Optimal maintenance decisions supported by SHM: A benchmark study

A. Kamariotis & D. Straub
Engineering Risk Analysis group
Technical University of Munich, Germany

E. Chatzi
Department of Civil, Environmental and Geomatic Engineering
ETH Zurich, Switzerland

ABSTRACT: Despite technological advancements, visual inspection still remains the primary, and oftentimes sole, means for condition-based assessment, in the current approach to infrastructure operation and maintenance. Structural Health Monitoring (SHM) may be exploited as a complementary source of information on the condition of a system. However, it is currently difficult to quantify the effect of SHM on optimal operation and maintenance and hence on the total life-cycle cost. As a step towards this goal, we employ a numerical benchmark for continuous monitoring under operational variability (Tatsis & Chatzi 2019). The numerical benchmark serves as a tool to create reference monitoring data from a two-span bridge system subject to deterioration (local stiffness reduction) over its lifespan. The benchmark is used as a simulator for extracting dynamic response data, i.e. simulated measurements (accelerations), corresponding to a typical deployment on the structure. At each time step, Bayesian updating of the deterioration model and the structural reliability is carried out, using the modal data stemming from an operational modal analysis. The reliability updating is the basis for a preposterior decision analysis, to evaluate the Value of Information (VOI) of the SHM system (Straub et al. 2017). For that, a decision time step, an action and the corresponding costs are defined. A heuristic-based approach using a simple decision rule is employed for life-cycle optimization. The resulting expected life-cycle costs are computed for the case of the deployed SHM system, and compared against the expected life-cycle costs obtained in the case of no information, thus enabling the quantification of the VOI of SHM.

1 INTRODUCTION

Structural deterioration is, among others, one of the main threats that structures and infrastructures are subjected to throughout their life-cycle. The technological advancements in developing sensors, capable of reliably measuring different quantities of structural response (e.g. accelerations, displacements, strains, temperatures, etc.), have lead to vast scientific and practical developments in the field of Structural Health Monitoring (SHM). Various techniques for translating the raw measurement data into indicators of structural "health" have been made readily available. The benefit of using SHM systems in supporting optimal maintenance decisions therefore remains to be investigated.

The question we are trying to answer is the following: How can information obtained from an SHM system provide optimal decision support and what is the value of this information? Preposterior Bayesian decision analysis can be used to quantify the VOI (Raiffa & Schlaifer 1961). Recent works (Pozzi & Der Kiureghian 2011; Zonta et al. 2014; Straub 2014; Thöns et al. 2015) use the VoI concept in an attempt to quantify the value of monitoring on idealized structural systems within a Bayesian framework. The works to date adopt simplifying assumptions regarding the type of information that the SHM system provides.

In recent years, significant research has been done in the field of operational modal analysis (Peeters & De Roeck 1999), Bayesian structural system identification and model updating using SHM modal data (Vanik et al. 2000; Papadimitriou et al. 2001; Simoen et al. 2015; Behmanesh et al. 2015), with these methods applied successfully on real structures (Behmanesh & Moaveni 2015). In this work, we employ state-of-the-art Bayesian model and structural reliability updating methods for incorporating the monitoring information coming from such an operational modal analysis within a sequential decision-making framework, following the roadmap to quantifying the benefit of SHM presented in (Straub et al.

2017). To the knowledge of the authors, no such work has been published before.

The structure of the paper is a follows: Section 2 introduces the two-span bridge system benchmark model, the creation of synthetic monitoring data, the damage scenario and the empirical stochastic deterioration model. Section 3 presents the proposed sequential Bayesian deterioration model updating framework in the presence of continuously obtained modal data. In Section 4, we address the reliability analysis of the deteriorating structural system and the updating of the reliability using the monitoring data. In Section 5, a heuristic-based solution to the decision problem is introduced, followed by the VoI results of our investigation in Section 6. Finally Section 7 concludes this work.

2 CONTINUOUSLY MONITORED BRIDGE SYSTEM SUBJECT TO DETERIORATION

Consider the two-span bridge model of Figure 1, with its reference behavior (Tatsis & Chatzi 2019) simulated by a Finite Element (FE) model of isoparametric plane stress quadrilateral elements. 200 elements are employed to mesh the x direction, and 6 elements are assumed per height (y direction). The beam dimensions form configurable parameters of the benchmark and are set as: height $h = 0.6$m, width $w = 0.1$m, while the lengths are L_1 = 12m for the first span and L_2 = 13m for the second span. A linear elastic material with Young's modulus E = 30GPa, Poisson ratio ν = 0.2, and material density ρ = 2000 kg/m^3 is assigned. For all the three support points elastic boundaries are assumed in both directions, in the form of translational springs with $K_x = 10^8$ N/m and $K_y = 10^7$ N/m.

It is assumed that the simulated two-span bridge is continuously monitored using a set of 18 sensors measuring vertical acceleration, whose locations are noted in red in Figure 1. A distributed Gaussian white noise excitation $F(x)$ is used as the load acting on the bridge. A dynamic time history analysis of the model, for a given realization of the load, results in the measured vertical acceleration signals at the assigned sensor locations.

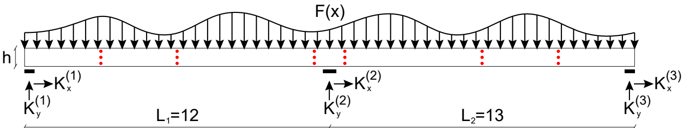

Figure 1: Benchmark model

2.1 *Synthetic monitoring data creation*

For the purpose of the present analysis, at every time instance for which we want to obtain data from the deployed SHM system, a dynamic time history analysis of the benchmark model is run and the "true" vertical acceleration signals $\ddot{x}$ at the sensor locations (FE nodes) are obtained. The noise-free acceleration time series data set is contaminated with Gaussian white noise of 2% root mean square noise-to-signal ratio, simulating a sensor measurement error. Subsequently the noisy accelerations $\tilde{\ddot{x}}$ are fed into an output-only operational modal analysis (OMA) scheme. Specifically, the stochastic subspace identification (SSI) (Peeters & De Roeck 1999) algorithm is used to identify a set of the lower eigenvalues (squares of natural frequencies) and mode shapes. A set of identified eigenvalues and mode shape vectors is referred to as SSI modal data herein.

