MgO 53.4CaO)	3.00		
	Feb II	Kieserite (25MgO 20S)			0.86
	Mar II	Calcium ammonium nitrate (27N 4Mg)			0.80
	Apr II	Water + Ammonium nitrate urea solution (9N)	2.1		
	Apr II	Calcium ammonium nitrate (27N 4Mg)			3.64
	Apr II	Urea (46N)		3.26	
	Apr II	Sulfuric acid ammonia (21N 24S)		0.92	
	May II	Urea (46N)	1.89		
	May II	Sulfuric acid ammonia (21N 24S)	0.87		
	May II	Water + Ammonium nitrate urea solution (9N)	2.50		
	May II	Calcium ammonium nitrate (27N 4Mg)		0.80	1.54
	May II	Sulfuric acid ammonia (21N 24S)		0.80	
	Jun II	Water + Ammonium nitrate urea solution (9N)	2.20		
	Sep II	Kieserite (25MgO 20S)			0.94
3rd rotation period			**Barley**	**Wheat**	**Maize**
	Feb II	Urea (46N)		1.51	
	Feb II	Diammon phosphate (18N 46P)	4.93	3.73	
	Feb II	Potash (40K 6MgO 5S)		4.19	
	Feb II	Carbonic lime (14MgO 53.4CaO)	6.34	5.25	
	Feb II	Kieserite (25MgO 20S)			1.51
	Mar II	Calcium ammonium nitrate (27N 4Mg)	1.61	0.80	
	Apr II	Calcium ammonium nitrate (27N 4Mg)			1.22
	Apr II	Diammon phosphate (18N 46P)			6.15
	May II	Sulfuric acid ammonia (21N 24S)	0.80		
	Jun II	Calcium ammonium nitrate (27N 4Mg)		0.91	
	Dec II	Potash (40K 6MgO 5S)		0.80	

5 IoFarm in Field Test: Does a Cost-Optimal Choice of Fertilization Influence Yield, Protein Content, and Market Performance in Crop Production?

This article is published as [Tröster, Michael Friedrich; Sauer, Johannes (2021): IoFarm in Field Test: Does a Cost-Optimal Choice of Fertilization Influence Yield, Protein Content, and Market Performance in Crop Production? *Agriculture*, 11, 571] under the terms and conditions of the Creative Commons Attribution (CC BY) license. The version of record is available online at: https://doi.org/10.3390/agriculture11060571.

Authors' contributions: Michael Tröster is the main author of this contribution. Michael Tröster: Conceptualization, methodology, validation, formal analysis, investigation, data curation, writing—original-draft preparation, visualization, project administration. Michael Tröster and Johannes Sauer: Resources, writing—review and editing. Johannes Sauer: Supervision.

Abstract

Decision-support system (DSS) IoFarm was developed to identify economically optimal fertilizer strategies on the farm level. The average cost savings are €66 ha^{-1}. This study aimed to determine whether this approach impacts yield, protein content, and market performance in crop production compared to usual farm-fertilization strategies. Few DSSs for fertilizer optimization consider multiple nutrients. DSSs with a clear focus on both fertilizer intensity and the least-cost combination of fertilizers are even rarer. To the best of our knowledge, there is no information in the literature on the impact of such DSSs on yield, protein content, and market performance for cereal–maize crop rotation. This study determines for the first time whether the financial benefits of using such an optimization tool are in conflict with important agronomic goals. In a three-year field trial, IoFarm was compared to standard farm-fertilization strategies. Results were evaluated with an analysis of variance followed by post hoc tests. No significant differences in yield, protein content, and market performance were found for comparable fertilization variants (with or without organic fertilization). However, differences exist in the selection of fertilizers and the timing of fertilization. Results show the agronomic comparability of IoFarm and usual farm-fertilizer strategies.

Keywords

Fertilizer recommendation; nutrient management; model validation; least-cost combination; decision support; field trial

5.1 Introduction

Agricultural goods are internationally traded on a large scale and are in global competition. The resulting price pressure requires steady adjustments by producers. Therefore, the optimal allocation of available production factors is necessary to achieve the entrepreneurial goal of profit maximization. Before farmers consider changing their production program, they usually first attempt to optimize their production technology. Fertilization is a vital component of production, as numerous current studies show (Tröster and Sauer, 2021b, Tian et al., 2020, Ransom et al., 2020, Hlisnikovský et al., 2020, Mi et al., 2019). About 29% of the variable costs of winter wheat production (WW = *Triticum aestivum* L.) in Bavaria in 2020 are related to fertilizers (Schätzl et al., 2019). Thus, the savings potential that can be achieved by fertilizer optimization is promising. Changing environmental conditions and dynamic changes in input and product prices greatly complicate decisions regarding fertilizer intensity and selection. Farmers are faced with this problem several times in a season. For an economically optimal solution, it is necessary to collect, update, and rationally process all relevant information. This results in high transactional costs that prevent farmers from thinking intensively about an economically optimal fertilizer strategy several times per season. Furthermore, due to the enormous number of combinations of fertilizer, fertilizer quantity, and timing, it is hardly possible to optimally solve this problem without assistance. Therefore, assistance from a decision support system (DSS) is extremely helpful to rationally and objectively deal with such complex decisions. DSS IoFarm (Tröster and Sauer, 2021b) was developed for this purpose. It enables rationally and objectively making complex decisions by taking into account changes in environmental conditions, input, product prices, and the associated application costs when searching for an optimal field-specific fertilization strategy. IoFarm considers a crop production function and is therefore able to regulate the output level in the case that the marginal cost of fertilization exceeds the marginal revenue of crop production. However, the focus of optimization is on identifying the least-cost combination of fertilizers. By simultaneously considering both input intensity and a least-cost combination, IoFarm represents the theoretical concept of the expansion path. A previous study showed that this approach can save both fertilizer costs (−19%) and valuable management time (Tröster and Sauer, 2021b). Of course, the growing conditions of crops are a key factor in the search for the optimal fertilizer strategy. A large number of agronomic restrictions in IoFarm represent these requirements, but verification in practice is still essential. According to the operations research requirements (Mariappan, 2013), this step is closely linked to the development of new models. Therefore, the literature also reports numerous field experiments in which DSSs were tested. For example, a study by Scharf et al. (2011) demonstrated positive effects on maize cultivation for the use of a sensor-based fertilizer system. Additionally, research was conducted on using a decision-support system for agrotechnology transfer (DSSAT) (Jame and Cutforth, 1996). Araya et al. (2019) calibrated a DSSAT system to simulate the effect of fertilization on wheat cultivation in Ethiopia, and Übelhör et al. (2015) developed the CROPGRO system on the basis of DSSAT to derive knowledge on the fertilization of white cabbage in Germany. Additionally, the Nutrient Expert for Wheat system (Chuan et al., 2013) was developed to optimize fertilizer intensity in Chinese wheat production. Successful tools were also developed and tested in other areas of crop production. One example is DSSHerbicide (Sønderskov et al., 2015), which is used to optimize herbicide use. All these DSSs were evaluated in practice or in field trials to show their utility for potential users. The question of economically efficient fertilization was also addressed by Mandrini et al. (2021). Their study focused on 10 different management strategies for corn cultivation in Illinois, which were investigated using the Agricultural Production System Simulator instead of field trials. As a result, they answered which of the tested strategies were preferable under different objectives (economics, ecology). IoFarm differs from the previously mentioned tools by its clear focus on the least-cost combination in fertilizer selection, simultaneous consideration of multiple nutrients, and the possibility of aggregated base fertilization within a crop rotation. The literature also includes several DSSs that have similar approaches and goals to those of IoFarm. These include Smart Fertilizer (Smart Fertilizer Management), Ecofert (Bueno-Delgado et al., 2016), and Optifer (Pagán et al., 2015). To date, no field trials have been published on any of these DSSs. Therefore, it is currently unclear whether the use of such DSS based on a pure economic objective function could be associated with undesirable effects on yield, protein content, and market performance. To address this gap, IoFarm was compared to a standard farm-fertilization strategy in a multiyear field trial. As competing variants in this field trial were based on the same system of nutrient requirements calculated by the Bavarian State Institute of Agriculture (Offenberger and Wendland), nutrient input was largely identical.

This article and the underlying field trial investigate the agronomic performance of IoFarm and highlight the utility of such an optimization tool for potential users. Additionally, the verification of the optimization model in practice is urgently needed to uncover its potential shortcomings and to initiate adaptation measures.

5.2 Materials and Methods

5.2.1 IoFarm Decision-Support System

IoFarm is a novel DSS to reduce fertilizer expenditure on the farm level (Tröster and Sauer, 2021b). The system provides precise guidance on fertilizer selection, application rate, and application timing for each field plot over an entire crop rotation cycle. Through regular updates of fertilizer and product prices, yield expectations, soil test results, and weather information, IoFarm is quickly adapted to changing conditions. To make the most of this ability, IoFarm should be used once a month during the growing season to recalculate the fertilization strategy. IoFarm falls into the category of mixed integer nonlinear problems. The objective function was designed to find the economically optimal fertilizer strategy that satisfies crop requirements. In addition to the market prices of the fertilizers, the application costs are also relevant in this choice. Within the model, marginal revenue and marginal cost are used to determine the optimal nutrient application, and hence yield level. Figure 5-1 provides a general overview of IoFarm’s data input, data processing, and output.

After this general overview of DSS IoFarm, some information on how IoFarm works and how it incorporates data from the biophysical environment is summarized below. The estimation of nitrogen dynamics in the soil is performed with the help of two annual soil tests, soil temperature and climatic water balance (CWB). The first soil test is performed in spring at the beginning of the growing season or before the first fertilization. The second soil test is carried out after harvest. Soil nitrogen content between these two sampling dates is derived from soil temperature (nitrogen mineralization) and CWB (leaching of nitrogen). Both of these measurements are typically recorded by local weather stations, and are therefore available as long-term monthly averages for forecasting purposes. Long-term monthly averages are replaced month by month by actual measured values. This approach to estimate nitrogen dynamics is highly simplified. It can only be justified by regular updates with real measured values for soil nitrogen content and by prioritizing a high level of user friendliness. Scientific models such as HERMES (Kersebaum, 1989), WAVE (Vanclooster et al., 1996), DAISY

(Abrahamsen and Hansen, 2000), or MONICA (Nendel, 2014) are available, but they are too complex for use in the context of a practical DSS.

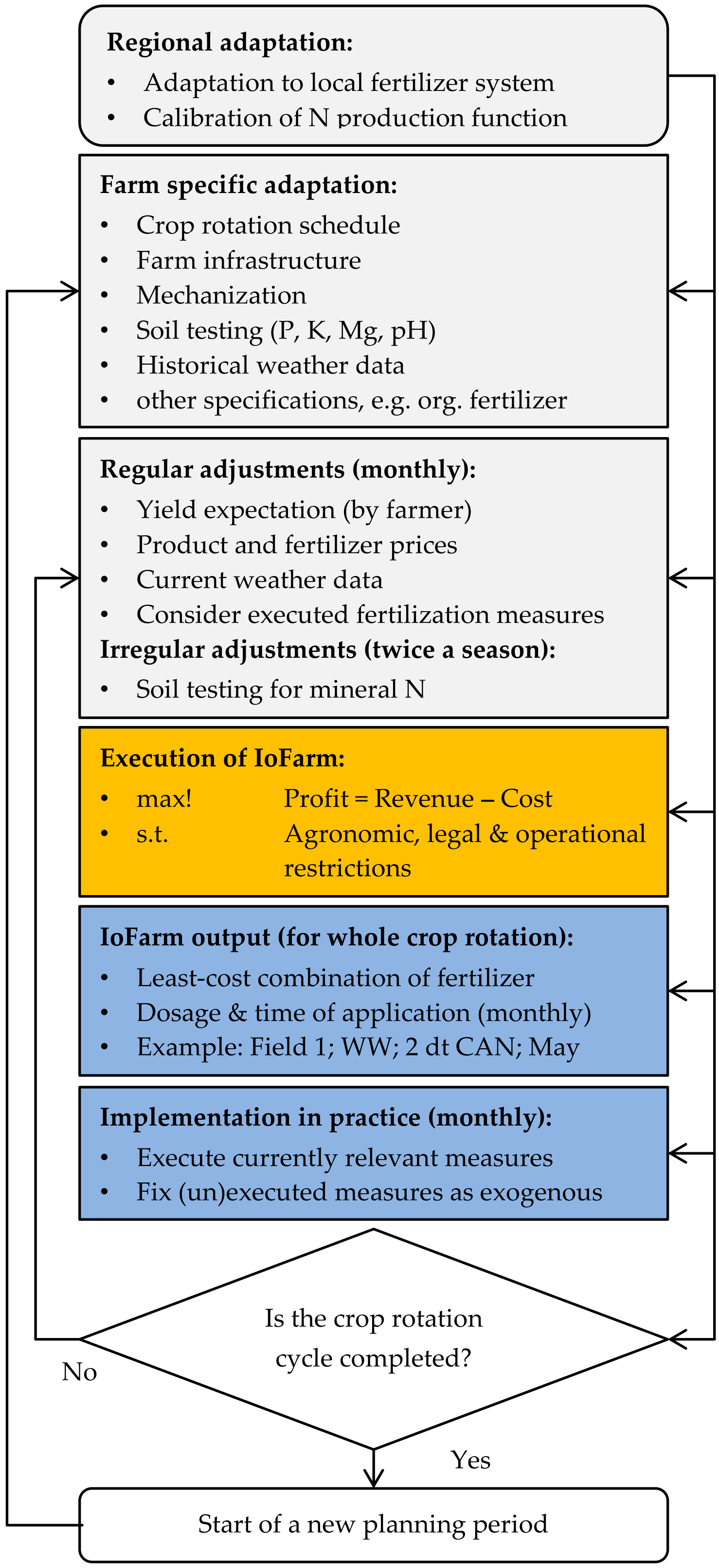


Figure 5-1: Workflow of DSS IoFarm: (grey) data input, (orange) execution, (blue) data output.

To determine the crop-specific nutrient requirements of N, P, K, Mg, and S, IoFarm must be adapted and calibrated to regionally common methods. This means that IoFarm is not an independent fertilizer system, but is based on the specifications of an externally specified fertilizer system, which is required in many countries for legal reasons. IoFarm could be said to just be a problem solver for a given fertilizer system. For the field trial, IoFarm was adapted to the usual nutrient requirement calculation of the trial sites. All three sites were located in Bavaria (southern Germany), so the calculation of requirements in our case was based on the guidelines of the Bavarian State Institute for Agriculture (Wendland et al., 2018).

We now present the basic features of this fertilizer system. The system is based on site-specific yield expectations, which are defined by the farmer or through statistical data. From this, the fertilizer requirement of the nutrients is derived. For nitrogen, the soil supply of available mineral nitrogen at the beginning of the season is deducted. Standard values offer the possibility of taking into account, for example, crop development or N mineralization with additions and deductions. The fertilizer requirement of nutrients P, K, and Mg is also adjusted depending on the location. This is conducted on the basis of soil-test results. If the respective nutrient content is low, additions are applied; the same applies in reverse for high nutrient contents. As a result, this system of determining fertilizer requirements provides information on the quantities of nutrients that can be used per hectare and year. The farmer uses this information to form their own fertilization strategy.

IoFarm largely follows this fertilization system, but additionally calculates an economically optimized fertilization strategy. It is taken into account that fertilization measures can also take place aggregated, in the course of a crop rotation (e.g., potash fertilization). IoFarm differs from the fertilization system described above only in the determination of N requirements. As already described above, nitrogen dynamics in the soil are taken into account in a simplified form within the model. In combination with fertilization, which is also internally determined in the model, it is thus known which N content is available to the plants month by month from the soil. Nitrogen must be allocated to the plants as close as possible to their temporal requirements. In order to account for this, the percentage nitrogen uptake of plants at distinctive developmental stages is estimated on the basis of literature data (Waldren and Flowerday, 1979, Reiner and Dörre, 1992, Lütke Entrup, 2000). In combination, this enables the identification of when and to what extent nitrogen fertilization is required. As crop-yield response function, IoFarm uses a linear function with different slopes depending on the nutrient. The maximum of this function is limited by the yield expectation of the farmers. In summary, the following applies: IoFarm is designed to meet important crop-management requirements for fertilization. An attempt is made to model the nitrogen dynamics in the soil and to synchronize fertilizer application as optimally as possible with the nutrient requirements of the plants over the growing season and the entire crop rotation. This ensures balanced nutrition and avoids overdosing of nutrients. For further details, please refer to the original manuscript (Tröster and Sauer, 2021b).

5.2.2 Site Description and Weather Conditions

The field experiment was conducted over three crop years (2016 to 2018) and at three locations within Bavaria (southern Germany): Geiselsberg (GB; 49°08′ N, 10°50′ E; altitude, 505 m), Triesdorf (TD; 49°11′ N, 10°39′ E; altitude, 430 m), and Roggenstein (RS; 48°11′ N, 11°20′ E; altitude, 514 m). The soil properties at the beginning of the experiment are shown in Table 5-1.

Table 5-1: Soil properties of the three field sites in Bavaria.

Site		**GB**			**TD**			**RS**		
Plots		**{1…9}**	**{10…18}**	**{18…27}**	**{1…15}**	**{16…30}**	**{31…45}**	**{1…9}**	**{10…18}**	**{19…27}**
Soil typ		Cambisol			Planosol			Cambisol		
Soil texture		Loam			Sandy Loam			Silty Clay		
Soil pH		6.6	6.6	6.9*	7.3*	7.3*	7.3*	6.1	6.0	6.0
Usable field capacity	%	17.5	16.2	16.2	12.7	15.5	16.0	24.5	21.8	23.7
Bulk density	g cm^{-3}	1.25	1.27	1.29	1.24	1.33	1.35	1.43	1.45	1.50
Organic matter	%	2.1	2.2	2.9	2.5	2.6	2.4	1.7	1.7	1.7
P_2O_5	mg100g^{-1}	12	6	8	17	19	24	7	7	7
K_2O	mg100g^{-1}	36	28	22	17	18	19	14	15	16
MgO	mg100g^{-1}	9	6	7	20	19	18	4	5	3

* Soil pH is above the desired level, therefore no liming was allowed.

Weather records are based on data from the nearest weather stations (Windsfeld, Triesdorf, and Roggenstein) of the German Weather Service (Agrarmeteorologie Bayern, 2019). Our own precipitation records were used for GB, as deviations were expected due to a distance of about 8000 m to the nearest weather station. Figure 5-2 provides a selection of relevant weather information.

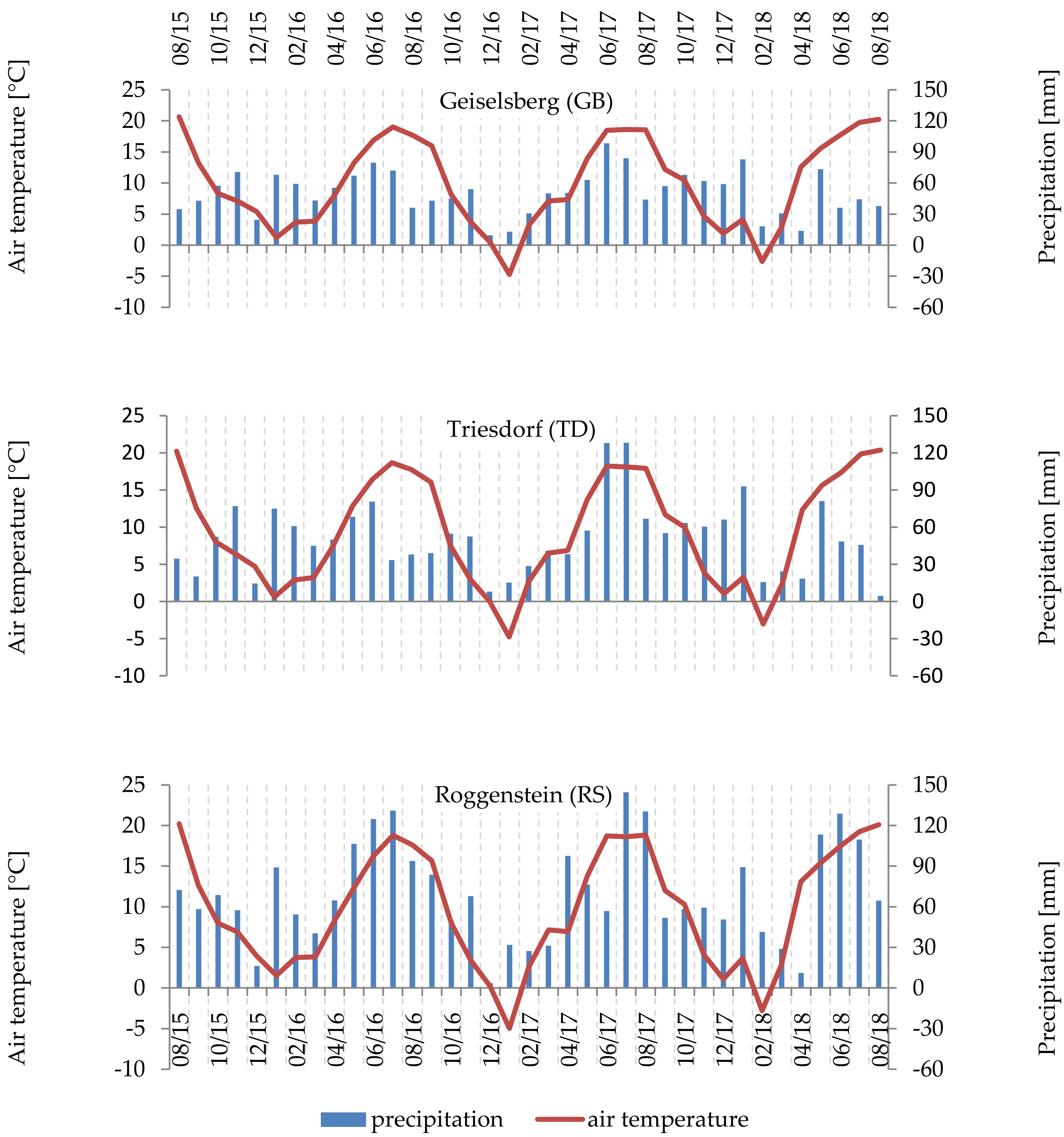


Figure 5-2: Weather conditions during trial period by location.

5.2.3 Field Experiment

The experiment was set up in a two-factorial design with three locations and three crop years. The first factor reflected the fertilization variant and was composed of a farm manager variant (FM), an IoFarm variant (IO), and a control variant (0V) without any fertilization. Additional organic fertilizer in the form of digestate was integrated into the trial at the TD site. Here, additional organomineral variants were created (oFM, oIO). The second factor was formed by different crops that were grown at each site in each year: winter wheat (WW; *Triticum* *aestivum* L.), silage maize (SM; *Zea mays* L.), and winter barley (WB; *Hordeum vulgare* L.). As both plots and variants were fixed over the entire experimental period, the experiment replicated a complete crop rotation cycle on each plot. The experiment was planned in a split-plot design with randomized replications. In GB and TD, the plots were laid out at 12 × 4.5 m, and the crop was harvested in a core area of 9 × 1.38 m for cereals, and 9 × 1.5 m for SM. Due to the different technology, the plot size in RS was 10 × 6 m. Here, the core area was harvested at 10 × 1.56 m for cereals and 10 × 1.5 m for SM. In RS, fertilizer was applied using a lifted drill. However, in GB and TD, fertilizers were applied using a plot spreader with a belt-head dispenser (own construction of the Educational Schools Triesdorf, Weidenbach, Germany). Digestate was only applied in TD using a slurry tank with a trailing shoe applicator (Gülle Zwerg constructed by Zunhammer in Traunreut, Germany), which was specially developed for plot trials. Digestate was applied according to a target in m^3 ha^{-1}. The maximal available amount of digestate was limited to 4000 m^3 per year for the assumed farm area of 150 ha. Repeated analysis of the digestate served to update the nutrient content.

Figure 5-3 presents how the tested fertilizer variants (FM, oFM, IO, oIO) were created. Both the farm managers and IoFarm used the same information.

Process by farm manager (Figure 5-3, left column): For the experiment, a yield expectation was assigned to each crop by the local farm managers. This value was based on empirical values (historical yield data). Subsequently, nutrient requirements were calculated for nitrogen, phosphorus, potash, and magnesium according to specifications of the Bavarian State Institute for Agriculture (Wendland et al., 2018). A description of this fertilizer system can be found in Section 5.2.1. The calculated quantitative and seasonal nutrient target values were passed on to the farm managers together with current fertilizer prices and other shared information. Then, with the help of a planning tool[10], the farm managers defined a ready-to-use fertilizer strategy for all three crops. The objective of the exercise was to select the most cost-effective option from the available fertilizers while satisfying the specified nutrient demands as much as possible. For phosphorus, potash, and magnesium, fertilization was freely allocable within crop rotation. However, lime fertilizers could not be applied to areas with a pH value above the site-specific optimum (compare Table 5-1). In the case of a fertilizer application, a minimal rate of 300 kg ha^{-1} was specified for lime fertilizer, 12.5 m^3 ha^{-1} for digestate, and 80 kg ha^{-1} for all other fertilizers. As application costs also play a role in fertilizer strategy selection, a hypothetical farm was specified with 50 ha of

[10] For more information on the used planning tool, please refer to the following link (accessed on 20 June 2021): https://drive.google.com/file/d/14rBHNKKDuBq8oyeeVUXuek2id1B9z_Dw/view?usp=sharing

WW, 50 ha of WB, and 50 ha of SM. The average field-to-farm distance was set as 7 min. Although this information is irrelevant to the evaluated parameters in this experiment, it is important for determining a particular fertilizer strategy, and is thereby mentioned here. This is how farm managers' mineral and organomineral fertilizer variants (FM and oFM) were generated. This procedure was repeated monthly, starting from the fall sowing season in 2015. The shared information, such as results from soil N_{min} testing and price changes, was constantly updated. Thus, farm managers had the opportunity to adjust their target yield and fertilizer strategy once a month at the beginning of the application period before the fertilizers were applied. However, the possibility to adjust the target yield was rarely used by the farm managers during the trial period. An overview of N_{min} values and adjustments to yield expectations can be found in appendix (Table A 5-2).

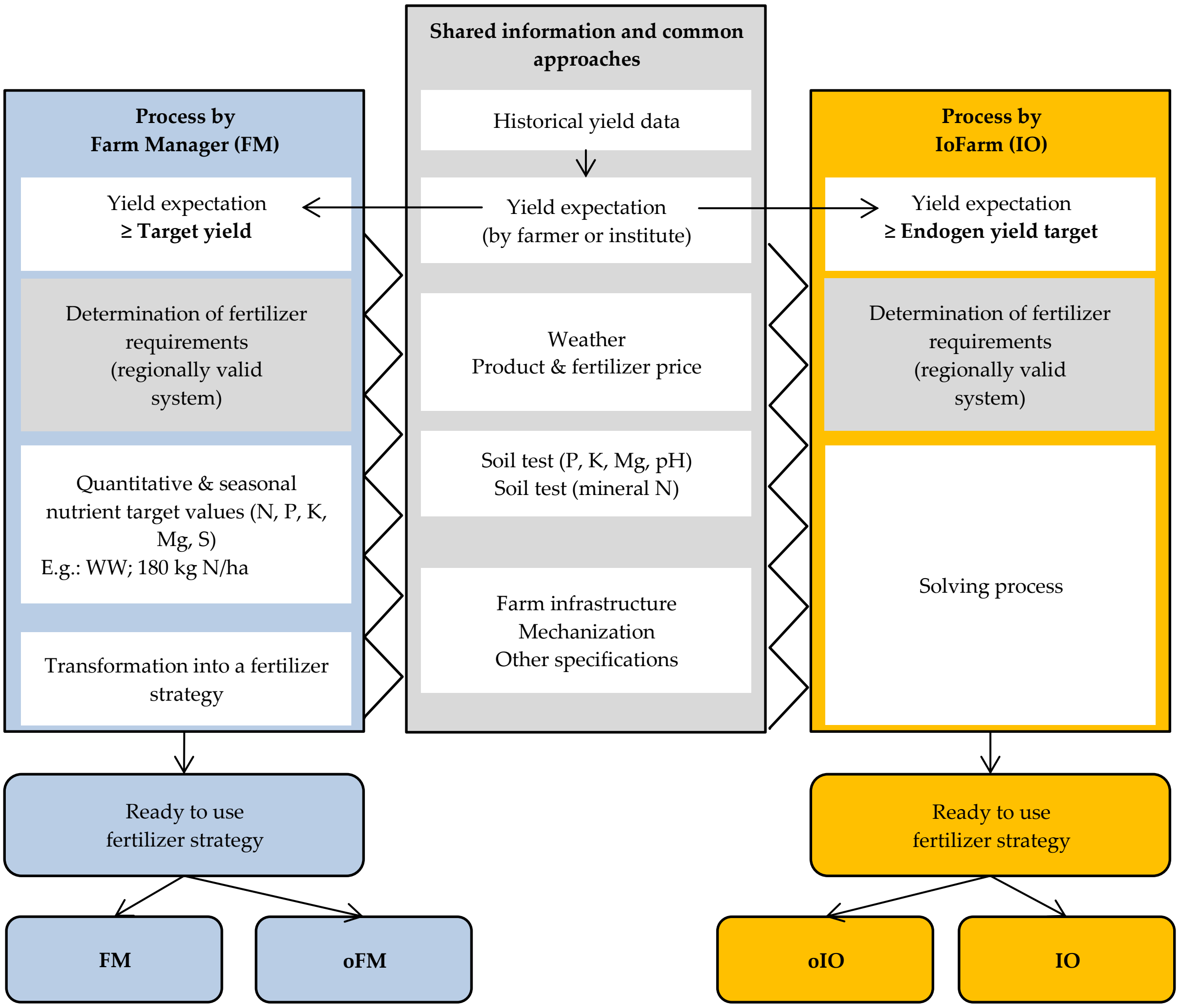


Figure 5-3: Similarities and differences in fertilizer strategy: Farm Manager and IoFarm.

Process by IoFarm (Figure 5-3, right column): Fertilization variants IO and oIO were defined with the help of IoFarm. The endogenous yield target of IoFarm was also limited by yield expectation (updated monthly by farm manager). Again, this was followed by determining the fertilizer requirement and the solving process in which IoFarm calculates the economically optimal fertilizer strategy. Regular updates of the input parameters (shared information) also require a regular repetition of this procedure.

Externally defined yield expectation has great influence on the intensity of fertilization in this system. Reliable yield prediction requires a lot of experience and is only possible relatively late in the growing season. Incorrect predictions lead to biases, but can probably be minimized, by integrating a validated plant-growth model into IoFarm in future. For the sake of usability and comparability, however, we decided to work with the farmers' yield expectation. This approach is quite common and is also used by extension services to achieve a regional differentiation of nutrient supply (Rajsic and Weersink, 2008).

