2017 IEEE Intelligent Vehicles Symposium (IV)
June 11-14, 2017, Redondo Beach, CA, USA

Multi Vehicle Trajectory Coordination for Automated Parking

Tobias Kessler[1] and Alois Knoll[2]

***Abstract*— Automated parking will probably be among the first tasks performed by fully autonomous vehicles. To smoothly integrate into the existing infrastructure and traffic scenarios an autonomous vehicle has to make assumptions on the intentions of human drivers and maneuver safely at low speeds in a highly structured environment without Car2X communication. This work presents a trajectory planning and coordination strategy for such situations. Based on the observed situation and an estimated intention of other participants in the scene their driving behavior is quantified and an optimized trajectory plan is calculated using mixed-integer linear programming. It balances own and other intentions and avoids collisions. The approach resolves conflicts occurring in parking situations as demonstrated in two simulated scenarios.**

I. Introduction

Fully or partially autonomous driving is currently under development and before handling all driving tasks automated driving functions will be present in a subset of driving situations as automated highway driving or automated parking. From a user perspective especially automated parking systems can serve as a gateway to autonomous driving [1]. However, for possibly very few automated vehicles the traffic infrastructure including most parking lots will not be updated soon and exclusive areas will be rare.

Therefore, the first autonomous vehicles have to be able to interact with human driven vehicles, especially in ambiguous and conflict situation without relying on Car2X communication. Apart from traffic rules, human drivers often rely on nonverbal communication intention interpretation to resolve such situations. An autonomous vehicle has to take part in this communication process by estimating the actions of other vehicles and performing clear and safe actions. Due to the low penetration, Car2Car communication is not a channel to rely on.

A simple traffic scene is depicted in Fig. 1. For human drivers the occurring conflict situations are easy or straightforward to resolve based on the behavior observation and simple communication clues. Also the exact goal (parking) poses does not matter. In this work, we contribute

- an approach for an autonomous vehicle to master parking situations in the presence of other communicating or non-communicating vehicles
- a formulation as symmetric mixed integer linear optimization problem
- a way to quantify intentions and driving styles and include those in the optimization problem.

[1]Tobias Kessler are with fortiss GmbH, affiliated institute of Technische Universität München, Munich, Germany
[2]Alois Knoll is with Robotics and Embedded Systems, Technische Universität München, Munich, Germany

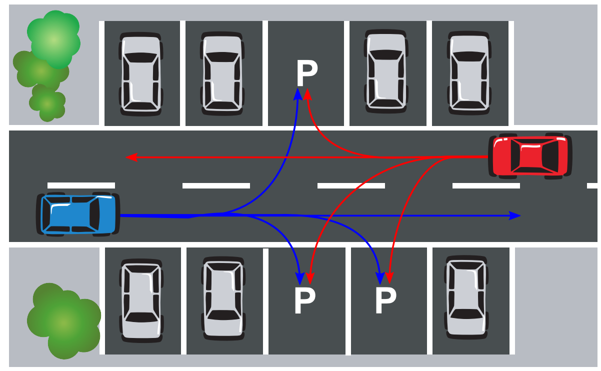


Fig. 1: A typical parking scenario: the possible vehicle trajectories are ambiguous and may result in collisions.

The presented approach is applied but not limited to parking situations.

This work is further organized as follows. Section II gives a literature overview on conflict resolution in automated driving scenarios with a focus on linear programming approaches and their differences to this work. The proposed algorithm is presented in section III and evaluated in section IV followed by a conclusion in section V.

II. Related Work

"Cooperative behavior" among autonomous vehicles can be quantified introducing an individual utility function for each vehicle and a joined utility for multiple vehicles. The joint utility of a behavior combination in relation to a reference behavior gives a classification into cooperative and uncooperative [2]. Without setting behaviors in relation cooperation can be seen as following the same (traffic) rules [3].

