Developing Dynamic Pricing Methods for Ride-sharing Services

Master's Thesis

A thesis presented in part fulfilment of the requirements of the Degree of Master of Science at the Department of Civil, Geo and Environmental Engineering, Technical University of Munich.

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I hereby confirm that this master's thesis was written independently by myself without the use of any sources beyond those cited, and all passages and ideas taken from other sources are cited accordingly.

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Abstract

Ride-sharing has become ubiquitous in many metropolises attributed to its affordability, convenience and flexibility. This thesis will develop dynamic pricing methods for ride-sharing services to improve its competitiveness in multi-modal transportation systems.

A market equilibrium model for ride-sharing services with a consideration of passenger preference in a multi-modal transportation system is built at the network level where network structure and origin-destination (OD) demand pattern are explicitly counted. Moreover, the method to calculate the system endogenous variables (e.g., ride-sharing demand, expected waiting time, and expected detour time) in the equilibrium is also deduced. A stated-preference survey data regarding the mode choice within private car, public transport and ride-sharing services are utilized to estimate the passenger preference. To reveal the operation difference of ride-sharing in different scale networks, a handmade network and the Munich network are adopted in the experiments.

We propose three different pricing strategies: 1) a unified pricing method (trip fare is a function of travel distance with the same unit price for all OD pairs over the network); 2) a spatial pricing method accounting for the spatial heterogeneity of the level of demand over the network by applying different unit prices for different OD pairs; 3) a utility-based compensation method compensating passengers based on their travel experiences to reduce variance/uncertainty for trip level-of-services (LOS) and add equity with or without (limited) sacrifice of the operation objectives. Gradient Descent (GD) algorithms are derived to optimize the operation strategy (trip fare and vehicle fleet size) for the monopoly optimum (MO) scenario and social optimum (SO) scenario for method 1 and 2, respectively. For method 3, a heuristic particle swarm algorithm (PSO) is applied. The results show that, in method 1, the optimal unit price for MO is greater than that for SO, while the optimal vehicle fleet size is smaller. And the difference between optimal vehicle fleet sizes in two scenarios becomes greater in the high demand level situation, while the difference between optimal unit prices almost keeps the same. In method 2, it is found that the optimal unit prices are linear to their distance and ride-sharing demand with negative slopes. Last but not least, the results illustrate that method 3 is effective to improve the LOS and equity without losses of profit or surplus as expected.

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Acronyms

CBD central business district. 10

CPT cumulative prospect theory. 9

CRF compensation reference factor. 49

CRP compensation reference point. 49

GD Gradient Descent. 25

IIA independence for irrelevant alternatives. 29

LOS level of service. 9

MNL Multinomial Logit. 9

MO monopoly optimum. 31

MooD mobility-on-demand. 24

OD origin-destination. 1

PSO Particle Swarm Optimization. 52

SAEV shared autonomous electric vehicles. 9

SMoDs shared mobility on demand services. 9

SO social optimum. 31

SUMO Simulation of Urban MObility. 21

TAZ traffic analysis zone. 3

TNC transportation network companies. 1

WGC wild goose chase. 7

1. Introduction

1.1. Motivation

Urban sprawl and the increase of urban population raise a higher requirement in the convenience and speed of people commuting and goods transportation. The urban transportation system is constantly updated and completed to satisfy the requirement. To date, many cities have constructed a multi-modal transport network supplemented with substantial transfers between different modes to enhance the communication between different regions. Nevertheless, private vehicle and public transport (including bus, metro, etc.) are still two dominant participants in the transportation system.

In addition, the taxi industry plays a more and more important role in people's movement. However, the taxi service is still unaffordable for many people. The most basic reason is that, most of the time, taxi only serves one passenger on a trip. Whereas, if there are more than one passenger (except the driver) on a trip and the bill is shared, passengers can finish their trips with much less fare compared to taking taxi separately. Substantially, this is the working mechanism behind ride-sharing (also known as ride-pooling, ride-sourcing, carpooling, etc.).

It must be noted that, ride-sharing is not a modern concept emerged in these years. Actually, it can be traced back to as early as World War II, when a Car-Sharing Club is established for fuel conservation in the US (Ferguson, 1997). However, the rapid development of mobile internet technologies and smartphones in the last decade has thoroughly facilitated the ubiquity of ride-sharing services. And many influential transportation network companies (TNCs) have launched their own ride-sharing services, such as UberPool, and DiDi ExpressPool. People can access the service via a dedicating function on the smartphone application of the corresponding platform. After being informed origin-destination (OD) of the trip in advance, the platform will match a vehicle and potential passengers with similar itineraries and time schedules for users, and the trip expenses will be shared by passengers.

It is widely envisioned that ride-sharing services will be very beneficial to individuals and the whole society in general. First of all, it is more convenient and flexible than public transport, and more economical than driving a car or hailing a taxi. It completes the urban transportation system, and enriches transport choices. Since it increases the number of passengers per vehicle trip, effective operation of ride-sharing services can reduce the number of vehicles running on the network to some extent, and therefore mitigate traffic congestion. Likewise, it can contribute to conserving fuel consumption, reducing emission pollution and other negative external effects.

Despite the potential benefits, to date, the understanding of ride-sharing services is still under exploration. The main challenges in ride-sharing are specifically summarized as follows.

1) The ride-share matching algorithm provides a matching solution for potential drivers and riders by optimizing an established objective, such as minimizing total system-wide vehicle-kilometers, or maximizing the total travel distance savings. The matching algorithm has a decisive influence on the operation of ride-sharing. Thus, it is essential to design an effective and stable matching algorithm that can maximize system performance.

2) After drivers and riders have been matched up, the platform needs to recommend an optimal route for each vehicle based on a predefined objective (e.g., minimizing distance, or minimizing travel time) via a routing algorithm. It is always desirable to create a more realistic and comprehensive routing algorithm with more accurate demand (for prepositioning) and traffic estimation algorithms.

3) For the competitiveness of ride-sharing services, it is necessary to implement appropriate pricing strategies, especially in a multi-modal transportation system. By applying a specific pricing method, the service provider wants to optimize the operational objective, such as maximizing profit or maximizing social welfare. This requires a better understanding of passenger preference and the interdependence of demand and supply.

The solution to these challenges still remains as open research questions. This thesis will have some explorations in the dynamic pricing methods with specific consideration of the characteristics of ride-sharing services.

1.2. Goal and Scope

1.2.1. Research goals

The primary objective of this thesis is to develop effective pricing methods for ride-sharing services. Specific objectives are listed below.

1) Construct an equilibrium model for a ride-sharing market in the context of a multi-modal transportation system.

2) Understand the impact of passenger preference on ride-sharing services operation.

3) Develop appropriate dynamic pricing methods in different operation contexts.

The outcome of this thesis will provide scientific reference and methodological support for the design of pricing methods for ride-sharing services.

1.2.2. Research scope

1) We model the ride-sharing market at the network level (i.e., OD-based). Trips attributes (distance, travel time, and trip fare) are aggregated based on their OD. In other words,

every traffic analysis zone (TAZ) is modeled as a mass point. Individual trip modeling is out of the scope of this thesis.

2) Service provider could be the service operator or a matching agency only in the ride-sharing operation. This thesis only considers the situation that the service provider is working as a service operator.

3) Detailed operation process (i.e., particular matching and routing algorithm) is not in the scope of this thesis.

4) We do not consider the impact of ride-sharing on traffic congestion in a single interval. In other words, the traffic situation of the network does not vary within a single studying interval.

1.3. Contributions

The major contributions of this thesis are three-fold.

1) This thesis is among the first to construct a ride-sharing market equilibrium considering the passenger preference in the multi-modal context. We also provide an approximation method to estimate the equilibrium model parameters with the service attributes (average detour time, average waiting time, vehicle fleet size, and network attributes) in actual operations of a market as prior knowledge, such that the equilibrium model can be used to model the corresponding market.

2) Develop different dynamic pricing methods including unified pricing, spatial pricing, and utility-based compensation method for ride-sharing services and derive applicable solution algorithms for each pricing method, respectively.

3) This thesis also explicitly explore the value of considering user heterogeneity on modeling the ride-sharing market, and its impact on the optimal solution for different pricing methods. This can provide a recommendation for operation regions and target user groups for the service provider.

More detailed contributions will be described in each chapter.

The pricing method is a very important component of the ride-sharing system. A suitable pricing method not only can improve the attractiveness of the service, but also can distribute the potential benefit to every user. To date, the main pricing methods applied in the market still have wide space for improvement. Thus our work is very timely, necessary and meaningful.

1.4. Thesis Structure

The overall methodologies adopted in this study and the corresponding findings are presented in the remainder of this thesis. Chapter 2 provides a comprehensive review of the most important and instructive works on the ride-sharing equilibrium model and dynamic pricing methods. Chapter 3 constructs the equilibrium in the ride-sharing market. Chapter 4 describes the stated-preference data used to model the passenger preference and the networks used to evaluate the proposed equilibrium model and pricing methods. Chapter 5 and Chapter 6 proposes the solution algorithms for the unified pricing method and spatial pricing method with respect to different objectives, respectively. Chapter 7 develops a novel utility-based compensation method considering the distinctive characteristics of ride-sharing service and provides the corresponding solution algorithms. Chapter 8 concludes the thesis and discusses future open questions. The overall structure of this thesis is illustrated in Figure 1.1. Notice that we maintain the same notation over the thesis and a list of the important variables is given in Table 1.1 for the reader's convenience. And these variables are defined in one single studying interval, which is one hour in this thesis.

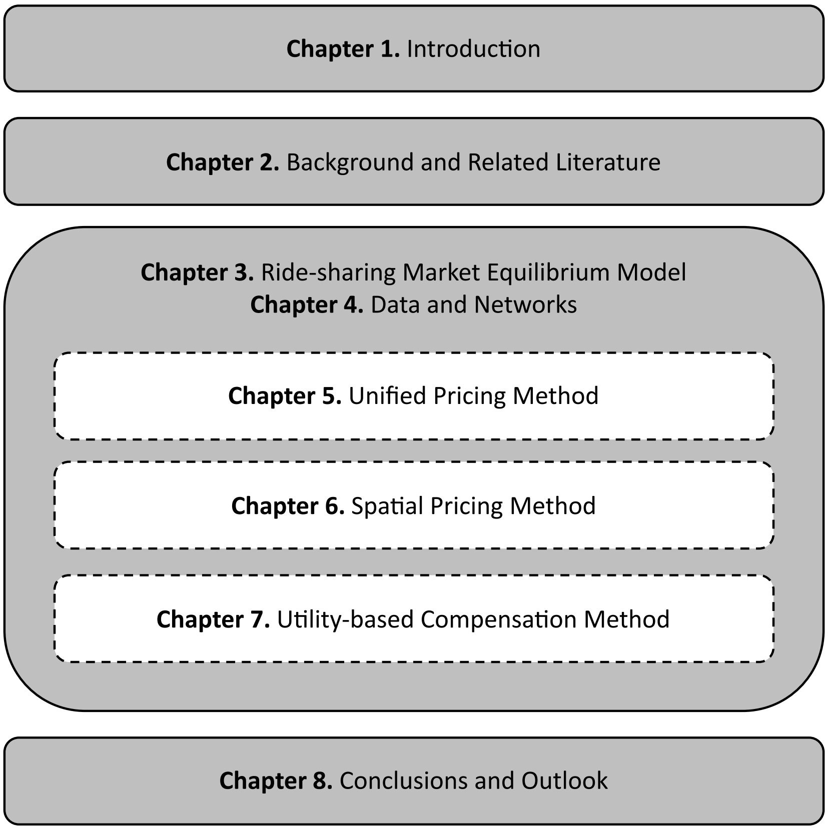


Figure 1.1.: Structure of the thesis.

Table 1.1.: Notations for the important variables.

Variable	Definition
	Network related variables
$\mathbb{Z}$	Set of OD pairs
i	OD pair i in $\mathbb{Z}$
i^o	Origin of OD pair i
i^d	Destination of OD pair i
D_i	Travel demand from i^o to i^d
d_i	Expected distance from i^o to i^d
	Service related variables
$P_{i,rs}$	Mode share of ride-sharing services for i
Q_i	Ride-sharing demand from i^o to i^d
N	Vehicle fleet size
n_s	Number of seats within one vehicle
H_v	Number of vacant seats
t_i	Expected travel time from i^o to i^d by ride-sharing
w_i	Expected waiting time for trips from i^o to i^d by ride-sharing
r_i	Expected trips fare for trips from i^o to i^d by ride-sharing
t_i^d	Expected direct trip time from i^o to i^d by ride-sharing
$\tilde{t}_i$	Expected detour time from i^o to i^d by ride-sharing
β_t	Preference coefficients with respect to travel time
β_w	Preference coefficients with respect to waiting time
β_r	Preference coefficients with respect to travel cost
$\tilde{t}^{(2)}$	Expected detour time between two passengers
$\tilde{t}^{(v)}$	The expected detour time of a vehicle
n_t	The maximum number of trips a vehicle can serve
n_a	The number of trips assigned to a vehicle
A	A positive exogenous parameter
B	A positive exogenous parameter
ϕ	Operating cost of a vehicle in one hour
Π	Total profit for ride-sharing
S	Social surplus (welfare) for ride-sharing
	Pricing methods related variables
p	Unit price for all trips (unified pricing)
p_i	Unit price for trips from i^o to i^d (spatial pricing)
a	Compensation reference point (compensation method)
α	Compensation reference factor (compensation method)
$r_{i,k}^a$	Utility for trip k of OD i after compensation (compensation method)
$c_{i,k}$	Amount of compensation for trip k of OD pair i (compensation method)

2. Background and Related Literature

In this chapter, we define basic concepts on ride-sharing services, present a comprehensive literature review, and identify the research gaps. Section 2.1 defines the basic concepts and assumptions that will be used throughout the thesis. Section 2.2 reviews the related literature on market equilibrium model and dynamic pricing methods. Section 2.3 summarizes the research gaps.

2.1. Basic Concepts and Assumptions

Trips are defined as instances of travel from one geographic location to another (Dailey et al., 1999). And each trip has its corresponding origin and destination.

In this thesis, ride-sharing refers to the service that potential passengers announced their origin and destination of the trips on a smartphone application in advance, and then the application program will inform them of the pick-up time if their requests have been matched up with any ride-sharing vehicles. As the name implies, by ride-sharing, it means individual riders with similar itineraries and time schedules share a vehicle for a trip and split the travel expenses.

Basically, platforms (service providers) can be categorized into two groups based on their functions in the operation. One is that the service provider is also the service operator, namely, it operates ride-sharing services with own vehicles and hired drivers. In this case, the service provider intrinsically works as a taxi company, but provides ride-sharing services. Another group is that the service provider is only a matching agency. In this case, the service provider only matches up individual drivers and potential passengers and has no capability to decide the vehicle fleet size and service scale. In this thesis, we only focus on the former situation and design dynamic pricing methods for such platforms.

We assume all riders make the choice objectively based on the perceived utilities of all transport modes of interest.

Besides, all potential requests are assumed to be matched and served, i.e., the successful pairing rate is set to 1.

2.2. Related Literature

Over the last decade, ride-sharing services have attracted enormous research interest, with a particular focus on one of the main challenges concluded in Section 1.1, i.e., matching algorithm (e.g., Agatz et al., 2011; Wang et al., 2018), routing algorithm (e.g., Ho et al., 2018; Li et al., 2019a), and dynamic pricing method (e.g., Sayarshad and Chow, 2015; Ke et al., 2020a). In this section, we focus on the service operator side and conclude related works on dynamic pricing methods in the ride-sharing services market. Furthermore, we also summarize the-state-of-the-art in market equilibrium model to provide references for our following equilibrium modeling.

2.2.1. Market equilibrium model

The prevalence of ride-sharing services in recent years attracts much attention from both academic and industry. Thus, it is urgent and desirable to formulate the equilibrium for the ride-sharing market. This section summarizes the works on the equilibrium model for the taxi market which has been thoroughly investigated in the literature. In some sense, due to their implicit resemblance, the exemplary works on the taxi market equilibrium can shed light on the research of equilibrium in the ride-sharing market. Some have extended them to the ride-sharing market with strict proofs for translating the discrepancies.

Cairns and Liston-Heyes (1996) developed an equilibrium model for the taxi market to understand the competition in the industry. It found that the unregulated industry does not satisfy the conditions of competition, and the existence of equilibrium depends on the regulation of price, entry, and intensity of use of licensed taxis. Besides, it also presented the models of monopoly, the social optimum and the second-best in the taxi industry. However, it did not consider the spatial difference in demand patterns. An initial attempt to model the taxi market at a network level considering the OD demand pattern was in Yang and Wong (1998). They constantly improved this model in a series of works by further incorporating demand elasticity and congestion effect (Wong et al., 2001), exploring the impacts of regulatory restraints on the equilibrium in regulated, competitive and monopoly markets (Yang et al., 2002). Furthermore, the improved equilibrium model was also applied to investigate the performance of nonlinear fare structures on perceived profitability. It was proposed to protect operators' businesses from the illegal phenomenon (e.g., discounts on metered fare) under a front-loaded flag-fall charging strategy (Yang et al., 2010a).

Adopting the modeling framework of previous works on the taxi industry, Ke et al. (2020a) presented an equilibrium model for ride-sharing markets and tried to elucidate the complex relationships between system endogenous variables and decision variables (trip fare, vehicle fleet size, and allowable detour time). It proved that the monopoly optimum, first-best and second-best social optimum are always in a normal regime instead of the wild goose chase (WGC) regime[1]. However, they restricted the problem in the situation with two passengers sharing a trip at most. In addition, the market was modeled as a whole in this study without considering the network structure and OD demand patterns.

Contrary to Ke et al. (2020a) where the service provider is the service operator, Bimpikis et al. (2019) formulated the equilibrium state for a matching agency. It pointed out that only

[1] An inefficient equilibrium where vehicles take substantial time to pick up riders. We refer the interested readers to Ke et al. (2020a) for the details of its definition and the corresponding analysis

when the demand pattern[2] across the network is balanced the benefit of applying spatial price discrimination can be observed. Leveraging the spatial pricing method can facilitate the pattern of the served demand becomes more balanced. The result of numerical experiments implied that total profit and consumers' surplus are maximized at the equilibrium with the optimal pricing policy when the demand pattern of network is balanced. However, in this study, the supply of potential drivers was assumed to be infinite and the relocation of drivers was assumed to be instantaneous which is overly idealistic and does not conform to reality.

Although the models developed in the aforementioned works perform well, there is still room for improvement. For example, none of them consider passenger preference in the presence of multiple transport modes. The value of time (or willingness to pay) of passengers is the only factor to be considered in their modeling framework regardless of how superior the service attributes of other transport options are. This will result in an inaccurate demand estimation when the service attributes of ride-sharing become incomparable with that of one of the other transport modes. The main literature on market equilibrium model is listed in Table 2.1.

Table 2.1.: Main literature on market equilibrium model.

Paper	Market	OD	MM	UH	Scenarios
Cairns and Liston-Heyes (1996)	Taxi				MO, SO, SSO
Yang and Wong (1998)	Taxi	+			
Yang et al. (2000)	Taxi	+			
Wong et al. (2001)	Taxi	+			
Yang et al. (2002)	Taxi	+			MO, CO, SO, SSO
Yang et al. (2010a)	Taxi	+			
Yang et al. (2010b)	Taxi	+			
Bimpikis et al. (2019)	MA	+		+	MO
Li et al. (2019b)	Ride-hailing				MO
Ke et al. (2020a)	Ride-sharing				MO, SO, SSO

Note: OD, OD-based; MM, multi-modal system; UH, user heterogeneity; MO, monopoly optimum; SO, social optimum; SSO, second-best social optimum; CO, competition optimum; MA, matching agency; '+' means the factor is considered.

2.2.2. Dynamic pricing method

Demand estimation is a main focus of all dynamic pricing strategies for ride-sharing services. Some aim to capture the temporal elasticity of demand to provide optimal solutions for a specific objective (e.g., profit maximization) (Sayarshad and Chow, 2015; Qian and Ukkusuri, 2017). Some try to improve the reliability of the proposed solution by considering the spatial

[2] Demand pattern of a network is defined as a combination of a demand vector for zones and a weighted adjacency matrix. And it is said to be balanced if, at each zone, the potential demand for rides weakly exceeds the available drivers in the same zone after completing rides.

heterogeneity of the level of demand over the network (Chen and Kockelman, 2016; Guo et al., 2017; Qiu et al., 2018; Bimpikis et al., 2019). Furthermore, the users heterogeneity which is represented by passenger preference/behavioral models is also an aspect that has been heavily researched in the literature (Chen and Kockelman, 2016; Qiu et al., 2018; Guan et al., 2019). Note that, some dynamic pricing methods are only applicable in specific operation context, e.g., in the monopoly (Qiu et al., 2018; Bimpikis et al., 2019), or duopoly (Sato and Sawaki, 2013). This section presents a short review of how these aspects are considered when developing the pricing method.

Sayarshad and Chow (2015) proposed a non-myopic pricing method for the non-myopic dynamic dial-a-ride problem to maximize social welfare under the assumption of elastic demand. It pointed out that ignoring the elasticity of demand can result in an overestimation of the improvement in level of service (LOS) with non-myopic considerations. Motivated by the demand elasticity among a day, Qian and Ukkusuri (2017) developed a time-of-day pricing scheme to maximize the profit for taxi service, where price multipliers are used to dynamically alter trip cost. It concluded that a strict pricing scheme should consider both temporal heterogeneity and spatial heterogeneity in demand, supply and traffic condition, together with additional consideration of users heterogeneity in price elasticity.

The Multinomial Logit (MNL) model was applied to estimate the mode share of shared autonomous electric vehicles (SAEV) in an agent-based framework in Chen and Kockelman (2016). It investigated the trade-offs between the revenue and mode share of SAEV under different pricing schemes including distance-based pricing, origin-based pricing, destination-based pricing, and combination pricing strategy. Guo et al. (2017) provided an elaborated demand analysis and dynamic pricing analysis of the ride-on-demand service provided by Shenzhou Ucar in Beijing, China. They adjusted the trip price dynamically by applying appropriate pricing multipliers for different regions based on the demand characteristics in both spatial and temporal dimensions. Knowing the passenger preference, demand distribution, and traffic information of the network, Qiu et al. (2018) proposed a dynamic programming framework to solve the profit maximization problem for a monopolistic private shared mobility-on-demand service operator. And the MNL model was used to model the passenger preference and was integrated into the price optimization model for the subproblem at the request level.

