Temporal Logic Formalization of Marine Traffic Rules

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Abstract— **Autonomous vessels have to adhere to marine traffic rules to ensure traffic safety and reduce the liability of manufacturers. However, autonomous systems can only evaluate rule compliance if rules are formulated in a precise and mathematical way. This paper formalizes marine traffic rules from the Convention on the International Regulations for Preventing Collisions at Sea (COLREGS) using temporal logic. In particular, the collision prevention rules between two power-driven vessels are delineated. The formulation is based on modular predicates and adjustable parameters. We evaluate the formalized rules in three US coastal areas for over 1,200 vessels using real marine traffic data.**

I. INTRODUCTION

Human error is the main contributing factor to half of the 1,801 marine accidents between 2014 and 2019 analyzed by the European Maritime Safety Agency [1]. Autonomous vessels are a potential solution to decreasing the number of accidents caused by humans. These autonomous vessels will have to consider marine traffic rules and act accordingly. Thus, formalizing coherent marine traffic rules for machines is necessary.

The Convention on the International Regulations for Preventing Collisions at Sea (COLREGS[1]) [2] describes the marine traffic rules for preventing collisions. The COLREGS became effective in 1972 and consists of 38 rules grouped in five parts. In international waters, the COLREGS are the sole collision avoidance rules[2]. For the collision avoidance of autonomous vessels, only the second part, which considers steering and sailing regulation (i.e., COLREGS rules 4 - 19), is relevant. The rules of this part define different encounters and how the encountering vessels should react to prevent a collision. However, the rules specified in the COLREGS are formulated for humans and are not directly applicable to and verifiable for an autonomous vessel.

In this paper, we formalize the marine traffic rules of the COLREGS which consider collision avoidance between power-driven vessels. Our main contributions are:

- To the best of our knowledge, we present the first formalization of COLREGS using temporal logic.
- Our predicates and functions are parameterizable and usable for additional marine traffic rules.
- We evaluate our formalized rules on real marine traffic data for over 1,200 vessels.

The remainder of this paper is structured as follows: Section II presents an overview of the related literature. Section III describes methodical concepts for rule formalization. Sections IV and V introduce the formalized rules and associated predicates in detail. Section VI presents the evaluation of the formalized rules on scenarios generated from automatic identification system (AIS). Section VII provides conclusions.

II. RELATED WORK

The COLREGS has been mainly considered implicitly for motion planning problems so that motion planners comply with the COLREGS. In general, the COLREGS rules for steering and sailing are integrated in planners through geometric thresholds [3]–[8], virtual obstacles [9], or cost functions [10], [11]. Further, most research does not comply with all COLREGS rules for steering and sailing but instead focuses on the rules for the give-way vessel in crossing, head-on, and overtaking situations (see Section IV) [3]–[6], [11]. Some research additionally considers the safe distance [7], [8], stand-on vessel [7], [8], [10], and last-minute maneuver rule [7], [8]. However, all of these approaches are directly integrated in problem representation or motion planner, and are difficult to extend to include additional marine traffic rules.

As soon as a collision is possible, the COLREGS section describing steering and sailing rules for collision prevention has to be enforced. Therefore, detecting future collision possibilities is imperative. The simplest approach to detect potential collisions is by checking if a specified distance is kept in relation to other traffic participants [6]. This is easy to implement, however, this disregards the direction in which the vessel moves and results in many false-positive collision warnings. Another option is by calculating the closest point of approach [3], [4], which returns the time when the shortest distance between two vessels is reached assuming constant velocity and heading. However, defining thresholds [3] or constructing a virtual obstacle from the closest point of approach [4] to determine the risk of collision, which is not trivial, remains necessary. Another common approach is using a velocity obstacle [7], [12]. Here, heading and velocity are also assumed to be constant, but a virtual collision obstacle is constructed instead of calculating scalar values. From this object, the ego ship can anticipate which headings and velocities would lead to a collision without the necessity of specific thresholds. Further, for crowded scenarios, various velocity obstacles can be superimposed, and static obstacles

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[1] We use this acronym in this work for the Convention on the International Regulations for Preventing Collisions at Sea. However, in the literature, there is no consistent orthography, and it is unclear how the acronym is built.

[2] In national territory, additional rules have to be satisfied, e.g., in German coastal waters the German Traffic Regulations for Navigable Maritime Waterways (SeeSchStrO).

can be included [12]. The most general approach to detect potential collisions is through reachability analysis, where detailed kinematic models and measurement uncertainties can be considered to obtain an over-approximate occupancy [13]. However, in this study, we focus on formalizing marine traffic rules for open-sea situations; thus, collision detection with velocity obstacles is deemed sufficient.

