Electron density modelling based on inequality constraints to study the impact of space weather events

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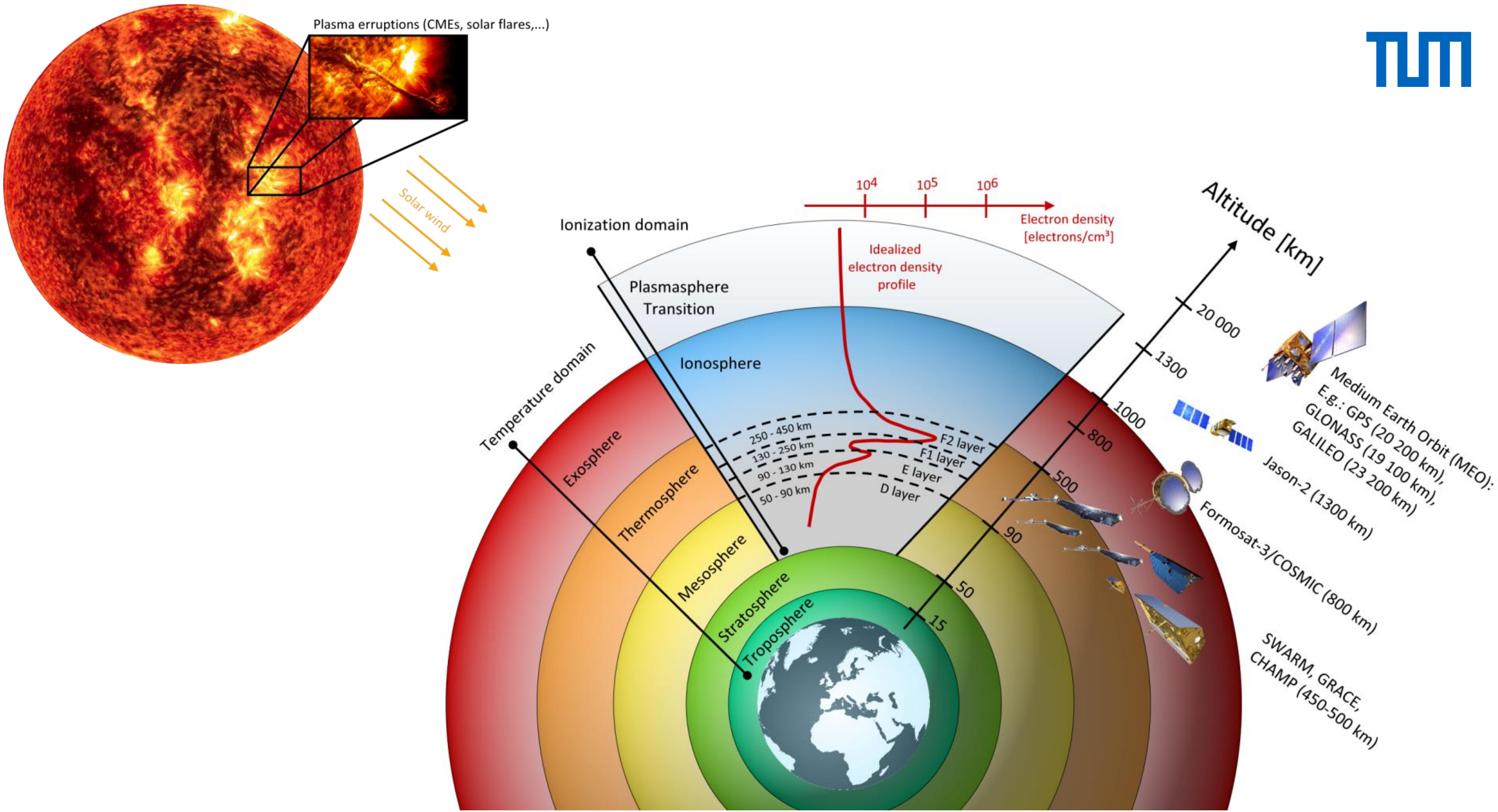
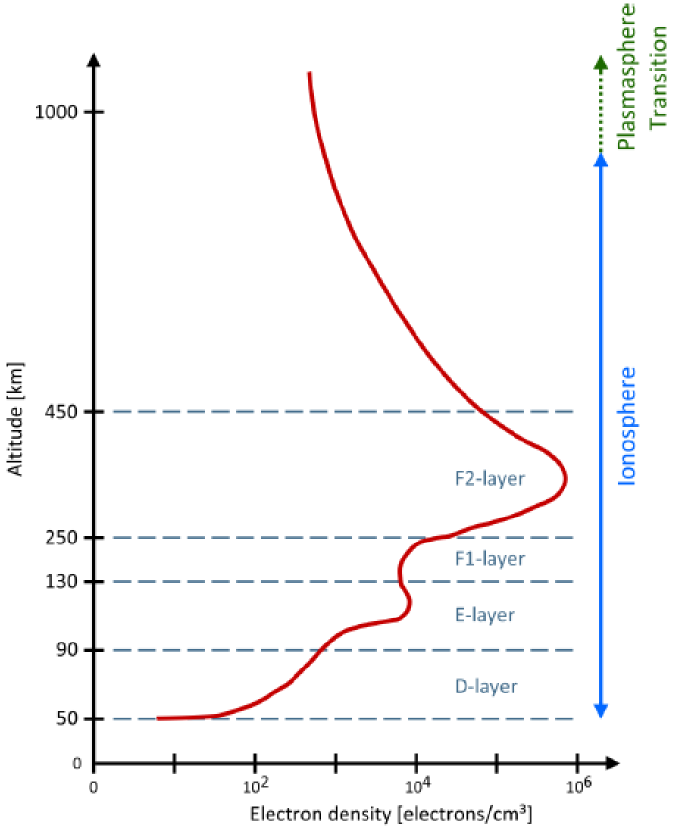