2.2 *Structural deterioration*

We assume that the support of the bridge structure is further subjected to gradual deterioration (e.g. from scour). Damage is introduced as a reduction of the stiffness in y-direction of the spring $K_y^{(2)}$ at the middle elastic support of the bridge. The evolution of the stiffness reduction of the vertical support spring over the lifespan of the bridge is described by employing the damage model of equation (1), where K_y is the initial undamaged value, and $D(t)$ is the simple stochastic deterioration model of equation (2). The uncertain parameters of the deterioration model are summarized in Table (1).

$$K_y^{(2)}(t) = K_y / \big(1 + D(t)\big) \tag{1}$$

$$D(t) = At^B \tag{2}$$

Table 1: Parameters of the stochastic deterioration model.

Parameter	Distribution	Mean	c.o.v.
A	Lognormal	2.435×10^{-4}	40%
B	Normal	2.0	15%

We consider a lifespan of the bridge T = 50 years. The mean and coefficient of variation of the parameters A and B are chosen to reflect sufficiently large uncertainty a-priori. They result in a 1% probability that $D(t = 50)$ >10 at the end of the lifespan.

3 BAYESIAN DETERIORATION MODEL UPDATING FRAMEWORK

In this section we present the Bayesian model updating framework that we employ for the sequential learning of the parameters of the deterioration model of equation (2). A detailed presentation of the framework can be found in Vanik et al. (2000), Simoen et al. (2015), Behmanesh & Moaveni (2015).

3.1 *Bayesian formulation*

The goal of the Bayesian inverse problem is to infer the deterioration model parameters $\boldsymbol{\theta} \in \mathbb{R}^2$ given noisy identified modal eigenvalues $\widetilde{\lambda}_m = (2\pi \widetilde{f}_m)^2$ and mode shape vector components $\widetilde{\mathbf{\Phi}}_m \in \mathbb{R}^{N_s}$ at the N_s DOFs which correspond to the sensor locations, where $m = 1, ..., N_m$ is the number of the observed modes.

Consider a linear FE model, which is parametrized through the deterioration model parameters $\boldsymbol{\theta} = [A, B]$. In this work, the model predicting the eigenvalues and mode shapes for the updating process is the same FE model as the one described in Section 2 for the creation of the noise-contaminated synthetic data. Despite of the noise being added, the use of the same model constitutes a so-called inverse crime (Wirgin 2004). However, it is not the purpose of this work to focus on the updating process, therefore for reasons of simplicity we consider this to be acceptable for the demonstration in this paper. The goal of the Bayesian probabilistic framework is to estimate the parameters $\boldsymbol{\theta}$, and their uncertainty, such that the FE model predicted modal eigenvalues $\lambda_m(\boldsymbol{\theta})$ and mode shapes $\mathbf{\Phi}_m(\boldsymbol{\theta}) \in \mathbb{R}^{N_s}$ best match the corresponding SHM modal data.

Using Bayes' theorem, the posterior probability density function (PDF) of the deterioration model parameters $\boldsymbol{\theta}$ given one identified modal data set $[\widetilde{\boldsymbol{\lambda}}, \widetilde{\mathbf{\Phi}}]$ is computed via equation (3) and is proportional to the likelihood function $L(\boldsymbol{\theta}; \widetilde{\boldsymbol{\lambda}}, \widetilde{\mathbf{\Phi}})$ multiplied with the prior PDF of the model parameters $\pi_{pr}(\boldsymbol{\theta})$ (Table 1). The proportionality constant is the so-called model evidence Z and requires the solution of a two-dimensional integral, shown in equation (4).

$$\pi_{\text{pos}}(\boldsymbol{\theta} \mid \widetilde{\boldsymbol{\lambda}}, \widetilde{\mathbf{\Phi}}) \propto L(\boldsymbol{\theta}; \widetilde{\boldsymbol{\lambda}}, \widetilde{\mathbf{\Phi}}) \pi_{\text{pr}}(\boldsymbol{\theta}) \tag{3}$$

$$Z = \int_{\Omega_{\boldsymbol{\theta}}} L(\boldsymbol{\theta}; \widetilde{\boldsymbol{\lambda}}, \widetilde{\mathbf{\Phi}}) \pi_{\text{pr}}(\boldsymbol{\theta}) d\boldsymbol{\theta} \tag{4}$$

The SHM modal data come with significant uncertainty, which should be taken into account within the Bayesian framework. According to (Simoen et al. 2015), one can separate between i) measurement uncertainty, including random measurement noise and variance or bias errors induced in the SSI procedure, and ii) model uncertainty. The combination of measurement and model uncertainty reflects the total prediction error. In order to construct the likelihood function, the eigenvalue and mode shape prediction errors for a specific mode m are defined as in equations (5) and (6).

$$\eta_{\lambda_m} = \widetilde{\lambda}_m - \lambda_m(\boldsymbol{\theta}) \tag{5}$$

$$\eta_{\mathbf{\Phi_m}} = \left\| \gamma_m \widetilde{\mathbf{\Phi}}_m - \mathbf{\Phi}_m(\boldsymbol{\theta}) \right\| \tag{6}$$

where γ_m is a normalization constant which is computed as in equation (7). $\mathbf{\Gamma}$ is a binary matrix for selecting the FE degrees of freedom, which correspond to the sensor locations.