Our approach uses temporal logic to model the COLREGS and explicitly checks rule compliance instead of only determining a contemporaneous traffic situation and rule-compliant actions. Temporal logic modeling allows for efficient extensions of rules, and implemented predicates can be reused. The COLREGS rules regarded in this work are the subset that regulates collision avoidance between power-driven vessels. The velocity obstacle concept is used to determine if a collision is possible.

III. METHODOLOGY

A. Metric temporal logic

Most marine traffic rules have preconditions; e.g., the possibility of a collision depends on the current relative position and speed of two vessels. These preconditions can be formalized by temporal logic. In particular, we use metric temporal logic (MTL) [14], which can define conditions that have to hold within a specified time interval.

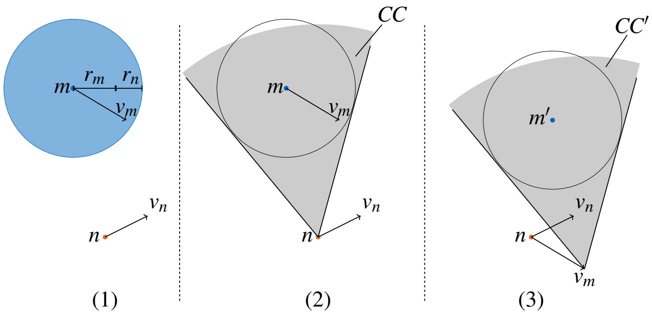
In this work, MTL is interpreted over finite traces of predicates. The used fragment of MTL can specify propositions for the future. The temporal operators used in this work are G, F, X, and U. The future globally operator $\mathrm{G}(\phi)$ indicates that the proposition ϕ has to hold true for all future time steps, whereas for the future operator $\mathrm{F}(\phi)$, ϕ has to be true only for at least one future time step. The next operator $\mathrm{X}(\phi)$ specifies that ϕ holds true for the next time step. The until operator $\phi_1 \mathrm{U} \phi_2$ specifies that ϕ_1 holds true for all time steps until ϕ_2 holds true. The formal semantics of the temporal operators are described in detail in [15]. Given atomic propositions ϕ_i, an MTL formula Φ can be constructed as follows:

$$\Phi := \phi_i | \neg\Phi | \Phi_1 \wedge \Phi_2 | \Phi_1 \vee \Phi_2$$
$$\Phi := \mathrm{G}_I(\Phi) \,|\, \mathrm{F}_I(\Phi) \,|\, \mathrm{X}(\Phi) \,|\, \Phi_1 \mathrm{U} \Phi_2$$

The subscript I indicates the time interval for which the temporal operator is applied relative to the current time step. If no interval is specified, the operator is applied for the whole trace. The traces regarded in this work are finite. Boolean operators $\neg, \vee, \wedge$ are also used. The Boolean implication operator $\implies$ is modeled by $\Phi_1 \implies \Phi_2 \equiv \neg\Phi_1 \vee \Phi_2$.

B. Velocity obstacle

The velocity obstacle concept was first introduced in [16] to extract relevant objects for an autonomous mobile robot to avoid collisions. A velocity obstacle is a geometric object from which the currently feasible velocities of robots can be determined by testing if a velocity intersects with a velocity obstacle. The basic velocity obstacle approach assumes that the current position, spatial extensions, and speed of the obstacles are known and that they move with constant speed and heading; there are extensions that eliminate these assumptions [17]. However, the basic velocity obstacle approach is sufficient for this study as these assumptions are valid for checking potential collisions for vessels on the open sea, where course and speed are usually kept constant.



(1) Initial configuration (2) Collision cone construction (3) Final velocity obstacle

Fig. 1. Construction of a velocity obstacle. The robot n and obstacle m potentially collide as the velocity of the robot v_n intersects with the collision cone CC'

The construction of the basic velocity obstacle from [16] follows the three-step process visualized in Fig. 1. First, the shapes of the robot n and obstacle m are over-approximated by circles. Since the shape of the robot and obstacle does not change, it suffices to initialize the robot as a point n and add the radius of the robot r_n to the obstacle m with radius r_m. The collision cone CC is drawn by constructing tangent lines on the enlarged obstacle, which intersect at point n. Finally, the cone is translated by the obstacle velocity vector v_m. A collision is possible when the velocity vector of the robot v_n intersects with the translated collision cone CC'.

IV. FORMALIZING COLREGS IN TEMPORAL LOGIC

As mentioned in the introduction, the steering and sailing rules are relevant for specifying collision prevention maneuvers and are crucial for safe motion planning of autonomous vessels. We assume the following conditions for our formalization of the steering and sailing rules:

- The water depth is sufficient for all vessels and does not restrict the possible maneuvers.
- Fairways and marine traffic marks are absent.
- There is a good visibility.
- All vessels are power-driven.