In the literature we find several conflict resolution algorithms, mainly focused on highway/merging scenarios [2], [4] or intersections [5]. The approaches differ in the amount of information exchanged and how a solution is computed in a distributed fashion. Schwarting and Pascheka [4] propose to first plan egoistic maneuvers neglecting other vehicles' interests, afterwards possible conflicts are detected and recursively resolved, starting from the furthest vehicle. By predicting the intention of other vehicles the need for inter-vehicle communication is avoided. Evaluating a cost function the best possible conflict free maneuver combination is selected. Likewise Krajewski *et al.* [5] propose a two step method, first an individual trajectory generation followed by cooperative behavior optimization for connected vehicles. In [2], Düring and Pascheka calculate a set of possible maneuvers including costs that are shared among all vehicles. This ensures a common environment understanding of all participants; the decision which action is taken is up to each vehicle individually.

Conflict situations in traffic scenarios have successfully been modeled as Mixed Integer Linear Problem (MILP). Examining the global traffic network Lin *et al.* [6] reformulate a model predictive control problem of an urban traffic network as MILP. Reghelin and De Arruda [7] solve highway scenarios with correct acceleration and overtaking behavior. An optimal vehicle to parking space assignment in a huge network is tackled by Geng and Cassandras [8] minimizing each driver's cost function and the proximity to the driver's destination. Concentrating on a local vehicle scope, Schouwenaars *et al.* [9] models a multi-vehicle path planning problem with obstacles as MILP. All vehicles are assumed to be cooperative. The vehicle dynamics are incorporated into the optimization problem in a linear and discrete fashion. Several algorithms for multi-vehicle motion planning, including a MILP-based method that minimizes the total amount of acceleration are compared in [10]. Constraints ensure a collision free motion inside the road boundaries. All vehicles are assumed to be cooperative. In the observed crossing and overtaking scenarios solving the MILP takes varying computation time, so the real-time applicability is questionable. Nevertheless the algorithm shows good results in terms of planning success, but tends to become infeasible due to conflicting constraints for more than three vehicles. Incorporating uncertain measurements, actuator disturbances and model uncertainties a MILP approach to optimize the trajectory of one vehicle in the presence of obstacles can be formulated [11]. A chance constrained framework keeps the failure probability below a threshold. With bounds on the collision probability the chance constrained non-convex problem is converted into a linear convex one.

In multi-aircraft trajectory planning and coordination we find several MILP formulations as e.g. optimizing a global fleet goal by solving small sub-problems in a decentralized way [12] or minimizing a total maneuver finishing time with collision avoidance and model dynamics constraints [13].

A substantial body of work is devoted to predicting other traffic participants actions. Bai *et al.* [14] use a partially observable Markov decision process to estimate actions of vehicles or pedestrians given noisy sensor measurements. As these information are incorporated into the path planning approach a safe motion under an uncertain environment is achieved. Bayesian networks can be applied to estimate the intention of drivers at intersections as well as the expectations on the drivers actions [15]. Tran and Firl [16] present a framework to predict a vehicle trajectory on an intersection with respect to the current traffic situation.

III. Algorithm Construction

In contrast to other work we do not assume all participants are interested in cooperation and a globally optimal solution but incorporate potentially egoistic behavior. Furthermore, we do not rely on a fixed computation order of all vehicles or other prioritization schemes among the vehicles. Each vehicle does not have a fixed predefined goal but a given set of possible goals. By design we handle communication enabled vehicles equivalent to non-communicating (human driven) vehicles. The difference is how the intention of the vehicle is gathered. In the communicating case we assume the other vehicle aims to find a cooperative solution and communicates its exact intention. In case of a non-communicating vehicle the intention has to be observed and quantified and this vehicle might not behave as estimated.

A. Motivation and Restrictions

We consider ambiguous traffic scenes at low speeds as the example depicted in Fig. 1. *We assume the environment is previously known from an offline map.* Especially lanes and parking spot positions are known. With bounded vehicle accelerations and top speeds this limits the number of possible trajectories. We aim to find an optimal matching of vehicles in all possible trajectories within the current traffic scene. Optimality is defined as the minimum of a global cost function each vehicle contributes its individual costs to.