$$\gamma_m = \frac{\widetilde{\mathbf{\Phi}}_m^T \mathbf{\Gamma} \mathbf{\Phi}_m}{\|\widetilde{\mathbf{\Phi}}_m\|^2} \tag{7}$$

The probabilistic model of the eigenvalue prediction error is a zero-mean Gaussian random variable with standard deviation assumed to be proportional to the measured eigenvalues:

$$\eta_{\lambda_m} \sim \mathcal{N}(0, c_{\lambda m}^2 \widetilde{\lambda}_m^2) \tag{8}$$

For the mode shape prediction error, which is defined as the L_2-norm of the difference of the mode shape vectors for a given mode m, a zero-mean Gaussian random variable is assigned with a standard deviation proportional to the L_2-norm of the measured mode shape vector:

$$\eta_{\mathbf{\Phi}_m} \sim \mathcal{N}\big(0, c_{\Phi m}^2 \|\widetilde{\mathbf{\Phi}}_m\|^2\big) \tag{9}$$

The factors $c_{\lambda m}$ and $c_{\Phi m}$ can be seen as assigned coefficients of variation, and their chosen value reflects the prediction error. In this work, a fixed value $c_{\lambda m} = c_{\Phi m} = 0.15$ is chosen for all the modes corresponding to a 15% coefficient of variation.

Assuming statistical independence among the identified modal data, the likelihood function for a given modal data set can be written as in equation (10).

$$L(\boldsymbol{\theta}; \widetilde{\boldsymbol{\lambda}}, \widetilde{\mathbf{\Phi}}) = \\ = \prod_{m=1}^{N_m} N(\eta_{\lambda_m}; 0, c_{\lambda m}^2 \widetilde{\lambda}_m^2) N\big(\eta_{\mathbf{\Phi}_m}; 0, c_{\Phi m}^2 \|\widetilde{\mathbf{\Phi}}_m\|^2\big) \tag{10}$$

The benefit of SHM is that the sensors can provide data in a continuous fashion, therefore resulting in an abundance of measurements received almost continually. Assuming independence among N_t modal data sets obtained at different time instances, the likelihood can now be expressed as:

$$L(\boldsymbol{\theta}; \widetilde{\boldsymbol{\lambda}}_1 ... \widetilde{\boldsymbol{\lambda}}_{N_t}, \widetilde{\mathbf{\Phi}}_1 ... \widetilde{\mathbf{\Phi}}_{N_t}) = \\ = \prod_{t=1}^{N_t} \prod_{m=1}^{N_m} N\big(\widetilde{\lambda}_{t_m} - \lambda_{t_m}(\boldsymbol{\theta}); 0, c_{\lambda m}^2 \widetilde{\lambda}_{t_m}^2\big) \\ N\Big(\big\|\gamma_{t_m} \widetilde{\mathbf{\Phi}}_{t_m} - \mathbf{\Phi}_{t_m}(\boldsymbol{\theta})\big\|; 0, c_{\Phi m}^2 \|\widetilde{\mathbf{\Phi}}_{t_m}\|^2\Big) \tag{11}$$

where the index t_m indicates the modal data of mode m identified at time instance t. The inclusion of data in a continuous fashion increases the level of accuracy of the Bayesian model updating procedure. However, when choosing how many data sets obtained at different time instances to consider for the updating, one should be aware of the fact that by increasing the number of data sets, the parameter estimation uncertainty will decrease, yet this might not properly reflect the full variability of the updated parameters (Vanik et al. 2000; Behmanesh et al. 2015).

3.2 *Adaptive MCMC sampling*

The solution of the Bayesian updating problem in the general case involves the solution of the n-dimensional integral for the computation of the model evidence, where n is the number of random variables. Analytic solutions to this integral are available only in special cases, and numerical integration or sampling methods are necessary. In this work, since the number of random parameters is limited to two, we are flexible in choosing the method to solve the Bayesian updating problem. However, to render this demonstration more generally applicable, we employ an adaptive Markov Chain Monte Carlo (MCMC) method (Haario et al. 2006), with the adaptation performed on the covariance matrix of the proposal PDF.

4 STRUCTURAL RELIABILITY

4.1 *Structural reliability analysis for the deteriorating structural system*

A detailed review can be found in Straub et al. (2020). In its simplest form, a failure event at time t can be described in terms of a structural system capacity $R(t)$ and a demand $S(t)$. Both R and S are random variables. In this investigation we are dealing with a problem in which the structural capacity $R(t)$ can be separated from the demand $S(t)$. At a time t, the structural capacity includes the effect of the deterioration process, as in Figure 2. This curve is obtained via a static analysis of the benchmark model of Section 2 for increasing values of the stiffness reduction of spring $K_y^{(2)}$ described by the damage rule of equation (1). For increasing values of the stiffness reduction, we evaluate the loss of load bearing capacity of the bridge structure relative to the undamaged state.

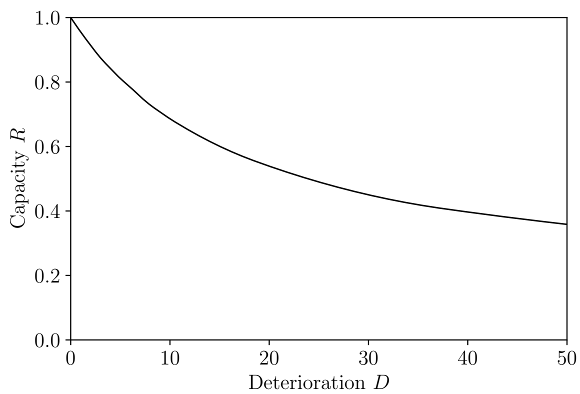

Figure 2: Structural capacity in function of the deterioration D.