Plasma erruptions (CMEs, solar flares,...)
Solar wind
Ionization domain
Plasmasphere
Transition
Ionosphere
10^4
10^5
10^6
Electron density
[electrons/cm³]
Idealized
electron density
profile
250 - 450 km
130 - 250 km
90 - 130 km
50 - 90 km
F2 layer
F1 layer
E layer
D layer
Temperature domain
Exosphere
Thermosphere
Mesosphere
Stratosphere
Troposphere
Altitude [km]
20 000
1300
1000
800
500
90
50
15
Medium Earth Orbit (MEO):
E.g.: GPS (20 200 km),
GLONASS (19 100 km),
GALILEO (23 200 km)
Jason-2 (1300 km)
Formosat-3/COSMIC (800 km)
SWARM, GRACE,
CHAMP (450-500 km)

Electron density profile modeling

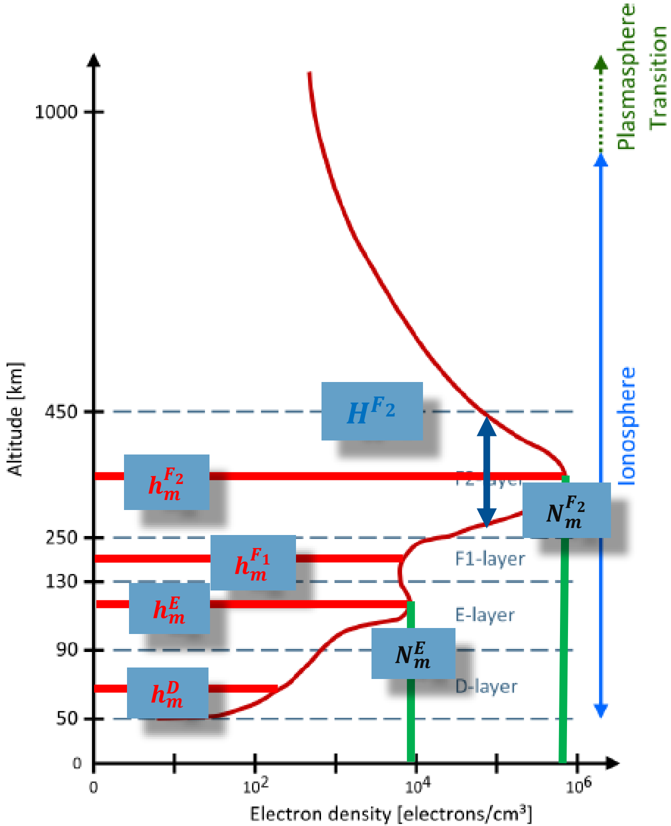


Limberger (2015)

„**Physics-motivated**" approaches for modelling the vertical electron density profile are based on

- Chapman function,
- Epstein layer,
- Multi-layer,
- other combinations

Electron density profile modeling



Limberger (2015)

„**Physics-motivated**“ approaches for modelling the vertical electron density profile are based on

- Chapman function,
- Epstein layer,
- Multi-layer,
- other combinations

The **multi-layer model** is mathematically described as the sum of several profile functions, one for each layer.

Electron density profile modeling

- Following Feltens (2007) our **multi-layer model** is defined as the sum of several **Chapman profile functions**, one for each layer, i.e.

$$\begin{aligned} N_e(h) &= N_e^D(h) + N_e^E(h) + N_e^{F_1}(h) + N_e^{F_2}(h) + N_e^P(h) \\ &= \sum_{Q=1}^{4} N_m^Q\, p^Q(h) + N_0^P\, p^P(h) \end{aligned}$$

with $Q \in \{D, E, F_1, F_2\}$ and $P =$ plasmasphere.

- The **profile function** $p^Q(h)$ represents the electron density distribution of the layer Q.
- It is characterized by the **peak height** h_m^Q, the **maximum** value N_m^Q and the **scale height** H^Q.
- The **plasmasphere profile function** $p^P(h)$ is characterized by the **plasmasphere scale height** H^P.

Electron density profile modeling

- The multi-layer Chapman model is characterized by a **set** $\mathcal{K}$ of **key parameters**, i.e.

$$
\begin{aligned}
\mathcal{K} &= \{N_m^D, h_m^D, H^D, N_m^E, h_m^E, H^E, N_m^{F_1}, h_m^{F_1}, H^{F_1}, N_m^{F_2}, h_m^{F_2}, H^{F_2}, N_0^P, H^P\} \\
&= \{\kappa_1, \kappa_2, \ldots, \kappa_r, \ldots, \kappa_R\}
\end{aligned}
$$

(This set can be extended, e.g. by considering different scale heights for the topside and the bottomside of Chapman layer functions.)

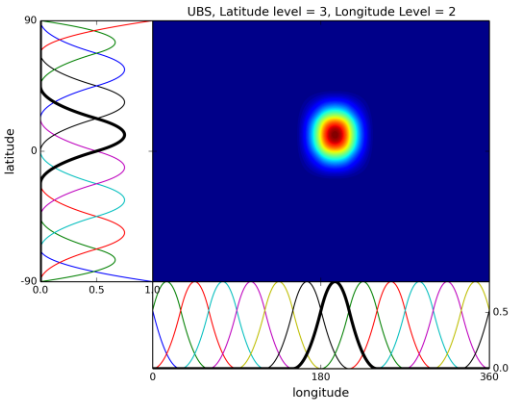
- For each key parameter κ_r we may set up a series expansion in terms of **2-D tensor products** of **polynomial** and **trigonometric B-Spline functions** depending on latitude φ and longitude λ, respectively.