However, our formalization can easily be extended to other vessel types by integrating a method (e.g., an automaton) that selects the rules applicable for the current vessel type combination as specified in rule 18. The assumptions limit the formalization to rules 4, 6, 8(d), 11, and 13 - 17.

We formulate the rules from the ego-ship perspective. The implementation is built on our previous work on road traffic rules [18], which developed a rule monitor for evaluating interstate traffic rules. The rule monitor first evaluates the predicates, which are described in detail in Section V, to create a finite predicate trace. This trace is then used to evaluate the specified MTL formulas.

TABLE I

OVERVIEW OF THE FORMALIZED MARINE TRAFFIC RULES

Rule	**COLREGS reference**	**MTL formula**
R_1	Rule 4, 8(d)	$\mathrm{G}\Big(\neg\text{collision\_possible}(x_{ego}, x_o, t^{coll}_{horizon})\Big)$
R_2	Rule 4, 6	$\mathrm{G}\Big(\text{safe\_speed}(x_{ego}, v_{max})\Big)$
R_3	Rule 11, 15, 16	$\mathrm{G}\Big(\big(\neg\text{crossing}(x_{ego}, x_o, *) \wedge \mathrm{G}_{[\Delta t, t_{react}]}\,(\text{crossing}(x_{ego}, x_o, *))\big) \implies$ $\big(\mathrm{F}_{[0, t_{react}+t_{maneuver}]}(\text{maneuver\_crossing}(x_{ego}, x_o, *)) \wedge \mathrm{F}_{[t_{react}, t_{react}+2t_{maneuver}]}(\neg\text{crossing}(x_{ego}, x_o, *))\big)\Big)$
R_4	Rule 11, 14, 16	$\mathrm{G}\Big(\big(\neg\text{head\_on}(x_{ego}, x_o, *) \wedge \mathrm{G}_{[\Delta t, t_{react}]}\,(\text{head\_on}(x_{ego}, x_o, *))\big) \implies$ $\big(\mathrm{F}_{[0, t_{react}+t_{maneuver}]}(\text{maneuver\_head\_on}(x_{ego}, x_o, *)) \wedge \mathrm{F}_{[t_{react}, t_{react}+2t_{maneuver}]}(\neg\text{head\_on}(x_{ego}, x_o, *))\big)\Big)$
R_5	Rule 11, 13, 16	$\mathrm{G}\Big(\big(\neg\text{overtake}(x_{ego}, x_o, *) \wedge \mathrm{G}_{[\Delta t, t_{react}]}\,(\text{overtake}(x_{ego}, x_o, *))\big) \implies$ $\big(\mathrm{F}_{[0, t_{react}+t_{maneuver}]}(\text{maneuver\_overtake}(x_{ego}, x_o, *)) \wedge \mathrm{F}_{[t_{react}, t_{react}+2t_{maneuver}]}(\neg\text{overtake}(x_{ego}, x_o, *))\big)\Big)$
R_6	Rule 11, 15, 17	$\mathrm{G}\Big(\text{keep}(x_{ego}, x_o, *) \implies \big(\text{no\_turning}(x_{ego}, *)\,\mathrm{U}\,\neg\text{keep}(x_{ego}, x_o, *)\big)\Big)$

Note: Additional arguments are abbreviated by $*$.

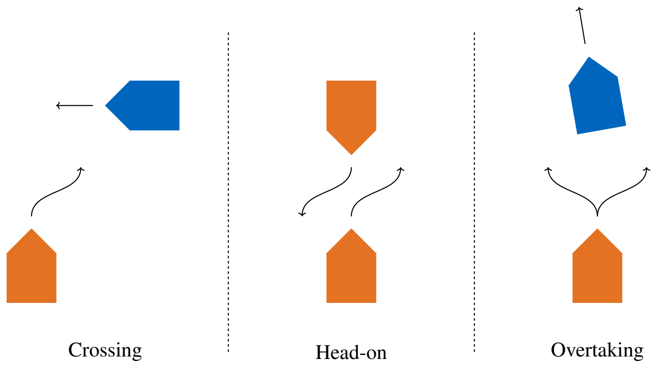


Fig. 2. Regulated situations of the COLREGS. Orange colors denote give-way vessels, and blue colors denote stand-on vessels. A give-way vessel has to evade the other vessel. A stand-on vessel has to keep its course during the maneuver of the other vessel.