The uncertain demand acting on the structure is modeled by the maximum load in a one-year time interval with a Gumbel distribution. The parameters of the Gumbel distribution are chosen such that the probability of failure in the initial undamaged state is equal to 10^{-6} and the coefficient of variation is 20%.

We discretize time in yearly intervals $j = 1,..,T$, where the j-th interval represents $t \in (t_{j-1}, t_j]$. For this type of problems, the time-variant reliability problem can be replaced by a series of time-invariant reliability problems. F_j^* is defined as the event of failure in interval $(t_{j-1}, t_j]$. For a given value of the capacity, the conditional interval probability of failure is

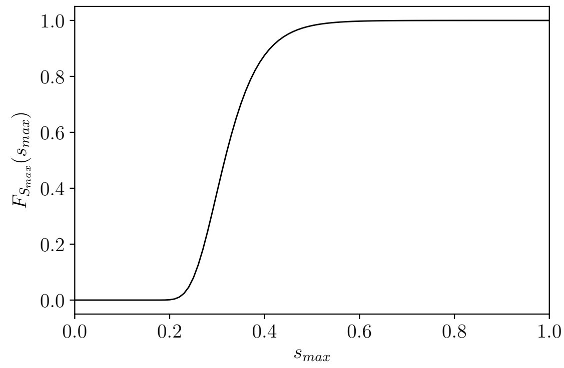

Figure 3: CDF of the Gumbel distribution for the load with location a_n = 0.0509, scale b_n= 0.297.

defined as:

$$\Pr\big(F_j^* \mid R(\boldsymbol{\theta}, t_j)\big) = 1 - F_{s_{max}}\big(R(\boldsymbol{\theta}, t_j)\big) \tag{12}$$

We define $\Pr[F(t_i)]$ =$\Pr(F_1^* \cup F_2^* \cup ...F_i^*)$ as the accumulated probability of failure up to time t_i. We can compute $Pr[F(t_i)]$ through the conditional interval probabilities $Pr(F_j^*|R(\boldsymbol{\theta}, t_j))$. The conditional accumulated probability of failure can be computed as:

$$\Pr\big[F(t_i) \mid \boldsymbol{\theta}\big] = 1 - \prod_{j=1}^{i}\Big[1 - \Pr\big(F_j^* \mid R(\boldsymbol{\theta}, t_j)\big)\Big] \tag{13}$$

Following the total probability theorem, the unconditional accumulated probability of failure is:

$$\Pr\big[F(t_i)\big] = \int_{\Omega_{\boldsymbol{\theta}}} \Pr\big[F(t_i) \mid \boldsymbol{\theta}\big]\pi_{\text{pr}}(\boldsymbol{\theta})d\boldsymbol{\theta} \tag{14}$$

The solution to the above integral is found using Monte Carlo simulation (MCS). We draw samples from the prior distribution $\pi_{\text{pr}}(\boldsymbol{\theta})$ of the two uncertain deterioration model parameters and the integral in (14) is approximated by:

$$\Pr\big[F(t_i)\big] \approx \frac{1}{n_{\text{MCS}}} \sum_{k=1}^{n_{\text{MCS}}} \Pr\big[F(t_i) \mid \boldsymbol{\theta}^{(k)}\big] \tag{15}$$

Having computed the probabilities $Pr[F(t_i)]$, one can compute the hazard function $h(t_i)$ for the different time intervals t_i, which expresses the failure rate of the structure conditional on survival up to time t_{i-1}:

$$h(t_i) = \frac{\Pr\big[F(t_i)\big] - \Pr\big[F(t_{i-1})\big]}{1 - \Pr\big[F(t_{i-1})\big]} \tag{16}$$

4.2 *Structural reliability updating using SHM modal data*

The goal of SHM is to identify structural damage. Data coming from the monitoring system can be employed in order to identify the parameters $\boldsymbol{\theta}$ of the deterioration model and obtain their posterior distribution. Consequently this affects the computation of the cumulative probability of failure at time t_i, which is now conditioned on data $\boldsymbol{Z} = [\widetilde{\boldsymbol{\lambda}}, \widetilde{\boldsymbol{\Phi}}]$ obtained up to time t_{i-1}.

$$\Pr\big[F(t_i) \mid \boldsymbol{Z}_{1:i-1}\big] = \Pr(F_1^* \cup F_2^* \cup ... F_i^* \mid \boldsymbol{Z}_{1:i-1}) \tag{17}$$

The cumulative probability of failure up to time t_i conditional on modal data obtained up to time t_{i-1} is:

$$\Pr\big[F(t_i) \mid \boldsymbol{Z}_{1:i-1}\big] = \int_{\Omega_{\boldsymbol{\theta}}} \Pr\big[F(t_i) \mid \boldsymbol{\theta}\big] \pi_{\text{pos}}(\boldsymbol{\theta} | \widetilde{\boldsymbol{\lambda}}_{1:i-1}, \widetilde{\boldsymbol{\Phi}}_{1:i-1}) d\boldsymbol{\theta} \tag{18}$$

In (18), one needs to integrate over the posterior distribution of the parameters $\boldsymbol{\theta}$. As described in Section (3.2), an adaptive MCMC algorithm is used for the Bayesian analysis, therefore at every step of the sequential updating we obtain the posterior distribution of the parameters in the form of posterior MCMC samples. Using these samples $\boldsymbol{\theta}^{(k)}$, the integral in equation (18) can be approximated:

$$\Pr\big[F(t_i) \mid \boldsymbol{Z}_{1:i-1}\big] \approx \frac{1}{n_{\text{MCMC}}} \sum_{k=1}^{n_{\text{MCMC}}} \Pr\big[F(t_i) \mid \boldsymbol{\theta}^{(k)}\big] \tag{19}$$

The hazard function conditional on the monitoring data can then be obtained via the following expression:

$$h(t_i \mid \boldsymbol{Z}_{1:i-1}) = = \frac{\Pr\big[F(t_i) \mid \boldsymbol{Z}_{1:i-1}\big] - \Pr\big[F(t_{i-1}) \mid \boldsymbol{Z}_{1:i-1}\big]}{1 - \Pr\big[F(t_{i-1}) \mid \boldsymbol{Z}_{1:i-1}\big]} \tag{20}$$

5 LIFE-CYCLE COST WITH SHM

5.1 *Life-cycle optimization based on heuristics*

We are interested in identifying optimal maintenance strategies for a deteriorating structure equipped with a monitoring system, and subsequently quantifying the Value of Information (VOI) of the SHM. Towards that goal, we set up a simple decision model. A prior deterioration model of equation (2) describing the dynamics of the system (deterioration) is available. The decision time horizon is the lifespan of the bridge structure T = 50 years.