A global optimum will often result in a sub-optimal motion of an individual vehicle, e.g. waiting for another vehicle to pass. A non-cooperating vehicle cannot be enforced to follow this individual sub-optimal plan and might take egoistic actions but does not behave fully destructive. *All participants behave meaningful.* We can rely on the structure of the environment and all vehicles following a set of (traffic) rules.

The sensor sight is ideal, each vehicle can estimate the own position and the positions of all other vehicles perfectly. This assumption is clearly non realistic and weakening this assumption is up to future research.

A sufficiently large subset of all possible trajectories for all vehicles can be computed. In a highly structured environment the set of possible actions and the set of goal poses is limited and the options each vehicle can take are few. In the parking case a vehicle is either looking for a suitable parking spot, maneuvering into/out of a parking spot, or leaving the parking site. The trajectory a vehicle will actually take is among the set of pre-calculated ones or close. No effort was put into the generation of the vehicle trajectories. Following the implementation of Reed-Shepps curves [17] a path drivable by a non-holonomic vehicle is generated and enriched with a velocity profile. Optimal trajectories regarding comfort can be found with a jerk minimizing trajectory generator, cf. [18].

Most multi-vehicle coordination approaches assume fixed goal positions for each vehicle. In this work, the exact goal is a degree of freedom and the goal choice is optimized. From the estimated vehicle intention a set of possible goal positions is computed. In a parking scenario this assumption is reasonable as a vehicle is not interested in taking a specific parking spot in case several are possible.

The vehicles are modeled as single track model with variable shape and turning radius. To speed up the collision check the vehicle geometry is approximated with two circles, cf. [19].

B. Cooperation algorithm

For each of the precomputed trajectories we compute a set of characteristic, quantified *properties*. Each vehicle can put *weights* on these properties according to it's high level intention. With an ideal knowledge of one vehicle's intention the trajectory it is willing to choose can be matched exactly. The estimation on the intentions is a non-trivial task (cf. section II) and will not be accurate. This results in the prediction of a wrong trajectory. Having computed a sufficiently large subset of all possible trajectories, the algorithm is constructed to find a close to optimal match with non-optimal intention predictions. All properties and weights are zero or positive. Any value that can be computed from a trajectory can serve as property, we use the following set:

- Spatial distance to the targeted goal
- Travelled distance on this trajectory if it is driven from start to goal
- Flag if the trajectory reaches its goal
- Amount of acceleration needed to follow the trajectory
- Amount of steering input needed to follow the trajectory
- Number of direction changes of the trajectory
- Deviation from the desired trajectory top speed

From the observed actions of a vehicle weights are extracted that fit the observed behavior most. By assumption the trajectory a vehicle took in the past is part of the set of pre-calculated trajectories. We find a set of weights that corresponds best to the choice of this trajectory. This estimation and the set of available goals is corrected if possible. As we will see in section III-C, the weights will directly influence the cost function of the optimization. Weights of different magnitude will force the optimization to stronger incorporate the intentions of the corresponding vehicle and property. This way a behavior from "egoistic" to "fair" to "altruistic" can be modeled. Interacting with other autonomous vehicles weights shall be exchanged via Car2Car communication. The set of weights of one vehicle is initialized with fixed reasonable values. According to the intention the weight set is biased towards (e.g.) minimum travel time, maximum smoothness or low consumption.

The proposed algorithm does not distinguish between the own and other vehicles. It is evaluated cyclic and at any time. With only one vehicle present it optimizes the trajectory choice, with more vehicles it also resolves possible conflicts and performs the following steps:

(1) Update step
   (a) Update environment model if necessary
   (b) Update vehicle intention estimates (weights)
   (c) Add new vehicles, assign best-guess intention
(2) Trajectory generation step
   (a) Generate trajectory set for all vehicles and goals
   (b) Calculate possible vehicle/trajectory assignments and trajectory properties
   (c) Check for trajectories for collisions
(3) Optimization step: find optimal vehicle/trajectory pairs
(4) Evaluation step: Perform own optimal plan, observe others behavior