With the consideration of demand difference over the network, Bimpikis et al. (2019) established an infinite-horizon, discrete-time model for ride-sharing services, and explored the impact of the demand pattern on platform's prices, profits, and the induced consumer surplus. It is worth noting that user heterogeneity is also considered with explicit consideration of their willingness to pay for receiving service.

Furthermore, considering the uncertainty nature of travel time and waiting time in shared mobility on demand services (SMoDs), Guan et al. (2019) applied the cumulative prospect theory (CPT) to capture the subjective decision making of passengers under uncertainty. A dynamic pricing strategy was proposed on the passenger behavioral model based on CPT, which incorporates a dynamic routing algorithm and thus can provide a complete solution to SMoDs.

Note that, most of the aforementioned works rely on the assumption of homogeneous users in terms of the value of time (or willingness to pay). However, such an assumption cannot capture the spatial difference of functional areas which has been presented in Guo et al. (2017). For example, the user group in the educational area and user group in the central business district (CBD) are very different in terms of affordability. The main literature on dynamic pricing method is listed in Table 2.2.

Table 2.2.: Main literature on dynamic pricing method.

Paper	Market	OD	DE	UH	Scenarios
Lin (2006)			+		MO
Dong et al. (2009)	Retail				MO
Sato and Sawaki (2013)					CO
Pfrommer et al. (2014)	Bicycle	+			
Sayarshad and Chow (2015)	DARP		+		SO
Banerjee et al. (2015)	Ride-sharing				MO
Chen and Kockelman (2016)	SAEV	+			MO
Qian and Ukkusuri (2017)	Taxi		+		MO
Guo et al. (2017)	MoD			+	MO
Qiu et al. (2018)	MoD		+		MO
Turan et al. (2019)	AMoD	+	+		MO
Li et al. (2019b)	Ride-hailing				MO
Guan et al. (2019)	SMoDs			+	IQ
Yang et al. (2010b)	Ride-sharing	+		+	MO

Note: OD, OD-based; DE, demand elasticity; UH, user heterogeneity; MO, monopoly optimum; SO, social optimum; IQ, individual request; CO, competition optimum; DARP, dial-a-ride problem; SAEV, shared autonomous electric vehicle; MoD, mobility on demand; AMoD, autonomous MoD; SMoDs, shared MoD; '+' means the factor is considered.

2.3. Literature Gaps

Based on the literature review in Section 2.2, we focus on the following research gaps.

1) Few works have considered passenger preference in a multi-modal transport context when constructing the market equilibrium model for ride-sharing services.

2) None of the existing works has developed a spatial pricing method considering spatial difference and user heterogeneity for the ride-sharing service operator.

3) There is no work that has studied a pricing method integrated with compensation based on the level of services in the ride-sharing context.

3. Ride-sharing Market Equilibrium Model

3.1. Objectives and Contributions

The intertwined relationship of the supply of and demand for taxi services distinguishes it from the traditional economic markets. Moreover, the complex demand-availability-utilization-supply[1] relation makes the attempts to analyze the taxi market within an inherently static supply-demand analysis framework impossible (Manski and Wright, 1976; Yang et al., 2002). An equilibrium in taxi markets is thus described as the system state when the supply-demand interaction eventually damps out under certain regulated conditions (e.g., trip fare). Analogously, the market equilibrium in the ride-sharing market is the ultimate stable state of the market, at which the relationships between the system endogenous variables (e.g., passenger demand, average detour time) can be satisfied, under a specific operation strategy (e.g., vehicle fleet size, trip fare). Mathematically, the demand-supply equilibrium is established when both demand and supply equations are satisfied simultaneously. It is worth noting that, the equilibrium model of taxi market has been systematically investigated in the literature (e.g., Manski and Wright, 1976; Cairns and Liston-Heyes, 1996; Wong et al., 2001; Yang et al., 2002; Yang et al., 2010b), while as described in Section 2.3, only a few works focus on the ride-sharing market since it just began attracting the attention of both academic and industry in recent years. On the other hand, to the best of our knowledge, there are no works that account for the passenger preference in a multi-modal transport network in the modeling framework. Furthermore, an effective equilibrium model at the network level for ride-sharing services is still an open question.

To address these research gaps, in this chapter, we propose a market equilibrium model for ride-sharing services in the multi-modal transport context by applying the Multinomial Logit (MNL) model to present the passenger preference. This equilibrium model will serve as the experiment bed for the pricing methods introduced in Chapter 5, Chapter 6, and Chapter 7. In particular, we will make the following contributions.

1) We explicitly construct the equilibrium model of ride-sharing markets with particular consideration of the passenger preference in the presence of multiple transport choices.

2) Instead of modeling the market as a whole, we attempt to understand the market at the network level (i.e., OD-based) by considering the specific network structure (i.e., OD demand pattern).

[1] Taxi availability (measured by expected waiting time) indirectly affects the taxi utilization (measured by the expected fraction of time a taxi is occupied) through its influence on taxi demand; taxi utilization, in turn, affects the taxi availability through its influence on the level of supply. Refer to Figure 1. in Manski and Wright (1976) for the details.

3) We improve the widely adopted assumption of the average waiting time in the existing literature by replacing its determinant from vehicle availability to seat availability.

This chapter is organized as follows. The methodology workflow is shown in Section 5.2. Section 3.3 and Section 3.4 describes how to model the supply of and demand for ride-sharing services in this study. Section 3.5 presents the formation of the equilibrium model. Section 3.6 and Section 3.7 presents the detailed model of the expected detour time and expected waiting time.

3.2. General Methodology

Figure 3.1 depicts the intertwined relationships between the variables in the ride-sharing market. The utilized pricing method decides the trip fare for ride-sharing services, while trip fare and the attributes of other transport modes, such as the average waiting time, average detour time and trip fare for public transport, serve as inputs for the passenger preference module. The passenger preference is working as the media for the interplay between the ride-sharing passenger demand, the expected detour time and waiting time.

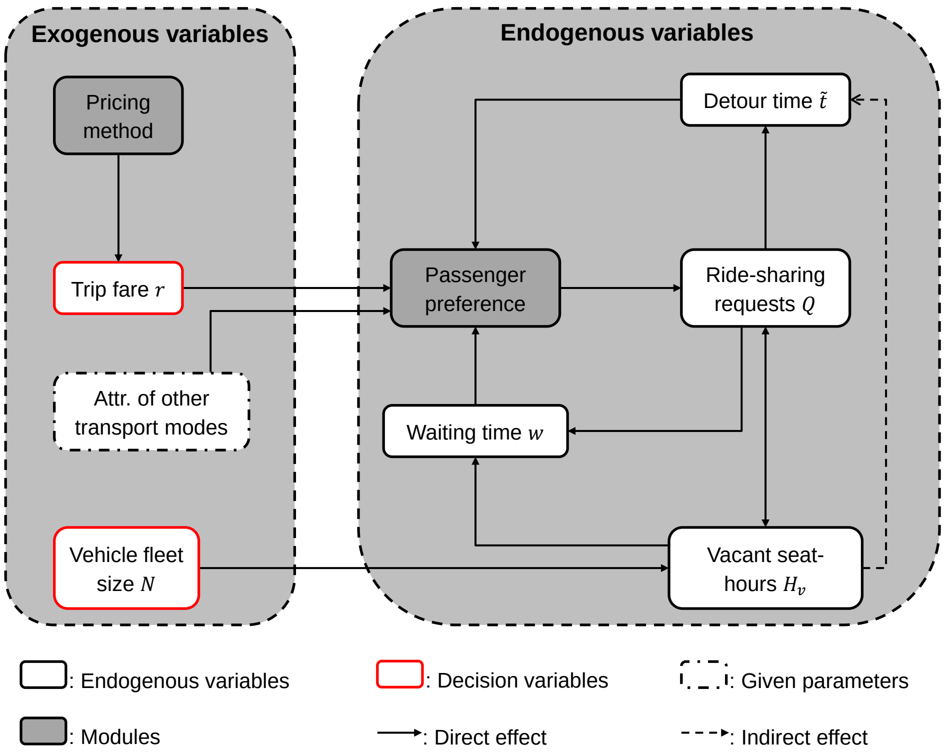


Figure 3.1.: Relationships between exogenous variables and endogenous variables in the ride-sharing market

Generally, the expected waiting time is deemed to be related to the number of available vehicles (Cairns and Liston-Heyes, 1996; Li et al., 2019b; Ke et al., 2020a). Considering the sharing nature of ride-sharing, we can modify this assumption as: expected waiting time is depending on the number of available seats. Note that, seat availability is decided by vehicle fleet size and ride-sharing demand. Moreover, due to the interdependence of the ride-sharing demand, waiting time, and detour time, seat availability also indirectly affects the expected detour time.

In the remainder of this chapter, how these variables interact with each other is interpreted in detail and the method to calculate the market equilibrium is presented. Concisely, market equilibrium means under a certain operation strategy of trip fare and vehicle fleet size, what the ultimate state of the market will be. In other words, given the exogenous variables, the values of system endogenous variables will be inherently derived by the market equilibrium model. Here we assume the service attributes of other transport modes are known.

Note that, the pricing methods proposed in Chapter 5, Chapter 6, and Chapter 7 are included in the pricing method module. Thus the equilibrium model of ride-sharing services presented in this chapter is actually the experimental bed for these pricing methods, which plays a critical role in the investigation of the effectiveness and benefits of them.

3.3. Supply of Ride-sharing Services

A trip is defined as the movement of an instance of travel from one geographic location to another (Dailey et al., 1999). Each trip has its own origin and destination. Different from the existing works based on abstract, simplified demand-supply models, in this study, we model the ride-sharing market at the network level with consideration of the network structure and traveler OD demand pattern.

Consider a network which allows the travel between OD pairs in set $\mathbb{Z}$. For an OD pair i in $\mathbb{Z}$, its origin and destination are denoted as i^o and i^d, respectively. For a given hour (the studying interval is set to one hour), the travel demand for i (i.e., the number of trips from i^o to i^d) is D_i. We denote $P_{i,rs}$ as the mode share of ride-sharing services of i under a certain operation strategy, where rs indicates ride-sharing. Then, the passenger demand for ride-sharing from i^o to i^d is estimated by

$$Q_i = D_i P_{i,rs} \tag{3.1}$$

where Q_i is the ride-sharing passenger demand for i.

We assume that each seat in ride-sharing vehicles can be either vacant or occupied. And the summation of vacant seats and occupied seats equals to the total number of seats of all vehicles. We define available seat capacity H_v as the number of vacant seats in stationary equilibrium, while utilized seat capacity H_c as the number of occupied seats. For a given hour, the conservation equation of seat capacity is thus given by

$$N n_s = H_v + H_c \tag{3.2}$$

where N is vehicle fleet size, n_s is the number of seats in a vehicle.

Note that, the travel time for a passenger in ride-sharing services consists of two components: direct trip time (equals to the time by driving private cars without detouring) and detour time (due to the detouring to pick-up and/or drop-off other passengers). Thus, the expected travel time of i is given by

$$t_i = t_i^d + \tilde{t}_i \tag{3.3}$$

where t_i is the expected travel time of i, t_i^d and $\tilde{t}_i$ denote the direct trip time and the expected detour time, respectively. As a result, the utilized seat capacity in one hour can be calculated as

$$H_c = \sum_{i \in \mathbb{Z}} Q_i t_i \tag{3.4}$$

Substitute Equation (3.4) into Equation (3.2) resulting in

$$N n_s = H_v + \sum_{i \in \mathbb{Z}} Q_i t_i \tag{3.5}$$

This seat capacity conservation equation bridges the demand for and supply of ride-sharing services and has to be satisfied at the market equilibrium.

3.4. Demand for Ride-sharing Services

To estimate the ride-sharing passenger demand, we assume that all travelers make decisions objectively based on perceived utilities of the available transport modes. In this study, we apply a classic discrete choice model, Multinomial Logit (MNL) model, to model the passenger preference in the multi-modal transport context.

Based on the random utility theory, the utility of a choice can be calculated by

$$U = V + \epsilon \tag{3.6}$$

where U is the utility, V is the systematic/deterministic component of the utility, ϵ is the disturbance.

Time cost and monetary cost are the main factors that influence passengers' choice among the available transportation options. Considering the difference of perceptions on travel time and waiting time, the utility function (i.e., the deterministic part of the utility) of taking one transport mode is constructed as

$$V = \beta_t t + \beta_w w + \beta_r r \tag{3.7}$$

where t, w and r denote the travel time (including direct trip time and detour time), waiting time and trip fare, respectively. β_t, β_w and β_r are preference coefficients for travel time, waiting time and trip fare, respectively.

If assume the error term ϵ follows the Gumbel distribution (also known as Extreme Value distribution)[2], then the probability of one from i^o to i^d choosing ride-sharing services is given by

$$P_{i,rs} = \frac{e^{V_{i,rs}}}{\sum_{j \in \mathbb{M}} e^{V_{i,j}}} \tag{3.8}$$

[2] We refer the interested reader to the Chapter 3 in Train, 2009 for the details of the derivation of Logit model.

where $\mathbb{M}$ is the set of available transport modes in the system, such as private vehicles, public transport and ride-sharing. Equation (3.8) gives the form of the MNL model.

As the easiest and the most extensively used discrete choice model for estimating the travel behavior of individuals (Train, 2009), MNL has been applied in many aspects of the transportation community including the transport mode choice behavior (e.g., Vrtic et al., 2010; Chen et al., 2013; Krueger et al., 2016). In this study, it is used to estimate the mode share of ride-sharing services in the multi-modal transport context. Substitute Equation (3.8) into Equation (3.1), we can estimate the ride-sharing demand for i by (omit the subscript rs in the following text)

$$Q_i = \frac{D_i e^{V_i}}{e^{V_i} + \mu_i} \tag{3.9}$$

where $\mu_i = \sum_{j \in \{\mathbb{M} - rs\}} e^{V_{i,j}}$.

3.5. Equilibrium in the Ride-sharing Market

From Figure 3.1, we know that both expected detour time $\tilde{t}_i$ and expected waiting time w_i for OD pair i are related to the vehicle fleet size N and ride-sharing passenger demand $\boldsymbol{Q}$ (the influence of H_v can be ultimately translated to N and $\boldsymbol{Q}$), where $\boldsymbol{Q}$ is the vector of ride-sharing demand for all OD pairs. Thus, we can rewrite them as $\tilde{t}_i(\boldsymbol{Q}, N)$ and $w_i(\boldsymbol{Q}, N)$, respectively. Recall that the expected travel time is the sum of direct trip time and expected detour time, such that we can rewrite the travel time as $t_i(\boldsymbol{Q}, N)$. Then the utility function for ride-sharing services is given by

$$V_i(\boldsymbol{Q}, N) = \beta_t t_i(\boldsymbol{Q}, N) + \beta_w w_i(\boldsymbol{Q}, N) + \beta_r r_i \tag{3.10}$$

Substitute Equation (3.10) into Equation (3.9), the ride-sharing passenger demand is thus an implicit function of itself.

$$Q_i = \frac{D_i e^{V_i(\boldsymbol{Q}, N)}}{e^{V_i(\boldsymbol{Q}, N)} + \mu_i} \tag{3.11}$$

Consequently, under certain operation strategies (i.e., given the value of vehicle flee size N and trip fare r_i for all OD pairs), an equilibrium in the market for ride-sharing services is a set of values for $\tilde{t}_i, w_i$ and Q_i that satisfies Equation (3.5), Equation (3.10) and Equation (3.11) for all i in $\mathbb{Z}$. It is worth pointing out that, Equation (3.5) and all Equation (3.11) for different i describes the supply of and demand for ride-sharing services, respectively.

In other words, the interplay between system endogenous variables (expected detour time, expected waiting time and ride-sharing demand) at equilibrium given the values of exogenous variables (vehicle fleet size and trip fare) is described by a simultaneous equations system written as below.

$$\begin{cases} Q_{i_1} = \frac{D_{i_1} e^{V_{i_1}(Q,N)}}{e^{V_{i_1}(Q,N)} + \mu_{i_1}} \\ Q_{i_2} = \frac{D_{i_2} e^{V_{i_2}(Q,N)}}{e^{V_{i_2}(Q,N)} + \mu_{i_2}} \\ \vdots \qquad \vdots \\ Q_{i_m} = \frac{D_{i_m} e^{V_{i_m}(Q,N)}}{e^{V_{i_m}(Q,N)} + \mu_{i_m}} \\ \vdots \qquad \vdots \end{cases} \tag{3.12}$$

where $\boldsymbol{Q} = [Q_{i_1}, Q_{i_2}, \cdots, Q_{i_m}, \cdots]^T$ is the vector of ride-sharing demand for all OD pairs. This equations system can be solved via a hybrid method for nonlinear equations proposed in Powell, 1970.

3.6. Expected Detour Time for Ride-sharing

Following the findings from substantial empirical data of real operations in several cities in Ke et al., 2020b, Ke et al., 2020a assumed the average detour time between two passengers is inversely proportional to the ride-sharing demand. Intuitively, more ride-sharing requests means the average distance between two passengers is closer. In this sense, it is plausible to take this assumption. However, in contrast to Ke et al., 2020a, which restricts the ride-sharing services of pairing at most two passengers, we consider the general operation situation in this study. In this case, more requests could denser the passengers manifested as a reduction in the average detour time on the one hand. On the other hand, it also increases the possibility of pairing more passengers and thus results in a longer detour time. Consequently, we make a modification to the assumption adopted in the aforementioned literature, and extent it to the general case.

Similarly, the average detour time between two passengers is given by

$$\tilde{t}^{(2)} = \frac{\tilde{A}}{\|\boldsymbol{Q}\|_1} \tag{3.13}$$

where $\|\boldsymbol{Q}\|_1$ is the L1 norm of $\boldsymbol{Q}$, i.e., $\|\boldsymbol{Q}\|_1 = \sum_{j \in \mathbb{Z}} Q_j$. $\tilde{A}$ is a parameter.

Let $\bar{t}^d$ denote the mean direct trip time of all trips, then $\bar{t}^d = \sum_{j \in \mathbb{Z}} Q_j t_j^d / \|\boldsymbol{Q}\|_1$. If there is no detouring, then the maximum number of trips a vehicle can serve in one hour is given by

$$n_t = \frac{1}{\bar{t}^d} = \frac{\|\boldsymbol{Q}\|_1}{\sum_{j \in \mathbb{Z}} Q_j t_j^d} \tag{3.14}$$

However, due to the limitation of vehicle fleet size, the number of trips assigned to a vehicle is given by

$$n_a = \|\boldsymbol{Q}\|_1 / N \tag{3.15}$$

As a result, the expected number of passengers in a vehicle at one moment is n_a/n_t. Then the detour time of a vehicle is given by

$$\tilde{t}^{(v)} = \left(\frac{n_a}{n_t} - 1\right)\tilde{t}_i^{(2)} \tag{3.16}$$

Recall that when we calculate n_t, we assume that there are no interruptions between the time vehicles drop off one passenger and pick up another, which does not comply with the reality. Thus we approximate $\tilde{t}^{(v)}$ by

$$\tilde{t}^{(v)} = \frac{n_a}{n_t}\tilde{t}_i^{(2)} \tag{3.17}$$

We want to point out that the accuracy of this approximation can be adjusted by the parameter $\tilde{A}$ in $\tilde{t}_i^{(2)}$. On the other hand, it can also guarantee the positive of detour time when the passenger demand is extremely small (e.g., when $n_a < n_t$). Thus, this approximation is plausible and necessary.

Moreover, we follows the assumption that the detour time of passengers is a fraction of the detour time of vehicles in Ke et al., 2020a, i.e., $\tilde{t}^{(p)} = \gamma\tilde{t}^{(v)}$. If we assume that trips with longer direct trip time are more likely to have a detour, then the detour time of i can be expressed as

$$\tilde{t}_i = \frac{t_i^d}{\bar{t}^d}\tilde{t}^{(p)} = \frac{t_i^d \gamma n_a}{\bar{t}^d n_t}\tilde{t}_i^{(2)} \tag{3.18}$$

Substitute Equation (3.13), Equation (3.14) and Equation (3.15) into Equation (3.18) resulting in

$$\tilde{t}_i = \frac{t_i^d \gamma \tilde{A} \sum_j Q_j t_j^d}{\bar{t}^d N \|\boldsymbol{Q}\|_1} \tag{3.19}$$

For simplicity of presentation, we define $A \triangleq \gamma\tilde{A}$ and $A_i \triangleq A t_i^d/\bar{t}^d$, such that

$$\tilde{t}_i = \frac{A_i \sum_j Q_j t_j^d}{N \sum_j Q_j} \tag{3.20}$$

Equation (3.20) is used to estimate the expected detour time of trips from i^o to i^d in this study.

3.7. Expected Waiting Time for Ride-sharing

Under a certain operation strategy, the supply of the ride-sharing market is also fixed. Nevertheless, analogous to the taxi market, the quantity of service supplied to the passengers (Nn_s) is always greater than the equilibrium quantity demanded (H_c) by a certain amount of slack (H_v) (Yang et al., 2002).

The assumption of the expected waiting time of taxi services and ride-sharing services is inversely proportional to the square root of the number of idle vehicles is widely applied in the literature[3] (e.g., Li et al., 2019b; Ke et al., 2020a). In this study, considering the sharing nature of ride-sharing services, we assume the waiting time is inversely proportional to the square root of the available seat capacity. Furthermore, due to the spatial difference of ride-sharing demand, we assume it is proportional to the ride-sharing demand for the corresponding OD as well. As a result, the expected waiting time of trips from i^o to i^d is estimated by

$$w_i = \frac{BQ_i}{\sqrt{H_v}} \tag{3.21}$$

where B is a parameter.