We introduce the terms policies and strategies (Bismut & Straub 2020), which we employ for the solution of the decision problem. A policy π_i can be thought as a decision rule that tells us which action to take at time step t_i, conditional on all the information at hand up to that time (Jensen and Nielsen 2007). A strategy S is the set of policies over all the decision problem time steps.

We introduce a simple heuristic for the solution of the sequential decision problem. A detailed presentation for the use of heuristics in optimal inspection and maintenance planning can be found in (Luque & Straub 2019) and (Bismut & Straub 2020). With the use of heuristics, the space of solutions to the decision problem is drastically reduced, but the problem is solved only approximately. The simple heuristic chosen here is to perform a repair action whenever the estimate of the hazard function is larger than a predefined threshold. The parameter $w = h_{thres}$ describing the heuristic is a parameter of the strategy S. We assume that performing a repair action means replacing the component (spring support) and bringing it back to its initial state, and that no failure will occur once a repair action has been performed.

With the use of heuristics, solving the sequential decision problem boils down to the solution of the optimization problem:

$$w^* = \text{argmin}_w \mathbf{E}[C_{\text{tot}} \mid w] \tag{21}$$

where C_{tot} is the total cost of maintenance and risk and w is the parameter describing the heuristic.

5.2 *Computation of the expected total cost in the prior case without any data*

In the prior case, where no monitoring data is available, the expectation in equation (21) is with respect to the system state, i.e. the deterioration model parameters $\boldsymbol{\theta}$.

$$\mathbf{E}_{\boldsymbol{\theta}}[C_{\text{tot}} \mid w] = \int_{\Omega_{\boldsymbol{\theta}}} C_{\text{tot}}(w, \boldsymbol{\theta}) f_{\boldsymbol{\theta}}(\boldsymbol{\theta}) d\boldsymbol{\theta} \tag{22}$$

The total cost of maintenance and risk $C_{\text{tot}}(w, \boldsymbol{\theta})$ is the sum of the repair costs $C_{\text{R}}(w)$ and the failure costs $C_{\text{F}}(w, \boldsymbol{\theta})$ over the lifetime of the bridge. Since we are interested in quantifying the value of information, as defined later in Section 5.4, the initial cost is not included in C_{tot}.

The integral of equation (22) is computed via MCS. One can draw samples $\boldsymbol{\theta}$ from $\pi_{\text{pr}}(\boldsymbol{\theta})$ to compute the accumulated probability of failure via equation (15), and subsequently compute the hazard function with equation (16). When the hazard function exceeds the threshold, i.e. when $h(t_i) \geq w$, we define $t_{\text{repair}} = t_{i-1}$ as the time that the repair takes place. The time of repair is thus a function of our chosen heuristic.

The cost of repair is given as:

$$C_{\text{R}}(w) = \hat{c}_R \gamma\big(t_{\text{repair}}(w)\big) \tag{23}$$

where $\hat{c}_R$ is the fixed cost of the repair, and $\gamma(t) = 1/(1+r)^t$ is the discount function, with r being the discount rate.

The risk of failure is given as:

$$C_{\text{F}}(w, \boldsymbol{\theta}) = \sum_{i=1}^{t_{repair}(w)} C_F(t_i, \boldsymbol{\theta}) \tag{24}$$

where:

$$C_F(t_i, \boldsymbol{\theta}) = \hat{c}_F \gamma(t_i) \Big\{ \Pr\big[F(t_i)\big] - \Pr\big[F(t_{i-1})\big] \Big\} \quad (25)$$

where $\hat{c}_F$ is the fixed cost of the failure event.

Following the solution of the optimization problem in (21), the expected life-cycle costs associated with the optimal decision, in the prior case without any monitoring data is $\mathbf{E}_{\boldsymbol{\theta}}[C_{\text{tot}} \mid w_0^*]$.

5.3 *Computation of the expected total cost in the SHM data-informed case*

$\boldsymbol{Z} = [\boldsymbol{Z}_1, ..., \boldsymbol{Z}_{n_T}]$ are the random vectors containing the identified modal data stemming from an SHM operational modal analysis. For our investigation, we obtain one set of identified modal data every year, and no optimization for the data acquisition time step or size is performed.

Under availability of identified modal data from the SHM system, the expectation in equation (21) is operating on both the system state $\boldsymbol{\theta}$ and on the monitoring outcomes $\boldsymbol{Z}$.

$$\mathbf{E}_{\boldsymbol{\theta},\boldsymbol{Z}}[C_{\text{tot}} \mid w] = \int_{\Omega_{\boldsymbol{\theta}}} \int_{\Omega_{\boldsymbol{Z}}} C_{\text{tot}}(w, \boldsymbol{\theta}, \boldsymbol{z}) f_{\boldsymbol{\Theta},\boldsymbol{Z}}(\boldsymbol{\theta}, \boldsymbol{z}) d\boldsymbol{z} d\boldsymbol{\theta} \quad (26)$$

The total cost of maintenance and repair $C_{\text{tot}}(w, \boldsymbol{\theta})$ is again the sum of the repair costs $C_{\text{R}}(w, \boldsymbol{Z})$ and the failure costs $C_{\text{F}}(w, \boldsymbol{\theta}, \boldsymbol{Z})$ over the lifetime of the bridge, which now depend also on the monitoring outcomes $\boldsymbol{Z}$.