Table I shows an overview of the formalized rules and indicates the corresponding rules of the COLREGS. The state of vessel i is denoted as x_i. We use the subscript *ego* for the ego vessel and o for the other traffic participants. Additional arguments of the predicates are abbreviated by $*$ to ease readability and are fully specified in Section V. The textual description of the formalized rules is as follows:

a) Safe distance R_1: Vessels always have to keep a safe distance from one another. This distance depends on the current speed and traffic scene. Therefore, we determine if the current distance between two vessels is safe by checking with the velocity obstacle concept if no collision is possible within the time horizon $t^{coll}_{horizon}$.

b) Safe speed R_2: A vessel shall always maintain a safe speed depending on the state of visibility, traffic density, and technical equipment on board.

c) Crossing R_3: When (a) two vessels are sailing on crossing paths in sight of each other, (b) there is a risk of collision, and (c) the other vessel is on starboard (i.e., right side), the ego vessel is the give-way vessel in the crossing situation. Therefore, when the ego vessel detects this situation and it is maintained until the reaction time t_{react}, it has to significantly change its course to starboard within the sum of the reaction time t_{react} and maneuver time $t_{maneuver}$. Further, the situation has to be resolved after another maneuver time $t_{maneuver}$. The detection of the changed situation has to happen within the time step Δt.

d) Head-on R_4: When two vessels approach each other in sight on opposing or near-opposing courses and there is a risk of collision, both vessels have to give way. Thus, similar to rule R_3, both vessels have to significantly change their course to starboard to resolve the situation.

e) Overtake R_5: When the ego vessel is faster and approaching another vessel in sight from its stern (i.e., from behind), it is in the give-way position of the overtaking situation. Therefore, the ego vessel has to significantly change its course to any side to avoid collision with the other vessel while overtaking.

f) Stand-on vessel R_6: When (a) two vessels are sailing in sight of each other, (b) the ego vessel has the other vessel on its port side (i.e., left side) or the ego vessel is overtaken, and (c) the risk of collision exists, the ego vessel is the stand-on vessel. Thus, the ego vessel has to keep its course until the situation is resolved.

Fig. 2 illustrates the situations and appropriate reactions for rules R_3 - R_6.

V. PREDICATES

Predicates are used to specify different conditions for marine traffic rules. We first specify necessary mathematical functions to model the predicates. Then, we group the predicates regarding position, velocity, and general conditions.

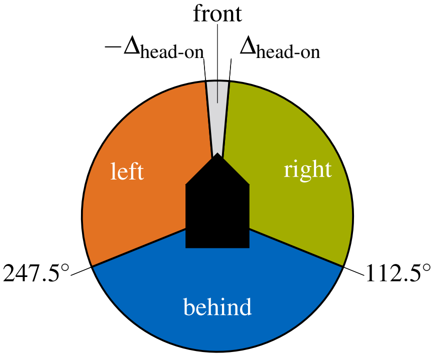


Fig. 3. Position regions relative to ego vessel.

A. Vessel movements and general functions

Vessel movements are specified through trajectories. Each trajectory consists of states at discrete time steps. The trajectory of vessel i is denoted as $\mathtt{traj}_i$.The state x_i of vessel i consists of the position $p \in \mathbb{R}^2$, heading $h \in [0, 2\pi]$, velocity $v \in \mathbb{R}$, and yaw rate $\dot{\theta} \in \mathbb{R}$. The operator $\mathrm{proj}_{\square}$ projects the state to the dimension specified by $\square$. We define a clock $\mathrm{cl}(\mathtt{traj}_i, x_i)$ which starts at the first time step of a trajectory and returns the passed time for a state x_i. In addition, we define a function $\mathrm{state}(\mathtt{traj}_i, t_k)$ which returns the state of a trajectory at time t_k. The Euclidean norm of a vector is denoted by $\|\cdot\|$ and the modulo operator $\mathrm{mod}(a,b)$ returns the remainder of a/b for $a, b \in \mathbb{R}$ using floored division.

B. Position predicates

The important relative position regions for the formalized rules are visualized in Fig. 3. We use halfspaces to determine in which sector the other vessel is located. A position p_i is within a halfspace if:

$$d_{hs}^T p_i - b_{hs} \leq 0$$

where b_{hs} is the offset to the origin, and d_{hs} is the normal vector of the halfspace.

The predicate $\mathrm{in\_front\_sector}(x_n, x_m)$ is true if and only if the center of vessel m is in the halfspace left of the $\Delta_{\text{head-on}}$ line and in the halfspace right of the $-\Delta_{\text{head-on}}$ line of Fig. 3 for vessel n. The parameter $\Delta_{\text{head-on}}$ specifies half of the front sector angle[3]. The predicates for the other three sectors can be analogously anticipated from Fig. 3. We consider the center of the other vessel instead of its entire occupancy because the spatial dimensions of vessels are negligibly small compared to the distance between the vessels.