According to Equation (3.5), the available seat capacity can be calculated from the seat conservation equation.

$$H_v = Nn_s - \sum_{j \in \mathbb{Z}} Q_j t_j \tag{3.22}$$

Substitute Equation (3.22) into Equation (3.21) resulting in

$$w_i = \frac{BQ_i}{\sqrt{Nn_s - \sum_j Q_j t_j}} \tag{3.23}$$

Equation (3.23) is used to estimate the expected waiting time of trips from i^o to i^d in this study.

3.8. Method to Estimate Equilibrium Model Parameters

Provably, the estimation accuracy of expected detour time and expected waiting time can significantly affect the effectiveness of the proposed ride-sharing equilibrium model, especially in the multi-modal transportation system. Therefore, it is critical to provide plausible values for A and B for the presented estimation models for detour time and waiting time. Obviously, A and B would be different for different markets.

In this section, we introduce an approximating method to calculate A and B for a specific market based on its real operation data. Specifically, if we have real operation data of a market (including the mode share, average detour time, average waiting time, vehicle fleet size, and the average trip fare) by applying this method, we can calculate applicable A and B for resulting in a reliable market equilibrium model which can reflect the reality.

For simplicity of presentation, the details will be omitted here and can be found in Appendix A.1.

[3] We refer the interested readers to Li et al. (2019b) for the proof for this assumption.

4. Experiment Setup

This section introduces the data and networks used in the experiments in the following chapters. In this study, we develop a market equilibrium model for ride-sharing services with a consideration of the passenger preference in the multi-modal transportation system. To model the passenger preference, we utilize the stated-preference data from the survey conducted in Tsiamasiotis (2019). To evaluate the performance of the proposed equilibrium model integrated with the proposed pricing methods under different traffic situations (congestion level) and network scales, we conduct experiments on two different networks, and consider two different demand levels for each network. One is a handmade test network, and the other is the Munich network. Two demand levels represent congested traffic and uncongested traffic, respectively. Furthermore, we also present the model of monopoly and social optimum scenario which have been widely used to evaluate the market in this chapter.

4.1. Stated-preference Survey Data

Tsiamasiotis (2019) designed and performed a web-based stated-preference survey to identify factors affecting the travel behavior of passengers due to the introduction of ride-sharing services in the transportation system. 27 hypothetical scenarios were created and divided into three blocks resulting in nine of each. And three alternatives were provided in each hypothetical scenario, including private car, public transport and dynamic ride-sharing. Respondents needed to state their preference in a 5-point rating scale for a scenario given the values of three attributes including in-vehicle travel time, travel cost and waiting time, which conforms to the requirement of the model proposed in this study.

The survey got 208 complete responses in total, and obtained 1,248 effective scenario answers. Apart from the scenarios, some respondents' demographics were required, including gender, age, income, education level, etc. We refer the interested readers to Tsiamasiotis (2019) for the design and data analysis of the survey.

4.2. Small Test Network

We construct a small test network for numerical experiments. Figure 4.1 depicts its layout. It consists of three zones, the related variables of the trips between zones at the low demand level are described in Table 4.1. And we assume at the high demand level, the travel demand is double and the direct trip time is 1.5 times compared to the situation at the low demand level.

Importantly, recall that we need to specify A and B for the equilibrium model. We apply the method presented in Appendix A.1 to calculate appropriate values of A and B for this network. We assume the mode share of ride-sharing $\hat{s} = 0.1$ when the unit price $\hat{p} = 1.8$ Euro/km (trip fare $\bar{r} = \sum_i pd_i/6$ Euro), the vehicle fleet size $\hat{N} = \sum_i D_i\hat{s}/2$ (and each vehicle has $n_s = 6$ seats) and the ratio of the average detour time $\bar{t}$ over the average waiting time $\bar{w}$ is $k = 2$. This market assumption leads to $A = 188.39, B = 0.22$, which will be applied in all thereafter experiments on the test network.

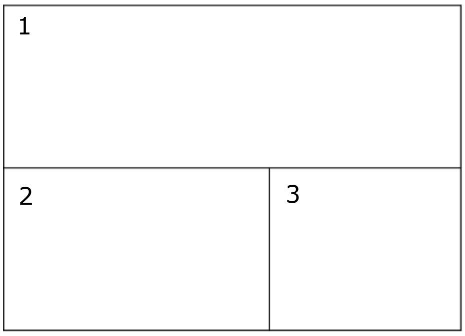


Figure 4.1.: Small test network

Table 4.1.: Attributes of the trips between two zones in the test network.

Origin i^o	Destination i^d	Travel demand D	Direct trip time t^d	Distance d
1	2	400	25 min	10 km
1	3	400	15 min	7 km
2	1	500	18 min	7 km
2	3	800	20 min	10 km
3	1	400	18 min	8 km
3	2	600	20 min	10 km

4.3. Munich Network

The layout of the Munich area used in the following experiments is shown in Figure 4.2. It is about 900 square kilometers area. This area is divided into 20 zones resulting in $20 \times 19 = 380$ possible OD pairs (internal trips of each zone are ignored in this study). The road traffic demand data partially calibrated with traffic counts of a Tuesday in May (on May 9th, 2017) are used. After screening, we decide to utilize the period from 5 a.m. to 6 a.m. as the low demand level scenario, while the period from 7 a.m. to 8 a.m. is the high demand level. It is worth noticing that both public transport and private transport (depending on the survey data source) are considered in this study. Whereas, road traffic demand data are nearly equal to the demand for private transport. Thus, we need to scale up the road traffic demand for the Munich network based on the modal split published on the official website of the relevant department of Munich[1].

Note that, in order to mitigate the randomness of the ride-sharing market, we only consider the OD pairs whose travel demand is greater than 100, rather than considering all OD pairs within the network. This restricts the services to 49 ODs. The total demand for these OD pairs between 5 a.m. and 6 a.m. is 8,264, while that between 7 a.m. and 8 a.m. is 15,254. What's more, the direct trip time and the distance of each OD pair are calculated as the mean of all trips of the same OD generated by Simulation of Urban MObility (SUMO) (Lopez et al., 2018). To eliminate the stochasticity in simulations, results from 10 replications are averaged. All simulations are implemented at the mesoscopic level through the trip-based (one-shot) stochastic user route choice assignment method.

We assume the mode share of ride-sharing $\hat{s} = 0.1$ when the unit price $\hat{p} = 1.8$ Euro/km (the trip fare $\bar{r} = \sum_i pd_i/49$ Euro), the vehicle fleet size $\hat{N} = \sum_i D_i\hat{s}/2$ (and each vehicle has $n_s = 6$ seats) and the ratio of the average detour time $\bar{t}$ over the average waiting time $\bar{w}$ is $k = 2$. This market assumption leads to $A = 697.36, B = 0.83$, which will be applied in all thereafter experiments on the Munich network.

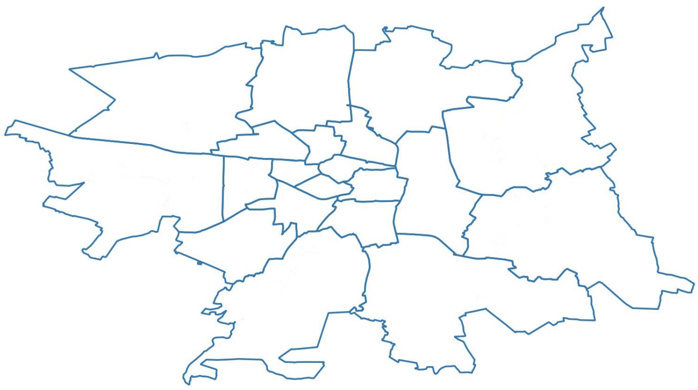


Figure 4.2.: Munich major region

4.4. operational objectives

In this section, we introduce two typical scenarios considered in the literature and shown in real operation. By analyzing the performance of the equilibrium model developed in Section 3.2 in different markets, we could have a in-depth understanding of ride-sharing services. Likewise, we will compare the optimal solutions of the pricing methods in different scenarios to deduce their application context in the following chapters. Specific scenarios are listed below.

1) Monopoly scenario. A monopolist aims to maximize its profit.

[1]We refers the readers to http://www.wirtschaft-muenchen.de/publikationen/pdfs/Businesslocation_Munich_e.pdf (available on Sep. 10th, 2020) for the published document.

2) Social optimum scenario. The platform aims to maximize the social welfare.

4.4.1. Monopoly scenario

As described in Chapter 2, the service provider of ride-sharing services can be either the service operator or a matching agency. In this study, we focus on the former situation, where the service provider operates ride-sharing services with its own vehicles and drivers. A ride-sharing monopolist, as usual in microeconomics problems, attempts to maximize its profit by optimizing the vehicle fleet size and trip fare. This problem can be formulated as

$$\textbf{(P1)} \quad \text{maximize} \quad \Pi(N, \boldsymbol{r}) = \textstyle\sum_{i \in \mathbb{Z}} D_i P_i r_i - \phi N \tag{4.1}$$

where ϕ is the operating cost of a vehicle in one hour, $\boldsymbol{r}$ is the vector of trip fare of all OD pairs.

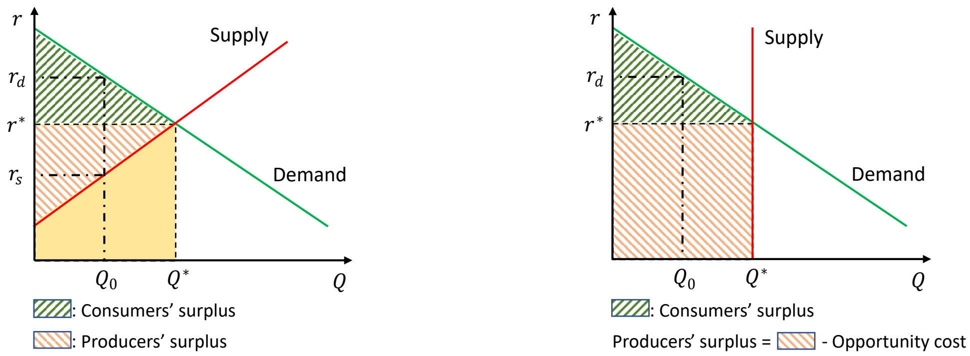
4.4.2. Social optimum scenario

Social welfare which is also known as social surplus equals the sum of consumers' and producers' surplus (Cairns and Liston-Heyes, 1996). Figure 4.3a shows the demand for and supply of a simple market, where the X-axis is the quantity and the Y-axis is the price. We assume the market is in the equilibrium, (Q^*, r^*). Consumer surplus is the difference between the market price and the maximum price the consumer is willing to pay. For instance, for the Q_0th consumer whose maximum acceptable price of purchasing the good is r_d, his or her surplus is $r_d - r^*$. In other words, the consumer surplus is the benefit the consumer receives by purchasing the good in the market. If we assume both r and Q are continuous variables, then the total consumers' surplus is the area between the marginal benefit curve (the demand curve) and the equilibrium price as denoted in the figure. On the other hand, producer surplus is the difference between the market price and the least the producer is willing to sell for. For instance, for the Q_0th good whose opportunity cost is r_s, the surplus of producer by selling this good is $r^* - r_s$. Concisely, the producer surplus is the benefit the producer receives by selling the good in the market. Thus, the total producers' surplus is the area between the equilibrium price and the marginal cost curve (the supply curve) as denoted in the figure.

Consider a special market with fixed supply as shown in Figure 4.3b. The consumers' surplus is calculated as in the market shown in Figure 4.3a. However, in this market, the opportunity cost is invisible. When we calculate the producers' surplus, we need to subtract the opportunity cost from the rectangle area formed by the origin and (Q^*, r^*). Thus, the social surplus equals to the area below the marginal benefit curve from the origin to Q^* minus the opportunity cost.

The ride-sharing market is similar to the market depicted in Figure 4.3b. Thus, the social welfare maximization problem can be constructed as

$$\textbf{(P2)} \quad \text{maximize} \quad S(N, p) = \textstyle\sum_i \int_0^{Q_i} F_i(x)dx - \phi N \tag{4.2}$$



(a) Surplus of a simple market

(b) Surplus of the fixed-supply

Figure 4.3.: Samples of consumers' surplus and producers' surplus

where $F_i(\cdot)$ is the inverse of the demand function given in Equation (3.9), i.e., marginal benefit function. According to Equation (3.9),

$$r_i = \frac{1}{\beta_r}\left[\ln Q_i - \ln(D_i - Q_i) + \ln \mu_i - \beta_t t_i - \beta_w w_i\right]$$

Thus, $F(x)$ is given by

$$F_i(x) = \frac{1}{\beta_r}\left[\ln x - \ln(D_i - x) + \ln \mu_i - \beta_t t_i - \beta_w w_i\right] \tag{4.3}$$

Besides, due to the inclusion of waiting time and detour time in the demand curve, the consumers' surplus must be carefully determined. The integral in (**P2**) is obtained by integrating under a (hypothetical) demand curve in which the service level (waiting time, detour time) is held fixed while the trip fare varies, rather than under the market demand curve[2].

[2]We refer the interested readers to Anderson and Bonsor (1974), Cairns and Liston-Heyes (1996) for the related analysis.

5. Unified Pricing Method

5.1. Objectives and Contributions

As one of the major challenges, dynamic pricing methods have been attracting increasing attention on ride-sharing services research. Some of the existing works account for the temporal elasticity of demand (e.g., Turan et al., 2019), or the spatial heterogeneity due to the network structure (e.g., Bimpikis et al., 2019), or both of them (e.g., Qiu et al., 2018). These pricing methods are always developed on a basic assumption of homogeneous users. However, as the importance and benefits of considering user heterogeneity have been proved in the related problems, such as assortment optimization problem (e.g., Rusmevichientong et al., 2010), parking choice (e.g., Ibeas et al., 2014), and route choice (e.g., Liu and Nie, 2011), it is desirable to investigate the optimal pricing strategies for ride-sharing with additional consideration of the user heterogeneity.

In this chapter, we aim to propose a unified pricing method, which has been widely applied in taxi markets and mobility-on-demand (MooD) markets, for the market developed in Chapter 3 and analyze its performance in the monopoly scenario and social optimum scenario. The contributions of this chapter are three-fold.

1) We propose a unified pricing method for ride-sharing services with a consideration of passenger preference in the multi-modal transport context, and prove its performance under different regulations.

2) We develop and derive solution algorithms for the proposed pricing method in different market scenarios, and investigate their effectiveness.

3) We explore the performance of the proposed market model in different user groups.

This chapter is organized as follows. Section 5.2 introduces the concept of unified pricing method. Section 5.3 and Section 5.4 presents the solution algorithm and its derivation, respectively. Section 5.5 describes simulation settings for the experiments in this chapter. Section 5.6 analyzes the results of different experiments and evaluates the proposed solution algorithms. Section 5.7 performs a sensitivity analyses with respect to passenger preference (user group). Section 5.8 concludes this chapter and proposes potential future directions.

5.2. Fare Structure

The fare structure used in this unified pricing method is such that trip fares are proportional to the travel distance. Mathematically, the trip fare of i is given by

$$r_i = pd_i \tag{5.1}$$

where p is the unit price for trips of all OD pairs.

Keller (2013) proved that under certain conditions, the local maximum of the non-convex problem (**P1**) in terms of the prices is also a global maximum. It means, though gradient-based algorithms ensure converge to a local minimum only, they can be used to solve the problems presented in this study. Thus, in this study, we apply the Gradient Descent (GD) algorithm (Cauchy, 1847; Curry, 1944) for solving (**P1**) and (**P2**) in terms of the unit price p and the vehicle fleet size N.

5.3. Solution Algorithm

Define the operation strategy as $O \triangleq (N, p)$. Let $\mathcal{J}(O)$ denote the objective function which can be either $\Pi(O)$ or $S(O)$. Then the GD for solving the relevant optimization problems can be described as Algorithm 5.1. By applying this algorithm, first, we need to initialize the starting point of the operation strategy $O^{(0)}$, the step size α, and the maximum iterations τ_{max} (line 1). Then at each iteration τ, we have to calculate the values of relevant variables in the market equilibrium under $O^{(\tau)}$ (line4). Followed with the computation of derivatives of travel time t_i, waiting time w_i with respect to p and N, respectively, for each OD pair (line 5). The derivatives of the objective $\mathcal{J}$ can then be calculated (line 6) based on the results of the previous step. The detailed derivations of the derivatives aforementioned and their interdependence are described in Section 5.4. At the end of each iteration, the operation strategy $O^{(\tau)}$ is updated based on the resulted gradient (line 12). This procedure will be repeated until either the maximum iteration is reached or the termination condition ($g^{(\tau)} < \varepsilon$) is met.

5.4. Partial Derivatives of Related Variables

As describe in Chapter 3, the interdependence among system endogenous variables (waiting time, detour time, ride-sharing demand) is very complicated. This leads to the difficulty of calculating the partial derivatives of relevant variables for p and N.

5.4.1. Partial derivatives of profit

Recall that $\Pi(N, p) = \sum_i D_i P_i r_i - \phi N$ and $r_i = pd_i$, then the derivative of Π with respect to p can be calculated as below.

$$\frac{\partial \Pi}{\partial p} = \sum_i (D_i \frac{\partial P_i}{\partial p} r_i + Q_i d_i) \tag{5.2}$$

Algorithm 5.1 Gradient Descent for solving unified pricing optimization problems

Initialize $O^{(0)}, \alpha, \tau_{max}$
$\tau = 0$
for $\tau < \tau_{max}$ **do**
    Compute the market equilibrium state: $r_i(O^{(\tau)}), w_i(O^{(\tau)}), \tilde{t}_i(O^{(\tau)}), P_i(O^{(\tau)}), Q_i(O^{(\tau)})$
    Compute $\frac{\partial t_i}{\partial p}, \frac{\partial w_i}{\partial p}, \frac{\partial t_i}{\partial N}, \frac{\partial w_i}{\partial N}, \forall i$ with $O^{(\tau)}$ as the operation condition
    Compute $\frac{\partial \mathcal{J}}{\partial p}, \frac{\partial \mathcal{J}}{\partial N}$ with $O^{(\tau)}$ as the operation condition
    $g^{(\tau)} = \nabla \mathcal{J}(O^{(\tau)})$
    if $g^{(\tau)} < \varepsilon$ **then**
        $O^* = O^{(\tau)}$
        break
    end if
    $O^{(\tau+1)} = O^{(\tau)} + \alpha g^{(\tau)}$
    $\tau = \tau + 1$
end for

By applying the chain rule, we know

$$\frac{\partial P_i}{\partial p} = \frac{\partial P_i}{\partial V_{i,rs}} \frac{\partial V_{i,rs}}{\partial p} \tag{5.3}$$

Since $P_i = e^{V_{i,rs}}/(e^{V_{i,rs}} + \mu_i)$ and $V_{i,rs} = \beta_t t_i + \beta_w w_i + \beta_r r_i$, so

$$\frac{\partial P_i}{\partial V_{i,rs}} = \frac{e^{V_{i,rs}}}{e^{V_{i,rs}} + \mu_i} - \frac{(e^{V_{i,rs}})^2}{(e^{V_{i,rs}} + \mu_i)^2} = \frac{e^{V_{i,rs}}}{e^{V_{i,rs}} + \mu_i}(1 - \frac{e^{V_{i,rs}}}{e^{V_{i,rs}} + \mu_i}) = P_i(1 - P_i) \tag{5.4}$$

$$\frac{\partial V_{i,rs}}{\partial p} = \beta_t \frac{\partial t_i}{\partial p} + \beta_w \frac{\partial w_i}{\partial p} + \beta_r d_i \tag{5.5}$$

Substitute Equation (5.4) and Equation (5.5) into Equation (5.3), we can get

$$\frac{\partial P_i}{\partial p} = P_i(1 - P_i)(\beta_t \frac{\partial t_i}{\partial p} + \beta_w \frac{\partial w_i}{\partial p} + \beta_r d_i)$$

As a result,

$$\frac{\partial \Pi}{\partial p} = \sum_i D_i P_i (1 - P_i)(\beta_t \frac{\partial t_i}{\partial p} + \beta_w \frac{\partial w_i}{\partial p} + \beta_r d_i) r_i + \sum_i Q_i d_i \tag{5.6}$$

Similarly, we can calculate the derivative of profit with respect to N as below.

$$\frac{\partial \Pi}{\partial N} = \sum_i \frac{D_i \partial P_i}{\partial N} r_i - \phi = \sum_i D_i P_i (1 - P_i)(\beta_t \frac{\partial t_i}{\partial N} + \beta_w \frac{\partial w_i}{\partial N}) r_i - \phi \tag{5.7}$$

Equation (5.6) and Equation (5.7) tell the derivatives of profit are functions of derivatives of detour time and waiting time. The derivatives of detour time and waiting time with respect to p and N are expounded in Section 5.4.3.

5.4.2. Partial derivatives of social welfare

We emphasize that the integral in (P2) is obtained by integrating under a (hypothetical) demand curve in which the service level (waiting time, detour time) is held fixed while the trip fare varies, rather than under the market demand curve.