The integral in equation (26) is computed with crude MCS. We draw samples from the uncertain deterioration model parameters, and for each of those samples, we have one deterioration history, as given by equation (2). For each deterioration history, we generate monitoring outcomes (one identified modal data set per year). In this way we are jointly sampling the system state space and monitoring data space, and equation (26) is approximated as:

$$\mathbf{E}_{\boldsymbol{\theta},\boldsymbol{Z}}[C_{\text{tot}} \mid w] = \\ = \frac{1}{n_{MCS}} \sum_{k=1}^{n_{MCS}} \big[C_{\text{R}}(w, \boldsymbol{z}^{(k)}) + C_{\text{F}}(w, \boldsymbol{\theta}^{(k)}, \boldsymbol{z}^{(k)}) \big] \quad (27)$$

For each of the sampled system states and corresponding monitoring data, when $h(t_i \mid \boldsymbol{z}_{1:i-1}^{(k)}) \geq w$, then $t_{\text{repair}}^{(k)} = t_{i-1}$.

The cost of repair is expressed as:

$$C_{\text{R}}(w, \boldsymbol{z}^{(k)}) = \hat{c}_R \gamma\big(t_{\text{repair}}(w, \boldsymbol{z}^{(k)})\big) \quad (28)$$

The risk of failure is:

$$C_{\text{F}}(w, \boldsymbol{\theta}^{(k)}, \boldsymbol{z}^{(k)}) = \sum_{i=1}^{t_{repair}(w, \boldsymbol{z}^{(k)})} C_F(t_i, \boldsymbol{\theta}^{(k)}) \quad (29)$$

where:

$$C_F(t_i, \boldsymbol{\theta}^{(k)}) \\ = \hat{c}_F \gamma(t_i) \Big\{ \Pr\big[F(t_i) \mid \boldsymbol{\theta}^{(k)}\big] - \Pr\big[F(t_{i-1}) \mid \boldsymbol{\theta}^{(k)}\big] \Big\} \quad (30)$$

Solving equation (21), we obtain the optimal expected life-cycle costs given the monitoring data, $\mathbf{E}_{\boldsymbol{\theta},\boldsymbol{Z}}[C_{\text{tot}} \mid w_{mon}^*]$.

5.4 *Value of information*

The VOI is given by equation (31) and is the difference of the optimal expected life-cycle costs between the prior case and the case where monitoring data are available.

$$VOI = \mathbf{E}_{\boldsymbol{\theta}}\big[C_{\text{tot}}(\boldsymbol{\theta}, w) \mid w_0^*\big] - \mathbf{E}_{\boldsymbol{\theta},\boldsymbol{Z}}\big[C_{\text{tot}}(\boldsymbol{\theta}, \boldsymbol{Z}, w) \mid w_{mon}^*\big] \quad (31)$$

6 RESULTS

6.1 *Sequential Bayesian deterioration model updating using monitoring data*

Initially we demonstrate how the Bayesian framework performs in learning the parameters of the deterioration model on the basis of the SHM modal data. We assume a scenario where the "true" deterioration model corresponds to parameters values A^*=3.2×10^{-4} and B^* = 2.34. Using equation (2), this corresponds to a value of the deterioration $D^*(t$ =50) = 3.03 and the stiffness at the mid support spring reduced to $K_y(t$ =50) = $K_0/$(1+3.03) at the end of the service life of the bridge. The "true" deterioration curve can be seen in black in all the subfigures of Figure 5.

For this "true" deterioration curve we create one monitoring history, i.e., we generate one set of SHM modal data ($N_m = 6$ identified modes) every year. In this simple example, the structural properties are not assumed influenced by environmental (temperature, humidity) and operation (non stationary effects due to traffic) variability. To this end, we assume it suffices to utilize one estimate of the modal properties set per year. This estimate is delivered upon processing of several datasets, which allows to further quantify the variance in the estimation of these values. Using this data, we employ the sequential Bayesian deterioration model updating framework of Section 3.

Figure 4 demonstrates how the distribution of the deterioration model parameters is updated, by comparing the prior PDF of A and B with the posterior filtering PDF of A and B at year 30, using modal data $\widetilde{\boldsymbol{\lambda}}_{1:30}$ and $\widetilde{\boldsymbol{\Phi}}_{1:30}$. The posterior PDF is given via a kernel density estimation using the 5000 posterior MCMC samples of A and B. It is observed that using one SHM data set per year up to year 30, the uncertainty in the deterioration model parameters has decreased, the filtering PDF is narrower (only slightly for A) and peaked around the true values A^* and B^*.

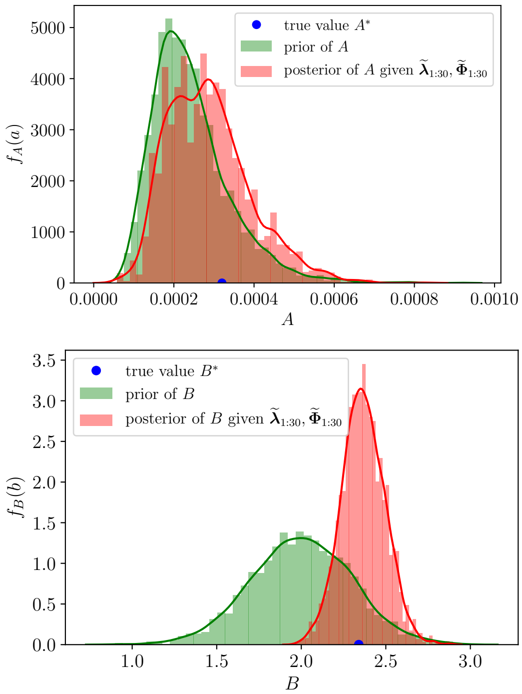

Figure 4: Prior PDF and posterior filtering PDF at year 30 for deterioration model parameters.