In addition to the occupied sector, the relative orientation is also relevant. The predicate orientation_delta evaluates if the heading difference of two vessels is larger than a specified difference Δ_{orient}. Additionally, a constant offset c_{o} is integrated to evaluate the heading difference between two vessels in a head-on situation:

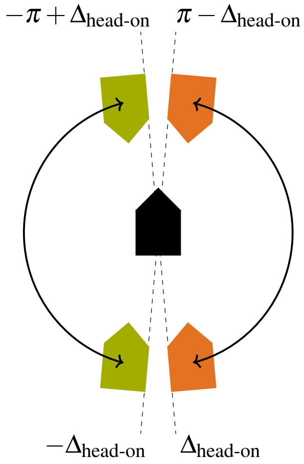


Fig. 4. Relative orientations to ego vessel. The black vessel is the ego vessel, the orange vessels indicate bounds for relative orientation toward left and the green vessels toward right.

$$\mathrm{orientation\_delta}(x_n, x_m, \Delta_{\text{orient}}, c_{\text{o}}) \iff$$
$$\mathrm{mod}(\mathrm{proj}_h(x_m) - \mathrm{proj}_h(x_n) + c_{\text{o}}, 2\pi) \in [\Delta_{\text{orient}}, 2\pi - \Delta_{\text{orient}}].$$

Checking if the other vessel is heading toward right or left with respect to the ego vessel is necessary for evaluating the crossing situation. Thus, we define the predicate $\mathrm{orientation\_towards\_right}(x_n, x_m, \Delta_{\text{head-on}})$ which is true when the relative orientation between the other vessel m and the ego vessel n is in $[-\pi + \Delta_{\text{head-on}}, -\Delta_{\text{head-on}}]$. Analogously, the predicate $\mathrm{orientation\_towards\_left}(x_n, x_m, \Delta_{\text{head-on}})$ is true for $[\Delta_{\text{head-on}}, \pi - \Delta_{\text{head-on}}]$. Fig. 4 illustrates the ranges in which the two predicates are evaluated to true.

C. Velocity predicates

The predicate drives_faster evaluates if vessel n drives faster than vessel m:

$$\mathrm{drives\_faster}(x_n, x_m) \iff \mathrm{proj}_v(x_n) > \mathrm{proj}_v(x_m).$$

For rule R_2, we need to determine if the ego vessel drives at a safe speed. As we assume the vessels to be on the open sea with a low traffic density, the maximal safe velocity v_{max} is the typical engine limit for power-driven vessels. Further, the minimal safe velocity is zero as backward driving is an unexpected and thus unsafe behavior on the open sea. Thus, the predicate that evaluates if a vessel is sailing at safe speed is as follows:

$$\mathrm{safe\_speed}(x_n, v_{max}) \iff 0 \leq \mathrm{proj}_v(x_n) \leq v_{max}.$$

D. General predicates

To determine if a collision between two vessels is possible, we use the velocity obstacle concept. Thus, we represent the velocity of a vessel i as vector v_i. The collision cone $CC'(x_n, x_m)$ for two vessels n and m is constructed as described in Section III-B and visualized in Fig. 1. Possible collisions are detected if the velocity v_n intersects with the collision cone $CC'(x_n, x_m)$ and if the lower bound of the time

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[3] Note that the COLREGS do not specify the angle for the front sector compared to the behind sector. In the literature, $\Delta_{\text{head-on}}$ is usually set to 5 deg or 10 deg.

to collision is less than the time horizon t, which is greater or equal than the time step size Δt:

$$\begin{aligned}&\text{collision\_possible}(x_n, x_m, t) \iff\\ &\qquad v_n \in CC'(x_n, x_m) \wedge\\ &\qquad \|v_n - v_m\| \leq \|\text{proj}_p(x_n) - \text{proj}_p(x_m)\|/t.\end{aligned}$$

We use this predicate for two purposes. First, if $t = t^{coll}_{horizon}$, we check if the distance between vessels is sufficient, so that the ego vessel can still avoid a collision by changing the course or stopping even if the other vessel does not react properly. Second, if $t = t^{check}_{horizon}$, the vessels sail on courses that lead to potential collisions, and a COLREGS collision avoidance maneuver has to be applied.