5.2.4 General Cultivation Management

To begin the trial in the first year with a neutral preceding crop, winter oilseed rape was grown in the preceding year in GB and TD. In RS, the preceding crop was spring barley. The soil was tilled with a cultivator, which was used several times if required. A rotary harrow was used to prepare the seedbed. Before sowing SM, an intercrop mixture (25 kg ha^{-1} "Terra Life Aqua pro") was sown in summer. The following seeding information applies to the main crops:

- WB: 320 tsr m^{-2}, KWS Meridian variety approx. 25 September, drill sowing.
- WW: 340 tsr m^{-2}, Patras variety, approx. 5 October, drill sowing.
- SM: 9 tsr m^{-2}, P8589 variety, approx. 25 April, precision seeding, row width 75 cm.

The used varieties are standard regional varieties. Plant protection measures were adapted to the conditions of the respective locations. Weed control was very successful. Fungicide and insecticide measures in the cereals were designed to keep the plants completely healthy. In the first year of cultivation, notable *Ramularia* infections of WB appeared at the RS site, and slight *Septoria tritici* infections of WW were detected at the GB site. Otherwise, disease and pest control was very successful. Fertilization measures were extremely diverse across all sites, crops, and varieties. An overview of all measures, including fertilizer choice, can be found in Table A 5-1 in Appendix. For a detailed differentiation of the fertilizer strategies themselves, we refer to (Tröster and Sauer, under review). To obtain an overview of the quantities of applied nutrients, individual measures are also summarized and compared in Section 5.3.1. Cereal plots were harvested using plot combines (Haldrup c65, Ilshofen

Germany). In GB and TD, SM plots were harvested using a two-row plot chopper equipped with rear container and weighing device. In RS, two maize rows were harvested by hand and then processed on-site with the above-mentioned plot chopper. Straw from the cereal plots remained on the harvested plots, and was afterwards chopped and incorporated. This procedure was repeated for three consecutive years until the crop rotation of WB-SM-WW was completed on each plot.

5.2.5 Crop and Soil Analysis

In fall 2015, detailed analysis of the soil conditions was conducted. All plots were analyzed for phosphorus, potash, magnesium, pH, organic matter, and soil type using standard methods, including calcium-acetate-lactate extraction. In parallel, nine undisturbed soil samples were collected at each site and analyzed for pore volume in the soil laboratory. Soil samples were annually taken at the beginning of the growing season or shortly before sowing SM to a depth of 0 to 30 cm and 30 to 60 cm to determine the supply of mineral nitrogen (N_{min}). This was separately performed for all variants. The results of the soil tests for mineral nitrogen only slightly differed among variants (Table A 5-2 in appendix). In WB and WW, yield structure was also surveyed (for details see Section 5.3.5). Samples of the harvested material from WW and WG were analyzed for water and protein content using near-infrared spectroscopy (Perten DA 7250, PerkinElmer, Waltham, MA). In the case of SM, only dry matter was determined. For this purpose, 200 g samples were taken from each plot and dried for 24 h at 105 °C in a drying oven.

5.2.6 Statistical Analysis

Statistical analysis was performed using STAT software (StataCorp, 2017). The two-factorial experiment (fertilization, crop) was evaluated using ANOVA for different dependent variables. In the case of significant F tests, multiple Tukey's post hoc tests were performed for primary-factor fertilization to determine statistical differences. The Tukey test was chosen because it corrects for alpha error accumulation and is considered to be moderate. The prerequisites of ANOVA were confirmed using the Shapiro–Wilk test to check for normal distribution of residuals, and Levene's test to check for homoscedasticity. In some groups, the data were not normally distributed ($p < 0.05$). However, with a sufficient number of observations per group (central-limit theorem), split-plot ANOVA was considered to be robust to the violation of this condition of a normal distribution of residuals (Salkind, 2010). Partial heteroscedasticity was also found. Various approaches used to transform the variables were inconclusive. The resulting consequences are discussed in Section 5.4, but they are not relevant to the post hoc tests that were performed.

ANOVA for the dependent variable yield (*Y*) was performed using Eq. (5-1).

$$Y_{ijk} = \mu + f_i + r_k + e(F)_{ik} + c_j + (fc)_{ij} + e(FC)_{ijk} \tag{5-1}$$

where *f* represents the *i*-th effect of fertilization, r represents the effect of the *k*-th replicate, and *e(F)* represents the associated *ik*-th error term. Variable *c* represents the *j*-th effect of crop, *fc* is the effect of the interaction of fertilization and crop in the *ij*-th combination, and *e(FC)* is the *ijk*-th error term. Random effects are indicated with capital letters.

In addition to yield, variance analyses were also performed for other dependent variables. Eq. (5-1) was adjusted accordingly:

$$P_{ijk} = \mu + f_i + r_k + e(F)_{ik} + c_j + (fc)_{ij} + e(FC)_{ijk} \tag{5-2}$$

$$MP_{ijk} = \mu + f_i + r_k + e(F)_{ik} + c_j + (fc)_{ij} + e(FC)_{ijk} \tag{5-3}$$

$$Y\_SM_{ik} = \mu + f_i + r_k + e(F)_{ik} \tag{5-4}$$

where the dependent variables from Eqs. (5-2) to (5-4) correspond to: (i) the protein content (*P*) of WB and WW; (ii) market performance (*MP*) taking into account the quality rating of WW; and (iii) the yield of each crop, here substituting *Y_SM* for SM. The underlying values of the variable *MP* are not measured, but were formed according to Eq. (5-5).

$$MP_{c,t,pl} = y_{c,t,pl} \times Py_{c,t} \tag{5-5}$$

where *y* represents the yield of the *c*-th crop in the *t*-th year on the *pl*-th plot. *Py* represents the crop- and year-specific price, which, in the case of wheat, additionally depends on protein content.

5.3 Results

The basis for the interpretation of the results is a comparison of the nutrient supply of the test variants (Table 5-2). Results themselves show the influence of the IoFarm DSS on yield, quality, and market performance compared with the standard fertilizer strategy of a farm manager. Only marginal differences were found.

5.3.1 Comparison of Nutrient Supply and Fertilizer Use

As Table 5-2 shows, the site-specific nutrient supply of the test variants only slightly differed.

Table 5-2: Comparative overview of nutrient supply by location, crop, and treatment.

Site:	Geiselsberg		Roggenstein		Triesdorf		Triesdorf	
Treatment:	**FM**	**IO**	**FM**	**IO**	**FM**	**IO**	**oFM**	**oIO**
Silage maize								
$N + N_{min}$	199	193	186	231	190	199	196	209
P_2O_5	146	116	140	149	46	85	72	80
K_2O	93	11	407	219	77	138	167	220
MgO	100	74	108	99	27	25	44	48
S	12	36	48	34	22	27	38	25
Winter barley								
$N + N_{min}$	201	204	188	211	206	209	217	223
P_2O_5	116	161	161	125	52	71	37	52
K_2O	0	73	0	83	141	95	81	109
MgO	28	57	92	103	27	30	19	15
S	5	19	22	22	90	22	31	21
Winter wheat								
$N + N_{min}$	235	234	247	242	236	220	234	190
P_2O_5	130	122	127	151	119	58	102	70
K_2O	67	79	0	111	199	194	167	65
MgO	73	67	73	84	43	44	44	28
S	30	28	25	26	105	34	70	25
Total crop rotation								
$N + N_{min}$	212	210	207	228	211	210	216	207
P_2O_5	131	133	143	142	72	71	70	67
K_2O	53	54	136	138	139	143	138	132
MgO	67	66	91	95	32	33	36	30
S	16	28	32	27	72	28	46	24

All average values in kg ha−1. FM = farm manager; IO = IoFarm; oFM and oIO were additionally treated with organic fertilizer. N + Nmin = nitrogen fertilization + soil nitrogen content (soil test in spring).

Larger deviations were found in the nitrogen fertilization of winter barley and silage maize at the Roggenstein site because of the different nitrogen sources: CAN was mainly used in the FM variant, while urea dominated in the IO variant due to relative price advantages. The higher gaseous losses of urea were taken into account by IoFarm through increased nitrogen fertilization. However, the emission targets of the European Union include a reduction in ammonia emissions (European Parliament, 2016). In this context, urea fertilization is problematic and could possibly be further restricted. There were also major differences in the nitrogen supply to wheat in the oFM and oIO variants. Here, significantly less nitrogen was applied in the oIO variant. In contrast to the oFM variant, in the oIO variant, organic fertilization in wheat was divided into two applications in two out of three years, whereby better nitrogen utilization of the organic fertilization could be assumed. Further deviations can be seen in the sulfur fertilization at the Triesdorf site. Here, fertilization in the FM variant was above the requirements, perhaps because of the complexity of the optimization problem itself. The farm manager had difficulties in defining a fertilization strategy in which all nutrients were applied in sufficient quantities.

The most important nitrogen fertilizer by volume in the IO variant was urea at all sites (39% to 52% of total N fertilization). In FM variants, farm managers relied on different nitrogen fertilizers, including CAN (GB and TD_{oFM}), DAP (RS), and urea (TD_{FM}). Phosphate supply was predominantly provided by DAP, while TSP or PK 16 + 16 was used to a greater extent at only two sites in the FM variant. Potash supply took place almost entirely with grain potash. For more detailed analysis of the fertilizer strategy of IoFarm, a separate study is planned (Tröster and Sauer, under review).

5.3.2 Analysis of Variance

ANOVA results are presented in Table 5-3 and considered in more detail below.

Table 5-3: Analyses of variance for yield, protein content, and market performance.

Dependent Variable	***Y* (Yield)**		***P* (Protein)**		***MP* (Revenue)**		***Y_SM* (Yield)**		***Y_WB* (Yield)**		***Y_WW* (Yield)**	
	F	***p***	***F***	***p***	***F***	***p***	***F***	***p***	***F***	***p***	***F***	***p***
Model	32.1	0.000	13.1	0.000	16.5	0.000	3.2	0.000	17.1	0.000	26.9	0.000
f	513.9	0.000	424.8	0.000	798.6	0.000	64.4	0.000	590.7	0.000	655.6	0.000
r	0.5	0.613	1.8	0.233	0.6	0.581	0.5	0.646	1.2	0.343	0.9	0.453
e(F#R)												
c	2772.2	0.000	3797.9	0.000	386.8	0.000	---	---	---	---	---	---
c#f	5.4	0.001	41.2	0.000	39.3	0.000	---	---	---	---	---	---
e(R#C#F)												
Obs.	297		198		297		99		99		99	
Adj R^2	0.822		0.640		0.697		0.240		0.697		0.788	

Column 1 shows the model structure. Fixed factors are in lower case, random factors are in upper case: *f*, fertilization; *r*, replications; and *c*, culture. Interactions are indicated by #. Dependent variable under investigation is defined by the column headings.

The influence of dependent variable *f* (fertilization) was significant in all models ($p < 0.001$). When factor *c* (crop) was included in the models, it was also significant ($p < 0.001$). As expected, variable *c* also explained a large part of the found variance, since differences in mean yield among SM (180.1 dt ha^{-1}), WB (80.4 dt ha^{-1}), and WW (80.6 dt ha^{-1}) were very high. In the first two models, the interpretation of the primary factors was biased due to a significant interaction term of fertilization and crop. Thus, there were interactions between these two factors that suggested that the fertilization factor was not equally effective in all crops. Closer data analysis (Figure 5-4) shows that SM responded with lower yield increases to the fertilization factor compared to the two other crops. This finding explains the significant interaction term. Additionally, the adjusted coefficient of determination indicates that the models were able to explain a large part of the found variance. Only the model for SM yield (*Y_SM*) was an exception, with a coefficient of determination of 0.240. Fertilization had a significant effect, but it is likely that unobserved effects, such as environmental influences, played a much larger role in this model than they did in the other models.

ANOVA confirmed the significance of fertilization in determining yield and differences in yield at different fertilization levels. A pairwise comparison of means (Tukey test) in combination with a box-plot diagram illustrates the yield differences within crops, differentiated by fertilizer level (see Figure 5-4).

Regardless of crop, there was a significant effect of fertilization compared with in the control (0V). However, for the evaluation of DSS IoFarm, direct comparisons of the variants were necessary. For FM and IO, the location and dispersion parameters in the box-plot diagram indicated that no significant differences were to be expected, which was also statistically proven (Table 5-4). For variants oFM and oIO in which additional organic fertilization was applied, slight negative yield effects were evident compared with those in purely mineral fertilization variants. In all cases, there were no significant differences between the two organomineral fertilized variants. For SM, only variant oIO was not significantly different from the control. For WW, there were significant differences between the oIO variant and the mineral variants, which was not the case for the oFM variant. These observations indicate that there could be slight disadvantages to using IoFarm in the case of organomineral fertilization. For a more detailed assessment of the results, the grouped mean values of the yields, the standard errors (SE), and the classification into Tukey groups are provided in Table 5-4.

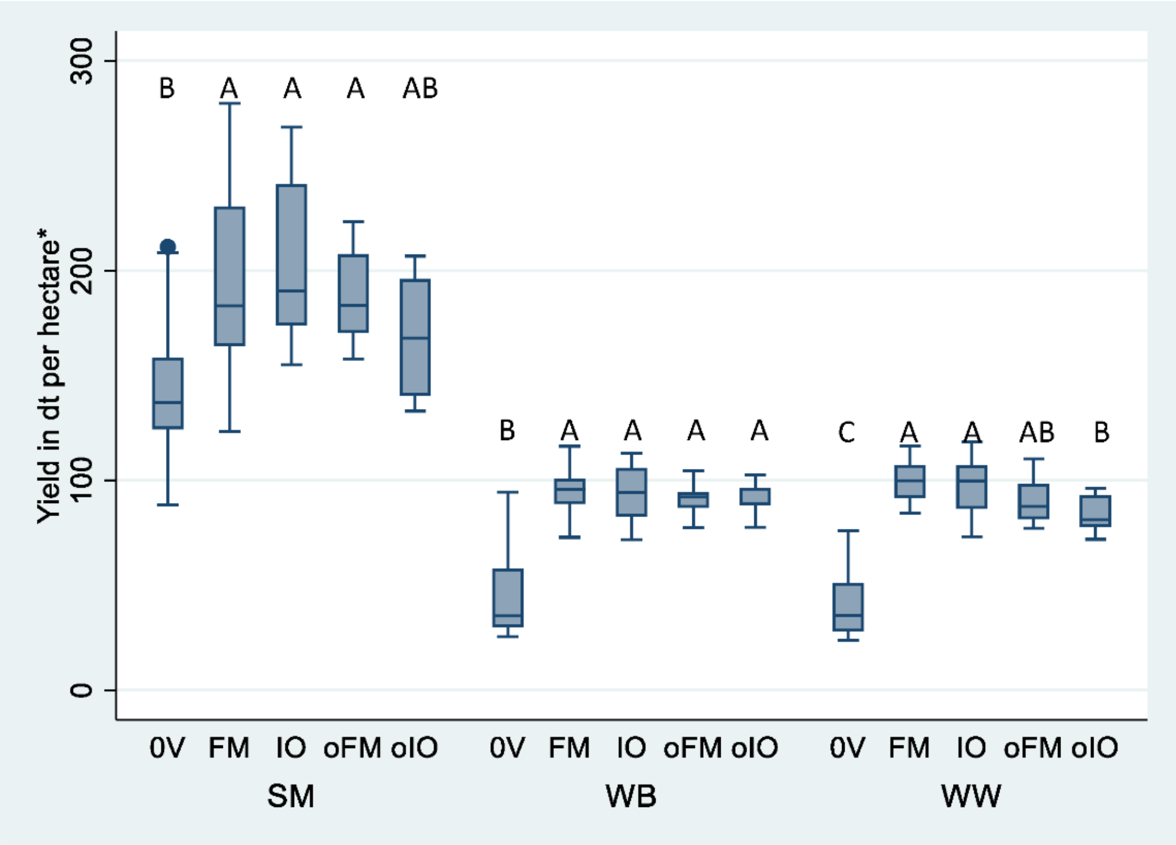


Figure 5-4: Location and dispersion measures for different crop yields grouped by level of fertilizer application (n = 297).

Levels of fertilization: 0V = no fertilization (n = 3 × 27); FM = farm manager variant (n = 3 × 27), IO = model variant (n = 3 × 27); oFM = FM + organic fertilization (n = 3 × 9); oIO = IO + organic fertilization (n = 3 × 9). Within a crop (SM, WB, and WW), the following applied: means sharing a letter in the group were not significantly different at the 5% level. * For SM, yield refers to dry matter; for WB and WW, yield was corrected to 86% of dry-matter content.

Table 5-4: Mean values (Ø), standard errors (SE), and Tukey groups (Gr.) for protein content (*P*), market performance (*MP*), and crop yields (*Y*) for five factor levels of fertilization.

	P			*MP*			*Y_SM*			*Y_WB*			*Y_WW*		
	Ø	**SE**	**Gr.**	**Ø**	**SE**	**Gr.**	**Ø**	**SE**	**Gr.**	**Ø**	**SE**	**Gr.**	**Ø**	**SE**	**Gr.**
0V	9.17	0.22	B	760.5	32.5	C	142.1	6.85	B	44.7	2.58	B	40.9	2.33	C
FM	12.13	0.23	A	1442.9	32.6	A	195.4	6.85	A	95.3	2.58	A	99.6	2.33	A
IO	11.96	0.24	A	1440.2	32.7	A	203.1	6.85	A	93.8	2.58	A	97.5	2.33	A
oFM	11.11	0.38	A	1338.0	56.2	AB	188.1	11.87	A	91.5	4.47	A	89.5	4.04	AB
oIO	11.03	0.38	A	1256.2	56.2	B	171.3	11.87	AB	91.7	4.47	A	83.7	4.04	B

Means sharing a letter in the group were not significantly different at the 5% level.

5.3.3 Effects on Protein Content in Cereals

Overall, protein content plays an important role in determining the market and feed value of cereals, and it is particularly influenced by nitrogen fertilization. Therefore, it is important to measure this quality parameter when comparing fertilizer systems. ANOVA (Table 5-3) showed that the primary effects that were tested (fertilization, crop, and their interaction) had significant influence on the protein content in cereal grains. A pairwise comparison of means (Tukey test) illustrates the differences in cereal protein content, differentiated by fertilizer level (Table 5-4, Column *P*). There were no significant differences in the protein content in the dry matter of all fertilized variants. They only differed significantly from the nonfertilized control. Nevertheless, cereal protein content tended to be somewhat lower in the IoFarm variants. Dilution effects could be excluded in view of the observed yields. Due to the comparable fertilization intensity of the treatments, a possible effect on the protein content is best sought in the dosage and timing of late fertilization.

5.3.4 Effects of IoFarm Decision Support System on Market Performance

From an economic point of view, it is useful to determine whether fertilization decisions made with the help of IoFarm can achieve comparable market performance to that of fertilization strategies decided by farm managers. This was determined by comparing market performance (calculated according to Eq. (5-5)). For WW, protein content was also used to indicate quality, which determines the market price. A complete overview of the underlying market prices is provided in Table 5-5.

Table 5-5: Overview of postharvest prices in 2016 to 2018 for silage maize (SM), winter barley (WB), and winter wheat (WW).

Crop	Year	2016	2017	2018
SM	€ (dt DM)$^{-1}$	8.13	8.00	8.20
WB	€ dt^{-1}	11.68	12.60	14.36
WW <12% XP	€ dt^{-1}	12.62	14.16	14.90
WW >12% XP	€ dt^{-1}	14.01	14.73	15.41
WW >13% XP	€ dt^{-1}	14.52	15.21	15.97
WW >14% XP	€ dt^{-1}	15.80	16.74	17.27

XP: Protein content in dry matter.

ANOVA indicated that fertilization had significant influence on market performance (Table 5-3). The market performance of variants FM and IO (Table 5-4, Column *MP*) could not be statistically distinguished from each other, indicating that IoFarm did not lead to any difference in market performance in the case of these two variants. However, the market performance of organomineral variant oIO was significantly lower than that of FM and IO, but not significantly different to oFM.

5.3.5 Effects on Yield Components

Less relevant for the economic evaluation of DSS IoFarm is its influence on the yield components in cereals. From a crop-production perspective, however, important relationships become visible with regard to the yield components. These are presented in Table 5-6 and thus enable a more detailed agronomic interpretation of the results.

Table 5-6: Yield components of winter barley and winter wheat.

Variant:	0V	FM	IO	oFM	oIO
Winter barley		Thousand-grain mass (g)			
GB	45	46	46		
RS	42	45	49		
TD	47	49	49	49	49
Winter wheat					
GB	55	50	51		
RS	47	51	51		
TD	53	54	56	55	56
Winter barley		Spikes per square meter (number)			
GB	484	721	756		
RS	357	609	622		
TD	392	732	657	665	611
Winter wheat					
GB	372	602	544		
RS	466	568	502		
TD	309	483	452	470	449
Winter barley		Grains per spike (number)			
GB	27	31	29		
RS	21	34	29		
TD	25	28	31	29	31
Winter wheat					
GB	29	34	37		
RS	16	37	41		
TD	22	36	35	35	34

Variants: 0V = control; FM = farm manager; IO = IoFarm; oFM and oIO additionally treated with organic fertilizer. Sites: GB = Geiselsberg; RS = Roggenstein; TD = Triesdorf.

Apart from the control, the differences in thousand-grain weight were moderate. The main differences between the FM and IO or oFM and oIO variants relate to the number of spikes per square meter and the grains per spike. On the basis of this observation, it can be concluded that the timing or synchronization between nitrogen fertilization and nitrogen uptake was different among test varieties. A model-internal consideration of variety characteristics in IoFarm could lead to significant improvements, and possibly contribute to the stabilization of the yield reliability of IoFarm.

5.4 Discussion

The main purpose of this study was to compare the agronomic performance of the IoFarm DSS with a standard farm-fertilization strategy in a field trial. From this, it was deduced whether fertilization strategies calculated by IoFarm or by similar DSSs could be expected to have agronomic impact. Results from Table 5-4 (compared Tukey groups) showed that there were no significant effects of yield, protein content, and market performance within comparable variants, with and without organic fertilization. Hence, IoFarm does not impair agronomic outcomes. The literature does not provide any studies on the agronomic effects of DSSs with similar objectives. DSSs with similar objectives are considered to be that by Pagán et al. (2015), by Bueno-Delgado et al. (2016), Smart Fertilizer (Smart Fertilizer Management), and by Villalobos et al. (2020). They also have a clear focus on the least-cost combination of fertilizers, and consider at least nutrients nitrogen, phosphorus, and potash in parallel. In contrast, numerous other studies mainly deal with the optimal intensity of fertilization and provide valuable knowledge in this area: For example, Wu and Ma (2015), who state in their review that integrated nutrient management is of great importance for global crop productivity, or Rajsic and Weersink (2008), and Mandrini et al. (2021), focusing on economically optimal nitrogen supply. More broadly, some field-tested DSSs that simulate or recommend the use of inputs in crop production were studied and found to be useful in enabling agronomic performance (Araya et al., 2019, Übelhör et al., 2015, Chuan et al., 2013, Sønderskov et al., 2015). These studies are based on crop-growth models, or apply ex ante versus ex post analysis. However, an estimation of potential agronomic effects caused by a primarily cost-optimized fertilization strategy (as, e.g., in IoFarm) is not possible with the help of these studies. This study fills that gap and shows that a primarily cost-optimized fertilization strategy can keep up the pace with a standard farm-fertilization strategy from an agronomic perspective.

Before further discussion of the results, some limitations should be noted: due to the relatively high variance of the dependent variable within the control variant, the requirements of ANOVA for homoscedasticity were not met in some groups. Efforts to reduce variance by different transformations were unfortunately not successful. If included in the respective model, interactions of the main factors of *f* fertilization and *c* crop were always significant. Strictly speaking, both observations led to an invalid interpretation of ANOVA. The problem of partial heteroscedasticity can be avoided by excluding the control variant from data analysis. However, our focus was on the comparison of the factor levels of fertilization. The Tukey test can be reliably used under these conditions, which is why we decided not to exclude the control variant. For comparison purposes, all group means were checked in parallel with an unadjusted least-significant-difference t-test. Even with this more liberal test, no significant differences were found between comparable variants FM and IO or oFM and oIO.

Predictably, the purely mineral fertilization variants (FM and IO) and the organomineral variants (oFM and oIO) only slightly differed from each other on the basis of their group mean values. Therefore, care was taken in the experimental design to test them under as many environmental conditions (year and location) as possible to obtain enough observations for comparison. The relative standard error indicated, among other things, whether the number of observations were sufficient to clarify the experimental question. In the case of the mineral-fertilized variants, the relative standard error of the yield across all crops was 2.3% to 3.5%. This allowed for a suitable estimation of the significance levels, which again confirmed that no yield effects were expected from using IoFarm instead of standard farm-manager decisions. For organomineral variants, the range of the relative standard error was significantly higher, at 4.5% to 6.9%. Therefore, additional observations are necessary to make a more robust estimation of the significance levels for these variants. This was not possible in the field trial because the necessary plot technology for digestate application was only available in TD. Comparing variants oFM and oIO was also affected by weather and possible fluctuations in the nutrient content in the used digestate: for the fictitious 150 ha farm, 4000 m^3 of digestate was available per year, which could be allocated to the crops almost freely in terms of quantity and timing. Thus, it was not possible to guarantee homogeneous weather conditions and homogeneous nutrient content in the digestate between the two variants, which inevitably led to unobserved influences on the nutrient supply. In sum, the comparison of the organomineral variants was significantly weakened. However, findings tend to indicate that farm managers were able to better integrate the digestate into their fertilization planning than in the IoFarm model. Therefore, it might make sense to leave the planning of organic fertilization to the farmer, and to consider this as an external specification in IoFarm, so that operational conditions, such as trafficability of the fields or storage capacities, can also be taken into account. Alternatively, it would be conceivable to adopt such restrictions in IoFarm and redefine the effectiveness of organic fertilizers within the model.

Market performance must be considered to evaluate the economic performance of IoFarm. However, volatile prices add another random factor: changes in price relations influence the contrast between group means, and could also influence whether there are significant differences between groups. It is also possible that farms use the entire grain yield for feed purposes and do not receive the market value, making it necessary to include a substitution value. In this case, the protein content of WB would also affect the substitution value. Analysis of yield, protein content, and market performance led to a largely consistent trend in differences between treatment groups. Therefore, despite the mentioned limitations, it could be assumed that moderate price or value changes did not have a significant influence on the assessment of market performance.

The financial-savings potential of using IoFarm was investigated in an independent experiment (Tröster and Sauer, 2021b). Results showed that the IoFarm DSS leads to an average cost saving of €66 ha^{-1}. This savings potential is mainly based on the least-cost combination of fertilizers, at largely identical nutrient inputs. In comparison, according to (Evangelou et al., 2020), the savings potential of sensor-based fertilizer systems ranges from €33 to €92 ha^{-1}, whereas manufacturing companies assume savings of €20 to €30 ha^{-1}. At least comparable results were obtained using IoFarm without additional technical equipment. As no significant differences were found in yield, protein content, and market performance for the mineral variants, the mentioned cost advantage could be fully attributed to using the IoFarm DSS. In the case of organomineral variants, the reliability of the results was less robust. In a direct comparison, variants oFM and oIO were not found to be significantly different. However, in contrast to oFM, oIO was somewhat behind the mineral-fertilized variants in terms of production. Therefore, if organic fertilizers are used, the oFM variant tends to have an advantage from an agronomic point of view. The actual extent of this difference and whether it is compensated for by the cost optimization of the fertilization strategy requires further investigation.

5.5 Conclusions

Our findings and the previous literature indicate that carefully developed DSSs are able to provide superior solutions in complex situations. When optimizing a fertilization strategy, IoFarm considers a large amount of information and restrictions, which is not possible for decision makers to process. Through this computation ability, IoFarm can save fertilizer costs without having to accept a reduction in yield and quality. Therefore, a cost-optimized fertilization strategy is not fundamentally in conflict with other agronomic objectives. The benefits for farmers and their advisors are evident: lower costs with the same levels of market performance. Since the search for a least-cost fertilization strategy is of global importance, the results of this study are also of international interest. However, by adapting the objective function, further objectives could also be achieved using IoFarm: instead of a least-cost fertilization strategy, minimizing the CO_2 footprint associated with fertilization could also be optimized. Therefore, CO_2-efficient fertilization strategies could be developed, which is important in the context of climate change, both socially and internationally. However, further research is needed to determine CO_2 emissions caused by individual fertilizers. Currently, it is necessary to expand the range of available crops in the IoFarm DDS to enable broad applicability for farmers and consultants. The final goal is to enable farmers to directly use IoFarm. For this purpose, the data exchange must be performed via an online platform. For a high level of user friendliness, it is important that digitally available farm data can be imported. The calculation of an optimal fertilizer strategy is then carried out via external servers with high computing capacity. The result is stored in the online platform and made digitally available to farmers in the form of a fertilization strategy.

5.6 Appendix

Table A 5-1: Detailed documentation of fertilizer application in dt per hectare (1 dt = 100 kg).

Geiselsberg: 2016		**IO**				**FM**		
	Fertilizer Code*	SM	WB	WW	Fertilizer Code	SM	WB	WW
Mar	12: 18,46,0,0,0,−36		2.6		02: 27,0,0,4,0,−9		2.5	2.5
	21: 0,0,40,6,5,0		3.3					
	26: 0,0,0,14,0,53		3.0					
Apr	12: 18,46,0,0,0,−36			1.8	02: 27,0,0,4,0,−9		1.0	
	24: 0,0,0,25,20,0			0.8	19: 0,0,46,0,0,−1		2.5	
	04: 46,0,0,0,0,−46		1.2		07: 21,0,0,0,24,−63			1.5
					12: 18,46,0,0,0,−36			2.0
May	04: 46,0,0,0,0,−46	2.1		1.7	02: 27,0,0,4,0,−9		2.0	
	21: 0,0,40,6,5,0		1.4	4.7	04: 46,0,0,0,0,−46	3.0		
	12: 18,46,0,0,0,−36	2.4	2.6		12: 18,46,0,0,0,−36	2.0		
	07: 21,0,0,0,24,−63	0.8						
	25: 0,0,0,0,2,50	3.0						
	26: 0,0,0,14,0,53	6.1						
Jun	04: 46,0,0,0,0,−46			1.1	02: 27,0,0,4,0,−9			2.0
Jul	12: 18,46,0,0,0,−36			1.1				

Geiselsberg: 2017		**IO**				**FM**		
	Fertilizer Code*	SM	WB	WW	Fertilizer Code*	SM	WB	WW
Aug	19: 0,0,46,0,0,−1		2.9	4.5				
Oct	26: 0,0,0,14,0,53	3.0	3.0					
Nov					26: 0,0,0,14,0,53	6.0	6.0	
Mar	02: 27,0,0,4,0,−9			1.3	02: 27,0,0,4,0,−9	2.5		
	21: 0,0,40,6,5,0		0.8	0.8	19: 0,0,46,0,0,−1	1.5		
	24: 0,0,0,25,20,0			0.8	13: 20,20,0,0,0,−31			3.5
Apr	07: 21,0,0,0,24,−63	1.0			13: 20,20,0,0,0,−31			3.5
	02: 27,0,0,4,0,−9			2.5	02: 27,0,0,4,0,−9	2.0		
					21: 0,0,40,6,5,0		2.0	
May	04: 46,0,0,0,0,−46	2.0	3.0		02: 27,0,0,4,0,−9	1.0		
	12: 18,46,0,0,0,−36	2.5			04: 46,0,0,0,0,−46		2.0	
	02: 27,0,0,4,0,−9			2.1	12: 18,46,0,0,0,−36		3.0	
	07: 21,0,0,0,24,−63		0.9					

Continuation Table of A5-1. Detailed documentation of fertilizer application in dt per hectare (1dt = 100kg).