Recall that $S(N,p) = \sum_i \int_0^{Q_i} F_i(x)dx - \phi N$, we can calculate the derivative of social welfare with respect to p as below.

$$\frac{\partial S}{\partial p} = \sum_i \left[\int_0^{Q_i} \frac{\partial F_i(x)}{\partial p} dx + \frac{\partial Q_i}{\partial p} F_i(Q_i) \right] \tag{5.8}$$

Since $F_i(x) = \frac{1}{\beta_r}\left[\ln x - \ln(D_i - x) + \ln \mu_i - \beta_t t_i - \beta_w w_i\right]$, so

$$\int_0^{Q_i} \frac{\partial F_i(x)}{\partial p} dx = \int_0^{Q_i} \frac{1}{\beta_r}(-\beta_t \frac{\partial t_i}{\partial p} - \beta_w \frac{\partial w_i}{\partial p})dx = \frac{Q_i}{\beta_r}(-\beta_t \frac{\partial t_i}{\partial p} - \beta_w \frac{\partial w_i}{\partial p})$$

$$\frac{\partial Q_i}{\partial p} = D_i P_i (1 - P_i)(\beta_t \frac{\partial t_i}{\partial p} + \beta_w \frac{\partial w_i}{\partial p} + \beta_r d_i)$$

Such that

$$\frac{\partial S}{\partial p} = \sum_i \left[\frac{Q_i}{\beta_r}(-\beta_t \frac{\partial t_i}{\partial p} - \beta_w \frac{\partial w_i}{\partial p}) + D_i P_i (1 - P_i)(\beta_t \frac{\partial t_i}{\partial p} + \beta_w \frac{\partial w_i}{\partial p} + \beta_r d_i) F_i(Q_i) \right] \tag{5.9}$$

Similarly, we can calculate the derivative of social welfare with respect to N as below.

$$\frac{\partial S}{\partial N} = \sum_i \left[\int_0^{Q_i} \frac{\partial F_i(x)}{\partial N} dx + \frac{\partial Q_i}{\partial N} F_i(Q_i) \right] - \phi \tag{5.10}$$

$$\int_0^{Q_i} \frac{\partial F_i(x)}{\partial N} dx = \int_0^{Q_i} \frac{1}{\beta_r}(-\beta_t \frac{\partial t_i}{\partial N} - \beta_w \frac{\partial w_i}{\partial N})dx = \frac{Q_i}{\beta_r}(-\beta_t \frac{\partial t_i}{\partial N} - \beta_w \frac{\partial w_i}{\partial N})$$

$$\frac{\partial Q_i}{\partial N} = D_i P_i (1 - P_i)(\beta_t \frac{\partial t_i}{\partial N} + \beta_w \frac{\partial w_i}{\partial N})$$

$$\frac{\partial S}{\partial N} = \sum_i \left[\frac{Q_i}{\beta_r}(-\beta_t \frac{\partial t_i}{\partial N} - \beta_w \frac{\partial w_i}{\partial N}) + D_i P_i (1 - P_i)(\beta_t \frac{\partial t_i}{\partial N} + \beta_w \frac{\partial w_i}{\partial N}) F_i(Q_i) \right] - \phi \tag{5.11}$$

Analogously, from Equation (5.9) and Equation (5.11), we can see that the derivatives of welfare are also functions of derivatives of detour time and waiting time.

5.4.3. Partial derivatives of detour time and waiting time

As shown in Section 5.4.1 and Section 5.4.2, the derivatives of the objectives are determined by the derivatives of detour time and waiting time.

Let $t'_{i|p}$ and $t'_{i|N}$ denote the derivatives of t_i with respect to p and N, respectively. And let $w'_{i|p}$ and $w'_{i|N}$ denote the derivatives of w_i with respect to p and N, respectively. Recall that $\tilde{t}_i = A_i \sum_j Q_j t_j^d / NQ$, and since $t_i = t_i^d + \tilde{t}_i$, then the derivative of t_i with respect to p can be calculated as

$$\frac{\partial t_i}{\partial p} = \frac{\partial \tilde{t}_i}{\partial p} = \frac{A_i}{N}\left[\frac{1}{Q}\sum_j \frac{\partial Q_j}{\partial p} t_j^d - \frac{1}{Q^2}\frac{\partial Q}{\partial p}\sum_j Q_j t_j^d\right]$$

$$t'_{i|p} = \frac{A_i}{NQ}\sum_j D_j P_j (1-P_j)(\beta_t t'_{j|p} + \beta_w w'_{j|p} + \beta_r d_j) t_j^d - \frac{A_i \sum_j Q_j t_j^d}{NQ^2}\sum_j D_j P_j (1-P_j)(\beta_t t'_{j|p} + \beta_w w'_{j|p} + \beta_r d_j) \quad (5.12)$$

Equation (5.12) is a linear combination of $t'_{i|p}, w'_{i|p}, \forall i$.

On the other hand, since $w_i = BQ_i / \sqrt{Nn_s - \sum_j Q_j t_j}$, so

$$\frac{\partial w_i}{\partial p} = \frac{B}{\sqrt{Nn_s - \sum_j Q_j t_j}}\frac{\partial Q_i}{\partial p} - \frac{BQ_i}{2\sqrt{(Nn_s - \sum_j Q_j t_j)^3}}\left[-\sum_j (\frac{Q_j}{\partial p} t_j + Q_j \frac{\partial t_j}{\partial p})\right]$$

$$w'_{i|p} = \frac{BD_i P_i (1-P_i)(\beta_t t'_{i|p} + \beta_w w'_{i|p} + \beta_r d_i)}{\sqrt{Nn_s - \sum_j Q_j t_j}} + \frac{BQ_i \sum_j \left[D_j P_j (1-P_j)(\beta_t t'_{j|p} + \beta_w w'_{j|p} + \beta_r d_j) t_j + Q_j t'_{j|p}\right]}{2\sqrt{(Nn_s - \sum_j Q_j t_j)^3}} \quad (5.13)$$

Equation (5.13) is also a linear combination of $t'_{i|p}, w'_{i|p}, \forall i$.

Combining Equation (5.12) and Equation (5.13) in terms of different i, we can get a linear equations system with $2n_z$ unknowns ($t'_{i|p}, w'_{i|p}, \forall i \in \mathbb{Z}$) and $2n_z$ equations (linear combinations). For a given p and N (at a specific iteration of the algorithm), this linear equations system can be easily solved via linear algebra.

Similarly, we can calculate the derivatives with respect to N as below.

$$\frac{\partial t_i}{\partial N} = \frac{\partial \tilde{t}_i}{\partial N} = \frac{A_i}{NQ}\sum_j \frac{\partial Q_j}{\partial N} t_j^d - \frac{A_i \sum_j Q_j t_j^d}{(NQ)^2}(Q + N\frac{\partial Q}{\partial N})$$

$$t'_{i|N} = \frac{A_i \sum_j D_j P_j (1-P_j)(\beta_t t'_{j|N} + \beta_w w'_{j|N}) t^d_j}{NQ} - \frac{A_i \left[Q + N \sum_j D_j P_j (1-P_j)(\beta_t t'_{j|N} + \beta_w w'_{j|N})\right] \sum_j Q_j t^d_j}{(NQ)^2} \tag{5.14}$$

$$\frac{\partial w_i}{\partial N} = \frac{B}{\sqrt{Nn_s - \sum_j Q_j t_j}} \frac{\partial Q_i}{\partial N} - \frac{BQ_i}{2\sqrt{(Nn_s - \sum_j Q_j t_j)^3}} \left[n_s - \sum_j (\frac{\partial Q_j}{\partial N} t_j + Q_j \frac{\partial t_j}{\partial N}) \right]$$

$$w'_{i|N} = \frac{BD_i P_i (1-P_i)(\beta_t t'_{i|N} + \beta_w w'_{i|N})}{\sqrt{Nn_s - \sum_j Q_j t_j}} - \frac{BQ_i \left[n_s - \sum_j D_j P_j (1-P_j)(\beta_t t'_{j|N} + \beta_w w'_{j|N}) t_j - \sum_j Q_j t'_{j|N} \right]}{2\sqrt{(Nn_s - \sum_j Q_j t_j)^3}} \tag{5.15}$$

Again, the linear equations system consists of Equation (5.14) and Equation (5.15) ($\forall i \in \mathbb{Z}$) can be solved via linear algebra.

5.5. Simulation Settings

In the experiments conducted in Section 5.6, the preference coefficients $\beta_r, \beta_t, \beta_w$ are estimated by utilizing the entire stated-preference survey data. Recall that respondents were asked to state their preferences in a 5-point rating scale in each scenario in the survey as introduced in Chapter 4. Therefore, the conventional MNL cannot be applied directly, since the ordered responses violate the independence for irrelevant alternatives (IIA) assumption of the logit model (Antoniou and Polydoropoulou, 2015). In this study, we utilize the ordered logit model presented in Antoniou et al. (2007) and Antoniou and Polydoropoulou (2015) to estimate the preference coefficients from the survey data. See the estimation result in Table 5.1. It can be seen that all p-values for the estimates are zero which indicates the estimation result is significant with a confidence level of 99%. Furthermore, as we assume passengers perceive travel time and waiting time differently, the value-of-waiting-time (VOT_w) and the value-of-travel-time (VOT_t) of passengers can be calculated as below.

$$VOT_w = \frac{\beta_w}{\beta_r} \cdot 60 = 11.51 \text{ Euro/h} \tag{5.16}$$

$$VOT_t = \frac{\beta_t}{\beta_r} \cdot 60 = 13.04 \text{ Euro/h} \tag{5.17}$$

Table 5.1.: Coefficient estimation for the entire survey.

Coefficient	Value	Standard error	t-test	p-value
β_r (/Euro)	-0.589	0.0509	-11.6	0
β_t (/min)	-0.128	0.0139	-9.18	0
β_w (/min)	-0.113	0.0134	-8.46	0

Besides, since the equilibrium model is developed in the context of multi-modal system, the attributes of different transportation options must be given. As explained in Chapter 4, we know the distance and direct trip time of all OD pairs within the networks. On the other hand, due to private car, public transport and ride-sharing are the options provided in the survey, thus in this study, we consider a transportation system consists of theses three transport modes. The attributes of public transport and private vehicles are given in Table 5.2. And the attributes of ride-sharing services are inherently decided by the market equilibrium model.

Table 5.2.: Attributes of public transport and private vehicles.

Mode	Waiting time (min)	Travel time (min)	Trip fare (Euro)
Public transport	10	$3t_i^d$	$1.5d_i$
Private vehicles	0	t_i^d	$2.5d_i$

Moreover, the operating cost per vehicle per hour is $\phi = 15$ Euro/h for all experiments thereafter.

The experiments and simulations are implemented using **Python**. The simultaneous equations system Equation (3.12) is solved using **Python** with **Scipy** library (Virtanen et al., 2020), an open source scientific computing library. The **fsolve** solver packaged in **SciPy.optimize** is used, which is a wrapper around the hybrid algorithm presented in Powell, 1970.

5.6. Case Study and Results

This section shows the performance of the proposed unified pricing method in the market constructed in Chapter 3. Section 5.6.1 and Section 5.6.2 shows the results for the handmade test network and the Munich network, respectively. Section 5.6.3 demonstrates the difference in the optimum for different scenarios. Section 5.6.4 evaluates the proposed GD algorithm in solving the optimization problems for the unified pricing method.

5.6.1. Results for the small test network

In this section, we show results for the test network at the low demand level, while results for the high demand level lead to the same conclusion and are omitted here.

Figure 5.1 shows the iso-profit contours and iso-welfare contours together with the monopoly optimum (MO) and social optimum (SO) in a two-dimensional space with vehicle fleet size on the X-axis and unit price on the Y-axis. And trip fares of different OD pairs can be calculated with $r_i = pd_i$. Obviously, the optimal unit price for a monopoly is higher than the optimal unit price at the social optimum, but the optimal vehicle fleet size at the monopoly optimum is less than that at the social optimum. Let (N_m^*, p_m^*) and (N_s^*, p_s^*) denote the coordinates of MO and SO. Then, $p_m^* > p_s^*$ and $N_m^* < N_s^*$. That is to say, a ride-sharing service operator needs to adjust its operation strategy based on its operational objective. If it aims to maximize its profit, it could refer to MO. However, if the service is operated by the government and serves as a supplement of public transport, its objective is always to maximize the social welfare in general. In this case, it should refer to SO.

Furthermore, clearly, both profit and social surplus first increase with the unit price and vehicle fleet size and then decrease. If the operational scale of the service is restricted by the limited source and funds, this figure can still provide suggestions for the unit price by referring to the vertical section of certain vehicle fleet size.

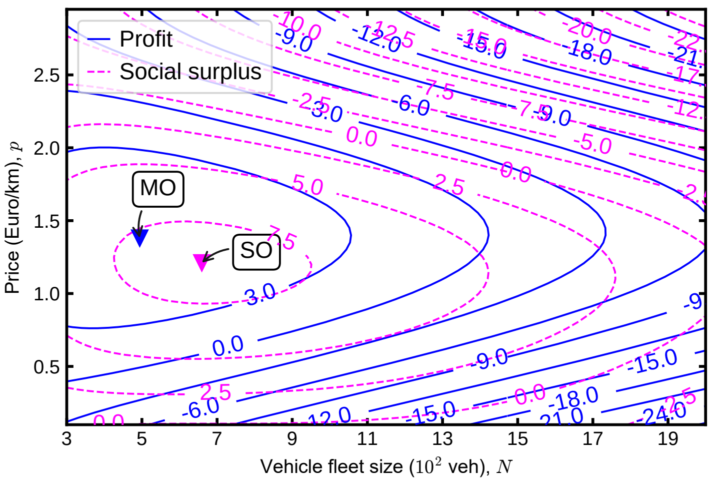


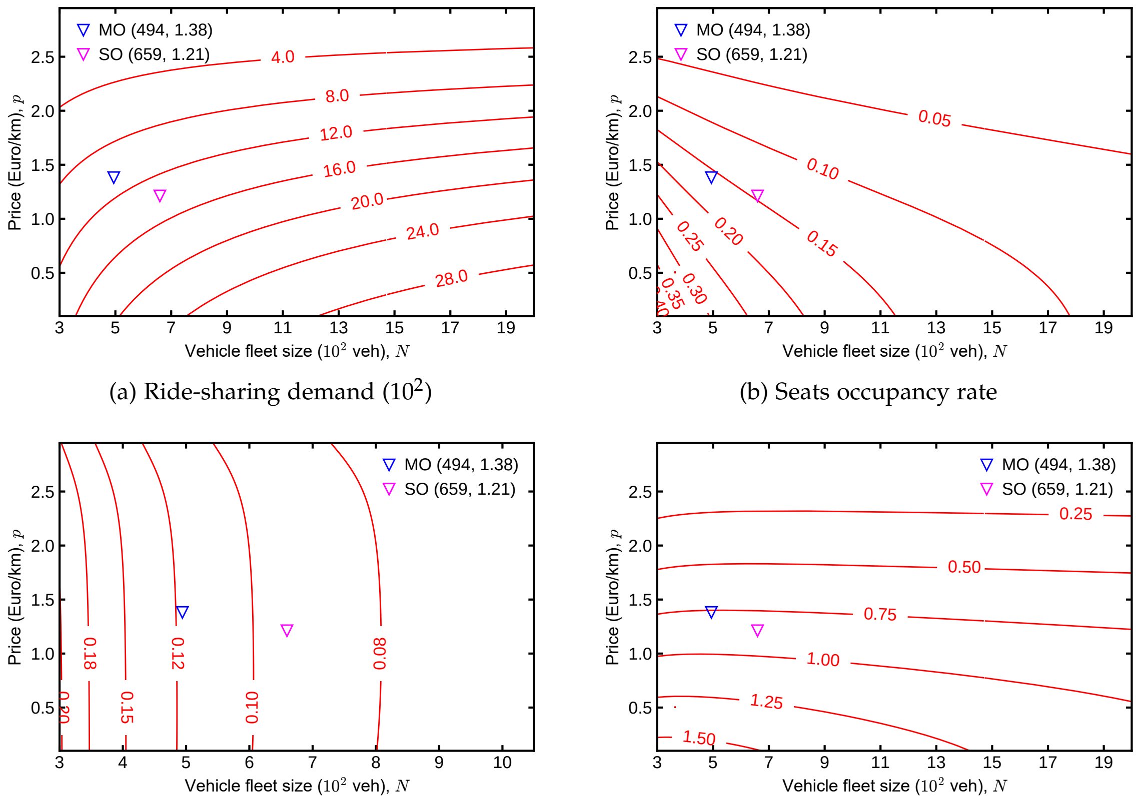
Figure 5.1.: Profit and social welfare of test market in a two-dimensional space of vehicle fleet size and unit price.

Figure 5.2a-Figure 5.2d depict the contours of ride-sharing demand, seats occupancy rate, network average detour time and network average waiting time of the test network, respectively. Let $\lambda, \bar{\tilde{t}}_i$ and $\bar{w}_i$ denote the seats occupancy rate, network average detour time and network average waiting time, then we have

$$\lambda = \frac{\sum_j Q_j t_j}{N n_s}, \bar{\tilde{t}} = \frac{\sum_i Q_i \tilde{t}_i}{\sum_i Q_i}, \bar{w} = \frac{\sum_i Q_i w_i}{\sum_i Q_i}$$

Figure 5.2a says that ride-sharing demand increases with vehicle fleet size, but decrease with unit price. Intuitively, more vehicles always means a higher serving quality, and therefore can attract more passengers from other transport modes. On the contrary, higher trip fares result in a demand loss due to the limited affordability of some passengers. Figure 5.2b shows that, different from ride-sharing demand, λ decreases with unit price and vehicle fleet size. Despite more vehicles can attract more passengers, the induced demand cannot meet the increase of seats supply.

Furthermore, it can be seen from Figure 5.2c and Figure 5.2d that, network average detour time is mainly influenced by vehicle fleet size, while the network average waiting time is mainly affected by the unit price. This conforms to their formulations. If we assume the direct trip time is the same for all OD pairs, then Equation (3.20) will become $\tilde{t}_i = At^d/N$. In other words, $\tilde{t}_i \propto 1/N$. Though this is an extreme case, it guilds the analysis of the contours of network average detour time if we do the approximation. Regarding the average waiting time, we have $w_i \propto Q_i/\sqrt{Nn_s - \sum_j Q_j t_j}$. The influence of N is weakened by the square root operator. Thus it mainly depends on the ride-sharing demand which is affected by unit price more as shown in Figure 5.2a. And this relationship would not change after averaging.



(a) Ride-sharing demand (10^2)

(b) Seats occupancy rate

(c) Network average detour time (h)

(d) Network average waiting time (h)

Figure 5.2.: System endogenous variables of test network in a two-dimensional space of vehicle fleet size and unit price.

5.6.2. Results for the Munich network

Similarly, in this section, we only present results for the Munich network at the low demand level for the purpose of presentation simplicity.

Figure 5.3 shows the iso-profit and iso-welfare contours together with MO and SO. We can see that SO is also in the lower right direction of MO, which complies with the phenomenon in the test network. And we also have $N_m^* < N_s^*$ and $p_m^* > p_s^*$. Both profit and welfare first increase with the unit price and vehicle fleet size and then decrease. Note that, when the unit price is relatively big, the joint influence of decision variables on profit and welfare is similar. However, when the unit price is relatively small, profit is influenced by both vehicle fleet size and unit price, while social welfare is mainly depending on the vehicle fleet size. This is not found in Figure 5.1. It implies that the operation of ride-sharing services in different markets could be very different. Thus, a reliable operation strategy in one market may be inappropriate in another. Every market should be specifically analyzed. What's more, for the same market, the optimum strategies should also be particularly identified in different operation phases due to distinctive operational objectives.

Noticing that there are some irregular spots in Figure 5.3, this is caused by the unsatisfactory solutions of Equation (3.12) when applying the hybrid algorithm to solve it. But this happens rarely. The equations system is solvable via the hybrid algorithm in nearly all cases.

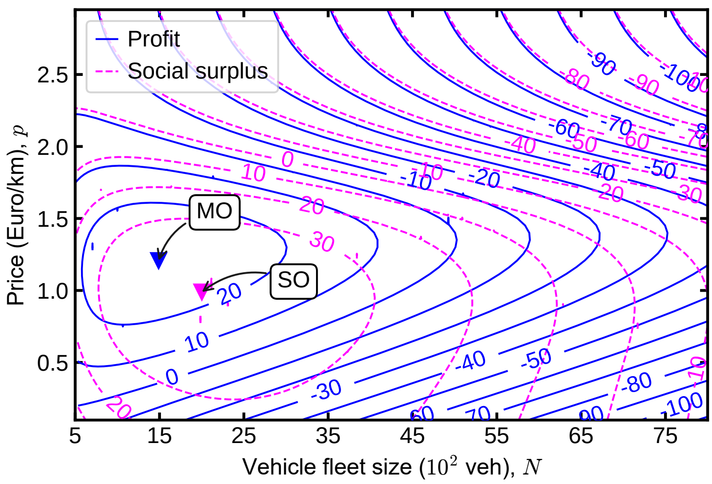
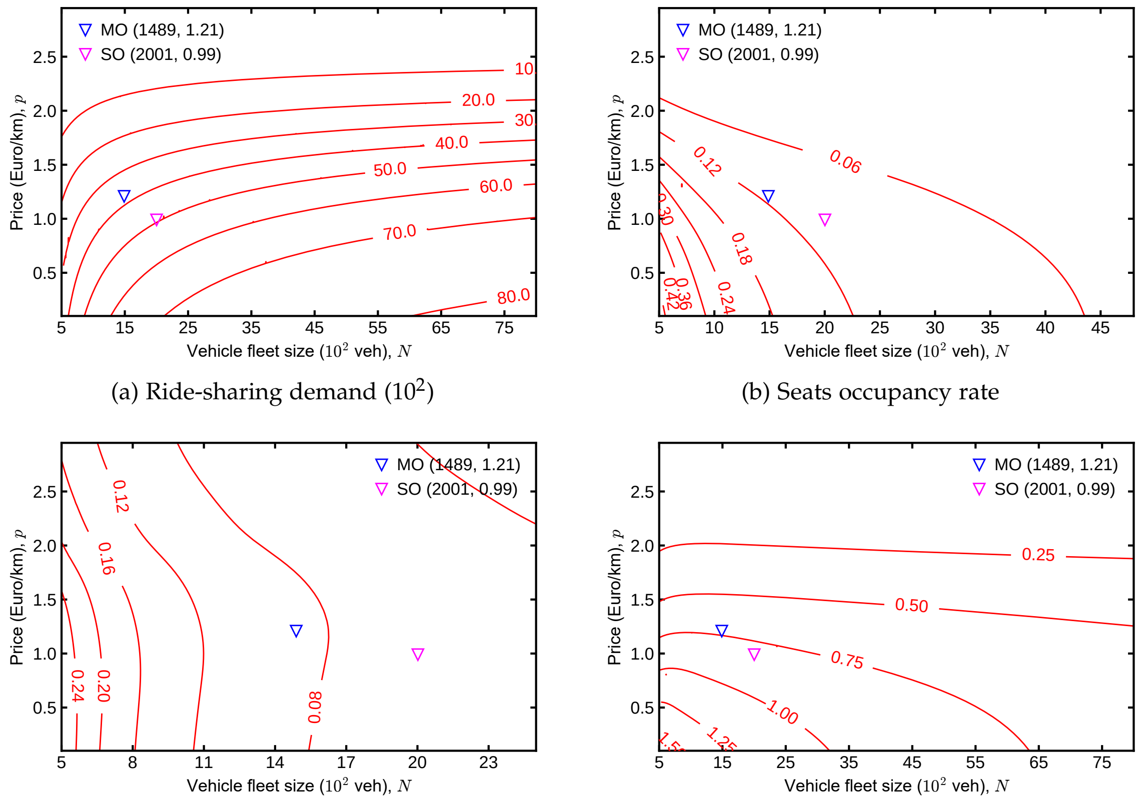


Figure 5.3.: Profit and social welfare of Munich market in a two-dimensional space of vehicle fleet size and unit price.