Electron density profile modeling

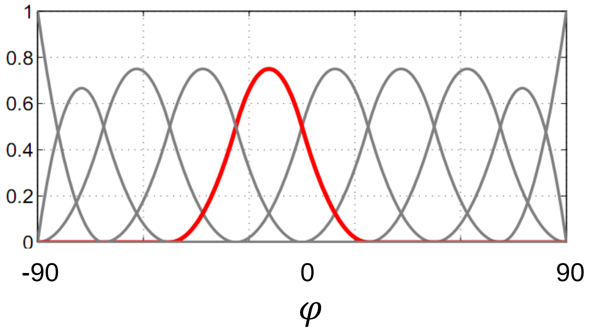
unknown coefficients

$$\kappa_r(\varphi,\lambda,t) = \sum_{k_1=0}^{K_{J_1}-1} \sum_{k_2=0}^{K_{J_2}-1} d_{k_1,k_2}^{J_1,J_2}(t)\; N_{J_1,k_1}^2(\varphi)\; T_{J_2,k_2}^2(\lambda)$$

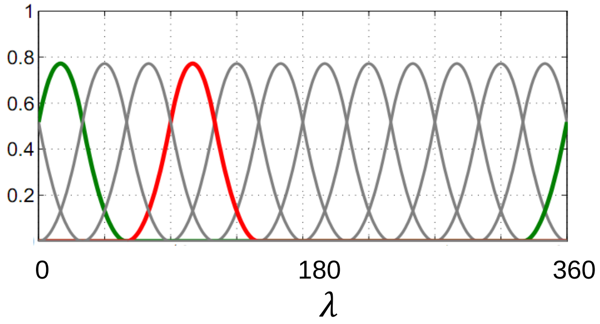
polynomial B-splines   trigonometric B-splines



The values $K_{J_1} = 2^{J_1} + 2$ and $K_{J_2} = 3 \cdot 2^{J_2}$ define the number of **polynomial** and **trigonometric** B-splines distributed along the latitude φ and the longitude λ.



$J_1 = 3, K_1 = 10, k_1 = 0,1,\dots,9$



$J_2 = 2, K_2 = 14, k_2 = 0,1,\dots,13$

Example

- As example we choose the **two-layer model**

$$
\begin{aligned}
N_e(h) &= N_e^{F_2}(h) + N_e^{P}(h) = N_m^{F_2}\, p^{F_2}(h) + N_0^{P}\, p^{P}(h) \\
&= N_m^{F_2} \exp\left(\frac{1 - z - \exp(-z)}{2}\right) + N_0^{P} \exp\left(-\frac{|h - h_m^{F_2}|}{H^{P}}\right)
\end{aligned}
$$

with

$$
z = \frac{h - h_m^{F_2}}{H^{F_2}}
$$

- We reduce the **set** $\mathcal{K}$ of the **key parameters** to $R = 3$, namely

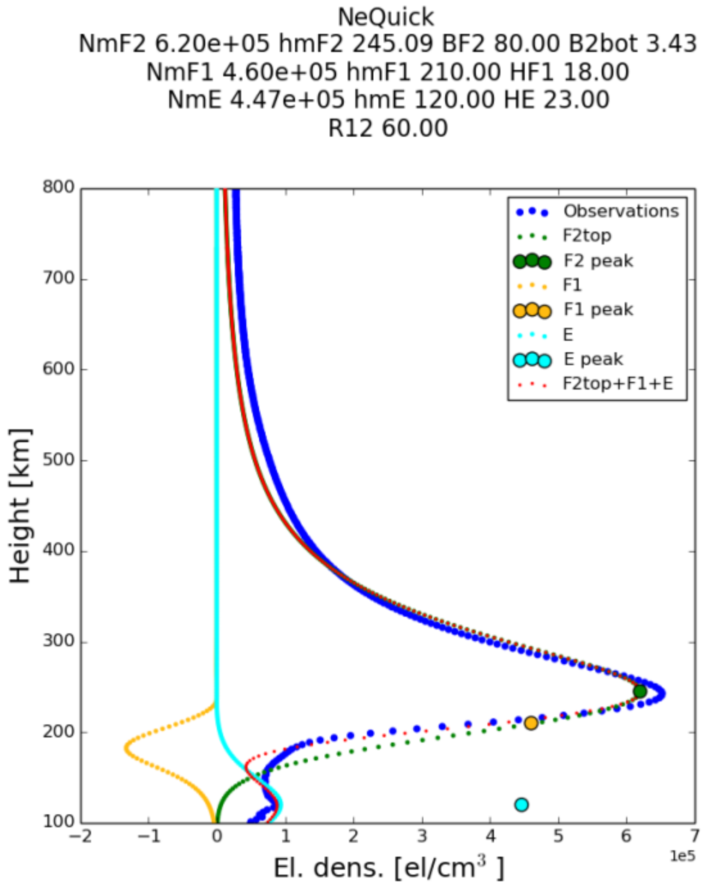
$$
\mathcal{K} = \{N_m^{F_2}, h_m^{F_2}, H^{F_2}\} = \{\kappa_1, \kappa_2, \kappa_3\}
$$

