# Haptic Shared Control for Human-Robot Collaboration: A Game-Theoretical Approach

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**Abstract:** Complementing human and robot capabilities is essential for many tasks, e.g. rehabilitation and collaborative manufacturing. However, it is still not clear how control between humans and robots should be shared in order to ensure efficient task execution and intuitive interaction. Game theory seems as a promising mathematical framework that allows: i) posing this challenge as a dynamic negotiation (game) among human and robot (players) and ii) solving it to obtain optimal solution. In this work, we propose a differential game-theoretic shared control approach for human-robot haptic collaboration with *Nash equilibrium optimal solution*. We validate the proposed approach experimentally in a scenario where human is physically coupled with a haptic device and interacts with a virtual reality to perform a trajectory tracking task.

*Keywords:* Human-robot interaction, haptic shared control, dynamic game theory, Nash equilibrium.

## 1. INTRODUCTION

*Shared control* is a general term for control approaches that enable human and semi-autonomous system to share responsabilities over task execution (Music and Hirche (2017)). Rehabilitation, telesurgery, elder care, exploration of inaccessible or dangerous environments, and collaborative manufacturing are just some of the application examples that require such control schemes. There are two main requirements of shared control: i) *efficiency* in terms of task performance and ii) *intuitiveness* of the interaction so that the human can interpret decisions of the semi-autonomous system and vice versa. While many existing shared-control approaches are efficient, intuitiveness is more difficult to achieve and requires an understanding of human decision making in control-sharing settings. In this work we consider a collaborative interaction where both partners know the task goal and jointly work to achieve it. We focus on continuous-time human-robot interaction for tracking tasks with partners tightly coupled through haptic channel.

Research shows that optimal control can be used to describe behavior of *trained* humans. According to Jagacinski and Flach (2018), optimal control can support both cognition (top-down) and information processing (bottom-up) components of human behavior. Therefore, it may be a suitable modeling approach not only for high-level human decision making but also for the motor control. For example, Flash and Hogan (1985) prove that humans minimize jerk in reaching tasks, Anderson and Pandy (2001) show that in locomotion tasks humans minimize energy, and Emken et al. (2007) provide a generalization of motor control as optimization of kinematic error and effort. Decision-making and motor control components of human behavior are unified in Todorov and Jordan (2002) with stochastic optimal feedback control. Analogously to optimal control, motor interaction between multiple humans can be modeled within game-theoretical framework. Braun et al. (2009) and Braun et al. (2011) show that in human-human motor interactions partners converge to Nash equilibria.

Designing an optimal control strategy of the robot autonomy w.r.t. the task requirements and the human control input has already been done in the context of haptic collaborative tasks, see e.g. Medina et al. (2015) and Rozo et al. (2015). Generalizing optimal control strategy to joint behavior of the human and the robot partners is possible within the game-theoretical framework. Therefore, Jarrassé et al. (2012) provide classification of interaction paradigms for two-agent haptic tasks according to the cost functions agents minimize. Game theory has been used in semi-autonomous driving use cases for assistance in wheel steering. For example, an open-loop and closed-loop solutions are provided by Na and Cole (2014) and Flad et al. (2014)), respectively. However, these works provide only exemplary simulation results. Inga et al. (2018) show experimentally that the cost function of the human changes in shared-control tasks compared to fully manual tasks. Therefore, it is important to consider the couplings between the agents and their effects on the decision making. Ji et al. (2018) propose a stochastic game-theoretical approach that mitigates conflicts in driver-machine interactions when the human and the autonomy perform different tasks; driving and obstacle avoidance, respectively. Recently, Li et al. (2019) proposed a Nash equilibria solution for reaching tasks in human-robot collaboration. The obtained results show the performance improvement compared to the fully manual task execution. However, the cost function assumed for the human partner is not supported by previous studies on human motor behavior in reaching tasks, see e.g. Flash and Hogan (1985). Therefore, it is still unclear if the human partner admits a Nash equilibrium solution in collaborative tasks.

In this paper, we propose a game-theoretical shared control approach for trajectory tracking tasks. For that purpose, we assume cost function of the human that is in line with previous research studies (Jagacinski and Flach (2018)). The control gains of the human control input are estimated with adaptive input observer proposed in Li et al. (2019) and extended to trajectory tracking tasks. The optimal control of the semi-autonomous partner is computed to obtain Nash equilibrium.

We pose the problem in Section 2. The shared control approach is detailed in Section 3. In Section 4 we present simulation and experimental results. Concluding remarks and future work are given in Section 5.

## 2. PROBLEM SETTING

In this section we pose the shared-control problem for trajectory tracking tasks within the game-theoretical framework. Let us assume the task is defined as a second-order linear time-invariant dynamical system

$$\begin{bmatrix}\dot{\boldsymbol{x}}\\ \ddot{\boldsymbol{x}}\end{bmatrix} = \underbrace{\begin{bmatrix}\boldsymbol{0} & \boldsymbol{I}\\ \boldsymbol{0} & -\boldsymbol{M}^{-1}\boldsymbol{D}\end{bmatrix}}_{\boldsymbol{A}} \underbrace{\begin{bmatrix}\boldsymbol{x}\\ \dot{\boldsymbol{x}}\end{bmatrix}}_{\boldsymbol{\xi}} + \underbrace{\begin{bmatrix}\boldsymbol{0}\\ \boldsymbol{M}^{-1}\end{bmatrix}}_{\boldsymbol{B}}(\boldsymbol{u}_h + \boldsymbol{u}_a) \tag{1}$$