In Figure 5 we compare the "true" deterioration model with the deterioration model estimated using MCS in the prior case, and with the ones estimated with filtering posterior MCMC samples at three different time instances. In the upper right subfigure we observe that at the first years of the deterioration, the deterioration model parameters are not sufficiently identified and there is still large uncertainty in the estimation, which is clearly reduced in later years, as seen in two lower subfigures.

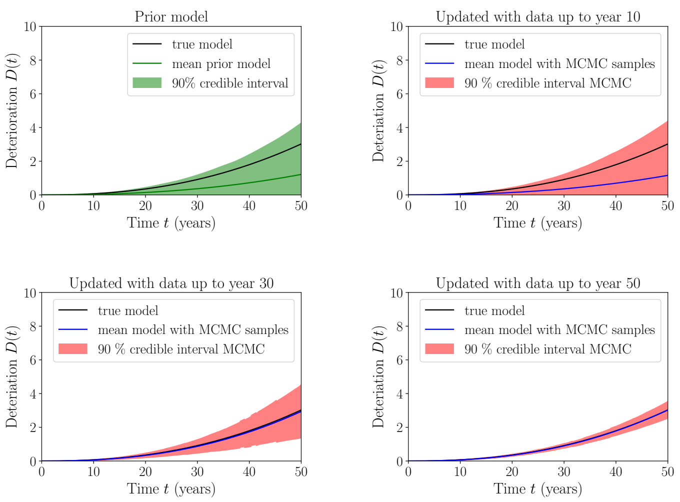

Figure 5: Sequential Bayesian learning of the deterioration model.

6.2 *Reliability updating using monitoring data*

As presented in Section 4.2, learning the deterioration model parameters and the reduction of the uncertainty in their estimation through the acquisition of SHM modal data affects the estimation of the reliability of the structure. In Figure 6, we show the accumulated failure probability $\Pr[F(t)]$ of the bridge structure, and compare it with the accumulated failure probability conditional on monitoring data $\Pr[F(t)|\boldsymbol{Z}(t)]$. The 90% credible interval of $\Pr[F(t)]$ is obtained from 1000 independent simulation runs. For the conditional case, we consider also here the case of a generated monitoring history from the "true" deterioration model described in Section 6.1. At each time t, $\Pr[F(t)|\boldsymbol{Z}(t)]$ is conditioned on monitoring outcomes up to time t.

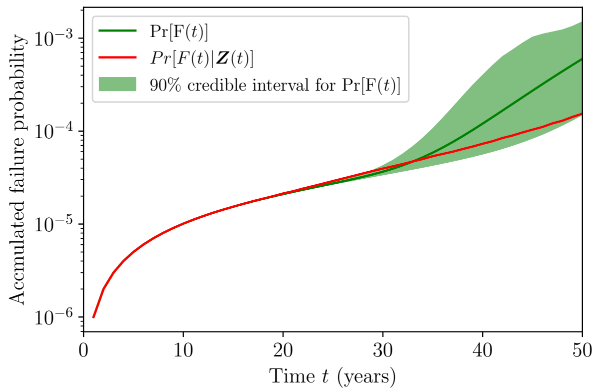

Figure 6: Probability of failure updating.

6.3 *VOI results*

The VOI is computed with equation (31) following the framework presented in Section 5.

The expected life-cycle cost $\mathbf{E}_{\boldsymbol{\theta},\boldsymbol{Z}}[C_{\text{tot}} \mid w]$ in the SHM data-informed case is calculated with MCS following equation (27). For this we draw 500 samples of $\boldsymbol{\theta}$, and for each of those we create one monitoring outcome history. In this way we are jointly sampling $\boldsymbol{\theta}$ and $\boldsymbol{Z}$. The expected life-cycle cost $\mathbf{E}_{\boldsymbol{\theta}}[C_{\text{tot}} \mid w]$ in the prior case without any data in equation (22) is also computed with MCS using the same 500 samples of $\boldsymbol{\theta}$ that are used in the SHM data-informed case.

We assume $\hat{c}_F = 10^6$, and for $\hat{c}_R$ we investigate different ratios $\hat{c}_R/\hat{c}_F = [10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}]$, and for each of those we calculate the VOI. The discount rate is $r = 2\%$.

The solution to the life-cycle optimization problem based on heuristics of equation (21) is performed with a standard optimization algorithm. Tables 2 and 3 summarize the results of this optimization and Table 4 documents the VOI for each cost ratio case.

As observed in Table 4, the VOI is 0 in the case when the costs have a ratio $\hat{c}_R/\hat{c}_F = 10^{-1}$, which means that one does not get any benefit from the data we obtain from our SHM system. This is related to the fact that, for this cost ratio, the optimal action is to not perform a repair in the lifespan of the bridge, in both prior and monitoring cases. However, for the further considered cost ratio cases, the VOI is positive, which indicates a potential benefit of installing an SHM system on the deteriorating bridge structure.

It is important to note that the expected costs are computed with a limited number of Monte Carlo samples and monitoring histories (500), thus resulting in significant uncertainty in the estimate of the expected costs. It is observed that for small $\boldsymbol{\theta}$, the variability in the estimation of the expected costs is high, which however is equally present in both prior and SHM data-informed cases (same samples $\boldsymbol{\theta}$). This still leads to high variability in the values that the VOI assumes.

Table 2: Life-cycle optimization using 500 samples in the prior case.