Geiselsberg: 2018		**IO**				**FM**		
	Fertilizer Code*	SM	WB	WW	Fertilizer Code*	SM	WB	WW
Mar	04: 46,0,0,0,0,−46		1.5		02: 27,0,0,4,0,−9	2.5	3	
	12: 18,46,0,0,0,−36		2.4		12: 18,46,0,0,0,−36	2		
	04: 46,0,0,0,0,−46	2			26: 0,0,0,14,0,53	3		
	07: 21,0,0,0,24,−63	0.8			19: 0,0,46,0,0,−1		5	
	12: 18,46,0,0,0,−36	0.8			22: 0,0,40,6,5,0		5	
	26: 0,0,0,14,0,53	3.7			24: 0,0,0,25,20,0		1.5	
Apr	07: 21,0,0,0,24,−63		0.8		04: 46,0,0,0,0,−46			1.7
	26: 0,0,0,14,0,53		7.3		12: 18,46,0,0,0,−36			4.5
	02: 27,0,0,4,0,−9			4.7	21: 0,0,40,6,5,0			5
	12: 18,46,0,0,0,−36			2.2	26: 0,0,0,14,0,53			13
May	04: 46,0,0,0,0,−46	0.8	1.3		02: 27,0,0,4,0,−9	2.3	2.5	
	21: 0,0,40,6,5,0		1.2					
	24: 0,0,0,25,20,0			2.9				
Jun					02: 27,0,0,4,0,−9		1.5	

Triesdorf: 2016		**IO**				**FM**		
	Fertilizer Code*	SM	WB	WW	Fertilizer Code*	SM	WB	WW
Mar	04: 46,0,0,0,0,−46		1.3	0.8	15: 15,15,15,0,2,−15			4.0
	12: 18,46,0,0,0,−36		0.8		17: 23,5,5,0,6,−23		2.5	
	21: 0,0,40,6,5,0		4.8					
Apr	21: 0,0,40,6,5,0	8.4		5.6	04: 46,0,0,0,0,−46	2.5		
	12: 18,46,0,0,0,−36		0.9		12: 18,46,0,0,0,−36	2.0		
					06: 26,0,0,0,13,−49		2.0	
May	04: 46,0,0,0,0,−46	2.1		0.8	06: 26,0,0,0,13,−49			1.5
	12: 18,46,0,0,0,−36	3.6	0.8	1.4				
Jun	04: 46,0,0,0,0,−46			1.1	06: 26,0,0,0,13,−49		2.0	2.7
Jul	12: 18,46,0,0,0,−36		2.2					

Continuation of Table A5-1. Detailed documentation of fertilizer application in dt per hectare (1dt = 100kg).

Triesdorf: 2017		IO				FM		
	Fertilizer Code*	SM	WB	WW	Fertilizer Code*	SM	WB	WW
Feb	21: 0,0,40,6,5,0	1.6	0.8					
Mar	02: 27,0,0,4,0,−9	1.1		2.5	05: 24,0,0,0,6,−34	2.5		
					20: 0,16,16,2,7,6	5.0		4.0
					18: 23,5,5,0,6,−23			2.5
Apr	11: 9,0,0,0,0,−9	2.4			05: 24,0,0,0,6,−34	2.0		1.5
	02: 27,0,0,4,0,−9			2.1				
	21: 0,0,40,6,5,0			2.4				
May	02: 27,0,0,4,0,−9	1,6	1.6	1.7	02: 27,0,0,4,0,−9	3.0		
	07: 21,0,0,0,24,−63	0.8	0.8	0.8	05: 24,0,0,0,6,−34			2.5
	11: 9,0,0,0,0,−9	2.3			04: 46,0,0,0,0,−46		3.0	
	12: 18,46,0,0,0,−36	1.4	0.8		12: 18,46,0,0,0,−36		1.0	
	04: 46,0,0,0,0,−46		1.8					
Jun	11: 9,0,0,0,0,−9	1.1						

Triesdorf: 2018		IO				FM		
	Fertilizer Code*	SM	WB	WW	Fertilizer Code*	SM	WB	WW
Mar	02: 27,0,0,4,0,−9		1.4		05: 24,0,0,0,6,−34	2.5		
	04: 46,0,0,0,0,−46	3.9	1.0		20: 0,16,16,2,7,6	1.7		
	24: 0,0,0,25,20,0	0.9			22: 0,0,40,6,5,0	3.7	6.4	
					06: 26,0,0,0,13,−49		2.5	
					19: 0,0,46,0,0,−1		3.6	
					24: 0,0,0,25,20,0		2.0	
Apr	02: 27,0,0,4,0,−9		1.0	1.8	14: 15,5,20,2,8,−14	8.0	10.0	
	24: 0,0,0,25,20,0		0.8		24: 0,0,0,25,20,0	1.3		1.8
	04: 46,0,0,0,0,−46			1.7	04: 46,0,0,0,0,−46			3.5
	07: 21,0,0,0,24,−63			0.8	22: 0,0,40,6,5,0			5.8
	12: 18,46,0,0,0,−36			1.2				
	21: 0,0,40,6,5,0			1.2				
May	04: 46,0,0,0,0,−46		1.1					
	12: 18,46,0,0,0,−36		0.9					
	21: 0,0,40,6,5,0		7.4					
Jun	02: 27,0,0,4,0,−9		0.8					

Continuation of Table A5-1. Detailed documentation of fertilizer application in dt per hectare (1dt = 100kg).

Roggenstein: 2016		**IO**				**FM**		
	Fertilizer Code*	SM	WB	WW	Fertilizer Code*	SM	WB	WW
Mrz	04: 46,0,0,0,0,−46		0.8	2.3	12: 18,46,0,0,0,−36		3	4
	07: 21,0,0,0,24,−63		0.8		24: 0,0,0,25,20,0		0.5	0.5
	12: 18,46,0,0,0,−36		0.8	1				
	26: 0,0,0,14,0,53		5.5	4.1				
	21: 0,0,40,6,5,0			2.8				
Apr	26: 0,0,0,14,0,53	5.1			02: 27,0,0,4,0,−9			2.6
May	04: 46,0,0,0,0,−46	3.3	1.2		02: 27,0,0,4,0,−9		3.1	1.7
	12: 18,46,0,0,0,−36	4	0.8	0.8	04: 46,0,0,0,0,−46	2.5		
	21: 0,0,40,6,5,0	8.4	1.5	1.8	12: 18,46,0,0,0,−36	3		
					21: 0,0,40,6,5,0	10		
					26: 0,0,0,14,0,53	10		
Jun	12: 18,46,0,0,0,−36		1.6	2.3	02: 27,0,0,4,0,−9		3.2	
Jul	03: 28,0,0,0,0,−28		1.7					
Sep					26: 0,0,0,14,0,53		12	12

Roggenstein: 2017		**IO**				**FM**		
	Fertilizer Code*	SM	WB	WW	Fertilizer Code*	SM	WB	WW
Feb	21: 0,0,40,6,5,0	1.9	0.8					
Mrz	02: 27,0,0,4,0,−9	2.8			02: 27,0,0,4,0,−9	2.2	2.2	
	26: 0,0,0,14,0,53	4.7		8.6				
	03: 28,0,0,0,0,−28		5.7					
Apr	11: 9,0,0,0,0,−9		0.8		01: 27,0,0,0,0,−15	2.8		
	26: 0,0,0,14,0,53		10		10: 46,0,0,0,0,−46			2.5
	07: 21,0,0,0,24,−63			1.1	07: 21,0,0,0,24,−63	1.5	1	
	10: 46,0,0,0,0,−46			2.7	12: 18,46,0,0,0,−36	3	3	2.5
	12: 18,46,0,0,0,−36			2.1	22: 0,0,40,6,5,0			10
May	04: 46,0,0,0,0,−46	1.2			01: 27,0,0,0,0,−15		2	
	07: 21,0,0,0,24,−63	0.8	0.8					
	12: 18,46,0,0,0,−36	3.1	2.5					
Jun	12: 18,46,0,0,0,−36	0.8						

Continuation of Table A5-1. Detailed documentation of fertilizer application in dt per hectare (1dt = 100kg).

Roggenstein: 2018		**IO**				**FM**		
	Fertilizer Code*	SM	WB	WW	Fertilizer Code*	SM	WB	WW
Mrz	02: 27,0,0,4,0,−9	1.1		1	06: 26,0,0,0,13,−49	2.5		2.3
	04: 46,0,0,0,0,−46	1.9		1.5				
	12: 18,46,0,0,0,−36			2.7				
	07: 21,0,0,0,24,−63	0.8						
Apr	26: 0,0,0,14,0,53	4.5		3.2	01: 27,0,0,0,0,−15	1.2		1.4
	02: 27,0,0,4,0,−9		2		02: 27,0,0,4,0,−9	0.7		
	04: 46,0,0,0,0,−46		2		12: 18,46,0,0,0,−36	3.5	3.7	2.3
	12: 18,46,0,0,0,−36		3.6		26: 0,0,0,14,0,53	4.5		
	21: 0,0,40,6,5,0		8.1		22: 0,0,40,6,5,0		11	
May	04: 46,0,0,0,0,−46			1.5	01: 27,0,0,0,0,−15		3.5	2.9
	21: 0,0,40,6,5,0	0.8		5				
	12: 18,46,0,0,0,−36	1.5						

Triesdorf: 2016			**oIO**				**oFM**	
	Fertilizer Code*	SM	WB	WW	Fertilizer Code*	SM	WB	WW
Mrz	28: Digestate			13	05: 24,0,0,0,6,−34		2.5	2.5
	04: 46,0,0,0,0,−46		1.3					
	21: 0,0,40,6,5,0	2.5	4.3					
Apr	12: 18,46,0,0,0,−36		1.7		28: Digestate		18	22
May	07: 21,0,0,0,24,−63			0.8	07: 21,0,0,0,24,−63	2		
	28: Digestate	48		20	28: Digestate	40		
	04: 46,0,0,0,0,−46		1					
	12: 18,46,0,0,0,−36	1.4						
Jun	04: 46,0,0,0,0,−46			1	05: 24,0,0,0,6,−34		2	2

Triesdorf: 2017			**oIO**				**oFM**	
	Fertilizer Code*	SM	WB	WW	Fertilizer Code*	SM	WB	WW
Mrz	03: 28,0,0,0,0,−28	1.1		3.5	05: 24,0,0,0,6,−34	2.5		2.5
	28: Digestate	13			20: 0,16,16,2,7,6	5		
Apr	28: Digestate	13	43	13	28: Digestate	25	35	20
May	07: 21,0,0,0,24,−63	0.8		0.8	12: 18,46,0,0,0,−36		1	
	11: 9,0,0,0,0,−9	1.9		1.3	05: 24,0,0,0,6,−34	1		2
	12: 18,46,0,0,0,−36	0.8		0.8				
	07: 21,0,0,0,24,−63		0.8					
Jun	11: 9,0,0,0,0,−9	0.8						

Continuation of Table A5-1. Detailed documentation of fertilizer application in dt per hectare (1dt = 100kg).

Triesdorf: 2018			**oIO**				**oFM**	
	Fertilizer Code*	SM	WB	WW	Fertilizer Code*	SM	WB	WW
Mrz	02: 27,0,0,4,0,−9		1.4		06: 26,0,0,0,13,−49	2		
	04: 46,0,0,0,0,−46	2.8			19: 0,0,46,0,0,−1	0.8		
	24: 0,0,0,25,20,0			0.8	05: 24,0,0,0,6,−34		2.5	
					20: 0,16,16,2,7,6		5	
					24: 0,0,0,25,20,0		2.2	
Apr	28: Digestate	13	24	44	04: 46,0,0,0,0,−46			1.4
	04: 46,0,0,0,0,−46			1.3	02: 27,0,0,4,0,−9	1	1.5	
					28: Digestate	20	20	40
					20: 0,16,16,2,7,6			1.2
					24: 0,0,0,25,20,0			1.5
May	02: 27,0,0,4,0,−9		2.2		02: 27.0.0.4.0.−9	2		
	12: 18,46,0,0,0,−36		0.8		15: 15,15,15,0,2,−15		4	
	24: 0,0,0,25,20,0		1.3					
	07: 21,0,0,0,24,−63	0.8						
Jun	12: 18,46,0,0,0,−36		1.7					

* First two digits of fertilizer codes are used to assign the fertilizer. Colon is followed by the respective composition of the fertilizers with the nutrient contents for N, P_2O_5, K_20, MgO, S, and their CaO effects. Fertilizers: 01, 02, and 05 = CAN, 03 = ammonium nitrate urea solution, 04 and 08 = urea, 06 = ammonium sulfate nitrate; 07 = sulfuric acid ammonia, 09 = ENTEC26, 10 = stabilized urea, 11 = ammonium nitrate urea solution + water, 12 = DAP, 13 = NP; 14 = ENTEC NPK, 15 to 18 = NPK, 19 = TSP, 20 = PK, 21 and 22 = potash, 23 = kainite, 24 = kieserit, 25 to 27 = lime, 28 = digestat. Variants: IO = IoFarm, FM = farm manager, oIO = IO + digestate, oFM = FM + digestate.

Table A 5-2: Results from soil testing (N_{min}) and farmers’ yield expectation (YEX).

Site →	**Geiselsberg**			**Triesdorf**					**Roggenstein**		
Variant →	**IO**	**FM**		**IO**	**FM**	**oIO**	**oFM**		**IO**	**FM**	
Crop and	N_{min}	N_{min}	YEX **	N_{min}	N_{min}	N_{min}	N_{min}	YEX **	N_{min}	N_{min}	YEX **
Date ↓	kg ha^{-1}		dt ha^{-1}	kg ha^{-1}		kg ha^{-1}		dt ha^{-1}	kg ha^{-1}		dt ha^{-1}
Winter Barley											
02/2016	41	41	75	46	46	46	46	75	26	26	80
04/2016			75					**70**			80
07/2016			H *					H *			H *
08/2016	82	83	75	65	69	72	67	**75**	62	74	80
02/2017	62	65	75	41	43	45	42	75	19	12	80
06/2017			**70**					**70**			**75**
07/2017			H *					H *			H *
08/2017	161	85	**75**	126	175	90	98	**75**	33		**80**
02/2018	39	44	75	31	33	34	35	75	23	23	80
07/2018			H *					H *			H *
Winter Wheat											
02/2016	52	52	85	50	50	50	50	85	30	30	89
04/2016			85					**70**			89
08/2016	106	74	H *	49	41	43	37	H *	24	21	H *
09/2016			85					**85**			89
02/2017	76	83	85	51	50	44	46	85	20	16	89
04/2017			85					**80**			89
06/2017			**75**					**75**			89
07/2017			H *					H *			H *
08/2017	108	116									
09/2017			**85**					**85**			89
10/2017			85	64	60	62	56	85	67		89
02/2018	49	44	85	45	34	43	41	85	32	32	89
07/2018			H *					H *			H *
Silage Maize											
04/2016	41	41	176	49	49	49	49	160	26	26	192
08/2016	89	95	176	91	85	95	92	160	50		192
09/2016			H *					H *			H *
03/2017	38	51	176	38	23	26	30	160	30	28	192
05/2017			176					160			**176**
08/2017	88	98	176	104	88	89	90	160			176
09/2017			H *					H *			H *
10/2017			176					160			**192**
03/2018	18	25	176	32	32	36	35	160	15	15	192
09/2018			H *					H *			H *

* Harvest; ** farmers yield expectation in dt ha^{-1} (1 dt = 100 kg). Only months in which new information or changes occurred compared to the previous month are shown. Changes highlighted in bold.

6 Characteristics of cost-efficient fertilization strategies at the farm level

This manuscript is coauthored with Johannes Sauer and submitted to NJAS: Impact in Agricultural and Life Sciences.

Authors' contributions: Michael Tröster is the main author of this contribution. Michael Tröster: Methodology, software, validation, formal analysis, investigation, data curation, writing—original draft; visualization. Michael Tröster and Johannes Sauer: Conceptualization, writing—review and editing. Johannes Sauer: Recourses, supervision.

Abstract

Context: Fertilization accounts for a significant share of the costs of crop production. Farmers therefore aim to find cost-efficient fertilization strategies. Due to numerous and partly volatile influencing factors, such as fertilizer price, yield expectation, product price, weather conditions, etc., this is a very complex and recurring optimization problem in crop production.

Objective: This study aims to analyze whole-farm fertilization strategies from an economic point of view. In the first part, differences between cost-efficient and inefficient fertilization strategies are analyzed. In the second part, the influence of different farm conditions on cost-efficient fertilization strategies is investigated. In summary, both parts contribute to generate a deeper economic understanding of cost-efficient fertilization strategies on farm level, thus extending the current knowledge.

Methods: First, an experiment was conducted in which participants had to plan a fertilizer strategy for a simplified farm. The data obtained were analyzed with linear regression analyses and t-test to reveal the characteristics of cost-efficient fertilizer strategies. For part two, a typical Bavarian farm was used to analyze extreme changes in farm conditions, following the *ceteris paribus* principle. The determination of the fertilization strategy was done with the decision support system IoFarm, which was used as a benchmark for cost-efficient fertilization strategies.

Results and conclusions: Our results show that certain fertilizers are more common in cost-efficient fertilizer strategies. The timing of application of base fertilizers is also important. Inefficient fertilizer strategies have surpluses of sulfur and potash, incurring costs and impacting sustainability. Application costs represent a significant portion of total costs, but play a minor role compared to other factors. Fertilizer prices or relative price differences were identified as the largest factor influencing the fertilizer strategy. Furthermore, this study provides an example of bounded rationality of human decision makers in complex situations. The decision support system IoFarm provides help and is able to determine cost-efficient fertilizer strategies.

Significance: This research analyzes the cost-efficiency of fertilizer strategies considering application costs. The focus is on the comparison and evaluation of: (i) fertilizer selection, (ii) timing of fertilizer measures, and(iii) the influence of farm conditions. The study provides a new and important contribution to the understanding of cost-efficient fertilizer strategies at the farm level. Farmers benefit significantly from this contribution, as it shows opportunities to increase cost-efficiency. The results show that cost-efficient fertilizer strategies are at the same time more sustainable, which also demonstrates the societal benefit of this study.

Keywords

IoFarm, cost efficiency, profit maximization, fertilization strategy, fertilizer application, sustainable intensification

6.1 Introduction

Economic efficiency is an objective that is generally pursued by rational actors. This objective requires technical efficiency and allocative efficiency. In the context of agricultural production, both the available production technology and the production program must be considered fixed when making short-term production decisions. Therefore, the most cost-efficient production possible is of great importance. Cost efficiency means that the combination and intensity of production factors and means of production are chosen in such a way that the resulting marginal profit does not become negative. Hence, cost efficiency follows a profit function, which in turn is based on at least one production function.

Since the fertilization of crops accounts for a significant proportion of variable production costs, a cost-efficient fertilization strategy contributes significantly to the economic efficiency of the farm. The isolated consideration of a cost-efficient fertilization strategy is already a complex optimization problem in itself, which in summary consists of two questions: Which fertilizer intensity (related to all relevant nutrients) promises the economically optimal, technical input/output ratio? Which combination of available fertilizers is able to provide the optimal fertilizer intensity at minimum cost? To answer these questions, all price information is relevant. In addition, growth conditions, as well as legal, operational and crop production requirements must be taken into account in order to develop the most cost-efficient fertilizer strategy. Fertilizer strategy is understood to be a farm-by-farm plan that includes, for each combination of field plot and crop, over the period of a crop rotation: (i) fertilizer selection; (ii) fertilizer application rate; (iii) timing of fertilizer measures. The fertilizer strategy must be adjusted several times over the course of a planning period (e.g., crop rotation cycle) to account for changes in prices, for example. In summary, this results in innumerable possible combinations of potential fertilizer strategies that differ significantly in terms of their cost-efficiency.

The characteristics of cost-efficient fertilizer strategies and their differences from inefficient fertilizer strategies are of particular interest to farmers and consultants. Thus, recommendations for their own fertilizer strategies can be derived. The differentiation between efficient and inefficient measures can be found in numerous scientific studies: Wimmer and Sauer (2020) analyze accounting data to identify efficient farm diversification strategies; Mollenhorst et al. (2020) train a machine-learning algorithm with organic fertilization management data to derive efficient organic fertilization decisions; Grassini et al. (2011) studied the effect of various management practices on corn production efficiency in the Western US corn belt. The topic of “fertilizer strategies” per se, is also heavily represented in the literature: Studies by Gil-Ortiz et al. (2020), Dimkpa et al. (2020), Mi et al. (2019) and Noellsch et al. (2009) look at the differences in fertilization strategies with conventional and slow release nitrogen fertilizers; Kozlovský et al. (2009) compare Cultan fertilization with conventional fertilization strategies; Song et al. (2021) and Koch et al. (2004) examine variable rate control as a possible fertilization strategy. These are primarily studies of technical efficiency. Studies that focus on cost-efficient fertilizer strategies often specifically consider the optimal intensity of nutrient supply (LI et al., 2021; Tabak et al., 2020; Sihvonen et al., 2018; Xu et al., 2017; Chuan et al., 2013) and, in rare cases, the least-cost combination of fertilizers (Villalobos et al., 2020; Bueno-Delgado et al., 2016; Pagán et al., 2015; Mínguez et al., 1988; Babcock, 1984). Instead of a very broad definition of “fertilizer strategy,” the studies mentioned focus on a specific aspect in each case and examine it mostly on the basis of trials in single crops. The situation is similar with production technology trials in the fertilizer industry, where in-house fertilizers are compared with competing products. Due to a lack of representativeness and validity, these competitive comparisons have no scientific value and are therefore not published accordingly

It should be noted that the literature provides a great amount of information on specific fertilization issues. This information can be used to draw conclusions about the benefits of different technologies or to derive suitable fertilizer intensities. Farmers or consultants need to convert this knowledge into a cost-efficient fertilizer strategy tailored to the farm. This requires defining the choice of fertilizers as well as the amount and timing of fertilization. To this end, there are currently no studies in the literature that specifically refer to the characteristics of cost-efficient and inefficient fertilizer strategies at farm level. Since fertilizer strategies in practice are influenced by the capabilities of decision makers and by natural and respective farm conditions, two research questions arise: (i) How do cost-efficient fertilizer strategies differ from inefficient ones in terms of fertilizer selection, dosage, timing and resource use? (ii) What is the influence of natural conditions and farm conditions on a cost-efficient fertilizer strategy?

To answer the first question, we refer back to a fertilizer quiz in which we had asked the participants to plan a fertilizer strategy. In this experiment, the natural and farm conditions were fixed. Despite uniform specifications and information, fertilizer strategies differed considerably in terms of design and cost-efficiency. These differences can be useful to improve cost-efficiency without using or having access to optimization tools, avoiding associated transfer costs. We address the second question using the decision support system (DSS) IoFarm (Tröster and Sauer, 2021b). IoFarm generates fertilizer strategies with optimal cost-efficiency on farm level. Application costs are also taken into account. Previous studies have shown that IoFarm, primarily through least cost combination, results in an average cost saving of €66 ha^{-1} (Tröster and Sauer, 2021b) at the same fertilizer intensity. Neither yield nor quality in crop production are affected to a significance level of 5% (Tröster and Sauer, 2021a). IoFarm is used to determine optimal fertilization strategies under different farm conditions. The main aim of this is to clarify whether different fertilization strategies arise for different farm types and what these potential deviations ultimately look like in concrete terms. Previous studies (Tröster and Sauer, 2021b; Tröster et al., 2019) have already pointed out relevant influencing factors in this context. Thus, we assume that the following factors will have an influence on the fertilization strategy: Farm size (hectares), internal infrastructure, organic fertilizer accumulation and heterogeneity of soil fertility.

This article shows characteristics in which cost-efficient and inefficient fertilization strategies differ and what influence varying farm conditions exert in this respect. This article thus contributes to a better understanding of cost-efficient fertilization strategies. Furthermore, it is clarified whether general recommendations for a cost-efficient fertilizer strategy can be derived from this information or whether the support of a DSS is indispensable. This information is particularly relevant for farmers and consultants who want to implement more economically efficient, but also resource-friendly fertilization strategies. In addition, the fertilizer industry benefits from the results of this study, e.g., in developing new products, or in connection with strategic decisions in the company.

6.2 Material and methods

6.2.1 The IoFarm decision-support system

IoFarm is a novel decision support system for identifying cost-efficient fertilizer strategies at the farm level (Tröster and Sauer, 2021b). In the context of this study, IoFarm is used, on the one hand, as a cost-efficient benchmark for comparing different fertilization strategies. On the other hand, IoFarm is based on a clear mathematical structure and is therefore well suited for scenario analyses in which a consistent solution path is important. Over an entire crop rotation cycle, the DSS IoFarm makes concrete specifications for selecting fertilizers, application rate and application time for each field plot. By regularly updating fertilizer and product prices, yield expectations, soil test results and weather information, IoFarm can dynamically adjust the fertilizer strategy. The mathematical structure of IoFarm belongs to the category of so-called “Mixed Integer Non-linear Problems.” The objective function is designed to find the most cost-efficient combination of fertilizers to meet crop requirements. In addition to the market prices of the fertilizers, the application costs of the fertilizers are also considered. Within the model, marginal revenues and marginal costs are also taken into account, which may limit the intensity of fertilization if it is economically reasonable.

6.2.2 An experiment as data source

To assess the economic performance of IoFarm, a fertilizer quiz was conducted as part of a previous study (Tröster and Sauer, 2021b). Participants were mainly reached via “mailing lists of alumni associations of higher agricultural education institutions and universities.” Participants were asked to define their experience level according to the following description: Expert = person possessing either scientific experience in plant nutrition or economic optimization models; Farmer = person with at least five years of professional experience in agriculture and plant nutrition; Student = student with advanced knowledge in economic optimization models and plant nutrition; “Others.” For further analysis, the last group was excluded.

The task was to define a complete fertilizer strategy for a 150-hectare farm, with three equal-sized field plots over a period of three years. To highlight differences in fertilizer selection and timing, we ensured that the participants and the IoFarm DSS followed identical guidelines for fertilizer intensity and timing. These guidelines are based on the fertilization standards valid in Bavaria (Sothern Germany) (Wendland et al., 2018), but are comparable to other state-specific standards in Germany (Zorn et al., 2007): N and S are allocated according to the yield expectation within a season. For N, soil test results are considered. The basic nutrients (P, K, and Mg) are applied according to the nutrient removal of the crop rotation, whereby soil nutrient content leads to additional increases or decreases as required. Seasonal requirements for basic fertilization only arise if the soil nutrient content falls below a critical level; otherwise these nutrients are freely allocable within a crop rotation. To keep up with these guidelines, the participants were provided with a planning tool that contained requirement specifications for the individual nutrients (N, P, K, Mg, S) as well as a selection of 25 fertilizers commonly available on the market. This allowed the participants to concentrate on selecting, dosing and timing the fertilization measures. The objective for the participants was to identify the most cost-efficient fertilizer strategy, and to help with this, a complete listing of all fertilizer prices covering the entire period was handed out along with the quiz. The relevance of application costs was also pointed out (The quiz is available online[11]). For comparability reasons, participants who were not able to follow the guidelines had to be excluded from further analysis. Thus, the data set for analysis contains only fertilizer strategies that meet uniform guidelines. It is therefore expected that there would be no significant differences in the output of crop production, which is supported by the results of a multi-year field trail (Tröster and Sauer, 2021a). The cost-efficiency of the fertilizer strategies can therefore be assessed based on total costs alone.

On average, it took the participants 81 minutes to complete the task. The best participant’s total cost for fertilizer and application is about €10 per hectare per year more expensive than IoFarm's fertilizer strategy. On average, this difference is as high as €66 per hectare per year. The data submitted by the quiz participants contain much more information than just the total costs: each data set represents a separate fertilization strategy that was more or less successful from a cost perspective. This allows a detailed characterization of the fertilization strategies.

[11] https://drive.google.com/file/d/14rBHNKKDuBq8oyeeVUXuek2id1B9z_Dw/view?usp=sharing

6.2.3 Classification of similar fertilizers

The participants could choose from 25 fertilizers on the fertilizer quiz, which is why participant fertilizer strategies differed considerably. To transparently compare the fertilizer strategies, it is therefore necessary to group similar fertilizers together in order to make potential solution patterns visible, which is why we distinguish between fertilizers with low (xL) and high (xH) nutrient content, e.g., 31% nutrient content is a high nutrient content. This limit was purposely chosen so as not to distort the group balance between these two categories too much. As a second characteristic, we distinguish between single-nutrient fertilizers (Sx) and compound fertilizers (Cx). A fertilizer is considered to be a single-nutrient fertilizer if it contains only N, P or K and, in parallel, no more than 20% of its total nutrient content comprises the nutrients S and Mg. The combination of these differentiation criteria results in the groups SL, SH, CL, and CH. Special lime fertilizers form their own group (CA). The fertilizers and their group allocation can be found in the appendix (Table A 6-1). Based on this grouping, our primary focus is to determine what proportion of the applied nutrient quantity a participant drew from each of the five fertilizer groups. This should allow conclusions to be drawn about the fertilizer strategy as well as the total costs of fertilization (including application).

6.2.4 Data preparation and comparison of fertilization strategies

Several steps were necessary to form an informative data set from the fertilization strategies of the individual quiz participants. First, the raw data that could be derived directly from the individual fertilization strategies was listed. This included, for example, the amount of fertilizer used, number of fertilizer applications, as well as the application rate of each nutrient. Based on this information, further variables were generated that are important for analyzing the fertilization strategy. These variables include costs for purchasing fertilizers and their application costs, nutrient losses or nutrient balances. A large number of the variables describe the proportion that fertilizers contribute to the total supply of each nutrient. In total, 675 variables comprise each fertilizer strategy. Relevant excerpts of these data can be found in the appendix (Table A 6-2, Table A 6-3).