See Figure 5.4a-Figure 5.4d for the contours for ride-sharing demand, seats occupancy rate, network average detour time and network average waiting time of the Munich network, respectively. Clearly, the contours of ride-sharing demand and seats occupancy rate are very similar to those in the test network. Therefore, we can draw the same conclusions for these variables as in the test network. However, in terms of the network average detour time, the influence of vehicle fleet size is strengthened. The possible reason may be the complicated network structure enhances the heterogeneity of the direct trip time such that ride-sharing demand plays a more important role in determining the detour time. And as we know, ride-sharing demand is obviously dominated by the unit price. Moreover, we can see from Figure 5.4d that when unit price is relatively small, the influence of vehicle fleet size is also been enlarged compared to the same contours in the test network. This analysis also complies with the findings in the comparison between Figure 5.3 and Figure 5.1, namely, the operation of ride-sharing services in different markets could be very different due to the distinctive characteristics, especially the difference in network structure.



(a) Ride-sharing demand (10^2)

(b) Seats occupancy rate

(c) Network average detour time (h)

(d) Network average waiting time (h)

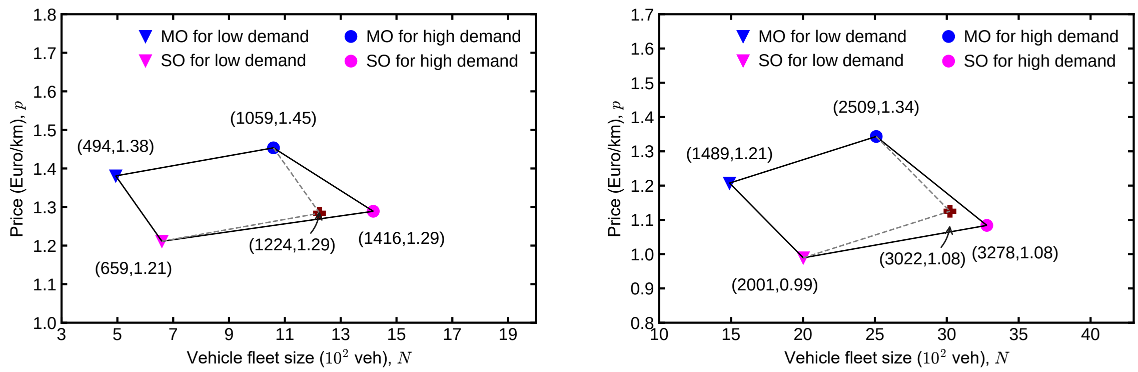
Figure 5.4.: System endogenous variables of Munich network in a two-dimensional space of vehicle fleet size and unit price.

5.6.3. Results comparison for different scenarios

To understand the relation of the optimums between different scenarios (different objectives, different demand levels), we make a comparison for test network and Munich network in Figure 5.5a and Figure 5.5b separately. Let $(N^*_{m,l}, p^*_{m,l})$, $(N^*_{s,l}, p^*_{s,l})$ denote the positions of MO and SO at the low demand level. Let $(N^*_{m,h}, p^*_{m,h})$, $(N^*_{s,h}, p^*_{s,h})$ be the positions of MO and SO at the high demand level. It can be seen that, at the high demand level, we also have $N^*_{m,h} < N^*_{s,h}$ and $p^*_{m,h} > p^*_{s,h}$ in both networks. What's more, in both networks, $N^*_{s,h} - N^*_{m,h} > N^*_{s,l} - N^*_{m,l}$

and $p^*_{m,h} - p^*_{s,h} \approx p^*_{m,l} - p^*_{s,l}$ are valid. That is to say, a ride-sharing provider attempting to maximize the social welfare needs to operate more incremental vehicles in high demand scenario than a provider who aims to maximize its profit.

Denote k_D as the ratio of total travel demand of the high demand scenario in Munich network over the total demand of the low demand scenario, then $k_D = 15254/8264 \approx 1.85$. From another point of view, in the Munich network, we can see that $N^*_{m,h} < k_D N^*_{m,l}$ and $N^*_{s,h} < k_D N^*_{s,l}$. Implicitly, it indicates that service providers can optimize resource deployment based on the demand distribution to reduce the requirement of vehicle fleet size rather than increasing vehicle fleet size in proportion with demand. However, this conclusion is not valid for the test network. A potential reason is we double the travel demand for all OD pairs in the test network which limits the possibility of optimizing the deployment of the resource. Differently, travel demand in the Munich network is from real measurements, and the demand for OD pairs is not increasing with the same proportion in the high demand scenario or even some are reducing.



(a) Test network

(b) Munich network

Figure 5.5.: Optimums comparison for different scenarios.

5.6.4. Convergence analysis

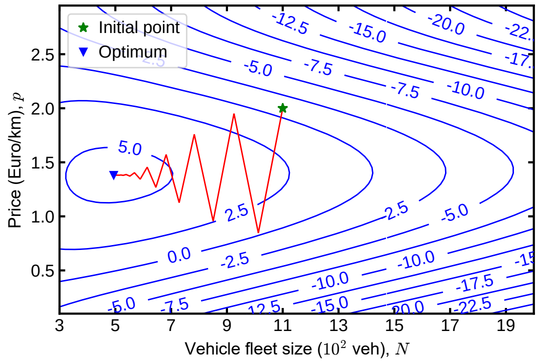
In this section, we evaluate the effectiveness of the GD algorithm on solving (**P1**) and (**P2**). The initialization is given in Table 5.3. As can be seen from Figure 5.6, the trajectory reaches the optimum after a few iterations in all scenarios. This indicates the effectiveness of the proposed GD algorithm (Algorithm 5.1) on solving the optimization problems for the unified pricing method. We can also observe the difference of two markets in terms of profit and social welfare more clearly through comparing the contours in Figure 5.6.

5.7. Sensitivity Analysis to Passenger Preference

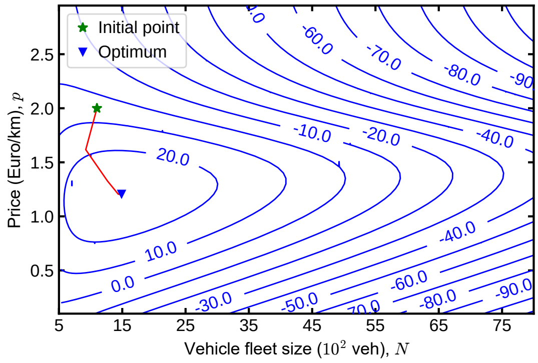
In this section, we analyze the performance of the ride-sharing market in different user groups with particular consideration of age level and income level. Note that, we only perform the

Table 5.3.: Initialization condition of GD for different scenarios.

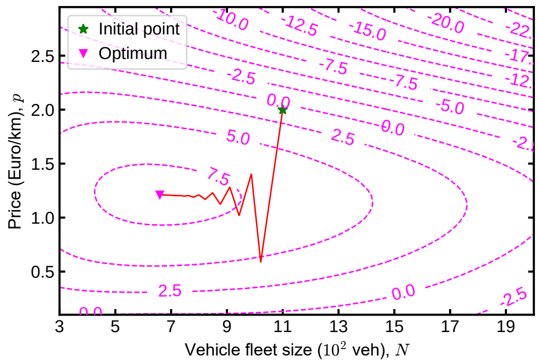
Scenario	$N^{(0)}$	$p^{(0)}$	α_N	α_p	τ_max
Test network (**P1**)	1,100	2.0	10	0.0001	200
Munich network (**P1**)	1,100	2.0	50	0.00001	200
Test network (**P2**)	1,100	2.0	50	0.00001	200
Munich network (**P2**)	1,100	2.0	50	0.000001	200



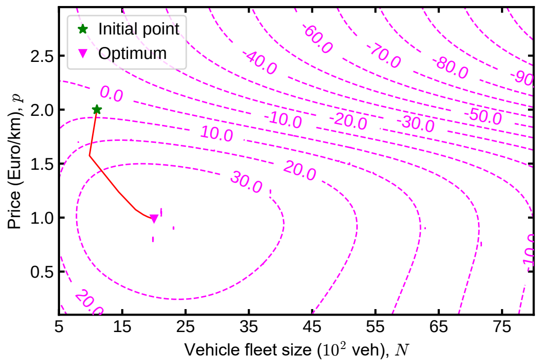
(a) Test network (**P1**)



(b) Munich network (**P1**)



(c) Test network (**P2**)



(d) Munich network (**P2**)

Figure 5.6.: Algorithm iteration process when solving optimization problems for the unified pricing method.

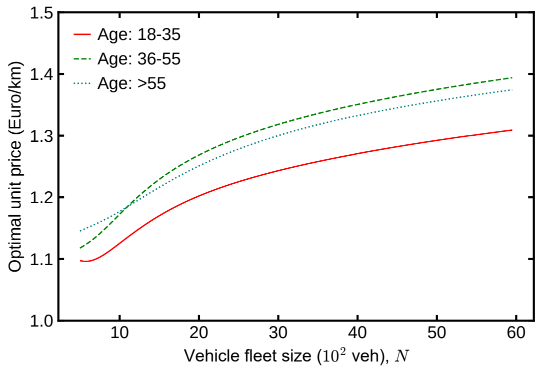
sensitivity analysis regarding age groups and income groups on the low-demand Munich network, while the other scenarios should lead to the same conclusions.

5.7.1. Different age groups

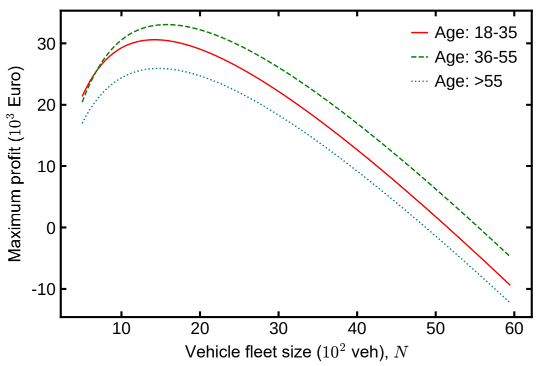
In this experiment, we divide the survey data into three groups based on respondents' age. The grouping rule and coefficient estimates are listed in Table 5.4. For the ease of reading, the estimation results are not detailed here and can be found in Appendix B.1. All estimates are significant with a 99% confidence level. If we assume value-of-time $VOT = (VOT_t + VOT_w)/2$, then we can see that VOT increases with age.

Table 5.4.: Coefficient estimation for different age groups.

Age	β_r	β_t	β_w	VOT_t	VOT_w	VOT	Samples
18-35	-0.716	-0.108	-0.126	9.05	10.56	9.805	216
36-55	-0.601	-0.139	-0.091	13.88	9.08	11.48	858
$\geq$55	-0.556	-0.128	-0.115	13.81	12.41	13.11	162



(a) Optimal unit price



(b) Maximum profit

Figure 5.7.: Performance of ride-sharing services on different age groups.

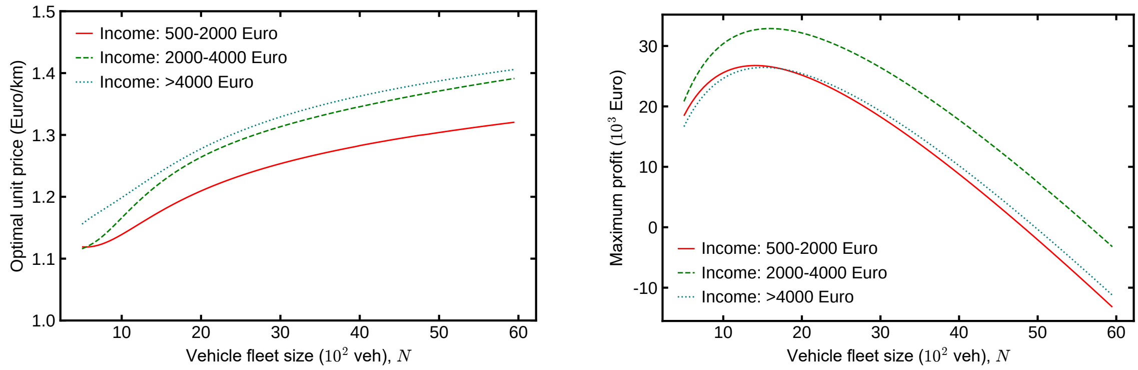
Figure 5.7 illustrates the development of optimal unit price (Figure 5.7a) and profit (Figure 5.7b) of different age groups under different vehicle fleet sizes. It can be seen that, though VOT of the oldest group is highest, the optimal unit price for the middle age group is slightly higher than that for the oldest group when the vehicle fleet size is bigger than 100. From Figure 5.7b, we can also see that service providers can earn more profit in the scenario of the middle-age group. This implies the importance and benefit of considering the passenger preference in the context of a multi-modal transportation system. We can see from Table 5.4 that VOT_w of the middle age group is much smaller than that of the oldest group, while their VOT_ts are very close. From Section 5.6, we know that the waiting time of ride-sharing is relatively high than driving private cars (0 min) but its detour time is still acceptable. As a result, ride-sharing is more popular in the middle-age group. Thus, the optimal unit price is relatively high. On the other hand, since the youngest group is most sensitive to the trip fare (β_r is biggest), thus the unit price is relatively small. Moreover, the maximum profit of the oldest group is smaller than that of the other groups, indicating its demand is smaller than the others.

5.7.2. Different income groups

In this experiment, we divide the survey data into three groups based on respondents' income. Again, for the purpose of presentation simplicity, the estimation results are not detailed here and can be found in Appendix B.2. All estimates are significant with a 99% confidence level. And as shown in Table 5.5, VOT increases with income as well.

Table 5.5.: Coefficient estimation for different income groups.

Income (Euro)	β_r	β_t	β_w	VOT_t	VOT_w	VOT	Samples
500-2000	-0.601	-0.106	-0.112	10.58	11.18	10.88	480
2000-4000	-0.781	-0.170	-0.151	13.06	11.60	12.33	282
$\geq$4000	-0.514	-0.135	-0.104	15.76	12.14	13.95	486



(a) Optimal unit price

(b) Maximum profit

Figure 5.8.: Performance of ride-sharing services on different income groups.

Figure 5.8 shows the optimal unit price and performance of ride-sharing in different user groups under different vehicle fleet sizes. It can be seen that the optimal unit price for the richest group is always the highest, while the maximum profit is much less than that of the middle-income group. From Table 5.5, we know the middle-income group is most sensitive to the trip fare. Nevertheless, its optimal unit price is still bigger than that for the poorest group, since it is also the most sensitive group to waiting time (ride-sharing always has a longest waiting time). Analogously, this can also explain why the optimal unit price for the middle-income group is slightly smaller than that for the richest group despite its sensitivity to travel time and waiting time is stronger than that of the richest group. Interestingly, we can see from Figure 5.8b that service providers can get more profit in the scenario of the middle-income group. This is also in accords with the findings in the age group experiments and can also explain in a similar way.

5.8. Conclusions

In this chapter, we propose a unified pricing method for ride-sharing services. Under this unified pricing method, the trip fare is linearly related to the travel distance, i.e., $r_i = pd_i$. This method has been widely used in taxi markets, ride-sharing markets, and other transport mobility markets, since it is the easiest method to implement. We evaluate the performance of the ride-sharing market installed with the unified pricing method under different operation strategies. Note that, the evaluation is based on the market equilibrium model presented in Chapter 3. The results show that the operation of ride-sharing services in different markets (cities) could be very different due to the distinctive network structure. This indicates the importance of considering network structure and OD demand pattern in the market equilibrium model, which is in accord with one of the motivations of this study.

Furthermore, a GD algorithm is developed to find the optimal solutions for the monopoly scenario and the social optimum scenario. The method to calculate the derivatives of the objectives is explicitly explained. By comparing the optimums of different scenarios, one can draw that, the unit price for the monopoly optimum is higher than that for the social optimum, while the vehicle fleet size for the monopoly optimum is smaller than that for the social optimum. Moreover, the difference between the optimal vehicle fleet sizes in MO and SO becomes greater in the high demand level scenario, while the difference between the optimal unit prices almost keeps the same. Besides, the GD algorithm is deemed to be effective in solving the optimization problems for the unified pricing method based on the convergence analysis presented in Section 5.6.4.

In addition, the performance of ride-sharing services in different user groups is evaluated. Particularly, we estimate the passenger preference for different age groups and income groups, and analyze the popularity and performance of ride-sharing in these groups. The result implies that ride-sharing operators can get more profit by providing services between middle-age communities and middle-income communities.

6. Spatial Pricing Method

6.1. Objectives and Contributions

Demand for and supply of ride-sharing services are allocated over the network, and thus equilibrium modeling of the problem should be conducted with a consideration of the network structure and a customer OD demand pattern (Yang et al., 2002). However, apart from accounting for the spatial structure in the equilibrium model, a specific trail for the spatial difference of pricing is also beneficial. Applying different pricing schemes for trips with different ODs has the potential to optimize the operation strategy and objective. Thus, in this chapter, the spatial structure of the market is explicitly considered for developing a spatial pricing method to improve the equilibrium model presented in Chapter 3 integrated with the pricing method introduced in Chapter 5.

The fare structure in the spatial pricing method is described in Section 6.2. Section 6.3 introduces the algorithm applied to optimize the operation strategy. Section 6.5 explains simulation settings for the experiments in this chapter. Section 6.6 analyzes the results of different experiments and evaluates the proposed solution algorithms. Section 6.7 concludes this chapter and suggests future works.

6.2. Fare Structure

By applying the spatial pricing method, the unit prices for different OD pairs are different. In other words, the trip fare is given by

$$r_i = p_i d_i \tag{6.1}$$

where p_i is the unit price for OD pair i.

6.3. Solution Algorithm

We also apply the GD algorithm to solve monopoly optimum and social optimum for the proposed spatial pricing method. The GD algorithm is given in Algorithm 6.1. Different from Algorithm 5.1, the derivatives with respect to p_z has to be specified for different z (line 4 and line 5), where z is another numbering for OD pairs in order to distinguish it from i.

Algorithm 6.1 Gradient Descent for solving MO and SO for spatial pricing

Initialize $O^{(0)}, \alpha, \tau_{max}$
$\tau = 0$
for $\tau < \tau_{max}$ **do**
  Compute the market equilibrium state: $r_i(O^{(\tau)}), w_i(O^{(\tau)}), \tilde{t}_i(O^{(\tau)}), P_i(O^{(\tau)}), Q_i(O^{(\tau)})$
  Compute $\frac{\partial t_i}{\partial p_z}, \frac{\partial w_i}{\partial p_z}, \frac{\partial t_i}{\partial N}, \frac{\partial w_i}{\partial N}, \forall i, z$, with $O^{(\tau)}$ as the operation condition
  Compute $\frac{\partial \mathcal{J}}{\partial p_z}, \frac{\partial \mathcal{J}}{\partial N}, \forall z$, with $O^{(\tau)}$ as the operation condition
  $g^{(\tau)} = \nabla \mathcal{J}(O^{(\tau)})$
  if $g^{(\tau)} < \varepsilon$ **then**
    $O^* = O^{(\tau)}$
    break
  end if
  $O^{(\tau+1)} = O^{(\tau)} + \alpha g^{(\tau)}$
  $\tau = \tau + 1$
end for

6.4. Partial Derivatives of Related Variables

Note that, the derivatives with respect to N are the same as that in the unified pricing method. However, we need to specify the derivatives for different p_z. Intrinsically, applying different unit prices for different OD pairs affects the derivatives of trip fare. Then the difference of the derivatives of the trip fare further influences derivatives of ride-sharing demand. In the spatial pricing method, the derivative of profit with respect to p_z is given by

$$\frac{\partial \Pi}{\partial p_z} = \sum_i \left(\frac{\partial Q_i}{\partial p_z} r_i + Q_i \frac{\partial r_i}{\partial p_z}\right) \tag{6.2}$$

Note that, the derivative of trip fare with respect to p_z is given by

$$\frac{\partial r_i}{\partial p_z} = \begin{cases} d_i & \text{if } i = z \\ 0 & \text{otherwise} \end{cases} \tag{6.3}$$

And the derivative of ride-sharing demand is given by

$$\frac{\partial Q_i}{\partial p_z} = \begin{cases} D_i P_i (1 - P_i)(\beta_t \frac{\partial t_i}{\partial p_z} + \beta_w \frac{\partial w_i}{\partial p_z} + \beta_r d_z) & \text{if } i = z \\ D_i P_i (1 - P_i)(\beta_t \frac{\partial t_i}{\partial p_z} + \beta_w \frac{\partial w_i}{\partial p_z}) & \text{otherwise} \end{cases} \tag{6.4}$$

Thus, we have

$$\frac{\partial \Pi}{\partial p_z} = \begin{cases} \sum_i D_i P_i (1 - P_i)(\beta_t \frac{\partial t_i}{\partial p_z} + \beta_w \frac{\partial w_i}{\partial p_z} + \beta_r d_z) r_i + \sum_i Q_i d_i & \text{if } i = z \\ \sum_i D_i P_i (1 - P_i)(\beta_t \frac{\partial t_i}{\partial p_z} + \beta_w \frac{\partial w_i}{\partial p_z}) r_i & \text{otherwise} \end{cases} \tag{6.5}$$

For the purpose of convenience, we repeat Equation (5.7) here.

$$\frac{\partial \Pi}{\partial N} = \sum_i \frac{\partial Q_i}{\partial N} r_i - \phi = \sum_i D_i P_i (1 - P_i)(\beta_t \frac{\partial t_i}{\partial N} + \beta_w \frac{\partial w_i}{\partial N}) r_i - \phi \tag{6.6}$$

Let $\boldsymbol{t}$ and $\boldsymbol{w}$ denote the vector of travel time of all OD pairs and the vector of waiting time of all OD pairs, respectively. It can be seen that, $\forall z \in \mathbb{Z}$, derivative of Π with respect to p_z are functions of derivative of $\boldsymbol{t}$ with respect to p_z and derivative of $\boldsymbol{w}$ with respect to p_z. And derivative of Π with respect to N are functions of derivative of $\boldsymbol{t}$ with respect to N and derivative of $\boldsymbol{w}$ with respect to N.