C. Optimization Problem

We use mixed-integer linear programming to assign each vehicle a trajectory and to optimize the assignment due to given criteria. The problem is formulated symmetrically in the sense that no distinction is made between the ego and other vehicles. To find an optimal assignment each trajectory is enhanced with N properties, denoted by p, and each vehicle can set weights, denoted by φ, on these properties. Defining the decision variables x with $x(v,t) = 1$ if vehicle v takes trajectory t and 0 otherwise the objective function aiming to minimize the total costs can be written as

$$\sum_{t\in\mathcal{T}}\sum_{v\in\mathcal{V}} x(v,t)\left(\sum_{i=1}^{N} p_i(t)\varphi_i(v)\right) \tag{1}$$

with the set of vehicles $\mathcal{V}$ with a unique start pose and possibly several goal poses and set of trajectories $\mathcal{T}$ from vehicle start to goal poses with different properties. By $\mathcal{G}$ we denote the set of possible goal poses. By the constraints

$$x(v,t) \in [0,1] \quad \forall \quad t \in \mathcal{T}, v \in \mathcal{V} \tag{2}$$

$$\sum_{t\in\mathcal{T}} x(v,t) = 1 \quad \forall \quad v \in \mathcal{V} \tag{3}$$

we ensure each vehicle out of $\mathcal{V}$ is assigned exactly one trajectory out of $\mathcal{T}$. With the pre-calculated collision matrix $C(t_1,t_2)$ that indicates if trajectory t_1 and t_2 collide (2 if no collision, 1 if collision) we exclude colliding trajectory combinations by

$$\sum_{v_1\in\mathcal{V}} x(v_1,t_1) + \sum_{v_2\in\mathcal{V}} x(v_2,t_2) \le C(t_1,t_2) \quad \forall \quad t_1 \in \mathcal{T}, t_2 \in \mathcal{T}. \tag{4}$$

We further define an assignment matrix A ensuring a vehicle v can take trajectory t as start and goal poses match (1 if possible, 0 if not) and a corresponding constraint

$$x(v,t) \le A(v,t) \quad \forall \quad v \in \mathcal{V}, t \in \mathcal{T}. \tag{5}$$

The matrix A is easily setup while computing the trajectories. As not the whole horizon of a trajectory might be checked for collision, constraint (6) sorts out collisions that occur if two vehicles target the same goal that will result in a collision in the future using an easy to compute goal assignment matrix G that indicates that a goal g is reached by a trajectory t

$$\sum_{t\in\mathcal{T}} G(g,t) \sum_{v\in\mathcal{V}} x(v,t) \le 1 \quad \forall \quad g \in \mathcal{G}. \tag{6}$$

The resulting optimization problem that computes an optimal vehicle/trajectory assignment for one step avoiding collisions and balancing individual interests is formulated as

$$\begin{aligned} &\text{minimize } (1) \\ &\text{subject to } (2),\ (3),\ (4),\ (5),\ (6). \end{aligned}$$

IV. Evaluation

We demonstrate the proposed method on two scenarios (section IV-C and section IV-D) evaluating the criteria introduced in section IV-B followed by a discussion of the results.

A. Implementation

The scene is build up and the trajectories are generated using MATLAB. As an optimization solver the GNU Linear Programming Kit[1] is used.

Non-cooperative driving behavior is modeled as follows. The corresponding vehicle targets a fixed goal location with a fixed speed profile. It follows this trajectory exactly and brakes to zero speed only if a collision is immanent. This can yield aggressive behavior which is compensated in the optimization as others react to this behavior defensively.

Parameters are chosen as follows. The optimization cycle time is chosen as 2 seconds, with a trajectory horizon of 10 seconds. The trajectories are checked for collision within a horizon of 6 seconds. This avoids immanent collisions but allows the vehicles to enter probably conflicting situations that are resolved later on. The safety distance is chosen as 20 centimeters. Target speeds from 0 to 5 meters per second are allowed. The optimization rewards choosing short and fast trajectories and penalize direction changes.