Statistical calculations were performed using STATA SE 13 software (StataCorp, 2017). The data set was analyzed for potential factors influencing the variable *FA_Cost* (total cost from fertilizer and application) using linear regression analysis. Eq. (6-1) represents the associated linear regression model. The coefficient β_0 represents a constant. The remaining coefficients

β_n represent the weighting factors of the independent variables x_n. The residual error is covered by the error term ε.

$$FA\_\mathrm{Cos}t = \beta_0 + \beta_1 x_1 + ...\beta_n x_n + \varepsilon \quad (6\text{-}1)$$

The variable *FA_Cost* was also used as the sole cluster variable to divide the fertilization strategies into clusters with different cost efficiencies. The median-linkage clustering method in combination with the Euclidean option as a continuous dissimilarity measure led, as desired, to a differentiation into three clusters: The cluster with the most cost-efficient fertilization strategies also includes the optimal IoFarm solution. We assign this cluster to the economically “efficient” fertilization strategies. The remaining two clusters can be described as “average” and economically “inefficient” fertilization strategies. This provided the opportunity to perform mean comparisons between clusters using t-tests in further analysis. Since more than two groups (efficient, average, and inefficient clusters) were compared, the Tukey test was used (Tukey, 1949). This post-hoc test is a multiple comparison of means that corrects for alpha error accumulation and is therefore considered to be conservative.

6.2.5 Scenario analysis using the DSS IoFarm

The DSS IoFarm was applied under different farm conditions to identify potential impacts on cost-efficient fertilization strategy. As a starting point for this study, data from an existing farm (“original farm”) was used. The farm is managed by one of the authors, therefore necessary details of on-farm infrastructure are well known. Complete information on all farm-to-field and field-to-field distances of other farms is not directly accessible. It is possible to generate such datasets (Machl et al., 2016), but these georeferenced data are highly sensitive, which is why we were unable to obtain access for this under data protection law.

The “original farm” (see Table 6-7, column 1) cultivates 63 hectares, of which one third each is winter barley, winter wheat and silage maize. The acreage and cropping structure correspond to an average Bavarian farm where, according to the Bavarian Agricultural Report (StMELF, 2020), cereals and fodder crops are cultivated on 60.4 hectares. No organic fertilizers are available. In order to be able to consider field-to-field distances, the total time required to reach all field pieces (in a circuit) was used according to Tröster et al. (2019). This amounts to 65 minutes. The field pieces were grouped into three management units (f1 to f3). This greatly facilitates the clarity and comparability of the results. Based on the size and the farm-to-field distances of the individual field pieces, a weighted average farm-to-field distance in minutes was determined for the three management units. The third of the plots close to the farm (f1) has a farm-field distance of only 0.75 minutes. Management unit f2 and f3 are 2.55 and 10.74 minutes away, respectively. Soil nutrient content (P, K, and Mg) of the management units was determined using representative farm plots and classified into categories “A” (very low) to “E” (very high) according to the guideline of the Bavarian State Institute of Agriculture (Wendland et al., 2018). Accordingly, a classification in categories “A” and “B” results in an increase in the respective nutrient requirement. Classification in categories “D” and “E” results in the respective nutrient requirement being halved or cancelled.

The initial situation of the original farm was now changed selectively in order to be able to represent the following scenarios: “small farm,” “big farm,” “nearby fields,” “faraway fields,” “homogeneous soil fertility,” “medium slurry accumulation” and “high slurry accumulation.” In addition, to test the influence of relative price changes on fertilizer strategy, fertilizer prices collected between August 2015 and October 2018 were artificially manipulated. The fertilizer prices can be viewed in conjunction with the fertilizer quiz online (link provided above). Using binary random numbers, a decision was made for each fertilizer at the beginning of each year whether to raise or lower the original prices by 10%. This results in a data set with annually changing price relations. Price trends of the individual fertilizers within the period under consideration, however, remain. The associated scenario is labeled “artificial price shift.” More detailed information on the scenarios, as well as an overview of the results, can be found in Table 6-7.

6.3 Results

The results show that cost-efficient fertilizer strategies are primarily influenced by relative changes in fertilizer prices. The farm-specific conditions investigated only partially influence the fertilizer strategy.

6.3.1 Differences between cost-efficient and inefficient fertilization strategies

In order to find out how cost-efficient fertilizer strategies differ from inefficient ones, a detailed analysis of the data from the fertilizer quiz is carried out in this point. The most cost-efficient fertilizer strategy of IoFarm is also shown separately in the group mean comparisons to enable cross-comparisons.

Fertilizer decision

Particularly relevant is identifying fertilizers with a high or low economic advantage. In order to be able to assess the importance of the fertilizers separated by nutrients, it is first calculated which share of a nutrient a fertilizer covers in total within the framework of the present fertilizer strategy. If, for example, the potash supply is covered exclusively by gr. potash, this fertilizer has a share of 100% in the potash supply. This shows what contribution each fertilizer has made to the respective quantities of nutrients applied. Numerous regression analyses were then carried out to determine the impact of these fertilizer variables on the total cost of fertilization, using the trial and error method. In the end, a model was created that is able to relate each nutrient to a specific fertilizer (see Table 6-1).

Table 6-1: Influence of important fertilizers on the total cost of fertilization.

FA_Cost	Coef.	SE	P>|t|
N%CAN+Mg	-18373	9805	0.074
P%DAP	-14852	4452	0.003
K%ENTEC NPK	13039	6580	0.060
Mg%Lime+Mg	-32785	9736	0.003
S%gr. potash	-76149	24935	0.006
constant	178999	5509	0.000
n= 29 | Prob > F = 0.000 | Adj. R-squared = 0.7365			

FA_Cost = total cost of fertilizer and application; Coef. = regression coefficient; SE = standard error; N%CAN+Mg = contribution of CAN+Mg to total nitrogen fertilization; P%DAP = contribution of DAP to total phosphorus fertilization, etc.

With the exception of the K source (K%ENTEC NPK), all variables contribute to lower total costs in the model from Table 1. It is therefore to be expected that low-cost fertilization strategies rely more on CAN+Mg as a nitrogen source, DAP as a phosphorus source, Lime+Mg as a magnesium source and gr. potash as a sulfur source and in parallel avoid ENTEC NPK as a potash source. Due to the uniform scaling of the independent variables, it is also possible to rank the influencing variable on the basis of the level of the coefficients. Accordingly, the use of gr. potash as a source of sulfur is of particular importance. A multiple mean comparison between the clusters formed in advance (cost-efficient, average and inefficient fertilizer strategies) provides additional insights into different frequencies of fertilizer use (see Table 6-2).

Table 6-2: Fertilizers with significant differences between group means of clusters.

Fertilizer	Io-Farm Ø %	Cluster 1 Ø %	←versus→ SE	P>|t|	Cluster 2 Ø %	←versus→ SE	P>|t|	Cluster 3 Ø %	←versus→ SE	P>|t|	Cluster 1 Ø %
N%AHL1to3	4.0	6.5	0.01	0.001	0.8	0.01	0.773	0.0	0.01	0.001	6.5
N%DAP	21.3	21.7	0.04	0.018	9.3	0.02	0.133	1.5	0.04	0.002	21.7
P%DAP	80.0	81.8	0.14	0.015	34.0	0.07	0.143	5.7	0.13	0.002	81.8
P%TSP	20.0	18.2	0.16	0.355	42.0	0.07	0.185	70.2	0.14	0.049	18.2
K%gr. potash	100	100	0.19	0.063	51.1	0.08	0.700	35.9	0.17	0.044	100
Mg%gr. potash	10.4	9.4	0.02	0.130	5.2	0.01	0.415	2.8	0.02	0.039	9.4
S%gr. potash	16.7	17.5	0.03	0.002	6.8	0.01	0.248	2.6	0.02	0.000	17.5
S%Kieserit	33.3	63.0	0.08	0.240	49.1	0.03	0.165	34.7	0.07	0.028	63.0

Notes: The optimal solution “IoFarm” is shown separately as a benchmark; Cluster 1 = cost-efficient fertilization strategies; Cluster 2 = average fertilization strategies; Cluster 3 = inefficient fertilization strategies; SE = standard error; N%AHL1to3 = amount of AHL1to3 to total nitrogen fertilization; P%_DAP = contribution of DAP to total phosphorus fertilization, etc.

Only a few of the combinations of nutrient source (e.g., N%) and fertilizer (e.g., DAP) differ significantly in their frequency of use between cluster 1 (cost-efficient fertilizer strategies) to cluster 3 (inefficient fertilizer strategies). Table 6-2 only shows combinations of nutrient and fertilizer for which significant differences in use frequency can be detected, at least when comparing clusters 1 and 3 (see right part of table). Significant differences in use frequency between clusters 1 and 2 are particularly relevant. Although AHL1to3 contributes only slightly (with 6.5%) to the nitrogen supply in cluster 1, it is still considered a success-determining factor for a cost-efficient fertilization strategy. Also relevant is DAP as a source of N and phosphorus, respectively, and gr. potash as a source of cost-efficient sulfur supply. Thus, comparing means essentially supports the results of the regression analysis. In looking at the raw data, we also notice some patterns that were not detectable, or not sufficiently detectable, using the statistical methods: Fertilizers with a combination of N, P and K, as well as stabilized nitrogen fertilizers are rarely used in cost-efficient solutions; gr. potash plays an important role for the supply of S, but the time of application must then be within the growing season; by far the greater part of the sulfur supply is via SSA in the context of the fertilizer quiz. The importance of SSA depends on its price, but also strongly on the pH value, as well as the K and Mg supply of the soil. Here is an example: A high pH value and a low Mg supply favor SSA, because due to the strong acidifying effect of this fertilizer, more Lime+Mg must be used to compensate for the acidifying effect. Lime+Mg is also the most economical source to ensure Mg supply. In contrast, at a low K supply and low pH, SSA becomes less relevant. In this case, larger portions of the S requirement are usually covered within the framework of potash fertilization via gr. potash.

Fertilizer categories

In a further model, it was investigated whether it might also be possible to derive conclusions about entire fertilizer groups. Fertilizer categorization was carried out as described in Section 6.2.3. Values between 0 and 1 were calculated for the proportional nutrient supply from the variables SL, SH, CL, CH, and CA. A value of 1 would mean that the respective quiz participant had obtained 100% of the total nutrient supply from one and the same fertilizer category. However, the total cost of fertilization (*FA_Cost*) cannot be explained by the nutrient proportions from the five fertilizer categories. The associated model (not shown) already fails at the F statistic (Prob > F 0.1475).

Timing of basic fertilization with K and P

P and K are nutrients that do not necessarily need to be spread every season, which presents the option of using potential low price periods to purchase these nutrients in order to save on costs. For this purpose, the total amount of applied nutrients in 2016, 2017, and 2018 was compared with the dependent variable *FA_Cost* in the form of a regression analysis for both P and K (see Table 6-3).

Table 6-3: Influence of the timing of basic fertilization on the total cost of fertilization.

FA_Cost	Coef.	SE	P>|t|
P2016	1.786	1.087	0.115
P2017	0.806	1.057	0.454
P2018	1.323	1.022	0.209
K2016	2.254	0.759	0.007
K2017	2.623	1.041	0.019
K2018	1.848	0.929	0.059
constant	36286.28	53581.99	0.505
n= 29 | Prob > F = 0.0238 | Adj. R-squared = 0.3095			

FA_Cost = total cost of fertilizer and application; Coef. = regression coefficient; SE = standard error; P2016 = phosphorus fertilization in 2016, K2018 = potash fertilization in 2018, etc.

For timing phosphorus fertilization (variables P2016 to P2018), no significant influence on the total costs can be detected. For timing potash fertilization, however, significant influences can be detected at least in the years 2016 and 2017. The coefficients of the variables K2016 to K2018 indicate that K had a comparatively high price in 2017. In contrast, in 2018, the price level for K was considerably lower. Thus, it can be expected that the basic supply of K in cost-efficient fertilizer strategies was preferentially provided in 2018.

The comparison of means (not shown) provides a significant difference only for the use of P in 2016. In cluster 1, significantly (P>|t| = 0.019) less P was used in this year compared to cluster 3. With this insight, more importance should be attached to the coefficient of variable P2016 in the above regression analysis (see Table 6-3). With the value of 1.786, it takes by far the highest value of variables P2016 to P2018, indicating high phosphorus prices in 2016. In contrast to the regression analysis, the t-tests (due to high standard errors) for the timing of potash fertilization between the clusters did not reveal any verified differences. A look at the raw data reveals that gr. potash was also used in cost-efficient solutions to some extent in 2016 and 2017, but then in reduced amounts and specifically in the spring to satisfy a proportion of the sulfur requirements in parallel. This confirms the importance of gr. potash and its dual function as a source of K and S.

Application costs and number of fertilization measures

On average, the application costs of all quiz participants account for 5.2% of the total costs. Despite this relatively low share of costs, a significant (P>|t| = 0.013) correlation between total costs (dependent variable: *FA_Cost*) and application costs (independent variable: *A_Cost*) was found in a linear regression analysis (Prob>F = 0.013; Adjusted R^2 = 0.1782). The coefficient for *A_Cost* is 7.35, indicating that high application costs are related to high total costs. The "Measures" variable indicates the number of all fertilization measures within the three-year period. With a positive correlation of 0.750, it is closely linked to application costs (*A_Cost*), which is why the "Measures" influence was tested separately to prevent autocorrelation. However, "Measures" as an independent variable is not suitable for drawing conclusions about the total costs of a fertilizer strategy (Model: Prob>F = 0.2745).

Table 6-4: Statistical comparison of means for application costs and number of measures.

	IoFarm	Cluster1	←versus→		Cluster2	←versus→		Cluster3	←versus→		Cluster1
Test- Variable	Ø	Ø	SE	P>|t|	Ø	SE	P>|t|	Ø	SE	P>|t|	Ø
A_Cost [€]	8323	7189	385	0.207	7926	172	0.272	8535	344	0.039	7189
Measures [No]	37	26.3	3.43	0.666	29.5	1.53	0.322	33.0	3.07	0.570	26.3

Cluster 1 = cost-efficient fertilization strategies; Cluster 2 = average fertilization strategies; Cluster 3 = inefficient fertilization strategies; SE = standard error; A_Cost = application cost; Measures = number of fertilization measures within 3 years.

The comparison of the cluster means (Table 6-4) confirms the result from the previous regression analysis. It is interesting, however, that the optimal solution of IoFarm, which was only presented here as a reference, stands out with high costs for application and many fertilization measures.

Nutrient losses and balances

Another factor that affects both the fertilization costs and the evaluation of the sustainability of this measure is a nutrient supply that is as close as possible to the requirements. For the basic nutrients P, K and Mg, balances were shown to the quiz participants during the processing of the experiment. In these balances, the nutrient requirements resulting from the withdrawals of the crop rotation were compared with the applied nutrients (taking into account the nutrient content of the soils). The corresponding names of the variables are "P_Bil," "K_Bil" and "Mg_Bil." For nutrients N and S, this balancing approach is not suitable due to the high potential for displacement in the soil. However, information on potential losses could be generated at least indirectly from the available data: For the required sulfur supply, crop-specific target values were considered demand. In addition, it was defined that effective sulfur fertilization can only take place in the time window from February to

May. Sulfur applications above the demand, or outside this time window were summarized in the variable "S_loss" as sulfur loss. N leaching losses unfortunately cannot be derived. However, since nitrogen fertilization was only allowed within reasonable time windows during the experiment and the fertilizer requirement was predefined, it is assumed that leaching losses do not differ significantly. However, theoretical conversion losses of the different nitrogen forms were considered during the course of the fertilizer quiz. The sum of these conversion losses is summarized in the variable N_loss. Table 6-5 shows the influence of the variables mentioned on the fertilizer costs (*FA_Cost*).

Table 6-5: Relationship between demand-based and cost-efficient fertilization.

FA_Cost	Coef.	SE	P>|t|
N_loss	0.272	1.502	0.858
S_loss	1.276	0.202	0.001
P_bil	0.787	0.739	0.296
K_bil	1.295	0.524	0.021
Mg_bil	-0.900	0.506	0.089
constant	134161	8918	0.001
n= 29 | Prob > F = 0.0001 | Adj. R-squared = 0.6960			

FA_Cost = total cost of fertilizer and application; Coef. = regression coefficient; SE = standard error; N_loss = theoretical conversion losses of nitrogen fertilizers; S_loss = displaced or overapplied sulfur fertilization; P_bil = balance of P fertilization and withdrawal, etc.

Despite the relatively high pure nutrient costs in the purchase of N and P, the associated variables (N_loss and P_bil) have no significant influence on fertilizer costs. This can only be explained by the fact that the quiz participants fertilized largely according to demand in this respect and that there are therefore no significant differences. There are significant differences for S (S_loss) and K (K_bil). In both cases, overfertilization leads to higher fertilizer costs and to a deterioration in resource efficiency. In Table 6-6, a statistical mean comparison is provided to show the differences between clusters 1 to 3.

Table 6-6: Statistical comparison of means with regard to demand-based fertilization.

Test-Variable	Io-Farm Ø	Cluster 1 Ø	←versus→ SE	P>|t|	Cluster 2 Ø	←versus→ SE	P>|t|	Cluster 3 Ø	←versus→ SE	P>|t|	Cluster 1 Ø
N_loss [kg]	6562	6388	552	0.206	5328	247	0.926	5534	494	0.492	6388
S_loss [kg]	3680	4036	2835	0.060	11486	1268	0.001	24486	2535	0.001	4036
P_bil [kg]	0	201	1048	0.344	1834	469	0.640	2786	937	0.177	201
K_bil [kg]	565	437	1334	0.498	2106	597	0.123	4825	1193	0.054	437
Mg_bil [kg]	1319	4456	1653	0.998	4347	739	0.243	7076	1478	0.474	4456

Cluster 1 = cost-efficient fertilization strategies; Cluster 2 = average fertilization strategies; Cluster 3 = inefficient fertilization strategies; SE = standard error; N_loss = theoretical conversion losses of nitrogen fertilizers; S_loss = displaced or overapplied sulfur fertilization; P_bil = balance of P fertilization and withdrawal, etc.

The differences in sulfur losses (S_loss) are very significant. Only between cluster 1 and cluster 2 there is no clear significant difference. The difference in the potassium balance (K_bil) between cluster 1 and cluster 3 is almost significant, confirming the result of the regression analysis in Table 6-5. However, the high nitrogen losses in the optimum solution (IoFarm) and in cluster 1 are surprising. The main reason for this is the somewhat greater use of urea as a nitrogen source. The higher conversion losses, which were used for urea fertilization, could apparently be tolerated due to relative price advantages. In cluster 3, the sum of nutrient surpluses is by far the highest. Cost-inefficient solutions thus also appear to be less sustainable and less resource-efficient. To test this, a new variable (NPKMgS) was formed from the sum of the five variables N_loss to Mg_bil. A regression analysis (Prob>F = 0.0000; adjusted R^2 = 0.5262) shows the highly significant (P>|t| = 0.001) influence of this variable on the increase in fertilizer costs. The differentiation between clusters 1 and 3, or clusters 2 and 3, is also highly significant (P>|t| = 0.001). This result is extremely relevant as it shows how cost efficiency and sustainability have a complementary objective at this point.

Identification and utilization of abrupt, relative price changes

In order to be able to put the fertilizer prices into perspective, a mean pure nutrient price was first derived for N (€0.81 kg^{-1}), P (€0.86 kg^{-1}) and K (€0.69 kg^{-1}) on the basis of the average prices of CAN, TSP and gr. potash. Subsequently, pure nutrient costs for these nutrients could be derived for all fertilizers. Continuous changes in the price relations as well as abrupt price changes were analyzed graphically. A particularly striking price drop was recorded for TSP from June to July 2016 (see Figure 6-1). This price drop was detected and used by the IoFarm model. Only six of the quiz participants also recognized this price drop and used TSP at that time. This shows that even clear price signals are often not recognized by human decision makers.

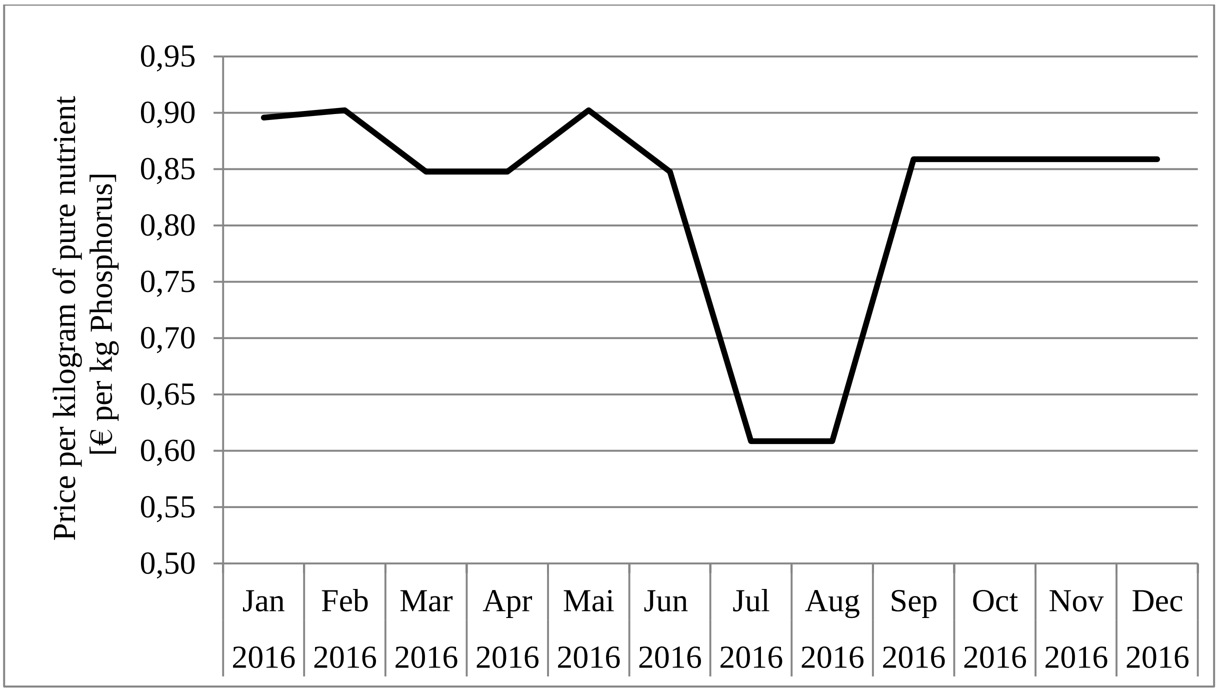


Figure 6-1: Pure nutrient price of P based on triple superphosphate in 2016.

6.3.2 Influence of farm conditions on cost-efficient fertilizer strategies

IoFarm is able to calculate cost-efficient fertilization strategies. Due to the clear mathematical structure of the DSS, the solution path is consistent. IoFarm is therefore well suited for investigating the influence of farm conditions on cost-efficient fertilization strategies. In the following sections, IoFarm is used to highlight the following farm conditions: Farm size, infrastructure, soil fertility, organic fertilizer availability, and price changes. Table 6-7 provides a central overview of all scenarios studied and their impact on the cost-efficient fertilizer strategy. The goal of this comparison is to identify possible trends in fertilizer selection in order to locate particularly relevant factors influencing a cost-efficient fertilizer strategy. This contributes to a deeper understanding of cost-efficient fertilization.

Influence of farm size (acreage)

To examine the influence of farm size, the farm size of the “original farm” was changed from 63 hectares to 6 hectares—“small farm”—or to 1,500 hectares—“big farm”—under otherwise identical conditions. Although it can be assumed that there is a correlation between farm size and on-farm infrastructure (e.g., farm-to-field distance), both aspects are examined separately.

In principle, it cannot be excluded that both large farms with surrounding fields and small farms with distant fields occur. The change in farm size was purposely chosen drastically in order to be able to clearly highlight potential effects on fertilizer strategy. Table 6-7, columns 1 through 3 compares the “original farm” with the two extreme variants of “small farm” and “big farm.” The total costs for fertilizer and application differ significantly (small farm: €324 ha^{-1}; original farm: €272 ha^{-1}; big farm €265 ha^{-1}). The share of application costs in the total costs, as well as the application costs per 100 kg fertilizer make clear that the cost differences are almost exclusively caused by changes in application costs. The reason for this is the nonlinear composition of application costs according to Tröster et al. (2019). Included in this: setup time per fertilizer application, costs for loading the spreader, farm-to-field and field-to-field trips, costs incurred during field work. Small farms are at a cost disadvantage compared to large farms, primarily due to setup time. By reducing the number of fertilizer measures from 32 (“original farm”) to 30 (“small farm”), an attempt is made to compensate for this cost disadvantage. However, effects on the selection of fertilizers cannot be identified due to the size of the farm. For example, the ratio of nutrient proportions from the different fertilizer categories (SL, SH, CL, CH and, CA) remains almost unchanged. However, a minor adjustment response of the “small farm” should be mentioned: the share of the fertilizer category SL is reduced by 2% compared to “big farm”, whereas slightly more of the higher concentrated fertilizer category SH is used. The main nutrient sources for N, P, K, Mg and S are identical for the various-sized farms: urea (N), DAP (P) and, gr. potash (K, Mg, S), each with approximately equal percentages of nutrient supply.

Overall, it can thus be stated that the farm size exerts only a minimal influence on the fertilization strategy.

Influence of the internal infrastructure

The on-farm infrastructure is the distance between farm and field, as well as the position of the fields in relation to each other (field-to-field distance) in combination with the existing mechanization. Changes in this area have a significant effect on transport and application costs. In order to test possible effects on fertilizer strategy selection, the farm-field distance of the management units (f1 to f3) was changed to one minute each for “nearby fields” and to 30 minutes each for “faraway fields.” In parallel, the duration for the complete approach of all field pieces (field-to-field distance) had to be adjusted to 6 minutes for “nearby fields” and to 180 minutes for “faraway fields.” Table 6-7, columns 4 and 5 compares the two scenarios. As expected, both scenarios differ in total fertilizer and application costs (nearby fields:

€266 ha^{-1}; faraway fields: €288 ha^{-1}). Also in this case, it can be seen that the difference in total costs is mainly caused by the application costs. However, the change in the fertilizer strategy itself is small: more fertilizer of the SL category is used in the “nearby fields” variant than in “original farm.” This is mainly a higher proportion of CAN+Mg. Due to the short transport distances, the nutrient density in the fertilizers used is less important, so this change is comprehensible. With regard to the other fertilizer categories and the main fertilizers used, the adjustments are insignificant compared to the “original farm.” It is interesting to note that even in the “faraway fields” scenario, significantly more of the lower concentrated SL fertilizers (10%) are used than in the “original farm.” This initially is contrary to the logic that fertilizers with high nutrient concentrations are preferred in case of long transport distances. However, in fact, part of the urea fertilization (SH) was replaced by CAN+Mg (SL) in this scenario (compare Main N source Table 6-7). The reason for this is probably the strong acidifying effect of urea, which in parallel also increases the need for compensatory liming. Partial replacement of urea with CAN+Mg can reduce fertilization measures and application costs. In the “faraway fields” scenario, the share of the CH fertilizer group is 3% higher than the original level. This increase is directly related to the growing importance of DAP as a P (and N) source (see Main P source Table 6-7). Since DAP has no other disadvantages, the benefits of the enormously high nutrient density of this fertilizer are fully realized.

As a result, on-farm distances play a minor role in the fertilizer strategy; the share of transport costs in the total costs of fertilization is too small to exert an influence on the fertilizer strategy.

Influence of soil nutrient content

In the “original farm” scenario, the soil content of the nutrients P, K and Mg is relatively heterogeneous. The classifications are between “B” low and “E” very high. In order to investigate the influence of soil nutrient content on the fertilization strategy, a scenario with an absolutely homogeneous soil nutrient content was created under otherwise identical conditions. For this purpose, it is assumed that the nutrients P, K and Mg are each present in optimal concentrations and can therefore be classified as “C.” This setup as well as the results for this variant can be found in Table 6-7, column 6, “Homogeneous soil fertility.” Due to the changes, slightly more P and Mg must be spread in total than in the “original farm” scenario. As a result, the total costs for fertilizer and application are higher than before (€287 ha^{-1}). Due to the homogeneous soil nutrient content of all management units, the fertilization strategy can be simplified. This reduces the fertilization measures from 32 to 28 measures, which also leads to a reduction in application costs. In total, the nutrient percentage shifts from the fertilizer category CH toward SL and SH. This is due to the fact that significantly less urea is used compared to "original farm.” Instead, more CAN+Mg is used. DAP (CH) is replaced by TSP (SH) as the main source of P. TSP has an advantage in homogeneous P soil conditions because it can be used simultaneously in all crops, largely independent of the season. This allows optimal use of periods of low prices and also enables fertilization measures to be combined to reduce application costs. Changes in terms of K, Mg and S fertilization are marginal. As in all previous scenarios, gr. potash is used for the most part as the fertilizer of choice for these nutrients.

It should be noted that differences in soil nutrient content affect the complexity of the optimization problem and therefore significantly influence the fertilization strategy.

Influence of the amount of organic fertilizer

The type and availability of organic fertilizers vary in practice from farm to farm. The "original farm", in which no organic fertilizer is available, was therefore compared with two scenarios with organic fertilizer (see Table 6-7): “medium slurry accumulation” with a nitrogen accumulation from livestock of 80 kg N ha^{-1} and “high slurry accumulation” with the maximum organic nitrogen fertilization currently permitted in Germany of 170 kg N ha^{-1}. Slurry fertilization covered a considerable proportion of the nutrient requirement in both variants. As a result, the cost of purchasing commercial fertilizer and the associated application costs drop drastically to €179 ha^{-1} (medium slurry accumulation), or to €55 ha^{-1}. The availability of organic fertilizer also has a significant effect on the number of mineral fertilization measures. In the “high slurry accumulation” scenario, the farm would manage with just 13 mineral fertilization measures in the 3-year period under consideration. Since slurry must be classified in the CL fertilizer group, the nutrient percentage of this fertilizer group increases from its original 0% to 44% or 78%. Due to this massive change, the percentages of the other fertilizer groups can no longer be directly compared with the previous scenarios. However, in both cases, a clear decrease in the SH fertilizer group is noticeable. This decrease is mainly due to a reduced use of urea and TSP. The evaluation of the main nutrient sources for the different nutrients is dominated by slurry in both scenarios. Only the main source of S stays inorganic and has changed from gr. potash to SSA. Further adjustments to the selection of purchased commercial fertilizers can only be detected when looking at the second-most important nutrient source after slurry: For the medium slurry accumulation scenario, CAN+Mg is the most important purchased N fertilizer, accounting for

27% of the N supply. For Mg supply, Lime+Mg is mainly used. DAP remains the main commercial fertilizer for P and gr. potash remains the main commercial fertilizer for K. In the “high slurry accumulation” scenario, neither P nor K is purchased externally. The second-most important N source besides slurry is urea (12%). Lime+Mg (8%) is used as a source for Mg. S needs are primarily covered by SSA.

In summary, the availability of organic fertilizers affects several issues at once: (i) external nutrient requirements; (ii) the cost of fertilizer purchase and application; (iii) the distribution and number of fertilizer applications; (iv) the choice of fertilizers themselves.