On the other hand, the derivative of welfare with respect to p_z is given by

$$\frac{\partial S}{\partial p_z} = \sum_i d_i \left[\frac{Q_i}{\beta_r d_i}(-\beta_t \frac{\partial t_i}{\partial p_z} - \beta_w \frac{\partial w_i}{\partial p_z}) + \frac{\partial Q_i}{\partial p_z} F_i(Q_i) \right] \tag{6.7}$$

Substitute Equation (6.4) into Equation (6.7) resulting in

$$\frac{\partial S}{\partial p_z} = \begin{cases} \sum_i d_i \left[\frac{Q_i}{\beta_r d_i}(-\beta_t \frac{\partial t_i}{\partial p_z} - \beta_w \frac{\partial w_i}{\partial p_z}) + D_i P_i (1 - P_i)(\beta_t \frac{\partial t_i}{\partial p_z} + \beta_w \frac{\partial w_i}{\partial p_z} + \beta_r d_z) F_i(Q_i) \right] & \text{if } i = z \\ \sum_i d_i \left[\frac{Q_i}{\beta_r d_i}(-\beta_t \frac{\partial t_i}{\partial p_z} - \beta_w \frac{\partial w_i}{\partial p_z}) + D_i P_i (1 - P_i)(\beta_t \frac{\partial t_i}{\partial p_z} + \beta_w \frac{\partial w_i}{\partial p_z}) F_i(Q_i) \right] & \text{otherwise} \end{cases} \tag{6.8}$$

For ease of reading, we also repeat Equation (5.11) at below.

$$\frac{\partial S}{\partial N} = \sum_i \left[\frac{Q_i}{\beta_r}(-\beta_t \frac{\partial t_i}{\partial N} - \beta_w \frac{\partial w_i}{\partial N}) + D_i P_i (1 - P_i)(\beta_t \frac{\partial t_i}{\partial N} + \beta_w \frac{\partial w_i}{\partial N}) F_i(Q_i) \right] - \phi \tag{6.9}$$

Clearly, analogous to the findings in the unified pricing method, derivatives of S are also functions of derivatives of $\boldsymbol{t}$ and $\boldsymbol{w}$.

Note that, the calculation of derivatives of $\boldsymbol{t}$ and $\boldsymbol{w}$ for the spatial pricing method follows the same procedure for the calculation of derivatives of $\boldsymbol{t}$ and $\boldsymbol{w}$ in the unified pricing method presented in Chapter 5. For the simplicity of reading, the details will be omitted here and can be found in Appendix A.2.

6.5. Simulation Settings

We conduct experiments on the test network to analyze the performance of ride-sharing on the travel for different OD pairs. As test network is simple and the difference (spatial difference and demand difference) between different OD pairs are apparent, it is easy to choose typical OD pairs for comparison.

In terms of Munich network, we exploit the relationship between optimal unit price and OD distance, as well as the relationship between optimal unit price and ride-sharing demand. Since we have more OD pairs in Munich network, it provides the possibility for us to draw statistical conclusions. What's more, we also try to analyze the importance of considering user heterogeneity when applying spatial pricing method. Figure 6.1 depicts the heterogeneous regions in the Munich network. The regions with read shadow are assumed to be cost-insensitive regions, and the preference of users from and to these regions is estimated with the coefficients from the higher-income (≥4000 Euro monthly) group. On contrary, trips between other regions will be estimated based on the lower-income (<4000 Euro monthly) group. For ease of reading, the coefficient estimation results are listed in Appendix B.2. Besides, we also make a comparison between unified pricing method and spatial pricing method on this network.

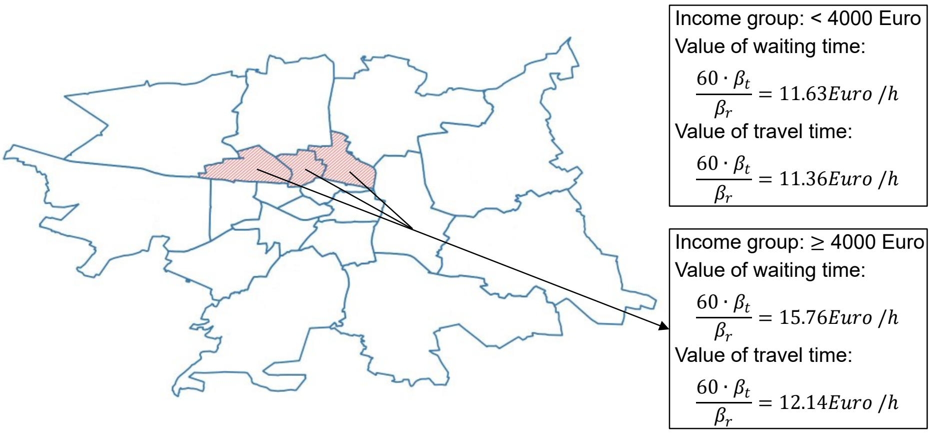


Figure 6.1.: Munich network with heterogeneous regions.

6.6. Case Study and Results

Section 6.6.1 evaluates the performance ride-sharing market using the spatial pricing method on the test network under different vehicle fleet sizes. Section 6.6.2 analyzes the relationship of the optimal unit price and the travel distance, ride-sharing demand for different OD pairs on the Munich network. Section 6.6.3 compares the operation in the Munich network with and without considering user heterogeneity, under unified pricing and spatial pricing. Section 6.6.4 provides a convergence analysis for the proposed GD algorithm on the optimization problems for the spatial pricing method.

6.6.1. Performance of ride-sharing on different OD configurations (test network)

This analysis depends on the results of the test network. Figure 6.2 shows the maximum profit or maximum welfare of operation of ride-sharing in different scenarios on the test network under different vehicle fleet size. In the monopoly scenarios, the difference in maximum profit in the low-demand period and high-demand period increases with the vehicle fleet size. In the social optimum scenarios, we can also observe that the difference in maximum welfare in the low-demand period and high-demand period increases with vehicle fleet size.

To understand the performance and popularity of ride-sharing services in different ranges of distance and travel time, we select three representative OD pairs from the test network for analysis. Figure 6.3a shows the ride-sharing demand for the selected OD pairs under different vehicle fleet sizes, while Figure 6.3b depicts the market share of ride-sharing. Figure 6.3b says that ride-sharing is more popular for long-distance and long-time trips. Besides, from both Figure 5.2a and Figure 6.3b, we can see that the slope of lines are reducing. It means

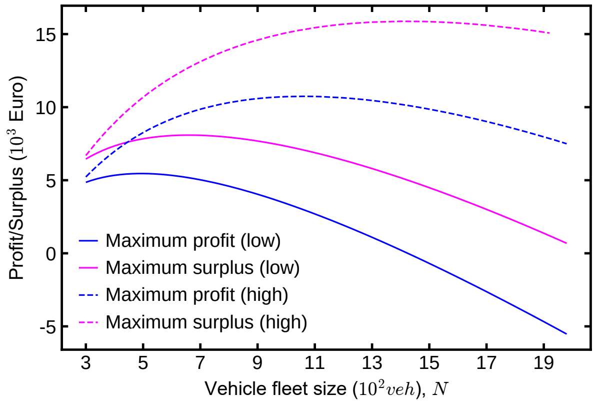
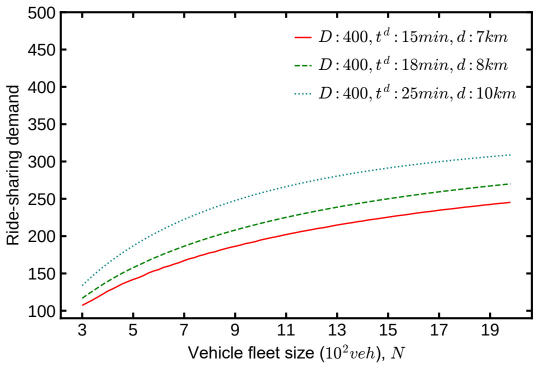
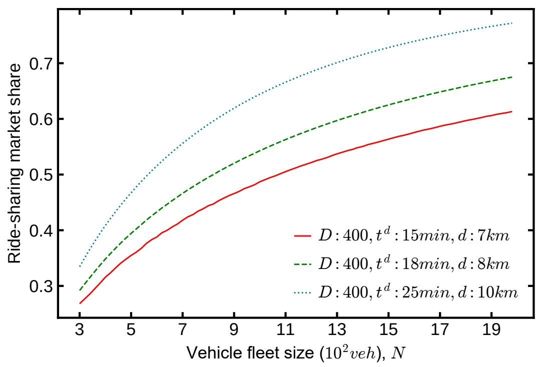


Figure 6.2.: Maximum profit and welfare with different vehicle fleet sizes.

providers need to operate more and more vehicles to increase the same range of market share of ride-sharing when its operating scale is relatively large (more vehicles).



(a) Ride-sharing demand



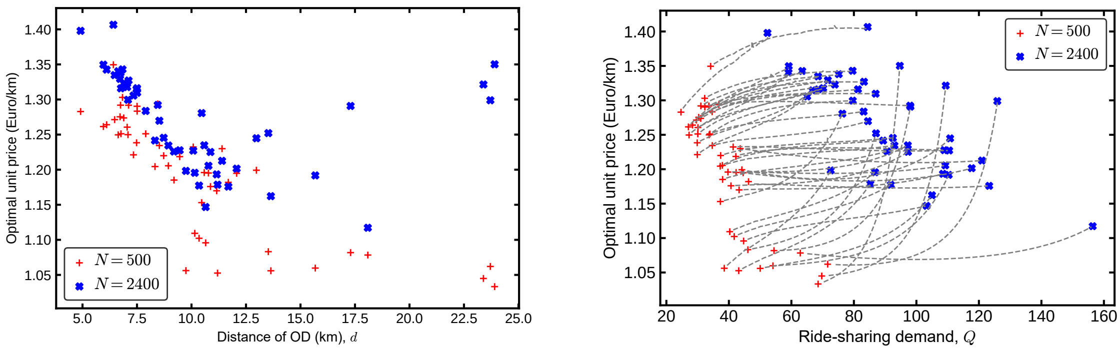
(b) Market share

Figure 6.3.: Ride-sharing performance on different OD pairs with different vehicle fleet sizes.

6.6.2. Relationship between optimal unit price and OD-related variables (Munich network)

Figure 6.4a shows the relationship between the optimal unit price and the distance of OD, where each point stands for an OD. We can see that when the distance less than 10 km, the optimal unit price is linearly reducing with the distance in both high supply level and low supply level. This somehow indicates the necessity of considering spatial heterogeneity in the market equilibrium model such that a more appropriate optimal unit price set can be applied on the network. But when the distance greater than 10 km, this relationship does not hold, especially in the high supply level. The potential reason could be demand exceeds supply for these ODs due to the popularity of ride-sharing in long-distance trips. Thus, the unit price could be increased to adjust the demand.

Figure 6.4b describes the relationship between the optimal unit price and ride-sharing demand. Clearly, the optimal unit price is also linearly related to the ride-sharing demand for the same OD in both the low supply level and high supply level. Moreover, if we draw the curve of optimal unit price along with the increase of supply, it can be seen that, there is an obvious rise of the optimal unit price for the ODs with high demand but the low unit price in the low supply level. And the unit price for ODs with low ride-sharing demand in the low supply level sees a slight increase. However, the unit price for ODs with middle-range ride-sharing demand almost keeps the same. It means if the service provider invests more to scale up its service, trips for ODs with high ride-sharing demand are influenced most, followed with that for ODs with low demand.



(a) Distance of OD

(b) Ride-sharing demand

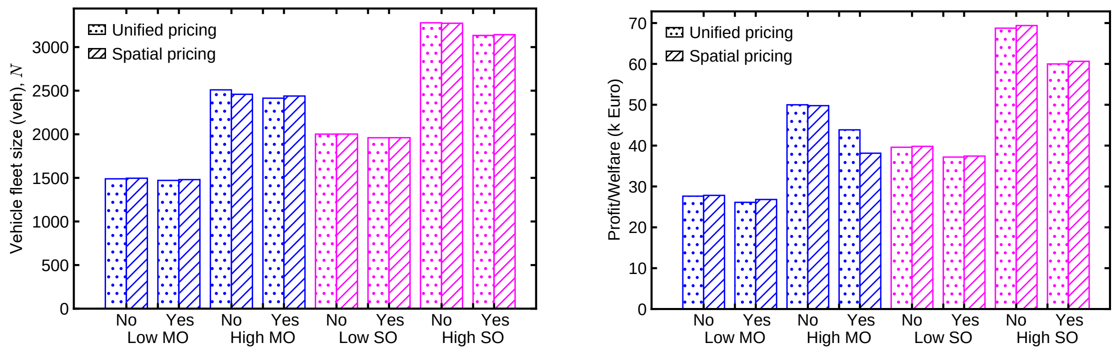
Figure 6.4.: OD-based optimal unit price for different vehicle fleet sizes.

6.6.3. Influence of considering user heterogeneity (Munich network)

In this section, we compare the performance of ride-sharing services utilizing unified pricing and spatial pricing with and without considering user heterogeneity.

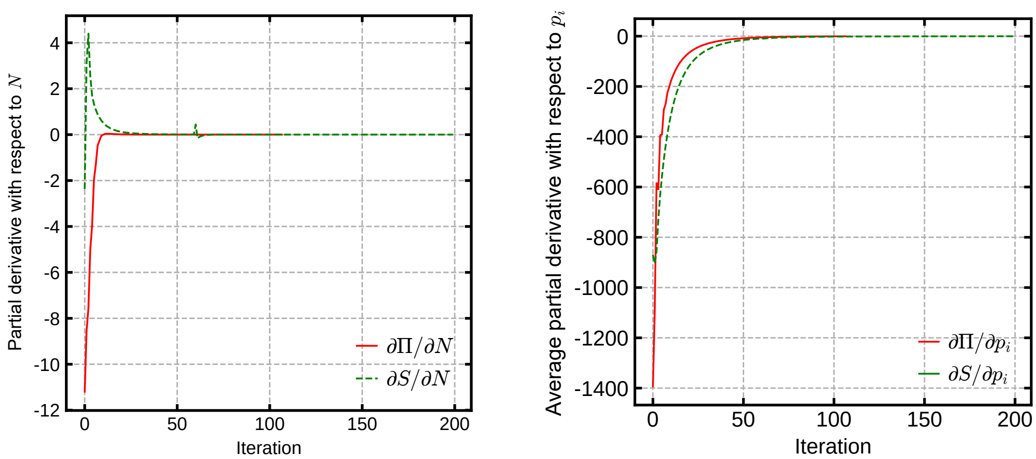
First, we compare the difference in the optimal vehicle fleet size in Figure 6.5a. Note that in the X-axis, *No* stands for without considering user heterogeneity, while *Yes* indicates with a consideration of user heterogeneity. *Low* is the low-demand period, while *High* means the high-demand period. Recall that MO is the monopoly optimum scenario, while SO denotes the social optimum scenario. From the figure, we can see a reduction of the optimal vehicle fleet size after considering user heterogeneity in all scenarios. This implies the necessity of considering user heterogeneity. If we can estimate the user preference more accurately, then the optimal solutions for scenarios are more reliable and thus could even reduce the operating budget. However, we cannot observe an evident change of the optimal vehicle fleet size by applying the spatial pricing method compared to the unified pricing method in these scenarios. Thus, it means the spatial pricing method can improve the pricing settings but have no influence on the solution for the supply side.

On the other hand, considering user heterogeneity can also help the service provider evaluate the risk and revenue of operating ride-sharing services in a more accurate and conservative way, as shown in Figure 6.5b. However, out of expectation, we can only see a very limited increase in the optimal objectives by applying the spatial pricing method in these scenarios, and even has an exception of a reduction in *High MO*. In our opinion, this is caused by the limitation of the proposed market equilibrium model. Recall that in Section 5.6.1, we proved $\tilde{t}_i \propto 1/N$ and $w_i \propto Q_i/\sqrt{Nn_s - \sum_j Q_j t_j}$. These relationships actually imply a hidden assumption of the proposed market equilibrium model—vehicles are evenly distributed to OD pairs. In other words, it assumes every OD is deployed almost the same number of vehicles. So a potential direction for future work is to consider the distribution of vehicles in a more accurate way.



(a) Optimal vehicle fleet size (b) Money

Figure 6.5.: Difference between before and after considering user heterogeneity.



(a) Derivatives with respective to fleet size (b) Derivatives with respective to unit prices

Figure 6.6.: Convergence of GD on solving MO and SO for spatial pricing method.

6.6.4. Convergence analysis

Figure 6.6 shows the performance of GD on solving the monopoly optimum and social optimum for the proposed spatial pricing method. Note that, in Figure 6.6b, the derivatives with respect to different p_i are averaged at each iteration for the purpose of simplicity of presentation. The initial parameters are the same as the ones used in the unified pricing method, which is listed in Table 5.3. The results show that GD is suitable for solving these problems. The derivatives with respect to N for both Π and S converge to zero after about 20 iterations. And the averages of derivatives with respect to p_i also converge to zero after around 50 iterations.

6.7. Conclusions

In this chapter, we propose a spatial pricing method for ride-sharing services. Under this spatial pricing method, the linear relationship between the trip fare and travel distance is measured by different unit prices in different OD pairs, i.e., $r_i = p_i d_i$. It shows that the discrepancy of the operation in low demand scenario and high demand scenario becomes greater with the increase of vehicle fleet size (operation scale), based on the results of experiments on the test network. Likewise, it indicates that ride-sharing is more popular for long-distance and long-time trips.

Furthermore, based on the experiment results on the Munich network, it is found that the optimal unit prices for different OD pairs are negatively linear to their distance and ride-sharing demand. However, the linear relationship of the optimal unit prices and OD distance does not hold when the distance becomes extremely long. From the trend of optimal unit prices for different OD pairs with the increase of vehicle fleet size, it recommends that service provider should increase the unit prices for the OD pairs with high ride-sharing demand, while that for the OD pairs with middle-range demand should keep the same.

User heterogeneity can influence the reliability of the solutions calculated from the experiments. This proof is observed in Section 6.6.3 by comparing the optimal vehicle fleet sizes and maximum profits. Considering user heterogeneity can also help the service provider better evaluate the risk and potential revenue of operating ride-sharing services. Though the benefits of applying the spatial pricing method have not been observed in the experiments conducted in this chapter due to the limitation of the proposed market equilibrium model, it is a candidate method for improving vehicle deployment (or vehicle rebalancing) and operation profit as pointed out in Bimpikis et al. (2019).

Similarly, we apply the GD algorithm to solve the optimal unit prices set and vehicle fleet size for the spatial pricing method. And the convergence analysis proves its effectiveness in these problems.

7. Utility-based Compensation Method

7.1. Objectives and Contributions

Due to the discrepancy of ODs and the uncertainty contained in ride-sharing services, passengers always spend different monetary costs and time costs (waiting time, detour time) for trips. This difference causes the inequity among the served passengers, represented by the variance of LOS. Since the utility function applied in this study is a linear combination of trip fare, waiting time, and detour time, it is plausible to hypothesize the utility is somehow tantamount to LOS. In other words, the LOS of ride-sharing can be captured and represented by the utility function applied in this study. As a result, in this chapter, we propose a utility-based compensation method for ride-sharing services. It aims to improve the equity among the passengers by reducing the standard deviation of trips' utilities. Apart from equity, it is expected to improve the LOS for ride-sharing by applying this method. The contributions of this chapter are two-fold.

1) We develop a utility-based compensation pricing method for ride-sharing services, which can improve the LOS and equity for passengers. To the best of our knowledge, this is the first study of compensation methods for ride-sharing services.

2) We proposed an algorithm to optimize the related variables for the proposed pricing method.

The rest of this chapter is organized as follows. Section 7.2 introduces the general methodology of the compensation method. Section 7.3 describes the algorithm for optimizing relevant variables in specific scenarios. Section 7.4 details the simulation environment and experiment settings for case studies. Section 7.5 presents and analyzes the results of the case study. Section 7.6 summarizes the main findings and provides potential directions for future study.

7.2. General Methodology

As described above, the utility function used in the proposed market equilibrium model can represent the LOS of ride-sharing services to some extent. In this sense, the LOS of the operation can be described by the mean utility of all trips, while the equity of the services can be captured by the standard deviation of utilities. And considering the spread of utilities due to difference in ODs and uncertainty of travel time and waiting time, we aim to enhance the equity among trips by compensating part of trips. On the other hand, this can attract more demand and probably result in longer waiting time and detour time, which may thus impede the improvement of LOS. However, generally, it can indeed improve the LOS of the entire operation. Figure 7.1 illustrates the proposition of the compensation method—compensate trips whose utility is below a predefined value. Further questions would be how much should be compensated for different trips (or the compensation principle).

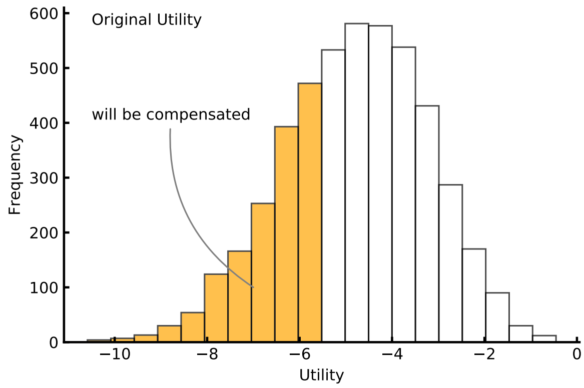


Figure 7.1.: Presentation of the proposed utility-based compensation method.