B. Evaluation Measures

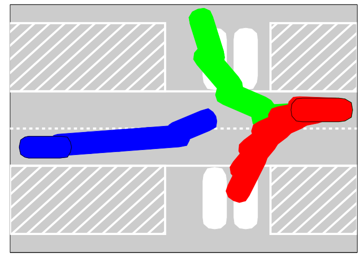
As a ground truth and a performance benchmark we resolve the scene assuming all vehicle intentions are known and each vehicle is forced to follow the optimized plan. This global optimal solution and the situation with potentially non-cooperative vehicles does not differ in the problem setup and the optimization problem formulation.

We further compare the solution of the coordination algorithm to the scenario if each participating vehicle would be alone in the scene. In case of a non-cooperating vehicle we compare both the optimal behavior with respect the optimization cost function and the costs by the fixed non-cooperative, potentially egoistic and sub-optimal behavior.

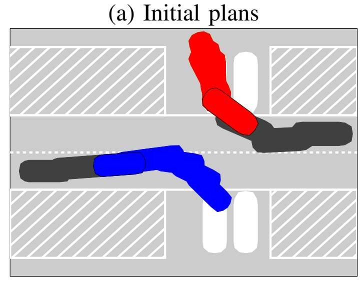
Most natural evaluation factors are already part of the optimization's objective function, and hence can be leveraged with appropriate parametrization. Therefore these may not serve as evaluation criteria; we instead define the following: By $\Phi(v,i)$ we denote the cost function value contributed by vehicle v in cycle i that was computed in the optimization. $\hat{\Phi}(v,i)$ is defined as the cost actual contribution of vehicle v in cycle i. In case of a non-cooperative vehicle the value is calculated (a posteriori) from the actually taken trajectory's properties and the estimated intentions. We define the fraction of both as $\Theta = \Phi/\hat{\Phi}$. An overline denotes the respective mean. We choose the following performance measures:

- Number of iterations $\#It$ for all vehicles to reach a goal as a measure of the maximum travel time
- Total $\sum\hat{\Phi}$ and average $\overline{\hat{\Phi}}$ cost function values giving a performance measure of the coordination algorithm in relation to the reference scenarios
- Ratio of egoistic costs to optimized costs Θ as an indicator how accurate the intention has been and how aggressive/defensice non-cooperative vehicles behave in relation to the ego-vehicle

[1] https://www.gnu.org/software/glpk/



(a) Initial plans



(b) Step 1, after 2s

Fig. 2: Evaluation of a 2 vehicles parking lot maneuvering scenario. The free parking spaces and goal poses are depicted as solid white, the white hatched patches are obstacle areas. The autonomous ego vehicle 1 to the left is colored in blue; the non-communicating vehicle 2 in red. In blue the respective optimized trajectories are drawn, in green the (unknown) trajectory vehicle 2 is going to take. Colored in gray are the already travelled trajectory.

C. A Simple Example

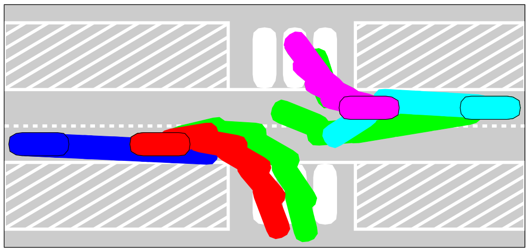
We consider an easy coordination situation as sketched in Fig. 1. The autonomous ego vehicle 1 (blue), another non-cooperative vehicle 2 (red), both having various reasonable driving options. Vehicle 1 wants to park and vehicle 2 has the fixed but unknown intention to park at its right.

From the cost function values presented in table I we can observe that the global optimal solution leverages the interests of both vehicles. In the non-cooperative case the total optimization costs are roughly the costs of both vehicles if they were alone in the scene. This is reasonable as the optimal solution is straightforward after the first iteration. $\sum_{v,i}\Theta(v,i) = 0.96$ and $\overline{\Theta(2,i)} = 0.79$ indicate that with a non-cooperative and suboptimal behaving vehicle a worse solution is found that is still close to the optimal result. The qualitative evolvement of the scene is depicted in Fig. 2.