Influence of relative price changes of fertilizers

So far, major changes in the fertilizer strategy have only occurred in the scenarios with organic fertilizer use and with changed soil nutrient conditions. In order to test whether relative price changes on the fertilizer market have a noticeable influence on the fertilizer strategy, fertilizer prices were artificially changed as described in Section 6.2.5.The result of this analysis can be found in Table 6-7, column 9. Despite moderate price adjustments of ±10%, different fertilizers are now selected for N (50% CAN+Mg) and P (71% TSP) than in the “original farm.” With regard to the origin of K, Mg and S, the fertilizer strategy remains relatively constant. Here, too, gr. potash is mainly used, although somewhat more use is made of other fertilizers for sulfur supply than in the original situation.

Relative changes in fertilizer prices thus have a major impact on cost-efficient fertilizer strategies.

Table 6-7: Differences in cost-efficient fertilization strategies under various farm conditions.

Setup		(1) Original Farm	(2) Small Farm	(3) Big Farm	(4) Nearby fields	(5) Faraway fields	(6) Homogen soil fertility	(7) Medium slurry accumulation	(8) High slurry accumulation	(9) Artificially shifted fert. price
I Farm operating data										
Farm size	ha	63	6	1500	63	63	63	63	63	63
Nitrogen from livestock	kg ha^{-1}	0	0	0	0	0	0	80	170	0
Field-field distance	min	65	65	65	6	180	65	65	65	65
II Farm unit data		f1 f2 f3	f1 f2 f3	f1 f2 f3	f1 f2 f3	f1 f2 f3	f1 f2 f3	f1 f2 f3	f1 f2 f3	f1 f2 f3
Farm-field distance	min	0.75 2.55 10.74	(as original farm)	(as original farm)	1.00 1.00 1.00	30 30 30		(as original farm)	(as original farm)	(as original farm)
Soil fertility Phosphor	Cat.	C D E	(as original farm)	(as original farm)	(as original farm)	(as original farm)	C C C	(as original farm)	(as original farm)	(as original farm)
Soil fertility Potash	Cat.	B B D	(as original farm)	(as original farm)	(as original farm)	(as original farm)	C C C	(as original farm)	(as original farm)	(as original farm)
Soil fertility Magnesium	Cat.	B C C	(as original farm)	(as original farm)	(as original farm)	(as original farm)	C C C	(as original farm)	(as original farm)	(as original farm)
III Differences in fertilization strategy										
Fertilizer & application costs (F&A)	€ ha^{-1}	272.5	323.6	265.5	266.4	287.9	286.6	178.9	54.72	264.2
Application costs relative to F&A	%	7%	22%	5%	6%	12%	6%	9%	13%	8%
Application costs	€ 100kg^{-1}	1.62	6.12	1.11	1.29	2.95	1.46	1.98	2.63	1.66
Number of measures	No	32	30	32	30	31	28	27	13	30
SL	% of nutrients	7%	6%	8%	11%	10%	11%	9%	2%	16%
SH	% of nutrients	23%	24%	23%	21%	18%	25%	7%	2%	21%
CL	% of nutrients	0%	0%	0%	0%	0%	0%	44%	78%	0%
CH	% of nutrients	61%	60%	60%	60%	64%	56%	35%	16%	56%
CA	% of nutrients	9%	10%	9%	9%	8%	8%	5%	2%	7%
Main N source	% of N	67% Urea	71% Urea	68% Urea	56% Urea	57% Urea	57% Urea	28% Slurry	69% Slurry	50% CAN+Mg
Main P source	% of P_2O_5	52% DAP	52% DAP	52% DAP	52% DAP	67% DAP	68% TSP	73% Slurry	100% Slurry	71% TSP
Main K source	% of K_2O	100% gr. potash	100% gr. potash	100% gr. potash	100% gr. potash	100% gr. potash	100% gr. potash	62% Slurry	100% Slurry	93% gr. potash
Main Mg source	% of MgO	46% gr. potash	43% gr. potash	46% gr. potash	47% gr. potash	46% gr. potash	54% gr. potash	48% Slurry	87% Slurry	46% gr. potash
Main S source	% of S	67% gr. potash	72% gr. potash	72% gr. potash	72% gr. potash	72% gr. potash	66% gr. potash	71% SSA	62% SSA	45% gr. potash

Field-field distance = time required to completely approach all fields in a circuit; Soil fertility: A = “very poor” to E = “very high”; SL = low concentrated single fertilizer; SH = high concentrated single fertilizer; CL = low concentrated compound fertilizer; CH = high concentrated compound fertilizer; CA = lime fertilizer; Nutrient composition of slurry (in kg per m³): N: 3.9; NH4+: 1.95; P2O5: 1.7; K2O: 4.7; MgO: 1.2; S: 0.25; CaO: 1.6

Discussion

This study investigated the characteristics of cost-efficient fertilizer strategies. For this purpose, an experiment in the form of a fertilizer quiz was conducted in which the participants had to set up a fertilizer strategy that was as cost-efficient as possible. On the other hand, this study also clarifies whether or how different farm conditions influence a cost-efficient fertilizer strategy. This study will also help to increase knowledge about cost-efficient fertilizer strategies and, if possible, derive general recommendations for action for farmers and advisors.

According to this and further work (Kiełbasa et al., 2018; Rajsic and Weersink, 2008), a nutrient supply that is as close to demand as possible is particularly relevant for the cost-efficiency of fertilizer strategies. Quiz participants had the supply of the N and P nutrients largely under control. Fertilizing with N and P is strictly regulated in Germany (Bundestag, 2017; Bundestag, 2009), which is why farmers pay particular attention to it. However, human decision makers find it difficult to meet all crop nutrient requirements in an equally balanced manner. As a result, inefficient solutions result in significant over-supply of S and K, which, of course, is associated with unnecessary costs. A look at the total nutrient surpluses (NPKMgS) also shows that cost-efficient fertilizer strategies have significantly lower nutrient surpluses. This in turn is evidence that an economically efficient fertilizer strategy makes an important contribution to resource efficiency and sustainable land use. This is also the conclusion of a study on the efficiency of mineral fertilizer use in Europe (Expósito and Velasco, 2020), as well as of a case study on sustainable agriculture by Kiełbasa et al. (2018). Although a general recommendation can be formulated at this point: “All crops should be fertilized according to nutrient demand,” this requirement is not new and was also known to the quiz participants. Therefore, it can be assumed that this recommendation simply cannot be fully implemented by the human decision maker. The mean time required (81 min) and the education level of the participants suggests carefully planned fertilization strategies on their part and reinforces this conclusion. The same is true for the recognition of clear price signals, which we could demonstrate with the example of the abrupt price drop of TSP (see Figure 6-1). Despite complete information (prices, weather, yield expectation, etc.) and a clear focus on profit maximization, the participants of the experiment could not keep a central assumption of production theory, namely: The assumption of rational behavior. Complex problems often cannot be fully understood by humans and limited rational behavior occurs (Simon, 1959). On the other hand, it is also possible that the transaction costs or costs of acquiring information to solve the problem optimally are so high for the decision maker that a suboptimal solution to the problem may be rational from their perspective (Simon, 1959). IoFarm is an important tool for overcoming these barriers. It allows cost-efficient fertilization strategies to be located and updated at regular intervals.

The analysis of the fertilizers used showed that significantly more DAP, gr. potash and lime+Mg were used in cost-efficient solutions (cluster 1). Here, DAP covers about 80% of the phosphorus demand and about 22% of the nitrogen demand and was thus an economically relevant source for both nutrients. Gr. potash also fulfills a significant dual function in cost-efficient solutions: 100% of the potash supply is realized via gr. potash, and in parallel around 17% of the sulfur supply is achieved. In order to take advantage of the dual function of both fertilizers, farmers and consultants must make sure to apply these fertilizers in spring. Also significant is the use of lime+Mg to ensure magnesium supply. In addition, the studies show that NPK fertilizers were not used in cost-efficient solutions. Other studies, however, come to different results here: Sayegh et al. (1981) found on poorly supplied soils in the Middle East that NPK fertilizers have a positive effect on yield at many locations and should therefore be used. There was no economic evaluation of the results in this context. If NPK fertilizers are evaluated with pure nutrient costs, they are at times definitely more favorable than single-nutrient fertilizers, which can also be shown for the period of this study (Schiebel, 2015 - 2018). The reason for avoiding NPK fertilizers in cost-efficient solutions lies rather in the fixed nutrient composition of these fertilizers. NPK fertilizers meet the exact farm requirements only in exceptional cases and therefore make it difficult to supply nutrients in line with requirements. However, compound fertilizers that are specifically tailored to the requirements of the farm are an interesting option. The perfect nutrient composition of such blended fertilizers can be identified with the help of IoFarm, which eliminates the disadvantage of a fixed nutrient composition of NPK fertilizers.

The benefits of the fertilizers mentioned at the beginning (DAP, gr. potash and lime+Mg) are clearly dependent on relative price changes in the fertilizer market. Lahmiri (2017) studied the price volatility of rock phosphate, DAP, TSP, urea, and potassium chloride before and after the global financial crisis in 2007, finding that external shocks lead to volatile fertilizer markets and are associated with relative price changes among fertilizers. External shocks are also to be expected in the future, as the current global COVID-19 pandemic teaches us. For this reason, relative price changes in the fertilizer market can also be expected in the future, which is why no long-term recommendations can be derived for farmers and consultants on the basis of the fertilizers currently considered to be beneficial.

During the analysis, it became apparent that the importance of application costs for a cost-efficient fertilizer strategy was often overestimated by the quiz participants. While the impact of application costs on the total cost of fertilization is significant, striving for the lowest possible application costs leads to several undesirable side effects. To achieve low application costs, the number of fertilization measures must be reduced. As a consequence, the nutrient quantities per measure are increased. This is done, for example, by using NPK fertilizers or by reducing the distribution of nitrogen fertilization to a few applications. All in all, a small number of fertilization measures leads to savings in application costs, but at the same time this makes it more difficult to combine fertilizers cleverly in the sense of demand-based fertilization. In addition, it is more difficult to benefit from the relative price advantages of individual fertilizers. Fertilizer systems that are designed to minimize the number of fertilization measures are more affected by these undesirable side effects. An example of this is CULTAN fertilization. This fertilization strategy is evaluated quite differently in the literature. For example, Kozlovský et al. (2009) and Sedlář et al. (2011) come to significantly higher, lower, and non-differentiable yield effects in different years compared to standard nitrogen fertilization. The effects of CULTAN fertilization or other aggregated N fertilization measures on the total fertilization costs of a crop rotation, however, remain unclear in the literature. Unfortunately, no recommendation for practice can be derived regarding prioritizing application costs. On the one hand, it has been shown that application costs have a relevant influence on the total costs of fertilization, on the other hand, a demand-based nutrient allocation is by far the most important factor to save costs. If both goals are not compatible, the demand-based nutrient allocation has to be prioritized.

To test the influence of farm size, infrastructure, soil fertility and the availability of organic fertilizers, a typical Bavarian farm was used, which was subjected to extreme changes in the respective categories according to the *ceteris paribus* principle. Contrary to the original assumption, the analysis showed that the factor farm size has no visible influence on the selection of a cost-efficient fertilizer strategy. Only a minor influence is caused by the on-farm infrastructure. Unfavorable infrastructure does increase application costs, but even under the conditions of the “faraway fields” scenario with a 30-minute farm-to-field distance, the influence of application costs, accounting for 12% of total costs, was not large enough to cause significant changes in fertilizer strategy. We therefore conclude that the factors of farm size and infrastructure (within realistic limits) do not have a significant impact on fertilizer strategy. Future versions of IoFarm may therefore be able to omit the consideration of transportation costs (farm-to-field and field-to-field), thereby saving considerable computational resources. In contrast, the factors of soil fertility and the availability of organic fertilizers must be evaluated differently. Both factors directly influence the need for nutrients. In the case of homogeneous soil fertility, fertilization measures can be saved, since no field-specific requirements have to be taken into account. Both factors change the selection of fertilizers and the timing of fertilization. In practice, the soil fertility factor differs even on a small scale within farmland. The availability and nutrient content of organic fertilizers also vary a great deal from farm to farm. Both factors are therefore of great importance for a cost-efficient fertilization strategy and must be taken into account.

The previous findings on cost-efficient fertilizer strategies (IoFarm) suggest that a large part of the optimization potential must come from the least-cost combination of fertilizers (type, quantity and timing). This assumption could be confirmed with the “artificial price shift” scenario. Even a slight manipulation of prices led to a recognizable adjustment of the fertilizer strategy. It also seems that a higher variability of fertilizer prices accommodates the optimization potential of IoFarm, because with total costs of €264 ha^{-1} the scenario “artificial price shift” was significantly cheaper than fertilization in the “original farm.” Overall, relative price changes in the fertilizer market are commonplace (Lahmiri, 2017), so regular recalculation is also required for a cost-efficient fertilizer strategy.

The results from the comparison of fertilizer strategies (Section 6.3.1) are based on a low number of quiz participants (n=31). Even with the greatest efforts, it was not possible to motivate more voluntary participants, which was also due to the large amount of time required to participate in the fertilizer quiz. A further simplification of the experiment or payment for participation was purposely rejected, since only intrinsically motivated participants show a real will to optimize and can thus serve as a reference for IoFarm (Stanley et al., 2020; Barge and Gehlbach, 2012; Göritz, 2006). The data set was analyzed using standard statistical methods. The optimal solution of IoFarm itself appears only once in this data set. Therefore, in the regression analyses and t-tests performed, the influence of the optimal solution is not accentuated. In the context of the comparisons of means (see Table 6-2, Table 6-4, Table 6-6), the optimal solution of IoFarm was additionally shown in order to be able to point out special features if necessary. However, statements made about cost-efficient fertilizer strategies can also be confirmed with regard to the optimal solution of IoFarm.

The second part of the analysis (Section 6.3.2) is based as described on a typical Bavarian farm, which was subjected to extreme changes by undergoing different scenarios. By consistently applying the *ceteris paribus* principle, it is possible to analyze the various farm conditions in the scenarios very precisely. This knowledge is helpful for deriving statements for farms that are subject to other conditions. For more applicability, however, it would be helpful to supplement the analysis with actual, but different types of farms. This might lead to combination effects that are suppressed in an analysis according to the *ceteris paribus* principle. However, due to the inaccessibility of information regarding the farm infrastructure, this consideration had to be postponed.

6.5 Conclusions

This study clarifies the distinguishing features between cost-efficient and inefficient fertilization strategies. The influences of different farm conditions were also demonstrated. Here, it is shown that the homogeneity of soil fertility and the availability of organic fertilizers have a far greater influence on fertilization strategy than a farm's size and infrastructure. Ultimately, however, the study also shows that relative price variations among fertilizers dominate cost-efficient fertilizer strategy design. Prices clearly influence the selection of fertilizers and the timing of fertilizer application. Nevertheless, some of the results of this study remain valid regardless of the fertilizer market and farmers and consultants should therefore consider them when thinking about cost-efficient fertilizer planning: nutrient surpluses should be avoided; application costs are not the primary issue in fertilizer planning; standard NPK fertilizers are difficult to integrate due to their fixed nutrient composition. Overall, it can be seen that the humans as decision makers are mentally unable or unwilling to optimally solve such a complex problem, or this inability or unwillingness could be due to transfer costs. The DSS IoFarm is a suitable tool to use to accomplish this task. IoFarm can help increase farm profits and, in parallel, help avoid redundant use of nutrients. In summary, IoFarm improves both profit and sustainability in agriculture and should therefore be used as widely as possible in the future. The contribution to sustainability justifies a subsidy to ensure widespread practical use. We will spend the coming months further developing IoFarm into an online-based DSS. This will generate the necessary computing capacities, which are a key requirement for widespread practical use.

6.6 Appendix

Table A 6-1: Allocation of fertilizers to fertilizer categories.

Abbreviation	Name	Content/Effect in kg per 100 kg					
		N	P_2O_5	K_2O	MgO	S	CaO
Category SL (single low)							
AHL	Ammonium nitrate urea solution	28					-28
AHL1to3	67%Water + 33%AHL	9					-9
CAN	Calcium ammonium nitrate	27					-15
CAN+Mg	Calcium ammonium nitrate	27			4		-9
CAN+S	Calcium ammonium nitrate	24				6	-34
Category SH (single high)							
TSP	Triple superphosphate		46				-1
U+Inhib	Alzon	46					-46
Urea	Urea	46					-46
Category CL (compound low)							
Kainite	Kainite			11	5	4	0
Slurry	Liquid organic fertilizer	3,9*	1,7	4,7	1,2	0,3	1,6
Category CH (compound high)							
ASS	Ammonium sulphate nitrate	26				13	-49
DAP	Diammon phosphate	18	46				-36
ENTEC+S	ENTEC	26				13	-49
ENTEC NPK	ENTEC	15	5	20	2	8	-14
gr. potash	Granular potash			40	6	5	0
Kieserit	Kieserite				25	20	0
NP 20;20	NP	20	20			2	-31
NPK 15;15;15	NPK	15	15	15		2	-15
NPK 20;8;8	NPK	20	8	8	3	4	-21
NPK 23;5;5	NPK	24	5	5		4	-23
PK 16;16	PK		16	16	2	8	6
SSA	Sulfuric acid ammonia	21				24	-63
Urea+S	Piamon S	33				12	-54
Category CA (special lime fertilizer)[3]							
Burned Lime	Burned lime						90
Lime+Mg	Carbonic lime				14		53
Lime+S	Carbonic lime					2	50

SL and SH: Single-nutrient fertilizers containing either N, P, or K and whose content of S and Mg does not exceed 20% of its total nutrient content. Fertilizers that do not meet this definition are called compound fertilizers (CL and CH). L stands for low nutrient content (<= 31%); H for high nutrient content (> 31%); CA = group of lime fertilizers.

Table A 6-2: Fertilizer Quiz Data Part 1 - Costs, Clusters, Quantities.

ID	FA_Cost [€]	Cluster	A_Cost [€]	Measures [No]	P2016 [kg]	K2016 [kg]	P2017 [kg]	K2017 [kg]	P2018 [kg]	K2018 [kg]	N_loss [kg]	S_loss [kg]	P_bil [kg]	K_bil [kg]	Mg_bil [kg]	NPKMgS [kg]
1	123376	1	8323	37	7531	9413	10435	0	34085	9977	6562	3680	0	565	1319	12125
2	127864	1	6776	27	12995	0	21919	9420	17158	9420	7655	4981	22	15	6555	19228
3	132756	1	7394	19	9200	0	34500	0	8625	19320	5087	2581	275	495	3736	12173
4	134985	1	6263	22	16100	0	12075	0	24380	19500	6250	4903	505	675	6215	18548
5	141114	2	7956	37	20470	8275	14720	4840	17250	6000	5720	4786	390	290	3510	14696
6	143540	2	6969	25	28750	0	16905	0	7360	19375	6357	8881	965	550	8855	25607
7	143793	2	8237	36	18170	0	28990	10000	10120	9600	5607	5976	5230	775	1655	19243
8	143828	2	7058	17	18676	12930	14973	7843	22678	0	3959	9496	4277	1948	4131	23810
9	145439	2	7988	30	16875	775	25610	14000	10015	4265	7004	15421	450	215	7165	30255
10	145631	2	8210	26	15870	4488	18630	7900	17940	6655	4747	4033	390	218	423	9811
11	145660	2	7700	32	18925	4525	17600	5550	16775	10125	4689	9861	1250	1375	7720	24895
12	146906	2	7723	24	16780	2750	8600	9735	27480	6525	6108	6701	810	185	6380	20183
13	148514	2	8957	40	17250	8000	23690	5000	11900	12600	5464	7916	790	6775	8930	29875
14	150062	2	6538	22	18545	7445	15600	1800	21530	13480	4063	8236	3625	3900	6565	26389
15	152787	2	8442	34	21575	8875	15650	4840	15575	6325	3897	10911	750	1215	700	17473
16	153753	2	7603	32	11660	7750	15425	1625	29050	18800	3720	15231	4085	9350	2850	35235
17	154487	2	7602	21	6200	0	7000	0	42500	19200	4451	18756	3650	375	3900	31132
18	155535	2	7832	32	22310	550	15220	2000	14870	19760	8638	7196	350	3485	4725	24393
19	155881	2	9629	37	15180	0	18860	13790	18630	8000	6002	19291	620	2965	1590	30468
20	156842	2	7220	25	10200	0	13000	10000	31450	10000	4302	12186	2600	1175	305	20568
21	159048	2	8902	42	17485	4325	18505	10580	17560	4200	5369	16921	1500	280	3535	27605
22	161504	2	8155	28	17250	0	18140	4800	17790	14240	5825	13386	1130	215	2205	22761
23	161825	2	7562	24	15600	6400	19600	10400	20250	8750	5650	14566	3400	6725	6400	36741
24	162500	2	8248	26	6400	6400	7425	4300	38640	8225	4981	19976	415	100	5405	30877
25	165821	3	8303	26	25300	2750	14950	9600	12650	6800	6456	37816	850	325	15235	60682
26	167323	3	8793	36	21200	500	22250	5000	18600	15400	5154	18761	10000	2075	9235	45225
27	170625	3	8843	41	28865	9000	15200	2800	8850	13500	5269	18471	865	6475	1190	32270
28	174442	3	7196	27	11385	25300	21150	0	20440	0	5282	16066	925	6475	2995	31742
29	180804	3	9539	35	22400	12300	16400	12550	14540	2750	5509	31316	1290	8775	6725	53615

ID: identification number; FA_Cost = total fertilizer and application costs; Cluster = classification according to FA_Cost; A_Cost = application costs; Measures = number of fertilizer measures; P2016 (and similar) = P fertilization in 2016; N_loss = N conversion losses; S_loss = S losses or wrong placement; P_bil (and similar) balance of P fertilization and withdrawal; NPKMgS = sum of N_loss to Mg_bil.

Table A 6-3: Data from the fertilizer quiz part 2 - Shares in nutrient supply.

ID	N% CAN+Mg	N% AHL1to3	N% DAP	P% DAP	P% TSP	K%-ENTEC NPK	K% gr. potash	Mg% gr. potash	Mg% Lime+Mg	S% gr. potash	S% Kiserit	SL	SH	CL	CH	CA
1	26%	4%	21%	80%	20%	0%	100%	10%	59%	17%	33%	8%	12%	0%	67%	12%
2	10%	0%	26%	100%	0%	0%	100%	9%	52%	16%	66%	3%	12%	0%	75%	10%
3	39%	9%	13%	47%	53%	0%	100%	10%	36%	20%	80%	16%	15%	0%	62%	7%
4	5%	13%	27%	100%	0%	0%	100%	9%	50%	17%	73%	12%	2%	0%	76%	10%
5	28%	0%	26%	100%	0%	0%	50%	5%	36%	8%	57%	15%	3%	18%	56%	8%
6	27%	0%	19%	74%	26%	0%	93%	8%	48%	12%	52%	9%	13%	2%	66%	10%
7	25%	0%	8%	28%	67%	0%	100%	10%	50%	16%	43%	12%	20%	0%	59%	10%
8	35%	0%	24%	82%	0%	0%	50%	5%	50%	5%	39%	9%	0%	0%	81%	10%
9	0%	3%	2%	7%	82%	0%	68%	6%	52%	7%	43%	4%	24%	0%	62%	10%
10	42%	6%	0%	0%	100%	0%	73%	8%	25%	13%	67%	23%	20%	10%	40%	6%
11	0%	0%	19%	71%	6%	0%	54%	5%	44%	7%	70%	13%	4%	2%	73%	9%
12	6%	0%	9%	32%	25%	0%	0%	0%	56%	0%	28%	5%	13%	32%	38%	11%
13	11%	1%	21%	80%	20%	0%	98%	11%	44%	17%	68%	11%	8%	0%	72%	9%
14	30%	0%	8%	27%	17%	0%	0%	0%	51%	0%	37%	8%	5%	0%	77%	10%
15	31%	0%	0%	0%	88%	0%	63%	7%	46%	7%	33%	24%	20%	2%	45%	9%
16	16%	0%	9%	32%	31%	0%	25%	4%	62%	4%	22%	19%	6%	0%	63%	12%
17	3%	0%	0%	0%	29%	0%	0%	0%	46%	0%	39%	15%	10%	0%	66%	9%
18	3%	4%	1%	5%	91%	9%	81%	9%	48%	14%	61%	2%	35%	1%	52%	10%
19	19%	0%	7%	28%	72%	0%	95%	11%	26%	9%	42%	9%	19%	1%	65%	6%
20	30%	0%	13%	48%	0%	0%	100%	11%	0%	12%	75%	12%	1%	0%	87%	0%
21	10%	3%	7%	25%	22%	40%	26%	2%	48%	2%	35%	7%	7%	3%	73%	10%
22	5%	0%	0%	0%	73%	32%	0%	0%	34%	0%	57%	19%	15%	0%	59%	8%
23	62%	0%	0%	0%	54%	0%	0%	0%	25%	0%	46%	17%	18%	0%	60%	5%
24	36%	0%	11%	42%	39%	0%	44%	4%	0%	4%	65%	18%	8%	7%	67%	0%
25	0%	0%	0%	0%	100%	0%	86%	6%	60%	4%	22%	9%	27%	4%	48%	12%
26	6%	0%	0%	0%	91%	4%	94%	8%	42%	9%	48%	8%	23%	0%	61%	8%
27	19%	0%	3%	12%	54%	47%	0%	0%	39%	0%	35%	8%	15%	2%	67%	8%
28	16%	0%	4%	16%	46%	100%	0%	0%	42%	0%	40%	5%	15%	0%	71%	9%
29	15%	0%	0%	0%	60%	0%	0%	0%	25%	0%	28%	6%	17%	19%	52%	6%

ID: identification number; N%CAN+Mg = share of calcium ammonium nitrate+Mg to N fertilization, etc.; SL to CA = share of fertilizer categories SL(single low), SH (single high), CL (compound low), CH (compound high), and CA (lime fertilizer) to total nutrient supply.

Part III

Discussion and Conclusions

7 Extended summary of embedded publications

For a quick overview, highlights and core findings on the individual studies are shown in Table 7-1, and detailed summaries are provided in Sections 7.1 to 7.4.

Table 7-1: Overview of highlights and core findings of embedded publications.

Chapter3: Effects of application costs on fertilizer application strategy
• Transport costs influence the fertilizer application strategy • Transport costs depend on-farm-specific infrastructure and route planning • Route planning is extremely resource-demanding and currently difficult to achieve • Farm- and measure-specific transport costs can be estimated • This is an effective approach that does not affect the fertilization strategy itself
Chapter4: IoFarm: A novel decision support system to reduce fertilizer expenditures at the farm level
• The selection of a cost-efficient fertilizer strategy is very complex • The least-cost combination and input level of fertilizer must be considered • The MINLP model, IoFarm, can solve this problem • In an experiment, IoFarm was 19% below the fertilizer costs of the participants • IoFarm is a useful DSS that increases farm profit and saves management time
Chapter5: IoFarm in Field Test: Does a cost-optimal choice of fertilization influence yield, protein content or market performance in crop production?
• The DSS, IoFarm, proved its agronomic performance in field trials • Agronomic performance was not negatively affected by least-cost fertilizer strategies • Cost advantages in fertilizer selection can be fully attributed to IoFarm
Chapter6: Characteristics of cost-efficient fertilization strategies at the farm level
• Certain fertilizers are used more often in cost-efficient strategies • Application costs are less relevant for cost-efficiency • Nutrient surpluses lead to inefficiency • Volatile fertilizer prices greatly influence cost-efficient fertilization strategies • Complexity requires farmers to use optimization software to increase profitability and sustainability

7.1 Summary of Chapter 3: Effects of application costs on fertilizer application strategy

To optimize production processes from an economic perspective, a comprehensive knowledge of the interrelationships and costs of production is necessary. Transport costs play an important role in agricultural production. However, in on-farm infrastructures, farms are heterogeneously structured. From large farms with nearby fields and small farms with widely dispersed field pieces, the most diverse constellations of on-farm infrastructure can be found. Therefore, transport distances are relevant to optimize transport-dependent production processes, such as the application of fertilizers. Currently, there is no resource-friendly way to integrate farm- and measure-specific application and transport costs into a mathematical optimization tool that helps improve the economic efficiency and sustainability of production.

This study presents the development of a cost function for fertilizer application. To integrate the influence of on-farm infrastructure in selecting a cost-efficient fertilizer strategy, the overall application cost function was first decomposed into individual components, and sub-functions were then formed for the following elements: setup, loading, fieldwork, farm-to-field transport, and field-to-field transport. To determine the extent of the differences in transportation costs between farms with different on-farm infrastructures, 70 random farms were generated. These farms are of different sizes and distances of field pieces. For each random farm, 28 fertilizer scenarios were examined based on the application rate per hectare and proportion of field pieces fertilized. An algorithm for solving the split delivery vehicle routing problem ("SDVRP method") was used to determine the optimal routing for fertilizer application. These results and the farm data could be used to identify important factors influencing farm- and measure-specific transport costs. Based on these influencing factors, a regression model was derived to estimate transport costs ("regression method"). Both possibilities in determining transport costs, SDVRP and regression method, were used to compare whether deviations in the selection of the optimal fertilization strategy are to be expected. Moreover, another possibility, which does not consider transport costs, was investigated. All three possibilities lead to largely consistent fertilization strategies. Wrong decisions caused by the regression method or the omission of transport costs are rare and cause marginal financial impact. Compared to the alternatively tested methods, input data and computational power requirements are by far the highest for the SDVRP method. Therefore, it is concluded that considering transportation costs as a sub-function of application costs in a mathematical optimization tool should be evaluated using the regression method, and a general disregard of transportation costs is not recommended. This study is relevant for issues related to agricultural logistics and similar cost functions, such as the application of pesticides or seeds.

7.2 Summary of Chapter 4: IoFarm: a novel decision support system to reduce fertilizer expenditures at the farm level

To maintain competitiveness and increase profits, producers focus on saving costs without reducing output. The expenses for fertilizers and their application play a major role in agricultural crop production and account for the largest share of variable costs in crop production, which also indicates a large degree of optimization potential. To fully achieve this optimization potential, the cost-efficient fertilizer strategy must be identified. Therefore, the following points are important: (i) optimal intensity and composition of nutrient supply; (ii) temporal and quantitative distribution in the course of a crop rotation; (iii) application costs; and (iv) selection of the most cost-efficient fertilizer combination. The described challenges on optimization represent the application of a well-known theory on expansion path. Changes in the field- and crop-specific nutrient requirements, weather effects, available fertilizers, volatile prices, and nonlinear structure of application costs suggest that a human decision maker may not solve this optimization problem alone. Therefore, various decision support systems (DSS) have been developed to assist farmers and consultants, but only a few DSS consider the least-cost combination of fertilizers. None of these DSS consider farm-specific application costs and plan fertilizer application over an entire crop rotation cycle. The development of IoFarm closes this gap, which can also achieve economic potential by taking a holistic view of this optimization problem.