where $\boldsymbol{x}, \dot{\boldsymbol{x}}, \ddot{\boldsymbol{x}} \in \mathbb{R}^n$ are the $n$-dimensional position, velocity, and acceleration vectors, $\boldsymbol{M}, \boldsymbol{D} \in \mathbb{R}^{n\times n}$ are the positive definite inertia and damping matrices, respectively, and $\boldsymbol{u}_h, \boldsymbol{u}_a \in \mathbb{R}^n$ are the human and the robot control inputs, respectively. Let us assume the desired trajectory and its first derivative are bounded, available online and given with the vectors $\boldsymbol{\xi}_d = [\boldsymbol{x}_d^\top, \dot{\boldsymbol{x}}_d^\top]^\top$ and $\dot{\boldsymbol{\xi}}_d = [\dot{\boldsymbol{x}}_d^\top, \ddot{\boldsymbol{x}}_d^\top]^\top \in \mathbb{R}^{2\times n}$, respectively. With the task tracking error being $\boldsymbol{\xi}_e = \boldsymbol{\xi} - \boldsymbol{\xi}_d$, the coordinates in (1) can be changed to obtain

$$\dot{\boldsymbol{\xi}}_e = \boldsymbol{A}\boldsymbol{\xi}_e + \boldsymbol{B}(\boldsymbol{u}_h + \boldsymbol{u}_a) + \underbrace{\boldsymbol{A}\boldsymbol{\xi}_d - \dot{\boldsymbol{\xi}}_d}_{\boldsymbol{c}}, \tag{2}$$

*Remark 1.* In the considered setting the control input of the human, $\boldsymbol{u}_h$, cannot be measured directly. Instead, its online estimator, $\hat{\boldsymbol{u}}_h$, is used.

The assumption within the game-theoretical framework is that both agents perform the task (2) according to their own cost functionals

$$J_i(\boldsymbol{\xi}_e, \boldsymbol{u}_h, \boldsymbol{u}_a) = \int_0^t g_i(\boldsymbol{\xi}_e, \boldsymbol{u}_h, \boldsymbol{u}_a)\mathrm{d}t, \tag{3}$$

where $i = \{h, a\}$, while we define the functions $g_i$ as follows:

$$\begin{aligned} g_h(\boldsymbol{\xi}_e, \boldsymbol{u}_h) &= \frac{1}{2}(\boldsymbol{\xi}_e^\top \boldsymbol{Q}_h \boldsymbol{\xi}_e + \boldsymbol{u}_h^\top \boldsymbol{u}_h),\\ g_a(\boldsymbol{\xi}_e, \boldsymbol{u}_h, \boldsymbol{u}_a) &= \frac{1}{2}(\boldsymbol{\xi}_e^\top \boldsymbol{Q}_a \boldsymbol{\xi}_e + \boldsymbol{u}_a^\top \boldsymbol{u}_a + \boldsymbol{u}_h^\top \boldsymbol{u}_h), \end{aligned} \tag{4}$$

where $\boldsymbol{Q}_h$ and $\boldsymbol{Q}_a$ are positive semidefinite matrices.

*Remark 2.* According to the cost functionals in (4), the human partner is expected to minimize the task error and the exerted effort as proposed and validated in Emken et al. (2007) for dynamic tasks. The robot partner simultaneously minimizes the task error, rendering the interaction *collaborative*. Additionally, the robot partner seeks to minimize its own effort as well as the effort of the human partner. The latter renders the posed interaction *assistive* to the human partner.

Let $\Gamma_i \in \mathbb{R}^n$ be a class of permissible strategies, $\boldsymbol{\gamma}_i$, such that $\boldsymbol{u}_i = \boldsymbol{\gamma}_i(\boldsymbol{\xi}_e)$. In order for the problem to be a well-defined differential game, uniqueness of the solution to (2) needs to be guaranteed. For that purpose, we introduce Assumption 1 and Lemma 1.

*Assumption 1.* The dynamics (2) and permissable strategies $\gamma_i \in \Gamma_i$ are uniformly Lipschitz in $\boldsymbol{\xi}_e$, $\boldsymbol{u}_h$, and $\boldsymbol{u}_a$.

*Lemma 1.* (Başar and Olsder (1998)). If Assumption 1 is satisfied, (2) has a unique solution for every $\boldsymbol{\gamma}_i$ so that $\boldsymbol{u}_i = \boldsymbol{\gamma}_i(\boldsymbol{\xi}_e)$, and this solution is continuous.

Assumption 1 does not impose restrictions on the considered problem since the task is continuous and the human input is smooth. In this work we consider the cases in which full state information is available to both partners.

*Assumption 2.* The task state $\boldsymbol{\xi}_e$ is measurable to both the human and the robot.

Now we can consider the human and the robot partners as two *players* which have their own cost functions given with (3) and (4). Consequently, the task defined with (2) is the *game* dynamics. The problem to be solved in the remainder of the paper is to design a shared control approach so that the robot (autonomy) control input $\boldsymbol{u}_a$ and the human control input $\boldsymbol{u}_h$ achieve a Nash equilibrium solution

$$\begin{aligned} &J_h(\boldsymbol{\xi}_e, \boldsymbol{u}_h^\star) \le J_h(\boldsymbol{\xi}_e, \boldsymbol{u}_h)\\ &J_a(\boldsymbol{\xi}_e, \boldsymbol{u}_h^\star, \boldsymbol{u}_a^\star) \le J_a(\boldsymbol{\xi}_e, \boldsymbol{u}_h^\star, \boldsymbol{u}_a), \end{aligned}$$

where $\boldsymbol{u}_h^\star, \boldsymbol{u}_a^\star$ are optimal control inputs. This solution will render the interaction optimal in the sense that no partner would benefit if they change their control strategy.