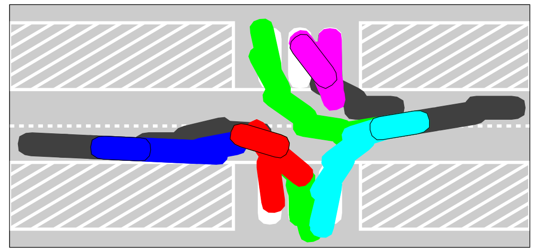
TABLE I: Results of the simple example

	#It	$\sum_{v,i}\hat{\Phi}(v,i)$	$\overline{\hat{\Phi}(v,i)}$
Vehicle 1 alone	4	421.8	105.5
Vehicle 2 alone, optimal	4	296.5	74.1
Vehicle 2 alone, egoistic	5	635.4	127.1
Global Optimum	4	716.1	69.5
Non-Cooperative	5	1178.9	117.9

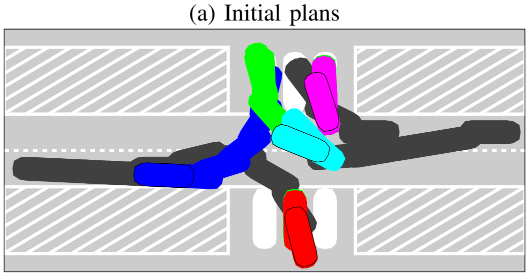
Comparable results are achieved if vehicle 2 has the intention to go straight instead of choosing a parking spot.



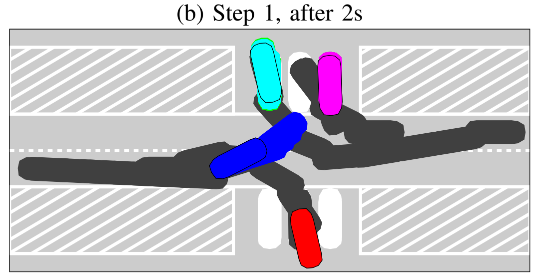
(a) Initial plans



(b) Step 1, after 2s



(c) Step 2, after 4s



(d) Step 3, after 6s

Fig. 3: Evaluation of the Parking Lot Scenario. The blue (leftmost) vehicle 1 is the ego vehicle, all others are non-communicating vehicles. The respectively colored trajectories are the optimized ones, the light ones the (unknown) trajectories the vehicles are going to take, gray the already travelled trajectory.

D. Simulation Results Exemplary Parking Lot Scenario

As a second example scene we pick a parking lot scenario with 3 free parking spaces to the left and 3 to the right of a two lane road. These are the possible goal positions. Vehicle 1 (leftmost, blue) is the autonomous ego vehicle under test, all other vehicles 2-4 are non-cooperative and potentially act egoistic. All four vehicles want to park. The initial scene is depicted in Fig. 3a including the vehicle plans in the first step (light colored) and the unknown actual plans (green). The evolvement of the scene is depicted in Fig. 3, this solution is compared to the references in table II. The higher total costs in the non-cooperative scenario are mainly due to the higher number of iterations to finish the scenario as the resulting mean values have a comparable magnitude. The mean cost function values $\overline{\hat{\Phi}}$ of the vehicles 2-4 in the optimized and egoistic cases indicate that the estimations on the trajectories differ from the actually taken trajectories. As these egoistic trajectories also may have non-optimal goals, more iterations and higher costs occur. Regarding the cost function ratios Θ, a deviation of optimized and non-cooperative value of $\sum_{v,i}\Theta(v,i) = 0.9$ is obtained. The mean values per vehicle v_j, $\overline{(\Theta(v_j,i))}$ vary between 0.84 and 0.97 depending on how much the optimized and the predefined egoistic trajectories deviate. Regarding the evolution of the costs over iterations both the egoistic and the optimal trajectories show similar trends. The optimized vehicle target speeds are lower on average than the egoistic vehicle top speed of $4m/s$. This on the one hand shows that the optimization leverages more costs like the needed amount of acceleration and on the other hand indicates that the non-cooperative behavior can be rated as egoistic. Slower top speeds are desirable from a global point of view.