For rules R_3 - R_6, we need to specify a collision avoidance maneuver. Therefore, we define a predicate that evaluates if the course has changed since a defined time:

$$\begin{aligned}&\text{change\_course}(x_n, \texttt{traj}_n, t_{start}, \Delta_{\text{course}}) \iff\\ &\qquad \Big|\sum_{t_i = t_{start}}^{\text{cl}(\texttt{traj}_n, x_n)} \text{proj}_{\dot{\theta}}(\text{state}(\texttt{traj}_n, t_i))\,\Delta t\Big| \geq \Delta_{\text{course}},\end{aligned}$$

where t_{start} is the staring time of the maneuver, and Δ_{course} is the change of heading that should be achieved. Further, for head-on and crossing situations, the give-way vessel has to evade to starboard. Therefore, we specify a predicate that indicates the turning direction:

$$\begin{aligned}&\text{turning\_to\_starboard}(x_n, \texttt{traj}_n, t_{start}) \iff\\ &\qquad \text{mod}\big(\text{proj}_h(\text{state}(\texttt{traj}_n, \text{cl}(\texttt{traj}_n, x_n))) -\\ &\qquad \text{proj}_h(\text{state}(\texttt{traj}_n, t_{start})), 2\pi\big) \in (\pi, 2\pi).\end{aligned}$$

For both previous predicates, we need the starting time of a maneuver. Therefore, let us define the operator $t_s(\Psi)$ that returns the time of the last rising edge of a predicate Ψ relative to the initial time of the trajectory, which can be formulated as $\neg\Psi \wedge \text{X}(\Psi)$. The arguments of the predicate are omitted for better readability. If Ψ remained constant until the current time step, $t_s(\Psi)$ returns zero.

An overtaking situation as specified in the COLREGS is defined by (a) a potential collision, (b) the overtaken vessel m has the regarded vessel n in its behind sector, (c) the regarded vessel n has to be faster than the other vessel, and (d) the heading difference has to be less than 67.5 deg:

$$\begin{aligned}&\text{overtake}(x_n, x_m, t^{check}_{horizon}) \iff\\ &\quad \text{collision\_possible}(x_n, x_m, t^{check}_{horizon}) \wedge\\ &\quad \text{in\_behind\_sector}(x_m, x_n) \wedge \text{drives\_faster}(x_n, x_m) \wedge\\ &\quad \neg\text{orientation\_delta}(x_n, x_m, 67.5\,\text{deg}, 0).\end{aligned}$$

The angle of 67.5 deg is half of the behind sector angle and specified in rule 13 of the COLREGS. The appropriate maneuver for an overtaking situation is significantly turning, so that the overtaken vessel can detect the maneuver.

$$\begin{aligned}&\text{maneuver\_overtake}(x_n, x_m, \texttt{traj}_n, t^{check}_{horizon}, \Delta_{\text{large\_turn}}) \iff\\ &\quad \text{change\_course}(x_n, \texttt{traj}_n, t_s(\text{overtake}), \Delta_{\text{large\_turn}}) \wedge\\ &\quad \text{overtake}(x_n, x_m, t^{check}_{horizon}),\end{aligned}$$

where $\Delta_{\text{large\_turn}}$ is a turning angle that is sufficiently large to be detected by other vessels. For head-on situations, the specification is similar, however, there is no velocity condition, and the vessels have to be on opposing courses with deviation of at most $\Delta_{\text{head-on}}$:

$$\begin{aligned}&\text{head\_on}(x_n, x_m, t^{check}_{horizon}, \Delta_{\text{head-on}}) \iff\\ &\quad \text{collision\_possible}(x_n, x_m, t^{check}_{horizon}) \wedge \text{in\_front\_sector}(x_n, x_m) \wedge\\ &\quad \neg\text{orientation\_delta}(x_n, x_m, \Delta_{\text{head-on}}, \pi).\end{aligned}$$

The appropriate maneuver in a head-on situation is similar to the overtaking maneuver, but the vessel has to turn to the starboard side. Therefore, we define that the maneuver is conducted as follows:

$$\begin{aligned}&\text{maneuver\_head\_on}(x_n, x_m, \texttt{traj}_n, t^{check}_{horizon}, \Delta_{\text{large\_turn}}, \Delta_{\text{head-on}})\\ &\quad \iff \text{change\_course}(x_n, \texttt{traj}_n, t_s(\text{head\_on}), \Delta_{\text{large\_turn}}) \wedge\\ &\qquad\quad \text{turning\_to\_starboard}(x_n, \texttt{traj}_n, t_s(\text{head\_on})) \wedge\\ &\qquad\quad \text{head\_on}(x_n, x_m, t^{check}_{horizon}, \Delta_{\text{head-on}}).\end{aligned}$$