## 3. SHARED CONTROL APPROACH

In this section we propose a game-theoretical shared control approach that determines the control input of the robot partner with the aim of establishing Nash equilibrium in human-robot haptic interaction. Since the control gain of the human is unknown to the robot partner, its estimate is obtained with an adaptive input observer.

### *3.1 Closed loop Nash equilibria - preliminaries*

We consider a differential game in which both the human and the robot partners have a *closed-loop perfect state information pattern (CLPS)*. Let $\boldsymbol{\eta}^i(t)$ determine the state information gathered by partner $i$ at time $t$. Then, a CLPS information patterns implies $\boldsymbol{\eta}^i(t) = \{\boldsymbol{\xi}_e(s), 0 \le s \le t\}$, $t \in [0, \infty)$, $i = \{h, a\}$ (Başar and Olsder (1998)). This information pattern assumes that the control input of each player depends causally on the system state at some point in time $s \in [0, t]$ and, consequently, the control input of the other player (Başar and Olsder (1998)). The consideration of CLPS information pattern in our case is reasonable because i) the state is measurable and ii) the interaction is continuous and achieved via haptic channel, so that both players receive information about the control input of another player online.

The following lemma provides a general Nash equilibrium solution for the game-theoretical problem with CLPS information pattern, posed with (2), (3), and (4).

*Lemma 2.* (Başar and Olsder (1998)). For a 2-person differential game, strategies $\gamma_h^\star \in \Gamma_h$ and $\gamma_a^\star \in \Gamma_a$ provide a CLPS Nash equilibrium solution if there exist functions $V_i : \mathbb{R}^{2n} \to \mathbb{R}, i = \{h, a\}$, satisfying

$$\begin{aligned} &-\frac{\mathrm{d}V_h(\boldsymbol{\xi}_e)}{\mathrm{d}t} = \min_{\boldsymbol{u}_h} \Big[\frac{\partial V_h(\boldsymbol{\xi}_e)}{\partial \boldsymbol{\xi}_e} \boldsymbol{f}(\boldsymbol{\xi}_e, \boldsymbol{u}_h, \gamma_a^\star),\\ &+ \boldsymbol{g}_h(\boldsymbol{\xi}_e, \boldsymbol{u}_h, \gamma_a^\star)\Big],\\ &-\frac{\mathrm{d}V_a(\boldsymbol{\xi}_e)}{\mathrm{d}t} = \min_{\boldsymbol{u}_a} \Big[\frac{\partial V_a(\boldsymbol{\xi}_e)}{\partial \boldsymbol{\xi}_e} \boldsymbol{f}(\boldsymbol{\xi}_e, \gamma_h^\star, \boldsymbol{u}_a)\\ &+ \boldsymbol{g}_a(\boldsymbol{\xi}_e, \gamma_h^\star, \boldsymbol{u}_a)\Big]. \end{aligned} \tag{5}$$

Lemma 2 determines Nash equilibrium solution with optimal control inputs $\boldsymbol{u}_h^\star = \gamma_h^\star(\boldsymbol{\xi}_e)$ and $\boldsymbol{u}_a^\star = \gamma_a^\star(\boldsymbol{\xi}_e)$.

### *3.2 Shared control with Nash equilibrium solution*

For the problem posed with (2), (3) and (4) it is possible to obtain an explicit solution to (5) with the following theorem.

*Theorem 1.* For a dynamic game defined by (2), (3) and (4), let there exist matrices $\boldsymbol{Z}_h$ and $\boldsymbol{Z}_a$ that satisfy

$$\begin{aligned}&\dot{\boldsymbol{Z}}_h + \tilde{\boldsymbol{F}}^\top \boldsymbol{Z}_h + \boldsymbol{Z}_h\tilde{\boldsymbol{F}} + \boldsymbol{Q}_h + \boldsymbol{Z}_h\boldsymbol{B}\boldsymbol{B}^\top\boldsymbol{Z}_h = \boldsymbol{0},\\ &\dot{\boldsymbol{Z}}_a + \tilde{\boldsymbol{F}}^\top \boldsymbol{Z}_a + \boldsymbol{Z}_a\tilde{\boldsymbol{F}} + \boldsymbol{Q}_a + \boldsymbol{Z}_a\boldsymbol{B}\boldsymbol{B}^\top\boldsymbol{Z}_a + \boldsymbol{Z}_h\boldsymbol{B}\boldsymbol{B}^T\boldsymbol{Z}_h = \boldsymbol{0},\end{aligned} \tag{6}$$

and vectors $\boldsymbol{k}_h$ and $\boldsymbol{k}_a$ such that

$$\begin{aligned}&\dot{\boldsymbol{k}}_h + \tilde{\boldsymbol{F}}^\top \boldsymbol{k}_h + \boldsymbol{Z}_h\boldsymbol{B}\boldsymbol{B}^\top\boldsymbol{k}_h + \boldsymbol{Z}_h\boldsymbol{h} = \boldsymbol{0},\\ &\dot{\boldsymbol{k}}_a + \tilde{\boldsymbol{F}}^\top \boldsymbol{k}_a + \boldsymbol{Z}_h\boldsymbol{B}\boldsymbol{B}^\top\boldsymbol{k}_h + \boldsymbol{Z}_a\boldsymbol{B}\boldsymbol{B}^\top\boldsymbol{k}_a + \boldsymbol{Z}_a\boldsymbol{h} = \boldsymbol{0},\end{aligned} \tag{7}$$

where

$$\begin{aligned}\tilde{\boldsymbol{F}} &= \boldsymbol{A} - \boldsymbol{B}\boldsymbol{B}^\top(\boldsymbol{Z}_h + \boldsymbol{Z}_a),\\ \boldsymbol{h} &= \boldsymbol{c} - \boldsymbol{B}\boldsymbol{B}^\top(\boldsymbol{k}_h + \boldsymbol{k}_a).\end{aligned} \tag{8}$$

