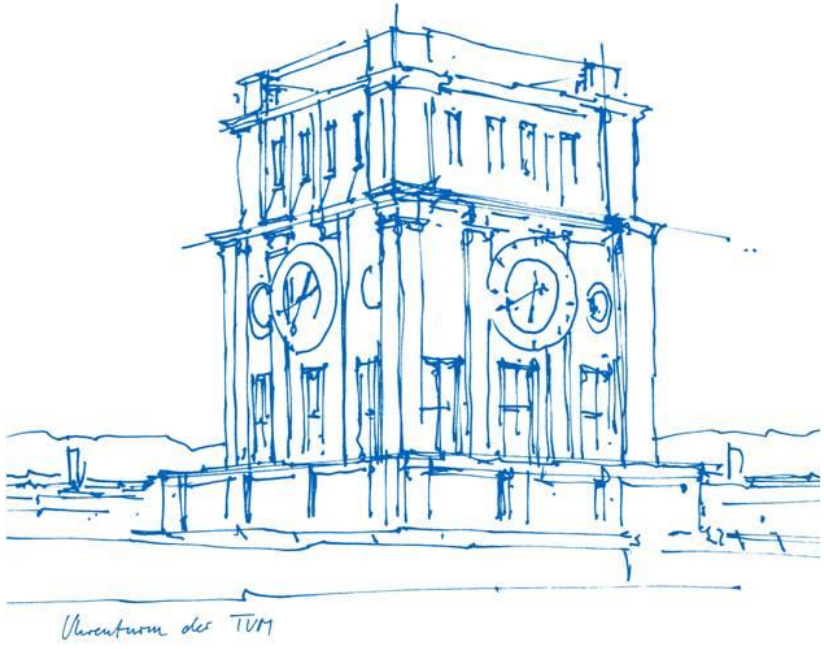
Unitary quantum process tomography by time-delayed measurements

Irene Lopez Gutierrez, Felix Dietrich and Christian B. Mendl

Technische Universität München
Faculty of Informatics
Chair of Scientific Computing
Garching, 16.03.2021



Our goal:

Given → $|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$

Our goal:

Given → $|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$

Find → H

Our goal:

Given ⟶ $|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$

Find ⟶ H

Direct approach is expensive in terms of required measurements:

$$2^{2N}(4^N - 1)$$ [1]

[1] M. NIELSEN, I. CHUANG. *Quantum Computation and Quantum Information.* (2000) p. 389

Our goal:

Given → $|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$

Find → H

Direct approach is expensive in terms of required measurements:

$$2^{2N}(4^N - 1) \quad [1]$$

Can we do better?

[1] M. NIELSEN, I. CHUANG. *Quantum Computation and Quantum Information.* (2000) p. 389

Answer: Yes. Use Takens' theorem

Given: $\mathcal{M} \subset \mathbb{R}^d$

Theorem: ***Generic delay embeddings*** *For pairs* (ϕ, y), $\phi : \mathcal{M} \to \mathcal{M}$ *a smooth diffeomorphism and* $y : \mathcal{M} \to \mathbb{R}$ *a smooth function, it is a generic property that the map* $\Phi_{(\phi,y)} : \mathcal{M} \to \mathbb{R}^{2d+1}$, *defined by*

$$\Phi_{(\phi,y)}(x) = \Big(y(x), y(\phi(x)), \ldots, y(\underbrace{\phi \circ \cdots \circ \phi}_{2d\ times}(x))\Big)$$

is an embedding of $\mathcal{M}$*; here, "smooth" means at least* C^2.

In our case:

$$x \longrightarrow e^{-iH}$$

$$\phi \longrightarrow \phi(x) = x^t = e^{-iHt}$$

$$y \longrightarrow y(U) = \langle\psi|U^\dagger M U|\psi\rangle$$

Answer: Yes. Use Takens' theorem

Given: $\mathcal{M} \subset \mathbb{R}^d$

Theorem: ***Generic delay embeddings*** *For pairs (ϕ, y), $\phi : \mathcal{M} \to \mathcal{M}$ a smooth diffeomorphism and $y : \mathcal{M} \to \mathbb{R}$ a smooth function, it is a generic property that the map $\Phi_{(\phi,y)} : \mathcal{M} \to \mathbb{R}^{2d+1}$, defined by*

$$\Phi_{(\phi,y)}(x) = \Big(y(x), y(\phi(x)), \ldots, y(\underbrace{\phi \circ \cdots \circ \phi}_{2d \ times}(x))\Big)$$

is an embedding of $\mathcal{M}$; here, "smooth" means at least C^2.