A crossing situation is defined by (a) a collision possibility; (b) the regarded vessel n in the give-way position, i.e., the other vessel m is in its right sector; and (c) the heading of the other vessel points toward the left side of the regarded vessel. Thus, the predicate for the crossing situation is specified as follows:

$$\begin{aligned}&\text{crossing}(x_n, x_m, t^{check}_{horizon}, \Delta_{\text{head-on}}) \iff\\ &\quad \text{collision\_possible}(x_n, x_m, t^{check}_{horizon}) \wedge \text{in\_right\_sector}(x_n, x_m) \wedge\\ &\quad \text{orientation\_towards\_left}(x_n, x_m, \Delta_{\text{head-on}}).\end{aligned}$$

The appropriate maneuver in a crossing situation is identical to the head-on maneuver:

$$\begin{aligned}&\text{maneuver\_crossing}(x_n, x_m, \texttt{traj}_n, t^{check}_{horizon}, \Delta_{\text{large\_turn}}, \Delta_{\text{head-on}})\\ &\quad \iff \text{change\_course}(x_n, \texttt{traj}_n, t_s(\text{crossing}), \Delta_{\text{large\_turn}}) \wedge\\ &\qquad\quad \text{turning\_to\_starboard}(x_n, \texttt{traj}_n, t_s(\text{crossing})) \wedge\\ &\qquad\quad \text{crossing}(x_n, x_m, t^{check}_{horizon}, \Delta_{\text{head-on}}).\end{aligned}$$

For the stand-on vessel, the predicate keep is introduced, which indicates if the vessel is in the stand-on position and, thus, has to keep its course. It either evaluates to true when the other vessel is in the left sector and driving toward the other vessel m or when the regarded vessel n is overtaken.

$$\begin{aligned}&\text{keep}(x_n, x_m, t^{check}_{horizon}, \Delta_{\text{head-on}}) \iff\\ &\quad \big(\text{collision\_possible}(x_n, x_m, t^{check}_{horizon}) \wedge \text{in\_left\_sector}(x_n, x_m) \wedge\\ &\quad \text{orientation\_towards\_right}(x_n, x_m, \Delta_{\text{head-on}})\big) \vee\\ &\quad \text{overtake}(x_m, x_n, t^{check}_{horizon})\end{aligned}$$

The maneuver of the stand-on vessel is keeping its course, which is described as follows:

$$\begin{aligned}&\text{no\_turning}(x_n, \texttt{traj}_n, \Delta_{\text{no\_turn}}) \iff\\ &\qquad \neg\text{change\_course}(x_n, \texttt{traj}_n, t_s(\text{keep}), \Delta_{\text{no\_turn}}),\end{aligned}$$

where $\Delta_{\text{no\_turn}}$ is the maximal heading deviation from the original course.

VI. NUMERICAL EXPERIMENTS

A. Dataset and preprocessing

Our dataset consists of recorded samples from the AIS, a radio system designed to improve the safety of marine traffic by providing information about surrounding vessels. AIS data consists of static vessel information (e.g., vessel name), dynamic information (e.g., current position), and voyage information (e.g., estimated time of arrival). All commercial vessels with gross tonnage over 300 and all passenger vessels are obligated to have AIS on board. As the AIS data providers track about 200,000 vessels per day, which is about four times the number of the worldwide merchant fleet, it is assumed that most larger vessels operating on the open sea are equipped with AIS.

Tu et al. [19] compare different AIS data sources with respect to their accessibility, time resolution, position precision, and live broadcasting possibility. For the purposes of this study, a high precision and time resolution is most important. Thus, we selected the Marine Cadastre dataset [20] for generating encounter scenarios. This dataset provides historic AIS data of US coastal waters from 2009 to 2020 of which we used data from January 2019. We preprocessed the data in two steps. First, we searched for encounters between vessels. Second, we generated the trajectories for the encountering vessels.

To ensure the validity of our assumptions in the chosen scenarios, we selected specific open-sea regions listed in Table II. For each of the locations, we search for vessels whose tracks have a distance lower than 0.03 degree of latitude or longitude, which is approximately 2000 m, within a 10 min time frame. As the rules have the implicit precondition that the vessels move, we excluded vessels that did not move.

We use the time stamp, longitudinal and lateral positions, speed over ground, and length and width of the vessel from the AIS data. For evaluation, each trajectory needs a fixed time step size, here 10 s, and the time steps between vessel have to be synchronous for evaluation. Therefore, we use linear interpolation to synchronize the time steps of different vessels. The longitudinal and lateral positions are converted in a metric coordinate system according to the Universal Transverse Mercator system. The generated scenarios are using the CommonRoad representation[4]. The scenarios include two, three, or four vessels and the duration is between 10 min and 106 min. The data and implementation are available online[5].