TABLE II: Results of the parking lot scenario compared to the reference behavior.

	#It	$\sum_{v,i}\hat{\Phi}(v,i)$	$\overline{\hat{\Phi}(v,i)}$
Vehicle 1 alone	4	658.4	164.6
Vehicle 2 alone, optimal	4	338.2	84.6
Vehicle 2 alone, egoistic	3	322.7	107.6
Vehicle 3 alone, optimal	3	335.2	111.7
Vehicle 3 alone, egoistic	4	259.2	64.8
Vehicle 4 alone, optimal	5	582.7	116.5
Vehicle 4 alone, egoistic	4	513.4	128.4
Global Optimum	5	1852.9	118.8
Non-Cooperative	7	2105.6	117.0

E. Runtime Discussion

The main runtime contributions are the trajectory generation and the optimization step. The number of generated trajectories is bounded by $|\mathcal{T}| = |\mathcal{V}| \times |\mathcal{G}| \times |\mathcal{S}|$, where $|\mathcal{S}|$ denotes the set of desired trajectory top speeds. The number of decision variables is bounded by $|\mathcal{T}|\times|\mathcal{V}|$. As well the trajectory generation time as the optimization runtime scales linearly with the number of trajectories. Collision checking of all trajectories is the most expensive part in the trajectory generation, this scales quadratically with the number of trajectories. A quantitative analysis shows that the optimization step scales quadratically with as well the number of vehicles and goals. The trajectory generation step scales linear in the number of vehicles. For few goals (≤ 10) this step scales quadratically, but exponentially for a higher number of goals.

No effort has been devoted improving the runtime. The simple example (section IV-C) with 2 vehicles and 3 goals

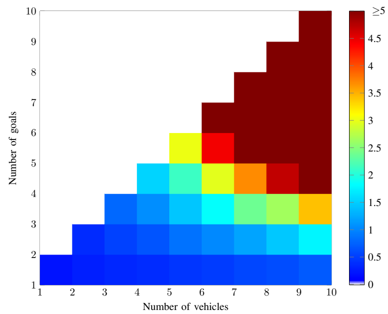


Fig. 4: Total runtimes in seconds with varying number of goals and vehicles in the same environment. White patches indicate infeasible combinations.

has a maximum step time of $0.53s^2$. In the parking lot example (section IV-D) the maximum cycle time is $2.1s$. In a benchmark scenario with a varying number of vehicles and goals the total runtime of one iteration is displayed in Fig. 4. With the current implementation only a local scope of few vehicles and goal positions can be addressed.

It shall be noted that there exist situations the mixed-integer optimization does not find the optimal solution in reasonable time but at least one feasible solution.

V. Conclusion and Future Work

The introduced algorithm is suitable for solving conflicts occurring in vehicle parking scenarios in mixed traffic with autonomous and non-cooperating human-driven vehicles. By quantifying the estimated intentions of non-communicating vehicles and taking these results into account in an optimization procedure a good conflict free solution can be found. The algorithm is symmetric in the vehicles it does not favor any vehicle. Fair, egoistic or altruistic behavior is modeled by appropriate weights in the optimization objective function.

To meet real-time requirements and to solve more sophisticated scenarios the trajectory generation step has to be improved and fully integrated into the optimization procedure without the here applied pre-generation. The model of the human driver tend to be too egoistic and too aggressive in the chosen driving scenarios.

To apply the approach in a real-world scenario a finer human driver model has to be applied or gathered from parking lot observations. Also the environment model is too idealistic.

The proposed method is not restricted to parking scenarios; also highway merging or crossing scenarios can be tackled.

References

[1] E. Fraedrich, R. Cyganski, I. Wolf, *et al.*, *User Perspectives on Autonomous Driving. A Use-Case-Driven Study in Germany*. 2016, pp. 1–111.

[2] M. Düring and P. Pascheka, “Cooperative decentralized decision making for conflict resolution among autonomous agents,” in *IEEE International Symposium on Innovations in Intelligent Systems and Applications, Proceedings*, 2014, pp. 154–161.