To address this optimization problem, a non-formal model that identifies important factors influencing the fertilization strategy and describes their effects was developed. It is discussed how these factors can be implemented in a DSS without limiting its usability. The implementation is presented in the form of a mathematical optimization model. Due to the complexity, a two-step solution procedure composed of a nonlinear problem (NLP) and a mixed-integer nonlinear problem (MINLP) is necessary. To evaluate the economic performance of the IoFarm DSS, an experiment was conducted. The participants of the fertilization experiment (experts, farmers, and students) were asked to plan the most cost-efficient fertilizer strategy possible for a highly simplified farm. All relevant information, such as prices, application costs and nutrient requirements, was provided. The results show on average that the fertilizer strategies of the participants were 19% more expensive compared to IoFarm. Additionally, participants spent on average 81 min of management time to solve this problem suboptimally. The best participant achieved significantly higher costs than IoFarm with a difference of €10 ha^{-1}. IoFarm is therefore a very promising tool for farmers and advisors that helps increase profit and save management time. The next research step is to examine potential agronomic impacts to identify possible output changes. The study also helps demonstrate the theory of the expansion path with a realistic case and is therefore also suitable for practical teaching of the production theory.

7.3 Summary of Chapter 5: IoFarm in field test: does a cost-optimal choice of fertilization influence yield, protein content, and market performance in crop production?

The IoFarm decision support system (DSS) was developed to identify cost-efficient fertilizer strategies for farms. Results show that significant cost savings can be obtained. For user acceptance and according to operations research specifications, it is essential to test such DSS in practice. Models, such as IoFarm, can only represent simplified interrelationships and have to be verified in more complex ones to assess their performance. Even though a small number of DSS with similar orientation as IoFarm are described in other studies, there have been no reports on their use in the field. Thus, the following question arises: does a least-cost fertilizer strategy potentially affect crop yield, quality, and market performance? This question is relevant because a cost-efficient fertilizer strategy is not based on low costs alone but also on the impact of output.

To answer this question, a two-factorial field trial with three replications was set up at three locations in Bavaria (Geiselsberg, Triesdorf, and Roggenstein) over a period of 3 cropping years (2016–2018). The first factor reflected the fertilizer variant, wherein IoFarm was compared with a common farm fertilization strategy and a control variant. The second factor was built by different crops, namely silage maize, winter wheat, and winter barley. Within the trial period, this crop rotation was cultivated once on each plot. At the Triesdorf location, additional variants of the first factor were built using organic fertilization. The evaluation of the field trial was conducted using a split-plot ANOVA and subsequent t-tests. The results show no significant differences in yield, protein content, and market performance between the common farm fertilization strategy and IoFarm. Thus, the cost benefits already demonstrated with the use of IoFarm are not achieved at the expense of market performance. This study highlights the benefits of IoFarm for farmers and consultants and thereby increasing the confidence of potential users. Therefore, according to the principle of least-cost combination, the selection of fertilizers has no significant impact on the output in crop production and thus should be widely applied.

7.4 Summary of Chapter 6: Characteristics of cost-efficient fertilization strategies at the farm level

Fertilization of agricultural crops represents a significant part of production costs in most farming systems. Through the selection of individual fertilization strategies, farmers have a great influence on costs and agronomically relevant parameters, such as yield and quality. Fertilizer strategy is based on intensity, timing, dosage, and selection of specific fertilizers. With this study, whole-farm fertilization strategies are analyzed to compare cost-efficient and inefficient fertilization strategies. This study aims to generate recommendations for farmers and consultants to improve competitiveness and sustainability.

To obtain information on individual fertilization strategies, a fertilization experiment was conducted. Participants were asked to plan a cost-efficient fertilizer strategy for a simplified three field, three crop farm for over 3 years. All relevant information, such as nutrient requirements (N, P, K, Mg, and S), fertilizer prices, product prices, weather information, and application cost information were provided. A series of linear regressions were used to evaluate the experiment. To generate additional information, the different fertilization strategies were divided into three clusters based on their total costs. Differences between clusters were tested using t-test. The results show that some fertilizers (e.g., DAP and granular potash) were more common in cost-efficient solutions, whereas NPK fertilizers were barely used. The application timing of P and K also played a role in the overall cost. Fertilization with K and S according to demand is a major challenge. In inefficient fertilization strategies, these nutrients were significantly overdosed, with a parallel negative impact on sustainability.

In the second part of the study, the IoFarm DSS was used under different farming conditions. With IoFarm, cost-efficient fertilization strategies can be identified. A typical Bavarian farm was used as the reference situation, which was then varied *ceteris paribus* in the following aspects: acreage, internal infrastructure, soil fertility, and organic fertilizer accumulation. In addition, artificially changed fertilizer prices were used to calculate another scenario. The results of this study highlight the differences between cost-efficient and inefficient fertilizer strategies and the influence of individual farm conditions. Interestingly, acreage had no effect, while internal infrastructure had very little effect on fertilizer strategy. Fertilizer prices and their relative price differences were found to be the largest factors influencing the design of the fertilizer strategy. Despite the large influence of fertilizer prices, the following recommendations for choosing a fertilizer strategy must be considered: (i) nutrient surpluses should be avoided; (ii) minimizing application costs is not effective; and (iii) NPK fertilizers are rarely used due to their fixed composition. Furthermore, the participants were unable to make optimal decisions despite being thoroughly informed. The study is thus providing an example of limited rational behavior in complex situations.

8 General discussion and conclusions

The objective of this thesis was to support farmers and consultants in choosing cost-efficient fertilization strategies at the farm level and better understand the interrelationships of cost-efficient fertilization strategies. This objective was addressed in a total of four consecutive studies (Chapters 3 to 6). In particular, this chapter provides an overview of the contribution of these studies. In each case, the background is briefly repeated, and the results and significance of the studies in relation to the existing literature are discussed. The succeeding paragraphs highlight extended stakeholder implications, methodological contributions, limitations of this thesis, and conclusions.

Discussion of the results from Chapter 3: Machine costs are a major factor in the cultivation of field crops. The share of variable machinery costs in winter wheat production in Bavaria was approximately 33% in 2017 to 2019 (Schätzl et al., 2019). Part of these costs is related to fertilizer application, including transport. Studies show that production decisions are related to transportation costs and thus application costs (Shamdasani, 2021; Damania et al., 2017). Therefore, application costs are expected to influence the selection of cost-efficient fertilizer strategies. Hence, the objective of Chapter 3 was to develop and implement methods that allow the consideration of farm- and measure-specific application costs within a mathematical optimization model. However, the transport costs for farm-to-field and field-to-field trips remain a challenge (Jensen et al., 2012).

The results show that, at least from a logistical perspective, the split delivery vehicle routing problem (SDVRP) (Dror et al., 1994; Dror and Trudeau, 1990) is perfectly suited to meet this objective. By incorporating the SDVRP, optimal transportation routes for fertilizer application can be identified, which can then be used to calculate transportation costs. Several studies show that the advantage of optimal routing is especially utilized in transportation-intensive economic sectors (Latiffianti et al., 2018; Eldrandaly and Abdallah, 2012; Basnet et al., 1996). However, it is unlikely that farmers can entirely follow the guidelines of such routing in practice. For example, the carrying capacity of the soil determines in which order and with which load a field can be driven. Although the SDVRP could still be used to estimate transportation costs, this study showed that high computational capacities are needed to solve an SDVRP. The same is also reported in a previous study, wherein state-of-the-art algorithms that can solve problems up to 288 subjects (field pieces) within 1.422 s (Archetti et al., 2011a). However, small problems with only 41 subjects may also not be solved even after

7.200 s (Hernández-Pérez and Salazar-González, 2019; Ozbaygin et al., 2018; Archetti et al., 2014). Therefore, there is no guarantee that an SDVRP can be reliably solved optimally. In addition, an SDVRP requires information on all on-farm travel distances, which includes all farm-to-field and field-to-field distances. As an example, a dataset of 24 field pieces including the farm location covers 325 distances that a farmer usually cannot provide. Although there are possibilities to generate this dataset (Machl et al., 2016), access to this possibility is strictly limited for data protection reasons.

Compared to the SDVRP method, two other approaches with low computational power and data input requirements were tested: (i) The "regression method," derived from the results of the SDVRP method, establishes a functional relationship between farm- and measure-specific parameters and thus estimates transportation costs without routing information; (ii) The "zero transportation cost method" (ZTC method). Here, transportation costs are neglected as part of the application costs. The results of a Monte Carlo simulation show that under the given circumstances, the ZTC method leads to much the same fertilization strategy as the SDVRP method. In cases where there is no agreement, the financial damage caused by the deviation is marginal (€0–€0.5 ha^{-1}). Minimal data requirements and computational capacity were in favor of the ZTC method, although with some weaknesses: with increasing importance of transport costs and inhomogeneous farm-field and field-field distances, the validity of the ZTC method wanes, as this information remains unobserved and cannot be considered in decision making; therefore, the regression method is more robust in this regard. Compared to the SDVRP method, the regression method leads to the same fertilization strategy with a percentage of 92% ("Nitrogen Experiment" Table 3-6) or 97% ("PK Experiment" Table 3-7). Wrong decisions result in minor cost disadvantages of €6.1 and €3.4 ha^{-1}.

The results of the study demonstrate why the SDVRP method cannot be combined with a complex optimization model. However, the increasing digitization of agriculture and developments of software and hardware might facilitate the provision and processing of the necessary data in the future. Until then, the regression method can be recommended as the second-best solution. By estimating transportation costs using farm- and measure-specific parameters, this method is useful for a wide range of farms. Due to the low demands on computing capacity and data requirements, the regression method is suitable for reflecting transport costs within the framework of a mathematical optimization model.

Discussion of the results from Chapter 4: This thesis assumes that farmers act according to the economic principle. Therefore, they are concerned with optimizing production processes to increase profit. Cost-efficient fertilization in crop production is a difficult task for farmers and consultants. The objective of Chapter 4 was to structure the complex optimization problem of cost-efficient fertilization, to develop a solution and to highlight potential economic advantages.

The results show that the optimization problem of cost-efficient fertilization can be represented using a mathematical model (IoFarm). In the operations research literature, this approach is quite recommended for solving problems with known relationships (Domschke et al., 2015, pp. 3–7; Suhl and Mellouli, 2013, pp. 8–20). To test the economic performance of IoFarm, a fertilization experiment was conducted in the form of a choice experiment. Participants were asked to plan cost-efficient fertilization strategies. Although the requirements of the individual nutrients were fixed, the participants' average cost of €66 $ha^{-1}a^{-1}$ was surprisingly well above the cost level of IoFarm. The participant with the best results was as close as €10 $ha^{-1}a^{-1}$ to IoFarm's solution, which still represents a considerable savings potential of €1000 $ha^{-1}a^{-1}$ for a 100-ha farm. In addition to IoFarm, other DSSs considering the least-cost combination of fertilizers can be found in previous studies (Villalobos et al., 2020; Jansen et al., 2013; Bueno-Delgado et al., 2016; Pagán et al., 2015; Mínguez et al., 1988; Babcock, 1984). However, none of these studies compare the cost-efficiency of the DSS solutions to a standard fertilization strategy. Therefore, a direct comparison with IoFarm is not possible. Smart Fertilizer (Smart Fertilizer Management) is a comparable commercial tool on the market. The company advertises a savings potential of 60% and an increase in income of 40%, although with insufficient details. Sensor-based and site-specific fertilization measures represent a completely different way of optimizing fertilization. Manufacturers of this technology present additional profits of €20 to €30 $ha^{-1}a^{-1}$, for example, by improving N efficiency. In a field trial using sensor-based N fertilization in silage maize, Evangelou et al. (2020) show potential savings of €33 to €92 $ha^{-1}a^{-1}$. Colaço and Bramley (2018) report an average profit increase in grain crops of \$30 $ha^{-1}a^{-1}$ (from −\$30 to +\$70 $ha^{-1}a^{-1}$) in their review on crop sensors. Given these numbers, the potential savings achieved with IoFarm are remarkable, especially since there is no investment in technology. In addition to the cost of fertilization, the time required for participants to plan a fertilization strategy was also considered. Despite highly simplified conditions (three field plots of three crops in 3 years), the average time spent on this task was unexpectedly high at 81 min per participant. The amount of management time a farmer uses annually to improve fertilizer cost-efficiency remains unclear from the fertilizer experiment. However, it can be assumed that real farms are more complex (StMELF, 2020) and the fertilizer strategy has to be readjusted several times per season. Using a DSS, this process can be formalized and largely automated (Blanco, 2020), which saves management time (González-Andújar, 2020, p. 27).

Taken together, IoFarm contributes significantly to reducing fertilizer costs and management time. Large farms benefit in particular because potential savings in management time and fertilizer costs increases with farm size. Since Chapter 4 suggests a constant output in crop production, this must be verified before providing statements about the cost-efficiency of IoFarm.

Discussion of the results from Chapter 5: Production theory demonstrates that changes in production inputs often affect the output (Debertin, 2012a, p. 82). Therefore, the cost reduction achieved by IoFarm must be examined as it may have consequences on the output. The objective of Chapter 5 was to compare the agronomic performance of IoFarm with a usual farm fertilization strategy, so the following fertilizer variants were tested in a field trial: (i) pure mineral IoFarm and farm manager variants (IO, BL); (ii) organic and mineral IoFarm and farm manager variants (oIO, oBL); and (iii) an unfertilized control.

The statistical analysis of the results shows that there were no significant differences in yield, quality, and market performance between comparable varieties, which include IO and BL, as well as oIO and oBL. Only a few DSS that show similarity with IoFarm are found in previous studies (Villalobos et al., 2020; Jansen et al., 2013; Bueno-Delgado et al., 2016; Pagán et al., 2015; Mínguez et al., 1988; Babcock, 1984). None of these studies tested the agronomic performance of the respective tools in a field trial; this is also true for the commercial tool of Smart Fertilizer Management. There were several studies on DSS in the field of fertilization (Mandrini et al., 2021; Mollenhorst et al., 2020; Kleinhenz et al., 2007), although it is rather the exception that these studies report results from own field trials. These exceptions include studies by Araya et al. (2019), Übelhör et al. (2015), Sønderskov et al. (2015), and Chuan et al. (2013). The DSS tested therein have been proven in field trials, although not comparable to IoFarm. Therefore, studies are lacking on whether fertilizer strategies that are trimmed for least-cost combination and cost-effectiveness may involve negative agronomic impacts. Chapter 5 makes an important contribution here, showing that a fertilizer strategy optimized primarily for cost-effectiveness does not necessarily have a significant impact on yield, quality, and market performance in crop production.

Before this statement can be accepted, possible errors in the design or execution of the field trial must be verified to exclude whether these are the cause for the non-significant differentiation of the fertilizer variants. A high variance within the measured values or an insufficient number of measured values can also be the cause for a statistically not reliable differentiation of the group mean values. Therefore, the relative standard error should be referred to at this point, which sets the standard error in relation to the mean value and ranges from 2.3% to 3.5% for the mineral fertilizer variants (IO and BL) across the yields of all crops. For a significant interpretation of field trials, the values should be <4% (Thomas, 2006, p. 61). The relative standard error revealed that there is no significant difference in yield between the IO and BL variants. For the organic and mineral fertilized variants, oIO and oBL, the relative standard error is 4.5% to 6.9%. This result is due to these variants were only tested at the Triesdorf site; therefore, significantly fewer observations are available. Additionally, the application of organic fertilizer varied between the variants in terms of time and quantity per application. Therefore, weather conditions and the variability of the nutrient content in the organic fertilizer led to unobserved influences, which are generally a problem in experiments with organic fertilizers (Tamburini et al., 2015; Chen et al., 2013). To further validate the comparison of the oIO and oBL variants, additional observations are required. Unfortunately, not all sites could be included in this comparison due to the lack of plot technology for organic fertilization.

Overall, it was demonstrated in Chapter 5 that IoFarm causes no significant agronomic effects. Future users can assume that with unchanged market performance, the cost savings will have a positive impact on the farm profit, as shown in Chapter 4.

Discussion of results from Chapter 6: The results of the previous chapters show that IoFarm leads to a significant reduction in fertilizer costs and management time without changes in agronomic performance. When discussing these results with farmers or consultants, the following question arises: What are the differences between a fertilizer strategy planned by IoFarm and a common fertilizer strategy? This question is also relevant from a scientific perspective because it simultaneously expands the knowledge on overall farm relations of cost-efficient fertilization strategies. There are many studies on various aspects of fertilization, e.g., specifically on intensity (LI et al., 2021; Tabak et al., 2020; Sihvonen et al., 2018; Xu et al., 2017; Chuan et al., 2013) or specifically on technologies (Song et al., 2021; Fulton et al., 2021; Gil-Ortiz et al., 2020; Dimkpa et al., 2020; Mi et al., 2019; Kozlovský et al., 2009; Koch et al., 2004). However, studies on perennial (crop rotation) and whole-farm cost-efficient fertilization strategies could not be found. Chapter 6 examines the different characteristics of cost-efficient and inefficient fertilizer strategies at the farm level, wherein data from the fertilization experiment are statistically analyzed. In addition, Chapter 6 will clarify whether and to what extent different farm conditions affect cost-efficient fertilization strategies. For this analysis, IoFarm is used as a simulation model, and the model output is compared.

The results from the first part of the study show a significant relationship between cost-efficiency and demand-based fertilization. Kiełbasa et al. (2018) and Rajsic and Weersink (2008) also established this relationship. In the fertilizer experiment, inefficient fertilization strategies are particularly noticeable due to surpluses of the nutrients K and S. It can be assumed that the participants focused more on demand-based N and P fertilization since both nutrients are strictly regulated under the German law (Bundestag, 2017; Bundestag, 2009). This results in a certain automatism that directs the focus on N and P. Cost-efficient fertilization strategies showed significantly lower nutrient surpluses in the experiment, suggesting that they are more resource efficient and sustainable. This conclusion is also found in a study by Expósito and Velasco (2020) on the efficiency of mineral fertilizer use in Europe, as well as in a case study by Kiełbasa et al. (2018) on sustainable agriculture. Therefore, one recommendation for fertilizer planning is to apply all nutrients as needed, although most participants were unable to do this. The same also applies to the recognition of clear price signals on the fertilizer market. For the fertilizer TSP, there was a massive and abrupt price decline in the summer of 2016. In cost-efficient fertilizer strategies, this time window was used for some phosphorus fertilization. However, 80% of participants did not take advantage of this low-price period, probably because it was not recognized. Both observations on nutrient surpluses and price signals suggest that human decision-makers are burdened with the multitude of information and complexity of this problem and, therefore, can only act rationally to a limited extent (Simon, 1959).

Chapter 6 also provides insights regarding the preferability of individual fertilizers. For example, the fertilizers DAP as a source of N and P, grain potash as a source of K and S, CAN as a source of N and Mg, and carbonic magnesium lime as a source of Ca and Mg were highly valued in cost-efficient fertilizer strategies. Urea frequently occurred in cost-efficient and inefficient fertilizer strategies. The aforementioned fertilizers combine well due to their nutrient composition, which allows a balanced distribution of nutrients. For the specific compensation of the nutrients P and S, TSP and SSA were applied selectively. NPK fertilizers and stabilized nitrogen fertilizers were not used in cost-efficient solutions. Studies on the cost-effectiveness of stabilized nitrogen fertilizers remain controversial statements: Mi et al. (2019) found both positive and negative effects on profit in different comparisons. Sikora et al. (2020b) show significant positive effects on profit in the cultivation of vegetables. In the present study, however, stabilized nitrogen fertilizers are avoided in cost-efficient solutions because of their high cost relative to pure nutrient content. Positive effects, such as reduction of nitrate leaching (Pack et al., 2006; Owens et al., 1999), mitigation of greenhouse gas emissions (Chen et al., 2021; Sikora et al., 2020a; Tang et al., 2018), and saving of labor inputs for the same expected yield (Wilson et al., 2009), justify a higher price. However, the fertilizer experiment suggests that the additional cost of stabilized fertilizers may not be acceptable in this case.

Broken down to their pure nutrient content NPK fertilizers often have competitive prices (Schiebel, 2015 - 2018). However, the fixed composition of these three nutrients in a fertilizer limits flexibility in choosing other fertilizers. Fertilizer industry advertises NPK fertilizers to have positive yield effects and a reduction in labor, which may be true. There is also evidence in the literature of the positive yield effect of NPK fertilizers (Sayegh et al., 1981). However, this effect is likely to occur even if the nutrients N, P, and K are applied separately in the same period.

Furthermore, application costs have a significant impact on the total cost of fertilization. However, this result can easily be misleading. The share of application costs in total costs is low across all participants, ranging from 4% to 7%; therefore, the savings potential in this respect is marginal. To achieve the lowest application cost, the number of fertilizer applications must be reduced by combining fertilization measures. Therefore, the participants specifically selected highly concentrated fertilizers (including NPK fertilizers) and summarized fertilization measures using stabilized fertilizers or increasing the N dose per application. In the previous paragraph, the effect of these fertilizers on cost-efficiency has already been discussed. A small number of fertilizer measures means that there are less time windows available for the selection of least-cost fertilizers. This also reduces the number of degrees of freedom in fertilizer selection, making it more difficult to allocate all nutrients according to demand. The results indicate the following: if the application costs in the total costs of fertilization are <7%, it is not advisable to focus on the lowest possible application costs. This statement may be true even for a significantly higher share of application costs, although this cannot be proven or generalized from the study.

The results from Chapter 6 show a significant correlation between cost-efficiency and timing of basic fertilization with P and K; for example, P was more expensive on average in 2016, whereas K was more expensive in 2017. The previous results lead to the assumption that relative price differences between fertilizers have the greatest influence on the selection of a cost-efficient fertilization strategy. This impression could be reinforced by a scenario analysis using artificially manipulated fertilizer prices, wherein the results show that even small changes in the relative price differences of fertilizers (±10%) lead to significant changes in fertilizer selection. Therefore, the volatility of fertilizer prices (Lahmiri, 2017) will permanently affect the selection of cost-efficient fertilizer strategies. Therefore, farmers and consultants cannot safely rely on previous statements about the preferability of individual fertilizers, and continuous adjustment of fertilizer strategy is mandatory for maximum cost-efficiency.

In addition to the characteristics of cost-efficient fertilizer strategies, the influence of farm conditions on the selection of the fertilizer strategy was also investigated. The size of the farm or the on-farm infrastructure had no significant influence on the selection of the fertilizer strategy. Both parameters mainly influence the share of application costs in the total costs. In the “small farm” scenario, this share is significantly increased to 22%, but nevertheless the expected effect on the choice of fertilizer strategy remains absent. This observation supports the secondary importance of application costs. As discussed in Chapter 3, a complete omission of the application costs or transport costs is nevertheless critical. A further increase in the share of application costs or strongly varying farm-field and field-field distances could affect the design of the cost-efficient fertilizer strategy.

Chapter 6 also shows a clear influence of the fertilizer strategy by the parameters “homogeneous soil fertility” and “slurry accumulation.” Changes in these two parameters directly influence the necessary mineral nutrient supply, which can even become obsolete with a correspondingly high proportion of organic fertilizers. As a result, the mineral nutrient input and the selection of fertilizers changes in the investigated scenarios. Of particular interest is the influence of overall homogeneous soil fertility. In this case, the complexity of the optimization problem is apparently diminished, since the “field-specific” requirements are reduced. This enables fewer fertilization measures (28 instead of 32 measures within the 3-year crop rotation) to be used in line with nutrient requirements and at lower overall costs. In the long term, farmers should strive to homogenize soil fertility to generate additional optimization potential.

Chapter 6 shows the complexity of the optimization problem itself and the dominant influence of volatile fertilizer prices on the cost-efficient fertilizer strategy. Farmers and consultants are strongly recommended to regularly use the appropriate DSS for a cost-efficient fertilization strategy.

Extended Stakeholder Implications: The implications addressed have focused on farmers and consultants. In addition, other stakeholders have a legitimate interest in the results of this thesis, including the fertilizer industry, trade companies, policymakers, and society.

Optimal routing of agricultural inputs via SDVRP also promises benefits from a social and political perspective, such as minimizing transport costs strengthens the competitiveness of agriculture and reduces road traffic and its consumption of fossil fuels. To accelerate application of the SDVRP in this context, policymakers should provide access to necessary infrastructural farm data. The tool of Machl et al. (2016) could be used to calculate farm-to-field and field-to-field distances and existing information systems (e.g., IBALIS for Bavaria) are helpful to exchange necessary data. With the widespread availability of these infrastructure data, the incentive for software developers to build measure-based route planning systems for agriculture increases, as it is already common in the logistics sector (Guo et al., 2021; Bortfeldt and Yi, 2020).

IoFarm enables farmers to evaluate the relative competitiveness of different fertilizers on a farm-specific basis. Therefore, farmers react more dynamically to price changes and demand different products at varying times, which impacts achievable trading margins and predictability of fertilizer production and sales for traders and the fertilizer industry. IoFarm users will selectively demand fertilizers that are beneficial to their farms; thus, the advisory function of the trading companies will be pushed back. It can be assumed that traders and industry will increasingly respond to individual farm requirements. However, there is also a great opportunity for agricultural traders who offer blended fertilizers on demand, as the output of IoFarm could be used to produce cost-efficient farm-specific blended fertilizers and generate real added value for both sides. The importance of this approach is also shown in previous studies (Benhamou et al., 2020; Cole et al., 2015; Aldeseit, 2014; Mínguez et al., 1988; Babcock, 1984). The fertilizer industry can also benefit and use IoFarm as part of product development. For example, compound fertilizers can be developed that are specific to crop rotations and farm types. In addition, IoFarm also offers an interesting option for pricing new products from the perspective of fertilizer manufacturers, wherein simulation runs can show compositions of fertilizer and the most acceptable maximum price compared to alternative fertilizers to succeed on the market.

The results from Chapter 6 showed that cost-efficient fertilization strategies have less nutrient surpluses and thus have a positive impact on sustainability. Together with the improvement of the competitiveness of agriculture, these are extremely relevant outcomes from a societal and political point of view, which can be expected from a widespread use of IoFarm.

Methodological contributions of this thesis: Currently, there are several studies on the optimization of the in-field logistics of agricultural operations that reduce nonworking distance (Vahdanjoo and Sorensen, 2021; Vahdanjoo et al., 2020; He et al., 2019; Utamima et al., 2019). However, route planning also plays a role in identifying shortest connections between farm and multiple field pieces. As far as known, this is the first time SDVRP was used in connection with the application of agricultural inputs. Using the SDVRP is particularly appropriate when transportation capacity is limited and under conditions where field-to-field trips occur. In these cases, farmers face the question of optimal route splitting ("spit delivery") and routing to minimize transportation costs. Ideal cases for the SDVRP are the application of mineral fertilizers and pesticides. However, this approach is not relevant for operations in which field-to-field travel is the exception, e.g., for the application of organic fertilizers or the transport of harvested crops. In these cases, the transport vehicle shuttles between two points (usually farm and field), which is why route optimization is not generally needed. In cases where field-to-field trips occur but no transportation capacity is required, e.g., tillage, reference is made to the more efficient solution methods of the traveling salesman problem (e.g. Zhang et al., 2021). The implementation of SDVRP in an agricultural context is currently limited mainly by the high demands on computational capacity and as already described, the availability of the necessary farm-specific distance data. Should it be possible to overcome these barriers in the future, e.g., through the advancing digitalization in agriculture, the SDVRP can additionally be used for the optimal allocation of crops on the farmland. This allows transport distances to be minimized and work processes to be optimized in advance, which is very relevant, especially from a landscape planning perspective (Harasimowicz et al., 2017).

The development of a farm- and measure-specific regression function for the estimation of farm-field and field-field trips, presented in Chapter 3, is also a new methodological contribution. This method allows estimating transportation costs for farm-field and field-field trips. In contrast to the SDVRP method, the result of the estimation function does not contain any information on the real routing. Instead, based on a small amount of information characterizing the farm and the measure carried out, an adapted estimation of the transport costs is made. This method is well suited to consider transport costs for the application of mineral fertilizers or pesticides within the framework of a mathematical optimization model.

The main advantage is the low computational demand, which is very important in combination with a possibly complex main problem.

Chapter 4 contains the conceptual framework of this thesis. It applies the usual expansion path theory to the optimization problem of a cost-efficient fertilization strategy at the farm level. The problem-specific features in the course of the isoquant and isocost lines are discussed in detail. It was shown that isoquants, due to technically efficient combinations of fertilizers , can be kinked (Mußhoff and Hirschauer, 2013, p. 167; Nicholson and Snyder, 2008, p. 113) and do not necessarily have to run in parallel (Nicholson and Snyder, 2008, p. 329). Isocost lines were also identified as nonlinear functions due to the influence of nonlinear application costs. Overall, this results in the following findings for cost-efficient fertilizer strategies: the expansion path can be erratic between different production levels, which means that the least-cost combination of fertilizers can be affected as production levels change. An additional methodological feature appears in the combination of the linear-limitational production function (Liebig, 1843) with erratic or nonlinear total costs of fertilization. In this combination, it is possible that the profit-maximizing fertilizer input is neither at the maximum nor at the minimum of the linear-limitational production function, but is defined by a certain input level between these two points. Usually the literature describes this observation only for production functions with decreasing slope. In test runs with IoFarm, however, it could be observed several times that the yield maximum given by the linear-limitational production function was deliberately not completely exhausted for economic reasons.

The mathematical optimization problem IoFarm developed in Chapter 4 combines a practical application of production theory with state-of-the-art economic modeling techniques. The underlying optimization problem is a real world MINLP problem. It thus extends the MINLP research area of operations research by an application with an agricultural context. MINLP's are currently at the threshold between theoretical research (e.g. Muts et al., 2020; Mauri et al., 2020) and application in practice (e.g. Ye et al., 2021; Kazi et al., 2021; Gao et al., 2021). Therefore, real applications tested in experiments are an important contribution to the existing literature. First promising approaches to solve this category of models were already found in the literature with the "outer-approximation" method (Duran and Grossmann, 1986). Despite continued development and improved solution methods, MINLP places high demands on computational power. However, MINLP allows for an almost uncompromized representation of real world requirements by combining nonlinear and discrete components (Bonami et al., 2012, p. 31). Only with this model category it was possible to define feasible fertilization strategies. An important aspect in this context is, for example, the minimum quantity in the application rate of fertilizers. To meet the high demand on computing power, a sequential process for decision making was additionally implemented in the model (Amann, 2019, pp. 19–20). Accordingly, IoFarm is an early agricultural application of a sequential MINLP problem. Didactically, IoFarm can be used as a bridge to connect the optimization potential of the expansion path theory, which is often considered abstract, with an everyday agricultural problem.

Limitations and need for research: Despite the greatest efforts and care, some limitations must also be addressed in this thesis. The method for farm- and measure-specific estimation of application costs developed in Chapter 3 is not based on infrastructure data of existing farms. For data protection reasons, access to this georeferenced data was denied. Instead, based on a real farm, infrastructure data were simulated (informed guess). Further research with data of representative and real existing farms is necessary to make the estimation more precise and more validated. For this project the data acquisition as well as the calculation of optimal routes by means of SDVRP could be a barrier, especially since there is currently no guarantee that every routing problem can be solved optimally.