In our case:

$$x \longrightarrow e^{-iH}$$

$$\phi \longrightarrow \phi(x) = x^t = e^{-iHt}$$

$$y \longrightarrow y(U) = \langle\psi|U^\dagger M U|\psi\rangle$$

Procedure

1. Start with known wavefunction (or density matrix)

Procedure

1. Start with known wavefunction (or density matrix)
2. Proceed with time evolution up to time Δt

Procedure

1. Start with known wavefunction (or density matrix)
2. Proceed with time evolution up to time Δt
3. Sample measurements to approximate $\langle\psi|U^\dagger M U|\psi\rangle$

Procedure

1. Start with known wavefunction (or density matrix)

2. Proceed with time evolution up to time Δt

3. Sample measurements to approximate $\langle\psi|U^\dagger MU|\psi\rangle$

4. Repeat 2-3 *2d* times for different times to create training data set

Procedure

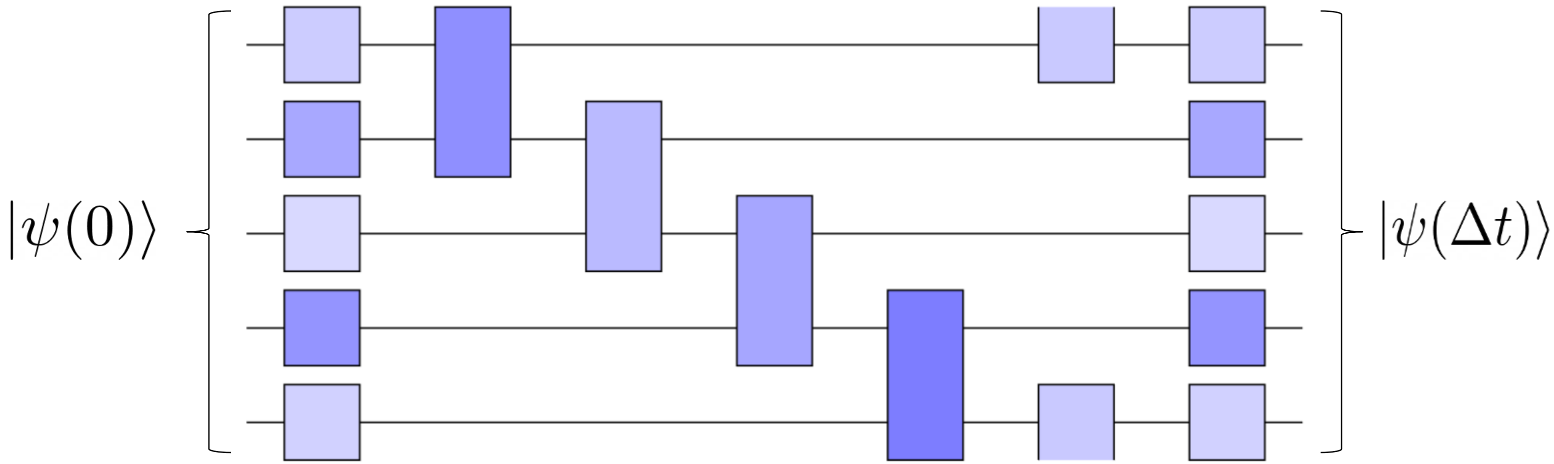
1. Start with known wavefunction (or density matrix)
2. Proceed with time evolution up to time Δt
3. Sample measurements to approximate $\langle\psi|U^\dagger M U|\psi\rangle$
4. Repeat 2-3 *2d* times for different times to create training data set
5. Optimize a variational model to find $U \in \mathcal{M}$