Then the differential game admits a closed-loop Nash equilibrium solution with the strategies

$$\begin{aligned}\gamma_h^\star &= -\boldsymbol{B}^\top(\boldsymbol{Z}_h\boldsymbol{\xi}_e + \boldsymbol{k}_h),\\ \gamma_a^\star &= -\boldsymbol{B}^\top(\boldsymbol{Z}_a\boldsymbol{\xi}_e + \boldsymbol{k}_a).\end{aligned} \tag{9}$$

**Proof.** Let us assume $V_i(\boldsymbol{\xi}_e)$, $i = h, a$, to be

$$V_i(\boldsymbol{\xi}_e) = \frac{1}{2}\boldsymbol{\xi}_e^\top \boldsymbol{Z}_i\boldsymbol{\xi}_e + \boldsymbol{\xi}_e^\top\boldsymbol{k}_i + \boldsymbol{n}_i \tag{10}$$

where

$$\dot{\boldsymbol{n}}_i + \boldsymbol{h}^\top\boldsymbol{k}_i + \frac{1}{2}\boldsymbol{k}_i\boldsymbol{B}\boldsymbol{B}^\top\boldsymbol{k}_i = \boldsymbol{0}$$

If we use (10) in (5), equations (6), (7), (8), and (9) are readily obtained. □

However, it is unknown if the human partner really admits the control strategy given with (9) in a human-robot haptic interaction for a tracking task. Therefore, in Section 4 we evaluate such a control scheme experimentally.

### *3.3 Estimation of the human control input*

Since we consider the problem in which the human control input cannot be directly measured, it is necessary to estimate it online. We assume the human control input to be given with (9). In order to obtain the estimate $\hat{\boldsymbol{Z}}_h$ of the human control gain $\boldsymbol{Z}_h$ and the estimate $\hat{\boldsymbol{k}}_h$ of the human feedforward term $\boldsymbol{k}_h$, we apply adaptive input observer (Ioannou and Fidan (2006))).

*Remark 3.* With the proposed input observer the gains and the feedforward term, $\boldsymbol{Z}_h$ and $\boldsymbol{k}_h$, are estimated. This is a particularity of the proposed approach, since adaptive input observer updates the input estimator, $\hat{\boldsymbol{u}}_h$, directly (Narendra and Annaswamy (2012)).

We start by formulating the estimate of the task dynamics (2) under the human control input estimator $\hat{\boldsymbol{u}}_h$ as

$$\dot{\hat{\boldsymbol{\xi}}}_e = \boldsymbol{A}\hat{\boldsymbol{\xi}}_e + \boldsymbol{B}\hat{\boldsymbol{u}}_h + \boldsymbol{B}\boldsymbol{u}_a + \boldsymbol{c} - \boldsymbol{\Gamma}\tilde{\boldsymbol{\xi}}_e, \tag{11}$$

where $\hat{\boldsymbol{\xi}}_e$ is the state estimate, $\tilde{\boldsymbol{\xi}}_e = \hat{\boldsymbol{\xi}}_e - \boldsymbol{\xi}_e$ is the estimate error and $\boldsymbol{\Gamma}$ is a block-diagonal matrix. We assume that the feedforward term in (9) depends on the desired trajectory; namely, $\dot{\boldsymbol{\xi}}_d$, and can be written as $\boldsymbol{k}_h = -\boldsymbol{K}_h\dot{\boldsymbol{\xi}}_d$. Therefore, $\hat{\boldsymbol{u}}_h$ is

$$\hat{\boldsymbol{u}}_h = -\boldsymbol{B}^\top(\hat{\boldsymbol{Z}}_h\boldsymbol{\xi}_e - \hat{\boldsymbol{K}}_h\dot{\boldsymbol{\xi}}_d). \tag{12}$$

Let the gain errors be $\tilde{\boldsymbol{Z}}_h = \hat{\boldsymbol{Z}}_h - \boldsymbol{Z}_h$ and $\tilde{\boldsymbol{K}}_h = \hat{\boldsymbol{K}}_h - \boldsymbol{K}_h$. By subtracting (11) from (2) the error dynamics is

$$\dot{\tilde{\boldsymbol{\xi}}}_e = \boldsymbol{A}\tilde{\boldsymbol{\xi}}_e - \boldsymbol{B}\boldsymbol{B}^\top(\tilde{\boldsymbol{Z}}_h\boldsymbol{\xi}_e - \tilde{\boldsymbol{K}}_h\dot{\boldsymbol{\xi}}_d) - \boldsymbol{\Gamma}\tilde{\boldsymbol{\xi}}_e. \tag{13}$$

In order to obtain the adaptation laws we propose the following Lyapunov function candidate:

$$V(\tilde{\boldsymbol{\xi}}_e, \tilde{\boldsymbol{Z}}_h, \tilde{\boldsymbol{K}}_h) = \frac{1}{2}\tilde{\boldsymbol{\xi}}_e^\top\tilde{\boldsymbol{\xi}}_e^\top + \frac{1}{2\alpha}\mathrm{tr}(\tilde{\boldsymbol{Z}}_h^\top\tilde{\boldsymbol{Z}}_h + \tilde{\boldsymbol{K}}_h^\top\tilde{\boldsymbol{K}}_h), \tag{14}$$

where $\alpha$ is a constant and tr(.) is a matrix trace. The formulation of the adaptive input observer is given in Proposition 1.