TABLE II

SELECTED LOCATIONS OF US COASTAL AREAS

Location	Degree latitude	Degree longitude	#Vessels
Florida	[27.51, 32.39]	[-80.18, -75.10]	364
Middle East Coast	[35.25, 38.89]	[-74.96, -73.92]	447
Upper West Coast	[37.32, 48.56]	[-126.91, -124.85]	487

TABLE III

USER-DEFINED PARAMETERS FOR TRAFFIC RULES

Parameter	Value	Parameter	Value
v_{max}	$20\,\mathrm{m\,s^{-1}}$	Δt	10 s
$\Delta_{\text{head-on}}$	5 deg	$t^{check}_{horizon}$	420 s
$\Delta_{\text{no\_turn}}$	10 deg	$t^{coll}_{horizon}$	300 s
$\Delta_{\text{large\_turn}}$	20 deg	t_{react}	60 s
		$t_{maneuver}$	60 s

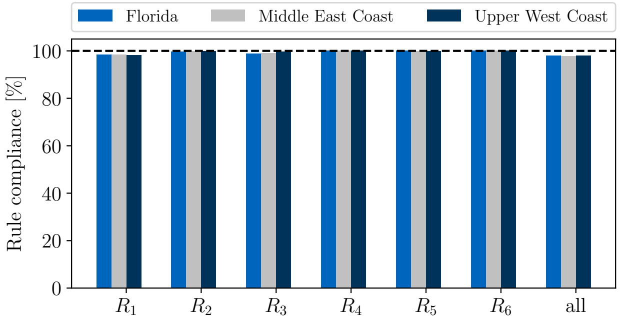

Fig. 5. Rule compliance for different geographical regions.

B. Results

For every location, we evaluate the number of vessels as indicated in Table II with the parameters specified by Table III. For each vessel, the rules are evaluated with respect to all other vessels in the scenario. Fig. 5 shows the rule compliance for different rules and regions. In general, the rule compliance at different locations is comparable. The individual rules R_1 - R_6 are mostly fulfilled with a compliance rate between 98 % and 100 %. On average, 98 % of all investigated vessels obey all formalized rules. As we investigate open-sea scenarios, where vessel often can easily keep a large distance to each other and have enough space to conduct collision avoidance maneuvers, the high rate of rule compliance confirms our expectations.

C. Discussion

The quantitative evaluation of the presented traffic rules has some limitations originating from the AIS data. In contrast to the vision-based reconstruction of scenarios, AIS data is less informative, and anomalies are difficult to detect. AIS data is asynchronously received, which makes interpolation necessary and, thus, can lead to deviations from the true path. Further, vessels without an AIS sender might sail in the area regarded as well, but are not visible in the data.

The user-defined parameters for evaluating the rules are based on the COLREGS and expert knowledge. For example, with the current value of $t^{check}_{horizon}$, the give-way vessel of two vessels sailing with 15 knots (i.e., $7.7\,\mathrm{m\,s^{-1}}$) in a rectangular crossing situation would have to conduct a crossing maneuver when the distance between both vessels is approximately 2.5 nautical miles (i.e., 4500 m). This parameter setting might

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[4] commonroad.in.tum.de/commonroad_io

[5] doi.org/10.24433/CO.8258454.v2

be too conservative for ship encounters close to shore but can be easily adapted if necessary.

We limited this study to encounters of power-driven vessels on the open sea. The data only includes encounters of two to four vessels, but the rules can be evaluated on any number of vessels. However, for these situations, the COLREGS are sometimes underspecified, e.g., a situation with three vessels where one vessel is overtaken by another one, whereas a third vessel crosses the path of the overtaken vessel from the left. Then, the overtaken vessel is in the stand-on position and the give-way position at the same time. Additionally, integrating the collision prevention hierarchy between vessel types (see rule 18 of COLREGS) would increase the applicability and could be done by switching between different rule sets depending on vessel types. Further, the last-minute-maneuver rule, which applies when one of the vessels violates the rules presented, highly depends on the situation (i.e., traffic, static obstacles, and weather conditions) and is insufficiently specified in the COLREGS to be readily formalizable. In general, MTL is very expressive and, thus, can be most likely used for further marine traffic rules as well. We will continuously update our formalized rule set and make it available online[6].

VII. Conclusions

We presented a temporal logic formalization of the COLREGS rules, which are essential for autonomous vessels. Without such a formalization, the rule compliance certification of autonomous vessels becomes much more difficult. The formalization is based on predicates and parameters that can be easily reused for specifying more marine traffic rules. The evaluation on real marine traffic data shows that most vessels in the investigated area and time obey the rules. The presented formalization allows the straightforward integration of marine traffic rules into motion planning of autonomous vessels. In addition, it can be used for the verification of marine traffic rule compliance independent of the used motion planner, thus demonstrating its general applicability.

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