[3] L. Pallottino, V. G. Scordio, A. Bicchi, *et al.*, “Decentralized cooperative policy for conflict resolution in multivehicle systems,” *IEEE Transactions on Robotics*, vol. 23, no. 6, pp. 1170–1183, 2007.

[4] W. Schwarting and P. Pascheka, “Recursive Conflict Resolution for Cooperative Motion Planning in Dynamic Highway Traffic,” in *International Conference on Intelligent Transportation Systems (ITSC)*, 2014, pp. 1039–1044.

[5] R. Krajewski, P. Themann, and L. Eckstein, “Decoupled Cooperative Trajectory Optimization for Connected Highly Automated Vehicles at Urban Intersections,” in *IEEE Intelligent Vehicles Symposium*, 2016, pp. 819–824.

[6] S. Lin, B. D. Schutter, Y. Xi, *et al.*, “Model Predictive Control for urban traffic networks via MILP,” *American Control Conference (ACC), 2010*, vol. 19, pp. 2272–2277, 2010.

[7] R. Reghelin and L. V. R. De Arruda, “A centralized traffic controller for intelligent vehicles in a segment of a multilane highway,” in *IEEE Intelligent Vehicles Symposium, Proceedings*, 2012, pp. 135–140.

[8] Y. Geng and C. G. Cassandras, “New ”Smart Parking” System Based on Resource Allocation and Reservations,” *IEEE Transactions on Intelligent Transportation Systems*, vol. 14, no. 3, pp. 1129–1139, 2013.

[9] T. Schouwenaars, B. D. Moor, E. Feron, *et al.*, “Mixed integer programming for multi-vehicle path planning,” in *European Control Conference*, 2001, pp. 2603–2608.

[10] C. Frese and J. Beyerer, “A comparison of motion planning algorithms for cooperative collision avoidance of multiple cognitive automobiles,” in *IEEE Intelligent Vehicles Symposium Proceedings*, 2011, pp. 1156–1162.

[11] L. Blackmore, M. Ono, and B. C. Williams, “Chance-constrained optimal path planning with obstacles,” *IEEE Transactions on Robotics*, vol. 27, no. 6, pp. 1080–1094, 2011.

[12] Y. Kuwata, “Trajectory Planning for Unmanned Vehicles using Robust Receding Horizon Control Yoshiaki Kuwata,” PhD thesis, 2003.

[13] A. Richards and J. How, “Aircraft trajectory planning with collision avoidance using mixed integer linear programming,” *Proceedings of the 2002 American Control Conference (IEEE Cat. No.CH37301)*, vol. 3, 1936–1941 vol.3, 2002.

[14] H. Bai, S. Cai, N. Ye, *et al.*, “Intention-aware online POMDP planning for autonomous driving in a crowd,” in *IEEE International Conference on Robotics and Automation*, 2015, pp. 454–460.

[15] S. Lefèvre, C. Laugier, and J. Ibanez-Guzmán, “Risk Assessment at Road Intersections: Comparing Intention and Expectation,” in *IEEE Intelligent Vehicles Symposium*, 2012, pp. 165–171.

[16] Q. Tran and J. Firl, “Online maneuver recognition and multimodal trajectory prediction for intersection assistance using non-parametric regression,” in *IEEE Intelligent Vehicles Symposium, Proceedings*, 2014, pp. 918–923.

[17] S. M. LaValle, *Planning Algorithms*. Cambridge University Press, 2006.

[18] M. Werling, J. Ziegler, and S. Thrun, “Optimal Trajectory Generation for Dynamic Street Scenarios in a Frenet Frame,” in *IEEE International Conference on Robotics and Automation*, 2010, pp. 987–993.

[19] D. Lenz, T. Kessler, and A. Knoll, “Stochastic Model Predictive Controller with Chance Constraints for Comfortable and Safe Driving Behavior of Autonomous Vehicles,” in *IEEE Intelligent Vehicles Symposium*, 2015, pp. 292–297.

[2]measured on a standard PC with Intel Xeon 3.7 GHz CPU