- The other key parameters are assumed to be **given**.

Constraint optimization

- For a joint estimation of the B-spline coefficients for different key parameters, it has to be stated that they are partly both
  - **highly correlated** and **exceeding bounds.**

  Consequently, the estimations could be **physically unrealistic**, e.g.
  - a negative value for the maximum value $N_m^{F_1}$ of the F1 layer or
  - the estimated F2 layer peak height $h_m^{F_2}$ is smaller than the corresponding value $h_m^{F_1}$ of the F1 layer, i.e. $h_m^{F_2} < h_m^{F_1}$.



Constraint optimization

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- Such problems can be avoided by introducing **equality** and **inequality** **constraints** to the modelling approach.
- We estimate the unknown parameters subject to **inequality constraints** using an **optimization approach** (motivated from the approaches in Roese-Koerner (2015)).

Types of inequality constraints

- Different types of inequality constraints could be applied to the electron density modelling.
  - **Absolute constraint** : Absolute lower bound or upper bound for key parameter, e.g. peak density of F1 layer: $N_m^{F_1} \geq 0$
  - **Relative constraint** : peak density of F2 larger than that of F1: $N_m^{F_2} > N_m^{F_1}$
  - **Bounded constraint** : $0.01 \leq N_m^{F_2} \leq 2.5$ (x 10^{12} elec/m^3)
  - **Unbounded constraint** : $0.01 \leq N_m^{F_2}$ or $N_m^{F_2} < 2.5$ (x 10^{12} elec/m^3)
  - **Active constraint** : An inequality constraint is active when the corresponding equality condition holds. The active set for $0.01 \leq N_m^{F_2}$ would be $N_m^{F_2} = 0.01$ (x 10^{12} elec/m^3)
- The constraints on key parameters could be transformed to constraints on the B-spline coefficients.