To define the compensation principle, we need to specify the threshold value, which is termed as compensation reference point (CRP) in this study, and the method to calculate the amount of compensation, which is termed as compensation function. Trips below CRP will be compensated with an amount of money computed by the compensation function. Moreover, in order to connect CRP with actual utilities of trips, we define CRP as a proportion of the mean of trips utilities, which can be written as

$$a = \alpha \bar{V} \tag{7.1}$$

where α is named as compensation reference factor (CRF).

7.2.1. Variables distributions

In unified and spatial pricing method, we compute system endogenous variables $Q_i, w_i, \tilde{t}_i, \forall i$, under certain operation strategy (N, p) or $(N, \boldsymbol{p})$ directly through the market equilibrium model, where all trips are considered at the network level and variables are aggregated based on OD. Differently, the compensation method developed in this chapter is individual-based. Thus, we make the following assumptions with respect to the related variables for ease of processing.

Assumption 7.1 *Attribute x of trips for OD pair i follows a normal distribution with μ_x and μ_x/ξ as the mean and standard deviation, respectively, where ξ is an exogenous positive parameter. Specifically,*

1) $t^d_{i,k} \sim \mathcal{N}(t^d_i, t^d_i/\xi)$;

2) $d_{i,k} \sim \mathcal{N}(d_i, d_i/\xi)$;

3) $w_{i,k} \sim \mathcal{N}(w_i, w_i/\xi)$;

4) $\tilde{t}_{i,k} \sim \mathcal{N}(\tilde{t}_i, \tilde{t}_i/\xi)$.

where k denotes the index of trips.

Under Assumption 7.1, we can calculate the utility of individual trips by

$$V_{i,k} = \beta_t t_{i,k} + \beta_w w_{i,k} + \beta_r r_{i,k} \tag{7.2}$$

where $t_{i,k} = \tilde{t}_{i,k} + t^d_{i,k}$. Note that, other appropriate distributions can also be applied and would not make any difference in the following analysis.

7.2.2. Compensation function

On the other hand, we emphasize that we do compensation based on the trip's utility. The utility of a trip after compensation must satisfy a predefined function termed as compensated utility function, which describes the relationship between the utility before and after compensation.

Note that, in this method, the fare structure is given by

$$r^a_{i,k} = pd_{i,k} + c_{i,k} \tag{7.3}$$

where i denotes OD, k denotes the index of trips, p is the unit price, $c_{i,k}$ is the amount of compensation, $r^a_{i,k}$ is the trip fare after compensation. Such that the compensation is given by

$$c_{i,k} = \frac{V^a_{i,k} - V_{i,k}}{\beta_r} \tag{7.4}$$

where β_r is the monetary preference coefficient, $V_{i,k}$ is the utility before compensation, and $V^a_{i,k}$ is the utility after compensation.

Furthermore, we state that compensated utility function should follow Prerequisite 7.1.

Prerequisite 7.1 *A compensated utility function is a function that describes the relationship between the utilities of trips before and after compensation. It is the base of the calculation of compensation for every trip. Every compensated utility function should satisfy the following conditions.*

1) *The utility after compensation should not bigger than the compensation reference point.*
2) *The order of trips sorted by utility should not change after compensation.*
3) *Trips with a utility farther below the compensation reference point should get more compensation than those closer.*

We apply the following compensated utility function to calculate the utility after compensation in this study. Its curve is shown in Figure 7.2. For the ease of reading, its derivation is omitted here and can be found in Appendix A.3.

$$V^a = \begin{cases} V & \text{if } V > a \\ -\sqrt{2aV - a^2} & \text{otherwise} \end{cases} \tag{7.5}$$

As a result, the compensation function is given by

$$c_{i,k} = \begin{cases} 0 & \text{if } V_{i,k} > a \\ \frac{1}{\beta_r}(-\sqrt{2aV_{i,k} - a^2} - V_{i,k}) & \text{otherwise} \end{cases} \tag{7.6}$$

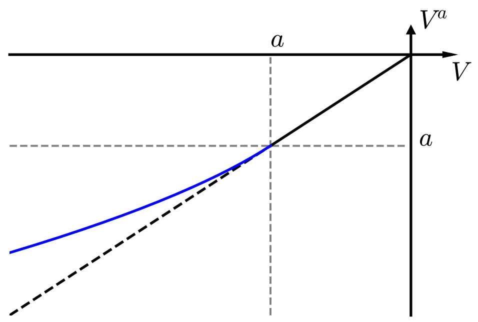


Figure 7.2.: The shape of the compensation function.

7.2.3. Market equilibrium

Furthermore, after compensation, trips need to be re-aggregated (averaged) again to the network level to calculate the new market equilibrium for the situation after compensation. The new trip fare is given by

$$\check{r}_i = \frac{1}{D_i} \sum_k r^a_{i,k} \tag{7.7}$$

Then the expected utility is given by

$$\check{V}_i(\check{\boldsymbol{Q}}) = \beta_t (t^d_i + \frac{A_i \sum_j \check{Q}_j t^d_j}{N \sum_j \check{Q}_j}) + \beta_w \frac{B\check{Q}_i}{\sqrt{Nn_s - \sum_j \check{Q}_j \check{t}_j}} + \beta_r \check{r}_i \tag{7.8}$$

And the ride-sharing demand can be calculated by

$$\check{Q}_i = D_i \frac{e^{\check{V}_i(\check{\boldsymbol{Q}})}}{e^{\check{V}_1(\check{\boldsymbol{Q}})} + \mu_i} \tag{7.9}$$

Finally, one can construct and solve another simultaneous equations system for the market with compensation according to Equation (7.10) to calculate ride-sharing demand at the market equilibrium. Then substitute these values into functions of other system endogenous variables to calculate the corresponding variables.

$$\begin{cases} \check{Q}_{i_1} &= \frac{D_{i_1} e^{\check{V}_{i_1}(\check{\boldsymbol{Q}},N)}}{e^{\check{V}_{i_1}(\check{\boldsymbol{Q}},N)} + \mu_{i_1}} \\ \check{Q}_{i_2} &= \frac{D_{i_2} e^{\check{V}_{i_2}(\check{\boldsymbol{Q}},N)}}{e^{\check{V}_{i_2}(\check{\boldsymbol{Q}},N)} + \mu_{i_2}} \\ \vdots & \vdots \\ \check{Q}_{i_m} &= \frac{D_{i_m} e^{\check{V}_{i_m}(\check{\boldsymbol{Q}},N)}}{e^{\check{V}_{i_m}(\check{\boldsymbol{Q}},N)} + \mu_{i_m}} \\ \vdots & \vdots \end{cases} \tag{7.10}$$

It is worth pointing out that, the market with a compensation scheme indeed will attract more demand for ride-sharing services. However, the CRP for a market is decided only by the origin trips. That is to say, the value of CRP (or CRF) only depends on the market without compensation.

7.3. Solution Algorithm

Since trips have an aggregation-disaggregation-reaggregation process in this method, thus it is very difficult to calculate the relevant derivatives of variables, so we apply a heuristic to solve the optimization problems. Note that, except for optimizing vehicle fleet size and unit price, we also need to optimize the CRF α to seek the optimal compensation scheme.

Define $\alpha_{max} \triangleq V_{min}/\bar{V}$, where $\bar{V} = \sum_i \sum_k V_{i,k} / \sum_i \sum_k 1$ denotes the mean of all trips, V_{min} is the lowest utility. Define the operation strategy as $O = (N, p, \alpha)$. Let $O_{max} = (N_{max}, p_{max}, \alpha_{max})$, where N_{max} and p_{max} are the upper bound of the search area of vehicle fleet size and unit price, respectively. We apply the Particle Swarm Optimization (PSO) algorithm (Kennedy and Eberhart, 1995; Reyes-Sierra and Coello Coello, 2006) to solve problem (**P1**) and (**P2**) in the corresponding scenario. Since both problems are maximization problem, we denote $\mathcal{J}(O)$ as the objective.

Algorithm 7.1 describes the procedure of applying PSO. First, we need to initialize the position, velocity and personal best for each particle in the swarm, together with the global best of the entire swarm (line 1 - line 9). At each iteration, we need to update the velocity of each particle based on their distance to the personal best and the global best (line 15). And the updated velocity is applied to update the position of the particle (line 16). Then the updated particle is evaluated based on the results of the market equilibrium (line 17). By comparing its performance on the objective with the personal best and global best, decide if need to update the personal best (line 18 - line 19) and global best (line 20 - line 21). In the algorithm, ω is the inertia weight employed to control the impact of the historical velocities on the current velocity. C_1, termed as cognitive learning factor, represents the attraction of the personal best. C_2, termed as social learning factor, represents the attraction of the global best (also known as leader). τ_{max} and n_p are the maximum iterations and the number of particles in the swarm, respectively. We refer the interested readers to Reyes-Sierra and Coello Coello (2006) for a comprehensive review of this algorithm.

7.4. Simulation Settings

Two experiments are conducted on the Munich network in this chapter. First, we do sensitivity analysis with respect to the CRF α under the MO operation strategy of the unified pricing method to analyze the influence of the introduction of the compensation method with different CRFs. The relevant results are presented in Section 7.5.1 and Section 7.5.2. Second, we apply the proposed PSO algorithm to optimize the vehicle fleet size, unit price and CRF simultaneously for the utility-based compensation method. A convergence analysis is

Algorithm 7.1 PSO for solving utility based compensation method optimization problems

Initialize the particle's position with a uniform distribution: $\boldsymbol{x}_n \sim U(0, \boldsymbol{O}_{max})$.
Initialize the particle's velocity with a uniform distribution: $\boldsymbol{v}_n \sim U(-\boldsymbol{O}_{max}/2, \boldsymbol{O}_{max}/2)$.
Initialize the particle's personal best to the initial position: $\boldsymbol{\rho}_n \leftarrow \boldsymbol{x}_n$.
$\boldsymbol{G}^* = \boldsymbol{x}_1$.
for $n < n_p$ **do**
    if $\mathcal{J}(\boldsymbol{x}_n) > \mathcal{J}(\boldsymbol{G}^*)$ **then**
        $\boldsymbol{G}^* = \boldsymbol{x}_n$
    end if
end for
$\tau = 0$
Initialize ω, C_1, C_2
for $\tau < \tau_{max}$ **do**
    for $n < n_p$ **do**
        $\boldsymbol{r}_1, \boldsymbol{r}_2 \sim U(0,1)$
        Update velocity: $\boldsymbol{v}_n^{(\tau+1)} = \omega \boldsymbol{v}_n^{(\tau)} + C_1 \boldsymbol{r}_1(\boldsymbol{\rho}_n - \boldsymbol{x}_n^{(\tau)}) + C_2 \boldsymbol{r}_2(\boldsymbol{G}^* - \boldsymbol{x}_n^{(\tau)})$
        Update position: $\boldsymbol{x}_n^{(\tau+1)} = \boldsymbol{x}_n^{(\tau)} + \boldsymbol{v}_n^{(\tau+1)}$
        Calculate the market equilibrium state after compensation.
        if $\mathcal{J}(\boldsymbol{x}_n^{(\tau+1)}) > \mathcal{J}(\boldsymbol{x}_n^{(\tau)})$ **then**
            Update personal best: $\boldsymbol{\rho}_n \leftarrow \boldsymbol{x}_n^{(\tau+1)}$
            if $\mathcal{J}(\boldsymbol{x}_n^{(\tau+1)}) > \mathcal{J}(\boldsymbol{G}^*)$ **then**
                Update leader: $\boldsymbol{G}^* \leftarrow \boldsymbol{x}_n^{(\tau+1)}$
            end if
        end if
    end for
    $\tau = \tau + 1$
end for

Table 7.1.: Parameters for PSO.

Parameter	Value	Meaning
n_p	100	The total number of particles within the swarm.
τ_{max}	50	Maximum number of iterations.
ω	0.4	Inertia weight.
C_1	0.4	Cognitive learning factor.
C_2	0.3	Social learning factor.

provided in Section 7.5.3 to analyze the performance of PSO on this problem. Parameters for PSO are listed in Table 7.1.

Actually, these two experiments reflect two different options of the service provider for applying the utility-based compensation method. One is to find an optimal CRF for the MO or SO solution of the unified/spatial pricing method based on the operational objective. Another option is to optimize the operation strategy directly based on the operational objective via an appropriate optimization algorithm (e.g., PSO). Provably, both methods are plausible and feasible.

7.5. Case Study and Results

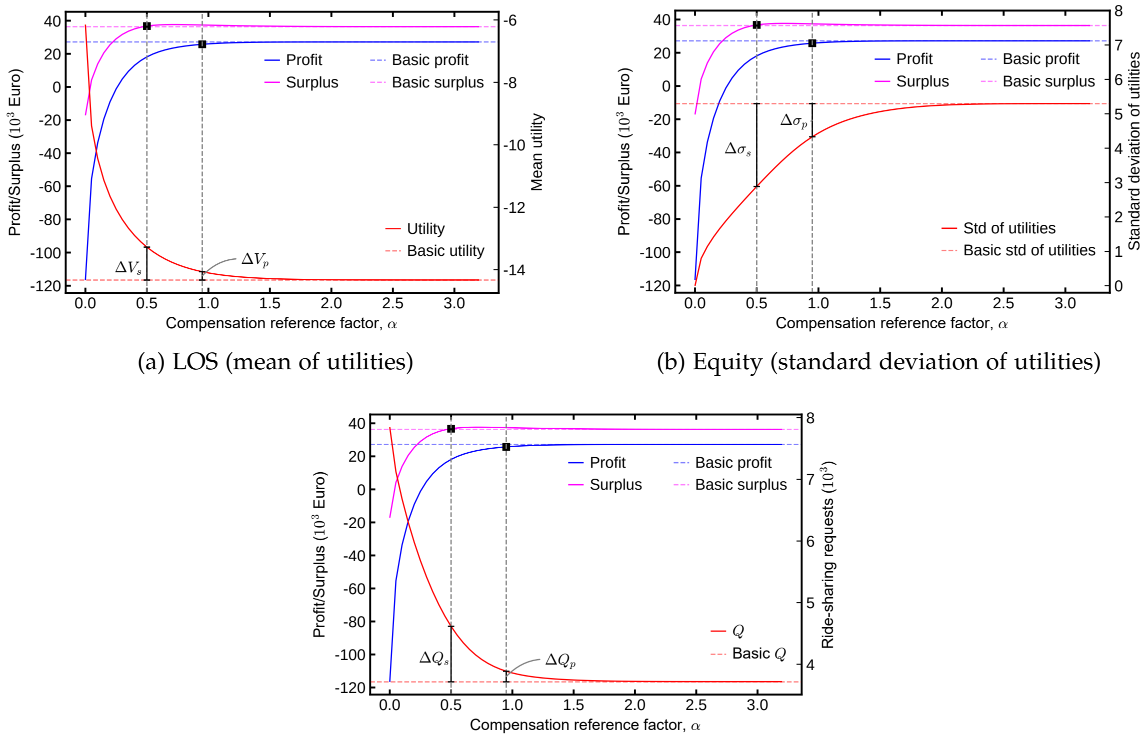
In this section, sensitivity analysis with respect to α is conducted on the monopoly optimum within the low demand period of the Munich network of the unified pricing method. The MO operation strategy of this scenario is $N^* = 1489, p^* = 1.21$. By analyzing the difference in relevant variables (e.g., LOS, equity, ride-sharing demand), we can better understand the impact and benefits of the compensation method. Section 7.5.1 describes the benefits of applying the utility-based compensation method with respect to LOS, equity and ride-sharing demand. Section 7.5.2 presents the relationship of the attracted new ride-sharing demand and the total amount of compensation, from which we can draw a subsidy scheme for the stakeholders (e.g., governments).

On the other hand, convergence analysis of applying PSO to optimize N, p and α for profit maximization within the low demand period of the test network is presented. This can enhance the understanding of the usefulness of PSO on this problem. The result is described in Section 7.5.3.

7.5.1. Benefits of utility-based compensation method

We emphasize that the motivation of the utility-based compensation method is to improve the equity of the services and the expected LOS perceived by passengers. As describes in Section 7.2, LOS and equity can be represented by the mean and standard deviation of utilities of trips, respectively. Figure 7.3a depicts the profit, social welfare, and mean of utilities under different CRFs with compensation reference factors on the X-axis, profit and welfare on the left Y-axis, and mean utility on the right Y-axis. Clearly, profit and welfare increase with CRF and converge to the corresponding basic values, where basic values are the values of the corresponding variables in the equilibrium of the MO solution for unified pricing method. Clearly, the proposed utility-based compensation pricing method cannot improve the overall profit and welfare considerably in general. However, we can see an obvious improvement in the LOS. And this improvement does not sacrifice the maximum profit and surplus in a range of α. Likewise, we can see the convergence point (termed as best point thereafter) for surplus is on the left of the best point for profit. It means we can compensate for more trips without seeing a loss of social surplus compared to profit. Denote X coordinate of the best point for surplus as α_s^*, and that for profit as α_p^*. Thus, $\alpha_s^* < \alpha_p^*$. And since the curve of mean utility is monotonically decreasing, so $\Delta V_s > \Delta V_p$, where ΔV_s and ΔV_p denote the improvement of LOS under α_s^* and α_p^*, respectively.

Furthermore, compensation under α_s^* can also improve service equity more than under α_p^*, i.e., $\Delta\sigma_s > \Delta\sigma_p$, as shown in Figure 7.3b. The curve of utility variance is monotonically increasing. And we can observe that the compensation method has more benefits on the improvement of equity compared to LOS. Actually, the compensation method influences LOS and equity from two directions. On the one hand, for the trips whose utility less than $\alpha\bar{V}$, their utilities will be increased by compensation. On the other hand, for the trips whose utility greater than $\alpha\bar{V}$, their utilities will be decreased due to the increasing of waiting time and detour time caused by the induced demand resulted from compensation (shown in Figure 7.3c). As a consequence, it is plausible to observe the improvement of equity. And the improvement of LOS indicates that the former effect is stronger than the latter one.



(a) LOS (mean of utilities)

(b) Equity (standard deviation of utilities)

(c) Attracted demand

Figure 7.3.: Benefits of utility-based compensation method.

7.5.2. Relationship of the attracted demand, profit and compensation amount

In order to understand the relationship between the quantity of attracted ride-sharing demand and compensation, we draw their relationship as depicted in Figure 7.4. X-axis is the total amount of compensation, left Y-axis is the number of attracted requests. Likewise, we also

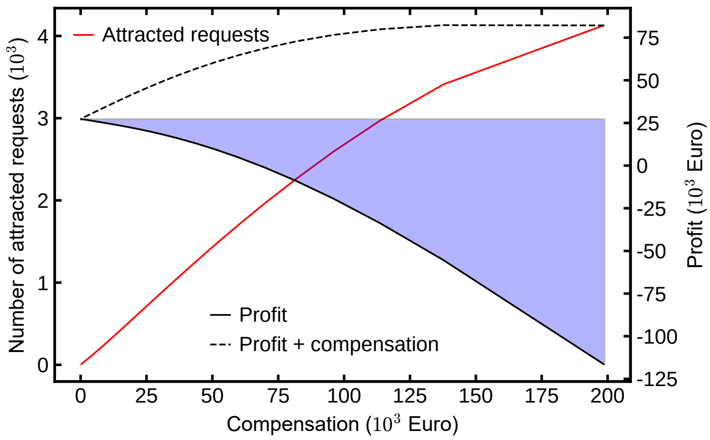


Figure 7.4.: Attracted demand and profit with different compensation amount

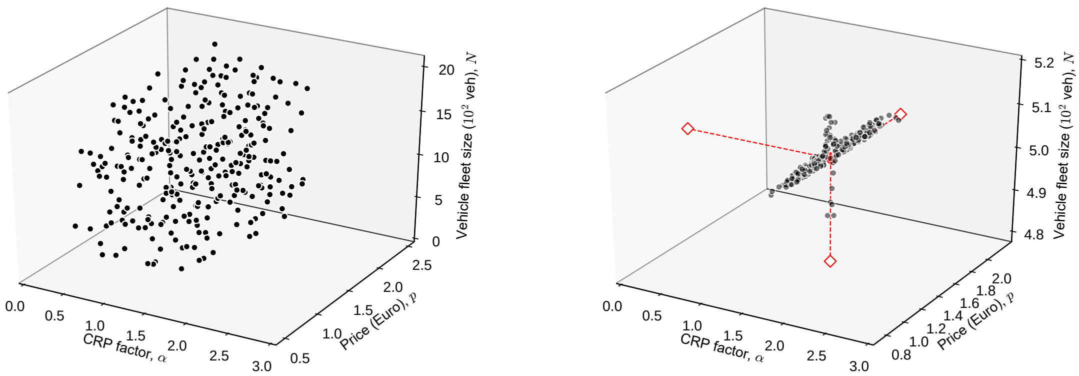
plot the relationship between compensation and profit in the same figure by adding another Y-axis to the right for profit.

Suppose that the governments want to promote the market share of ride-sharing by subsidizing the ride-sharing company. If the utility-based compensation method is utilized as a pricing strategy by the company, then Figure 7.4 could be subsidy guidance or reference for the government. The solid black line represents profit, which is decreasing with compensation. The dashed line is the sum of profit and compensation, which indicates the potential profit for the company if all compensation were subsidized by the government. However, basically, governments always want to subsidize an amount such that there is nearly no profit reduction for the company. In this sense, the blue area could provide a reference to the government regarding the amount of subsidy.

On the other hand, the solid reared line implies a nearly linear relationship between the attracted demand and compensation. In our opinion, this is depending on the compensation function applied in the proposed method. It is possible to see an increasing curve with reducing slopes when specific compensation functions are used. Under different compensation functions, there may be different optimal strategies for service providers based on their operational objectives.