*Proposition 1.* If $\boldsymbol{\Gamma}$ is chosen so that $(\boldsymbol{\Gamma} - \boldsymbol{A}) \succ \boldsymbol{0}$ and the rates of the adaptive error estimators, $\tilde{\boldsymbol{Z}}_h$ and $\tilde{\boldsymbol{K}}_h$, are updated as follows

$$\begin{aligned}\dot{\hat{\boldsymbol{Z}}}_h &= \frac{\alpha}{2}\boldsymbol{B}\boldsymbol{B}^\top\tilde{\boldsymbol{\xi}}_e\boldsymbol{\xi}_e^\top,\\ \dot{\hat{\boldsymbol{K}}}_h &= -\frac{\alpha}{2}\boldsymbol{B}\boldsymbol{B}^\top\tilde{\boldsymbol{\xi}}_e\dot{\boldsymbol{\xi}}_d^\top,\end{aligned} \tag{15}$$

then $\dot{V} \le 0$, $\hat{\boldsymbol{Z}}_h, \hat{\boldsymbol{K}}_h, \tilde{\boldsymbol{\xi}}_e$ are bounded, and $\tilde{\boldsymbol{\xi}}_e \to 0$ as $t \to \infty$.

**Proof.** The time derivative of (14) is

$$\begin{aligned}\dot{V} = &-\tilde{\boldsymbol{\xi}}_e^\top(\boldsymbol{\Gamma} - \boldsymbol{A})\tilde{\boldsymbol{\xi}}_e - \frac{1}{2}\tilde{\boldsymbol{\xi}}_e^\top\boldsymbol{B}\boldsymbol{B}^\top\tilde{\boldsymbol{Z}}_h\boldsymbol{\xi}_e + \frac{1}{2}\tilde{\boldsymbol{\xi}}_e^\top\boldsymbol{B}\boldsymbol{B}^\top\tilde{\boldsymbol{K}}_h\dot{\boldsymbol{\xi}}_d\\ &+ \frac{1}{\alpha}\mathrm{tr}(\tilde{\boldsymbol{Z}}_h^\top\dot{\hat{\boldsymbol{Z}}}_h + \tilde{\boldsymbol{K}}_h^\top\dot{\hat{\boldsymbol{K}}}_h).\end{aligned} \tag{16}$$

Since

$$\begin{aligned}&\tilde{\boldsymbol{\xi}}_e^\top\boldsymbol{B}\boldsymbol{B}^\top\tilde{\boldsymbol{Z}}_h\boldsymbol{\xi}_e = \mathrm{tr}(\boldsymbol{\xi}_e^\top\tilde{\boldsymbol{Z}}_h^\top\boldsymbol{B}\boldsymbol{B}^\top\tilde{\boldsymbol{\xi}}_e) = \mathrm{tr}(\tilde{\boldsymbol{Z}}_h^\top\boldsymbol{B}\boldsymbol{B}^\top\tilde{\boldsymbol{\xi}}_e\boldsymbol{\xi}_e^\top),\\ &\tilde{\boldsymbol{\xi}}_e^\top\boldsymbol{B}\boldsymbol{B}^\top\tilde{\boldsymbol{K}}_h\dot{\boldsymbol{\xi}}_d = \mathrm{tr}(\dot{\boldsymbol{\xi}}_d^\top\tilde{\boldsymbol{K}}_h^\top\boldsymbol{B}\boldsymbol{B}^\top\tilde{\boldsymbol{\xi}}_e) = \mathrm{tr}(\tilde{\boldsymbol{K}}_h^\top\boldsymbol{B}\boldsymbol{B}^\top\tilde{\boldsymbol{\xi}}_e\dot{\boldsymbol{\xi}}_d^\top),\end{aligned}$$

we can cancel the last three terms in (16) by imposing (15). Then (16) can be simplified to

$$\dot{V} = -\tilde{\boldsymbol{\xi}}_e^\top(\boldsymbol{\Gamma} - \boldsymbol{A})\tilde{\boldsymbol{\xi}}_e. \tag{17}$$

□

*Remark 4.* We assume that for $t \to \infty$ in (3) $\dot{\boldsymbol{Z}}_h \to 0$ and $\dot{\boldsymbol{Z}}_a \to 0$ in (6) and $\dot{\boldsymbol{K}}_h \to 0$. Therefore, $\dot{\tilde{\boldsymbol{Z}}}_h \to \dot{\hat{\boldsymbol{Z}}}_h$ and $\dot{\tilde{\boldsymbol{K}}}_h \to \dot{\hat{\boldsymbol{K}}}_h$ and the proposed adaptation law (15) can be used to update $\hat{\boldsymbol{Z}}_h$ and $\hat{\boldsymbol{K}}_h$. The convergence of $\hat{\boldsymbol{Z}}_h$ and $\hat{\boldsymbol{K}}_h$ to their actual values, $\boldsymbol{Z}_h$ and $\boldsymbol{K}_h$, respectivaly, can be achieved if $\boldsymbol{\xi}_e$ and $\boldsymbol{\xi}_d$ are persistently exciting signals (Narendra and Annaswamy (2012)).

The block structure of the proposed control scheme is depicted in Fig. 1. The human and the robot are coupled and communicate their control inputs, $\boldsymbol{u}_h$ and $\boldsymbol{u}_a$, via haptic channels. The state of the task, $\boldsymbol{\xi}_e$, is measurable to both partners, and the desired trajectory and its derivative, $\boldsymbol{\xi}_d$ and $\dot{\boldsymbol{\xi}}_d$, are known to both partners.