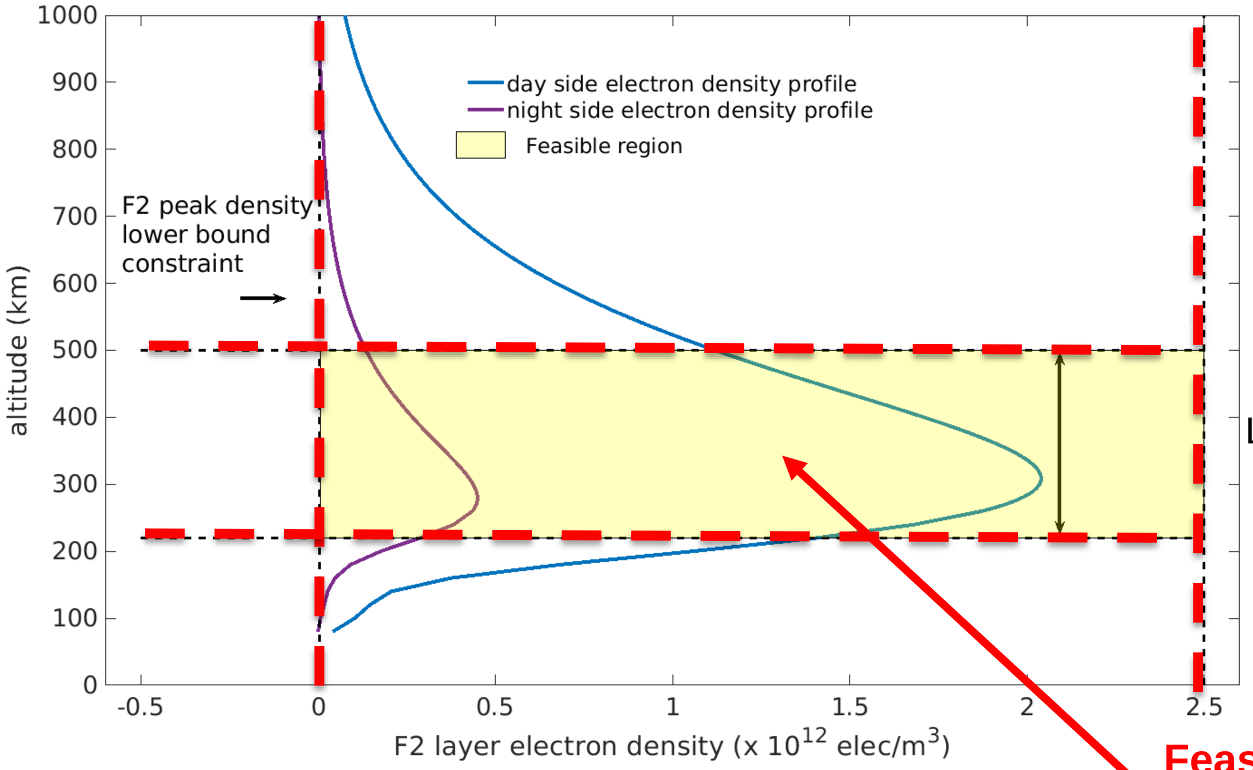
Optimization – imposition of constraints

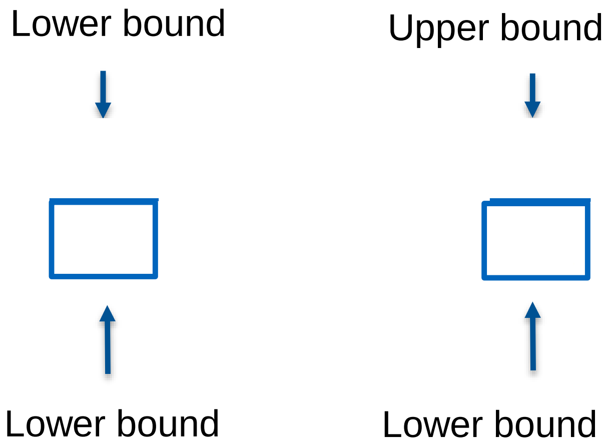
Key parameter	Lower bound	Upper bound
F2 peak density $N_m^{F_2}$ (x 10^{12} elec/m^3)	0.01	2.5
F2 peak height $h_m^{F_2}$ (km)	220	500

- We impose bound constraints on **two key parameters**: F2 peak density and peak height.