7.5.3. Convergence analysis

In this section, we evaluate the proposed PSO algorithm on solving the profit maximization problem for the test network at the low demand period. The parameters used in this experiment are listed in Table 7.1. Figure 7.5a shows the initial positions of particles, while Figure 7.5b demonstrates the positions of particles at the final iteration. It can be seen that, compensation reference factor α converges to a point, while the other two variables N and p are still scattered, especially the unit price p. This implies that particles easily converge to the personal best in the dimension of unit price, and thus cannot follow the trajectory of the global best (leader). One potential reason is parameters listed in Table 7.1 are only appropriate for the convergence on other dimensions. Moreover, it is also possibly caused by the narrow search space of unit price. To avoid negative values, we restrict its search space and the initial velocity in a small range, which may result in slow movements. However, the optimum from the swarm is still almost the same as the solution we got through the first method. Thus, PSO is appropriate for this problem.



(a) Positions of initial particles within the swarm

(b) Final positions of particles within the swarm

Figure 7.5.: Convergence of PSO.

7.6. Conclusions

In this chapter, we propose a utility-based compensation method for ride-sharing services. With this pricing method, the LOS and equity of the service can be improved without sacrificing the performance of an operation on the objectives. And this method is more beneficial under the consideration of social welfare with clearer improvement on LOS and equity compared to the situation of considering profit. Besides, an additional benefit of the increase from ride-sharing demand can also be observed.

For a ride-sharing operator applying the proposed compensation method, the subsidy scheme for the governments to stimulate/support the operator doing the compensation to serve more requests is provided. Without subsidy, profit for the service operator is reducing with the amount of compensation in general. However, if all compensation is subsidized by the stakeholders, the operator can see a profit increase. In more general cases, the stakeholders want to subsidize the service operator such that the operator can receive the same profit after compensation. Thus, in this sense, the stakeholders can refer to Figure 7.4 for the subsidy scheme under different amounts of compensation. It is worth pointing out that the improvement of LOS and equity, as well as the subsidy scheme, would be different when different compensation function is applied.

Note that, apart from applying sensitivity analysis to seek the optimal operation strategy, we also propose a PSO algorithm to optimize the operation strategy for the utility-based compensation method. This algorithm is proved to be effective based on the convergence analysis conducted in Section 7.5.3.

8. Conclusions and Outlook

8.1. Summary

This thesis is devoted to developing dynamic pricing methods for ride-sharing services in a multi-modal transportation system. Efforts are therefore made to handle the challenges associated with the ride-sharing market equilibrium and the optimal operation strategy for certain pricing methods. The main contributions and findings of this thesis are summarized below.

8.1.1. Completing the market equilibrium model for ride-sharing services

The first main contribution of this thesis is the development of a more comprehensive market equilibrium model for ride-sharing by considering the passenger preference in a multi-modal transport network (Chapter 3). In the existing literature, the demand for ride-sharing services is determined by a function of the value of time (or willingness to pay) of passengers and the individual attributes of ride-sharing services. However, the fact is that passengers typically compare different modes of transportation when they need to choose one. This means that it is important to consider passengers' preference for multiple modes of transportation when modeling the market for ride-sharing services. Otherwise, it will result in an inaccurate demand estimation. In this study, the MNL model is applied to estimate the probability of choosing ride-sharing services. This improvement can also make the consideration of user heterogeneity in the experiments becomes easier and more efficient.

What's more, the network structure and OD demand pattern are explicitly integrated into the modeling framework. A market equilibrium modeled at the network level can enhance the reliability of its application as indicated in Yang et al. (2002). On the other hand, considering that spatial difference does exist in networks, it is important to estimate the OD-based demand for ride-sharing specifically in the market model.

More importantly, we explicitly present the method to calculate the system endogenous variables (detour time, waiting time and ride-sharing demand) in the equilibrium for the proposed market model. Under a certain operation strategy (trip fare and vehicle fleet size), the relationship between system endogenous variables at equilibrium can be described by a simultaneous equations system, which can be addressed by a hybrid method presented in Powell (1970).

8.1.2. Developing solution algorithms for the proposed pricing methods

It has been proved theoretically (Keller, 2013) and empirically (Ke et al., 2020a) that the local maximums of the non-convex objective functions considered in this thesis are also global maximums, which means the gradient-based optimization algorithms can be applied to solve these problems. Consequently, we propose a GD algorithm to solve the optimums for the unified pricing method (Chapter 5) and spatial pricing method (Chapter 6). The complexity of the derivatives of relevant variables with respect to the operation strategy also demonstrates the intertwined relationship between these variables. It is evident that the GD algorithm is effective and efficient for these problems based on the results of the convergence analysis.

In terms of the utility-based compensation method (Chapter 7), we apply a heuristic—PSO to optimize the operation strategy as the aggregation-disaggregation-reaggregation chain included in the method makes it impossible to calculate the exact derivatives. Nevertheless, the convergence analysis shows that the heuristic can also result in a reliable optimum.

8.1.3. Constructing an utility-based compensation method to improve LOS and equity of ride-sharing services

The third main contribution of this thesis is the development of a novel utility-based compensation method for ride-sharing services (Chapter 7). By applying this method, trips whose utility below a threshold (i.e., CRP) are compensated based on a predefined utility compensated function (describing the relationship between the utility before and after compensation). The results show that it can improve the LOS (mean utility) and equity (variance of utility) of the service. Moreover, the compensation can also attract more demand and increase the market share for ride-sharing services.

A useful and practical subsidy scheme is provided to the stakeholders. By applying this scheme, the stakeholders can motivate the ride-sharing provider to adopt this utility-based compensation method to offer services to more passengers.

8.2. Limitations

This thesis develops dynamic pricing methods for ride-sharing services. A market equilibrium model, which incorporates passenger preference in a multi-modal urban transportation system, works as the testbed for the pricing methods. Effective algorithms are proposed to optimize the operation strategies for the pricing methods. The pricing methods are also evaluated based on both numerical and real-world case studies. Nevertheless, there are a few limitations that require further investigation.

1) When modeling the expected detour time, we simply utilize the assumption widely applied in the existing literature that the average detour time between two passengers is inversely proportional to the ride-sharing demand, and extend it to the general case (arbitrary passengers) through a multiplier. Besides, we also simply assume that the expected detour time for different OD pairs is proportional to the direct trip time which lacks evidence

from the real operation. It is essential to incorporate a more accurate estimation model for expected detour time into the proposed market equilibrium model in future work.

2) In the market equilibrium model, vehicles are distributed evenly to the OD pairs as implied by the models of expected detour time and waiting time. This should be improved by explicitly accounting for the vehicle distribution mechanism.

3) The survey data available for the experiments result in $\beta_w > \beta_t$ in MNL, which does not conform to the reality and general cognition despite the estimates are significant. Normally, passengers are deemed to be more sensitive to waiting time than in-vehicle time. Thus, it is important to utilize a more reliable dataset for estimating passenger preference.

4) In the chapter for spatial pricing method (Chapter 6), we implement an experiment considering user heterogeneity by differentiating three regions from the others. However, the user heterogeneity over the network, in reality, could be more random and chaotic.

8.3. Outlook

The dynamic pricing method for ride-sharing services is a relatively new and promising research area. Given the stochasticity and the uncertainty in ride-sharing services, there are many directions to explore. Specifically, potential future research directions are listed as follows.

1) **Comprehensive ride-sharing market equilibrium model.** In this thesis, we propose a market equilibrium model for ride-sharing services with particular consideration of passenger preference in a multi-modal transportation system. And the network structure and OD demand pattern have also been incorporated. An important future direction is to explicitly consider the vehicle distribution in detour time estimation and waiting time estimation. Furthermore, another possible future direction is to introduce subjective decision making to the passenger preference model to estimate passengers' behavior under uncertainty in ride-sharing, such as a preference model based on Cumulative Prospect Theory (CPT, Wakker and Tversky, 1993).

2) **Reliable survey data.** We utilize the survey data from Tsiamasiotis (2019) to estimate the preference coefficients for all user groups analyzed in the experiments conducted in this thesis. But the estimation result seems to be unrealistic. Thus, it is important to conduct a survey on a larger scale and with effective design.

3) **Consideration of temporal elasticity of demand.** In this thesis, we focus on optimizing the operation strategies within a single studying interval. However, due to the demand elasticity, the optimum may be suboptimum in a wider time interval, which should be overcome in future work.

4) **Exploration in compensation functions.** In the utility-based compensation method (Chapter 7), an explicit compensation function is adopted and instructs the analysis of the benefits

of this method. It is desirable to explore the proposed utility-based compensation method with different compensation functions (e.g., linear or nonlinear, univariate or multivariate) in the future work.

5) **Incorporation with a simulator.** The utility-based compensation method is proved to be beneficial on the LOS and equity of ride-sharing services based on the experiments implemented on the proposed equilibrium model. However, to prove its applicability in reality, a future direction to incorporate it with a simulator is needed.

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A. Detailed Calculations

A.1. Method to Calculate Market Equilibrium Model Parameters

This appendix elaborates the procedure for calculating plausible A and B for the proposed market equilibrium model. Assuming that we can obtain the following data from the real operation of the market of interest.

- $\hat{N}$: the vehicle fleet size.
- $\bar{r}$: the average trip fare .
- $\hat{s}$: the mode share under $(\hat{N}, \bar{r})$.
- k: the quotient by dividing average detour time by average waiting time.

If we assume the probabilities of choosing ride-sharing are the same for all OD pairs in the network, we can simply get $\hat{s} \approx \bar{P}$, where $\bar{P}$ is the expected probability of choosing ride-sharing under $(\hat{N}, \bar{r})$. According to the MNL model, we can simply get

$$\hat{s} = \frac{e^{\bar{V}_{rs}}}{\sum_j e^{\bar{V}_j}} \Rightarrow \hat{s}(e^{\bar{V}_{pt}} + e^{\bar{V}_{car}} + e^{\bar{V}_{rs}}) = e^{\bar{V}_{rs}} \Rightarrow e^{\bar{V}_{pt}} + e^{\bar{V}_{car}} = \frac{1-\hat{s}}{\hat{s}} e^{\bar{V}_{rs}} \tag{A.1}$$

where $\bar{V}_{rs} = \beta_t \bar{t} + \beta_w \bar{w} + \beta_r \bar{r}$. $\bar{V}_{car}$ and $\bar{V}_{pt}$ are the mean utility of private car and public transport, respectively. And

$$\bar{t} = \frac{\sum_i D_i t_i}{\sum_i D_i} \text{ and } \bar{w} = \frac{\sum_i D_i w_i}{\sum_i D_i} \tag{A.2}$$

We omit the superscript for mean value (i.e., bar marker) in the remainder of this section. We take logarithm for both sides of the last equation in Equation (A.1) resulting in

$$\ln(e^{V_{pt}} + e^{V_{car}}) = \ln(1-\hat{s}) - \ln(\hat{s}) + V_{rs} \tag{A.3}$$

Since $V_{rs} = \beta_t t + \beta_w w + \beta_r r$, so

$$\beta_t t + \beta_w w = \ln(e^{V_{pt}} + e^{V_{car}}) - \ln(1-\hat{s}) + \ln(\hat{s}) - \beta_r r \tag{A.4}$$

Substitute $t = t^d + \tilde{t}$ into the equation above resulting in

$$\beta_t \tilde{t} + \beta_w w = \ln(e^{V_{pt}} + e^{V_{car}}) - \ln(1 - \hat{s}) + \ln(\hat{s}) - \beta_r r - \beta_t t^d \tag{A.5}$$

We know that $\tilde{t} = kw$, so

$$(k\beta_t + \beta_w)w = \ln(e^{V_{pt}} + e^{V_{car}}) - \ln(1 - \hat{s}) + \ln(\hat{s}) - \beta_r r - \beta_t t^d \tag{A.6}$$

If we define $R \triangleq \ln(e^{V_{pt}} + e^{V_{car}}) - \ln(1 - \hat{s}) + \ln(\hat{s}) - \beta_r r - \beta_t t^d$, then

$$w = \frac{R}{k\beta_t + \beta_w} \tag{A.7}$$

$$\tilde{t} = kw = \frac{kR}{k\beta_t + \beta_w} \tag{A.8}$$

And according to the proposed equilibrium model, the expected waiting time is estimated by

$$w_i = \frac{BQ_i}{\sqrt{Nn_s - \sum_j Q_j t_j}} \tag{A.9}$$

If we approximate $Q_i \approx \hat{s} \sum_i D_i / n_z, \sum_j Q_j t_j \approx \hat{s} \sum_j D_j t_J^d$, where n_z is the number of OD pairs, then we can get,

$$B = \frac{R\sqrt{Nn_s - \hat{s} \sum_j D_j t_J^d}}{(k\beta_t + \beta_w)\hat{s} \sum_i D_i / n_z} \tag{A.10}$$

According to the proposed equilibrium model, the detour time is estimated by

$$\tilde{t}_i = \frac{At_j^d \sum_j Q_j t_j^d}{\bar{t}^d N \sum_j Q_j} \tag{A.11}$$

Similarly, we approximate $t_j^d \approx \bar{t}^d, \sum_j Q_j t_j^d \approx \hat{s} \sum_j D_j t_j^d, \sum_j Q_j \approx \hat{s} \sum_j D_j$, then we can get,

$$A = \frac{kRN\hat{s} \sum_j D_j}{(k\beta_t + \beta_w)\hat{s} \sum_j D_j t_j^d} \tag{A.12}$$

Equation (A.10) and Equation (A.12) provide a reliable value of B and A, respectively. Note that, sometimes we may still need to tune the result from this calculation procedure to make the experiment with the same input conditions (i.e., vehicle fleet size and price) result in a similar modal split.

A.2. Calculation of Derivatives of Expected Detour Time and Expected Waiting Time for Spatial Pricing Method

Let $t'_{i|p_z}$ and $t'_{i|N}$ denote the derivatives of t_i with respect to p_z and N, respectively. And let $w'_{i|p_z}$ and $w'_{i|N}$ denote the derivatives of w_i with respect to p_z and N, respectively. Recall that $t_i = t_i^d + A_i \sum_j Q_j t_j^d / N \sum_j Q_j$ and $w_i = BQ_i / \sqrt{Nn_s - \sum_j Q_j t_j}$.

On the one hand, if $i = z$, we have

$$\frac{\partial Q_i}{\partial p_z} = D_i P_i (1 - P_i)(\beta_t \frac{\partial t_i}{\partial p_z} + \beta_w \frac{\partial w_i}{\partial p_z} + \beta_r d_z) \tag{A.13}$$

Thus, we have

$$\begin{aligned} t'_{i|p_z} = {} & \frac{A_i}{N \sum_j Q_j} \sum_j D_j P_j (1 - P_j)(\beta_t t'_{j|p_z} + \beta_w w'_{j|p_z} + \beta_r d_j) t_j^d \\ & - \frac{A_i \sum_j Q_j t_j^d}{N (\sum_j Q_j)^2} \sum_j D_j P_j (1 - P_j)(\beta_t t'_{j|p_z} + \beta_w w'_{j|p_z} + \beta_r d_j) \end{aligned} \tag{A.14}$$

$$\begin{aligned} w'_{i|p_z} = {} & \frac{BD_i P_i (1 - P_i)(\beta_t t'_{i|p_z} + \beta_w w'_{i|p_z} + \beta_r d_i)}{\sqrt{Nn_s - \sum_j Q_j t_j}} \\ & + \frac{BQ_i \sum_j \left[D_j P_j (1 - P_j)(\beta_t t'_{j|p_z} + \beta_w w'_{j|p_z} + \beta_r d_j) t_j + Q_j t'_{j|p_z} \right]}{2 \sqrt{(Nn_s - \sum_j Q_j t_j)^3}} \end{aligned} \tag{A.15}$$

On the other hand, if $i \neq z$, we have

$$\frac{\partial Q_i}{\partial p_z} = D_i P_i (1 - P_i)(\beta_t \frac{\partial t_i}{\partial p_z} + \beta_w \frac{\partial w_i}{\partial p_z}) \tag{A.16}$$

Thus, we have

$$\begin{aligned} t'_{i|p_z} = {} & \frac{A_i}{N \sum_j Q_j} \sum_j D_j P_j (1 - P_j)(\beta_t t'_{j|p_z} + \beta_w w'_{j|p_z}) t_j^d \\ & - \frac{A_i \sum_j Q_j t_j^d}{N (\sum_j Q_j)^2} \sum_j D_j P_j (1 - P_j)(\beta_t t'_{j|p_z} + \beta_w w'_{j|p_z}) \end{aligned} \tag{A.17}$$

$$\begin{aligned} w'_{i|p_z} = {} & \frac{BD_i P_i (1 - P_i)(\beta_t t'_{i|p_z} + \beta_w w'_{i|p_z})}{\sqrt{Nn_s - \sum_j Q_j t_j}} \\ & + \frac{BQ_i \sum_j \left[D_j P_j (1 - P_j)(\beta_t t'_{j|p_z} + \beta_w w'_{j|p_z}) t_j + Q_j t'_{j|p_z} \right]}{2 \sqrt{(Nn_s - \sum_j Q_j t_j)^3}} \end{aligned} \tag{A.18}$$

Note that regardless of the relation of i and z, we have

$$t'_{i|N} = \frac{A_i \sum_j D_j P_j (1-P_j)(\beta_t t'_{j|N} + \beta_w w'_{j|N}) t_j^d}{N \sum_j Q_j} - \frac{A_i \left[Q + N \sum_j D_j P_j (1-P_j)(\beta_t t'_{j|N} + \beta_w w'_{j|N}) \right] \sum_j Q_j t_j^d}{(N \sum_j Q_j)^2} \quad \text{(A.19)}$$

$$w'_{i|N} = \frac{B D_i P_i (1-P_i)(\beta_t t'_{i|N} + \beta_w w'_{i|N})}{\sqrt{N n_s - \sum_j Q_j t_j}} - \frac{B Q_i \left[n_s - \sum_j D_j P_j (1-P_j)(\beta_t t'_{j|N} + \beta_w w'_{j|N}) t_j - \sum_j Q_j t'_{j|N} \right]}{2\sqrt{(N n_s - \sum_j Q_j t_j)^3}} \quad \text{(A.20)}$$

A.3. Derivation for the Compensated Utility Function

By applying the utility-based compensation method, we do compensations for the trips whose utility is less than a predefined threshold $a (a < 0)$. In this study, we assume the form of the compensated utility function is

$$f(x) = l\sqrt{-x+b} \quad \text{(A.21)}$$

with following mild assumptions.

Assumption A.1 *$f(x)$ is continuous and smooth on (a, a), such that: 1) $f(a) = a$; 2) $f'(a) = 1$.*

One can prove that this function satisfies the rules stated in Chapter 7. Based on Assumption A.1, we have

$$\begin{cases} f(a) = l\sqrt{-a+b} = a & \text{(A.22)} \\ f'(a) = -\frac{1}{2} l(-a+b)^{-\frac{1}{2}} = 1 & \text{(A.23)} \end{cases}$$

Because utility is negative, i.e., $a < 0, f(a) < 0$, so from Appendix A.3, we have $l < 0$. From Appendix A.3, we have

$$\frac{1}{-a+b} = \frac{-2}{l}$$

$$\Rightarrow b = (\frac{l}{2})^2 + a \quad \text{(A.24)}$$

And from Appendix A.3, we have

$$b = (\frac{a}{l})^2 + a \quad \text{(A.25)}$$

Then from Equation (A.24) and Equation (A.25), we have

$$(\frac{a}{l})^2 = (\frac{l}{2})^2$$

$$\Rightarrow l = -\sqrt{-2a} \text{ (since } l < 0) \tag{A.26}$$

Substitute Equation (A.26) into Equation (A.24) resulting in

$$b = \frac{a}{2} \tag{A.27}$$

Thus, substitute Equation (A.26) and Equation (A.27) into Equation (A.21) resulting in

$$f(x) = -\sqrt{2ax - a^2} \tag{A.28}$$

Consequently, let V^a denote the utility after compensation, then the full formulation of the compensated utility function is given by

$$V^a = \begin{cases} V & \text{if } V > a \\ -\sqrt{2aV - a^2} & \text{otherwise} \end{cases} \tag{A.29}$$

B. Coefficient Estimation of Passenger Preference

B.1. Age Groups

Table B.1.: Coefficient estimation for age group: 18-35.

Coefficient	Value	Standard error	t-test	p-value
β_r (/Euro)	-0.716	0.1480	-4.84	0
β_t (/min)	-0.108	0.0381	-2.84	0.00447
β_w (/min)	-0.126	0.0358	-3.51	0.000443

Table B.2.: Coefficient estimation for age group: 36-55.

Coefficient	Value	Standard error	t-test	p-value
β_r (/Euro)	-0.556	0.0585	-9.51	0
β_t (/min)	-0.128	0.0163	-7.86	0
β_w (/min)	-0.115	0.0158	-7.28	0

Table B.3.: Coefficient estimation for age group: >55.

Coefficient	Value	Standard error	t-test	p-value
β_r (/Euro)	-0.601	0.1510	-3.98	0
β_t (/min)	-0.139	0.0402	-3.48	0.000529
β_w (/min)	-0.091	0.0407	-2.23	0.0258

B.2. Income Groups

Table B.4.: Coefficient estimation for income group: 500-2000.

Coefficient	Value	Standard error	t-test	p-value
β_r (/Euro)	-0.601	0.0861	-6.98	0
β_t (/min)	-0.106	0.0227	-4.65	0
β_w (/min)	-0.112	0.0224	-5.00	0

Table B.5.: Coefficient estimation for income group: 2000-4000.

Coefficient	Value	Standard error	t-test	p-value
β_r (/Euro)	-0.781	0.125	-6.23	0
β_t (/min)	-0.170	0.0329	-5.17	0
β_w (/min)	-0.151	0.0321	-4.69	0

Table B.6.: Coefficient estimation for income group: $\geq$4000.

Coefficient	Value	Standard error	t-test	p-value
β_r (/Euro)	-0.514	0.0746	-6.89	0
β_t (/min)	-0.135	0.0216	-6.25	0
β_w (/min)	-0.104	0.0201	-5.18	0

Table B.7.: Coefficient estimation for income group: <4000.

Coefficient	Value	Standard error	t-test	p-value
β_r (/Euro)	-0.660	0.0707	-9.34	0
β_t (/min)	-0.128	0.0186	-6.86	0
β_w (/min)	-0.125	0.0183	-6.82	0