## 4. RESULTS

In this section we first present simulation results for a one-dimensional tracking task. Then we show the experimental results of the proposed shared control approach.

### *4.1 Simulation results*

The simulation results provide a comparison of two cases: i) the system is controlled only by the human operator, $u = u_h$, and

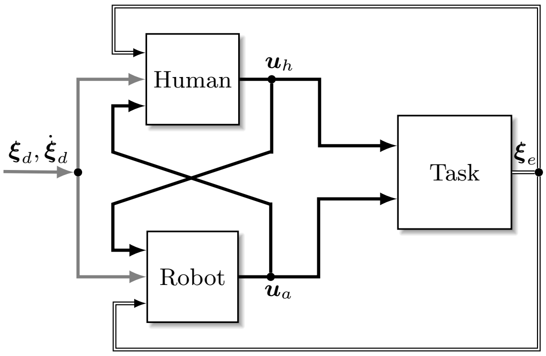



Fig. 1. Block structure.

ii) the system is controlled by the human and the autonomous partners, $u = u_h + u_a$. The parameters of the human input, $\boldsymbol{Z}_h$ and $\boldsymbol{K}_h$, are chosen so that in the case i) successful trajectory tracking cannot be achieved. The desired trajectory is a sum of sine waves, where the number of frequencies is twice as large as the number of unknown parameters, which guarantees convergence of the adaptive laws (15) (Astrom (1987))

$$x_d = \sin(0.1t) + \sin(0.5t) + \sin(0.7t) + \sum_{i=1}^{4} \frac{1}{\omega_i} \sin(\omega_i t),$$

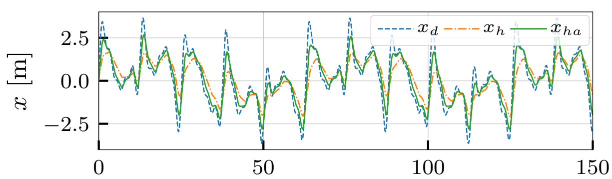
where $\omega_i = 0.5(1 + i)$. The sampling time is $t_s = 1$ ms. The relevant parameters are listed in Table 1.

Table 1. Simulation parameters.

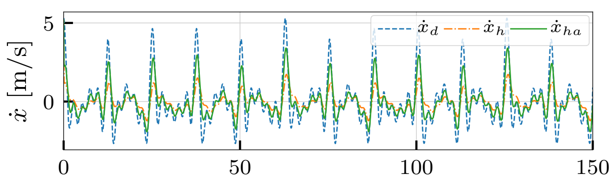
| $m$ | $d$ | $\boldsymbol{Z}_h$ | $\boldsymbol{K}_h$ | $\boldsymbol{Q}_a$ | $\boldsymbol{\Gamma}$ | $\alpha$ |
|---|---|---|---|---|---|---|
| 1 | 20 | $[1\,0; 10\,5]$ | $[1\,0; 0\,1]$ | 500 $\boldsymbol{I}_2$ | 800 $\boldsymbol{I}_2$ | $10^4$ |

The constant $\alpha$ is chosen high in order to achieve faster convergence of the unknowns parameters. The goal is to apply proposed shared control while estimating $\boldsymbol{Z}_h$ and $\boldsymbol{K}_h$ online. Note that the relevant elements of the gain matrices that are estimated are $Z_{h(2,1)} = 10$, $Z_{h(2,2)} = 5$, and $K_{h(2,2)} = 1$.

Fig. 2 shows the tracking performance when the task is controlled only by the human partner, $x_h$, and when the task is controlled by both the human and the robot partners, $x_{ha}$, with the proposed shared control scheme. It can be observed that the shared-control approach improves trajectory tracking performance.




(a) Position trajectories.




(b) Velocity trajectories.

Fig. 2. Simulation results. Tracking performance.

The actual human control input, $u_h$, and the human control input estimator, $\hat{u}_h$, are depicted in Fig. 4b. Convergence of the human control input estimator to the actual human control input, by applying the adaptation law (15), is achieved. Therefore, the approach enables an online estimation of the human input. Fig. 4b also depicts the total control input applied to obtain trajectories in Fig. 2.

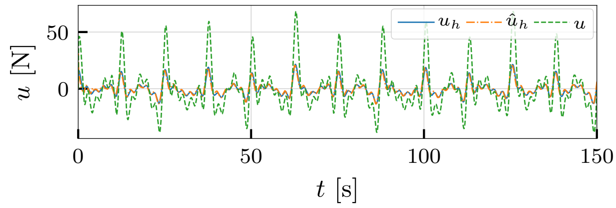



Fig. 3. Simulation results. Actual control input of the human is denoted with $u_h$, its estimator with $\hat{u}_h$, and the total control input with $u$. All trajectories are recorded for the case ii).

The top row of Fig. 4 depicts a successful convergence of the relevant human input parameters to the desired values using the proposed adaptation law. The bottom row of Fig. 4 depicts relevant autonomous control gains, $Z_{a(2,1)}$, $Z_{a(2,2)}$, as well as the feedforward term $k_{a(2)}$. It can be observed that the gains and the feedforward term of the autonomy converge to the expected values that ensure Nash equilibrium. Therefore, the solutions of (6) and (7) converge even if the control gains of the partner are initially unknown and estimated online, as long as $\boldsymbol{\xi}_e$ and $\dot{\boldsymbol{\xi}}_d$ are sufficiently exciting trajectories.