The implementation of biological, chemical, and agronomic processes and requirements in IoFarm is simplified in many cases. For instance, the internal modeling of the nitrogen stock in the soil is strongly affected by this. Recurrent soil tests are used to correct potential misestimates through updates. This approach allows to cope with a minimum of standard data, which is highly relevant for applicability in practice (Rose et al., 2016). In principle, however, there are quite interesting options to combine IoFarm with other models or technologies. For example, the use of sensor technology or remote sensing would be a promising alternative for determining timing-related nutrient requirements (Pedersen et al., 2021; Lu et al., 2020). Specialized models to estimate soil N dynamics, e.g., MONICA (Nendel, 2014), SNAP (Paul et al., 2002), or DAISY (Abrahamsen and Hansen, 2000) could also increase the accuracy of IoFarm. Thus, further research is needed to potentially improve IoFarm in this regard. However, it is important not to lose track on the trade-off between practical benefits and the requirements for data acquisition and computational power.

In terms of computational power requirements, the MINLP problem IoFarm is highly demanding. Therefore, a sequential method is required to solve the problem (Amann, 2019, pp. 19–20), wherein the solution space is constrained by temporary upper and lower limits for fertilizers to find a solution to the MINLP. The fertilizers concerned are highly unlikely to be considered for an optimal solution. Nevertheless, this cannot be said with certainty and optimization potential may be lost. With the further development of efficient solution procedures for MINLP problems, future versions of IoFarm may offer the possibility to replace the sequential solution procedure.

A fertilizer experiment was used to evaluate the economic performance of IoFarm. Due to the limited number of participants (31), the evidence of the results is restricted. Incentives were purposefully not used to recruit participants. The large amount of time required to participate in the experiment, most likely results in only intrinsically motivated participants who contribute credible fertilization strategies. Several studies (Stanley et al., 2020; Barge and Gehlbach, 2012; Göritz, 2006) show the feared negative relationship between incentives and data quality, supporting the chosen approach. The selection experiment is available online. A further expansion of the number of participants is thus possible and will hopefully lead to a better validation of the results in the future.

The evaluation of the agronomic performance of IoFarm was carried out in field trials. Due to the location of the trial sites, only a regional validity of the results for Bavaria (Southern Germany) can be given. Another limitation concerns the interpretation of the trial results of the organically fertilized variants (oIO, oBL). With a relative standard error of 4.5% to 6.9%, variation within the variants is above the threshold of 4% (Thomas, 2006, p. 61), and thus only of limited validity. Therefore, it is important to investigate the agronomic performance of IoFarm in the future under many different environmental conditions, with and without the use of organic fertilizers.

In Chapter 6, the dominant influence of volatile fertilizer prices on the design of the fertilizer strategy was established. As a result, the statements made regarding the preferability of individual fertilizers cannot be generalized. Since this study is also based on the fertilizer experiment, the basis for analyzing fertilizer strategies is currently limited to 31 participants. To investigate the influence of different farm conditions on cost-efficient fertilization strategies, a real existing initial farm was used and selectively modified according to the *ceteris paribus* principle. This approach allows a targeted evaluation of these selective changes. A repeated analysis with a representative set of farms would provide additional insights. The challenge of such an empirical study, however, is likely to be in the collection of infrastructure data from the farms.

Final Conclusions: In the course of this thesis, it becomes visible what degree of complexity the identification of the cost-efficient fertilizer strategy entails for the individual farm. Cost-efficient fertilizer strategies are primarily influenced by relative price differences between fertilizers. Volatile fertilizer prices and other volatile influencing factors, such as growing conditions, result in the need for frequent readjustment of fertilizer strategy. To optimize the solution of this time-consuming management task, farmers and consultants should rely on the help of appropriate optimization tools, such as IoFarm. With IoFarm, average cost benefits of €66 $ha^{-1}a^{-1}$ could be generated with unchanged market performance. In addition, IoFarm largely automates the fertilizer strategy planning process, reducing the need for valuable management time. From an individual farm perspective, the benefits are thus assured. However, it has also been shown that cost-efficient fertilization strategies are additionally more resource efficient. IoFarm therefore also contributes to sustainability, supporting important social and political goals. By simply adapting the overall target function, IoFarm will also be used in the future to develop CO_2-efficient fertilization strategies. From an economic perspective, IoFarm contributes to increasing the competitiveness of agriculture and improving sustainability. In addition, CO_2-efficient fertilizer strategies offer the opportunity to make a global contribution to climate protection. This justifies the subsidization of IoFarm and policymakers should consider this option.

The short-term goal is now to prepare IoFarm for a wide range of crops and to provide farmers with direct access to IoFarm. Due to the increasing digitalization of agriculture, numerous farm-related data are now available in digital form. Many applications, e.g. field mapping, from external providers show that the exchange of this data already works in practice. It is therefore realistic to use this approach for IoFarm in the future to provide farmers and consultants with an online platform for data exchange. This platform is to be connected to a powerful external computing center to solve large optimization problems efficiently.

Acknowledgments

I would like to thank all those who encouraged me to work on this dissertation and to all those who supported me in carrying it out or keeping it up. Many thanks to my supervisor Prof. Dr. Johannes Sauer, who had confidence in me and who always motivated but also challenged me. Many thanks to my mentor Dr. Hubert Pahl, who gave me advice and had an open ear for me. I would also like to thank my esteemed colleagues at the chair, which included me in their team as an external doctoral student. I cannot put into words the gratitude I would like to express to my wife Iryna, my two children Juliane and Elly my parents Friedrich and Irmgard, my brother Bernd, and grandma Lies. It was only through the support of my family that I found the time and strength to devote to this dissertation in addition to my job and farming business. Further, I would like to thank all my friends and explicitly to Christian Schuh for valuable feedback and Alisher Qurbanov, who among other things encouraged me to do this work and actively supported me during the field trials. I would also like to thank to my colleagues, in Roggenstein and in Triesdorf, without their help, the field trial would not have been possible.

9 References

365FarmNet GmbH. 365FarmNet. https://www.365farmnet.com/ (accessed 14 October 2019).

Abrahamsen, P., Hansen, S., 2000. Daisy: an open soil-crop-atmosphere system model. Environ. Modell. Softw. 15, 313–330. https://doi.org/10.1016/S1364-8152(00)00003-7.

Adnan, N., Jian, S.D., 2006. Product Cost Estimation: Technique Classification and Methodology Review. Journal of Manufacturing Science and Engineering, 563–575.

Agrarmeteorologie Bayern, 2019. Weather data in Bavaria. Institut für Pflanzenschutz. https://www.wetter-by.de/Internet/global/inetcntr.nsf/dlr_web_full.xsp?src=98OE9BL691&p1=6BH7UJ4826&p3=S313638Z32&p4=YNDMXE6MAN# (accessed 17 November 2019).

Aldeseit, B., 2014. Linear Programming-Based Optimization of Synthetic Fertilizers Formulation. JAS 6. https://doi.org/10.5539/jas.v6n12p194.

Amann, E., 2019. Entscheidungstheorie. Springer Fachmedien Wiesbaden, Wiesbaden.

Andrei, N., 2013. Nonlinear Optimization Applications Using the GAMS Technology. Springer US, Boston, MA.

Anger, M., Hüging, H., Huth, C., Kühbauch, W., 2002. Nitrat-Austräge auf intensiv und extensiv beweidetem Grünland, erfasst mittels Saugkerzen- und Nmin-Beprobung I Einfluss der Beweidungsintensität. J. Plant Nutr. Soil Sci. 165, 640–647. https://doi.org/10.1002/1522-2624(200210)165:5<640:AID-JPLN640>3.0.CO;2-F.

Araya, A., Prasad, P., Gowda, P.H., Afewerk, A., Abadi, B., Foster, A.J., 2019. Modeling irrigation and nitrogen management of wheat in northern Ethiopia. Agr. Water Manage. 216, 264–272. https://doi.org/10.1016/j.agwat.2019.01.014.

Archetti, C., Bianchessi, N., Speranza, M.G., 2011a. A column generation approach for the split delivery vehicle routing problem. Networks 58, 241–254. https://doi.org/10.1002/net.20467.

Archetti, C., Bianchessi, N., Speranza, M.G., 2014. Branch-and-cut algorithms for the split delivery vehicle routing problem. European Journal of Operational Research 238, 685–698. https://doi.org/10.1016/j.ejor.2014.04.026.

Archetti, C., Feillet, D., Gendreau, M., Grazia Speranza, M., 2011b. Complexity of the VRP and SDVRP. Transportation Research Part C: Emerging Technologies 19, 741–750. https://doi.org/10.1016/j.trc.2009.12.006.

Archetti, C., Speranza, M.G., Hertz, A., 2006. A Tabu Search Algorithm for the Split Delivery Vehicle Routing Problem. Transportation Science 40, 64–73. https://doi.org/10.1287/trsc.1040.0103.

ARKI Consulting and Development A/S, 2016a. CONOPT. ARKI Consulting and Development A/S, Bagsvaerd, DK, Bagsvaerd, DK.

ARKI Consulting and Development A/S, 2016b. SBB: Simple Branch and Bound. ARKI Consulting and Development A/S, Bagsvaerd, Denmark, Bagsvaerd, Denmark.

Babcock, B., 1984. Identifying least-cost sources of required fertilizer nutrients. American Journal of Agricultural Economics 66, 385–391. https://doi.org/10.2307/1240806.

Bäckman, S.T., Vermeulen, S., Taavitsainen, V.-M., 1997. Long-term fertilizer field trials: Comparison of three mathematical response models. Agricultural and Food Science in Finland 6, 151–160. https://doi.org/10.23986/afsci.72778.

Baey-Ernsten, H. de, 2011. Methodik zur KTBL online Anwendung Verfahrensrechner Pflanze, 3 pp. http://daten.ktbl.de/downloads/vrpflanze/KalkulationVerfahrensrechner.pdf (accessed 7 January 2017).

Barge, S., Gehlbach, H., 2012. Using the Theory of Satisficing to Evaluate the Quality of Survey Data. Res High Educ 53, 182–200. https://doi.org/10.1007/s11162-011-9251-2.

Basnet, C., Foulds, L., Igbaria, M., 1996. FleetManager: a microcomputer-based decision support system for vehicle routing. Desision Support Systems, 195–207.

Baule, B., 1954. Eine physikalische Analogie zum Pflanzenertragsgesetz. Zeitschrift für Acker- und Pflanzenbau.

Belotti, P., Bonami, P., Wächter, A., 2006. Couenne, an exact solver for nonconvex MINLPs: Couenne. IBM and Carnegie Mellon University.

Benhamou, L., Giard, V., Khouloud, M., Fenies, P., Fontane, F., 2020. Reverse Blending: An economically efficient approach to the challenge of fertilizer mass customization. International Journal of Production Economics 226, 107603. https://doi.org/10.1016/j.ijpe.2019.107603.

Bielecke, H., 1934. Die Geschichte der künstlichen Düngung und der Kunstdüngerversorgung.

Blanco, A.M., 2020. Optimization in Decision Support Systems, in: Chantre, G.R., González-Andújar, J.L. (Eds.), Decision Support Systems for Weed Management. Springer International Publishing, Cham, pp. 39–57.

BMEL, 2006. Verordnung über die Anwendung von Düngemitteln, Bodenhilfsstoffen, Kultursubstraten und Pflanzenhilfsmitteln nach den Grundsätzen der guten fachlichen Praxis beim Düngen: DüV, 26 pp.

Bonami, P., Kilinç, M., Linderoth, J., 2012. Algorithms and Software for Convex Mixed Integer Nonlinear Programs, in: Lee, J., Leyffer, S. (Eds.), Mixed Integer Nonlinear Programming, vol. 154. Springer New York, New York, NY, pp. 1–39.

Borealis L.A.T GmbH. Product information. https://www.borealis-lat.com/ (accessed 26 June 2021).

Bortfeldt, A., Yi, J., 2020. The Split Delivery Vehicle Routing Problem with three-dimensional loading constraints. European Journal of Operational Research 282, 545–558. https://doi.org/10.1016/j.ejor.2019.09.024.

Brandes, W., Woermann, E., 1982. Theorie und Planung des landwirtschaftlichen Betriebes, 2nd ed. Parey, Hamburg, Berlin, 234 S.

Büchert, M., Wachdorf, M., Taube, F., 2001. Nitratauswaschung unter Silomais in Abhängigkeit von der Bewirtschaftungsformund der N-Düngungsintensität – Ergebnisse aus dem N-Projekt Karkendamm, in: Mitteilungen der Arbeitsgemeinschaft für Grünland und Futterbau, pp. 75–77.

Bueno-Delgado, M.V., Molina-Martínez, J.M., Correoso-Campillo, R., Pavón-Mariño, P., 2016. Ecofert: An Android application for the optimization of fertilizer cost in fertigation. Comput. Electron. Agr. 121, 32–42. https://doi.org/10.1016/j.compag.2015.11.006.

Bundestag, 2009. Düngegesetz: DüngeG.

Bundestag, 2017. Verordnung über die Anwendung von Düngemitteln, Bodenhilfsstoffen, Kultursubstraten und Pflanzenhilfsmitteln nach den Grundsätzen der guten fachlichen Praxis beim Düngen: DüV.

Bussiek, M.R., Pruessner, A., 2003. Mixed-Integer Nonlinear Programming. GAMS Development Corporation. http://www.gamsworld.org/minlp/siagopt.pdf (accessed 3 July 2021).

Byrd, R.H., Nocedal, J., Waltz, R.A., 2006. Knitro: An Integrated Package for Nonlinear Optimization, in: Pardalos, P., Di Pillo, G., Roma, M. (Eds.), Large-Scale Nonlinear Optimization, vol. 83. Springer US, Boston, MA, pp. 35–59.

Chen, L., Liu, X., Hua, Z., Xue, H., Mei, S., Wang, P., Wang, S., 2021. Comparison of Nitrogen Loss Weight in Ammonia Volatilization, Runoff, and Leaching Between Common and Slow-Release Fertilizer in Paddy Field. Water Air Soil Pollut 232. https://doi.org/10.1007/s11270-021-05083-6.

Chen, L., Xing, L., Han, L., 2013. Review of the application of near-infrared spectroscopy technology to determine the chemical composition of animal manure. Journal of Environmental Quality 42, 1015–1028. https://doi.org/10.2134/jeq2013.01.0014.

Chen, Q., Qi, L., Bi, Q., Dai, P., Sun, D., Sun, C., Liu, W., Lu, L., Ni, W., Lin, X., 2015. Comparative effects of 3,4-dimethylpyrazole phosphate (DMPP) and dicyandiamide (DCD) on ammonia-oxidizing bacteria and archaea in a vegetable soil. Applied microbiology and biotechnology 99, 477–487. https://doi.org/10.1007/s00253-014-6026-7.

Chuan, L., He, P., Pampolino, M.F., Johnston, A.M., Jin, J., Xu, X., Zhao, S., Qiu, S., Zhou, W., 2013. Establishing a scientific basis for fertilizer recommendations for wheat in China: Yield response and agronomic efficiency. Field Crop Res. 140, 1–8. https://doi.org/10.1016/j.fcr.2012.09.020.

Colaço, A.F., Bramley, R., 2018. Do crop sensors promote improved nitrogen management in grain crops? Field Crops Research 218, 126–140. https://doi.org/10.1016/j.fcr.2018.01.007.

Cole, B.M., Bradshaw, S., Potgieter, H., 2015. An optimisation methodology for a supply chain operating under any pertinent conditions of uncertainty - an application with two forms of operational uncertainty, multi-objectivity and fuzziness. IJOR 23, 69180, 200. https://doi.org/10.1504/IJOR.2015.069180.

Cummings, C., Miao, Y., Paiao, G.D., Kang, S., Fernández, F.G., 2021. Corn Nitrogen Status Diagnosis with an Innovative Multi-Parameter Crop Circle Phenom Sensing System. Remote Sensing 13, 401. https://doi.org/10.3390/rs13030401.

Damania, R., Berg, C., Russ, J., Federico Barra, A., Nash, J., Ali, R., 2017. Agricultural Technology Choice and Transport. American Journal of Agricultural Economics 99, 265–284. https://doi.org/10.1093/ajae/aav073.

Dantzig, G.B., Ramser, J.H., 1959. THE TRUCK DISPATCHING PROBLEM. Management Science, 80–91.

Debertin, D.L., 1986. Agricultural production economics. Macmillan; Collier Macmillan, New York, London, 366 pp.

Debertin, D.L., 2012a. Agricultural production economics, 2nd ed. Univ. of Kentucky, Lexington, 413 pp.

Debertin, D.L., 2012b. Applied Microeconomics: Consumption, Production and Markets, 2nd ed. Lexington, KY.

Dimkpa, C.O., Fugice, J., Singh, U., Lewis, T.D., 2020. Development of fertilizers for enhanced nitrogen use efficiency - Trends and perspectives. The Science of the total environment 731, 139113. https://doi.org/10.1016/j.scitotenv.2020.139113.

Dirkse, S.P., Ferris, M.C., 1995. The path solver: a nommonotone stabilization scheme for mixed complementarity problems. Optimization Methods and Software 5, 123–156. https://doi.org/10.1080/10556789508805606.

DOMO Caproleuna GmbH. Product information. https://www.domogran.de/ (accessed 26 June 2021).

Domschke, W., Drexl, A., Klein, R., Scholl, A., 2015. Einführung in Operations Research. Springer Berlin Heidelberg, Berlin, Heidelberg.

Dror, M., Laporte, G., Trudeau, P., 1994. Vehicle routing with split deliveries. Discrete Applied Mathematics 50, 239–254. https://doi.org/10.1016/0166-218X(92)00172-I.

Dror, M., Trudeau, P., 1990. Split delivery routing. Naval Research Logistics 37, 383–402. https://doi.org/10.1002/nav.3800370304.

DüKa Düngekalkgesellschaft mbH. Product information. https://www.dueka.de/index.php?id=9 (accessed 26 June 2021).

Duran, M.A., Grossmann, I.E., 1986. An outer-approximation algorithm for a class of mixed-integer nonlinear programs. Math. Program. 36, 307–339. https://doi.org/10.1007/BF02592064.

Eldrandaly, K.A., Abdallah, A.F., 2012. A novel GIS-based decision-making framework for the school bus routing problem. Geo-spatial Information Science 15, 51–59. https://doi.org/10.1080/10095020.2012.708151.

EuroChem Agro GmbH. Product information. https://www.eurochemdach.com/produkte/ (accessed 26 June 2021).

European Parliament, 2016. Directive (EU) 2016/2284 of the European Parliament and of the Council on the reduction of national emissions of certain atmospheric pollutants: NEC Directive.

Evangelou, E., Stamatiadis, S., Schepers, J.S., Glampedakis, A., Glampedakis, M., Dercas, N., Tsadilas, C., Nikoli, T., 2020. Evaluation of sensor-based field-scale spatial application of granular N to maize. Precision Agric 21, 1008–1026. https://doi.org/10.1007/s11119-019-09705-2.

Expósito, A., Velasco, F., 2020. Exploring environmental efficiency of the European agricultural sector in the use of mineral fertilizers. Journal of Cleaner Production 253, 119971. https://doi.org/10.1016/j.jclepro.2020.119971.

FarmFacts GmbH. NextFarming: NEXT Düngeplanung Office. FarmFacts GmbH.

Fechner, W., 2014. Einfluss der Hauptbearbeitungsrichtung auf die Arbeitszeit im Feldbau am Beispiel eines mitteldeutschen Großbetriebs, in: Bornimer Agrartechnische Berichte. 19. Arbeitswissenschaftliches Kolloquium des VDI-MEG Arbeitskreises Arbeitswissenschaften im Landbau, Potsdam-Bornim Dresden. 11. - 12.03.2014, pp. 22–34.

Finck, A., 1991. Düngung: 41 Tabellen ; [ertragssteigernd, qualitätsverbessernd umweltgerecht]. Ulmer, Stuttgart, 174 S.

Finck, A., 2007. Pflanzenernährung und Düngung in Stichworten. Borntraeger, Berlin [u.a.].

Fitzgerald, G., Rodriguez, D., O'Leary, G., 2010. Measuring and predicting canopy nitrogen nutrition in wheat using a spectral index—The canopy chlorophyll content index (CCCI). Field Crops Research 116, 318–324. https://doi.org/10.1016/j.fcr.2010.01.010.

Floudas, C.A., 2011. Deterministic global optimization: Theory, methods andaApplications, 1st ed. Kluwer, Dordrecht, 739 pp.

Frank, M.D., Beattie, B.R., Embleton, M.E., 1990. Crop response Modells Liebig, Mitscherlich, Quadratic. American Journal of Agricultural Economics, 597–603.

Frisk, M., Jonsson, A., Sellman, S., Flisberg, P., Rönnqvist, M., Wennergren, U., 2018. Route optimization as an instrument to improve animal welfare and economics in pre-slaughter logistics. PloS one 13, e0193223. https://doi.org/10.1371/journal.pone.0193223.

Fulton, J.P., Colley, R., Shearer, S.A., Gauci, A., Hawkins, E.M., 2021. 72. High definition fertilizer as-applied maps for pneumatic applicators, in: Precision agriculture '21. 13th European Conference on Precision Agriculture, Budapest, Hungary. 19-22 July, 2021. Wageningen Academic Publishers, The Netherlands, pp. 601–607.

Gamrath, G., Fischer, T., Gally, T., Gleixner, A., Hendel, G., Koch, T., Maher, S.J., Miltenberger, M., Müller, B., Pfetsch, M., Puchert, C., Rehfeldt, D., Schenker, S., Schwarz, R., Serrano, F., Shinano, Y., Vigerske, S., Weninger, D., Winkler, M., Witt, J.T., Witzig, J., 2016. The SCIP Optimization Suite 3.2. https://opus4.kobv.de/opus4-zib/frontdoor/index/index/docId/5767.

GAMS Development Corporation, 2016. General Algebraic Modeling System (GAMS) Release 24.8.1. Washington D.C. , USA, Washington D.C. , USA.

Gao, X., Zhao, Y., Wang, Y., Zuo, X., Chen, T., 2021. A Lagrange Relaxation Based Decomposition Algorithm for Large-Scale Offshore Oil Production Planning Optimization. Processes 9, 1257. https://doi.org/10.3390/pr9071257.

Gill, P.E., Murray, W., Saunders, M.A., 2013. SNOPT. Stanford University, Stanford, California, Stanford, California.

Gil-Ortiz, R., Naranjo, M.Á., Ruiz-Navarro, A., Atares, S., García, C., Zotarelli, L., San Bautista, A., Vicente, O., 2020. Enhanced Agronomic Efficiency Using a New Controlled-Released, Polymeric-Coated Nitrogen Fertilizer in Rice. Plants (Basel, Switzerland) 9. https://doi.org/10.3390/plants9091183.

González-Andújar, J.L., 2020. Introduction to Decision Support Systems, in: Chantre, G.R., González-Andújar, J.L. (Eds.), Decision Support Systems for Weed Management. Springer International Publishing, Cham, pp. 25–38.

Göritz, A.S., 2006. Incentives in Web Studies: Methodological Issues and a Review. International Journal of Internet Science 1, 58–70.

Grassini, P., Thorburn, J., Burr, C., Cassman, K.G., 2011. High-yield irrigated maize in the Western U.S. Corn Belt: I. On-farm yield, yield potential, and impact of agronomic practices. Field Crops Research 120, 142–150. https://doi.org/10.1016/j.fcr.2010.09.012.

Grimm, S.S., Paris, Q., Williams, W.A., 1987. A Von Liebig Model for Water and Nitrogen Crop Response. Western Journal of Agricultural Economics 12, 182–192.

Guerrero, A., Neve, S. de, Mouazen, A.M., 2021. Current sensor technologies for in situ and on-line measurement of soil nitrogen for variable rate fertilization: A review, in: Elsevier.

Guo, F., Huang, Z., Huang, W., 2021. Heuristic approaches for a vehicle routing problem with an incompatible loading constraint and splitting deliveries by order. Computers & Operations Research 134, 105379. https://doi.org/10.1016/j.cor.2021.105379.

Haber, F., 1908. Verfahren zur synthetischen Darstellung von Ammoniak aus den Elementen. https://depatisnet.dpma.de/DepatisNet/depatisnet?action=bibdat&docid=DE235421A.

Hansen, S., Jensen, H.E., Nielsen, N.E., Svendsen, H., 1991. Simulation of nitrogen dynamics and biomass production in winter wheat using the Danish simulation model DAISY, in: Groot, J.J.R., Willigen, P., Verberne, E.L.J. (Eds.), Nitrogen Turnover in the Soil-Crop System: Modelling of Biological Transformations, Transport of Nitrogen and Nitrogen Use Efficiency. Proceedings of a Workshop held at the Institute for Soil Fertility Research, Haren, the Netherlands, 5-6 June 1990. Springer Netherlands; Imprint; Springer, Dordrecht, pp. 245–259.

Harasimowicz, S., Janus, J., Bacior, S., Gniadek, J., 2017. Shape and size of parcels and transport costs as a mixed integer programming problem in optimization of land consolidation. Computers and Electronics in Agriculture 140, 113–122. https://doi.org/10.1016/j.compag.2017.05.035.

He, P., Li, J., Qin, H., He, Y., Cao, G., 2019. Using hybrid algorithm to reduce non-working distance in intra- and inter-field logistics simultaneously for heterogeneous harvesters. Computers and Electronics in Agriculture 167, 105065. https://doi.org/10.1016/j.compag.2019.105065.

Herbst, F., Gans, W., Merbach, W., 2006. Einfluss eines Urease-Inhibitors bei Harnstoff-Düngung auf den Stickstoff-Umsatz im Boden, die Ammoniak-Verflüchtigung und die Verwertung des Stickstoffs durch Hafer. Pflanzenbauwissenschaften, 37–43.

Hernández-Pérez, H., Salazar-González, J.-J., 2019. Optimal Solutions for the Vehicle Routing Problem with Split Demands, in: Paternina-Arboleda, C., Voß, S. (Eds.), Computational Logistics, vol. 11756. Springer International Publishing, Cham, pp. 189–203.

Heumann, S., Fier, A., Haßdenteufel, M., Höper, H., Schäfer, W., Eiler, T., Böttcher, J., 2013. Minimizing nitrate leaching while maintaining crop yields: Insights by simulating net N mineralization. Nutr Cycl Agroecosyst 95, 395–408. https://doi.org/10.1007/s10705-013-9572-y.

Hlisnikovský, L., Vach, M., Abrhám, Z., Mensik, L., Kunzová, E., 2020. The effect of mineral fertilisers and farmyard manure on grain and straw yield, quality and economical parameters of winter wheat. Plant Soil Environ. 66, 249–256. https://doi.org/10.17221/60/2020-PSE.

Holzworth, D.P., Huth, N.I., deVoil, P.G., Zurcher, E.J., Herrmann, N.I., McLean, G., Chenu, K., van Oosterom, E.J., Snow, V., Murphy, C., Moore, A.D., Brown, H., Whish, J.P., Verrall, S., Fainges, J., Bell, L.W., Peake, A.S., Poulton, P.L., Hochman, Z., Thorburn, P.J., Gaydon, D.S., Dalgliesh, N.P., Rodriguez, D., Cox, H., Chapman, S., Doherty, A., Teixeira, E., Sharp, J., Cichota, R., Vogeler, I., Li, F.Y., Wang, E., Hammer, G.L., Robertson, M.J., Dimes, J.P., Whitbread, A.M., Hunt, J., van Rees, H., McClelland, T., Carberry, P.S., Hargreaves, J.N., MacLeod, N., McDonald, C., Harsdorf, J., Wedgwood, S., Keating, B.A., 2014. APSIM – Evolution towards a new generation of agricultural systems simulation. Environmental Modelling & Software 62, 327–350. https://doi.org/10.1016/j.envsoft.2014.07.009.

Hoogenboom, G., Porter, C.H., Boote, K.J., Shelia, V., Wilkens, P.W., Singh, U., White, J.W., Asseng, S., Lizaso, J.I., Moreno, L.P., Pavan, W., Ogoshi, R., Hunt, L.A., Tsuji, G.Y., Jones, J.W., 2019. The DSSAT crop modeling ecosystem, in: Boote, K. (Ed.), Advances in crop modelling for a sustainable agriculture. Burleigh Dodds Science Publishing, pp. 173–216.

Hujer, T., 2011. Design and Development of a Compound DSS for Laboratory Research, in: Jao, C. (Ed.), Efficient Decision Support Systems - Practice and Challenges From Current to Future. InTech.

Hutchings, N., Webb, J., Amon, B., 2019. MEP/EEA air pollutant emission inventory guidebook: Technical guidance to prepare national emission inventories. https://www.eea.europa.eu/publications/emep-eea-guidebook-2019/part-b-sectoral-guidance-chapters/4-agriculture/3-d-crop-production-and/view (accessed 17 November 2019).

IBM Corporation, 2017. CPLEX 12. o.O., o.O.

IFA Productions Inc., 2012. AgPhD: Fertilizer Removal by Crop. IFA Productions Inc.

Innovation 24, 2016. Local Solver. Innovation 24, Paris, France, Paris, France.

Irigoyen, I., Muro, J., Azpilikueta, M., Aparicio-Tejo, P., Lamsfus, a.C., 2003. Ammonium oxidation kinetics in the presence of nitrification inhibitors DCD and DMPP at various temperatures. Aust. J. Soil Res. 41, 1177. https://doi.org/10.1071/SR02144.

Jame, Y.W., Cutforth, H.W., 1996. Crop growth models for decision support systems. Can. J. Plant Sci. 76, 9–19. https://doi.org/10.4141/cjps96-003.

Jansen, J., Wortmann, C., Stockton, M., Kaizzi, C., 2013. Maximizing Net Returns to Financially Constrained Fertilizer Use. Agronomy Journal 105, 573–578. https://doi.org/10.2134/agronj2012.0413.

Jensen, M.A.F., Bochtis, D., Sørensen, C.G., Blas, M.R., Lykkegaard, K.L., 2012. In-field and inter-field path planning for agricultural transport units. Computers & Industrial Engineering 63, 1054–1061. https://doi.org/10.1016/j.cie.2012.07.004.

Jin, M., Liu, K., Bowden, R.O., 2007. A two-stage algorithm with valid inequalities for the split delivery vehicle routing problem. International Journal of Production Economics 105, 228–242. https://doi.org/10.1016/j.ijpe.2006.04.014.

K+S Minerals and Agriculture GmbH. Product information. http://www.ks-minerals-and-agriculture.com/dede/fertiliser/products/ (accessed 26 June 2021).

Kantorovich, L.V., 1960. Mathematical methods of organizing and planning production (Translation). Management science : journal of the Institute for Operations Research and the Management Sciences 6, 366–422.