Results

We test the procedure on the Ising Model

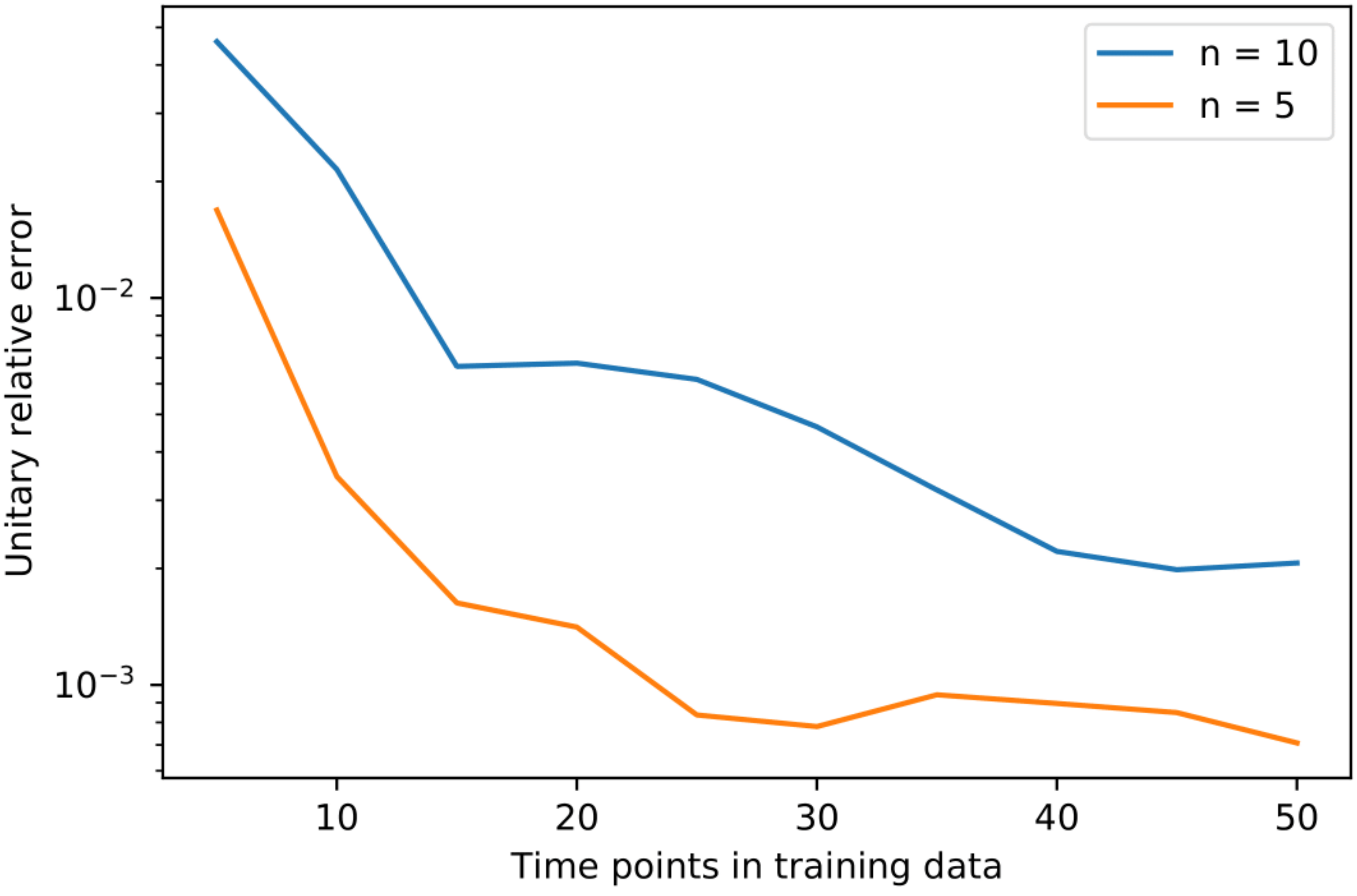
$$H = -J \sum_{\langle i,j \rangle} \sigma_i^z \sigma_j^z - h \sum_i \sigma_i^x$$