### *4.2 Experimental setup*

We evaluate experimentally if the proposed shared control approach improves the task performance in terms of tracking. Additionally, we evaluate if the estimated control gains of the robot partner converge to constant values, i.e. if the partners reach a Nash equilibrium for $t \to \infty$. We show the results for the cases i) and ii) from Subsection 4.1.

Fig. 5a depicts the experimental setup. The human partner interacts with the *sigma.7* haptic device (robot partner) from Force Dimension to collaboratively track the desired trajectory in the virtual environment (Fig. 5b) which is implemented using *chai3d* framework (Conti et al. (2003)). The task dynamics (2) is assigned to the tool, see Fig. 5b. The human and the robot partners jointly track a one-dimensional trajectory along $y$ direction. The desired trajectory is the sum of eight sine waves

$$x_d = 0.01(\sin(0.5t)+\sin(0.7t)+\sin(t))+0.02 \sum_{i=1}^{5} \frac{1}{\omega_i} \sin(\omega_i t),$$

where $\omega_i \in \{1.2, 1.5, 1.7, 2, 2.5\}$. The sampling time of the haptic device is $t_{s,h} = 0.5$ ms, while the controller sampling time is $t_{s,c} = 1$ ms. The experiment parameters are listed in Table 2.

Table 1. Experiment parameters.

| $m$ | $d$ | $\boldsymbol{Q}_a$ | $\boldsymbol{\Gamma}$ | $\alpha$ |
|---|---|---|---|---|
| 0.2 | 10 | 150 $\boldsymbol{I}_2$ | 500 $\boldsymbol{I}_2$ | $i)1000, ii)10$ |

Parameter $\alpha$ is considerably reduced compared to the simulation and in the case ii) due to its noise sensitivity.

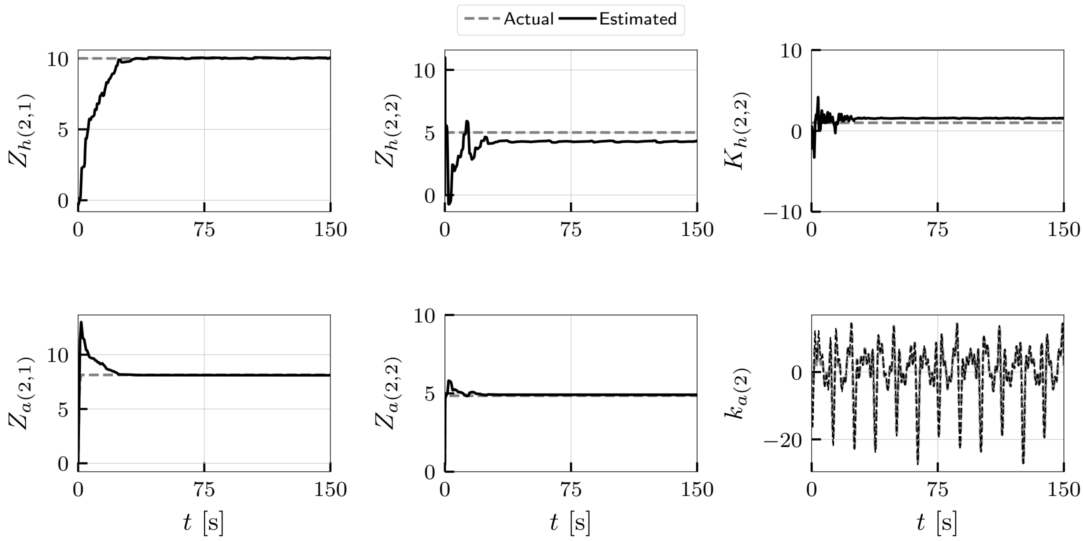
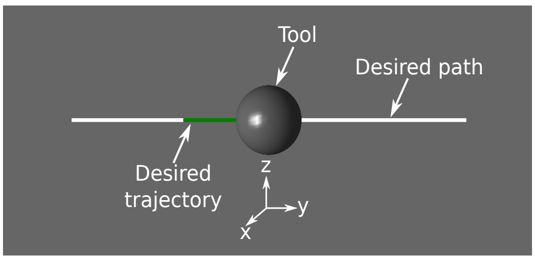



Fig. 4. Simulation results. Unknown parameters of the human input $u_h$ converge to their true values (upper row). Consequently, the parameters of the autonomous input converge to their expected values to ensure Nash equilibrium.

(a) The experimental setup.




(b) The virtual environment.

Fig. 5. The experimental setup. The human and the robot partner (the haptic device) collaborate to accomplish the task, (a). In (b) the desired path is a line, marked with white. The desired trajectory is marked with green. The goal is to track the desired trajectory as precisely as possible with the tool (the sphere).

### 4.3 Experimental results

Fig. 6 depicts desired and actual trajectories along $y$ axis, $y_d$ and $y_h$, as well as the corresponding velocities $\dot{y}_d$ and $\dot{y}_h$ without the robot assistance (case i)). Even though the task can be accomplished, it can be observed that the human operator cannot track the trajectory well in the fully manual case. Fig. 7

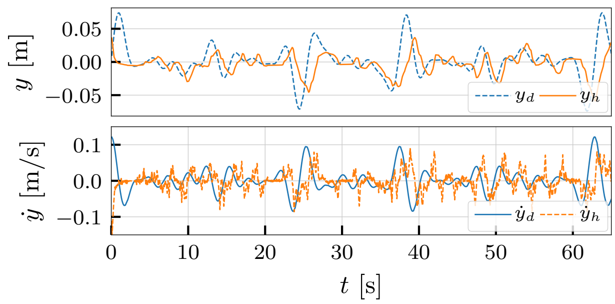



Fig. 6. Experimental results. The desired and actual position and velocity trajectories. The tracking task is controlled by the human operator through the haptic device.

shows the estimated human control input $\hat{u}_h$. Convergence of human input gains is depicted in Fig. 8.