$\hat{c}_R/\hat{c}_F$	w_0^*	$\mathbf{E}[C_{\text{tot}}\|w_0^*]$	t_{repair}
10^{-1}	$\geq 8 \times 10^{-5}$	178.21	no repair
10^{-2}	$\geq 8 \times 10^{-5}$	178.21	no repair
10^{-3}	$\geq 8 \times 10^{-5}$	178.21	no repair
10^{-4}	2.1×10^{-6}	81.23	year 31

Table 3: Life-cycle optimization using 500 samples and monitoring histories in the SHM data-informed case. t_{repair} is varying for each sample and monitoring history.

$\hat{c}_R/\hat{c}_F$	w_{mon}^*	$\mathbf{E}[C_{\text{tot}}\|w_{mon}^*]$
10^{-1}	$\geq 1.95 \times 10^{-2}$	178.21
10^{-2}	5×10^{-4}	95.34
10^{-3}	9.9×10^{-5}	57.88
10^{-4}	8.6×10^{-6}	45.86

Table 4: Value of information

$\hat{c}_R/\hat{c}_F$	$\mathbf{E}[C_{\text{tot}}\|w_0^*]$	$\mathbf{E}[C_{\text{tot}}\|w_{mon}^*]$	VOI
10^{-1}	178.21	178.21	0.00
10^{-2}	178.21	95.34	82.87
10^{-3}	178.21	57.88	120.33
10^{-4}	81.23	45.86	35.37

7 CONCLUSIONS

In this paper, we present an investigation on optimal decision support with SHM and on quantifying the VOI of SHM, with the use of a simple benchmark bridge-type deteriorating structure. We simulate sequential monitoring acceleration data, which would be generated by an SHM system in a realistic scenario, and input them in an operational modal analysis, to identify the modal characteristics of the system. On the basis of this sequential modal data, we employ Bayesian model updating methods for system identification. The stochastic decision problem for life-cycle optimization and computation of the VOI is solved with the use of a simple heuristic. For this simplified hypothetical example, and for the particular choices we make, we find that the VOI assumes positive values for some of the cost ratios, thus quantifying the benefit of the SHM system in taking optimal maintenance actions in evidence of deterioration. The ideas presented here serve as a first step towards the quantification of the VOI of SHM for more realistic applications with fewer simplifying assumptions.

ACKNOWLEDGEMENTS

The support of the TUM Institute for Advanced Study through the Hans Fischer Fellowship is gratefully acknowledged. We thank Konstantinos Tatsis for his help in the operational modal analysis.

REFERENCES

Behmanesh, I. & B. Moaveni (2015). Probabilistic identification of simulated damage on the Dowling Hall footbridge through Bayesian finite element model updating. *Structural Control and Health Monitoring 22*, 463–483.

Behmanesh, I., B. Moaveni, G. Lombaert, & C. Papadimitriou (2015). Hierarchical Bayesian model updating for structural identification. *Mechanical Systems and Signal Processing 64-65*, 360–376.

Bismut, E. & D. Straub (2020). Optimal adaptive inspection and maintenance planning for deteriorating structural systems. *Reliability Engineering and System Safety*, under review.

Haario, H., M. Laine, A. Mira, & E. Saksman (2006). DRAM: Efficient adaptive MCMC. *Statistics and Computing 16*, 339–354.

Jensen, F. & T. Nielsen (2007). *Bayesian Networks and Decision Graphs, 2nd Edition*. New York: Springer.

Luque, J. & D. Straub (2019). Risk-based optimal inspection strategies for structural systems using dynamic Bayesian networks. *Structural Safety 76*, 68–80.

Papadimitriou, C., J. Beck, & L. Katafygiotis (2001). Updating robust reliability using structural test data. *Probabilistic Engineering Mechanics 16(2)*, 103–113.

Peeters, B. & G. De Roeck (1999). Reference-based stochastic subspace identification for output-only modal analysis. *Mechanical Systems and Signal Processing 13(6)*, 855–878.

Pozzi, M. & A. Der Kiureghian (2011). Assessing the Value of Information for long-term structural health monitoring. In *SPIE Conference on Health Monitoring of Structural and Biological Systems*, San Diego, California, USA.

Raiffa, H. & R. Schlaifer (1961). *Applied statistical decision theory.* Harvard University, Boston: Division of Research, Graduate School of Business Administration.

Simoen, E., G. De Roeck, & G. Lombaert (2015). Dealing with uncertainty in model updating for damage assessment: a review. *Mechanical Systems and Signal Processing 56*, 123–149.

Straub, D. (2014). Value of information analysis with structural reliability methods. *Structural Safety 49*, 68–80.

Straub, D., E. Chatzi, E. Bismut, et al. (2017). Value of information: A roadmap to quantifying the benefit of structural health monitoring. In *ICOSSAR – 12th International Conference on Structural Safety*, Vienna, Austria.

Straub, D., R. Schneider, E. Bismut, & H. Kim (2020). Reliability analysis of deteriorating structural systems. *Structural Safety 82*.

Tatsis, K. & E. Chatzi (2019). A numerical benchmark for system identification under operational and environmental variability. In *8th International Operational Modal Analysis Conference (IOMAC 19)*, Copenhagen, Denmark.

Thöns, S., R. Schneider, & M. Faber (2015). Quantification of the value of structural health monitoring information for fatigue deteriorating structural systems. In *Proceedings of the 12th International Conference on Applications of Statistics and Probability in Civil Engineering*, Vancouver, Canada.

Vanik, M., J. Beck, & S. Au (2000). Bayesian probabilistic approach to structural health monitoring. *Journal of Engineering Mechanics 126(7)*, 738–745.

Wirgin, A. (2004). The inverse crime. *arXiv e-prints*, math–ph/0401050.

Zonta, D., B. Glisic, & S. Adriaenssens (2014). Value of information: impact of monitoring on decision making. *Structural Control and Health Monitoring 21(7)*, 1043–1056.