- Bounded constraint results in a finite set of candidate points to evaluate the **cost function** and to determine the next set of feasible directions and descent directions.
- The region of space occupied by constraints is called the **feasible region.**
- The residual sum of squares of the electron density becomes the optimization **cost function.**

Optimization – imposition of constraints





Feasible region between the constraints

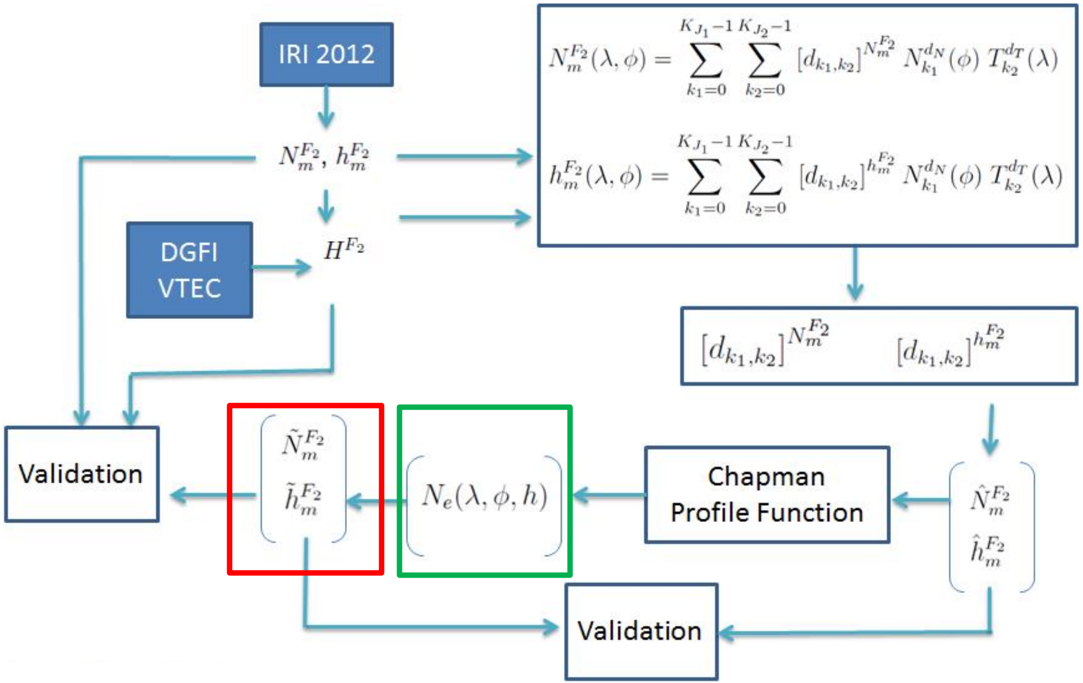
Inequality constraint optimization - solution

- The main idea in inequality constraint optimization is to seek a certain direction to **traverse** the **feasible region** to reach an optimal state.
- This involves the determination of a set of direction vectors, called **feasible directions**. In addition the direction of the gradient of the cost function is determined. This is used to determine the **descent direction**.
- The algorithm looks for the **descent direction and feasible direction** until it is no longer possible to traverse along the descent direction within the feasible region and yet **reduce the cost function**
- At this point, the algorithm has found the **global minimum** and solved the optimization problem

Electron density modelling – simulation

- We used a standard 5° x 5° uniform grid to create **pseudo observations of electron density** along latitude and longitude.
- We have also used a variable sampling along the vertical to generate the **electron density profile**. The nominal altitude step side is 10 km in the D, E and F1 layer. We have used a relatively denser sampling rate (5 km step size) for the F2 layer. Above the F2 layer we have used a step size of 20 km.
- We have set the **lower limit** and **upper limit** of ionosphere at 90 km and 1000 km respectively.
- The **inequality constraints** are applied consistently at all locations (grid points) globally.

Electron density modelling – simulation



$$\Delta\tilde{N}_m^{F_2}(\lambda,\phi) = \tilde{N}_m^{F_2}(\lambda,\phi) - \hat{N}_m^{F_2}(\lambda,\phi)$$

$$\Delta\tilde{h}_m^{F_2}(\lambda,\phi) = \tilde{h}_m^{F_2}(\lambda,\phi) - \hat{h}_m^{F_2}(\lambda,\phi)$$

Deviations

B-spline series for ionosphere key parameters

“True set” of B-spline coefficients (reference)