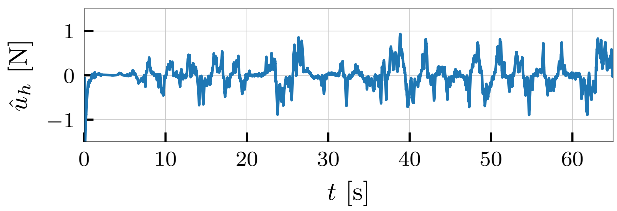



Fig. 7. Experimental results. The estimated human control input, $\hat{u}_h$.

Fig. 9 shows the tracking performance when the proposed shared control scheme is applied. It can be seen that the tracking is improved compared to Fig. 6. The control force applied by the robot assists the human operator to track the desired trajectory. The total control input from the human and the robot is depicted in Fig. 10.

Fig. 11 depicts the gains of the robot control input and its feedforward term. The convergence of the feedback gain to constant values is achieved. Therefore, the solution of the Riccati equation is feasible. Therefore, the players do find

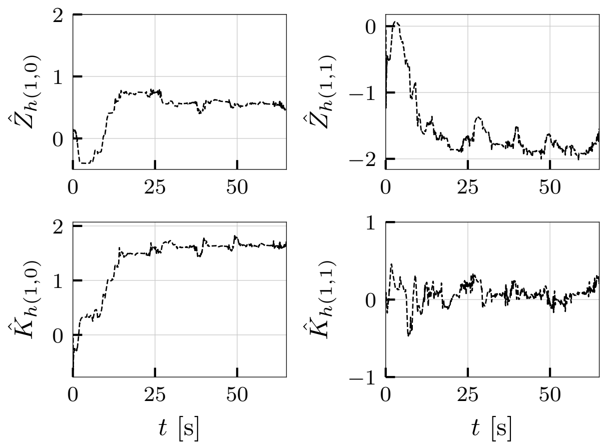



Fig. 8. Experimental results. The estimated parameters of the human input, obtained with the proposed adaptive input observer.

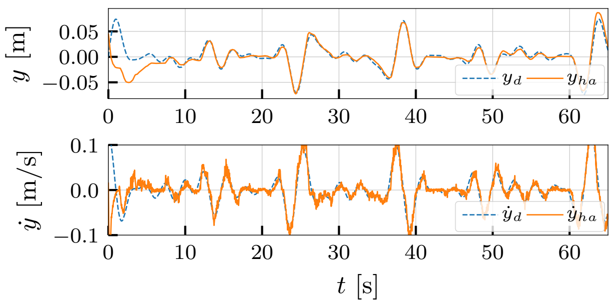



Fig. 9. Experimental results. The tracking performance. The task is controlled by the human and the robot partners.

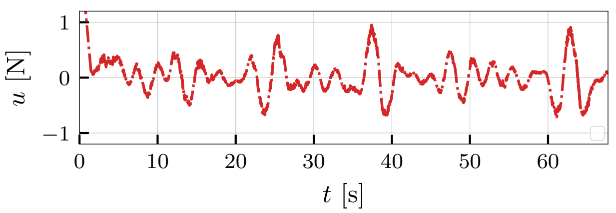



Fig. 10. Experimental results. The total control input as the sum of the human and the robot inputs.

an equilibrium solution during the task execution. The errors in position and velocity tracking for the cases i) and ii) are depicted in Fig. 12. Evidently, the shared control approach

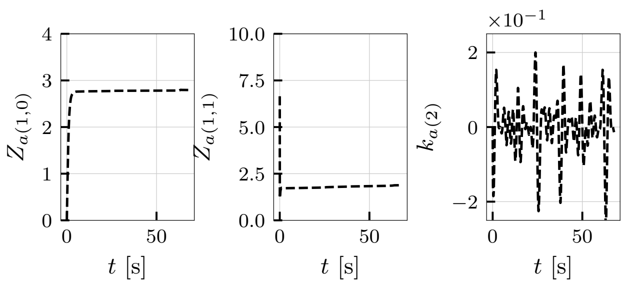



Fig. 11. Experimental results. The parameters of the autonomous agent control input gains.

improves the task performance in terms of tracking and reduced effort.

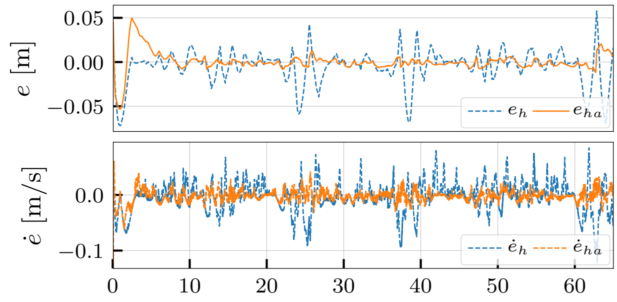



Fig. 12. Experimental results. The position and velocity tracking errors.

## 5. CONCLUSION

In this paper we propose a shared control approach for tracking tasks that is based on the differential game-theoretical framework. We assume the human partner admits an optimal control strategy and the control strategy of the robot is chosen so that the Nash equilibrium can be achieved. We evaluate experimentally the tracking performance of the proposed controller and the convergence of control gains to constant values. The experimental results show that the proposed control approach outperforms the manual case (the task performed only by the human operator). However, comprehensive user study analysis is needed to evaluate the suitability of the proposed control approach for human-robot cooperative tracking tasks.

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