Kazi, S.R., Short, M., Biegler, L.T., 2021. A trust region framework for heat exchanger network synthesis with detailed individual heat exchanger designs. Computers & Chemical Engineering 153, 107447. https://doi.org/10.1016/j.compchemeng.2021.107447.

Kersebaum, K.C., 1989. Die Simulation der Stickstoff-Dynamik von Ackerböden. Hannover.

Kiełbasa, B., Pietrzak, S., Ulén, B., Drangert, J.-O., Tonderski, K., 2018. Sustainable agriculture: The study on farmers' perception and practices regarding nutrient management and limiting losses. Journal of Water and Land Development 36, 67–75. https://doi.org/10.2478/jwld-2018-0007.

Kleinhenz, B., Falke, K., Kakau, J., Rossberg, D., 2007. SIMBLIGHT1 ? a new model to predict first occurrence of potato late blight. EPPO Bulletin 37, 339–343. https://doi.org/10.1111/j.1365-2338.2007.01135.x.

Kling, A., 1985. Optimale Stickstoffdüngung ausgewählter Ackerfrüchte aus ökonomischer Sicht. Dissertation. München.

Kluge, E., Jörg, E., Rossberg, D., 2006. SIMSEPT: Eine Entscheidungshilfe zur Bekämpfung von Septoria tritici und Septoria nodorum. Archives Of Phytopathology And Plant Protection 39, 79–92. https://doi.org/10.1080/03235400500289577.

Koch, B., Khosla, R., Frasier, W.M., Westfall, D.G., Inman, D., 2004. Economic Feasibility of Variable-Rate Nitrogen Application Utilizing Site-Specific Management Zones. Agron.j. 96, 1572–1580. https://doi.org/10.2134/agronj2004.1572.

Kozlovský, O., Balík, J., Černý, J., Kulhánek, M., Kos, M., Prášilová, M., 2009. Influence of nitrogen fertilizer injection (CULTAN) on yield, yield components formation and quality of winter wheat grain. Plant Soil Environ. 55, 536–543. https://doi.org/10.17221/165/2009-PSE.

Kreuter, T., Ni, K., Gaßner, M., Schmidhalter, U., Döhler, J., Pacholski, A., 2014. Ammonia loss rates from urea and calcium ammonium nitrate applied to winter wheat on three different sites in germany, in: Cordovil, C.S.M.d.S. (Ed.), The Nitrogen challenge: Bulding a blueprint for nitrogen use efficiency and food security: Proceedings of the 18th Nitrogen workshop, Lisboa, Portugal, 30th June - 3rd July 2014. ISA Press, Lisboa, pp. 453–455.

Kronqvist, J., Bernal, D.E., Lundell, A., Grossmann, I.E., 2019. A review and comparison of solvers for convex MINLP. Optim Eng 20, 397–455. https://doi.org/10.1007/s11081-018-9411-8.

KTBL (Ed.), 2005. Faustzahlen für die Landwirtschaft, 13th ed. Kuratorium für Technik und Bauwesen in der Landwirtschaft e.V.(KTBL), Darmstadt, XXXIII, 1095 S.

KTBL (Ed.), 2016. Betriebsplanung Landwirtschaft 2016/17: Daten für die Betriebsplanung in der Landwirtschaft, 25th ed. Kuratorium für Technik und Bauwesen in der Landwirtschaft, Darmstadt, 768 Seiten.

KTBL, 2019. Feldarbeitsrechner. Kuratorium für Technik und Bauwesen in der Landwirtschaft e.V.(KTBL).

Lahmiri, S., 2017. Asymmetric and persistent responses in price volatility of fertilizers through stable and unstable periods. Physica A: Statistical Mechanics and its Applications 466, 405–414. https://doi.org/10.1016/j.physa.2016.09.036.

Latiffianti, E., Siswanto, N., Firmandani, R.A., 2018. Split delivery vehicle routing problem with time windows: A case study. IOP Conf. Ser.: Mater. Sci. Eng. 337, 12012. https://doi.org/10.1088/1757-899X/337/1/012012.

Lee, J., Leyffer, S., 2012. Mixed Integer Nonlinear Programming. Springer New York, New York, NY.

LI, G., CHENG, G., LU, W., LU, D., 2021. Differences of yield and nitrogen use efficiency under different applications of slow release fertilizer in spring maize. Journal of Integrative Agriculture 20, 554–564. https://doi.org/10.1016/S2095-3119(20)63315-9.

Liebig, J.¬., 1843. Die Chemie in ihrer Anwendung auf Agricultur und Physiologie. Vieweg, Braunschweig.

Lu, J., Miao, Y., Shi, W., Li, J., Hu, X., Chen, Z., Wang, X., Kusnierek, K., 2020. Developing a Proximal Active Canopy Sensor-based Precision Nitrogen Management Strategy for High-Yielding Rice. Remote Sensing 12, 1440. https://doi.org/10.3390/rs12091440.

Lu, N., Wang, W., Zhang, Q., Li, D., Yao, X., Tian, Y., Zhu, Y., Cao, W., Baret, F., Liu, S., Cheng, T., 2019. Estimation of Nitrogen Nutrition Status in Winter Wheat From Unmanned Aerial Vehicle Based Multi-Angular Multispectral Imagery. Frontiers in plant science 10, 1601. https://doi.org/10.3389/fpls.2019.01601.

Lütke Entrup, N. (Ed.), 2000. Lehrbuch des Pflanzenbaues. Verlag Th. Mann, Gelsenkirchen.

Machl, T., 2018. Digitale Karte mit Hof-Feld Entfernungen des Betriebes Tröster. E-Mail.

Machl, T., Ewald, W., Donaubauer, A., Kolbe, T.H., 2016. Entwicklung eines Werkzeugs zur landesweit flächendeckenden Analyse landwirtschaftlicher Transportbeziehungen in Bayern. Zeitschrift für Geodäsie, Geoinformation und Landmanagement 141, 197–205. https://doi.org/10.12902/zfv-0118-2016.

Mandrini, G., Bullock, D.S., Martin, N.F., 2021. Modeling the economic and environmental effects of corn nitrogen management strategies in Illinois. Field Crop Res. 261, 108000. https://doi.org/10.1016/j.fcr.2020.108000.

Mariappan, P., 2013. Operations research: An introduction. Dorling Kindersley (India), New Delhi, 1 online resource (1 volume).

Mauri, M. de, Gillis, J., Swevers, J., Pipeleers, G., 2020. A proximal-point outer approximation algorithm. Comput Optim Appl 77, 755–777. https://doi.org/10.1007/s10589-020-00216-9.

Meier, U., 2018. Growth stages of mono- and dicotyledonous plants: BBCH Monograph. Open Agrar Repositorium, 204 pp.

METRAC Handelsgesellschaft mbH. Product information. https://www.metrac.de/duengemittel/ (accessed 26 June 2021).

Mi, W., Gao, Q., Guo, X., Zhao, H., Xie, B., Wu, L., 2019. Evaluation of Agronomic and Economic Performance of Controlled and Slow-Release Nitrogen Fertilizers in Two Rice Cropping Systems. Agron.j. 111, 210–216. https://doi.org/10.2134/agronj2018.03.0175.

Mínguez, M.I., Romero, C., Domingo, J., 1988. Determining Optimum Fertilizer Combinations through Goal Programming with Penalty Functions: An Application to Sugar Beet Production in Spain. Journal of the Operational Research Society 39, 61–70.

Misener, R., Floudas, C.A., 2013. ANTIGONE: Algorithms for coNTinuous/Integer Global Optimization. Computer-Aided Systems Laboratory, Princeton USA, Princeton USA.

Mitscherlich, E.A., 1909. Das Gesetz des Minimums und das Gesetz des abnehmenden Bodenertrages. Landwirtschaftliche Jahrbücher, 537–552.

Mollenhorst, H., Haan, M.H. de, Oenema, J., Kamphuis, C., 2020. Field and crop specific manure application on a dairy farm based on historical data and machine learning. Computers and Electronics in Agriculture 175, 105599. https://doi.org/10.1016/j.compag.2020.105599.

Murtagh, B.A., Saunders, M.A. MINOS. Department of Management Science and Engineering, Stanford University.

Mußhoff, O., Hirschauer, N., 2013. Modernes Agrarmanagement: Betriebswirtschaftliche Analyse- und Planungsverfahren, 3rd ed. Vahlen, München, XX, 471 S.

Muts, P., Nowak, I., Hendrix, E.M.T., 2020. The decomposition-based outer approximation algorithm for convex mixed-integer nonlinear programming. J Glob Optim 77, 75–96. https://doi.org/10.1007/s10898-020-00888-x.

Nendel, C., 2014. MONICA: A Simulation Model for Nitrogen and Carbon Dynamics in Agro-Ecosystems, in: Mueller, L., Saparov, A., Lischeid, G. (Eds.), Novel Measurement and Assessment Tools for Monitoring and Management of Land and Water Resources in Agricultural Landscapes of Central Asia. Springer International Publishing, Cham, pp. 389–405.

Ni, K., Pacholski, A., Kage, H., 2014. Ammonia volatilization after application of urea to winter wheat over 3 years affected by novel urease and nitrification inhibitors. Agriculture, Ecosystems & Environment 197, 184–194. https://doi.org/10.1016/j.agee.2014.08.007.

Nicholson, W., Snyder, C.M., 2008. Microeconomic theory: Basic principles and extensions, 10th ed. Thomson Business and Economics, Belmont, CA, 740 pp.

Noellsch, A.J., Motavalli, P.P., Nelson, K.A., Kitchen, N.R., 2009. Corn Response to Conventional and Slow-Release Nitrogen Fertilizers across a Claypan Landscape. Agron.j. 101, 607–614. https://doi.org/10.2134/agronj2008.0067x.

Offenberger, K., Wendland, M. Düngebedarfsermittlung: NPK 1.13.0. https://www.stmelf.bayern.de/npk/app/demo?3 (accessed 17 October 2019).

Owens, L.B., Edwards, W.M., van Keuren, R.W., 1999. Nitrate Leaching from Grassed Lysimeters Treated with Ammonium Nitrate or Slow-Release Nitrogen Fertilizer. Journal of Environmental Quality 28, 1810–1816. https://doi.org/10.2134/jeq1999.00472425002800060017x.

Ozbaygin, G., Karasan, O., Yaman, H., 2018. New exact solution approaches for the split delivery vehicle routing problem. EURO J Comput Optim 6, 85–115. https://doi.org/10.1007/s13675-017-0089-z.

Pack, J.E., Hutchinson, C.M., Simonne, E.H., 2006. Evaluation of Controlled-Release Fertilizers for Northeast Florida Chip Potato Production. Journal of Plant Nutrition 29, 1301–1313. https://doi.org/10.1080/01904160600767633.

Pagán, F.J., Ferrández-Villena, M., Fernández-Pacheco, D.G., Rosillo, J.J., Molina-Martínez, J.M., 2015. Optifer: AN application to optimize fertiliser costs in fertigation. Agr. Water Manage. 151, 19–29. https://doi.org/10.1016/j.agwat.2014.11.007.

Paul, K.I., Polglase, P.J., O'Connell, A.M., Carlyle, J.C., Smethurst, P.J., Khanna, P.K., 2002. Soil nitrogen availability predictor (SNAP): a simple model for predicting mineralisation of nitrogen in forest soils. Soil Res. 40, 1011. https://doi.org/10.1071/SR01114.

Pedersen, M.F., Gyldengren, J.G., Pedersen, S.M., Diamantopoulos, E., Gislum, R., Styczen, M.E., 2021. A simulation of variable rate nitrogen application in winter wheat with soil and sensor information - An economic feasibility study. Agricultural Systems 192, 103147. https://doi.org/10.1016/j.agsy.2021.103147.

Power, D.J., 2002. Decision support systems: Concepts and resources for managers. Quorum Books, Westport, Conn., London, xvi, 251 sayfa.

Power, D.J., 2013. Decision support, analytics, and business intelligence, 2nd ed. Business Expert Press, [New York, N.Y.] (222 East 46th Street, New York, NY 10017), 162 pp.

Ragsdale, C.T., 2001. Teaching Management Science with Spreadsheets: From Decision Models to Decision Support. INFORMS Transactions on Education 1, 68–74. https://doi.org/10.1287/ited.1.2.68.

Rajsic, P., Weersink, A., 2008. Do farmers waste fertilizer? A comparison of ex post optimal nitrogen rates and ex ante recommendations by model, site and year. Agr.Syst. 97, 56–67. https://doi.org/10.1016/j.agsy.2007.12.001.

Ransom, C.J., Kitchen, N.R., Camberato, J.J., Carter, P.R., Ferguson, R.B., Fernández, F.G., Franzen, D.W., Laboski, C.A.M., Nafziger, E.D., Sawyer, J.E., Scharf, P.C., Shanahan, J.F., 2020. Corn nitrogen rate recommendation tools' performance across eight US midwest corn belt states. Agron.J. 112, 470–492. https://doi.org/10.1002/agj2.20035.

Reiner, L., Dörre, R., 1992. Weizen aktuell, 2nd ed. DLG-Verlag, Frankfurt am Main, 269 pp.

Rose, D.C., Sutherland, W.J., Parker, C., Lobley, M., Winter, M., Morris, C., Twining, S., Ffoulkes, C., Amano, T., Dicks, L.V., 2016. Decision support tools for agriculture: Towards effective design and delivery. Agricultural Systems 149, 165–174. https://doi.org/10.1016/j.agsy.2016.09.009.

Salkind, N.J., 2010. Encyclopedia of research design. SAGE Publications, Thousand Oaks, Calif., 1719 pp.

Sayegh, A.H., Jaloud, A., am Osman, 1981. The effect of compound versus single fertilizers on the productivity of some crops in the middle eastern-countries. LANDWIRTSCHAFTLICHE FORSCHUNG 34, 60–66.

Scharf, P.C., Shannon, D.K., Palm, H.L., Sudduth, K.A., Drummond, S.T., Kitchen, N.R., Mueller, L.J., Hubbard, V.C., Oliveira, L.F., 2011. Sensor-Based Nitrogen Applications Out-Performed Producer-Chosen Rates for Corn in On-Farm Demonstrations. Agron.j. 103, 1683–1691. https://doi.org/10.2134/agronj2011.0164.

Schätzl, R., Reisenweber, J., Schägger, M., Frank, J., 2019. LfL Deckungsbeiträge und Kalkulationsdaten. Bayerische Landesanstalt für Landwirtschaft. https://www.stmelf.bayern.de/idb/winterweizen.html (accessed 14 October 2019).

Schiebel, D., 2015 - 2018. Fertilizer sales prices. e-Mail. Herrieden.

Sedlář, O., Balík, J., Kozlovský, O., Peklová, L., Kubešová, K., 2011. Impact of nitrogen fertilizer injection on grain yield and yield formation of spring barley (Hordeum vulgare L.). Plant Soil Environ. 57, 547–552. https://doi.org/10.17221/429/2011-PSE.

Seith, T., 2015. N_Mineralisation_nach Kulturen_Jahrestagung_der_DBG. DBG. DBG Jahrestagung Unsere Böden Unser Leben, 5 September 2015, München.

Shamdasani, Y., 2021. Rural road infrastructure & agricultural production: Evidence from India. Journal of Development Economics 152, 102686. https://doi.org/10.1016/j.jdeveco.2021.102686.

Sihvonen, M., Hyytiäinen, K., Valkama, E., Turtola, E., 2018. Phosphorus and Nitrogen Yield Response Models for Dynamic Bio-Economic Optimization: An Empirical Approach. Agronomy 8, 41. https://doi.org/10.3390/agronomy8040041.

Sikora, J., Niemiec, M., Szeląg-Sikora, A., Gródek-Szostak, Z., Kuboń, M., Komorowska, M., 2020a. The Impact of a Controlled-Release Fertilizer on Greenhouse Gas Emissions and the Efficiency of the Production of Chinese Cabbage. Energies 13, 2063. https://doi.org/10.3390/en13082063.

Sikora, J., Niemiec, M., Tabak, M., Gródek-Szostak, Z., Szeląg-Sikora, A., Kuboń, M., Komorowska, M., 2020b. Assessment of the Efficiency of Nitrogen Slow-Release Fertilizers in Integrated Production of Carrot Depending on Fertilization Strategy. Sustainability 12, 1982. https://doi.org/10.3390/su12051982.

Simon, H.A., 1959. Theories of decision-making in economics and behavioral science. The American economic review 49, 253–283.

SKW Stickstoffwerke Piesteritz GmbH. Product information. https://www.skwp.de/produkte/agrochemie/ (accessed 26 June 2021).

Smart Fertilizer Management. Smart Fertilizer. London, London.

Smit, A.B., Stoorvogel, J.J., Wossink, G., 2000. A methodology to support the decision to invest in spatially variable nitrogen fertilisation. NJAS - Wageningen Journal of Life Sciences 48, 273–290. https://doi.org/10.1016/S1573-5214(00)80018-4.

Sommer, S.G., Jensen, C., 1994. Ammonia volatilization from urea and ammoniacal fertilizers surface applied to winter wheat and grassland. Fertilizer research 37, 85–92. https://doi.org/10.1007/BF00748549.

Sønderskov, M., Fritzsche, R., Mol, F. de, Gerowitt, B., Goltermann, S., Kierzek, R., Krawczyk, R., Bøjer, O.M., Rydahl, P., 2015. DSSHerbicide: Weed control in winter wheat with a decision support system in three South Baltic regions – Field experimental results. Crop Prot. 76, 15–23. https://doi.org/10.1016/j.cropro.2015.06.009.

Song, C., Zhou, Z., Zang, Y., Zhao, L., Yang, W., Luo, X., Jiang, R., Ming, R., Zang, Y., Le Zi, Zhu, Q., 2021. Variable-rate control system for UAV-based granular fertilizer spreader. Computers and Electronics in Agriculture 180, 105832. https://doi.org/10.1016/j.compag.2020.105832.

Sprague, R.H., Carlson, E.D., 1982. Building effective decision support systems. Prentice-Hall, Englewood Cliffs, N.J., 329 pp.

Sprengel, C., 1828. Von Den Substanzen Der Ackerkrume Und Des Untergrundes (About the Substances in the Plow Layer and the Subsoil). Journal for Technische and Okonomische Chemie 2, 397–421.

Stamatiadis, S., Schepers, J.S., Evangelou, E., Tsadilas, C., Glampedakis, A., Glampedakis, M., Dercas, N., Spyropoulos, N., Dalezios, N.R., Eskridge, K., 2018. Variable-rate nitrogen fertilization of winter wheat under high spatial resolution. Precision Agric 19, 570–587. https://doi.org/10.1007/s11119-017-9540-7.

Stanley, M., Roycroft, J., Amaya, A., Dever, J.A., Srivastav, A., 2020. The Effectiveness of Incentives on Completion Rates, Data Quality, and Nonresponse Bias in a Probability-based Internet Panel Survey. Field Methods 32, 159–179. https://doi.org/10.1177/1525822X20901802.

StataCorp, 2017. Stata Statistical Software. StataCorp LLC, College Station, TX, College Station, TX.

StMELF, 2020. Bayrischer Agrarbericht 2020. https://www.agrarbericht.bayern.de/ (accessed 11 April 2021).

Stöckle, C.O., Donatelli, M., Nelson, R., 2003. CropSyst, a cropping systems simulation model. European Journal of Agronomy 18, 289–307. https://doi.org/10.1016/S1161-0301(02)00109-0.

Stone, K.C., Camp, C.R., Sadler, E.J., Evans, D.E., Millen, J.A., 2010. CORN YIELD RESPONSE TO NITROGEN FERTILIZER AND IRRIGATION IN THE SOUTHEASTERN COASTAL PLAIN. Applied Engineering in Agriculture, 429-428.

Suhl, L., Mellouli, T. (Eds.), 2013. Optimierungssysteme. Springer Berlin Heidelberg, Berlin, Heidelberg.

Tabak, M., Lepiarczyk, A., Filipek-Mazur, B., Lisowska, A., 2020. Efficiency of Nitrogen Fertilization of Winter Wheat Depending on Sulfur Fertilization. Agronomy 10, 1304. https://doi.org/10.3390/agronomy10091304.

Taha, H.A., 2017. Operations Research: An Introduction, Global Edition, 10th ed. Pearson Education, Limited, Harlow, 1 Online-Ressource.

Tamburini, E., Castaldelli, G., Ferrari, G., Marchetti, M.G., Pedrini, P., Aschonitis, V.G., 2015. Onsite and online FT-NIR spectroscopy for the estimation of total nitrogen and moisture content in poultry manure. Environmental technology 36, 2285–2294. https://doi.org/10.1080/09593330.2015.1026287.

Tang, Y., Li, X., Shen, W., Duan, Z., 2018. Effect of the Slow-Release Nitrogen Fertilizer Oxamide on Ammonia Volatilization and Nitrogen Use Efficiency in Paddy Soil. Agronomy 8, 53. https://doi.org/10.3390/agronomy8040053.

Tawarmalani, M., Sahinidis, N.V., 2005. A polyhedral branch-and-cut approach to global optimization. Math. Program. 103, 225–249. https://doi.org/10.1007/s10107-005-0581-8.

Thaer, A.C., 1881. Die alt-ägyptische Landwirthschaft: Ein Beitrag zur Geschichte der Agricultur. Paul Parey, Berlin, 1 online resource (36.

Thaer, A.D., 1809. Grundsätze der rationellen Landwirthschaft: Erster Band. Begründung der Lehre des Gewerbes. Oekonomie oder die Lehre von den landwirtschaftlichen Verhältnissen, 1st ed. Realschulbuchhandlung, Berlin.

Thomas, E., 2006. Feldversuchswesen: 196 Tabellen. Verlag Eugen Ulmer, Stuttgart, 387 pp.

Tian, C., Zhou, X., Liu, Q., Peng, J., Zhang, Z., Song, H., Ding, Z., Zhran, M.A., Eissa, M.A., Kheir, A.M.S., Fahmy, A.E., Abou-Elwafa, S.F., 2020. Increasing yield, quality and profitability of winter oilseed rape (Brassica napus) under combinations of nutrient levels in fertiliser and planting density. Crop Pasture Sci. 71, 1010. https://doi.org/10.1071/CP20328.

Tröster, M.F., Pahl, H., Sauer, J., 2019. Effects of application costs on fertilizer application strategy. Computers and Electronics in Agriculture 167, 105033. https://doi.org/10.1016/j.compag.2019.105033.

Tröster, M.F., Sauer, J., under review. Characteristics of cost efficient fertilization strategies at farm level. Agr.Syst. (Agricultural Systems).

Tröster, M.F., Sauer, J., 2021a. IoFarm in Field Test: Does a Cost-Optimal Choice of Fertilization Influence Yield, Protein Content and Market Performance in Crop Production? Agriculture 11, 571. https://doi.org/10.3390/agriculture11060571.

Tröster, M.F., Sauer, J., 2021b. IoFarm: A novel decision support system to reduce fertilizer expenditures at the farm level. Comput. Electron. Agr. 188. https://doi.org/10.1016/j.compag.2021.106322.

Tukey, J.W., 1949. Comparing Individual Means in the Analysis of Variance. Biometrics 5, 99. https://doi.org/10.2307/3001913.

Turban, E., Sharda, R., Delen, D., Aronson, J.E., Liang, T.-P., King, D., 2011. Decision support and business intelligence systems, 9th ed. Prentice Hall, Boston, Massachusetts, Amsterdam, 696 pp.

Übelhör, A., Munz, S., Graeff-Hönninger, S., Claupein, W., 2015. Evaluation of the CROPGRO model for white cabbage production under temperate European climate conditions. Sci. Hortic.-Amsterdam 182, 110–118. https://doi.org/10.1016/j.scienta.2014.11.019.

Utamima, A., Reiners, T., Ansaripoor, A.H., 2019. Evolutionary Estimation of Distribution Algorithm for Agricultural Routing Planning in Field Logistics. Procedia Computer Science 161, 560–567. https://doi.org/10.1016/j.procs.2019.11.156.

Vahdanjoo, M., Sorensen, C.G., 2021. Novel Route Planning Method to Improve the Operational Efficiency of Capacitated Operations. Case: Application of Organic Fertilizer. AgriEngineering 3, 458–477. https://doi.org/10.3390/agriengineering3030031.

Vahdanjoo, M., Zhou, K., Sørensen, C.A.G., 2020. Route Planning for Agricultural Machines with Multiple Depots: Manure Application Case Study. Agronomy 10, 1608. https://doi.org/10.3390/agronomy10101608.

Valencia-García, R., Paredes-Valverde, M.A., Del Salas-Zárate, M.P., Alor-Hernández, G. (Eds.), 2018. Exploring Intelligent Decision Support Systems: Current State and New Trends, 1st ed. Springer International Publishing; Imprint: Springer, Cham, 238 pp.

Vanclooster, M., Viaene, P., Diels, J., Christiaens, k., 1996. WAVE: a mathematical model for simulating water and agrochemicals in the soil and vadose environment. Institute for Land and Water Management, Leuven, Belgium, Leuven, Belgium.

VanRaden, P.M., 2008. Efficient methods to compute genomic predictions. Journal of dairy science 91, 4414–4423. https://doi.org/10.3168/jds.2007-0980.

Vecchietti, A., Grossmann, I.E., 2016. DICOPT. Engineering Design Research Center Carnegie Mellon University, Pittsburgh, Pennsylvania, Pittsburgh, Pennsylvania.

Villalobos, F.J., Delgado, A., López-Bernal, Á., Quemada, M., 2020. FertiliCalc: A Decision Support System for Fertilizer Management. Int. J. Plant Prod. 14, 299–308. https://doi.org/10.1007/s42106-019-00085-1.

Vilsmeier, K., Amberger, A., 1980. Umsetzung von Cyanamid, Harnstoff und Ammonsulfat in Abhängigkeit von Temperatur und Bodenfeuchtigkeit. Z. Pflanzenernaehr. Bodenkd., 47–54.

Vinzent, B., Fuß, R., Maidl, F.-X., Hülsbergen, K.-J., 2018. N 2 O emissions and nitrogen dynamics of winter rapeseed fertilized with different N forms and a nitrification inhibitor. Agriculture, Ecosystems & Environment 259, 86–97. https://doi.org/10.1016/j.agee.2018.02.028.

Vougioukas, S.G., Spanomitros, Y., Slaughter, D., 2012. Dispatching and Routing of Robotic Crop-transport Aids for Fruit Pickers using Mixed Integer Programming, in: . American Society of Agricultural and Biological Engineers Annual International Meeting, Dallas, Texas. July 29 - August 1, 2012. American Society of Agricultural and Biological Engineers, St. Joseph, MI.

Wächter, A., Biegler, L.T., 2006. On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming. Math. Program. 106, 25–57. https://doi.org/10.1007/s10107-004-0559-y.

Waldren, R., Flowerday, A., 1979. Growth Stages and Distribution of Dry Matter, N, P and K in Winter Wheat. Agron.j. 71, 391–397. https://doi.org/10.2134/agronj1979.00021962007100030004x.

Waugh, F.V., 1951. The Minimum-Cost Dairy Feed (An Application of "Linear Programming"). Journal of Farm Economics 33, 299. https://doi.org/10.2307/1233608.

Wendland, M., Diepolder, M., Offenberger, K., Raschbacher, S., 2018. Leitfaden für die Düngung von Acker- und Grünland. Institut für Ökologischen Landbau, Bodenkultur und Ressourcenschutz, Freising, 97 pp. https://www.lfl.bayern.de/mam/cms07/publikationen/daten/informationen/leitfaden-duengung-acker-gruenland_gelbes-heft_lfl-information.pdf (accessed 20 October 2019).

Westerlund, T., Lundqvist, K., 2001. Alpha-ECP, version 5.01: An interactive MINLP-solver based on the extended cutting plane method. Åbo Akademi, Åbo, 59, [2] s.

Widmann, E., 2007. DIE ARCHÄOLOGIE DES HAUSHALTS IN DER KRETISCHEN NEUPALASTZEIT: ARCHÄOLOGISCHE UND IKONOGRAPHISCHE ZEUGNISSE ZUM ALLTÄGLICHEN BEREICH, 191 pp.

Wilson, M.L., Rosen, C.J., Moncrief, J.F., 2009. Potato Response to a Polymer-Coated Urea on an Irrigated, Coarse-Textured Soil. Agron.j. 101, 897–905. https://doi.org/10.2134/agronj2008.0192x.

Wimmer, S., Sauer, J., 2020. Diversification economies in dairy farming – empirical evidence from Germany. European Review of Agricultural Economics 47, 1338–1365. https://doi.org/10.1093/erae/jbaa001.

Wu, W., Ma, B., 2015. Integrated nutrient management (INM) for sustaining crop productivity and reducing environmental impact: a review. The Science of the total environment 512-513, 415–427. https://doi.org/10.1016/j.scitotenv.2014.12.101.

Xu, X., He, P., Yang, F., Ma, J., Pampolino, M.F., Johnston, A.M., Zhou, W., 2017. Methodology of fertilizer recommendation based on yield response and agronomic efficiency for rice in China. Field Crops Research 206, 33–42. https://doi.org/10.1016/j.fcr.2017.02.011.

YARA GmbH & Co. KG. Product information. https://www.yara.de/pflanzenernaehrung/duengemittel/ (accessed 26 June 2021).

YARA GmbH & Co. KG. YARA Plan. YARA GmbH & Co. KG.

Yara International ASA. ImageITA. Yara International ASA.

Ye, Z., Mo, X., Zhao, L., 2021. MINLP Model for Operational Optimization of LNG Terminals. Processes 9, 599. https://doi.org/10.3390/pr9040599.

Zámečníková, E., Kreslíková, J., 2016. Business Rules Definition for Decision Support System Using Matrix Grammar. AIP 5, 72–81. https://doi.org/10.18267/j.aip.86.

Zerulla, W., Barth, T., Dressel, J., Erhardt, K., Horchler von Locquenghien, Klaus, Pasda, G., Rädle, M., Wissemeier, A., 2001. 3,4-Dimethylpyrazole phosphate (DMPP) - a new nitrification inhibitor for agriculture and horticulture. Biology and Fertility of Soils 34, 79–84. https://doi.org/10.1007/s003740100380.

Zhang, J., Hong, L., Liu, Q., 2021. An Improved Whale Optimization Algorithm for the Traveling Salesman Problem. Symmetry 13, 48. https://doi.org/10.3390/sym13010048.

Zhou, M., Butterbach-Bahl, K., 2014. Assessment of nitrate leaching loss on a yield-scaled basis from maize and wheat cropping systems. Plant Soil 374, 977–991. https://doi.org/10.1007/s11104-013-1876-9.

Zorn, W., Heß, H., Albert, E., Kolbe, H., Kerschberger, M., Franke, G., 2007. Düngung in Thüringen 2007 nach ”Guter fachlicher Praxis„. `Landwirtschaft und Landschaftspflege in Thüringen 7/2007, Hohenleuben. http://www.tll.de/www/daten/pflanzenproduktion/duengung/dung0108.pdf (accessed 16 October 2020).