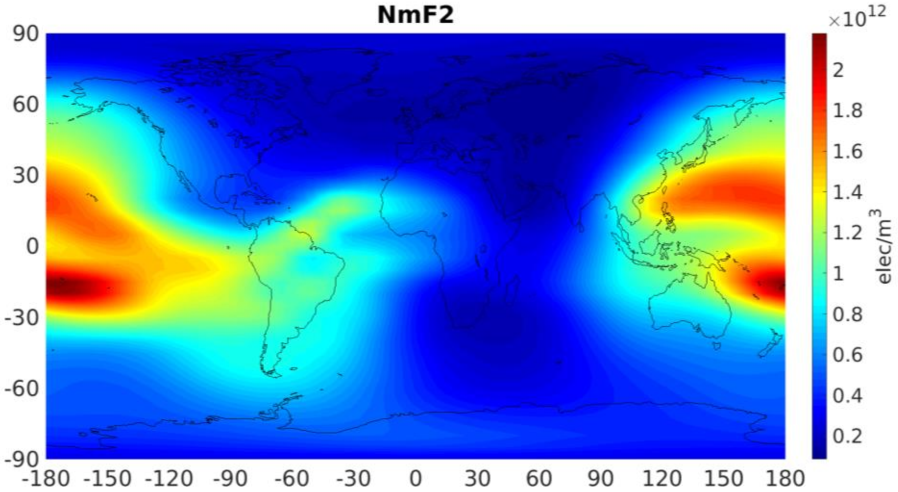
“True set” of key parameters

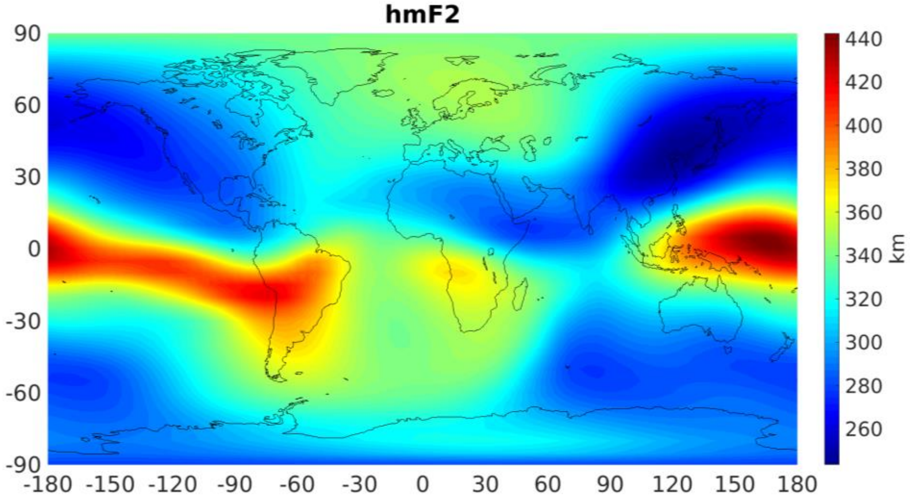
Electron density observations

Final set of estimated key parameters

Electron density modelling – simulation

- The **differences** between the estimated and the “true values” for the two key parameters are in the range of **computational accuracy**.
- Comparing the **constrained** with the **unconstrained** solution it has to be stated that the latter needs **more iterations** as the constrained one.





Summary

- The estimation of B-spline coefficients of ionosphere key parameters is performed using the **constraint optimization** approach.
- In order to validate the optimization procedures, we use pseudo observations (based on the IRI model). This allows a **close loop validation**.
- The technique has been extended for additional analysis for **modeling 5 key parameters** (including F1 layer) and with realistic observation noise (see back up slides).
- The approach is especially well suited for modelling (mutually correlated) ionosphere key parameters for **multiple layers** simultaneously.
- The main benefit of the approach is **to avoid the physically incomprehensible values** for the key parameters (such as ensuring F2 layer peak height above F1 layer peak height and ensuring non negative peak densities).

References

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