We use a circuit as Ansatz



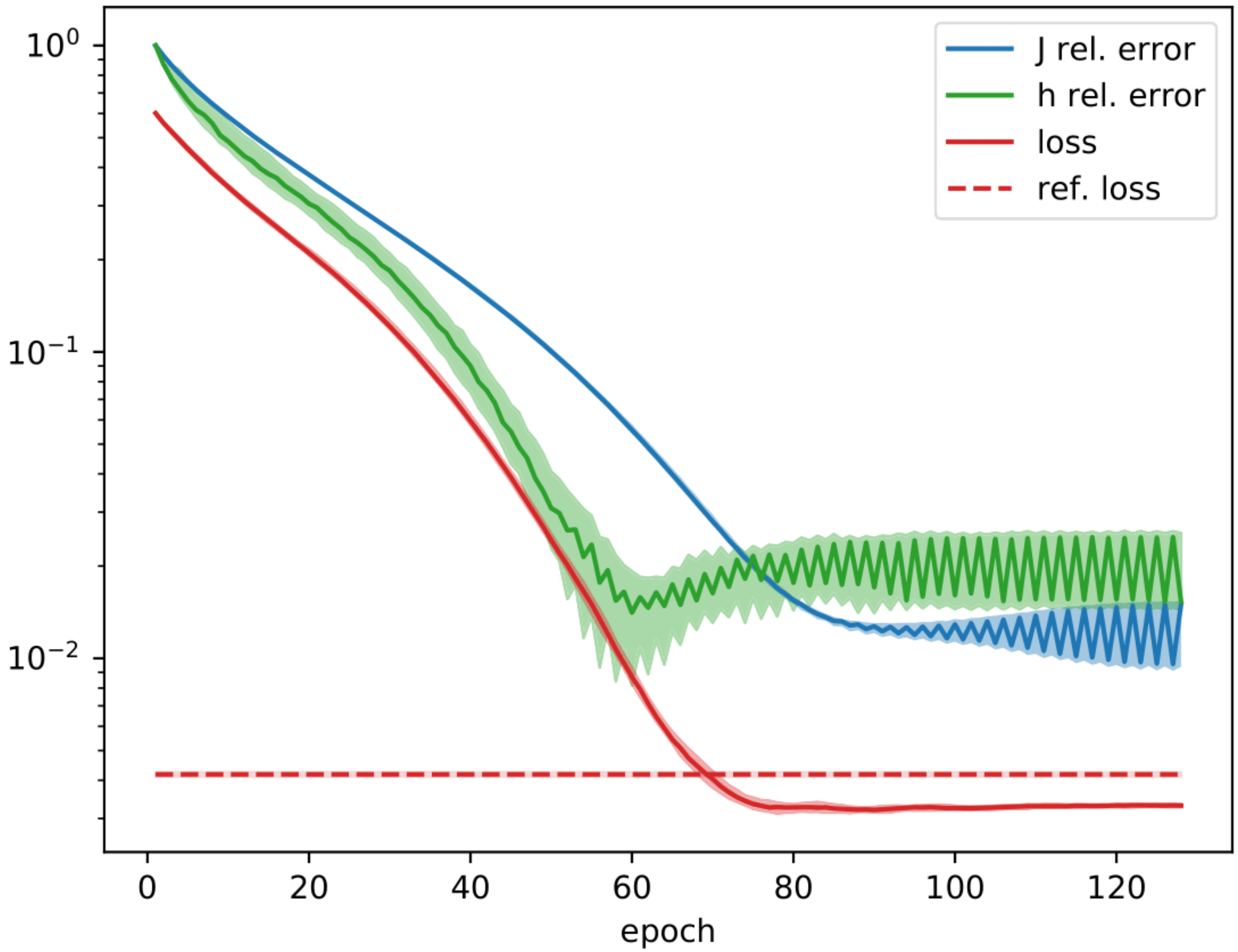
Results

1D system - Learning full gates



Results

2D system (3x4) - Learning J and h



Any questions?

Relevant literature:

D. RUELLE AND F.TAKENS, *On the nature of turbulence*, Commun. Math. Phys., 20 (1971), pp. 167-192

F. TAKENS, *Detecting strange attractors in turbulence*, Lecture Notes in Mathematics, (1981), pp. 366-381

H. WHITNEY, *Differentiable manifolds*, The Annals of Mathematics, 37 (1936), p. 645